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SCHOOL ON PHYSICS AT LHC: "EXPECTING LHC"
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QCD at the LHC - Part I

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These are preliminary lecture notes, intended only for distribution to participants.

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QCD at the LHC

Expecting the LHC

ICTP, Trieste, Sept 12-13 2006

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Preface

- **The reason we build the LHC is NOT to study QCD!**
  - We already know that QCD IS the theory of strong interactions
- **QCD is however a crucial tool to understand what the LHC data are telling us:**
  - QCD drives all production processes for known and new particles
  - QCD is required to extract valuable information about the properties of new physics to be detected at the LHC
- Anyone who wants to interpret the findings of the LHC experiments must have some basic understanding of how QCD works and of how it determines the properties of the LHC final states

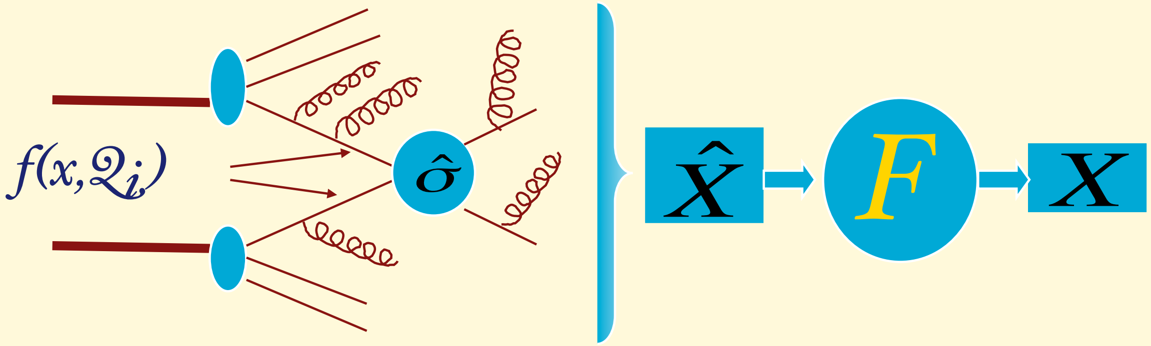
Contents

- **Lecture I:** Define the framework and basic rules
  - Factorization theorem
  - Parton densities
  - Evolution of final states
  - Hard processes
- **Lecture II:** Benchmark applications and examples, relevant to the search of BSM phenomena at the LHC:
  - jets
  - leptons
  - b-quark jets
  - W+multijets
  - top quark

LECTURE I

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1,Q_i)\, f_k(x_2,Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i,Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i,Q_f)$$



$f_j(x,Q)$ Parton distribution functions (PDF)

- sum over all initial state histories leading, at the scale Q, to:

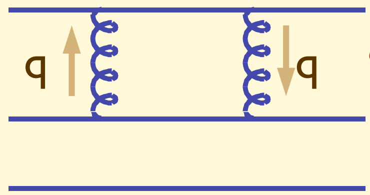
$$\vec{p}_j = x\,\vec{P}_{proton}$$

$F(\hat{X} \to X; Q_i, Q_f)$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

Universality of parton densities and factorization, an intuitive view

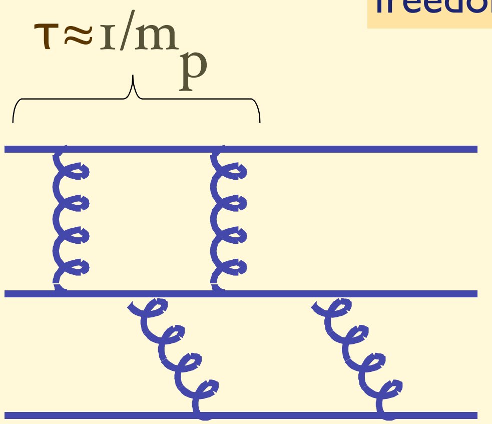
1) Exchange of **hard gluons** among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\stackrel{q>Q}{\sim} \int_Q^\infty \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

Assuming asymptotic freedom!

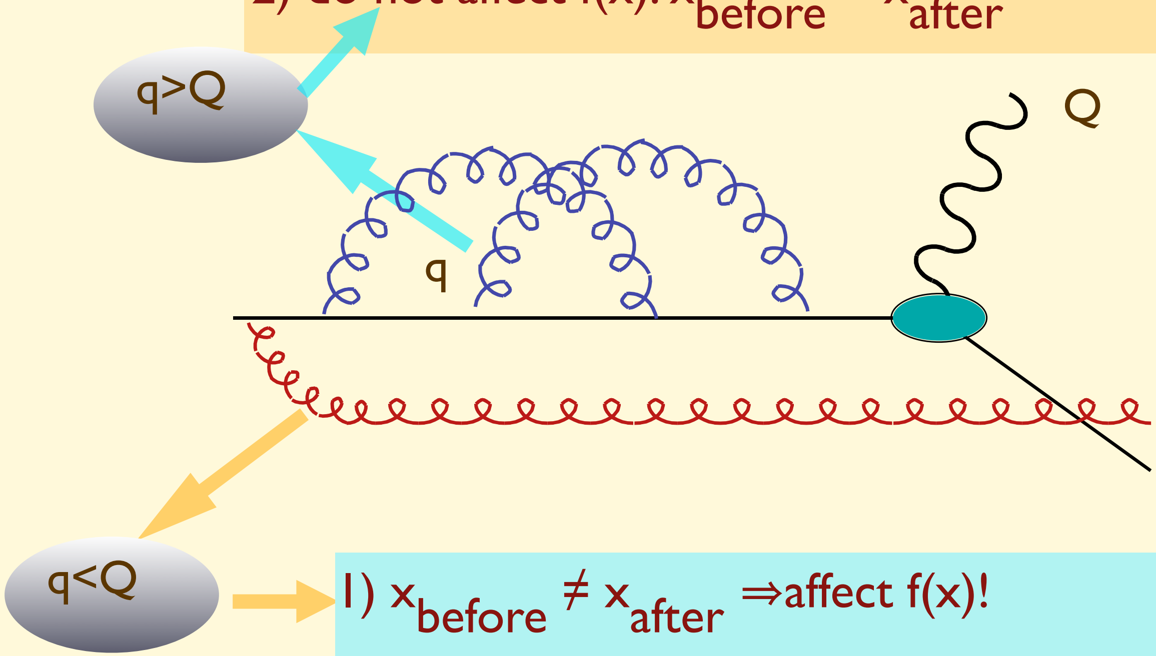
2) Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau = \gamma/m_p = E/m_p^2$)



