The Abdus Salam
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United Nations
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SMR 1773 - 9

SCHOOL ON PHYSICS AT LHC: "EXPECTING LHC"
11 - 16 September 2006

Extra Dimensions and other (non-Susy)
New Physics at LHC
"Physics Beyond the Standard Model at the LHC"

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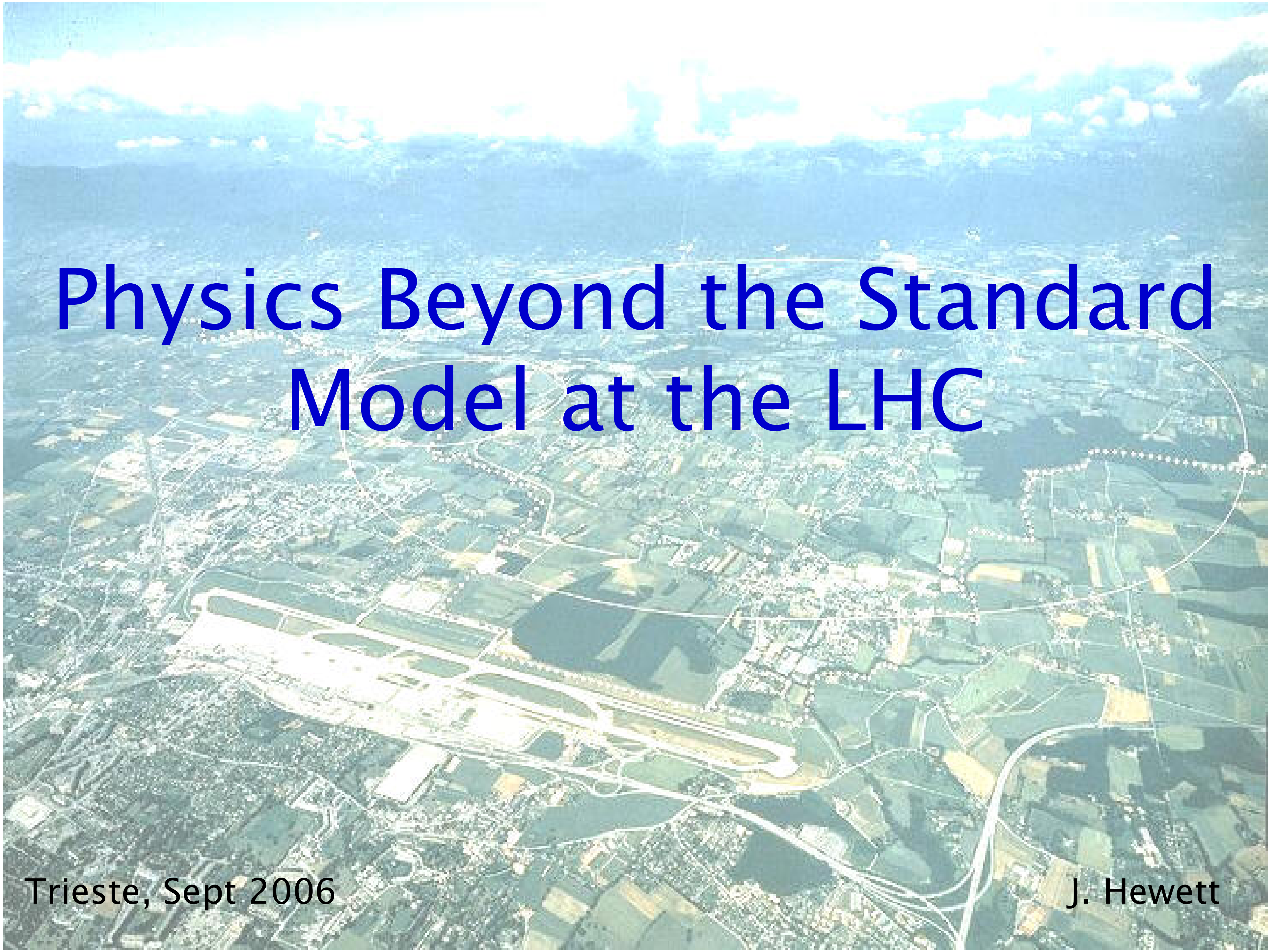


Physics Beyond the Standard Model at the LHC

Trieste, Sept 2006

J. Hewett

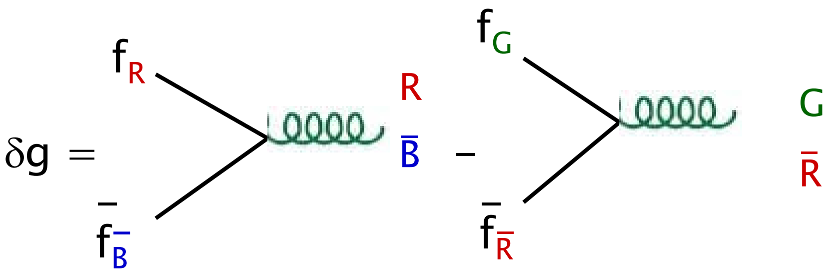
Theory

- Reflects our understanding of the universe
- Provides a simplifying framework to interpret data

Examples:

Implicit:

We take for granted that SU(3)$_C$ is exact – nobody probes hard breaking



Explicit:

LHC Collision rate 10^9 Hz
LHC Event writing rate 10^2 Hz

Selection bias has theoretical input

Both: Data analysis use Monte Carlo programs which employ various levels of theoretical assumptions from the Model to Showering to Hadronization

The Standard Model

Brief review of features which guide & restrict BSM physics

The Standard Model on One Page

$$S_{Gauge} = \int d^4x \, F^Y_{\mu\nu} F^Y_{\mu\nu} + F^\alpha_{\mu\nu} F^\alpha_{\mu\nu} + F^a_{\mu\nu} F^a_{\mu\nu}$$

$$S_{Fermions} = \int d^4x \sum_{Generations} \sum_{\substack{f = Q,u,d,\\ L,e}} fDf$$

$$S_{Higgs} = \int d^4x \, (D_\mu H)^\dagger (D_\mu H) - m^2|H|^2 + \lambda|H|^4$$

$$S_{Yukawa} = \int d^4x \, Y_u Q u^c H + Y_d Q d^c H^\dagger + Y_e L e^c H^\dagger$$

$$\left(S_{Gravity} = \int d^4x \sqrt{g} \, [M_{Pl}{}^2 R + \Lambda_{CC}{}^4] \right)$$

