The Abdus Salam
International Centre for Theoretical Physics

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SMR.1771 - 32

Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

4 - 8 Sept., 2006, ICTP, Trieste, Italy

Wind and its boundary layers in Rayleigh-Benard convection

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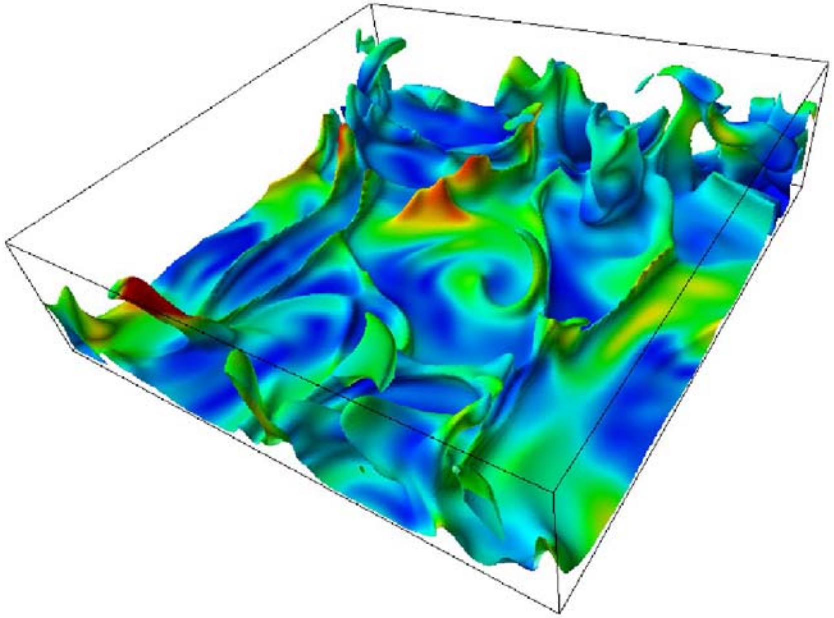
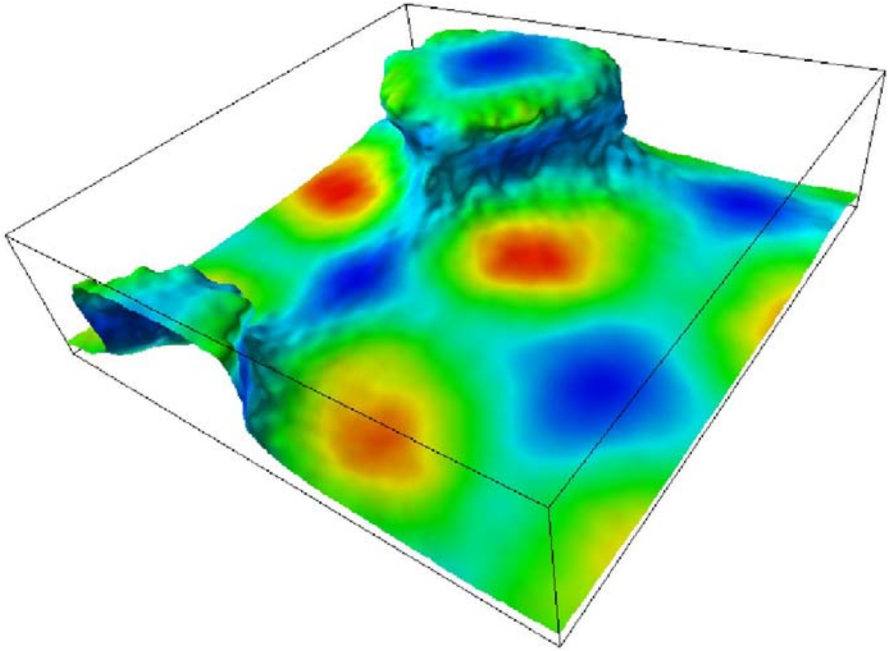
Delft
The Netherlands

These are preliminary lecture notes, intended only for distribution to participants

Wind and boundary layers in turbulent Rayleigh-Bénard convection





Maarten van Reeuwijk

Harm Jonker, Kemo Hanjalic

September 27, 2006

Department of Multi-Scale Physics
Faculty of Applied Sciences

FOM

TU Delft
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Boundary layers laminar?

Evidence in favor:

- Re is very low (1500 at $Ra=10^8$ and $Pr=1$)
- Friction factor scales as $C_f = Re^{-1/2}$ (Chavanne *et. al.* PRL 1997, POF 2001; Amati *et. al.* POF 2005).
- G-L correctly predicts Ra-Nu over large range of Ra and Pr

Evidence against:

- BL is highly unsteady due to constant impingement and detachment of plumes.

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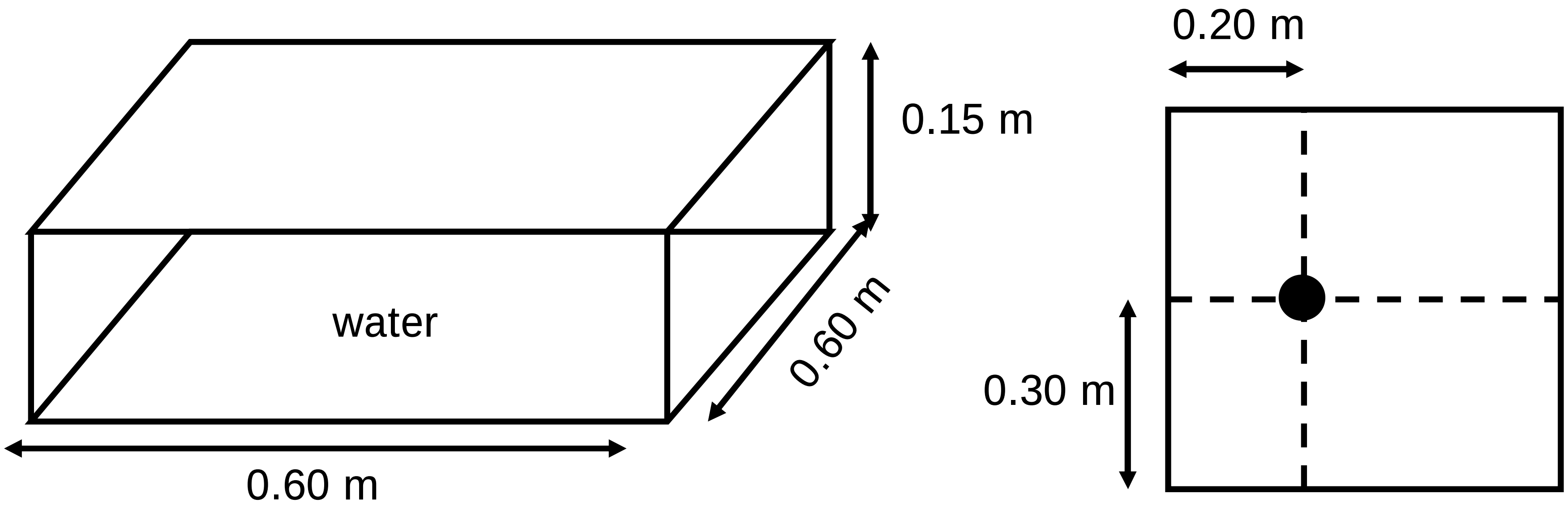
Turbulence indicators for BL

- Spectrum
- Velocity profile
- Friction factor
- Shape factor
- Momentum budgets

Turbulence indicator: spectra

- Simultaneous bulk-BL measurements with LDV at Ra=$2x10^7$ – $2x10^9$.
- DNS at Ra=$7x10^4$ – $7.7x10^7$ (stress-free sidewalls).