3) If a hard probe ($Q >> m_p$) hits the proton, on a time scale $= 1/Q$, there is no time for quarks to negotiate a coherent response. The struck quark receives no feedback from its pals, and acts as a free particle

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{before} = x_{after}$



This gluon cannot be reabsorbed because the quark is gone

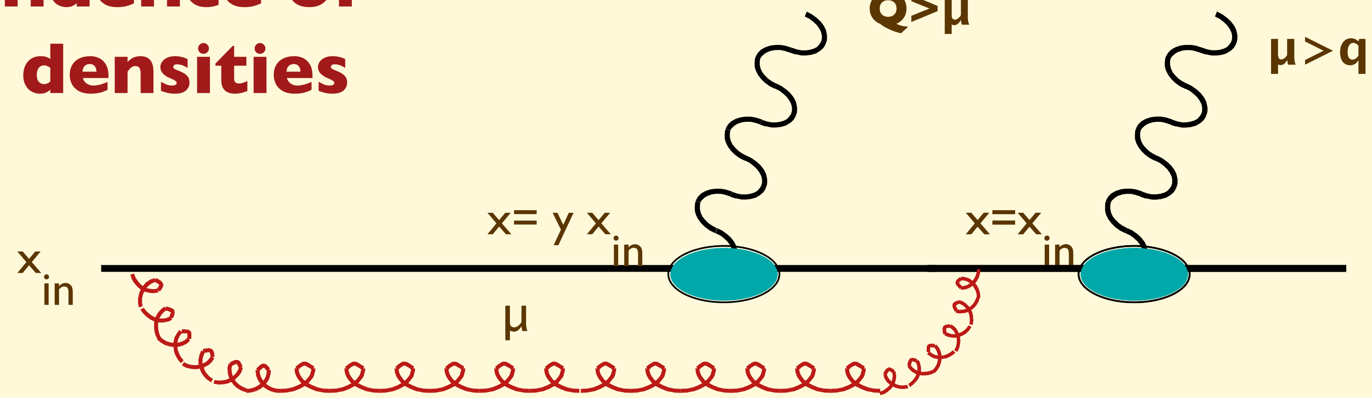
1) $x_{before} \neq x_{after}$ ⇒affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere

➡ **Universality of f(x)**

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - yx_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - yx_{in})$$

f(x,Q) should be independent of the intermediate scale μ considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y,\mu^2)$$

One can prove that:

$$P(x,Q^2) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} P(x)$$

calculable in pQCD

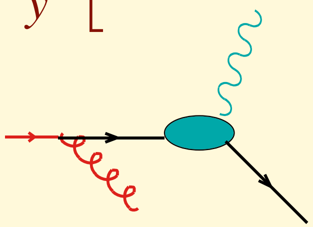
and therefore (Altarelli-Parisi equation):

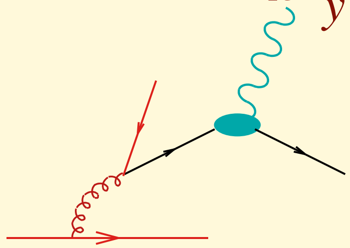
$$\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y)$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q ($t = \log Q^2$):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}(\frac{x}{y}) + g(y,Q)\, P_{qg}(\frac{x}{y}) \right]$$

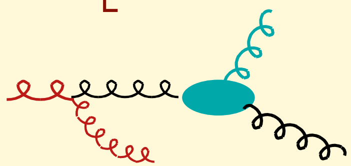


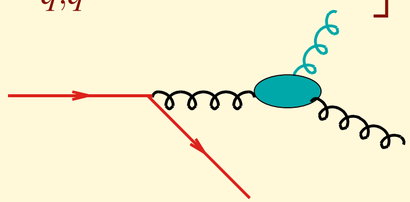


$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}(\frac{x}{y}) + \sum_{q,\bar{q}} q(y,Q)\, P_{gq}(\frac{x}{y}) \right]$$

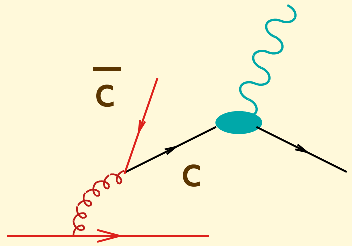




$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Example: charm in the proton



$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density: $g(x,Q) \sim A/x$

and using $P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$ we get:

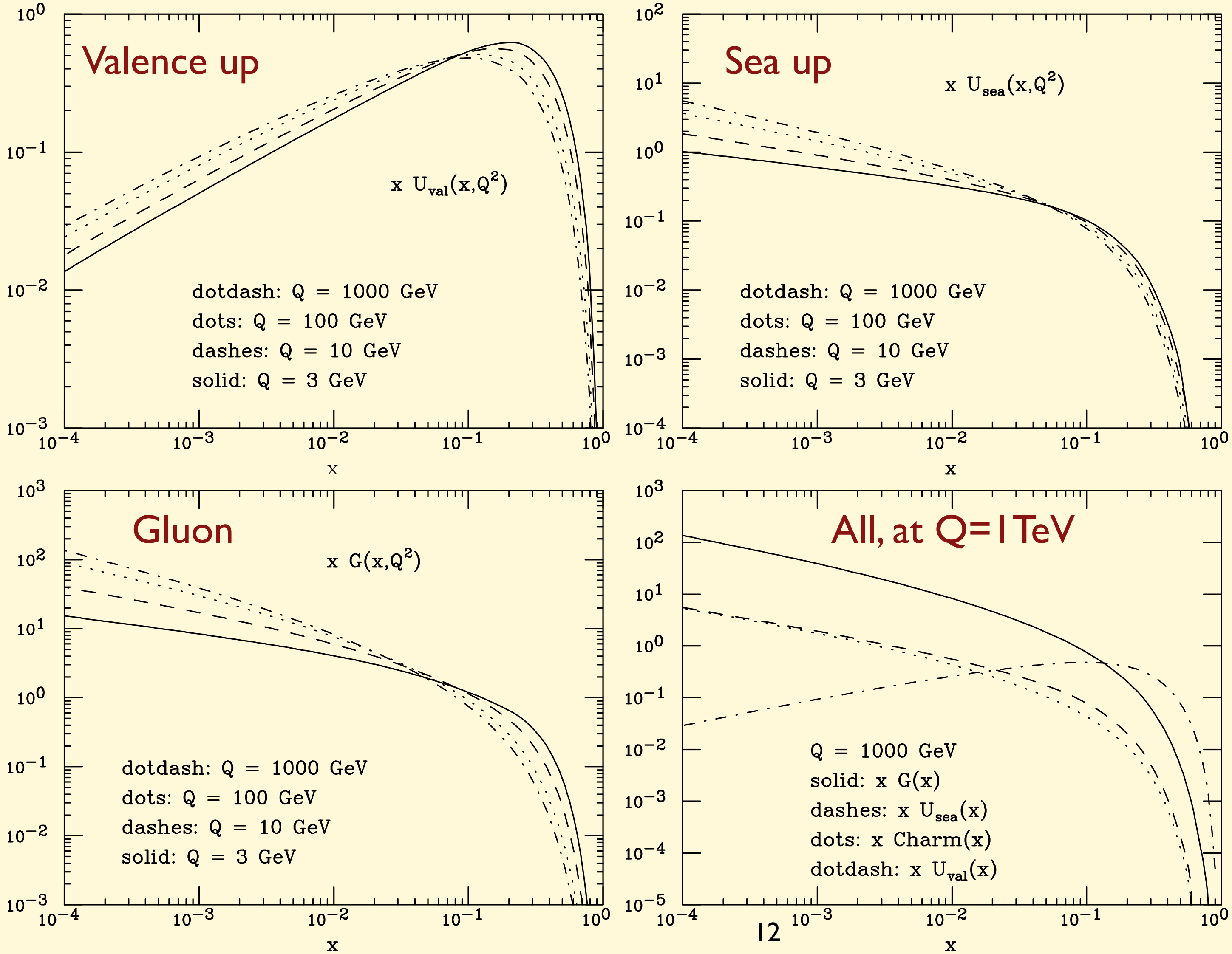
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}[y^2 + (1-y)^2] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