Gauged Symmetries

	Color	Electroweak	
Matter Fermions	$SU(3)_C$ x	$SU(2)_L$ x	$U(1)_Y$
Q	3	2	$+1/6$
u^c	3	1	$-2/3$
d^c	3	1	$+1/3$
L	1	2	$-1/2$
e^c	1	1	$+1$

Global Flavor Symmetries

SM matter secretly has a large symmetry:

$$U(45) \qquad \begin{pmatrix} Q_1 \\ u_1 \\ d_1 \\ L_1 \\ e_1 \\ \cdot \\ \cdot_{\,2} \\ \cdot \\ \cdot_{\,3} \end{pmatrix}$$

Rotate 45 fermions into each other

$$\downarrow$$

Explicitly broken by gauging 3x2x1

$$\underbrace{U(3)_Q \text{ x } U(3)_u \text{ x } U(3)_d} \text{ x } \underbrace{U(3)_L \text{ x } U(3)_e}$$

Rotate among generations

Explicitly broken by quark Yukawas + CKM

$$\downarrow$$

$$U(1)_B$$

Baryon Number

Explicitly broken by charged lepton Yukawas

$$\downarrow$$

$$\underbrace{U(1)_e \text{ x } U(1)_\mu \text{ x } U(1)_\tau}$$

Explicitly broken by neutrino masses

$$\downarrow$$

Lepton Number

$$U(1)_L$$

(or nothing)

(Dirac)

(Majorana)

Global Symmetries of Higgs Sector

Higgs Doublet: $\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$

Four real degrees of freedom

Secretly transforms as a

4 of SO(4) $\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$

Decomposes into subgroups

(2,2) SU(2) x SU(2)

SU(2)$_L$ of EW

Remaining Global Symmetry

Gauging U(1)$_Y$ explicitly breaks

SU(2)$_{\text{Global}}$ → Nothing

Size of this breaking given by Hypercharge coupling g'

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + (g')^2} \rightarrow 1 \text{ as } g' \rightarrow 0$$

New Physics may excessively break SU(2)$_{\text{Global}}$

Custodial Symmetry

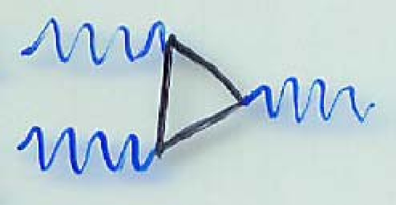
Standard Model Fermions are Chiral

Fermions cannot simply ‘pair up’ to form mass terms
i.e., $m\bar{f}_L f_R$ is forbidden Try it!

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
(Qu^c)	1	2	$-1/2$
(Qd^c)	1	2	$+1/2$
(QL)	3	1	$-1/3$
(Qe)	3	2	$+7/6$
$(u^c d^c)$	$\bar{3}\mathrm{x}\bar{3}$	1	$-1/3$
$(u^c L)$	$\bar{3}$	2	$-7/6$
$(u^c e)$	$\bar{3}$	1	$+1/3$
$(d^c L)$	$\bar{3}$	2	$-5/6$
$(d^c e)$	$\bar{3}$	1	$+4/3$
(Le)	1	2	$+1/2$

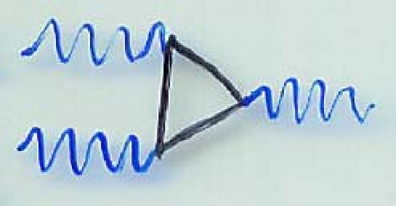
Fermion masses must be generated by Dimension-4 (Higgs) or higher operators to respect SM gauge invariance!

Anomaly Cancellation



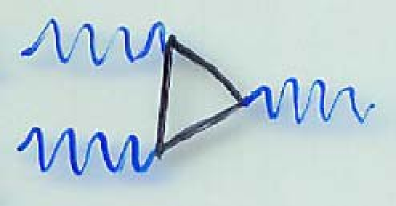
$3[\ 2\cdot(1/6) - (2/3) + (1/3)] = 0$

Q u^c d^c → $Q \quad u^c \quad d^c$

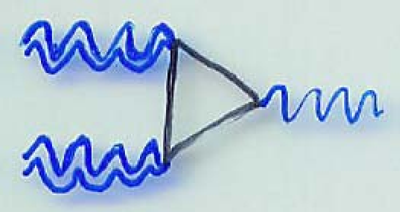


$3[3\cdot(1/6) - (1/2)] = 0$

$Q \quad L$



$3[\ 6\cdot(1/6)^3 + 3\cdot(-2/3)^3 + 3\cdot(1/3)^3$
$+ 2\cdot(-1/2)^3 + 1^3] = 0$



$3[(1/6) - (2/3) + (1/3) - (1/2) + 1]$
$= 0$

$Q \quad u^c \quad d^c \quad L \quad e$

Can't add any new fermion ⇒ must be chiral or vector–like!

Standard Model Summary

- Gauge Symmetry

  $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$

  Exact Broken to $U(1)_{QED}$

- Flavor Symmetry

  $U(3)^5 \to U(1)_B$ x $U(1)_L$ (?)