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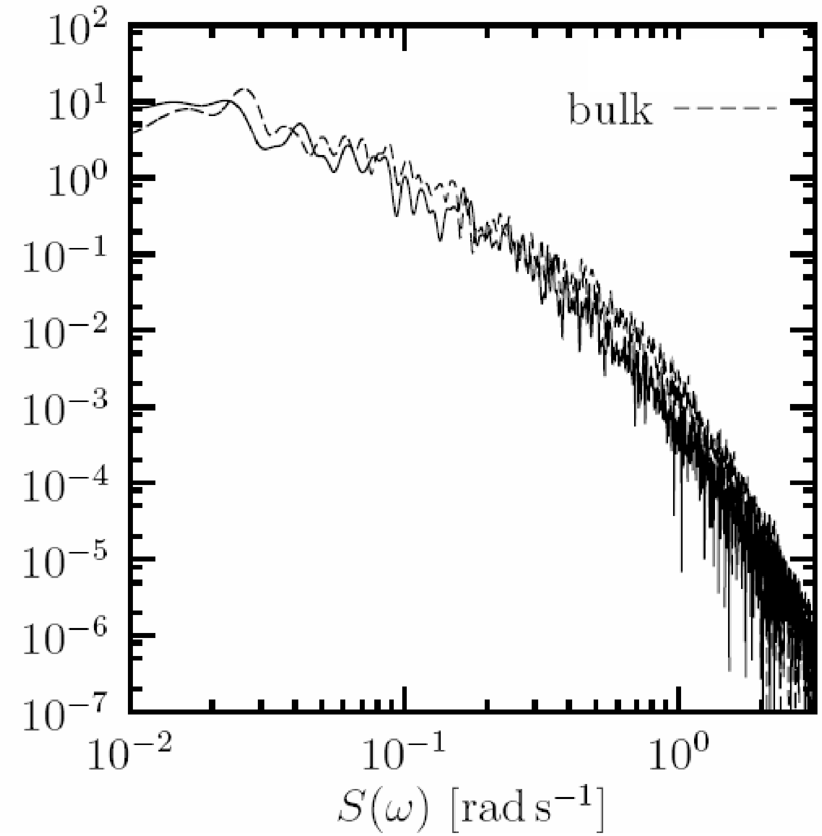
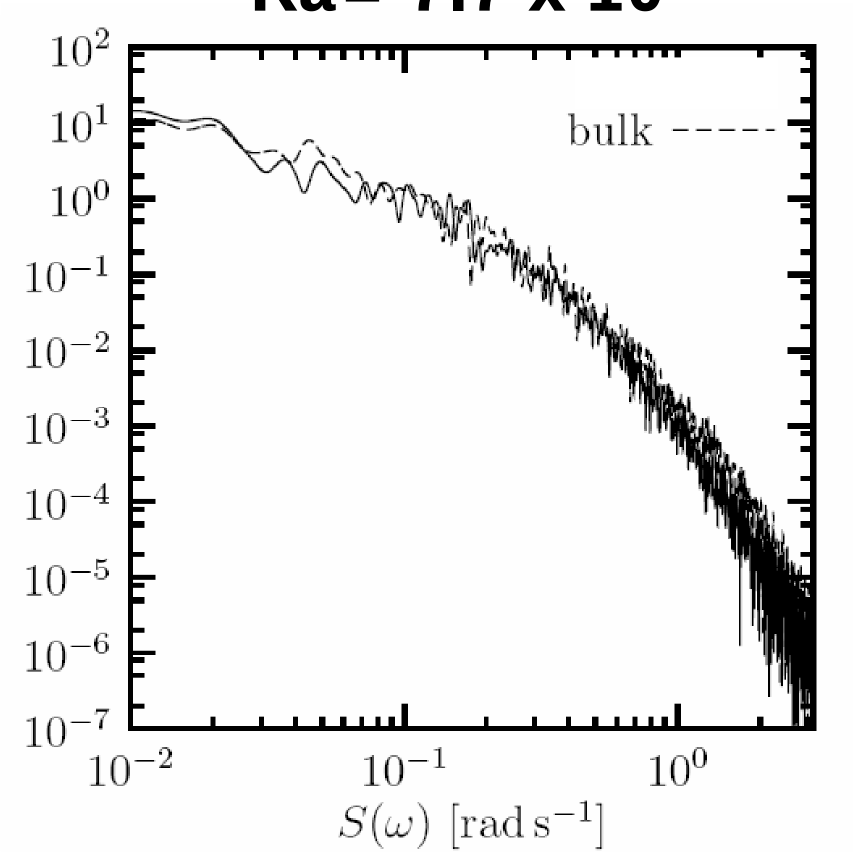
Turbulence indicator: spectrum

Ra= 2.0 x 10^7



Ra= 7.7 x 10^7



Verdoold, van Reeuwijk, Tummers, Jonker and Hanjalic (2006), *in preparation*

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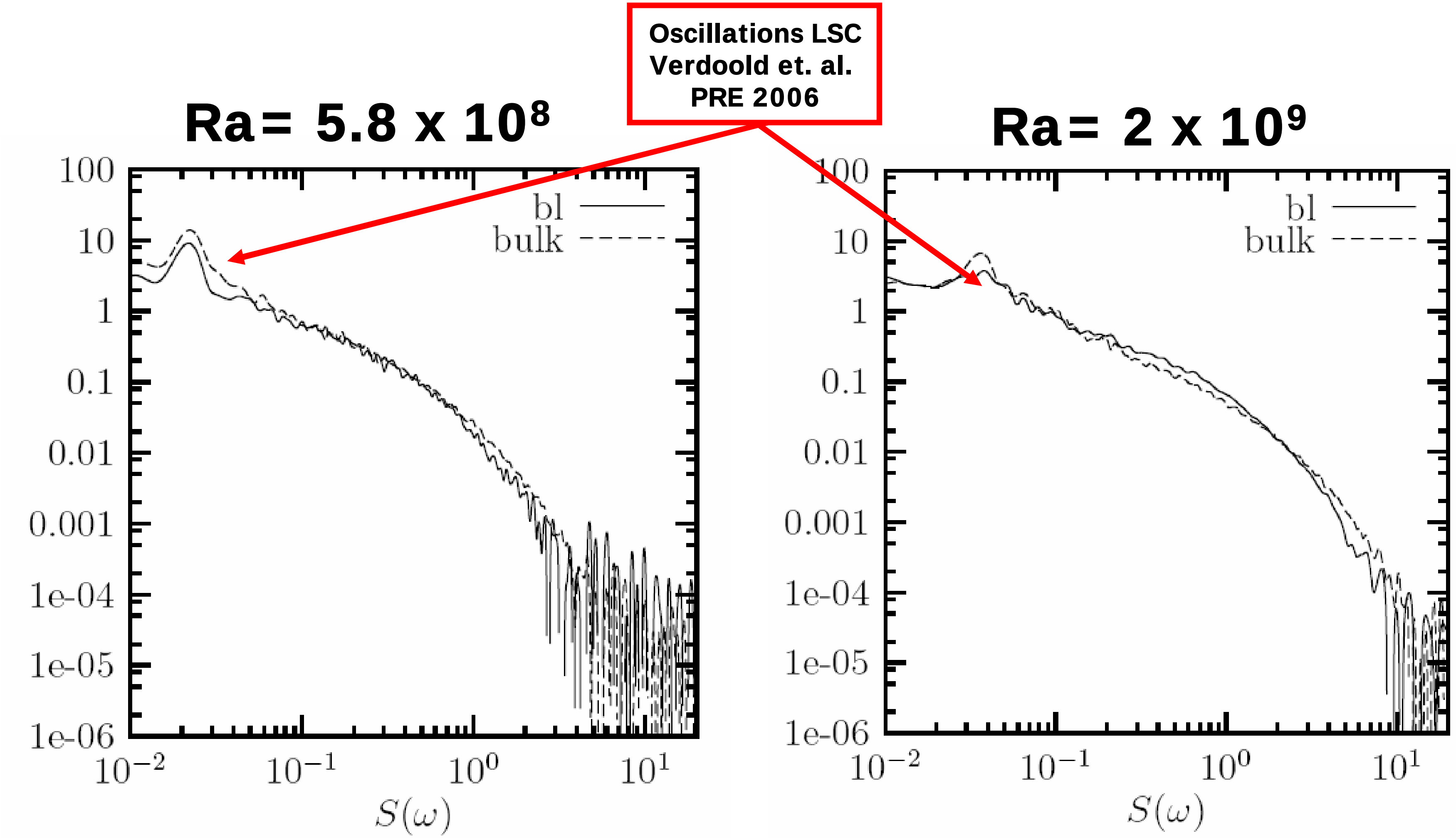
LDV measurements



Verdoold, van Reeuwijk, Tummers, Jonker and Hanjalic (2006), *in preparation*

DNS with periodic sidewalls; Ra=10^8

Ra	Pr	grid	Time (hr / t^*)	#procs
10^8	1	640 x 640 x 320	2000	128

2DV slice of Θ-field (5x speed)

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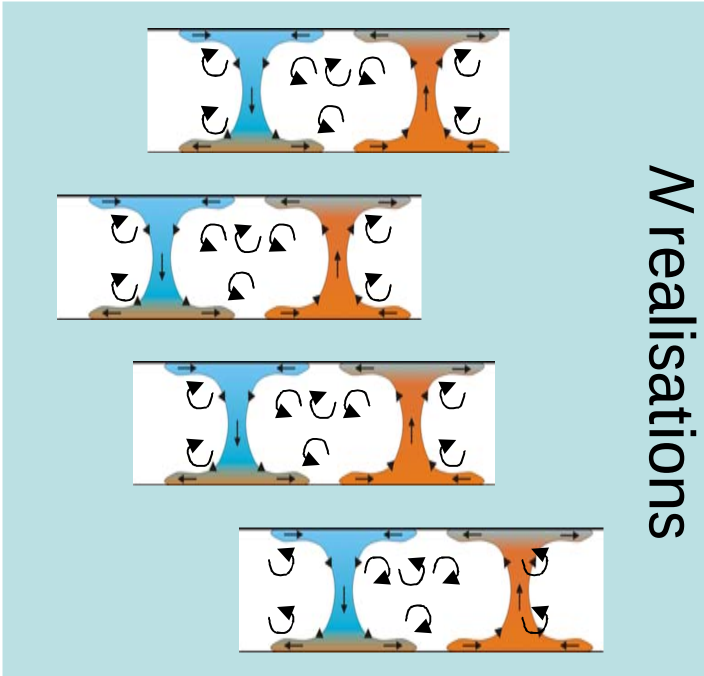
Classical ensemble averaging

$$\overline{u}(\vec{\mathbf{x}}) = \frac{1}{N}\sum_{N} u^{(n)}(\vec{\mathbf{x}})$$



