The Abdus Salam
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United Nations Educational, Scientific and Cultural Organization

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Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

4 - 8 Sept., 2006, ICTP, Trieste, Italy

High Rayleigh number convection: helium experiments

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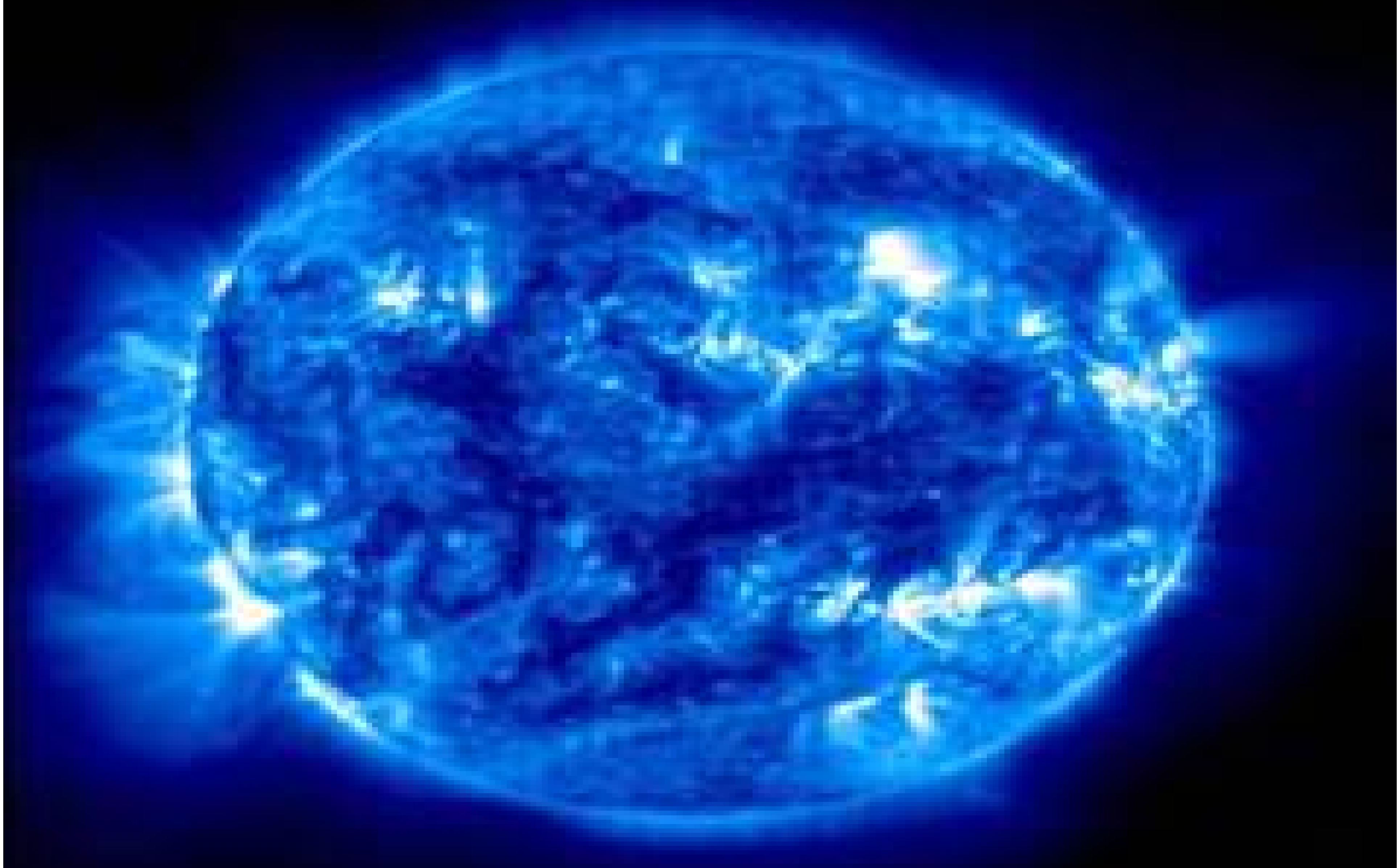
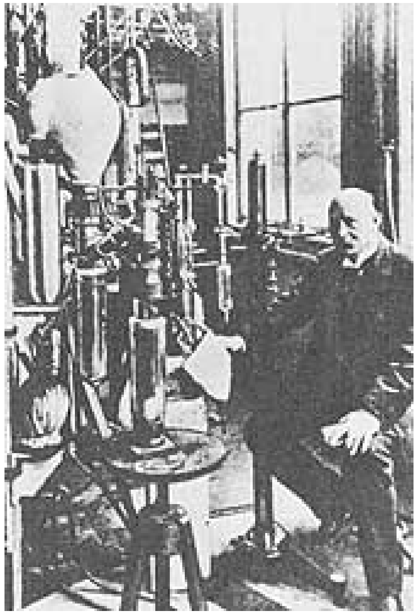
Trieste
Italy

These are preliminary lecture notes, intended only for distribution to participants

High Rayleigh Number Convection: Helium Experiments



J. Niemela,   ICTP Trieste

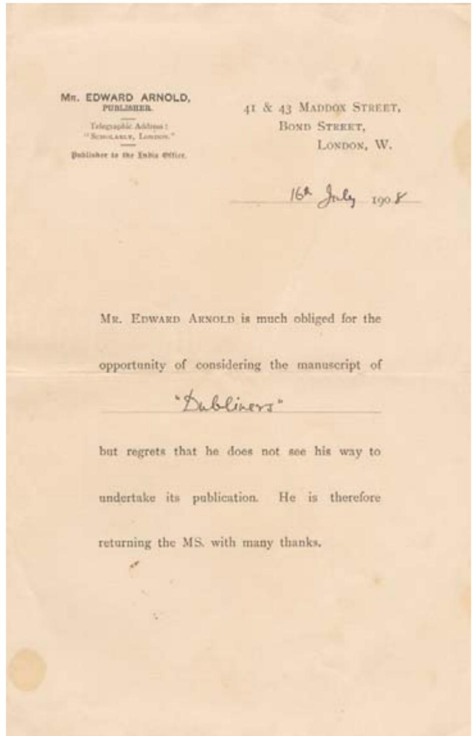


A brief historical digression….

Leiden, 1908: Kamerlingh Onnes succeeds in liquifying helium, an element first identified spectroscopically in India in 1868 during a total eclipse of the sun. This leads to the discovery of superconductivity a few years later, superfluidity a few decades later, and many diverse applications in science and engineering. Applications evolve to turbulence (both classical and quantum), including thermal convection problems at high Ra and also near onset (Ahlers, Threlfall…).

That same year in Trieste….

Trieste, 1908: Trieste resident James Joyce receives a rejection letter for what would become the first of his major works: "Dubliners." [It was finally published 6 years later].



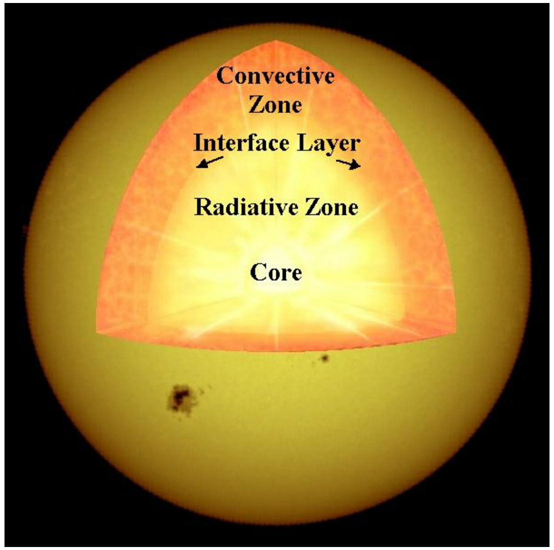
MR. EDWARD ARNOLD,
PUBLISHER.
Telegraphic Address:
"SCHOLARLY, LONDON."
Publisher to the India Office.

41 & 43 MADDOX STREET,
BOND STREET,
LONDON, W.

16th July 1908

MR. EDWARD ARNOLD is much obliged for the opportunity of considering the manuscript of "Dubliners" but regrets that he does not see his way to undertake its publication. He is therefore returning the MS. with many thanks.

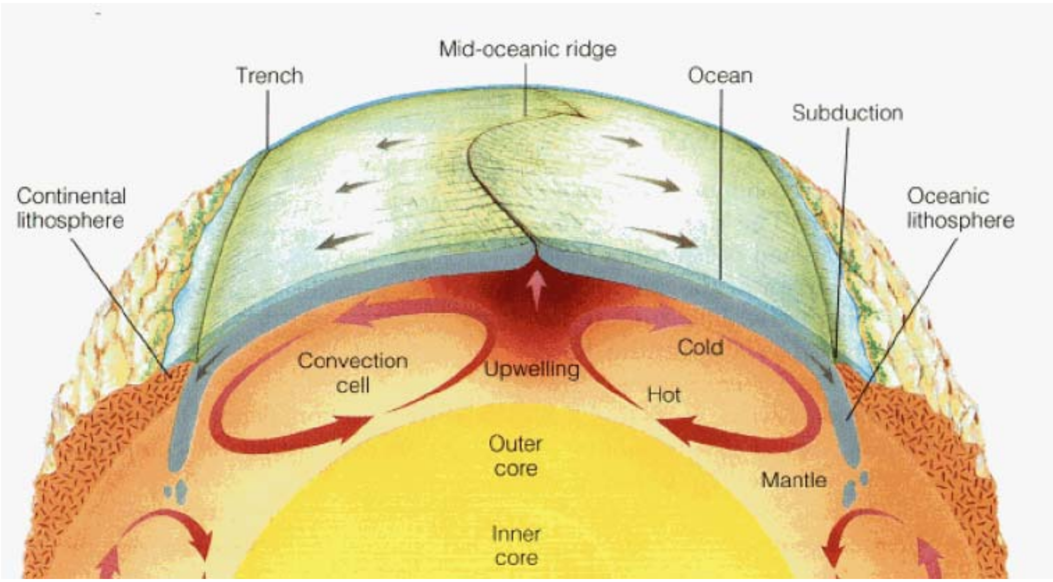
Some examples of thermal convection at limiting values of the control parameters in Nature



Sun

Ra ~10^{22}

10^{-3}<Pr<10^{-10}



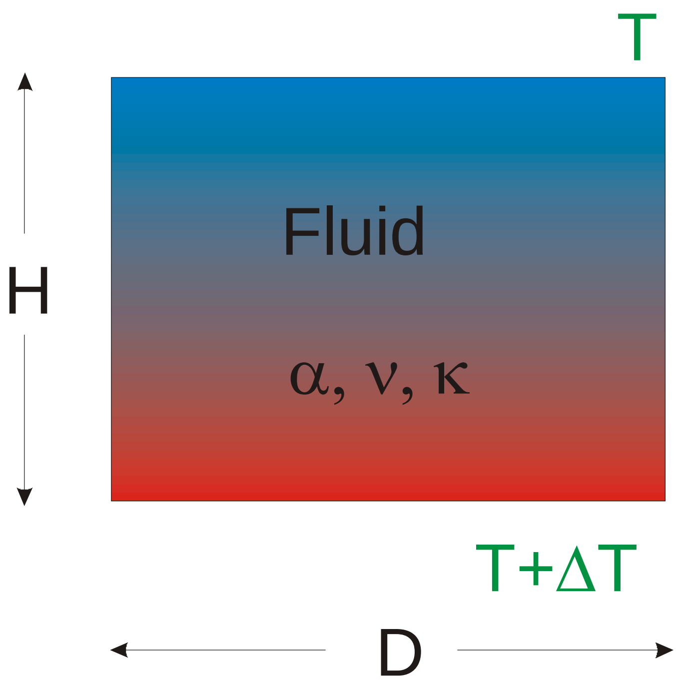
Mantle

Ra ~ 10^{6}

Pr~10^{21}

(Pr~10^{3},magma)

Rayleigh-Benard Convection



α fluid thermal expansion coefficient

ν fluid kinematic viscosity

κ fluid thermal diffusivity

Control parameters for convection

$$Ra = \frac{g\alpha\Delta T H^3}{\nu\kappa}$$

$$Pr = \nu/\kappa$$

$$\Gamma = D/H$$

Global heat transfer: Nusselt number

$$Nu = Q \cdot \left(\frac{k_f \Delta T}{H} \right)^{-1}$$ = applied heat flux normalized by that calculated for conduction

where Q = applied heat flux; k_f= fluid thermal conductivity

$$\mathrm{Nu} = f(\mathrm{Ra}; \mathrm{Pr}; \Gamma; \ldots)$$

In the limit of large (infinite) Ra (and holding other parameters constant):