  Explicitly broken by Yukawas

- Custodial Symmetry

  $SU(2)_{Custodial}$ of Higgs sector

  Broken by hypercharge so $\rho = 1$

- Chiral Fermions

  Need Higgs or Higher order operators

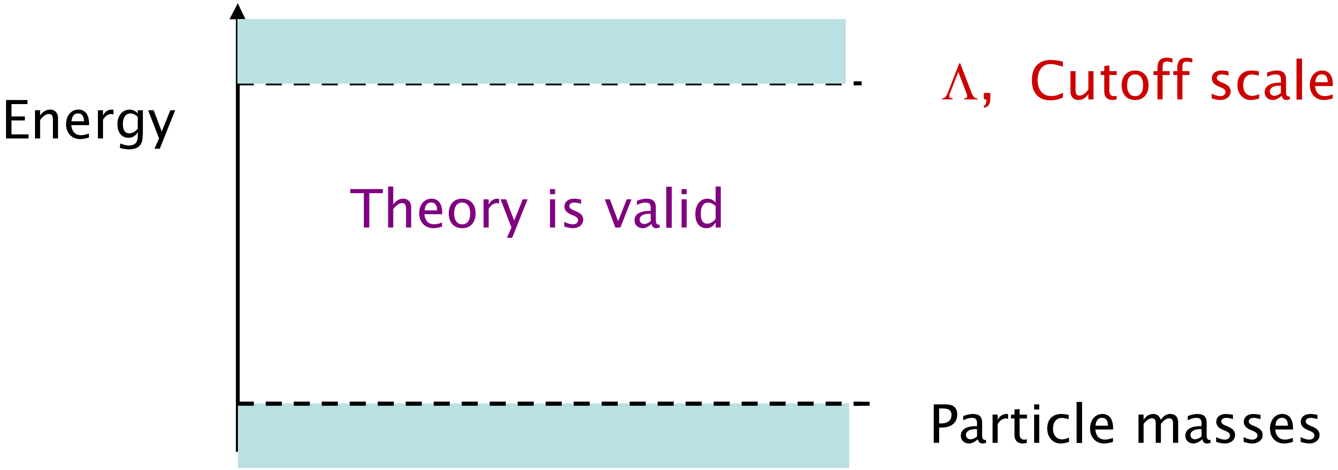
- Gauge Anomalies

  Restrict quantum numbers of new fermions

Any model with New Physics must respect these symmetries

Standard Model is an Effective field theory

An effective field theory has a finite range of applicability in energy:



Energy

Λ, Cutoff scale

Theory is valid

Particle masses

All interactions consistent with gauged symmetries are permitted, including higher dimensional operations whose mass dimension is compensated for by powers of Λ

- What sets the cutoff scale Λ ?
- What is the theory above the cutoff?

New Physics, Beyond the Standard Model!

Three paradigms:

1. SM parameters are unnatural
   $\Rightarrow$ New physics introduced to "Naturalize"
2. SM gauge/matter content complicated
   $\Rightarrow$ New physics introduced to simplify
3. Deviation from SM observed in experiment
   $\Rightarrow$ New physics introduced to explain

How unnatural are the SM parameters?

Technically Natural

- Fermion masses
  (Yukawa Couplings)
- Gauge couplings
- CKM

Logarithmically sensitive to the cutoff scale

Technically Unnatural

- Higgs mass
- Cosmological constant
- QCD vacuum angle

Power-law sensitivity to the cutoff scale

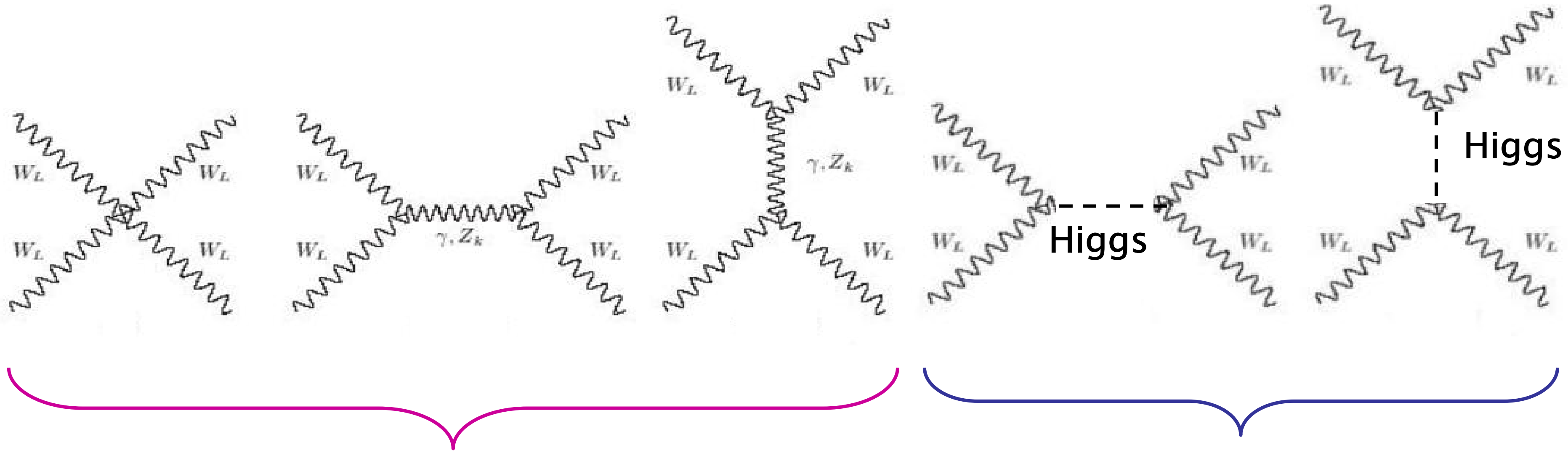
The naturalness problem that has had the greatest impact on collider physics is:

The Higgs (mass)^2 problem

or

The hierarchy problem

Do we really need a Higgs?



