The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR.1771 -31

Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

4 - 8 Sept., 2006, ICTP, Trieste, Italy

Power-Law scaling of Laminar natural convection with horizontal temperature gradient

H. Yu
Los Alamos National Laboratory
USA

LA-UR-06-2485

Power-law Scaling of Laminar Natural Convection with Horizontal Temperature Gradient

Huidan Yu, Ning Li, Robert E. Ecke
Los Alamos National Laboratory, New Mexico, USA

Conference and Euromech Colloquium #480 on High Rayleigh Number Convection
4-8 September 2006, Miramare, Trieste, Italy

Outline

- Problem description
- Numerical technique -- lattice Boltzmann method
- Power-law scaling and physical interpretation
- Summary

Dimensionless Boussinesq Equations

$$\nabla \cdot \vec{u} = 0$$

$$\partial \vec{u} / \partial t + \vec{u} \cdot \nabla \vec{u} = -\nabla p + \nu_d \nabla^2 \vec{u} + Ra\,Pr\,\Theta \hat{z}$$

$$\partial \Theta / \partial t + \vec{u} \cdot \nabla \Theta = \kappa_d \nabla^2 \Theta$$

Control parameters:

Pr (=ν/κ) : Prandtl number

Ra (=GrPr=$g\beta(T_h - T_l)L^3/\nu\kappa$): Rayleigh number

AR (=H/L): Aspect ratio

RBC vs LHC

Rayleigh-Bernard convection (vertical temperature gradient)

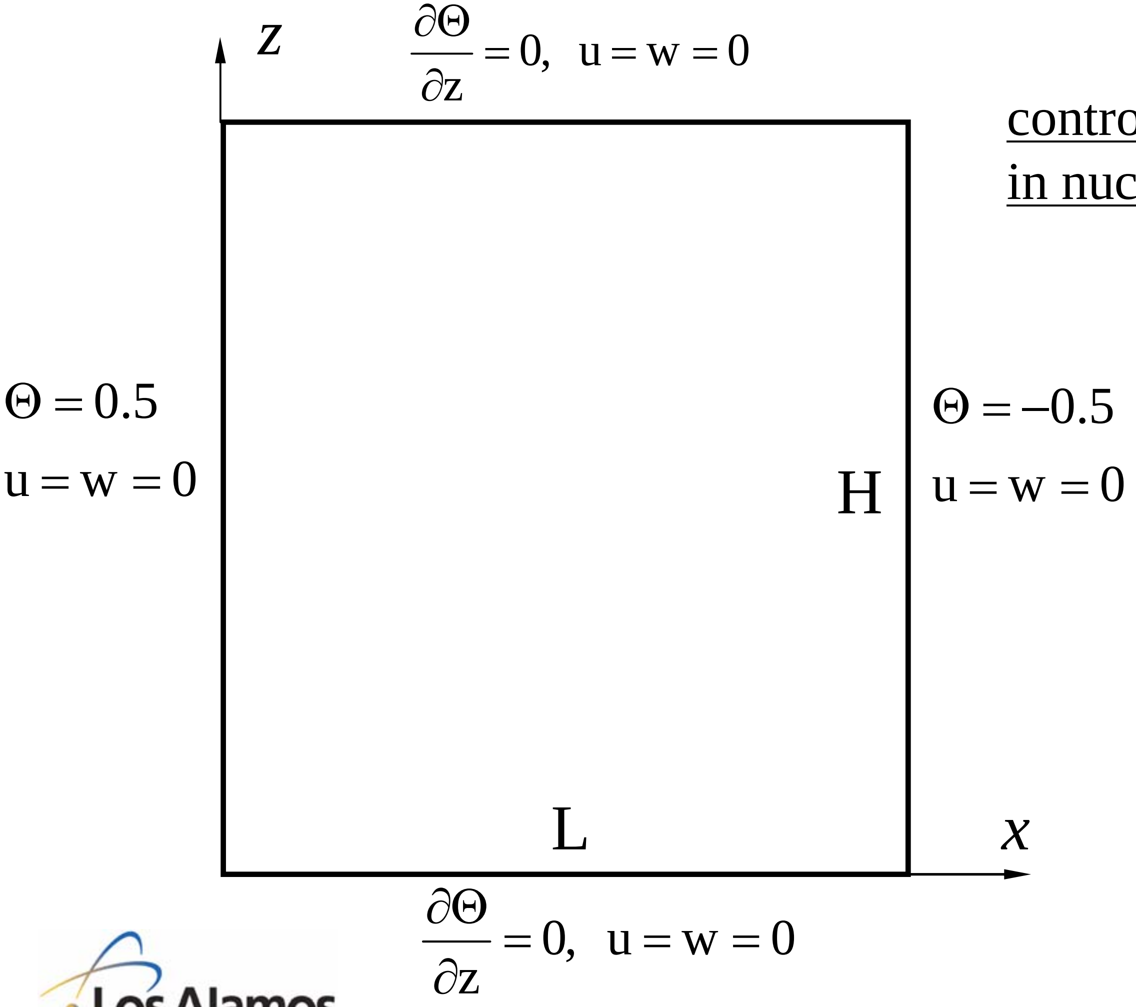
- Important in oceanography, geophysics, meteorology,astrophysics, energy process
- Paradigm system to study nonlinear physics, turbulence properties
- Much effort expended and significant insight gained in addressing fundamental aspects of heat transfer and flow characteristics
- Heat-transfer (Nu) /turbulence (Re) scaling with Ra, Pr, AR