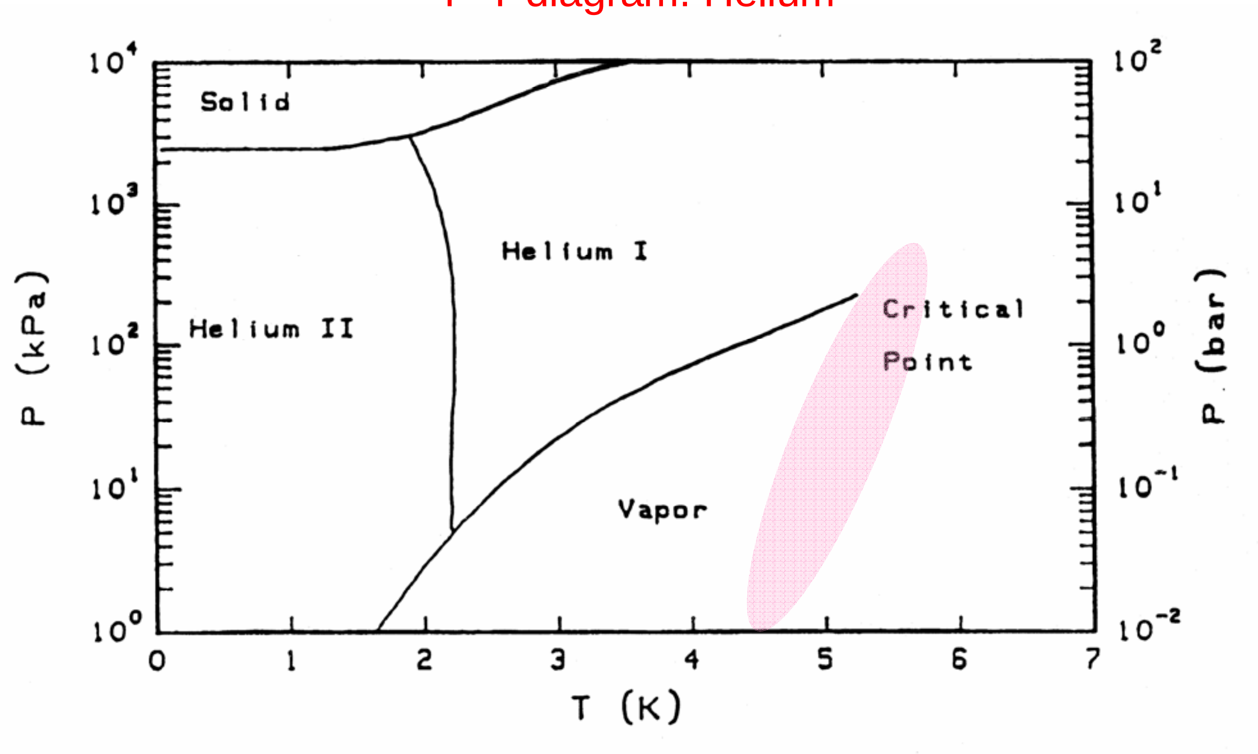
$$\mathrm{Nu} = C\, \mathrm{Ra}^{\beta} + \ldots$$ (perhaps with logrithmic corrections, etc.…)

Persistent asymptotic predictions:

β=1/3: in the limit of infinite Pr appears to be a rigourous result (with log corrections) (e.g., Doering, et al., Constantin, et al).

β=1/2 , "ultimate" regime (Kraichnan, with log corrections); observed in numerical simulations under certain conditions (e.g., Toschi & Lohse).

P-T diagram: Helium



$$Ra = g \cdot \left(\frac{\alpha}{\nu\kappa} \right) \cdot \Delta T \cdot H^3$$

4.4 K , 2 mbar:

$\alpha / \nu\kappa = 5.8 \times 10^{-3}$

5.25 K, 2.4 bar:

$\alpha / \nu\kappa = 6.5 \times 10^{9}$

Ra ~ const. x ($\rho^2 \alpha C_P$). Ra increases as ρ^2 away from critical point and as αC_P in its vicinity.

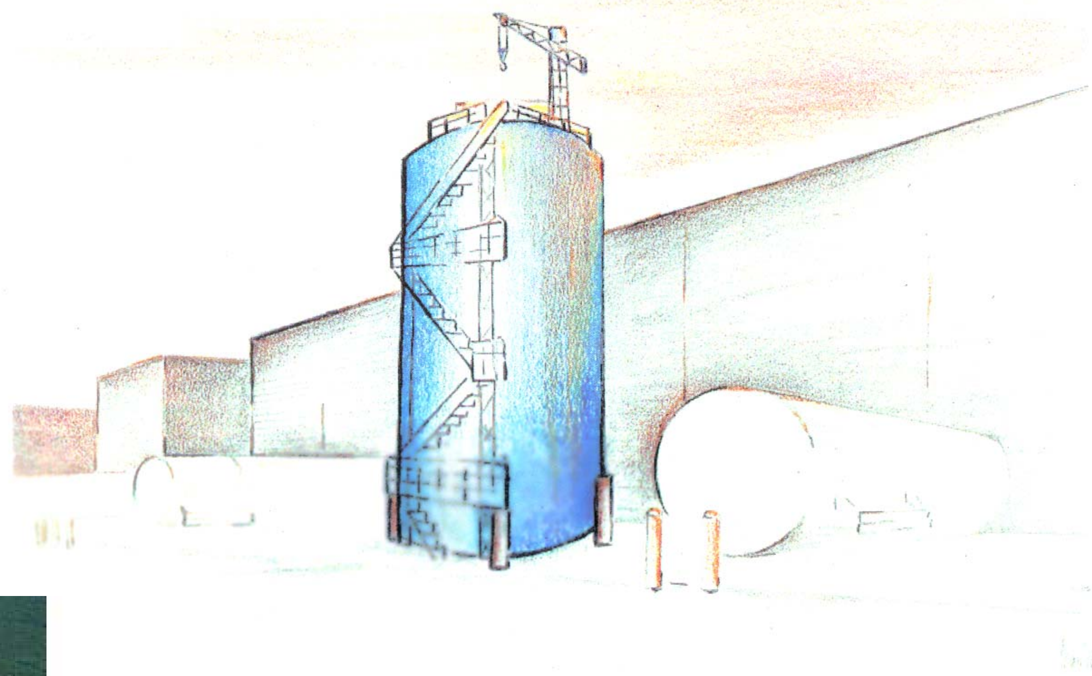
In the not-so-distant past…there was the proposed “Barrel of Donnelly”

Inside cell dimensions
D = 5m, L = 10m,
0.5-10^5 liters of liquid helium needed.

Outside dimensions
~7 m dia and ~20 m high

Refrigeration needed
< 200 W at 4K

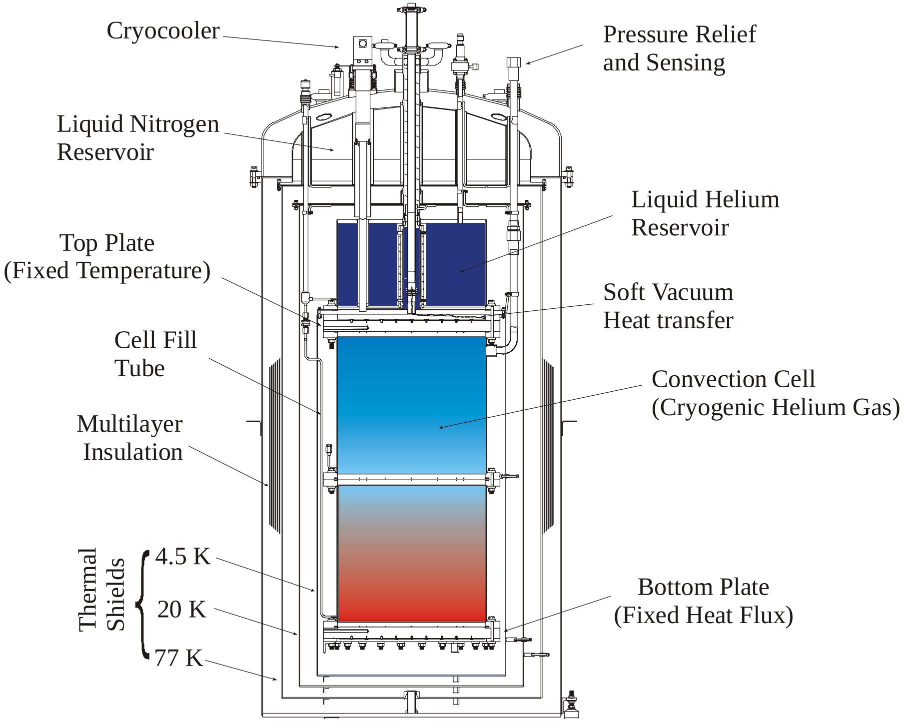
Ra: up to 10^{21}



Huge accelerator facilities like CERN or BNL would have plenty of liquid helium in circulation (used for cooling superconducting magnets).