Bad violation of unitarity

$\mathcal{A} = a\ s/M_W^2 \ + \ldots$

Restores unitarity

$\mathcal{A} = \ -a\ (s - m_h^2)/M_W^2$
$+ \ldots$

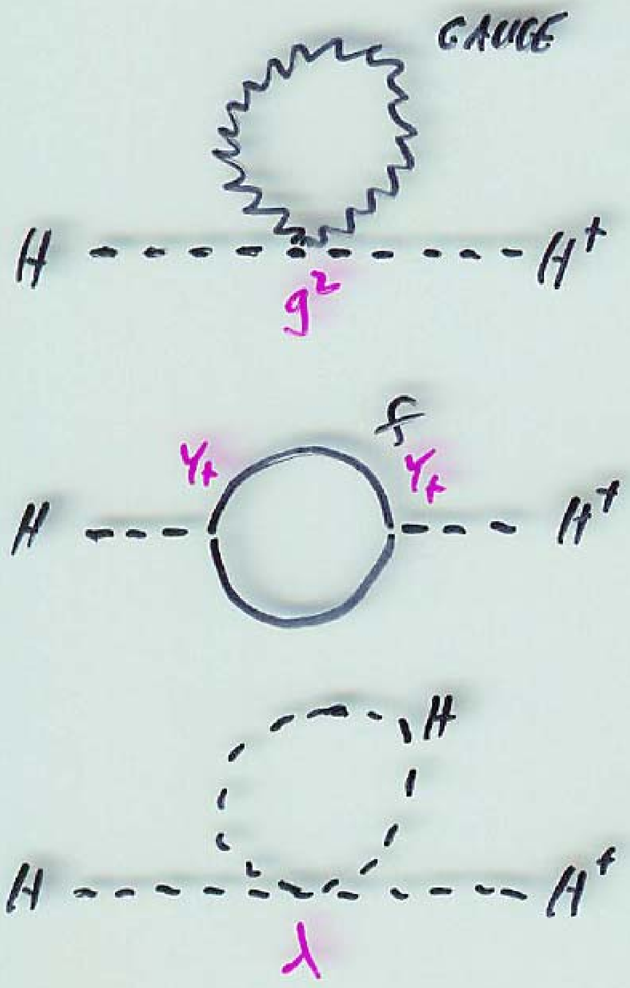
Expand cross section into partial waves
Unitarity bound (Optical theorem!) $\Rightarrow$ Gives $m_h < 4\pi M_W$

LHC is designed to explore this entire region!

Electroweak Hierarchy Problem

- Higgs $(\text{mass})^2$ is quadratically divergent

  Diagrammatically:

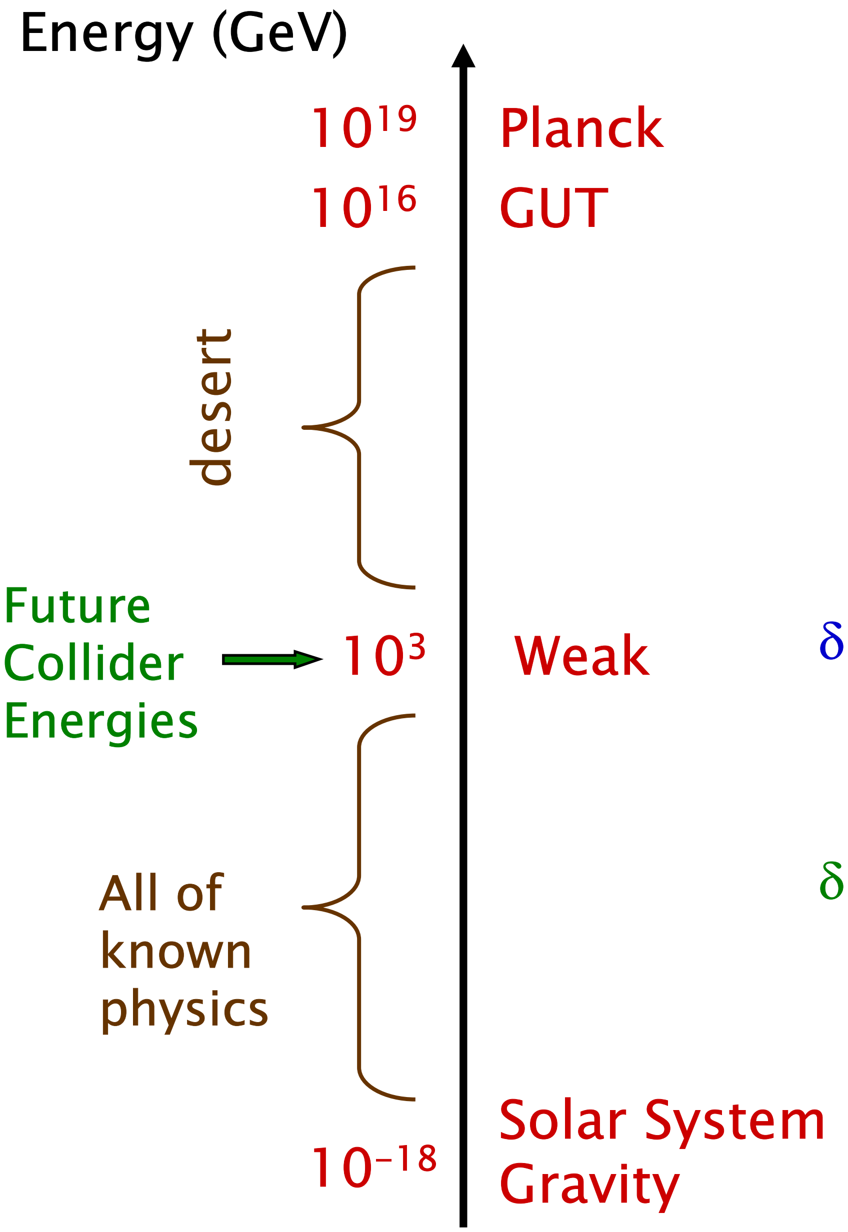


$$\frac{g^2}{16\pi^2}\int_0^{\Lambda} d^4q \, \frac{g_{\mu\nu}}{q^2 - M_G^2 + i\varepsilon} \sim \frac{g^2}{16\pi^2}\Lambda^2$$

$$\frac{Y_f^2}{16\pi^2}\int_0^{\Lambda} d^4q \, \frac{1}{(\not{q} - M_f)^2} \sim \frac{Y_f^2}{16\pi^2}\Lambda^2$$

$$\frac{\lambda}{16\pi^2}\int_0^{\Lambda} d^4q \, \frac{1}{q^2 - M_H^2 + i\varepsilon} \sim \frac{\lambda}{16\pi^2}\Lambda^2$$