Laterally heated convection (horizontal temperature gradient)

- Important in industrial and engineering applications
- Less attention from physics community
- Validation example to compare numerical algorithms
- Flow structure, temperature distribution, AR effect

— stop well short of physical fundamentals

Schematic Configuration



<u>Extracted from oxygen activity control technique to control corrosion in nuclear liquid metal reactors</u>

- Pr = 0.71(air)
- AR = H/L = 1
- Nusselt number:

$$Nu = \int_0^1 \left(u\Theta - \frac{\partial \Theta}{\partial x}\right)\Bigg|_{x=0.5} dz$$

(Nu=1 for conduction)

- Maximum velocity v_{max}

Mesoscopic Computation Technique — Lattice Boltzmann Method

$$g_\alpha(\vec{x}+c\vec{e}_\alpha\delta t, t+\delta t) = g_\alpha(\vec{x},t) - [g_\alpha(\vec{x},t) - g_\alpha^{(eq)}(\vec{x},t)]/\tau_f + \delta t F_\alpha$$

$$g_\alpha^{(eq)} = \lambda_\alpha p + s_\alpha(\vec{u}) \qquad \tau_f = 0.5 + \frac{\nu_d}{3c\delta x}$$

Discrete buoyant force

$$\vec{F} = \mathrm{Ra}\,\mathrm{Pr}\,\Theta\hat{z}$$

$$p = (\sum_{\alpha>0} g_\alpha + s_0(\vec{u}))/\lambda_0 \qquad \vec{u} = \sum_\alpha c\vec{e}_\alpha g_\alpha$$

$$\Theta_\alpha(\vec{x}+c\vec{e}_\alpha\delta t, t+\delta t) = \Theta_\alpha(\vec{x},t) - [\Theta_\alpha(\vec{x},t) - \Theta_\alpha^{(eq)}(\vec{x},t)]/\tau_T$$

$$\Theta_\alpha^{(eq)} = \Theta[\omega_\alpha + s_\alpha(\vec{u})] \qquad \tau_T = 0.5 + \frac{\kappa_d}{3c\delta x}$$

$$\Theta = \sum_\alpha \Theta_\alpha$$

λ_a, s_a, ω_a are lattice related

Shi, et al, Prog. Comp. Fluid Dyn., **5**, 50 (2005)