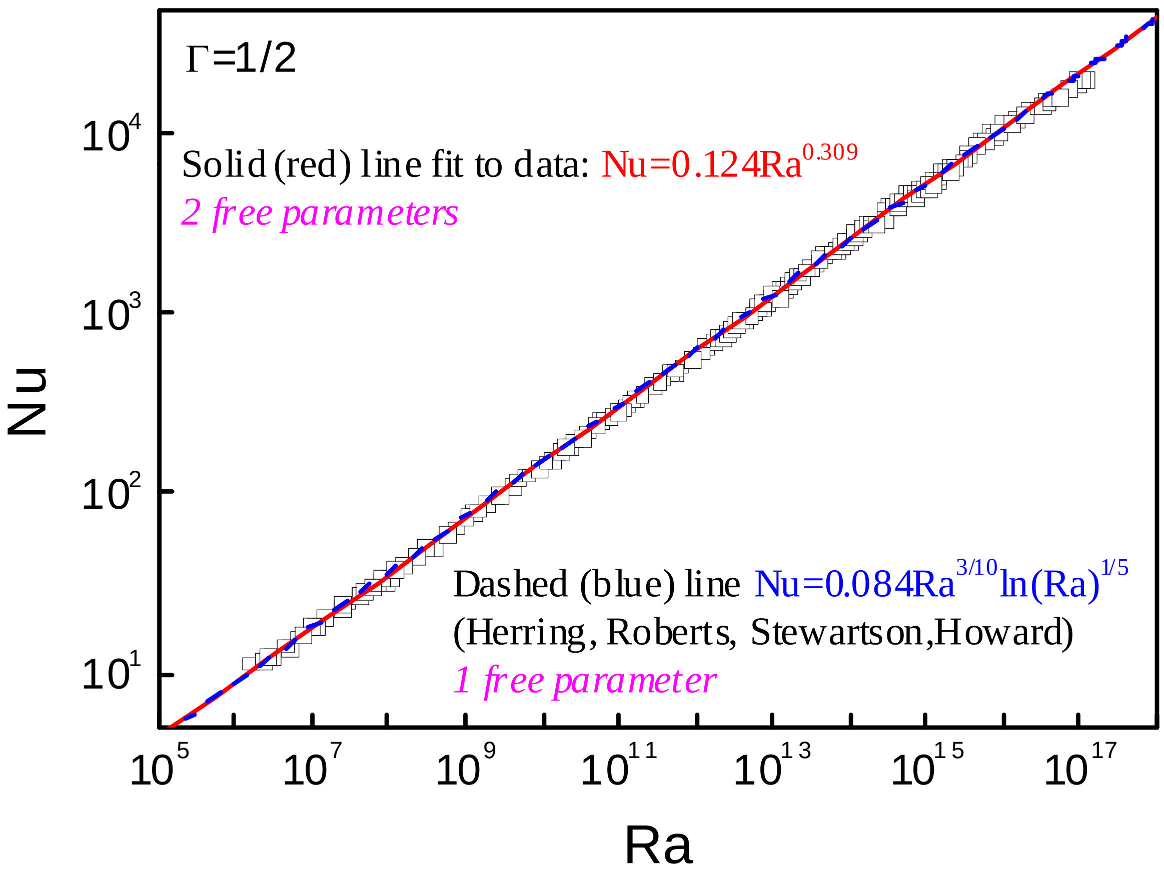
A cryogenic apparatus for very high Ra

Maximum (minimum) sample height: 1 m (0.125 m)
Fixed diameter= 0.5 m

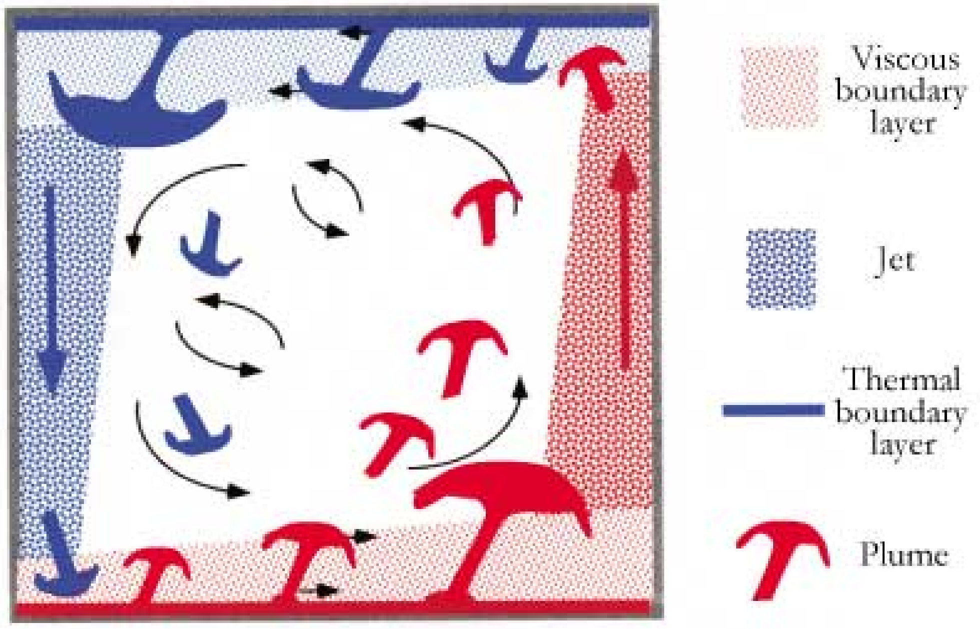


11 decades of Ra possible. Larger sample heights move *entire range of accessible Ra* into turbulent regime and indirectly extends conditions of constant Pr to higher Ra.

Observed Nu vs Ra for 1 meter tall , Γ=1/2
(original data)



An average picture of turbulent convection, showing in particular an organized advection of plumes

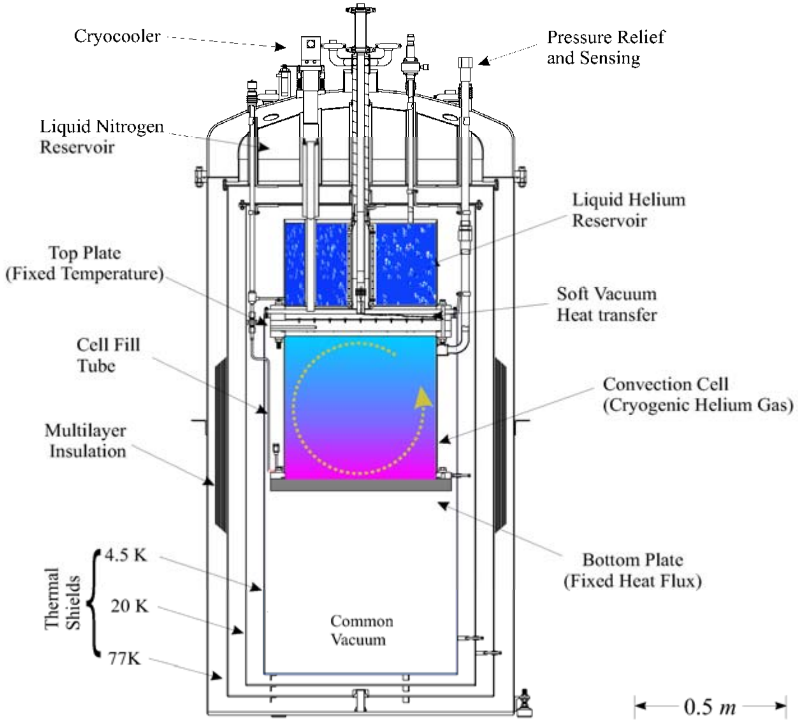


(from L. Kadanoff, *Physics Today*, August 2001)

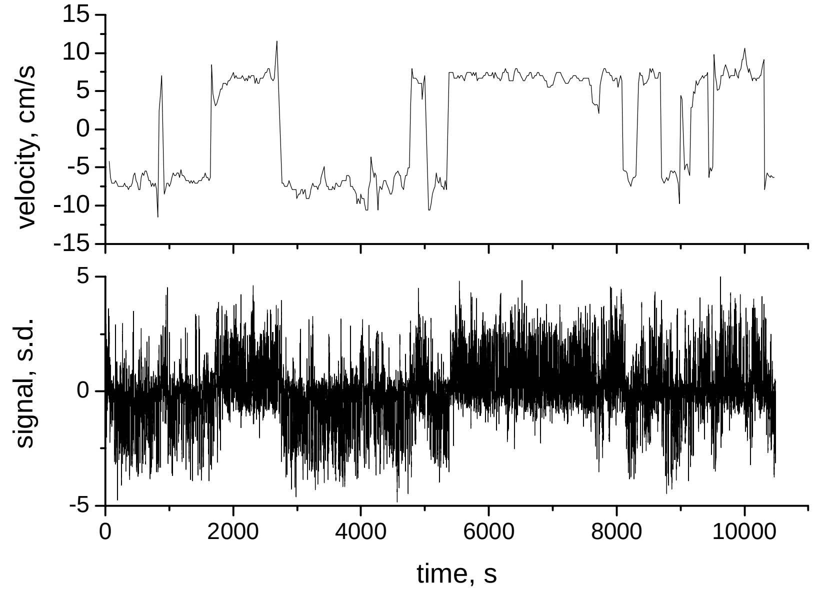
Some issues for consideration:

- *coupling of top and bottom boundary layers due to wind*
- *importance of thermal conditions on sidewalls in presence of wind*
- *ability of heating plates to supply needed rate of plume formation*
- *What does it mean to have a laminar or turbulent boundary layer in the presence of plume activity?*

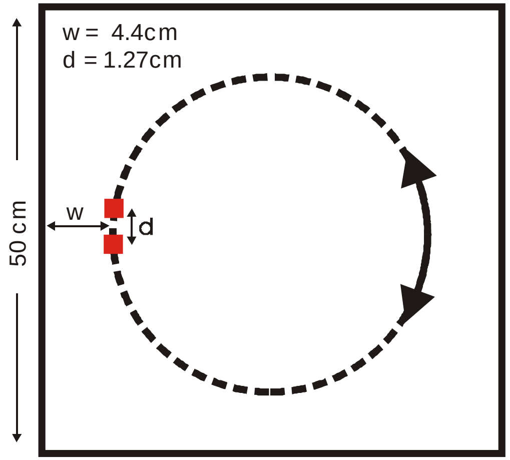
An aspect ratio unity cell assembled for maximizing the mean wind



Stabilization: **10^5 turn-over times of the wind**
Max run times: **10^4 turn-over times of the wind**

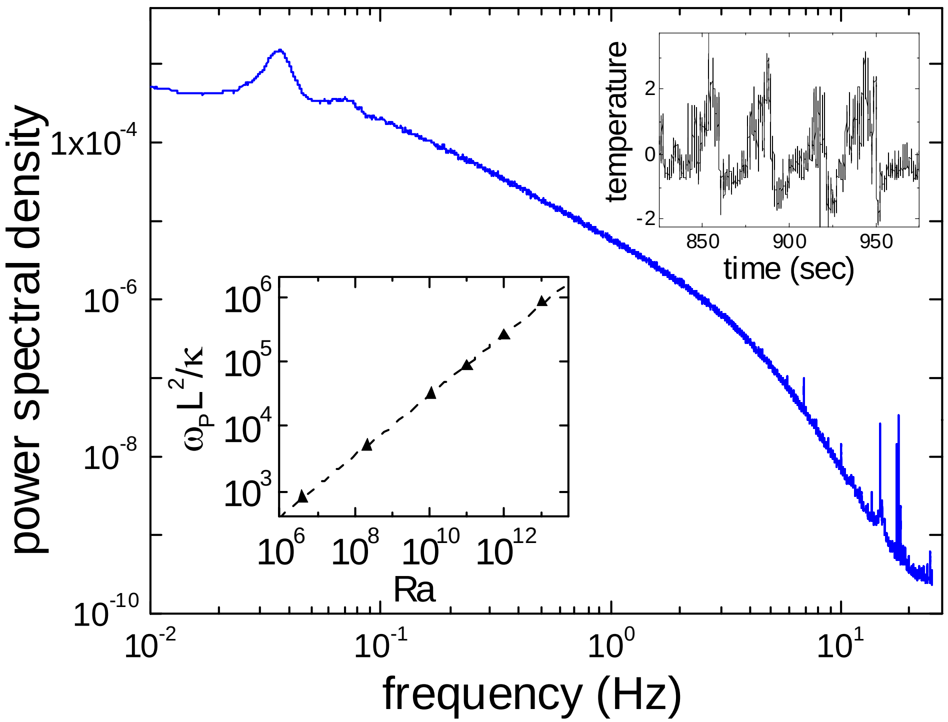


Maximizing the correlation between temperature signals gives the magnitude and direction of a large scale circulation.



Reversals of flow direction are observed at points in the fluid (sensors 180 azimuthal degrees apart show simultaneous flipping of the wind direction).

250-micrometer NTD-doped Ge sensors are placed in various positions in the flow. Shown are two sensors vertically separated near the sidewall.

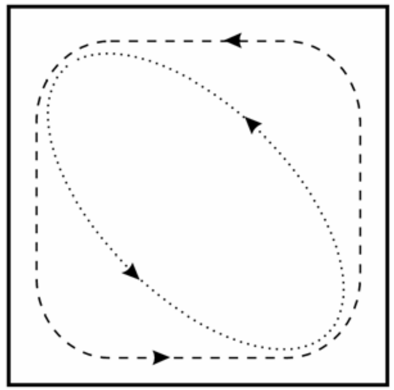


Coherent advection of plumes indicated by peak in PSD.

Note a **crude** estimate for the circulation frequency:

$$\frac{V_M}{4H} \sim \frac{7\,cm/s}{200\ cm} = 0.035\,Hz$$

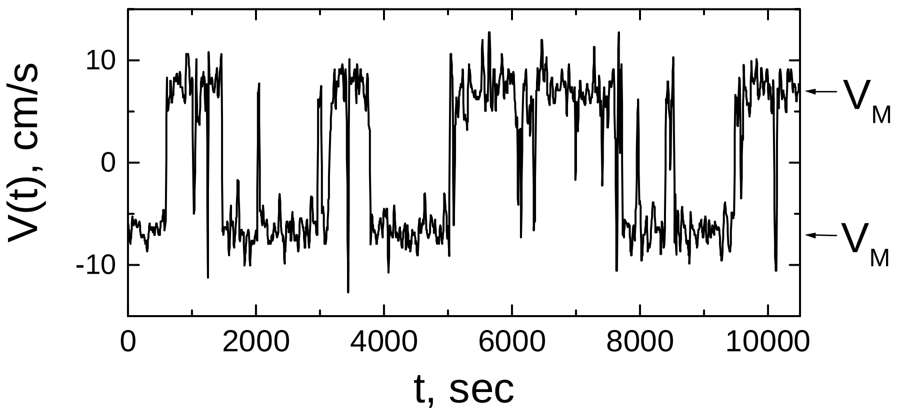
(If "4H" is interpreted as an effective path length, it could be smaller by a factor of, say, 2 depending on Ra. Consider also model of Villermaux, 1995)