+

$$\overline{u} = \overline{v} = \overline{w} = 0$$

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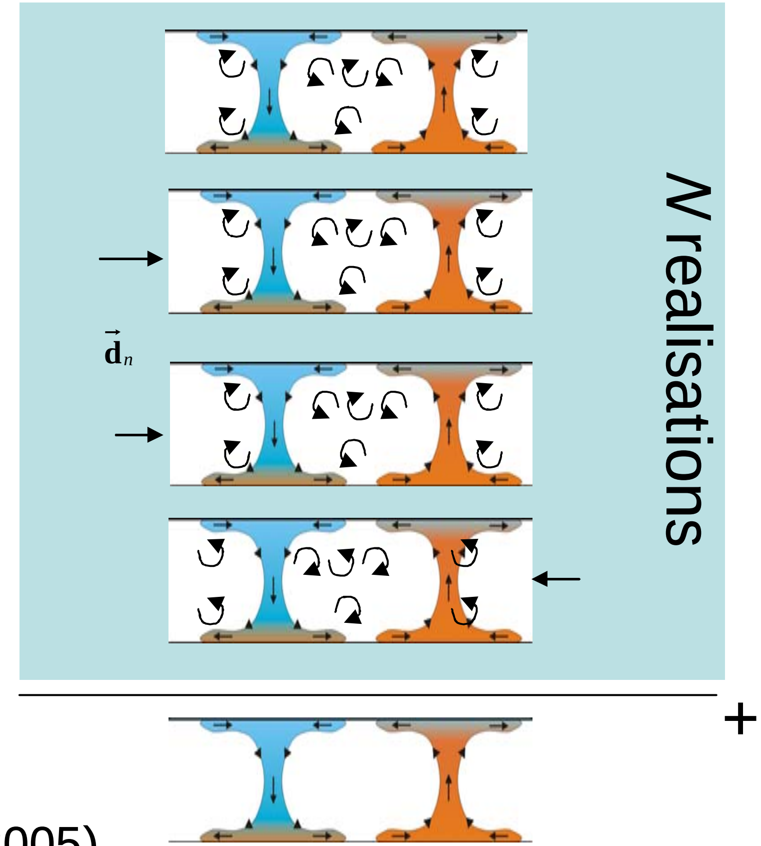
Symmetry-accounting ensemble averaging

$$\bar{u}(\vec{\mathbf{x}}) = \frac{1}{N}\sum_{N} u_n(\vec{\mathbf{x}} - \vec{\mathbf{d}}_n)$$

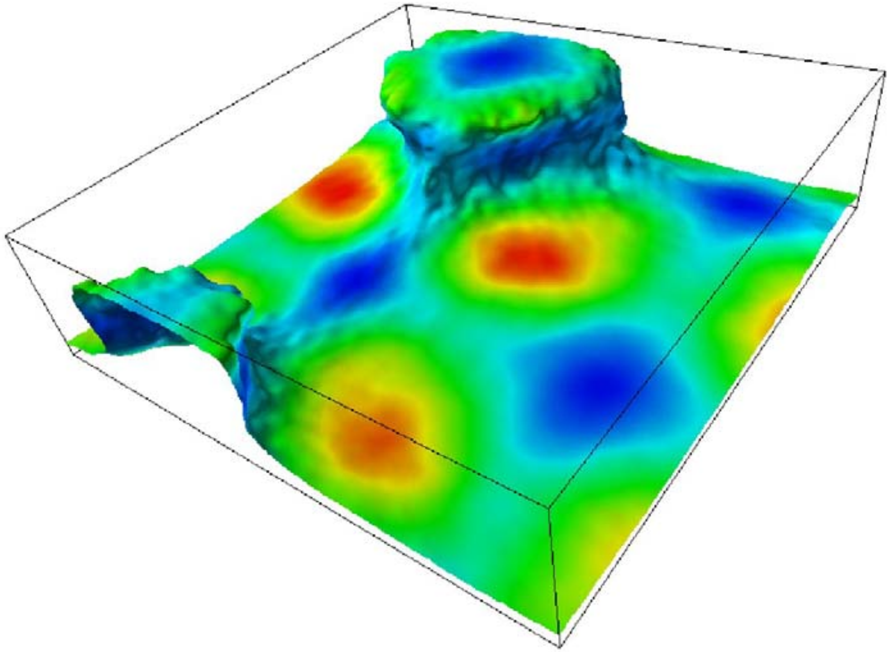


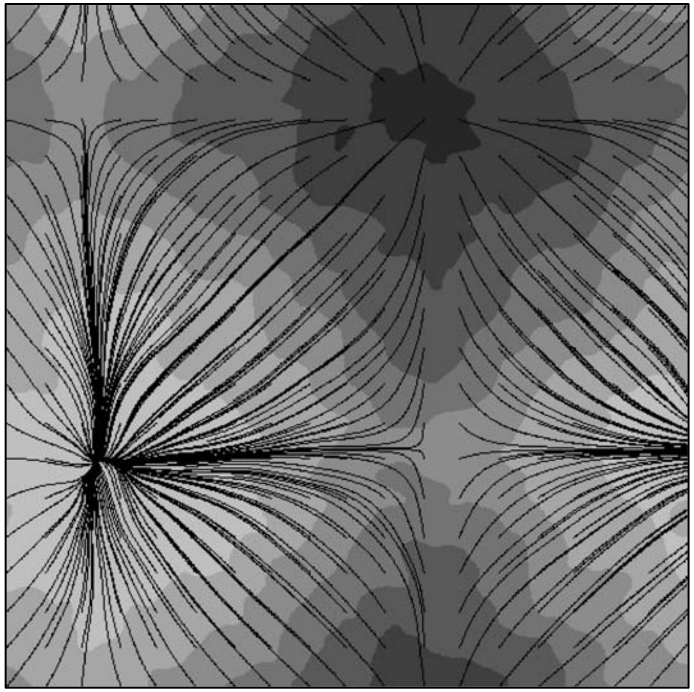
(van Reeuwijk *et. al.*, *Phys. Fluids* **17**, 2005)

Simulations with symmetry averaging

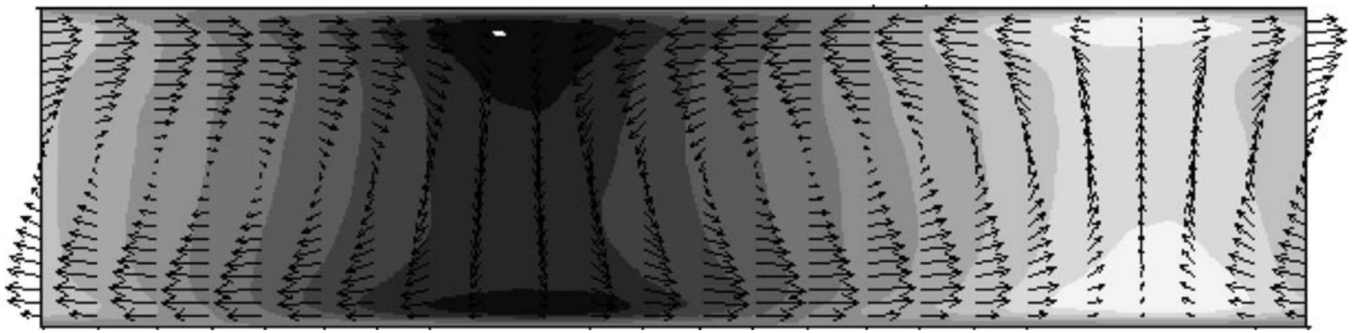
Ra	mesh	$\Delta t/t^* \times 10^3$	T/t^*	#sims	Nu	Re	Re_τ
10^5	$128^2 \times 64$	1.13	68	10	4.7	54	6
10^6	$192^2 \times 128$	0.57	20	10	8.5	154	14
10^7	$256^2 \times 256$	0.45	20	10	16.2	497	35

Wind structure





$Ra=10^6$
$Pr=1$



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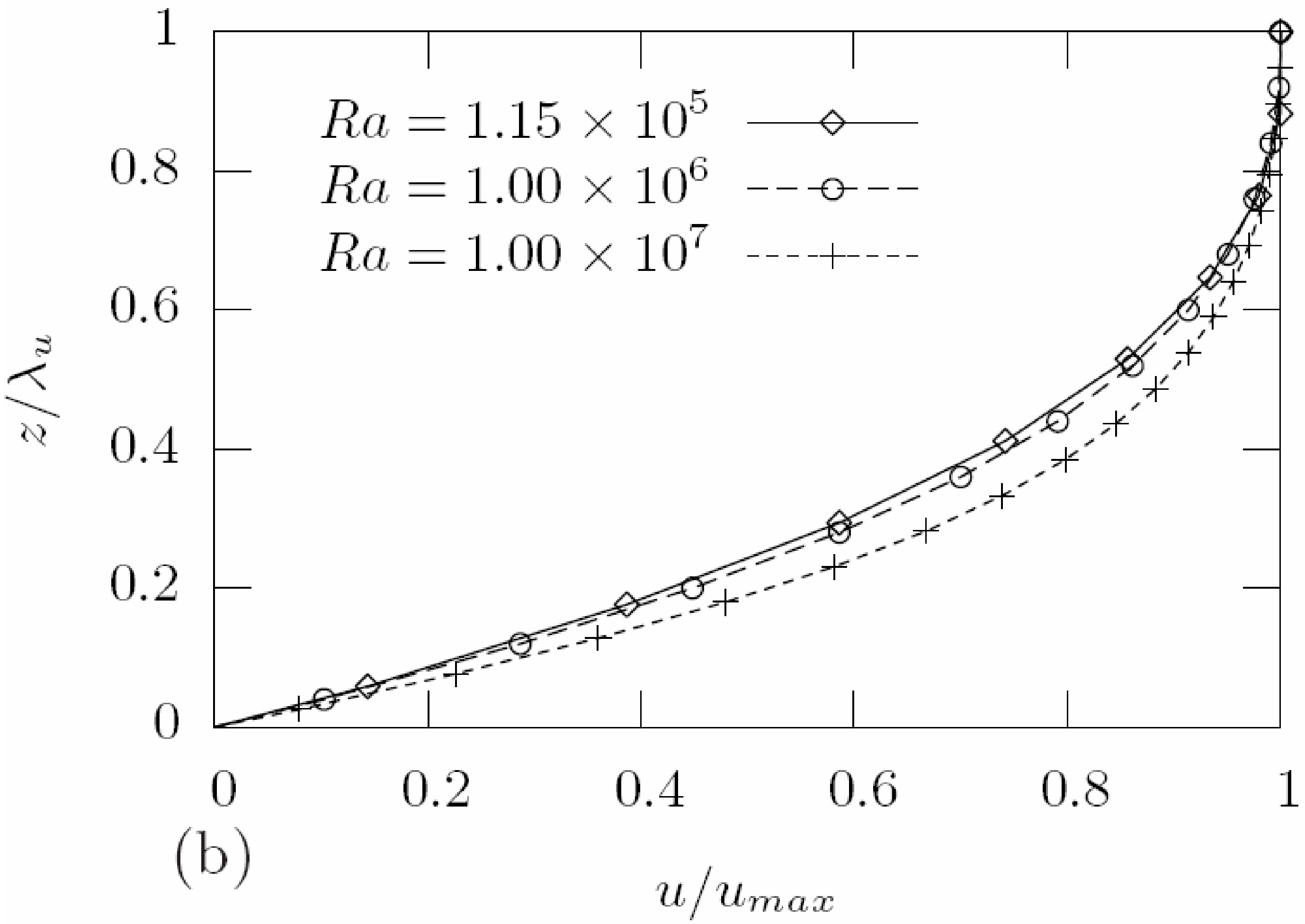
Turbulence indicator: velocity profile