Validation: LBM vs. NS-based

Ra=10^3

Values	**Davis (bench-mark) (1983)**	**LBM (current)**	Ismail (2000)	D & J (1983)
$u_{max}(0.5,z)$	**3.65**	**3.64**	3.65	3.59
z	**0.81**	**0.81**	0.81	0.81
$w_{max}(x,0.5)$	**3.70**	**3.70**	3.70	3.63
x	**0.18**	**0.18**	0.18	0.18
Nu	**1.12**	**1.12**	1.12	1.11

Ra=10^4

Values	**Davis (bench-mark) (1983)**	**LBM (current)**	Ismail (2000)	Ding (2004)	D & J (1983)
$u_{max}(0.5,z)$	**16.18**	**16.15**	16.16	16.06	16.19
z	**0.82**	**0.82**	0.82	—	0.82
$w_{max}(x,0.5)$	**19.62**	**19.70**	19.83	19.27	19.20
x	**0.12**	**0.12**	0.12	—	0.13
Nu	**2.24**	**2.24**	2.26	2.17	2.21

Validation: LBM vs. NS-based(cont'd)

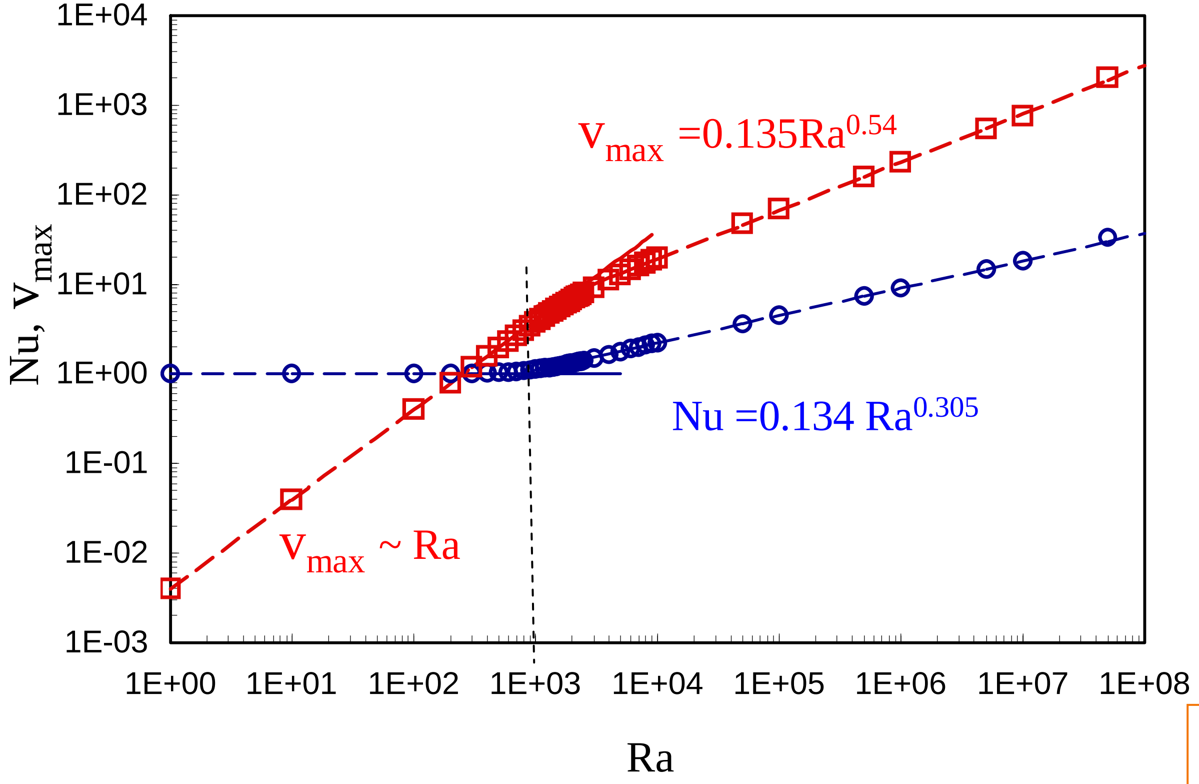
Ra=10^5

Values	**Davis (bench-mark)**	**LBM**	Ismail	Ding	D&J
$u_{max}(0.5,z)$	**34.73**	**35.24**	33.42	34.60	36.46
z	**0.86**	**0.85**	0.85	—	0.85
$w_{max}(x,0.5)$	**68.59**	**69.10**	70.44	69.18	62.79
x	**0.066**	**0.069**	0.067	—	0.075
Nu	**4.52**	**4.50**	4.65	4.58	4.45