Niemela & Sreenivasan, EPL 2003

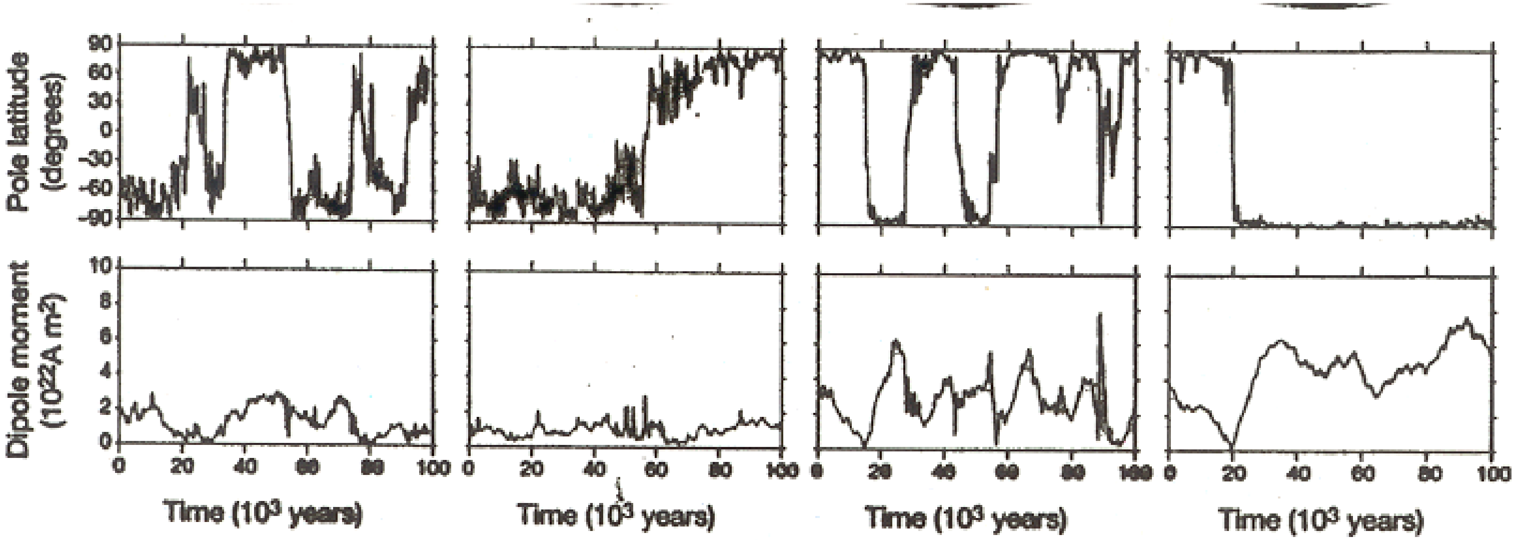
The mean wind and its reversals

Thermal convection



Segment of continuous 5.5-day time series.

Geomagnetic polarity reversals: range of time scales~ 10^3-10^5 years.



Glatzmaier, Coe, Hongre and Roberts *Nature* **401**, p. 885-890**,** 1999

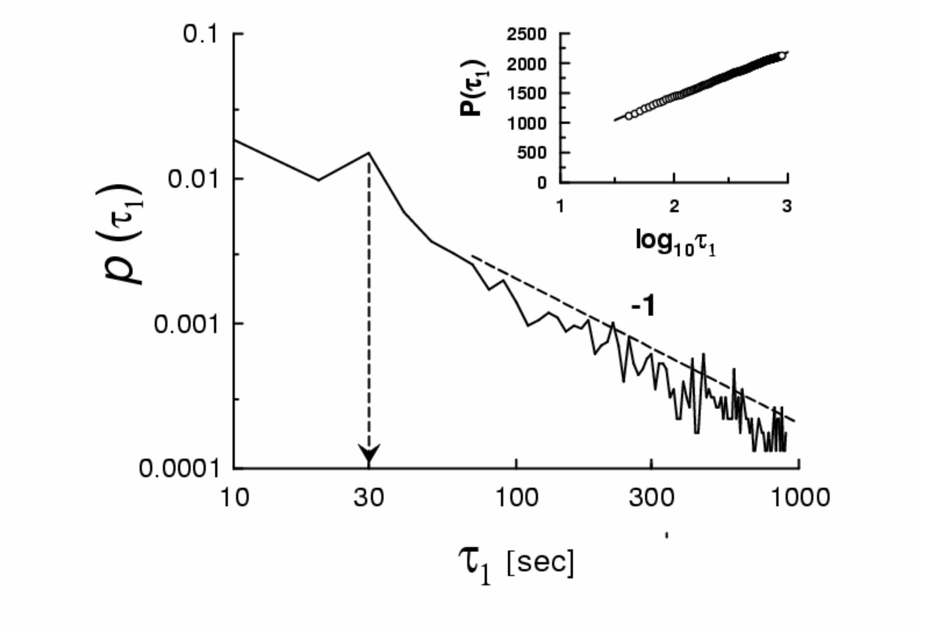
How are the reversals distributed?

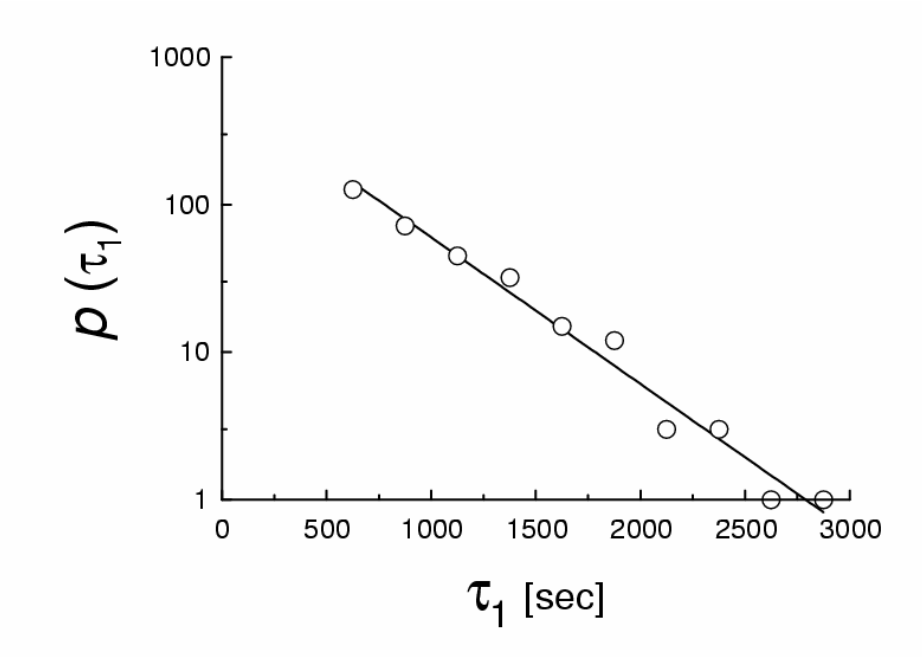
Define τ_1 = time between the nth and (n+1)th switch in flow direction: $\tau_1 \equiv T_{n+1} - T_n$

power-law scaling for small τ:

For large τ_1: $p(\tau_1) \sim \exp[-(\tau_1 / \tau_m)]$

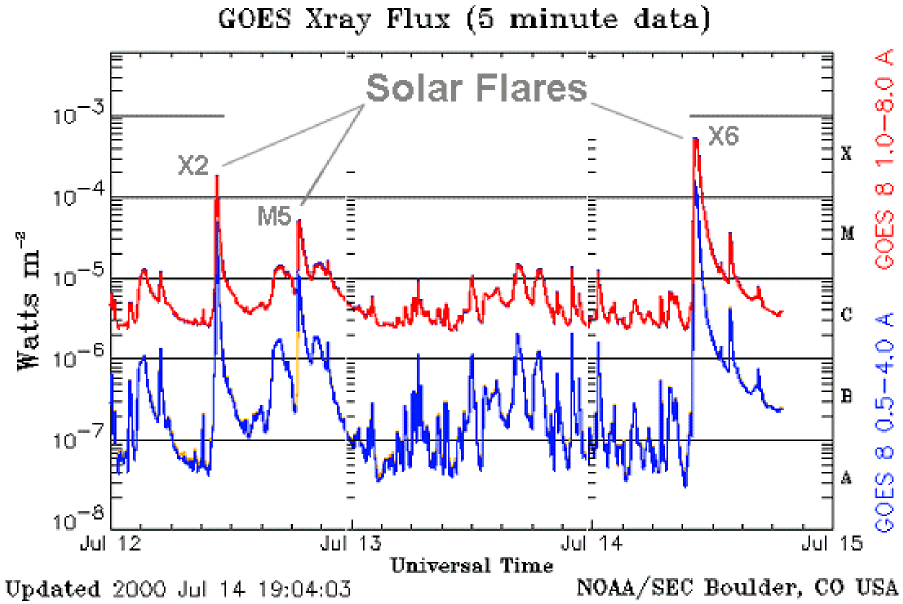
$\tau_m \simeq 400s$





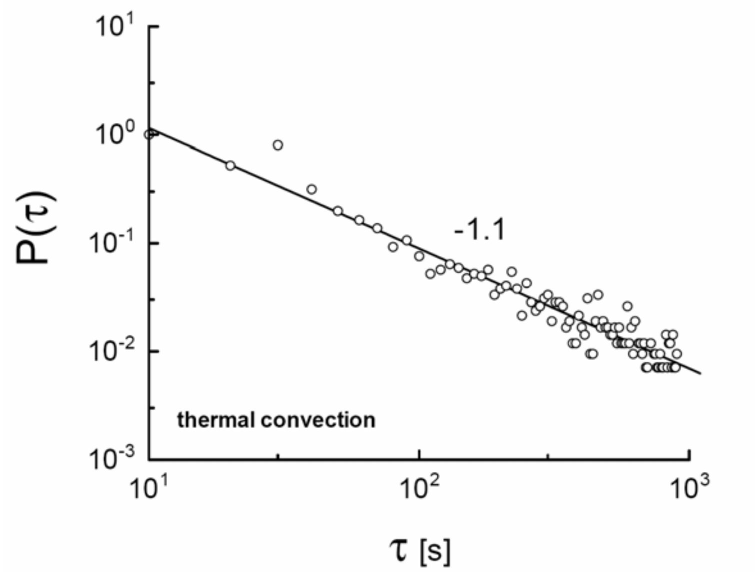
K.R. Sreenivasan, A. Bershadskii & J.J. Niemela, *Phys. Rev. E,* **65**, 056306 (2002)

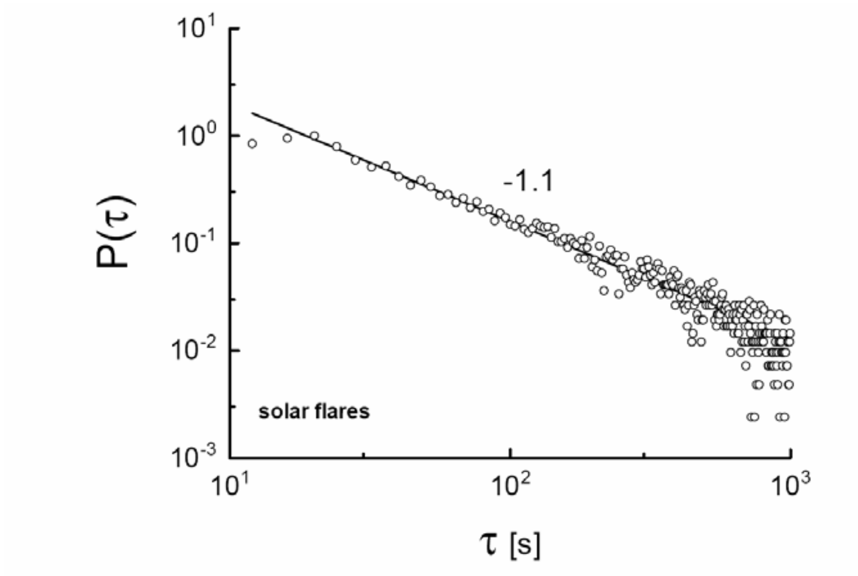
Medium energy solar flares owe their duration to turbulent convective motions in the convective zone of the sun which shuffle footprints of the magnetic coronal loops (Parker, 1994).