The Hierarchy Problem: Supersymmetry



Energy (GeV)

10^{19} Planck

10^{16} GUT

desert

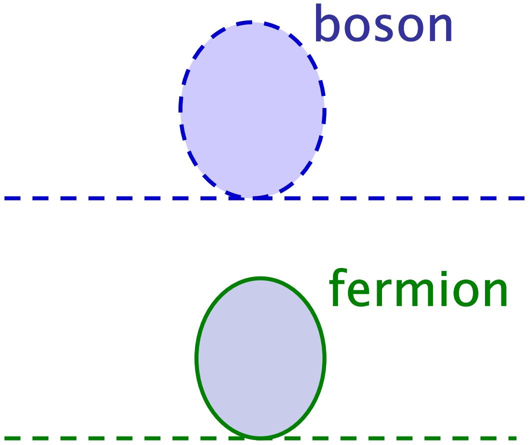
Future Collider Energies → 10^{3} Weak

All of known physics

10^{-18} Solar System Gravity

Quantum Corrections:

Virtual Effects drag Weak Scale to M_{Pl}

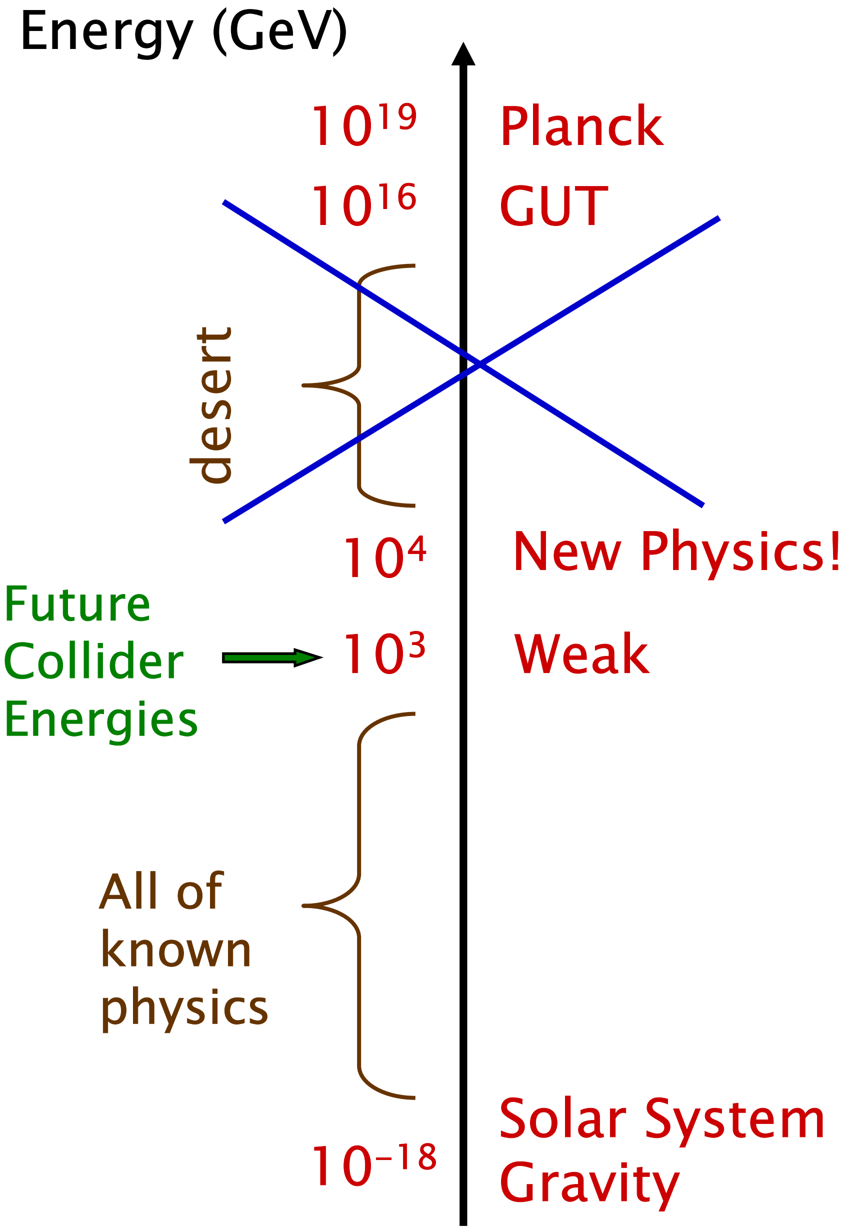


$\delta m_H^2 \sim$ $\sim M_{Pl}^2$

$\delta m_H^2 \sim$ $\sim - M_{Pl}^2$

Large virtual effects cancel order by order in perturbation theory

The Hierarchy Problem: Little Higgs



Energy (GeV)

10^{19} Planck

10^{16} GUT

desert

10^{4} New Physics!