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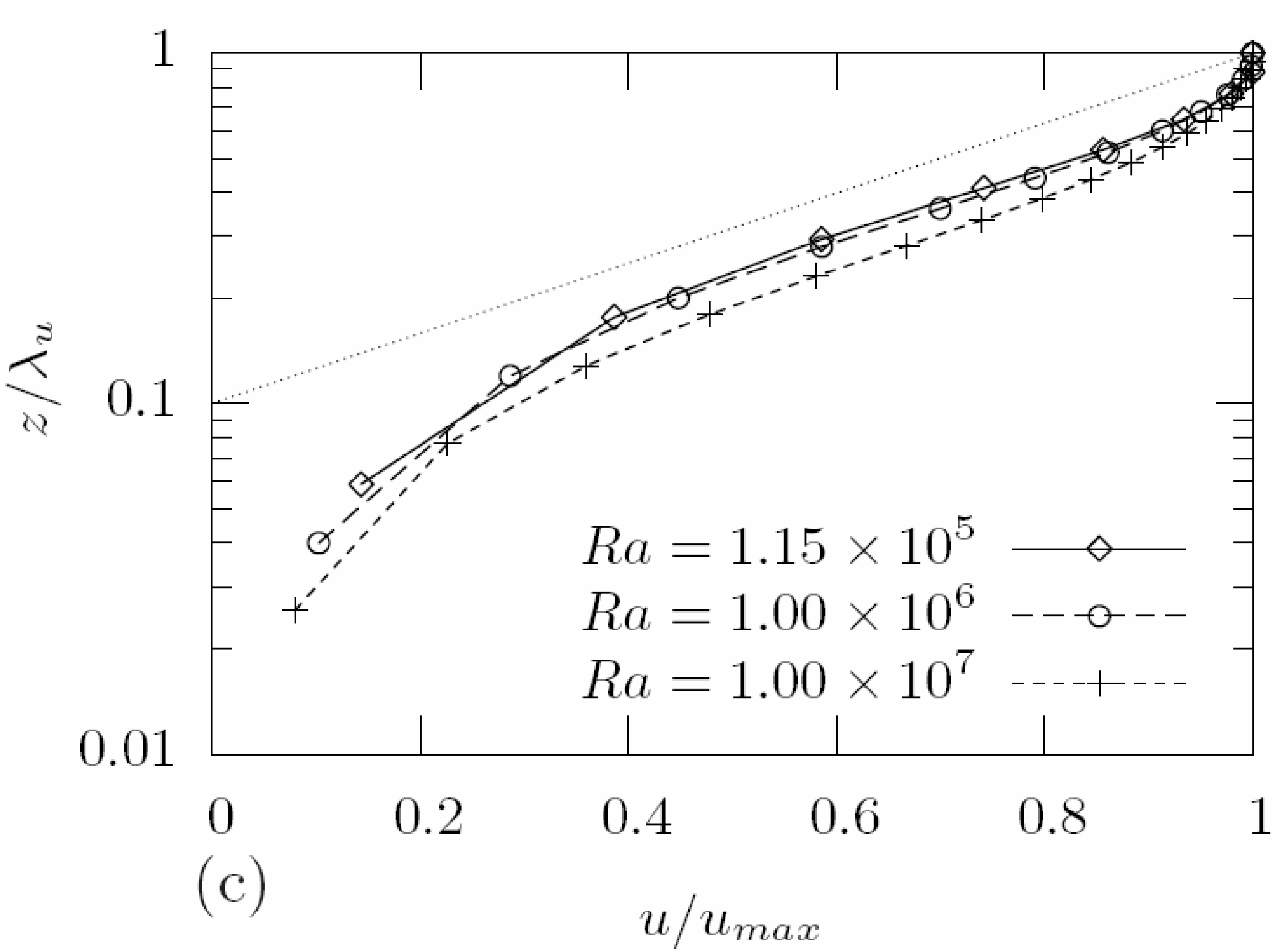
Turbulence indicator: velocity profile



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Turbulence indicator: friction factor

$$C_f = \frac{u_\tau^2}{\frac{1}{2}U^2}$$

Poisseuille: 8 / Re

Blasius flat plate : 0.664 $Re_x^{1/2}$

Turbulent channel: 0.073 $Re^{-0.25}$

Ra	Re_τ	C_f	S	L_O/H
1.15×10^5	26	1.02	2.37	-8.0×10^{-3}
1.00×10^6	52	0.51	2.35	-4.1×10^{-3}
1.00×10^7	119	0.23	2.27	-2.5×10^{-3}

Turbulence indicator: shape factor

Displacement thickness: $$\delta_1 = \int_0^{\lambda_u} \left(1 - \frac{u}{u_{\max}}\right) dz$$

Momentum thickness: $$\delta_2 = \int_0^{\lambda_u} \frac{u}{u_{\max}} \left(1 - \frac{u}{u_{\max}}\right) dz$$

Shape factor

$$S = \frac{\delta_1}{\delta_2}$$

Ra	Re_τ	C_f	S	L_O/H
1.15×10^5	26	1.02	2.37	-8.0×10^{-3}
1.00×10^6	52	0.51	2.35	-4.1×10^{-3}
1.00×10^7	119	0.23	2.27	-2.5×10^{-3}

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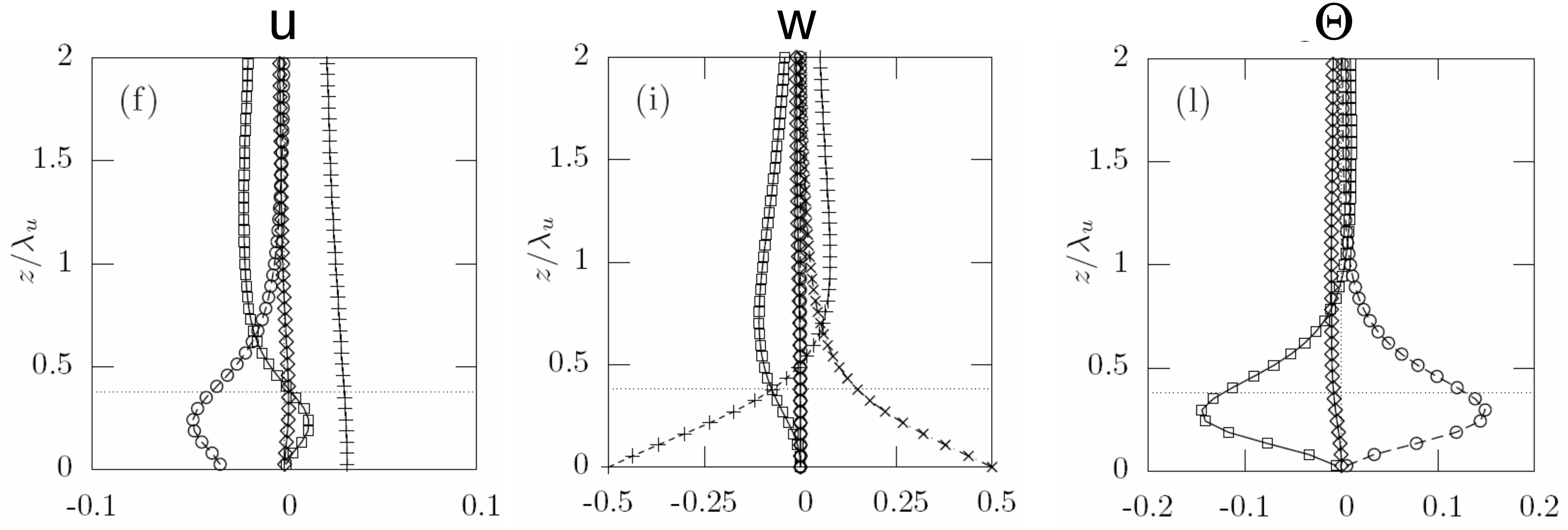
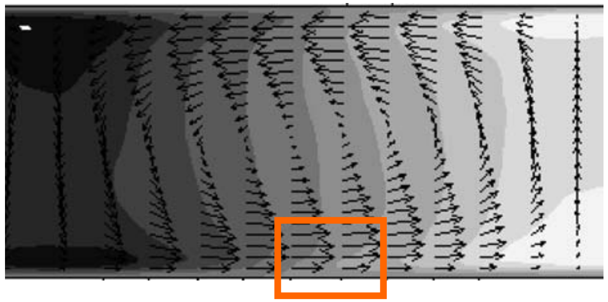
BL momentum budgets Ra=10^7



$$
\begin{array}{lllllll}
 & & \mathcal{A} & \mathcal{D} & \mathcal{P} & \mathcal{B} & \mathcal{R} \\
\partial_t \widetilde{u}_i & = & -\partial_j \widetilde{u}_j \widetilde{u}_i & +\nu \partial_j^2 \widetilde{u}_i & -\partial_i \widetilde{p} & +\beta g \widetilde{\Theta} \delta_{i3} & -\partial_j \widetilde{u'_j u'_i} \\
\partial_t \widetilde{\Theta} & = & -\partial_j \widetilde{u}_j \widetilde{\Theta} & +\kappa \partial_j^2 \widetilde{\Theta} & & & -\partial_j \widetilde{u'_j \Theta'}
\end{array}
$$