and therefore:

$$c(x,Q) \sim \frac{\alpha_s}{6\pi} \log(\frac{Q^2}{m_c^2})\, g(x,Q)$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of αs

Examples of PDFs and their evolution



Note:
sea ≈10% glue

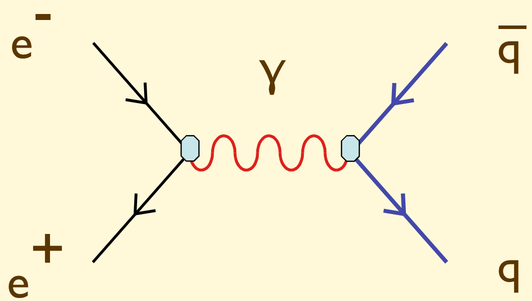
Note:
charm≈up at high Q

Evolution of hadronic final states

Asymptotic freedom implies that at E_{CM} >> 1 GeV

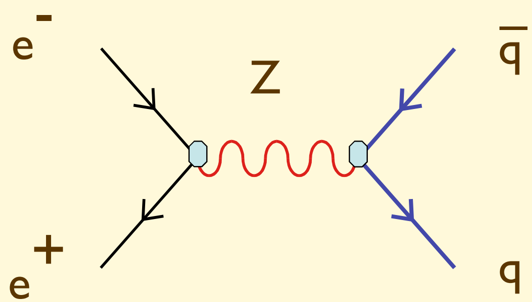
$\sigma(e^+ e^- \rightarrow \text{hadrons})$ ⟷ $\sigma(e^+ e^- \rightarrow \text{quarks/gluons})$

At the Leading Order (LO) in PT:



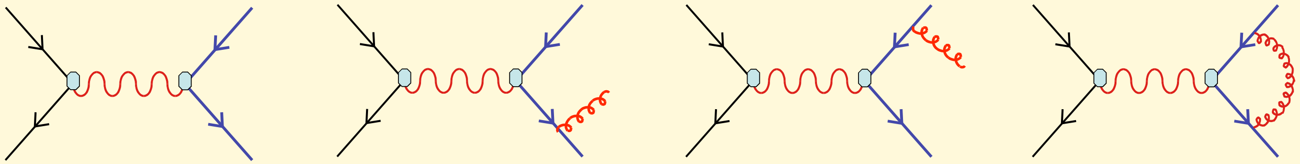
$$\sigma_0(e^+e^- \to q\bar{q}) = \frac{4\pi\alpha^2}{9s} N_c \sum_{f=u,d,\ldots} e^2_{q_f}$$

$$\frac{\sigma_0(e^+e^- \to q\bar{q})}{\sigma_0(e^+e^- \to \mu^+\mu^-)} = N_c \sum_{f=u,d,\ldots} e^2_{q_f}$$



$$\frac{\sigma_0(e^+e^- \to Z \to q\bar{q})}{\sigma_0(e^+e^- \to Z \to \mu^+\mu^-)} = N_c \frac{\sum_{f=u,d,\ldots}\left(v^2_{q_f} + a^2_{q_f}\right)}{\left(v^2_{\mu} + a^2_{\mu}\right)}$$

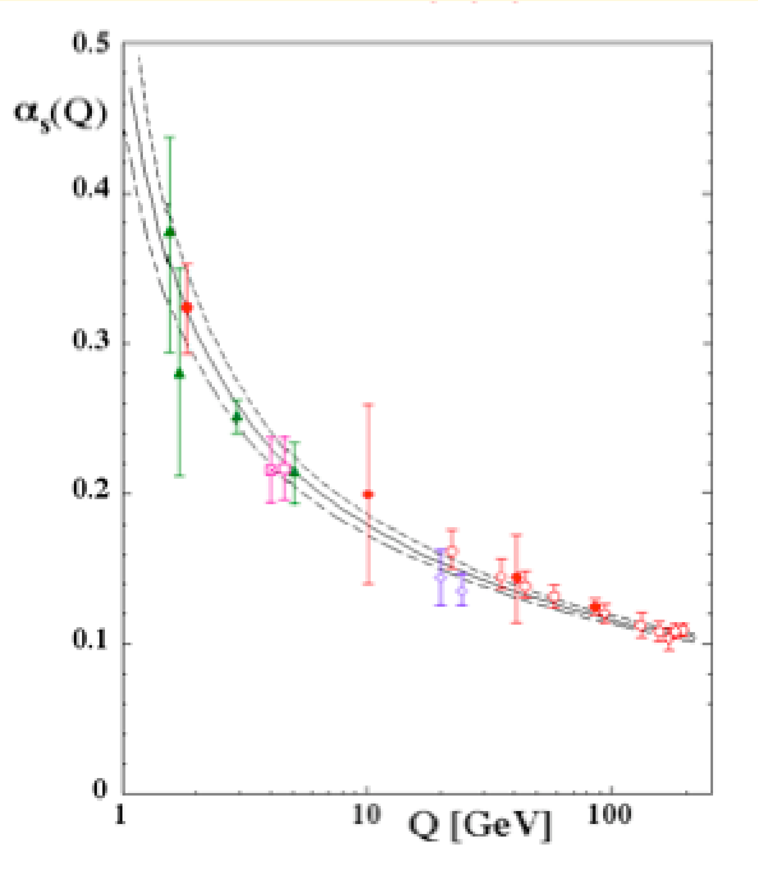
Adding higher-order perturbative terms:



+ ≥2-gluon emissions

$$\sigma_1(e^+e^- \to q\bar{q}(g)) = \sigma_0(e^+e^- \to q\bar{q})\left(1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2)\right)$$

O(3%) at M_Z

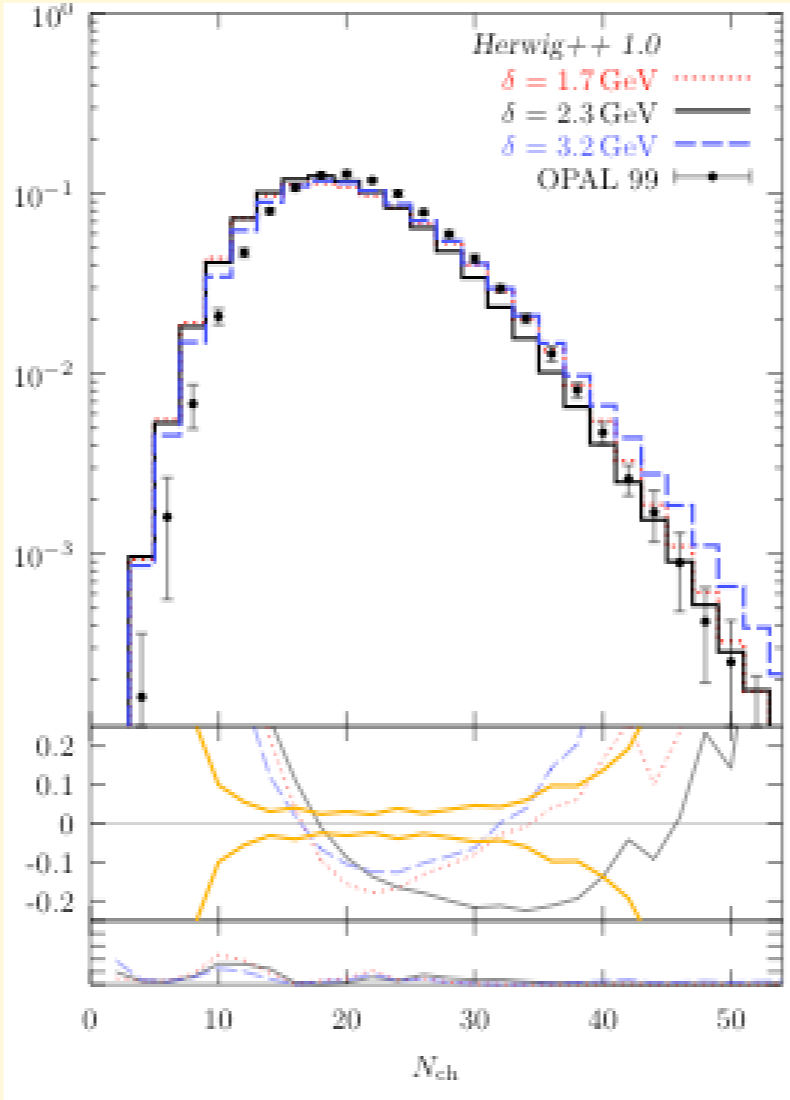


Excellent agreement with data,
provided N_c=3
Extraction of α_s consistent with the Q evolution predicted by QCD

However, the final states contain a large number of particles, not 2 or 3 as apparently predicted by the perturbative calculation.

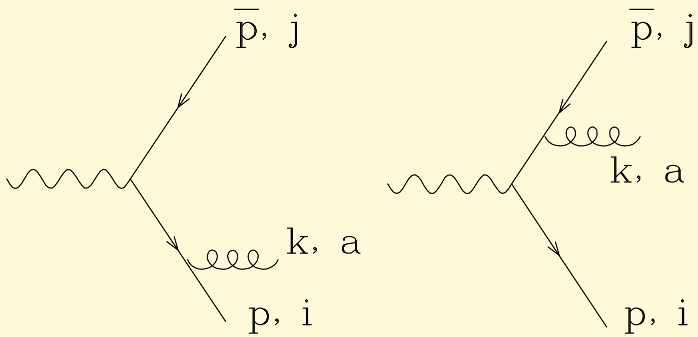
Experimental multiplicity distribution

$\langle n_{charged} \rangle = 20.9$



What's going on??

Soft gluon emission



$$A = \bar{u}(p)\epsilon(k)(ig)\,\frac{-i}{\not p + \not k}\,\Gamma^\mu\, v(\bar p)\ \lambda^a_{ij} + \bar u(p)\,\Gamma^\mu\,\frac{i}{\not{\bar p} + \not k}\,(ig)\epsilon(k)\,v(\bar p)\ \lambda^a_{ij}$$

$$= \left[\frac{g}{2p\cdot k}\bar u(p)\epsilon(k)\,(\not p + \not k)\Gamma^\mu\, v(\bar p) - \frac{g}{2\bar p\cdot k}\bar u(p)\,\Gamma^\mu\,(\not{\bar p} + \not k)\epsilon(k)\,v(\bar p)\right]\lambda^a_{ij}$$

$p\cdot k = p_0\, k_0\, (1-\cos\theta) \Rightarrow$ singularities for collinear ($\cos\theta \to 1$) or soft ($k_0 \to 0$) emission

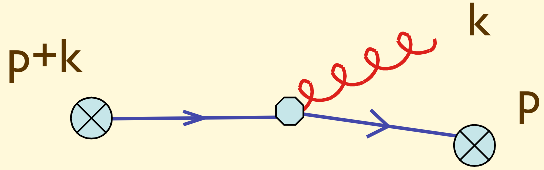
Collinear emission does not alter the global structure of the final state, since its preserves its "pencil-like-ness". **Soft emission** at large angle, however, could spoil the structure, and leads to strong interferences between emissions from different legs. So soft emission needs to be studied in more detail.