Future Collider Energies → 10^{3} Weak

All of known physics

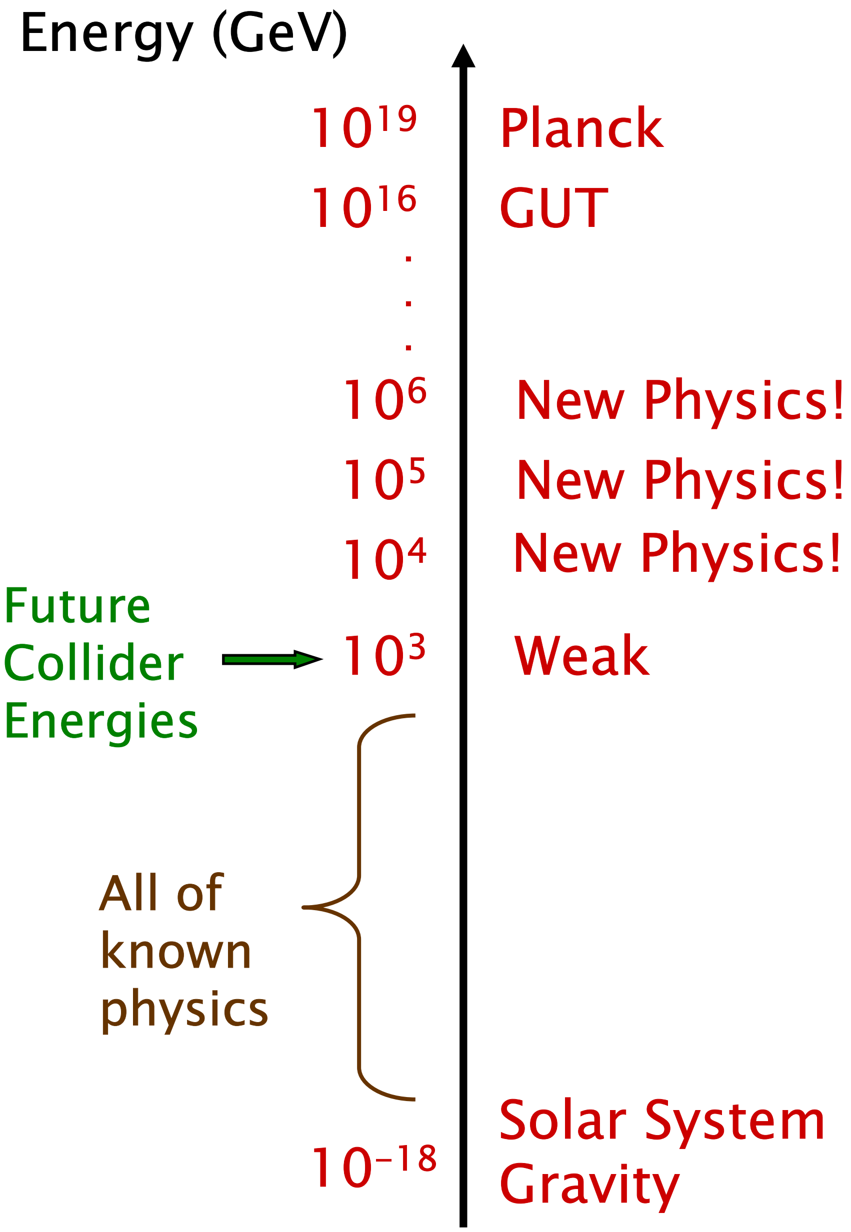
10^{-18} Solar System Gravity

Little Hierarchies!

Simplest Model:
The Littlest Higgs with
$\Lambda \sim 10$ TeV

No UV completion

The Hierarchy Problem: Little Higgs



Energy (GeV)

10^{19} Planck

10^{16} GUT

.
.
.

10^{6} New Physics!

10^{5} New Physics!

10^{4} New Physics!

Future Collider Energies → 10^{3} Weak

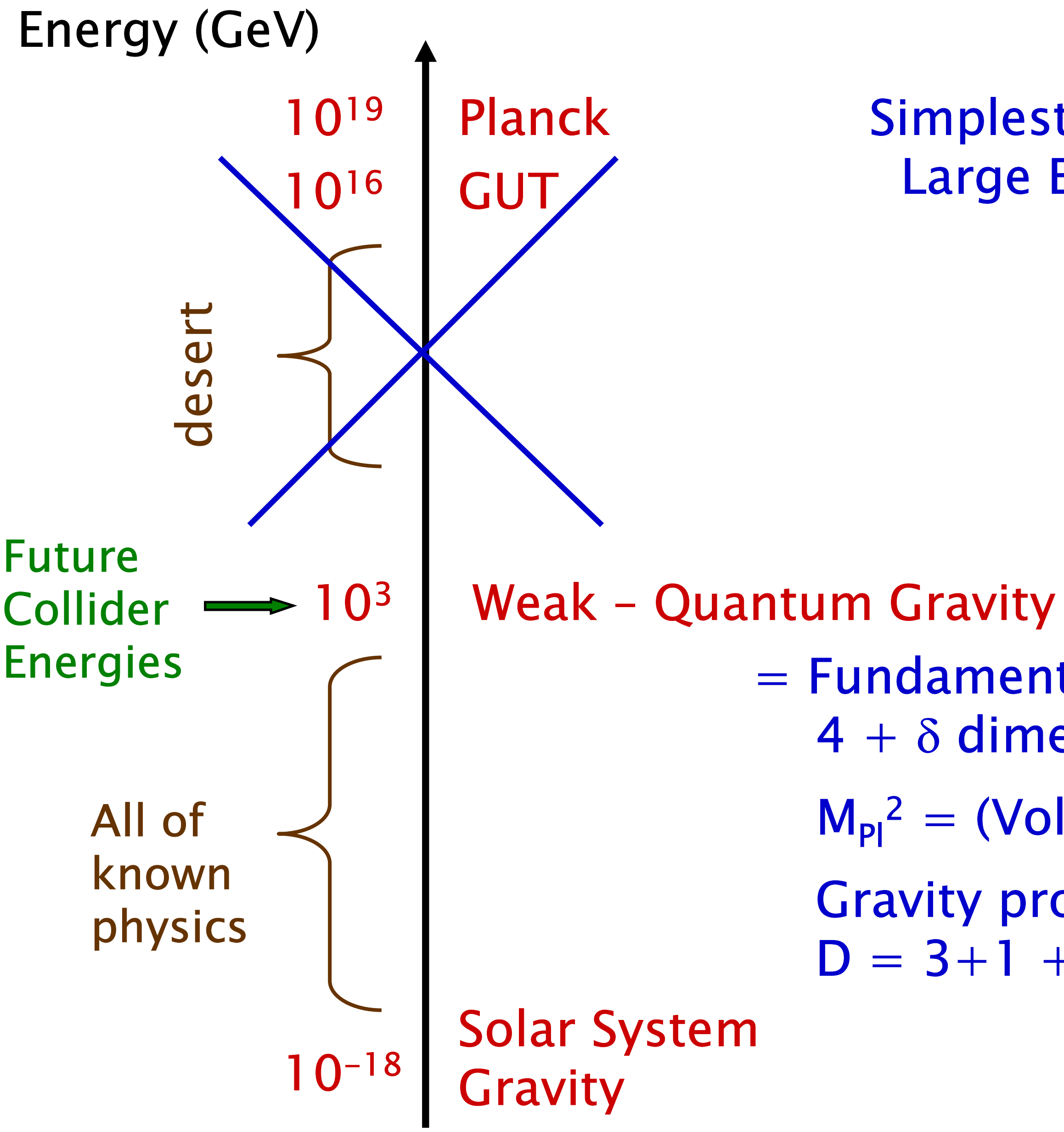
All of known physics

10^{-18} Solar System Gravity

Stacks of Little Hierarchies

Simplest Model:
The Littlest Higgs with
Λ_1 ~ 10 TeV
Λ_2 ~ 100 TeV
Λ_3 ~ 1000 TeV
…..