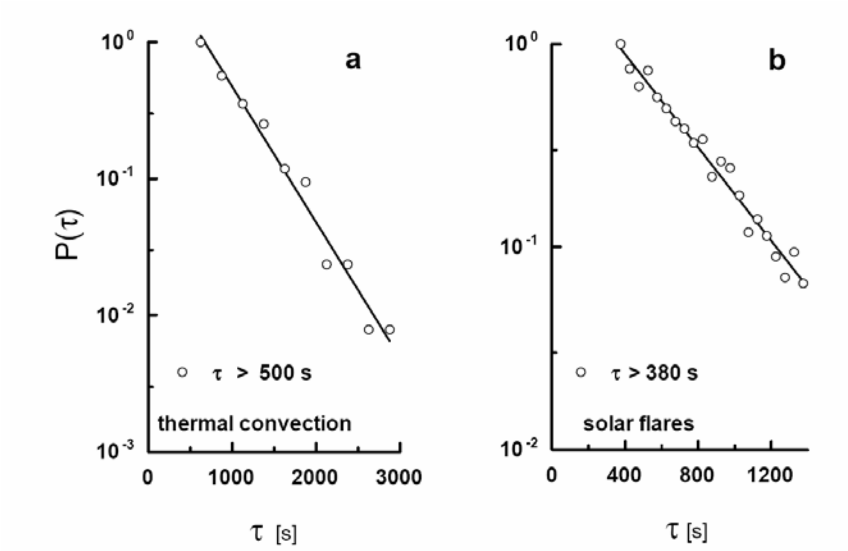
Reuven Ramaty High Energy Solar Spectroscopic Imager
(RHESSI)

Comparison of the duration of single-direction wind in RBC experiments to the duration of solar flares observed by RHESSI





PDFs of duration times for (left) the maintenance of one direction of the wind in confined thermal convection and (right) lifetimes of medium energy solar flares. Both exhibit the same power law scaling with -1.1 exponent.



…and exponential decay tails at long times

(Bershadskii, Niemela and Sreenivasan, Physics Letters A 2004)

Convection in a loop

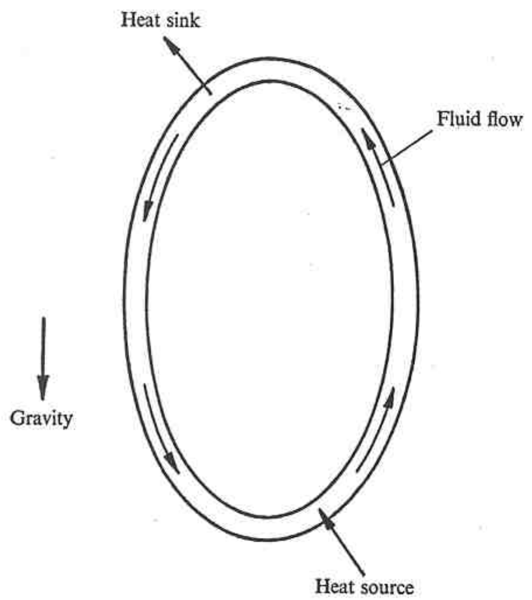
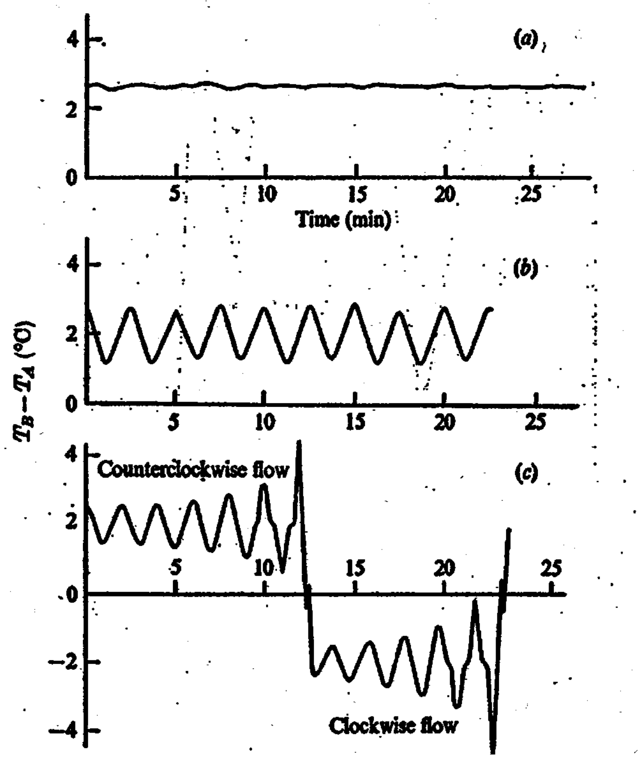


FIGURE 1. Free convection loop.

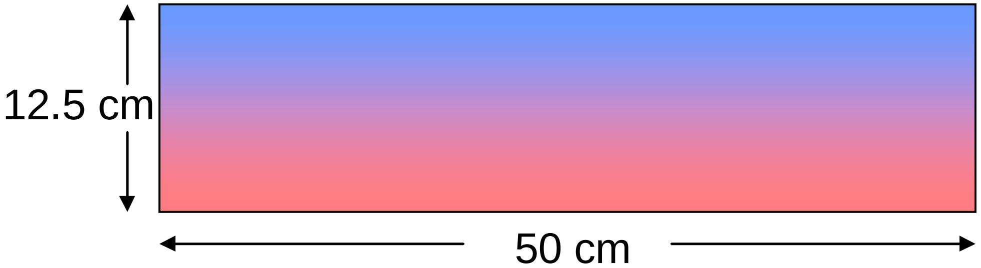
Stability of a single-phase free convection loop

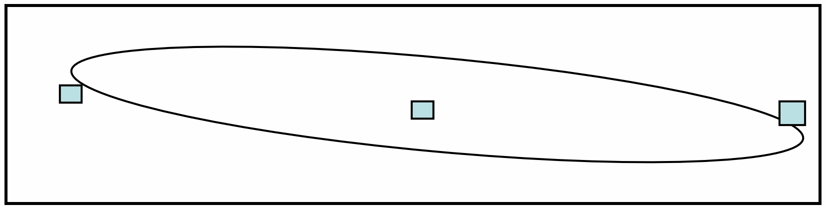


Next: an Γ=4 cell capable of attaining high Ra*

* J.J. Niemela & K.R. Sreenivasan, *J. Fluid Mech*. **557**, 411-422 (2006)

Elettra synchrotron light laboratory





Side view/sensor arrangement, $\Gamma = 4$ cell

Various corrections are applied to the data for effects associated with non-ideal conditions. Here, we consider the finite conductivity of the plates

Verzicco (2004):

$Nu = f(X)Nu_{inf}$, where Nu_{inf} is achieved with "ideal" plates

$X=R_f/R_p= k_pH/(k_f*Nu*e)$ →

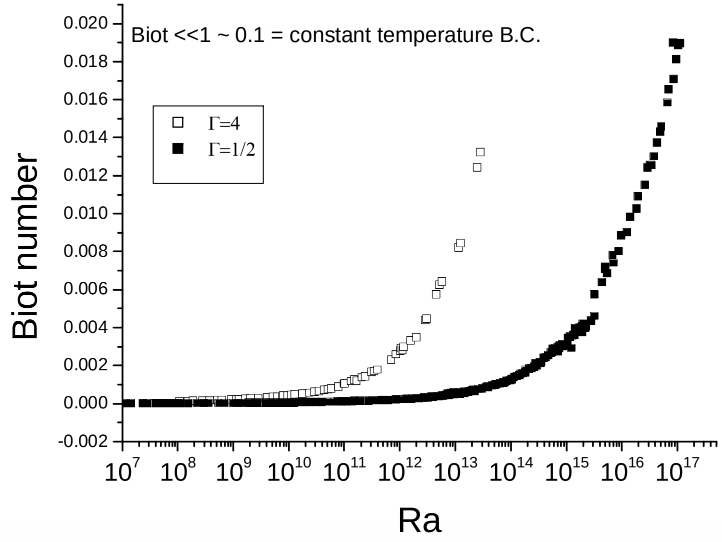
k_p= thermal conductivity of plates
k_f= thermal conductivity of fluid
e= thickness of the plates.

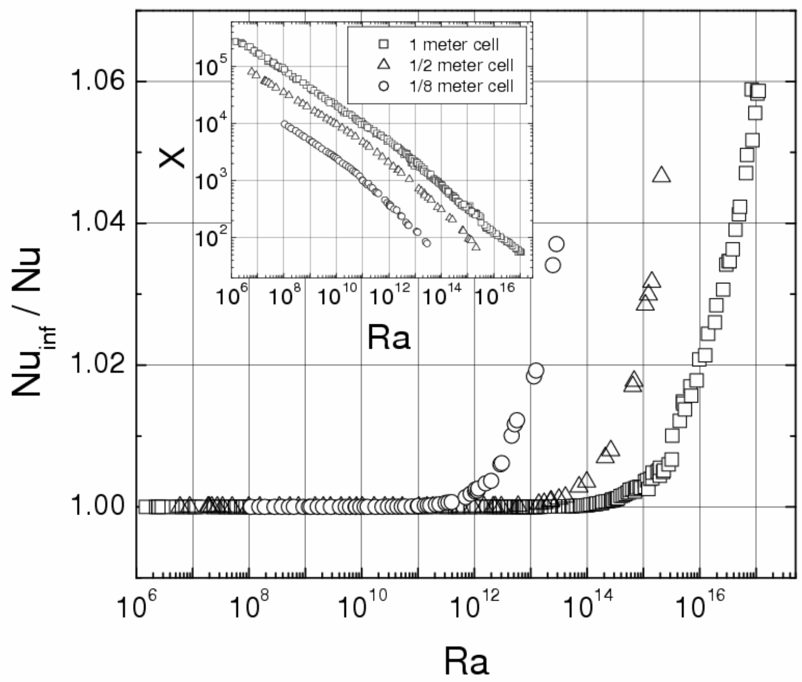
$F(X) =1-exp[-(X/4)^{1/3}] X/(X-2)$

An empirical relation was derived subsequently by Brown, et al (2005):

$F(X) =1-exp[-aX^b]$ with a=0.275, b=0.39

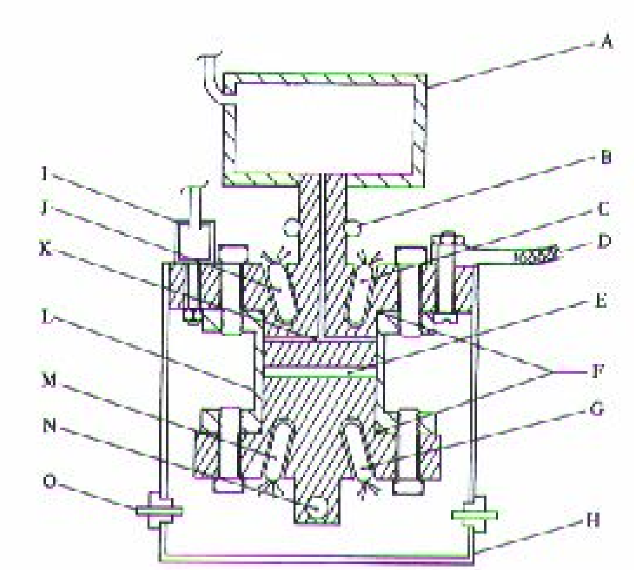
Effect is small for helium experiments and affects *only the last half decade of results* no matter what the height—this is because lowering H lowers X in a way that just compensates for the reduced range of Ra. We may consider that this effect is relatively negligible in helium experiments.





A digression on thermal boundary conditions

Sparrow, et al (JFM, 1964) argued that a fixed heat flux at one or the other plate should result in measureable differences, particularly in the onset Ra which should be reduced significantly. [Busse argued that relaxation oscillations could be seen in certain circumstances beyond the initial bifurcation].



A set of measurements (unpublished) was performed to test the Sparrow hypothesis empirically (Niemela & Donnelly, 1986). This was done by fabricating most of the bottom plate out of an OFHC copper “sponge” of 60% porosity and small, nearly micronic, pore size. Thus, the thermal conductivity retained its same high value, but the heat capacity of the plate increased by several orders of magnitude, being essentially that of the entrained fluid, thus providing an effectively constant temperature boundary condition (high thermal conductivity *and* heat capacity). **No measurable difference could be detected, however, in the observed critical temperature difference to a level better than 0.1%.**

Sidewall conduction effects

[Ahlers (2001), Roche (2001), Roche et al (2002), Verzicco (2002), Niemela and Sreenivasan, (2002)],

Numerical exp's by Verzicco (2002) show the LSC is *forced* by the sidewall conduction and is changed by it. This is more significant at low Ra and becomes negligible for Nu >>1.