In the soft ($k_0 \to 0$) limit the amplitude simplifies and factorizes as follows:

$$A_{soft} = g\lambda^a_{ij}\left(\frac{p\cdot\varepsilon}{p\cdot k} - \frac{\bar p\cdot\varepsilon}{\bar p\cdot k}\right) A_{Born}$$

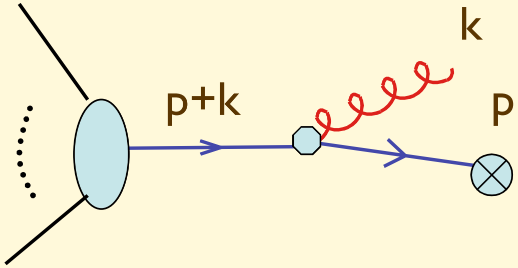
Factorization: it is the expression of the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

Another simple derivation of soft-gluon emission rules



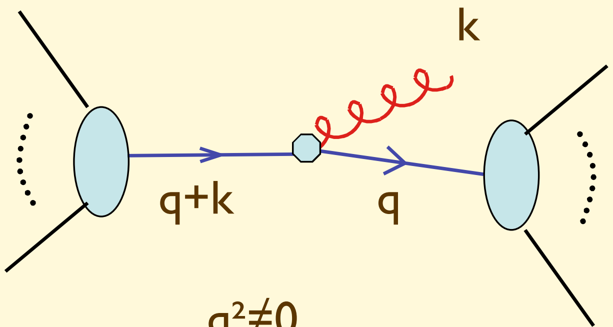
charge current of a free fermion

$$\bar{\psi}(p)\gamma_\mu\psi(p+k)\,\varepsilon^\mu(k) \;\xrightarrow{k\to 0}\; \bar{\psi}(p)\gamma_\mu\psi(p)\varepsilon^\mu(k) \;=\; 2p\cdot\varepsilon$$



$$\frac{1}{\not{p}+\not{k}}\gamma_\mu\psi(p+k)\,\varepsilon^\mu(k) \;\xrightarrow{k\to 0}$$

$$\frac{1}{2p\cdot k}\not{p}\gamma_\mu\psi(p)\varepsilon^\mu(k) \;=\; \frac{p\cdot\varepsilon}{p\cdot k}$$

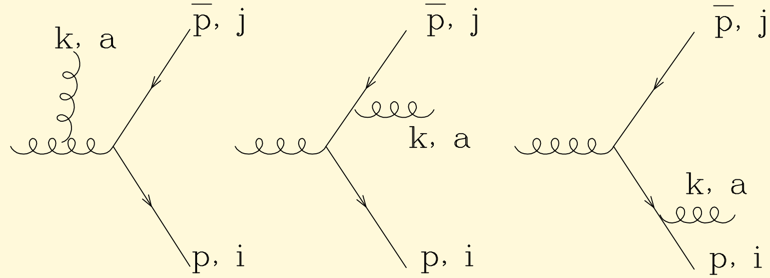


$q^2 \neq 0$

$$\frac{1}{\not{q}+\not{k}}\gamma_\mu\frac{1}{\not{q}}\,\varepsilon^\mu(k) \;\xrightarrow{q^2\neq 0,\; k\to 0}\; \frac{1}{q^2}\not{q}\gamma_\mu\not{q}\frac{1}{q^2}\varepsilon^\mu(k)$$

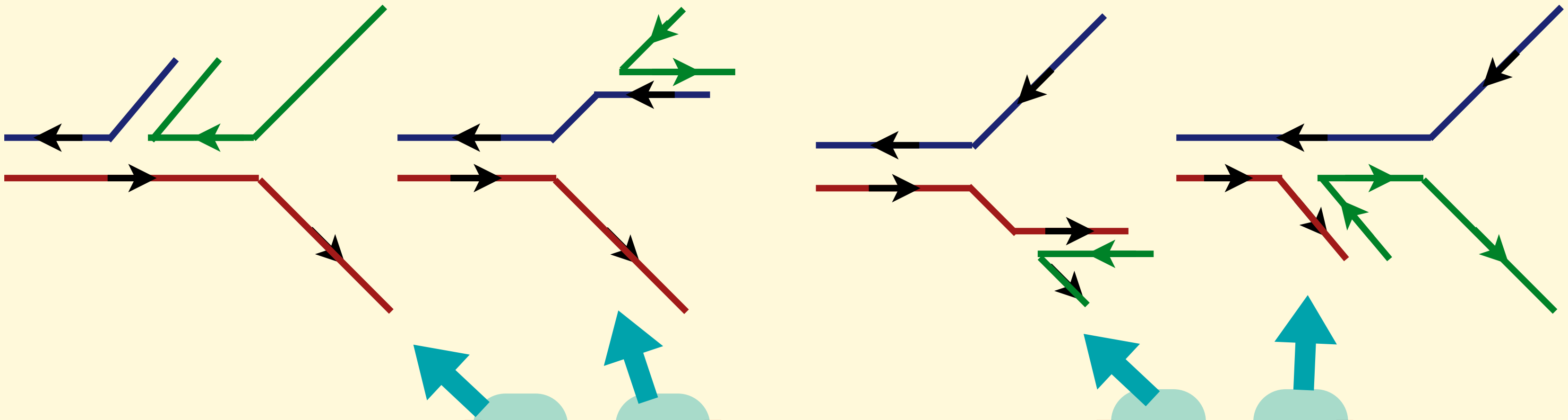
=> finite

Similar, but more structured, result in the case of a fully coloured process:



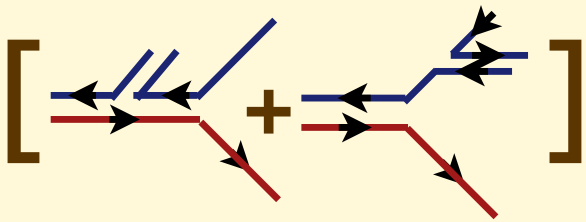
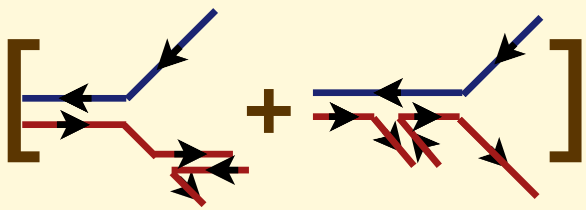
$$A_{soft} = g\,(\lambda^a\lambda^b)_{ij}\left[\frac{Q\varepsilon}{Qk} - \frac{\bar{p}\varepsilon}{\bar{p}k}\right] + g\,(\lambda^b\lambda^a)_{ij}\left[\frac{p\varepsilon}{pk} - \frac{Q\varepsilon}{Qk}\right]$$

The four terms correspond to the two possible ways colour can flow, and to the two possible emissions for each colour flow:



$$A_{soft} = g\,(\lambda^a\lambda^b)_{ij}\left[\frac{Q\varepsilon}{Qk} - \frac{\bar{p}\varepsilon}{\bar{p}k}\right] + g\,(\lambda^b\lambda^a)_{ij}\left[\frac{p\varepsilon}{pk} - \frac{Q\varepsilon}{Qk}\right]$$