The Hierarchy Problem: Extra Dimensions



Energy (GeV)

10^{19} Planck

10^{16} GUT

desert

Future
Collider
Energies

10^{3} Weak – Quantum Gravity

All of
known
physics

10^{-18} Solar System
Gravity

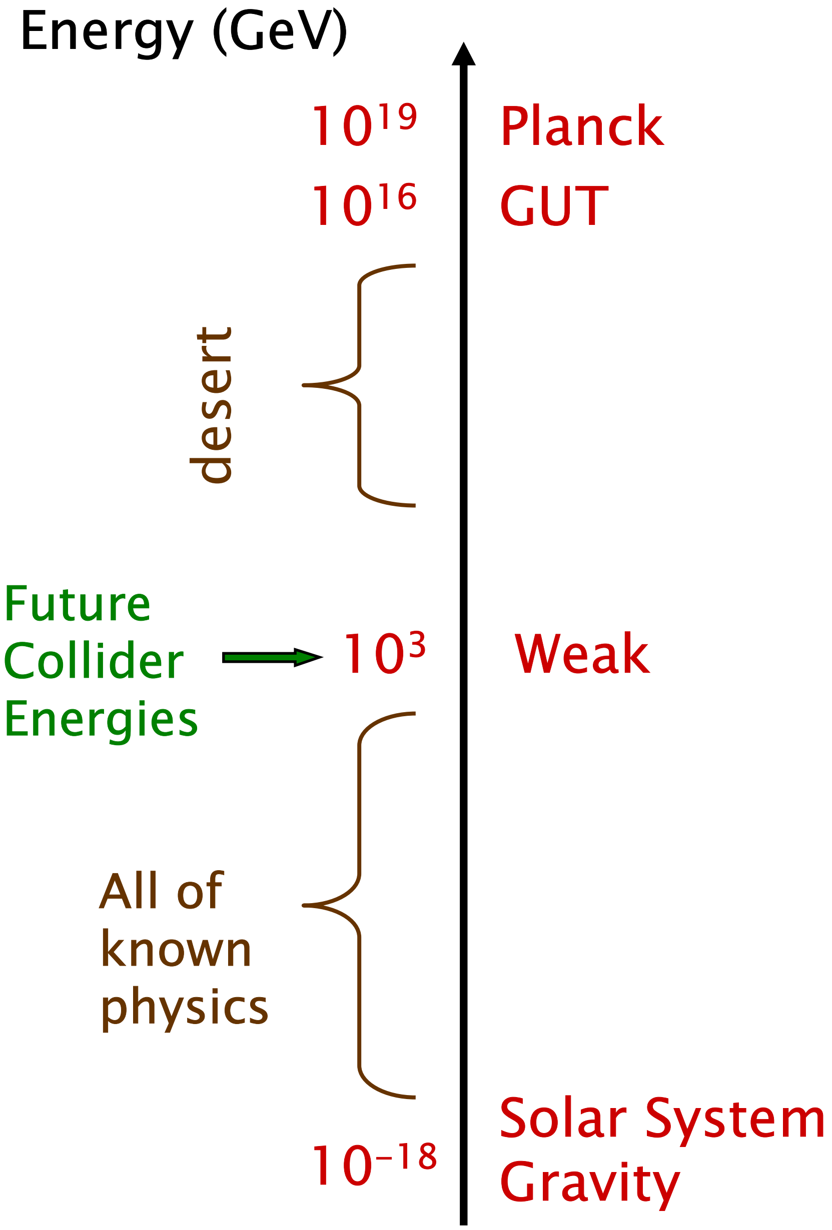
Simplest Model:
Large Extra Dimensions

= Fundamental scale in
4 + δ dimensions

$M_{Pl}{}^{2} = (\text{Volume})_{\delta}\, M_{D}{}^{2+\delta}$

Gravity propagates in
D = 3+1 + δ dimensions

The Hierarchy Problem: Extra Dimensions



Energy (GeV)