Ra=10^6

Values	**Davis**	**Quéré (spectral)**	**LBM**	Ismail	D&J
$u_{max}(0.5,z)$	**64.63**	**64.83**	**73.00**	57.22	79.27
z	**0.85**	**0.85**	**0.85**	0.87	0.86
$w_{max}(x,0.5)$	**219**	**220**	**229**	220	195
x	**0.038**	**0.038**	**0.042**	0.045	0.045
Nu	**8.8**	**8.8**	**9.0**	8.9	9.0

Nu, v_{max} vs. Ra



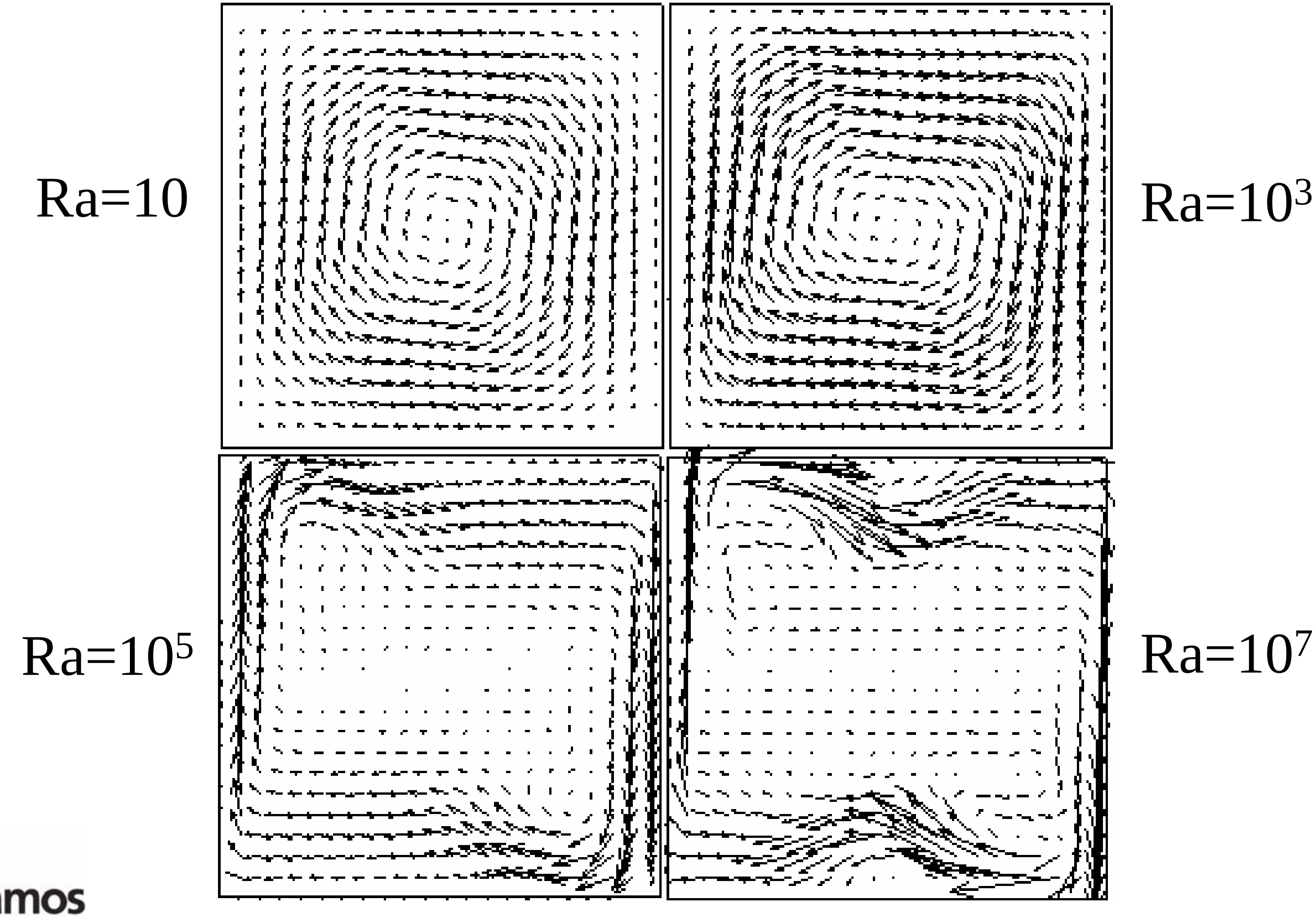
Paolucci & Chenoweth
JFM (1989)
Ra = 2x10^8 , turbulence