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Making up the score

Turbulent	Laminar
• Momentum budgets • Spectra • Velocity profiles	• Friction factor • Shape factor

The boundary layers are turbulent

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Turbulent boundary layer equations

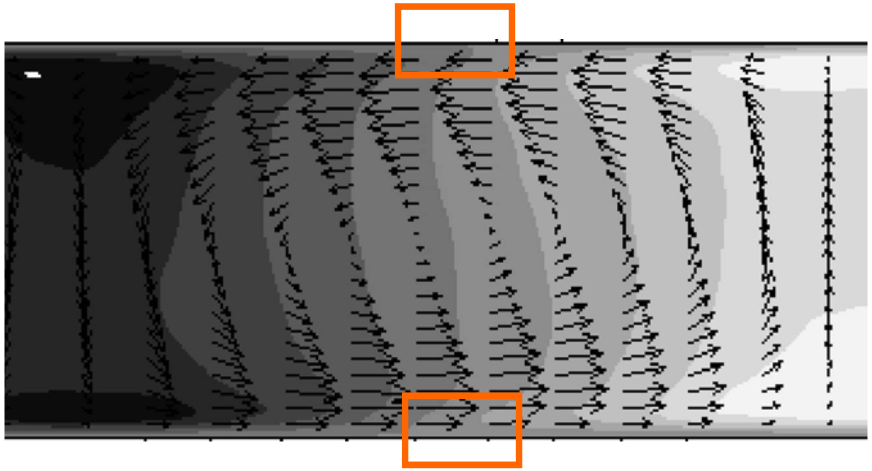
$$\partial_x \widetilde{p} + \partial_z \widetilde{w'u'} = \nu \partial_z^2 \widetilde{u},$$

$$\partial_z \widetilde{p} + \partial_z \widetilde{w'w'} = \beta g \widetilde{\Theta},$$

$$\partial_z \widetilde{w'\Theta'} = \kappa \partial_z^2 \widetilde{\Theta}.$$

Particularly, $\lambda_u \sim Re^{-1/2}$ Does not hold for wide AR domains. We find:

$$\lambda_u \propto Ra^{-0.13} \propto \mathrm{Re}^{0.26}$$



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Monin-Obukhov theory

- Mixed convection – combined effects of shear and buoyancy
- Overlap layer theory

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Obukhov length

$$L_0 = -\frac{u_\tau^3}{K\beta g \overline{w'\theta'}_w} = -\frac{H}{K}\frac{\mathrm{Re}_\tau^3}{\mathrm{Ra}\,\mathrm{Nu}\,\mathrm{Pr}^{-2}}$$

$$\frac{L_0}{H} = -2.40 Ra^{-0.30}$$

Ra	Re_τ	C_f	S	L_O/H
1.15×10^5	26	1.02	2.37	-8.0×10^{-3}
1.00×10^6	52	0.51	2.35	-4.1×10^{-3}
1.00×10^7	119	0.23	2.27	-2.5×10^{-3}

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... Scales at same rate as λ_θ

$$\frac{L_0}{\lambda_\theta} = -0.77\, Ra^{-0.01}$$

$$\mathrm{Re}_\tau^3 \approx \mathrm{Ra}\,\mathrm{Pr}^{-2}$$

Interplay of turbulent shear and buoyancy production

$$\frac{\lambda_u}{H} = \mathrm{Ra}^{-2/3}\,\mathrm{Re}$$

→ $$\lambda_u \propto \mathrm{Ra}^{-1/6}$$

→ $$\mathrm{Re}_\tau \propto \mathrm{Ra}^{1/3};\ \tau_w \propto \mathrm{Ra}^{2/3}$$

Compatible with Xin et. al (1996); Qiu & Xia (1998)

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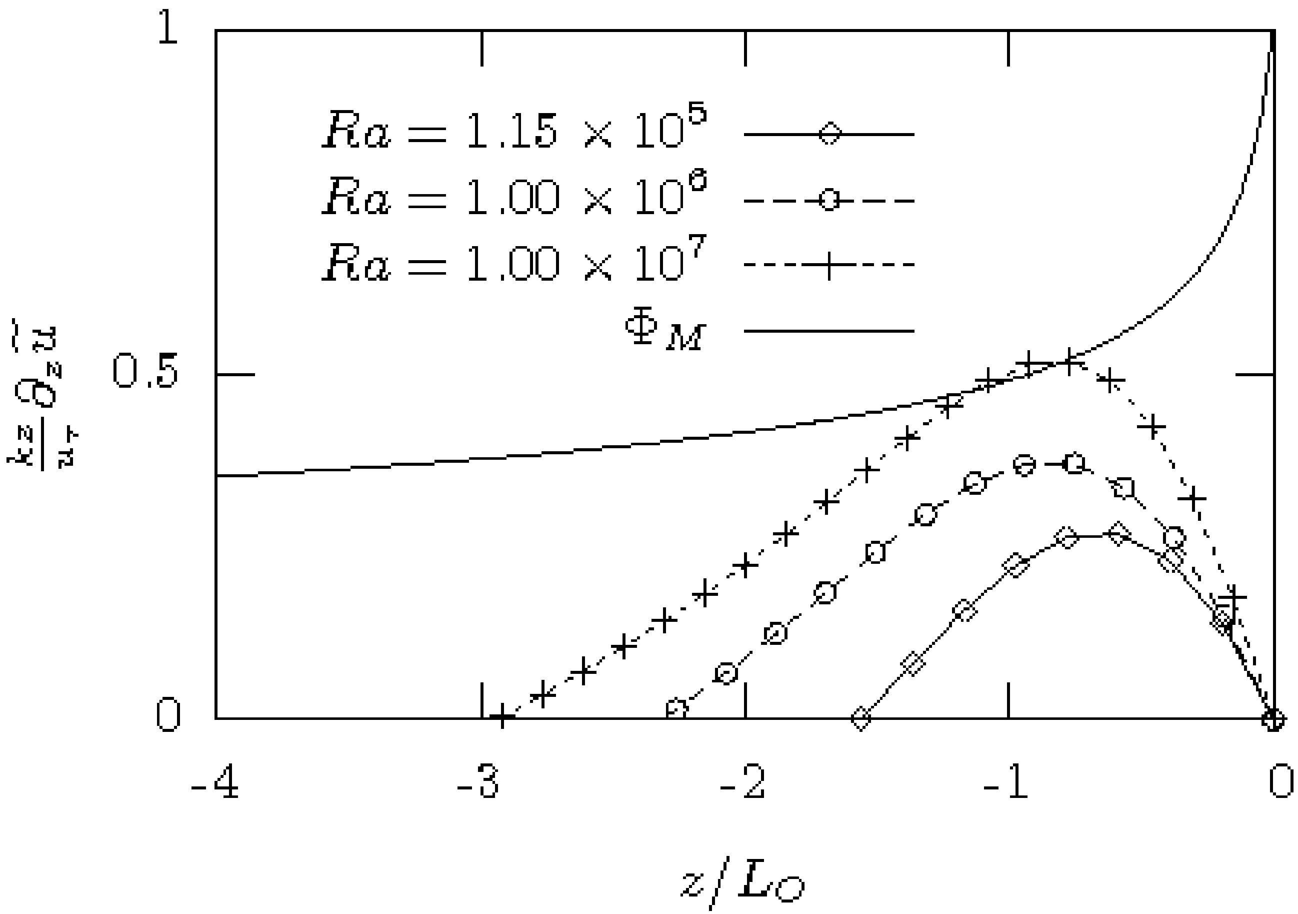
MO-similarity



Unstable situation ($L_O < 0$)

$$\Phi_M = \left(1 - \gamma \frac{z}{L_O}\right)^{-1/4}$$

e.g. Garrat (1994)

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Conclusions

- Boundary layers are turbulent
- Shape and friction factor point towards laminar-like behavior.
- The flow is firstly buoyancy-driven; the wind is a secondary effect.
- Obukhov length scales in the same way as λ_θ
- MO similarity theory does not match very well at these low Ra.