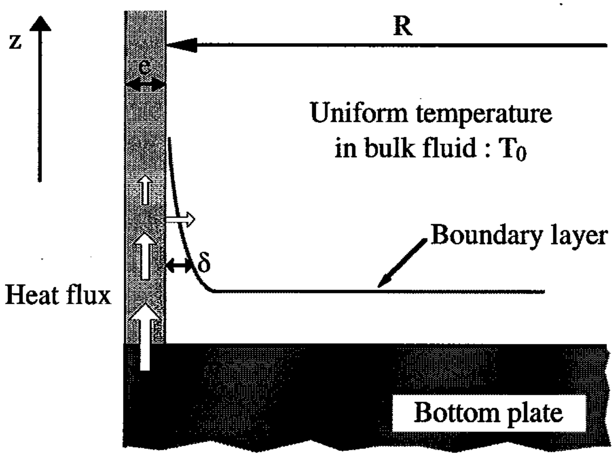
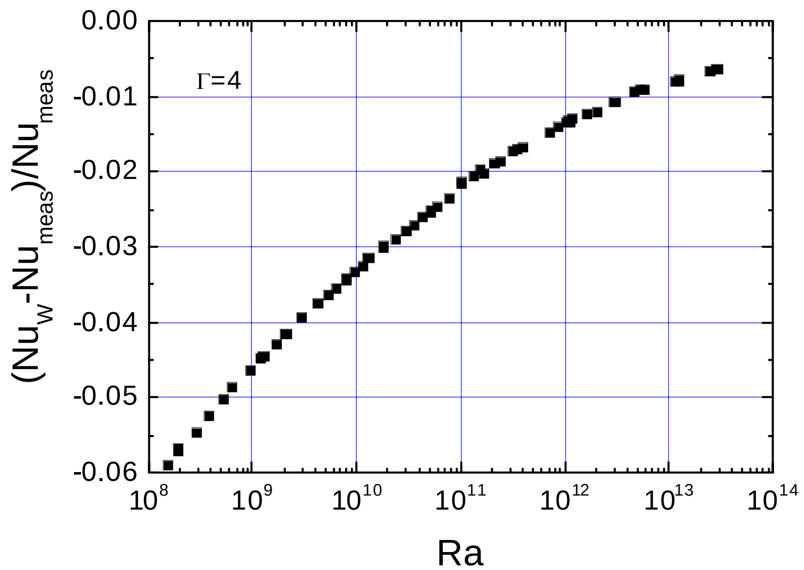
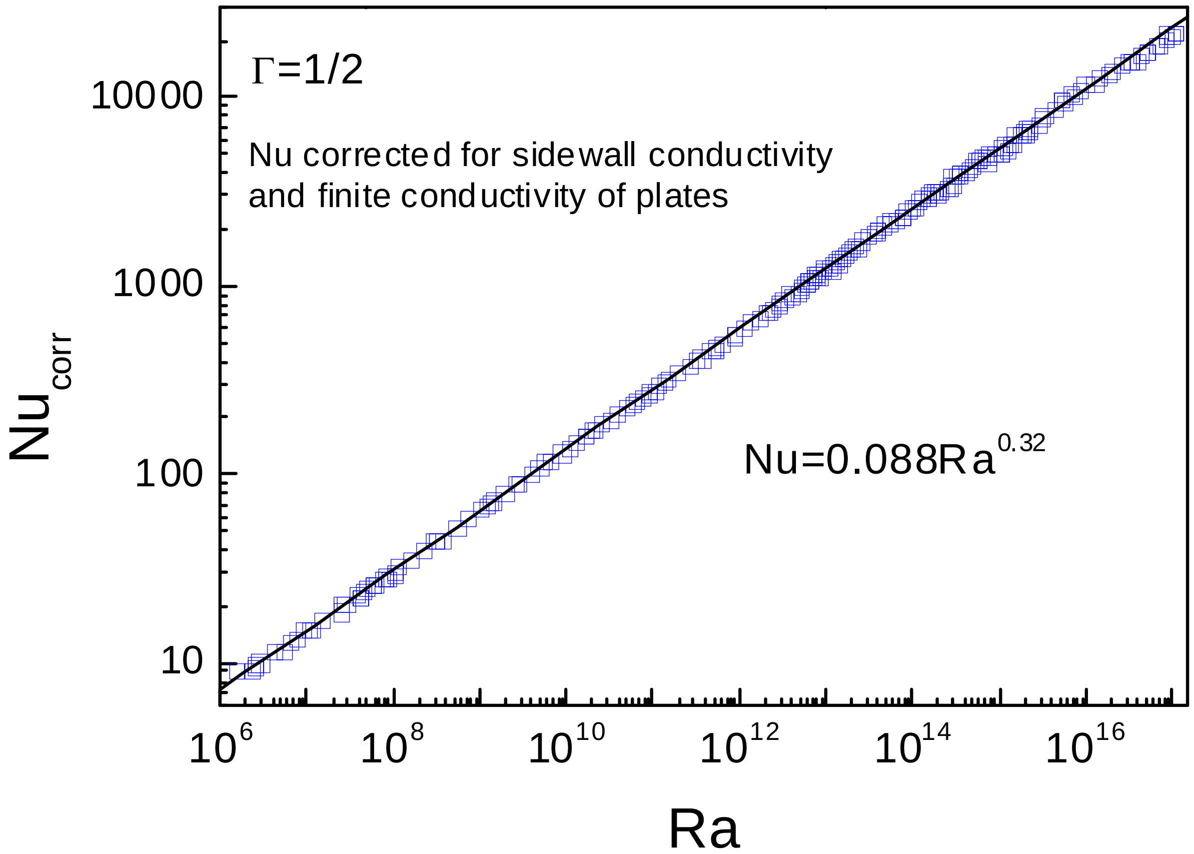


Fig. 1. Heat balance in the wall



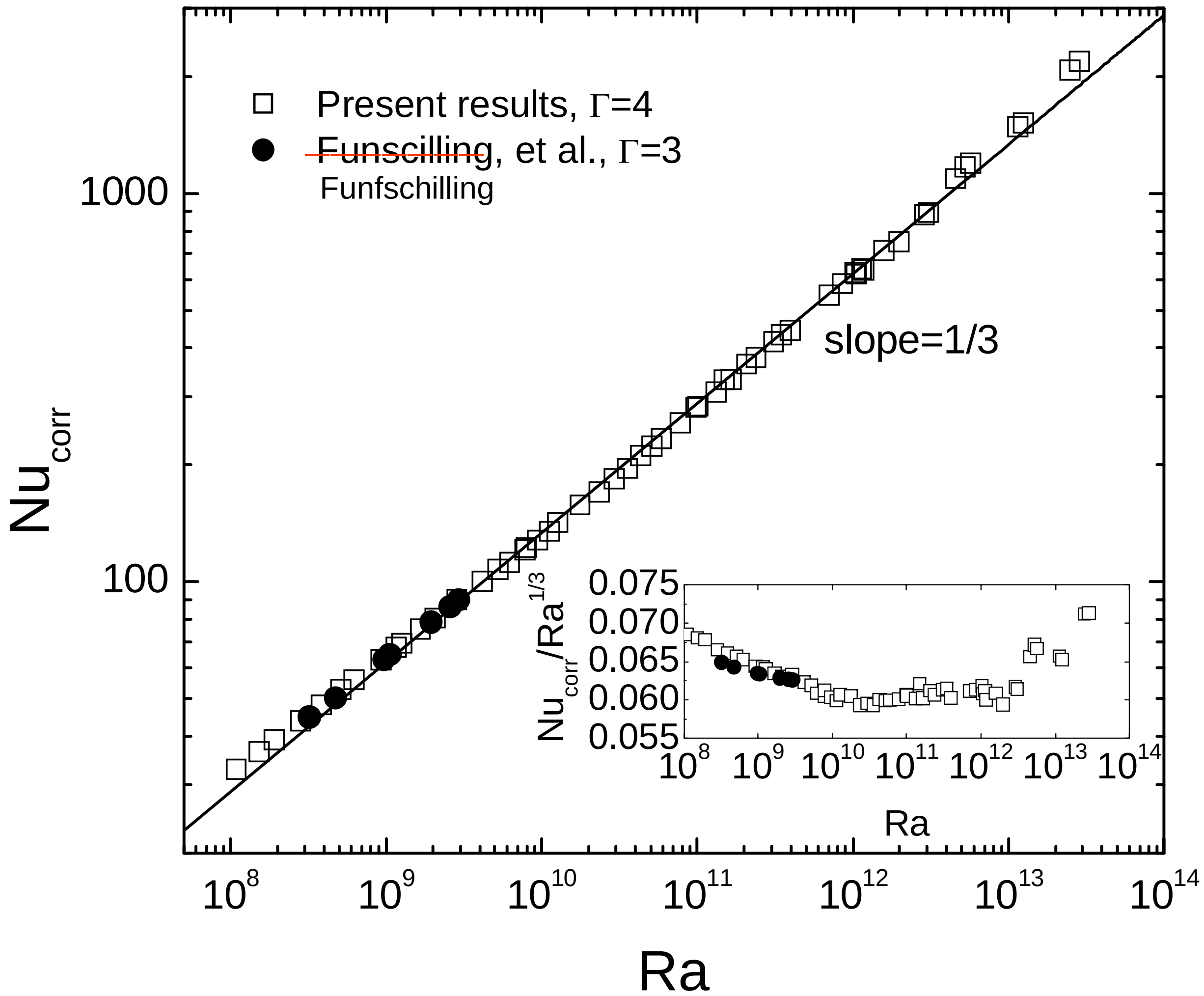
Data corrected using the correction proposed by Roche, et al.

Combining both corrections for sidewalls, and those for the plate conductivity, we revisit the original results for $\Gamma=1/2$:



Taking again a linear fit as the lowest order approximation to the trend, the log-log slope is not altered much by the application of these corrections: from 0.31 to 0.32.

Similarly corrected heat transfer results for Γ=4



J.J. Niemela & K.R. Sreenivasan, *J. Fluid Mech*. **557**, 411-422 (2006)

We shall look more closely at the issue of the heat flux rise at high Ra later

Temperature fluctuations $\Gamma=4$

The mean wind survives as a structure correlated over the entire container for moderate Ra, at least on average

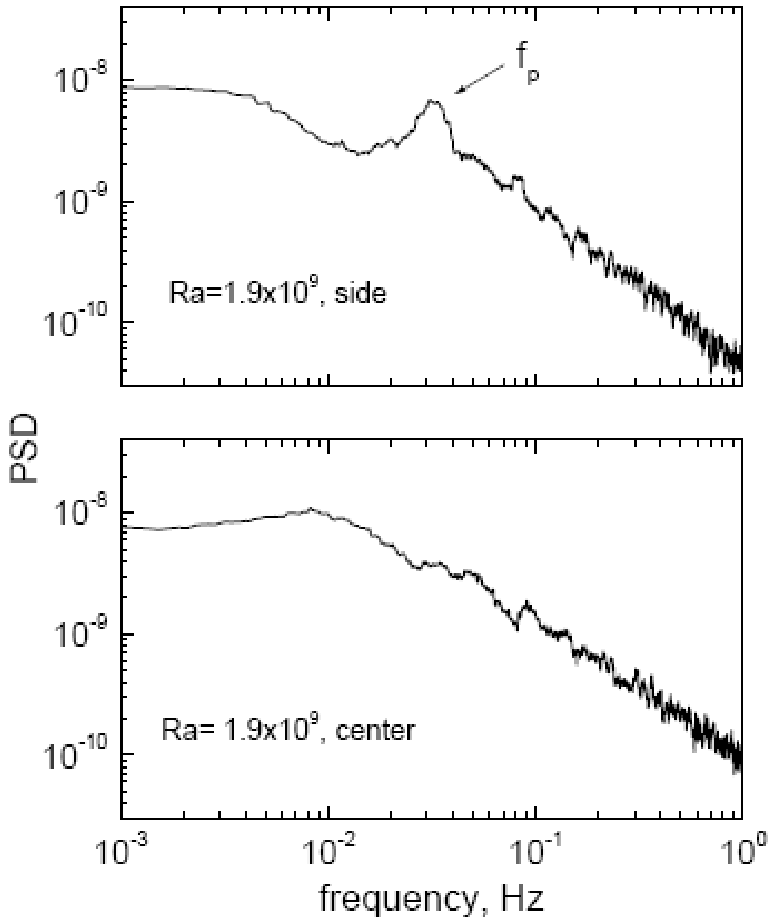


FIGURE 4. The power spectral density (PSD) for temperature fluctuations measured at the horizontal midplane of the apparatus along the sidewall (top) and in the center (bottom). The peak in the sidewall data, labelled as f_p, indicates the advection of plumes by a large-scale coherent wind. The broad and weak peak in the lower panel is roughly centered at $f_p/4$.