The interference between the two colour structures

 $\propto (\lambda^a \lambda^b)_{ij}$  $\propto (\lambda^b \lambda^a)_{ij}$

is suppressed by $1/N_c^2$:

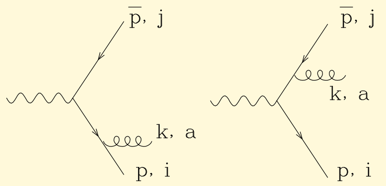
$$\sum_{a,b,i,j} |(\lambda^a \lambda^b)_{ij}|^2 = \sum_{a,b} tr\left(\lambda^a \lambda^b \lambda^b \lambda^a\right) = \frac{N^2-1}{2} C_F = O(N^3)$$

$$\sum_{a,b,i,j} (\lambda^a \lambda^b)_{ij} [(\lambda^b \lambda^a)_{ij}]^* = \sum_{a,b} tr(\lambda^a \lambda^b \lambda^a \lambda^b) = \frac{N^2-1}{2} \underbrace{(C_F - \frac{C_A}{2})}_{-\frac{1}{2N}} = O(N)$$

As a result, the emission of a soft gluon can be described, to the leading order in $1/N_c^2$, as the incoherent sum of the emission from the two colour currents

What about the interference between the two diagrams corresponding to the same colour flow? ➥

Angular ordering in soft-gluon emission



$$d\sigma_g = \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)}\, g^2 \sum \varepsilon_\mu \varepsilon^*_\nu \frac{d^3k}{(2\pi)^3 2k^0}$$

$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} d\cos\theta$$

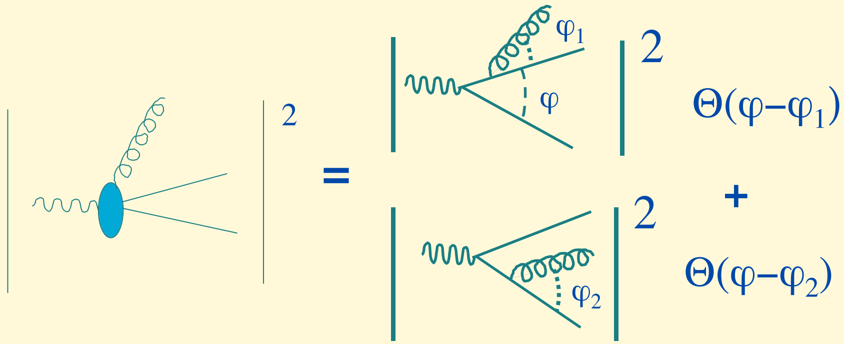
You can easily prove that:

$$\frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} = \frac{1}{2}\left[\frac{\cos\theta_{jk}-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ik}}\right] + \frac{1}{2}[i \leftrightarrow j] \equiv W_{(i)} + W_{(j)}$$

where:

$$W_{(i)} \rightarrow finite\ if\ k \parallel j\ (\cos\theta_{jk} \rightarrow 1)$$
$$W_{(j)} \rightarrow finite\ if\ k \parallel i\ (\cos\theta_{ik} \rightarrow 1)$$

The probabilistic interpretation of W(i) and W(j) is a priori spoiled by their non-positivity. However, you can prove that after azimuthal averaging:

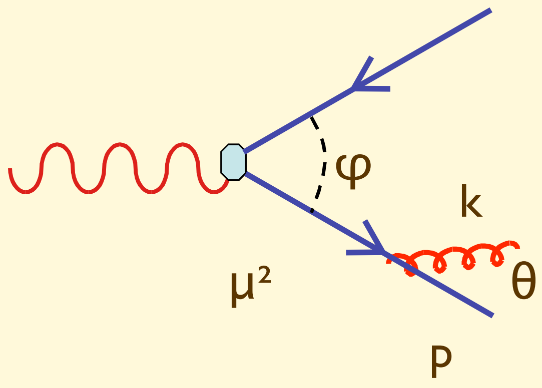


$$\int \frac{d\phi}{2\pi} W_{(i)} = \frac{1}{1-\cos\theta_{ik}} \text{ if } \theta_{ik} < \theta_{ij}\,, \quad 0 \text{ otherwise}$$

$$\int \frac{d\phi}{2\pi} W_{(j)} = \frac{1}{1-\cos\theta_{jk}} \text{ if } \theta_{jk} < \theta_{ij}\,, \quad 0 \text{ otherwise}$$

Further branchings will obey angular ordering relative to the new angles. As a result emission angles get smaller and smaller, squeezing the jet

An intuitive explanation of angular ordering



Lifetime of the virtual intermediate state:

$$\tau < \gamma/\mu = E/\mu^2 = 1 / k_0\, \theta^2 = 1/k_\perp\theta$$

$\mu^2 = (p+k)^2 = 2E\, k_0\, (1-\cos\theta)$
$\sim E\, k_0\, \theta^2 \sim E\, k_\perp\, \theta$

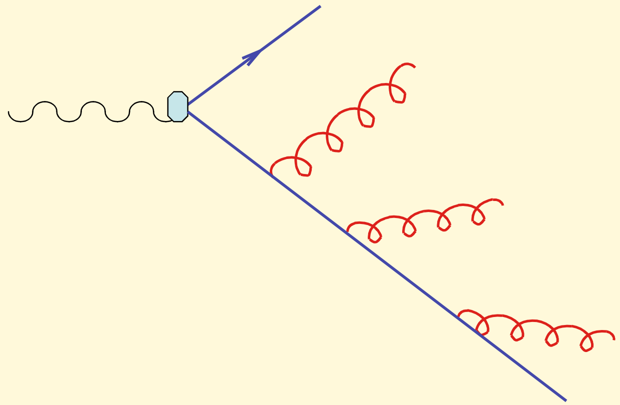
Transverse separation between q and qbar after τ:

$$d = \varphi\tau = (\varphi/\theta)\ 1/k_\perp$$

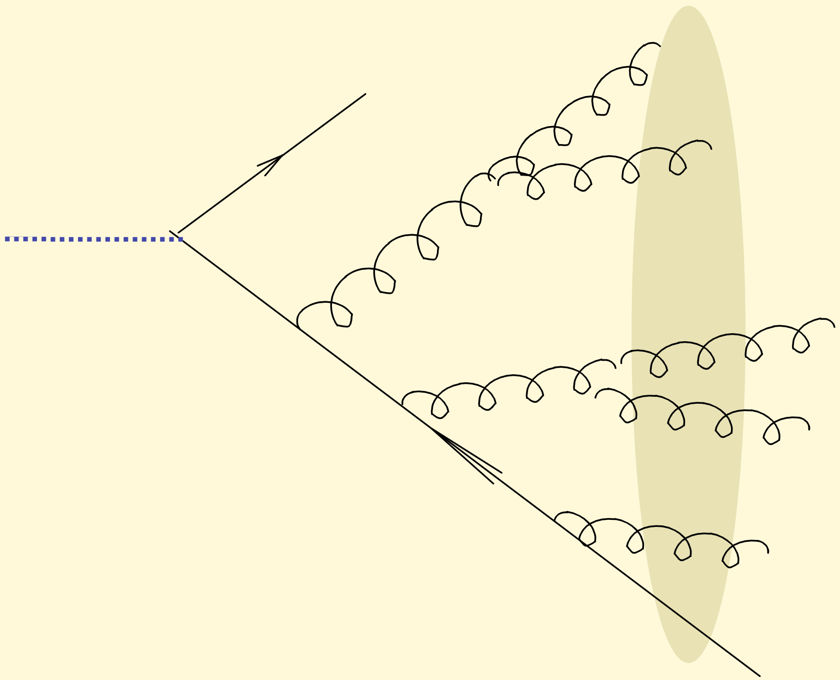
If the transverse wavelength of the emitted gluon is longer than the separation between q and qbar, the gluon emission is suppressed, because the q qbar system will appear as colour neutral (=> dipole-like emission, suppressed)

Therefore $d > 1/k_\perp$, which implies

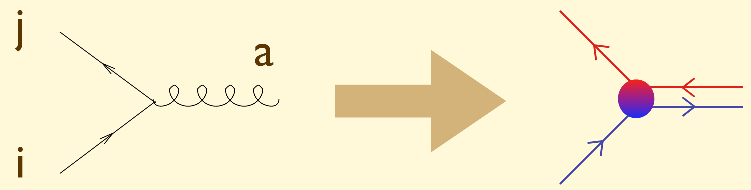
$$\theta < \varphi$$



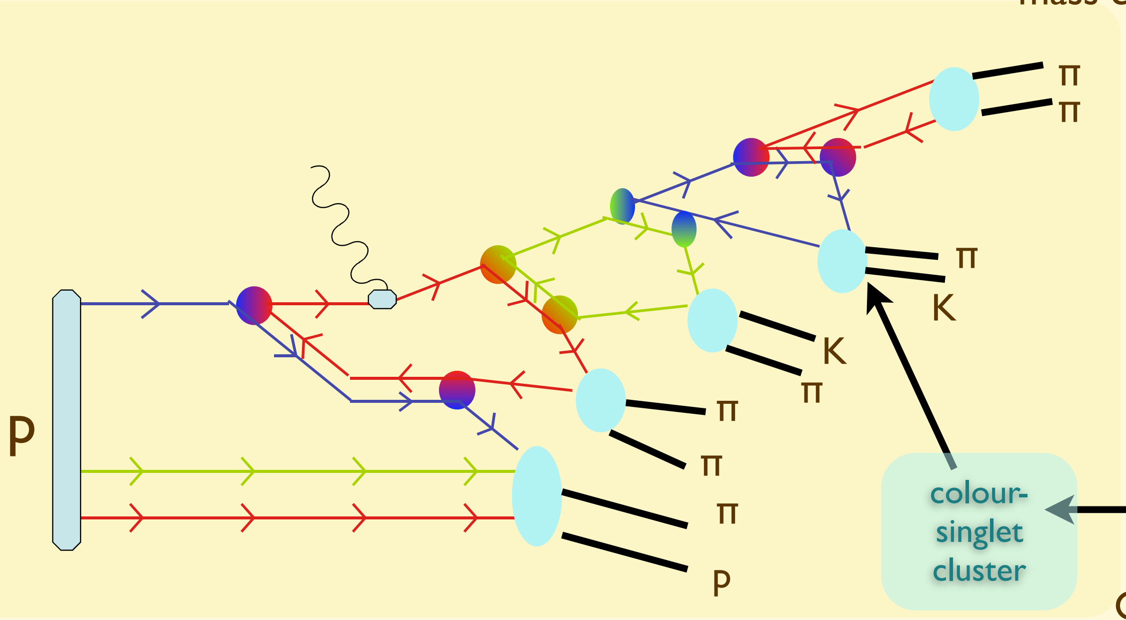
The construction can be iterated to the next emission, with the result that emission angles keep getting smaller and smaller => **jet structure**

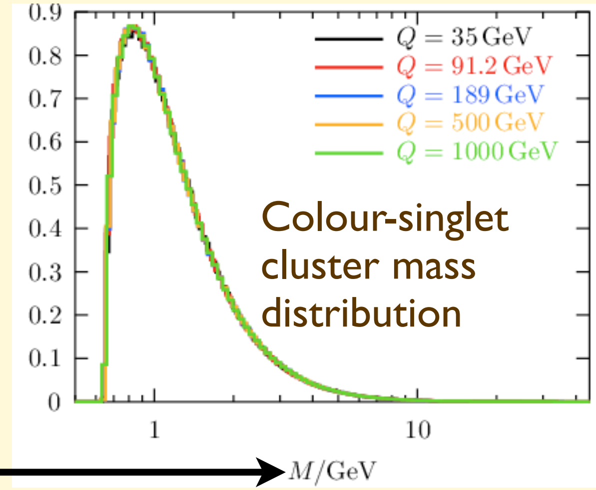


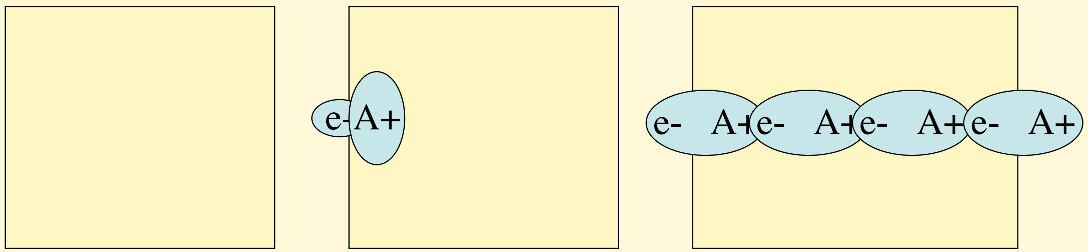
Total colour charge of the system is equal to the quark colour charge. Treating the system as the incoherent superposition of N gluons would lead to artificial growth of gluon multiplicity. Angular ordering enforces coherence, and leads to the proper evolution with energy of particle multiplicities.