van Reeuwijk, Jonker & Hanjalic (2006), *submitted to JFM*

Preprint available at maarten.vanreeuwijk.net/publications

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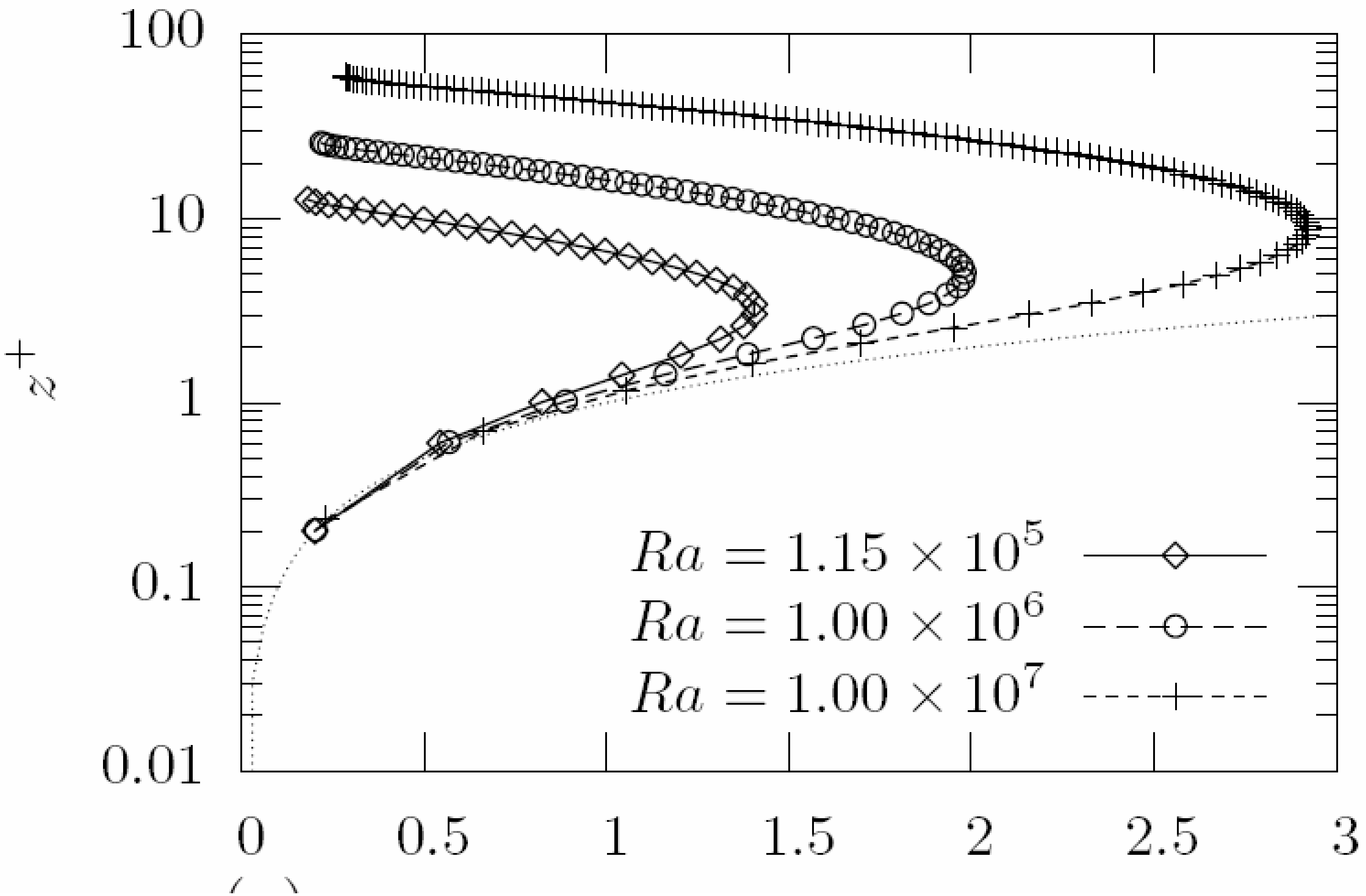
Outlook

- Seek for improved conceptual picture of the boundary layers.
- Move to higher Ra (10^8 and 10^9).
- Study the budgets of kinetic energy and Reynolds stress.
- Study smaller (AR=1,2) boxes with sidewalls.

Velocity profile in plus-units



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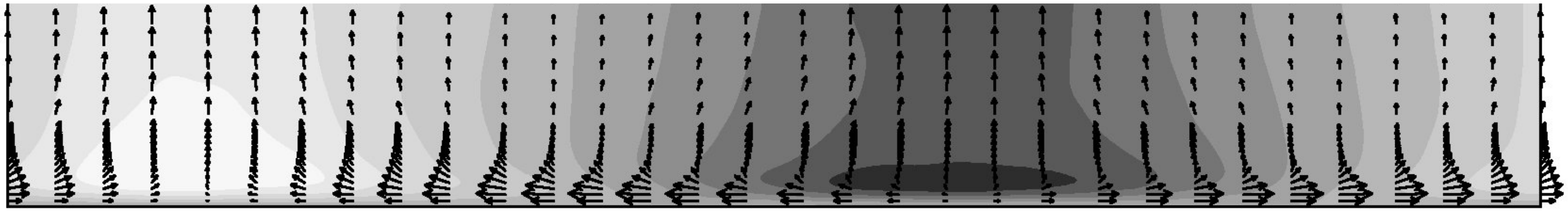
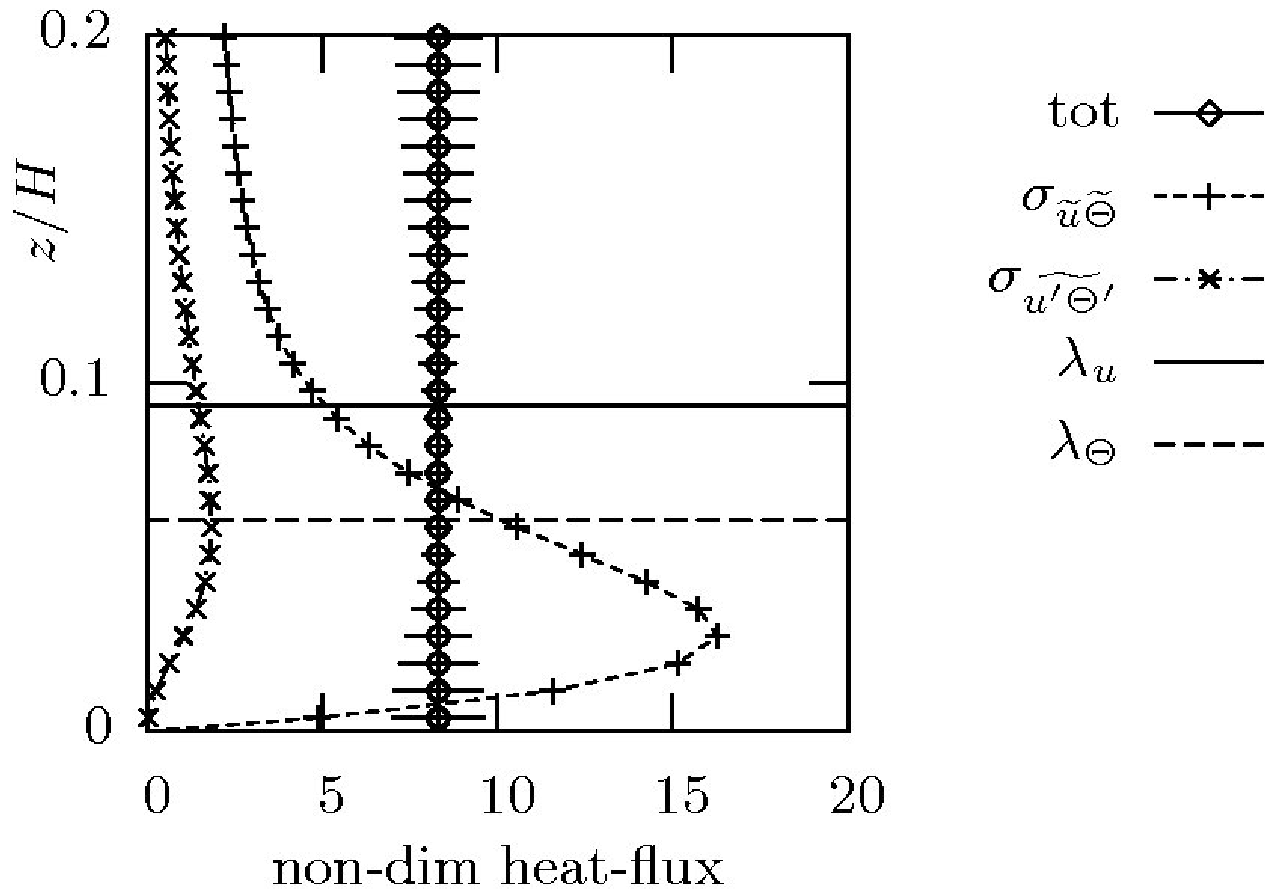
Horizontal heat-flux is zero

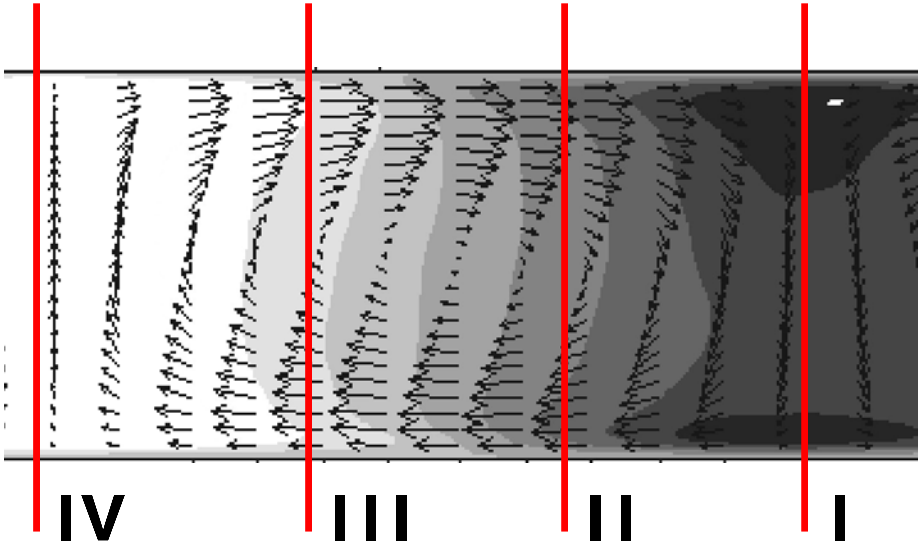
Horizontal heat-flux is zero by definition due to reflectional symmetry.