Aspect ratio scaling of the wind

From the peak in the PSD (or equivalently the dominant periodicity of the autocorrelation) we can extract the wind or plume advection frequency f_P.

Conventionally, we nondimensionalize this frequency using the vertical diffusion time scale:

$$\tilde{\omega} = \omega_P H^2 / \kappa \qquad \text{where} \qquad \omega_P \equiv 2\pi f_P$$

However, the peak frequencies for $\Gamma=4$ normalized in this way do not scale with similar wind frequencies obtained in aspect ratio unity experiments. In fact, in the present case there is an ambiguity that does not exist for larger aspect ratios—i.e., the vertical and horizontal measures of the container are no longer similar, and the largest dimension is along a direction in which there is essentially only inertia. Instead we consider:

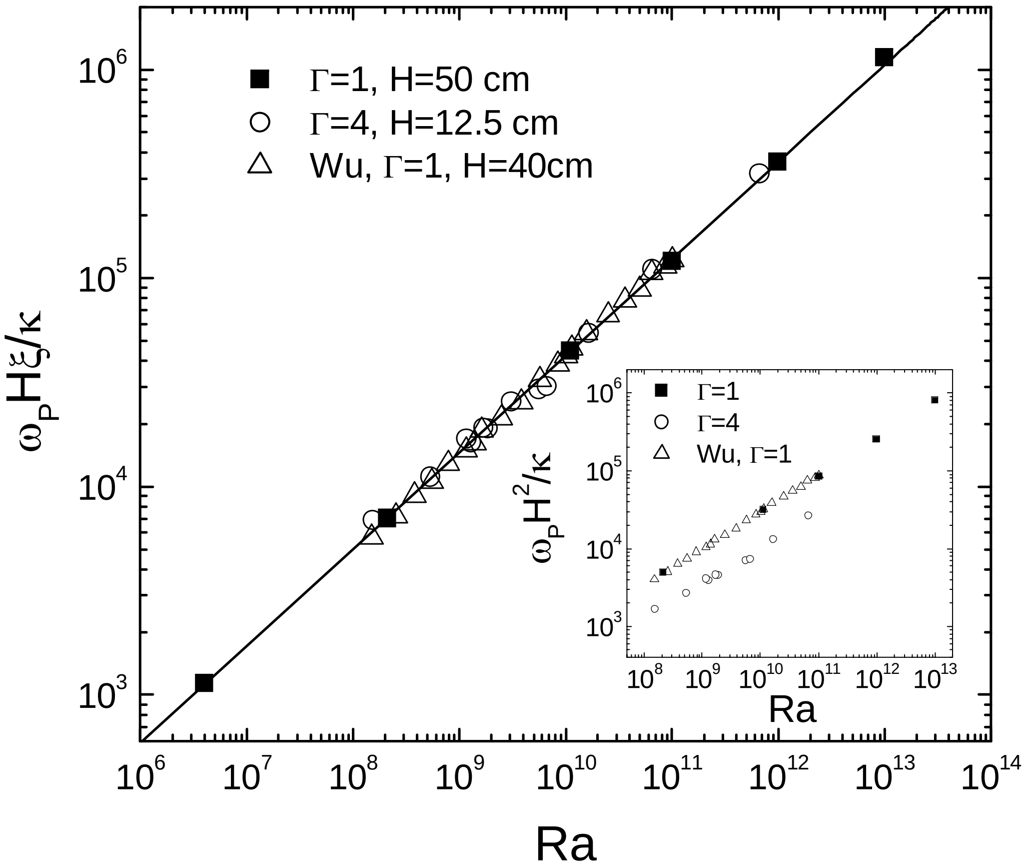
$$\tilde{\omega}^* = \tilde{\omega}\left(\frac{\xi}{H}\right) = \tilde{\omega}[1+\Gamma^2]^{1/2}$$

where $\xi = (H^2 + D^2)^{1/2}$ is a "diagonal" measure and is suggested by the tilted nature of the wind (we could equivalently use a perimeter measure (H+D)).

A modifed Reynolds number*, derived from the measured oscillation frequency is then defined as

$$Re^*{}_f \equiv (1/\pi)\tilde{\omega}^* Pr^{-1}$$

*see, e.g., Lam, et al 2002; Qui & Tong, 2001; Niemela et al. 2001…



From the least squares fit (solid line) we find:

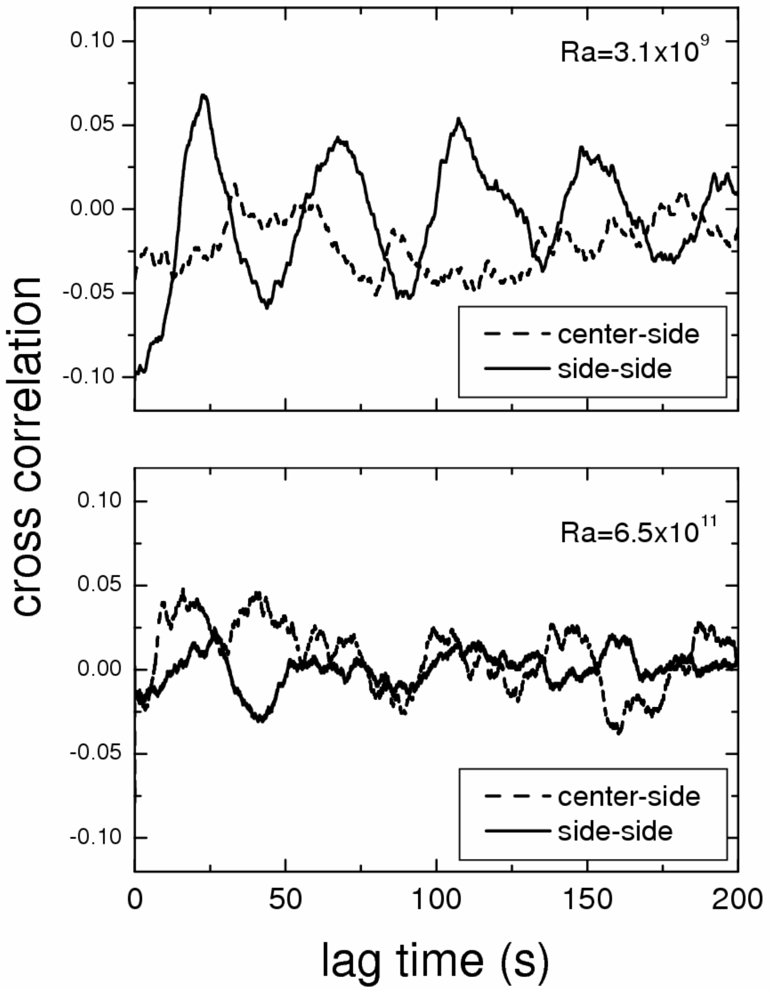
$$Re^*{}_f Pr^{2/3} = 0.44 Ra^{0.453}$$

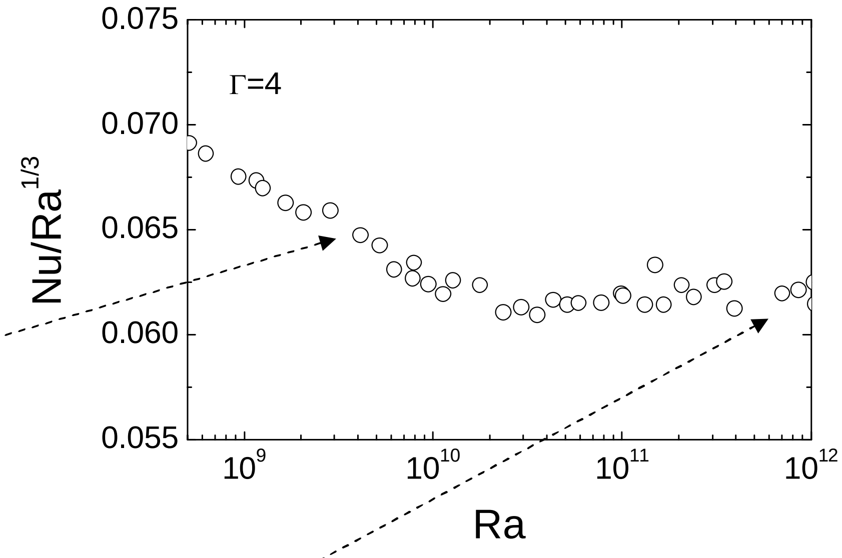
which is very similar to the theoretical result of Grossmann & Lohse for the conventionally defined Reynolds number:

$$Re Pr^{2/3} \sim Ra^{4/9}$$

This scaling is also similar to the empirical results of Qui & Tong (2001) and Lam, et al. (2002). Using the $Pr^{-0.76}$ dependence of the latter authors does not affect the results much (the prefactor becomes 0.39 and the Ra exponent 0.456). [There is no resolvable Ra-dependence of the Pr exponent, and essentially none is predicted from Grossman & Lohse (2002) for Pr of order unity.]

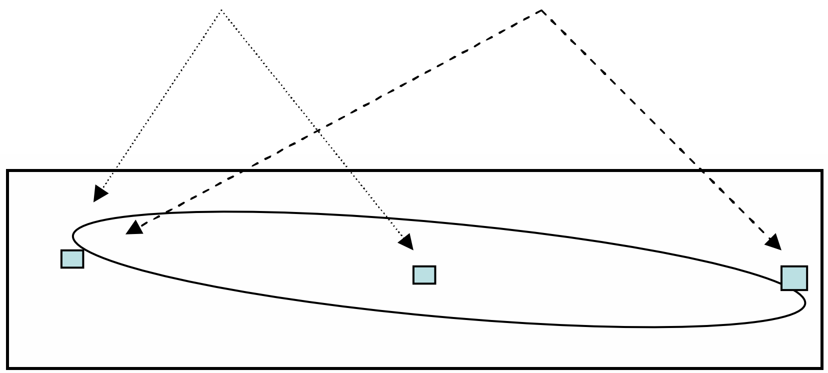
Loss of coherence of the mean wind





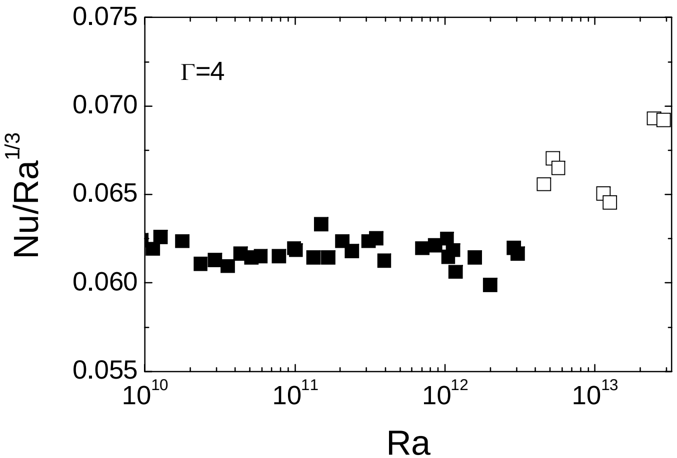
Cross-correlations between:

side and center/ opposite side

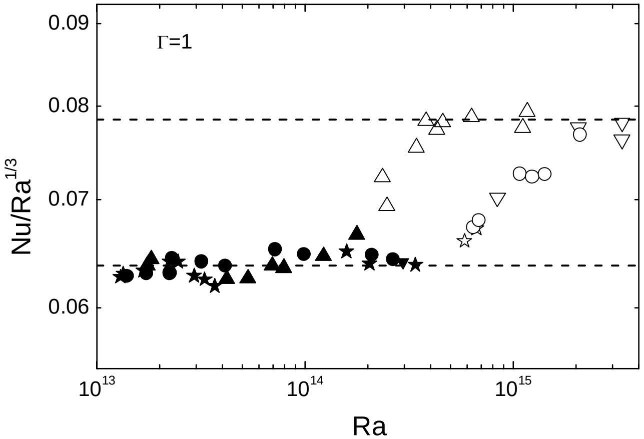


At higher Ra, in the region of 1/3 power law scaling of the heat transfer, there is a “randomization” of the mean wind