conduction | Power-law scaling

steady flow (laminar) | turbulence

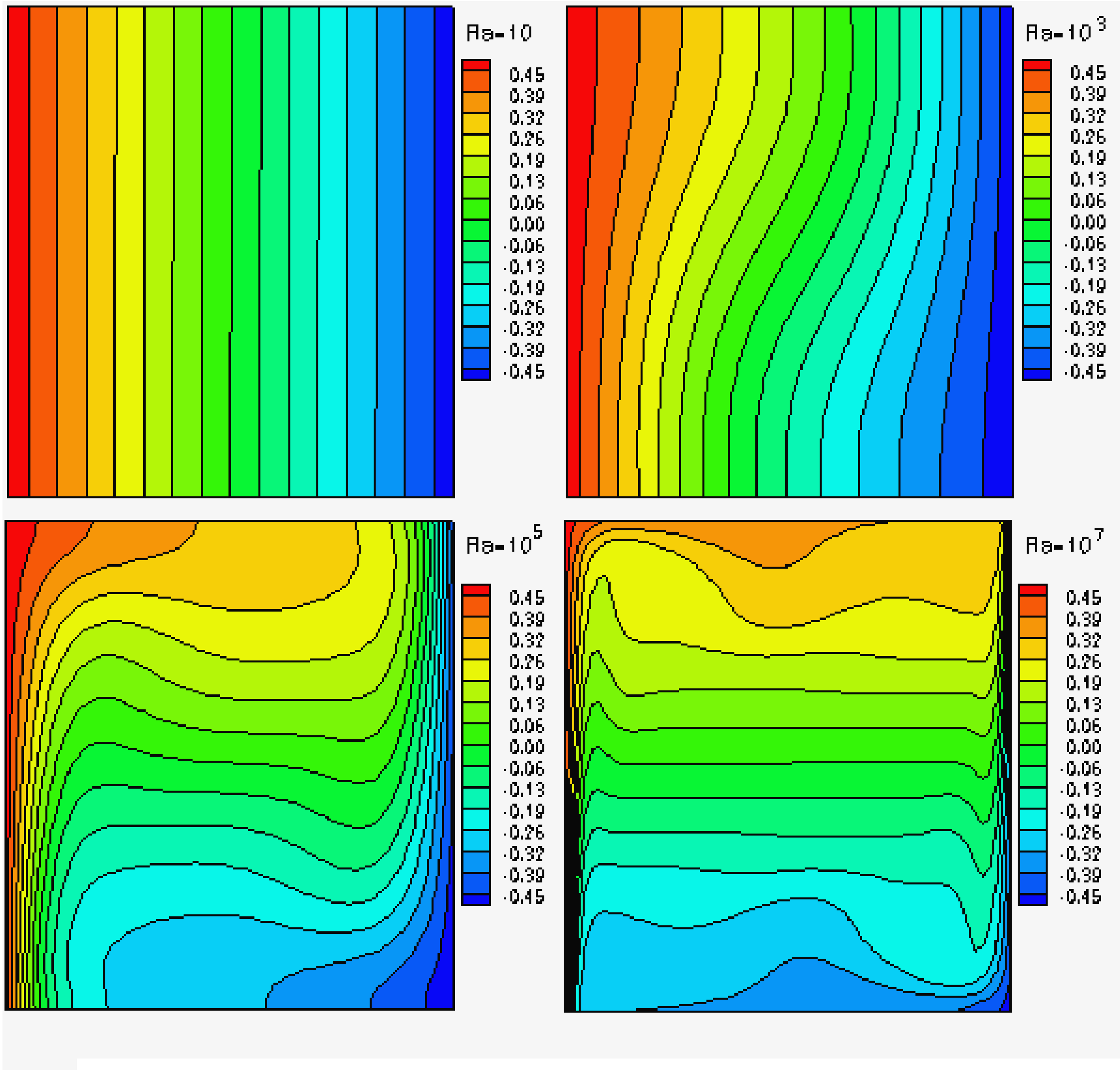
What is the physics behind the scaling?