Horizontal heat-flux is not zero!



A look inside the wind engine



$$\mathcal{A} \quad \mathcal{D} \quad \mathcal{P} \quad \mathcal{B} \quad \mathcal{R}$$

$$\partial_t \widetilde{u}_i = -\partial_j \widetilde{u}_j \widetilde{u}_i + \nu \partial_j^2 \widetilde{u}_i - \partial_i \widetilde{p} + \beta g \widetilde{\Theta} \delta_{i3} - \partial_j \widetilde{u'_j u'_i}$$

$$\partial_t \widetilde{\Theta} = -\partial_j \widetilde{u}_j \widetilde{\Theta} + \kappa \partial_j^2 \widetilde{\Theta} - \partial_j \widetilde{u'_j \Theta'}$$

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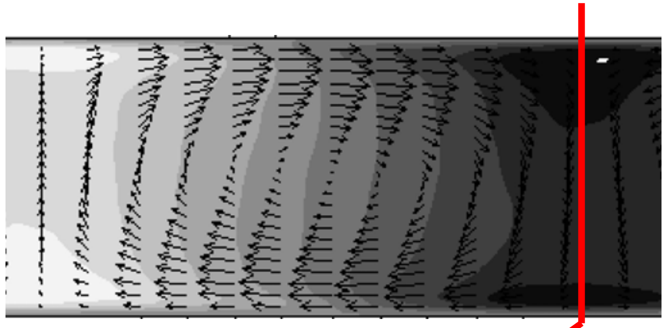
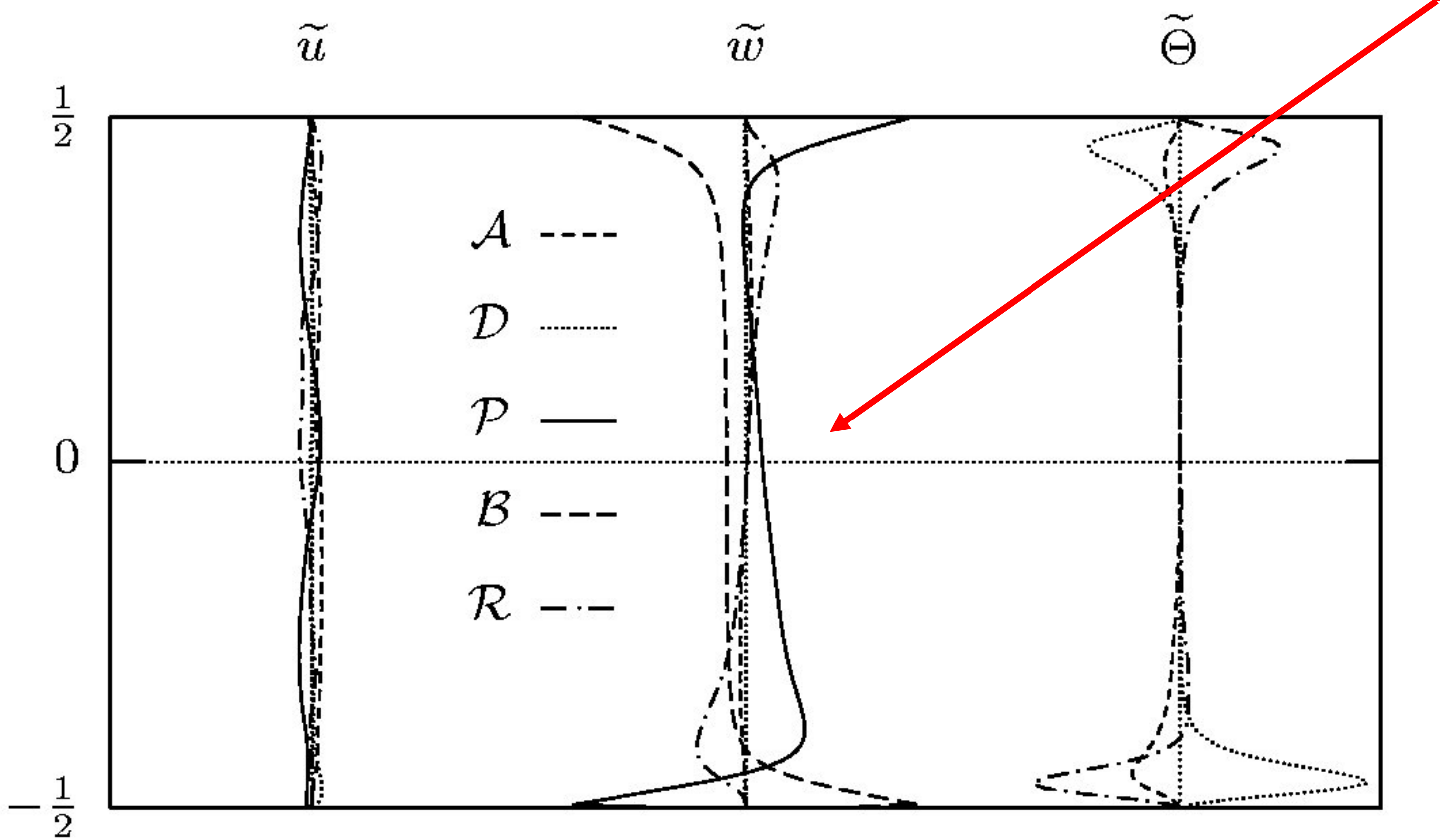
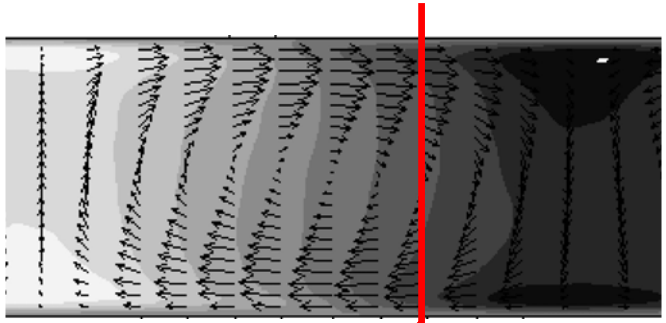
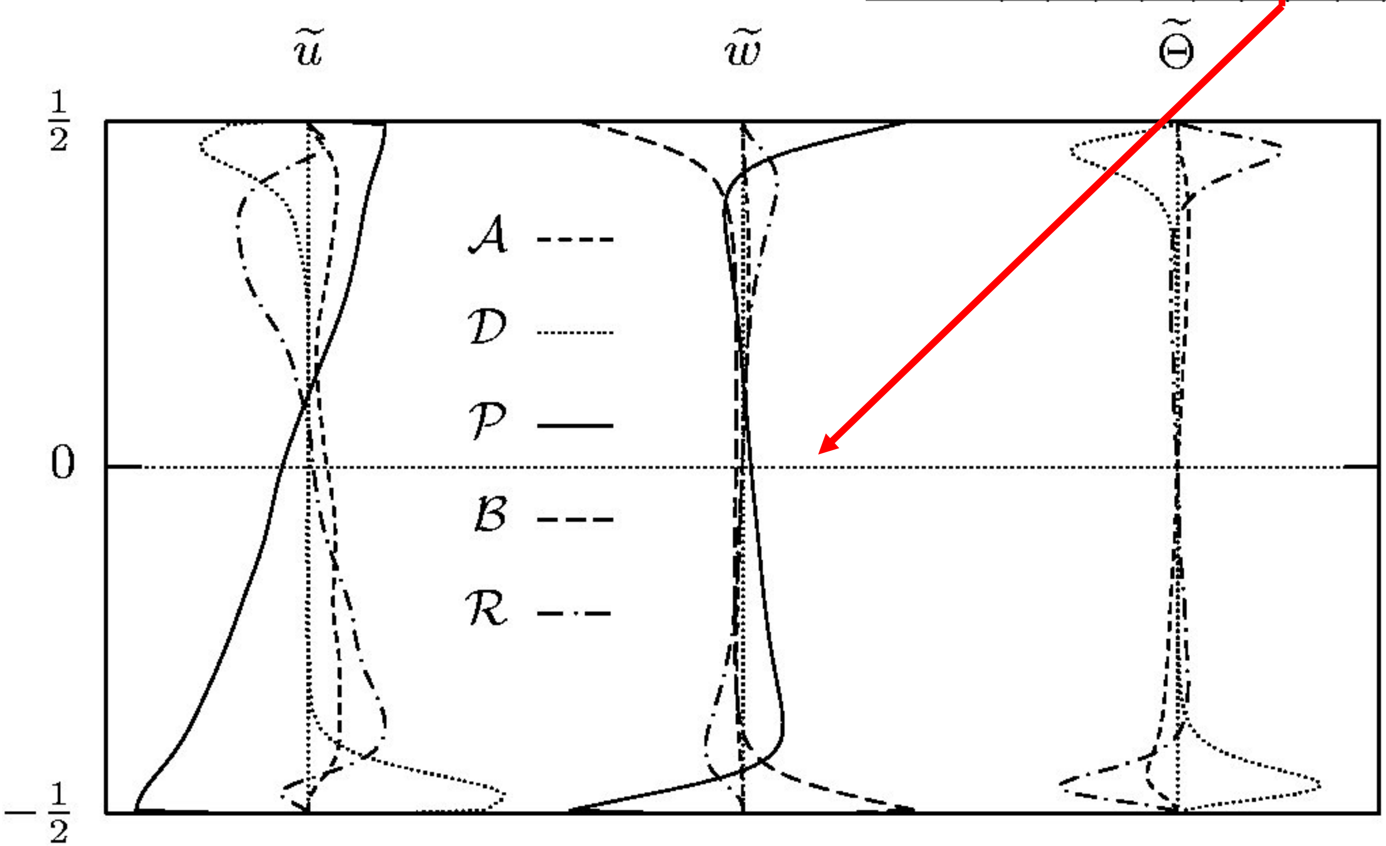
Momentum balance I