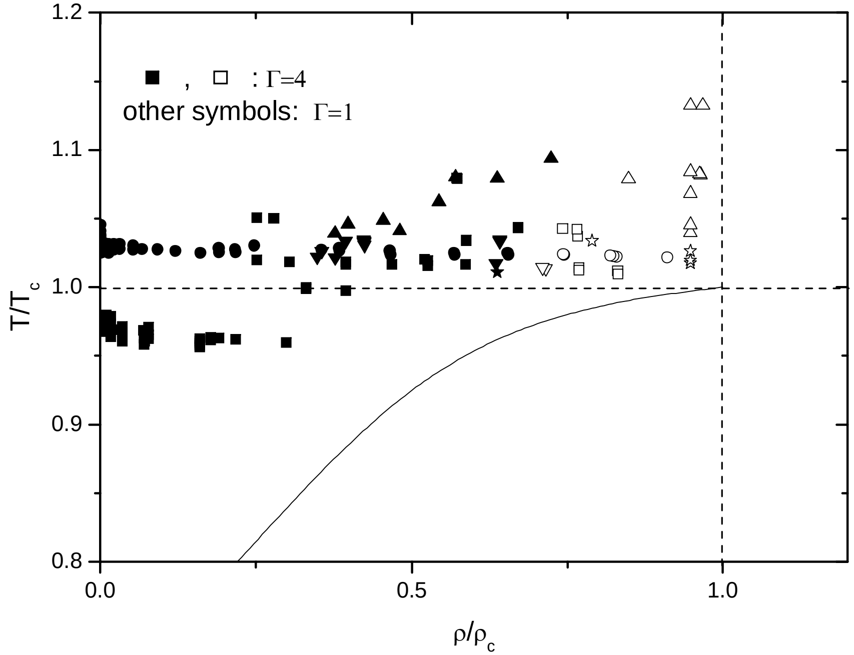
Returning to the observed, anomalous rise in heat transfer: comparing several independent experiments this transition **does not scale with Ra**, **but rather appears to depend on distance from critical point.**



From additional Γ=1 experiments



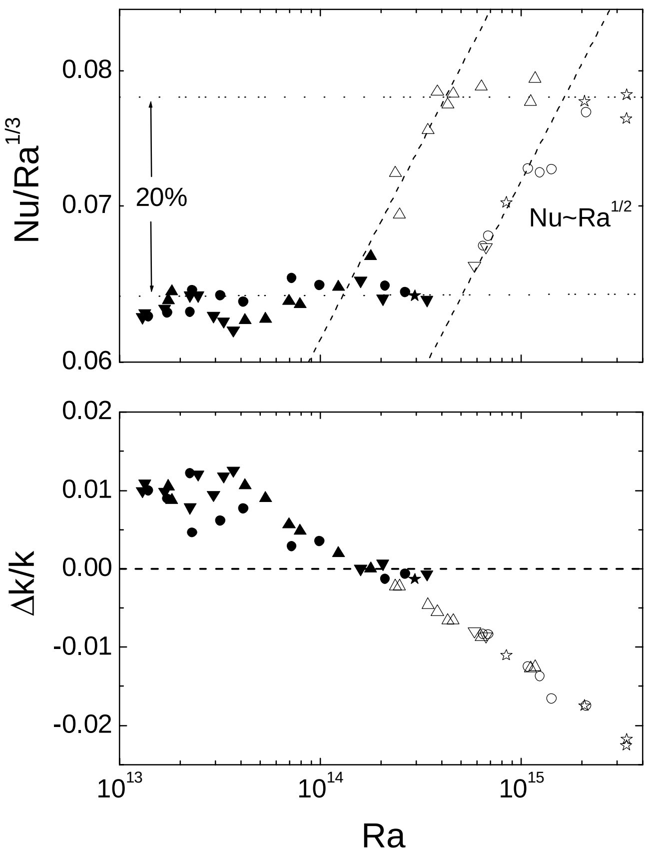
Circles represent published data (JJN, KRS, JFM, 2003). Other data either reproduce the original path through the P-T diagram or not.



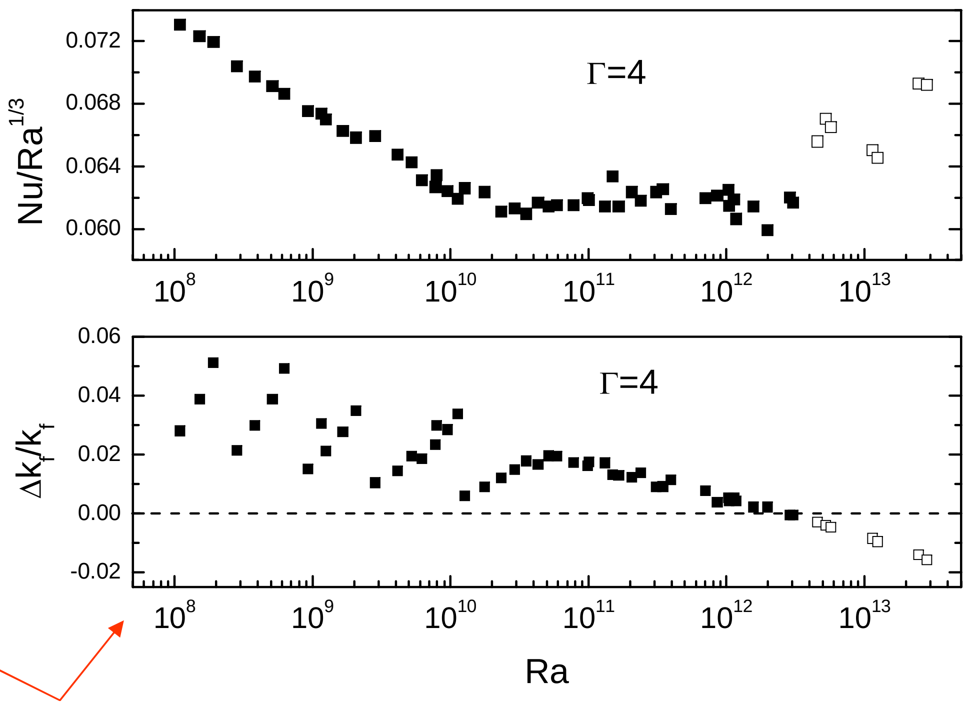
Open and solid symbols correspond to those in the Nu-Ra plots at left, associated with the anomalous increase in heat transfer .

Taking one Boussinesq parameter, Δk/k,* we observe a strong correlation. However, the magnitude of this quantity is very low. Note that there are various pairs of data points at the same high Ra for which the heat transfer can differ by 20%, and these points appear to correlate significantly to different signs of the thermal conductivity gradient in the boundary layers.This is true in three independent experiments (two for $\Gamma=1$ in which different paths through the phase space were taken and one experiment for $\Gamma=4$.

$\Gamma=1$



$\Gamma=4$



Here, open symbols refer to the data in which Nu is anomalously enhanced in the top plots

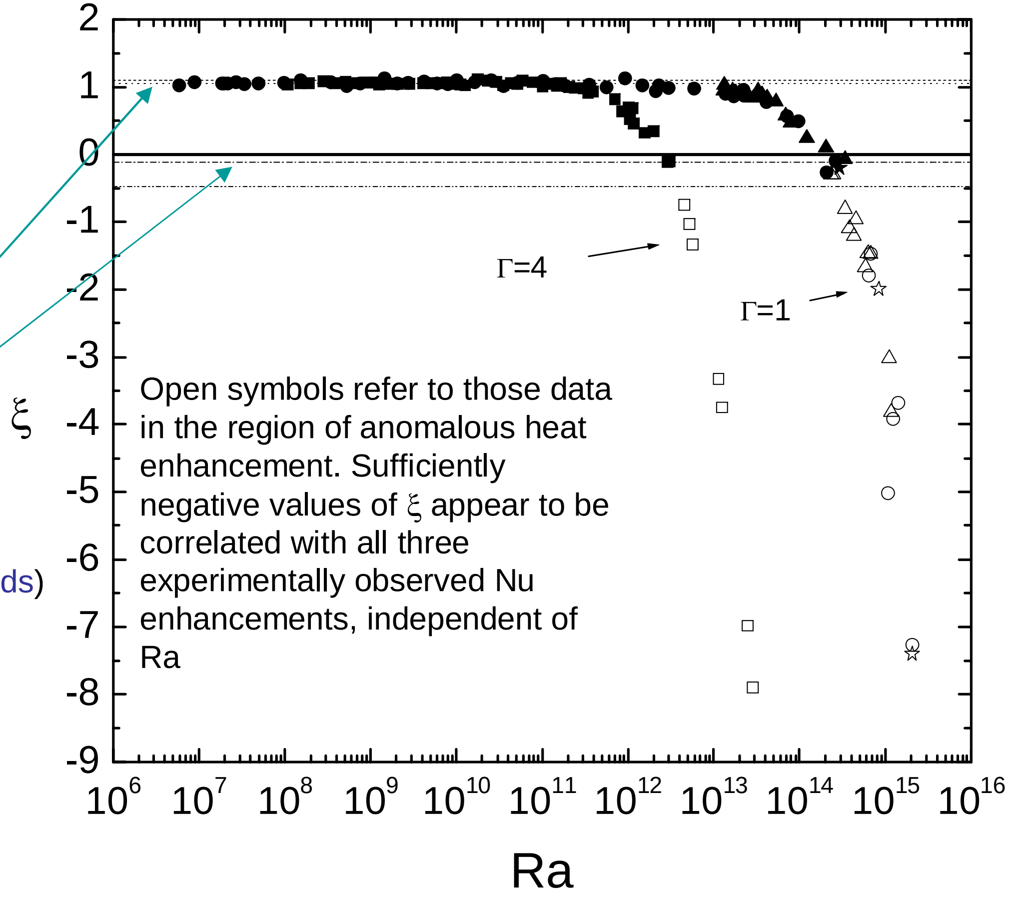
*Here and elsewhere, "ΔX/X" means $(X_{hotter}-X_{colder})/X_{mean}$

The viscosity is also observed to have some positive correlation on the heat transfer enhancement. We consider the following:

$$\xi = \frac{\mu}{k}\frac{\Delta k}{\Delta \mu}$$

k: thermal conductivity
μ: shear viscosity

$$\xi \longrightarrow \frac{Pr \cdot \left(\frac{\Delta k}{\Delta \nu}\right)}{\rho C_P}$$



ξ values in conventional fluids (both gases and liquids)

Ethanol (40 deg C): $\xi = +1.1$ (but Δk small, <0)
Air (40 deg C): $\xi = +1.05$
Water (40 deg C): $\xi = -0.11$
Hg (40 deg C): $\xi = -0.5$

We note that the mercury experiments of Cioni, et al, who did report an anomalous heat transfer rise, also have a value of ξ negative at high Rayleigh number, although there may be more obvious reasons for this, connected to boundary conditions.

Summary

•In these helium experiments, ranges of high Ra can be investigated where it is difficult to apply numerical techniques. This is especially true of large aspect ratio experiments, also for which little data exists of any kind at high Ra.

•We have presented new results from an $\Gamma=4$ cylindrical cell, up to Ra~10^{13}.

•log Nu- log Ra slope atttains a limiting value of nearly 1/3 over a wide range of Ra.

•A transition to enhanced heat transport occurs beyond a few times 10^{12}. **This enhancement is also seen in two other independent experiments and does NOT scale with Ra, but DOES correlate significantly with the distance from the critical point.**

•In aspect ratio 4, the wind survives throughout the entire container, at least on average, although it loses its coherence at moderately high Ra as evidenced by a loss of correlation between opposite wall sensors. This loss of coherence is consistent with the relatively large observed 1/3 power law scaling region, indicative of the randomization generally prescribed for its attainment. In general the wind is significantly less organized than in $\Gamma=1$ and it is difficult to describe its behavior precisely.

•The scaling of the predominant oscillation peak in the PSD of the sidewall sensors scales with $\Gamma=1$ results only if we adopt a different normalization that includes a dependence on aspect ratio.

•A high Ra experiment under constant Pr and Boussinesq conditions is possible: with a 5 meter tall sample, Ra > 10^{15} could be achieved in the laboratory under these ideal conditions. Having small aspect ratio may be less complicating at high enough Ra, given that the coherent circulation becomes sufficiently weak (no effects due to sidewall conductivity). It was shown that known effects due to plate conduction will only affect those Ra for which the conditions of constant Pr and Boussinesq behavior are no longer satisified.