The structure of the perturbative evolution leads naturally to the clustering in phase-space of colour-singlet parton pairs (”preconfinement”). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass colour-singlet clusters.



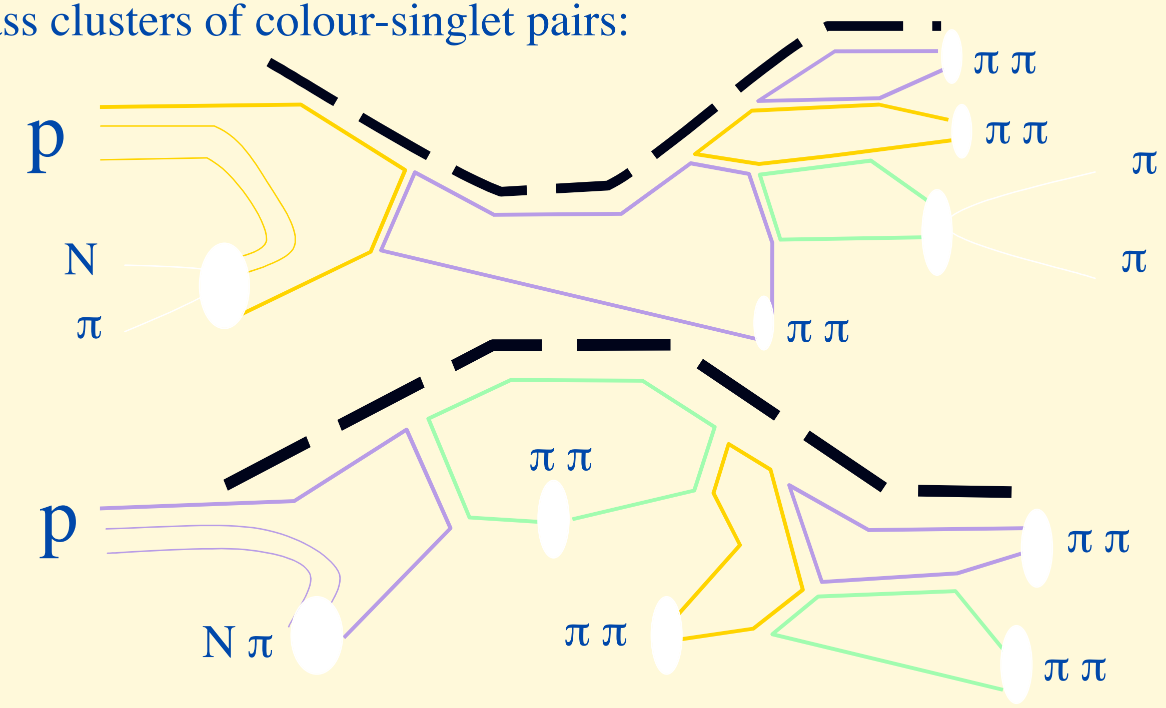




Colour is left “behind” by the struck quark. The first soft gluon emitted at large angle will connect to the beam fragments, ensuring that the beam fragments can recombine to form hadrons, and will allow the struck quark to evolve without having to worry about what happens to the proton fragments.

Hadronization

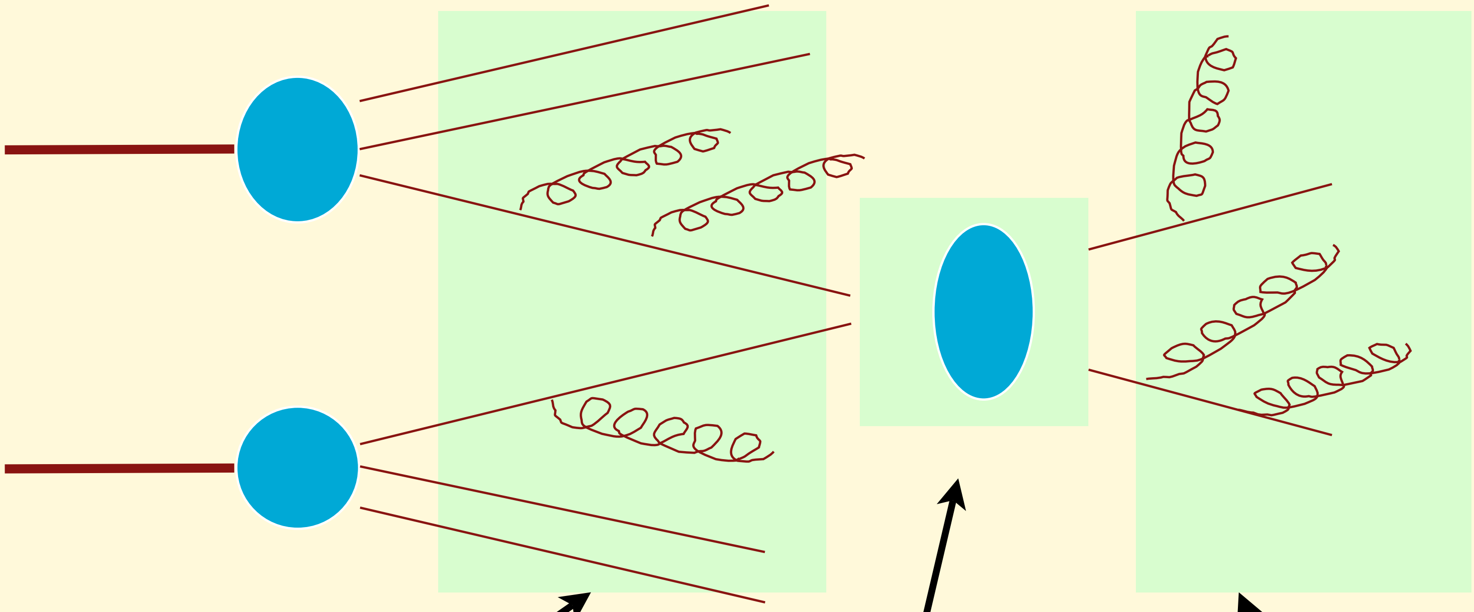
At the end of the perturbative evolution, the final state consists of quarks and gluons, forming, as a result of angular-ordering, low-mass clusters of colour-singlet pairs:



Thanks to the cluster pre-confinement, hadronization is local and independent of the nature of the primary hard process, as well as of the details of how hadronization acts on different clusters. Among other things, one therefore expects:

N(pions) = C N(gluons),
C=constant~2

Summary



Parton distribution functions (PDF): AP fits of available data, and evolution to high Q^2, and soft/ collinear gluon emission

Hard process: ME calculations

Final state evolution: shower MCs, soft/ collinear gluon emission, hadronization