Momentum balance II





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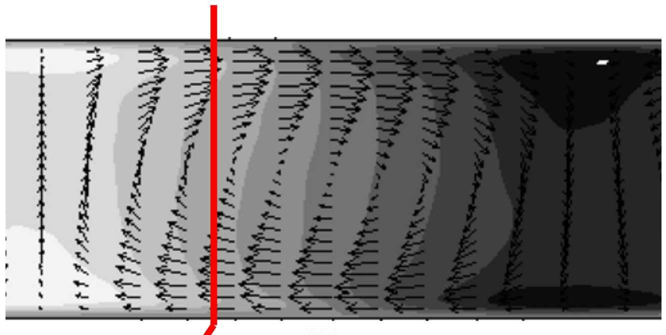
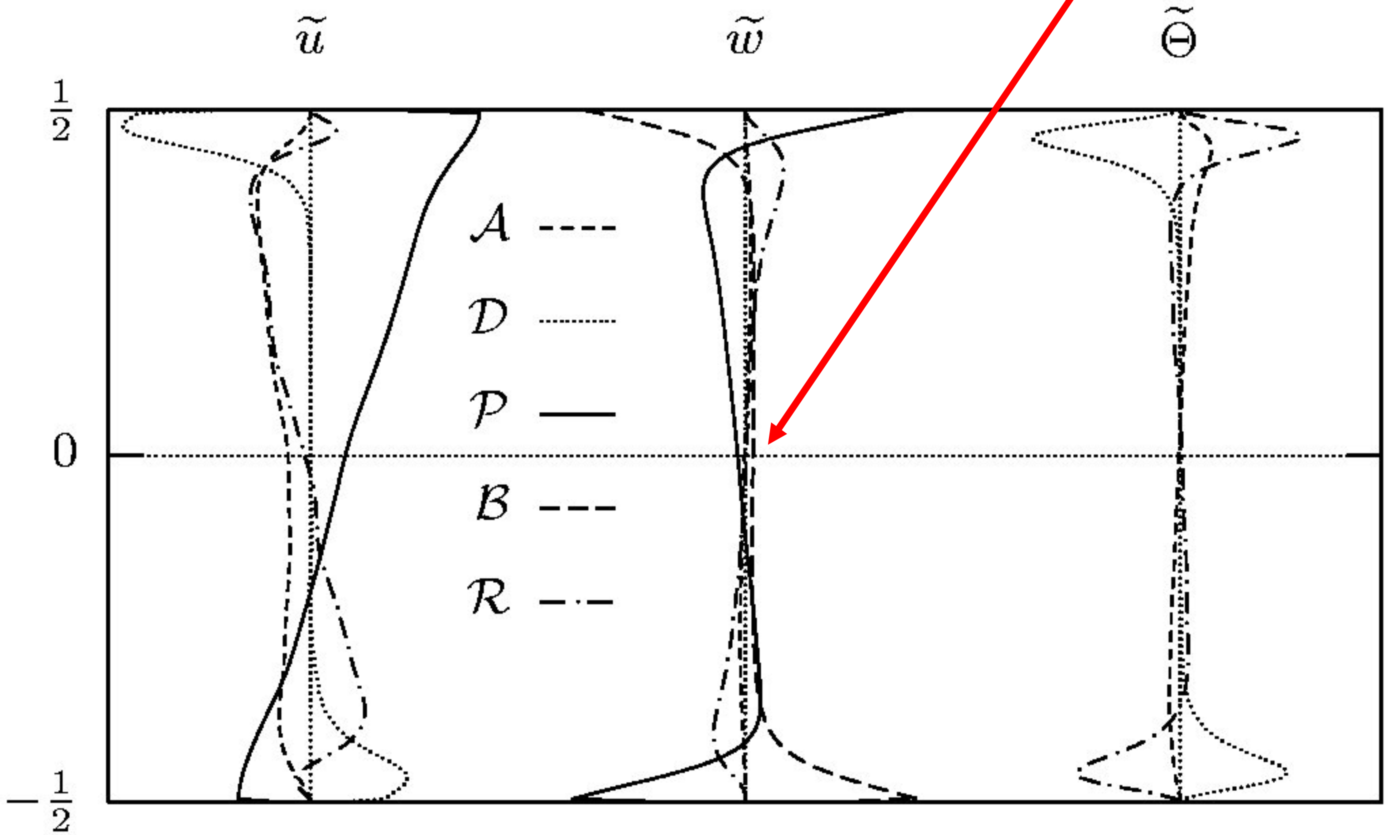
Momentum balance III





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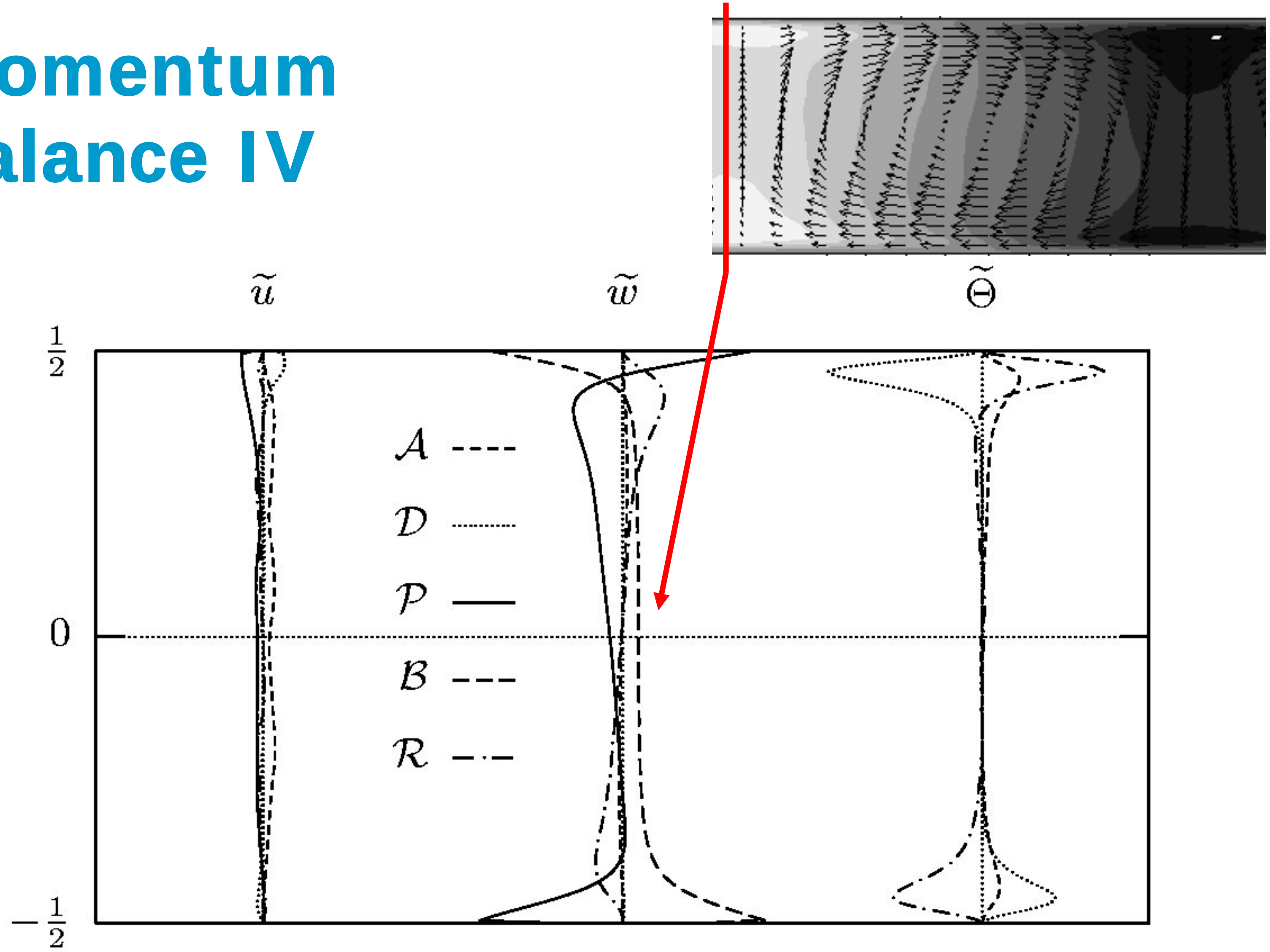
Momentum balance IV



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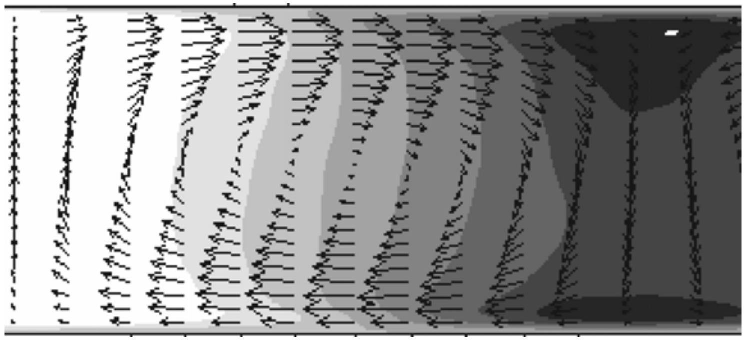
The wind feedback cycle

Momentum equation

Wind

Heat transport through BL's

Spatial variations in pressure



Spatial variations in Nu

Buoyancy

Spatial variations in temperature

$$Nu(x) = \frac{H}{\Delta} \partial_z \tilde{\theta}(x)$$

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