Steady Flow Structures (magnitude not scaled)

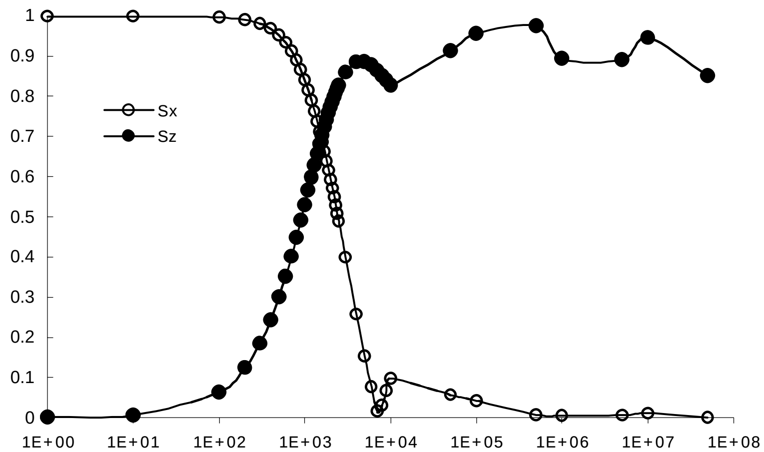


Large scale circulation inherently exists

Steady Temperature Contours



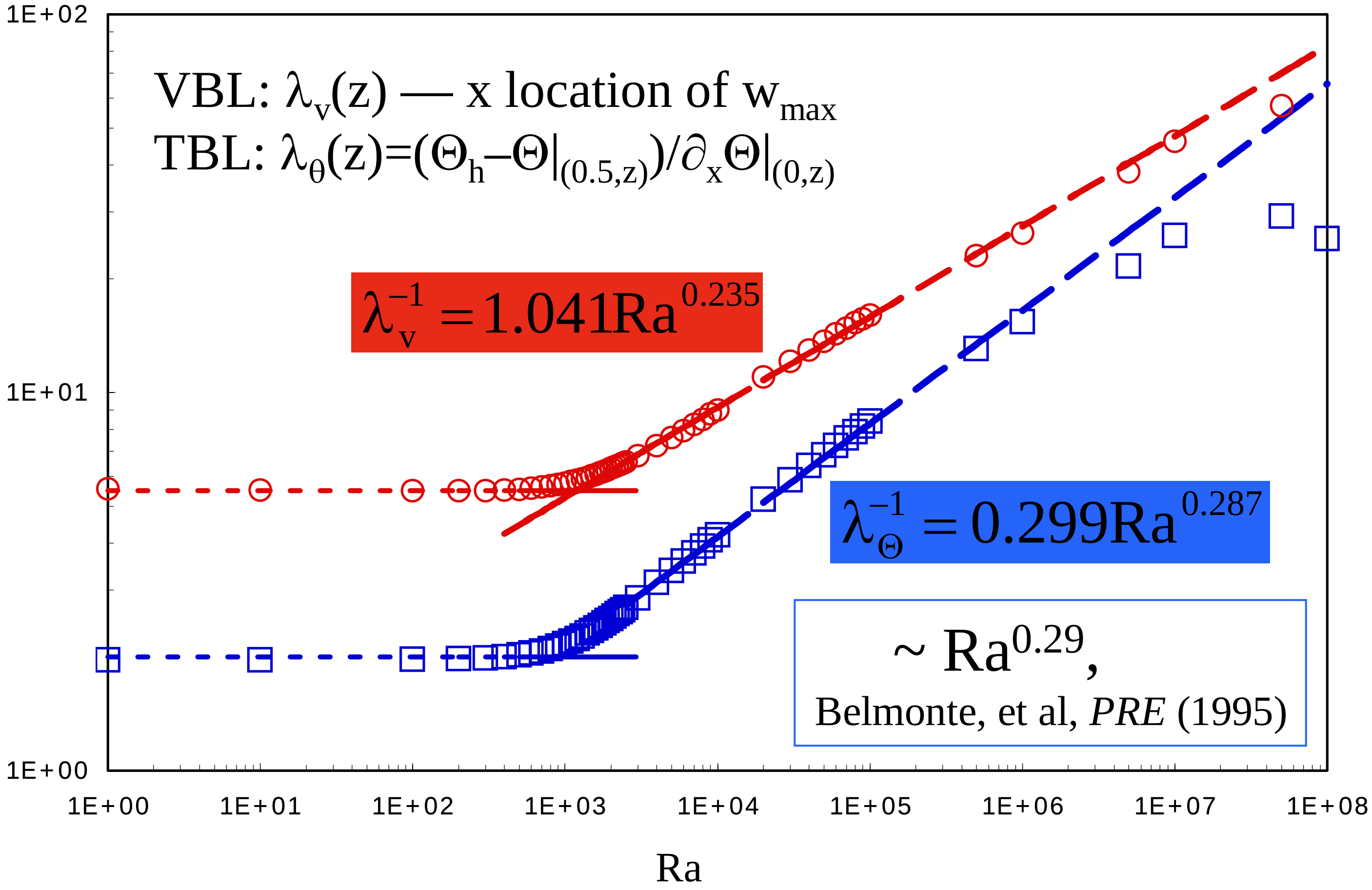
Core Temperature Gradients
(Interior Stratification)



$$Sx = \frac{L}{\Delta T} \frac{\partial \Theta}{\partial x}\bigg|_{\substack{x=0.5\\z=0.5}} \qquad Sz = \frac{L}{\Delta T} \frac{\partial \Theta}{\partial z}\bigg|_{\substack{x=0.5\\z=0.5}}$$

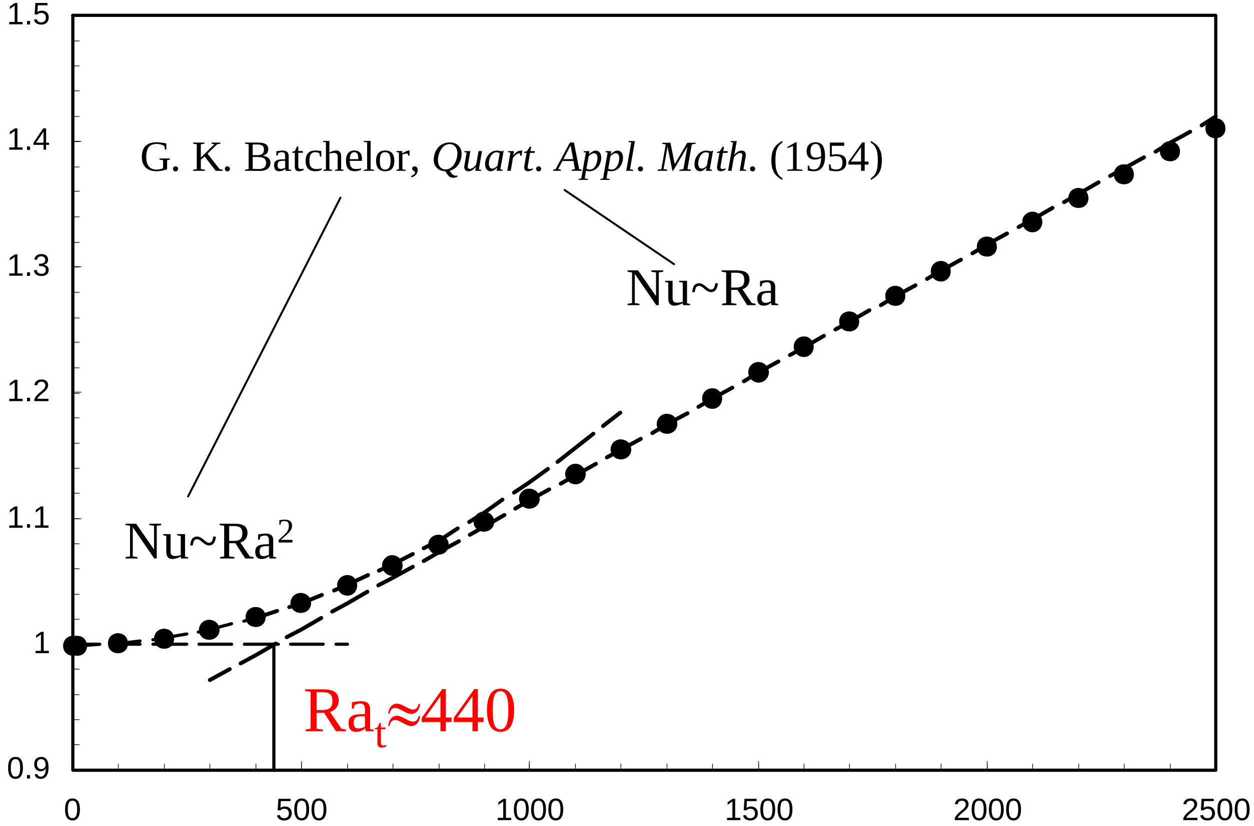
Thermally stratified, conduction dominates in thin boundary layers

Mean Inverse Boundary Layer Thickness



- TBL precedes VBL, true for Pr<1
- BL meaningless till fluid starts to stratify
- Nu grows faster than $(\lambda_\theta)^{-1}$, conduction dominates, convection still contributes

Threshold Ra_t



AR	Ra_t
1	440
5	435
10	449
12	441
14	434
16	432
18	436
20	455

- Ra_t analogous to Ra_c in RBC
- Ra_t characterizes the onset of convection significance

Interpretation of 1/3 Power Law

- $Ra < Ra_t$. conduction
- $Ra_t = 440$, convection takes place, stratification starts
  - Mechanism, buoyancy? shear ?
  - Relation with thermal boundary layer thickness?

- Higher enough Ra (stratified), boundary layer/core region apparent, significant horizontal temperature gradients are in two boundary layers with thickness λ

$$j_{heat} = (T_h - T_l)\, k/\lambda, \quad j_{cond} = (T_h - T_l)\, k/L$$

$$Ra = g\beta(T_h - T_l)L^3/\nu\kappa, \quad Ra_t = g\beta(T_h - T_l)\, \lambda^3/\nu\kappa$$

$$Nu = j_{heat}/j_{cond} = L/\lambda = (Ra/Ra_t)^{1/3} = 0.13Ra^{1/3}$$

Mixing length theory, Spiegel, *Annu. Rew. Astron. Astrophys.* (1971)

- This work:
  $Nu = 0.134Ra^{0.305}$ (Ra $10^{3.5}$-10^7)
- Turbulent RBC:
  $Nu = 0.134Ra^{0.309}$ (Ra 10^6-10^{17})
  Niemela et al. *Nature* (2000)

Summary: Factors Driven Near 1/3 Power-law

$$Nu \sim aRa^{\beta}$$

- Large scale circulation present
- Interior thermal stratification $\rightarrow \beta$
- Threshold Rayleigh number Ra_t identified $\rightarrow a$

Further Work

- More sophisticated or rigorous analysis
- Nu dependency on AR
- Nu dependency on Pr
- Power-law scaling in turbulent convection
- ……..
- Experiments?