10^{19} Planck

10^{16} GUT

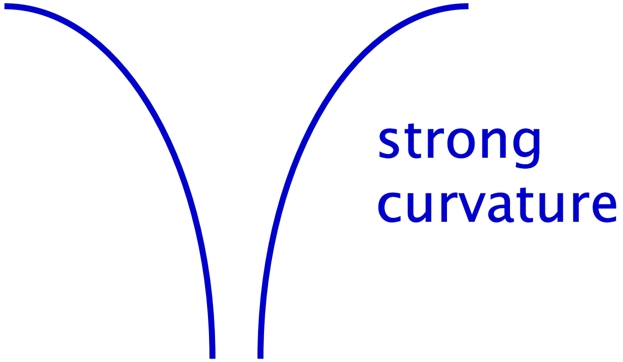
desert

Future Collider Energies → 10^3 Weak

All of known physics

10^{-18} Solar System Gravity

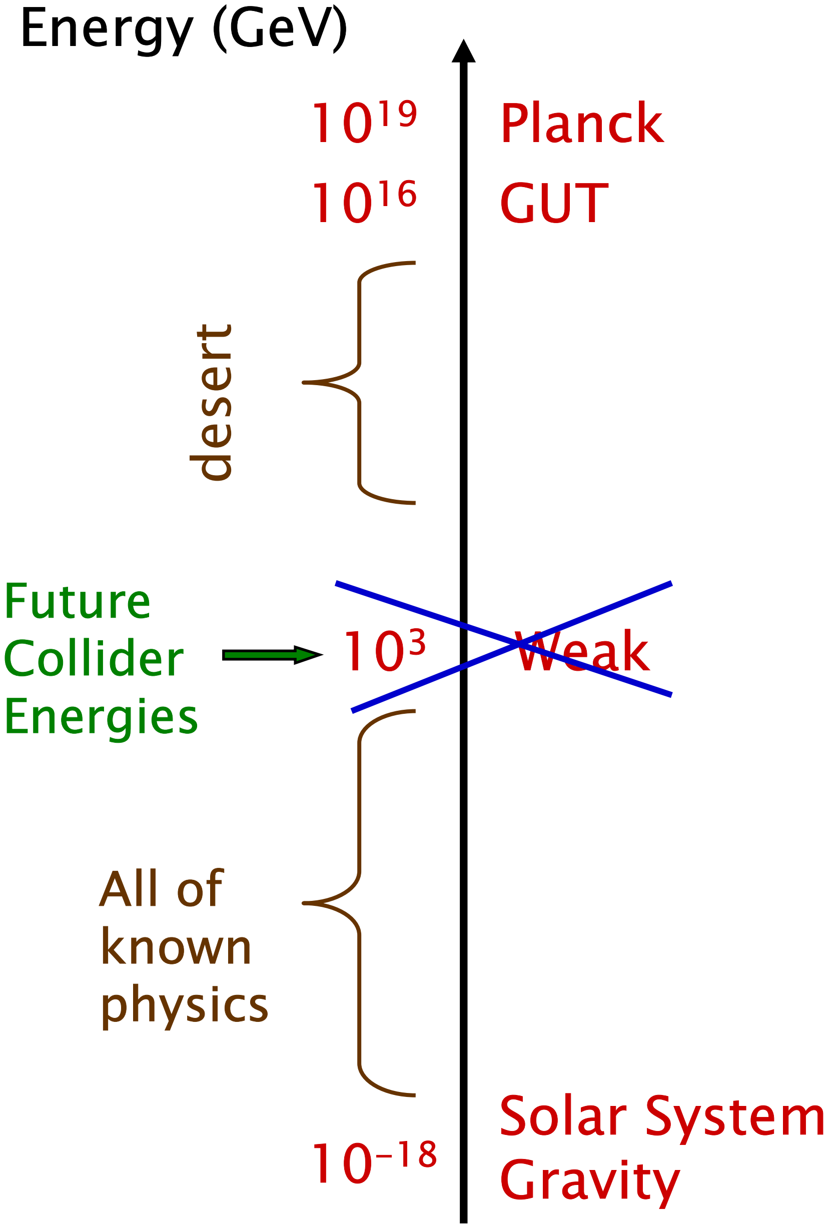
Model II:
Warped Extra Dimensions



strong curvature

$$\Lambda_{wk} = M_{Pl}\ e^{-kr\pi}$$

The Hierarchy Problem: Higgsless



Energy (GeV)

10^{19} Planck

10^{16} GUT

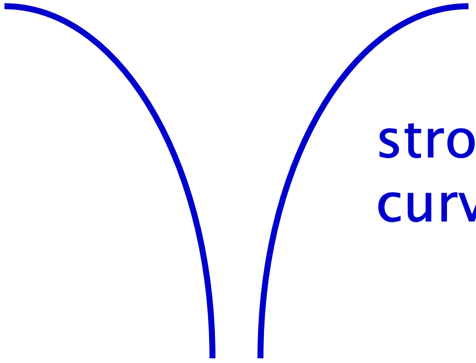
desert

Future Collider Energies → ~~10^3 Weak~~

All of known physics

10^{-18} Solar System Gravity

Warped Extra Dimensions



strong curvature

$\Lambda_{wk} = M_{Pl}\, e^{-kr\pi}$

With NO Higgs boson!

Extra Dimensions

A fifth dimension? Some History:

Gunnar Nordstrom
1881-1923

- Finnish physicist Nordstrom showed in 1914 that gravity and electromagnetism could be unified in a single theory with 5 dimensions

- However, this theory incorporated Nordstrom's theory of gravity – in competition with Einstein's at the time – and was largely ignored

A fifth dimension?

Theodor Kaluza
1885-1954

- Polish mathematician Kaluza showed in 1919 that gravity and electromagnetism could be unified in a single theory with 5 dimensions – using Einstein's theory of gravity

"The idea of achieving a unified theory by means of five-dimensional world would never have dawned on me…At first glance I like your idea tremendously"

The fifth dimension

Oskar Klein
1894–1977

- Swedish physicist Klein proposed in 1926 that the fifth dimension was real, but too tiny to be observed
- Computed it had a size of

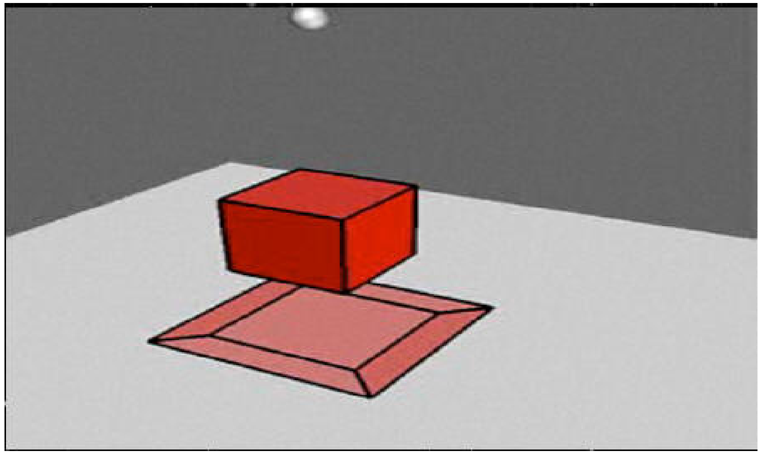
0.000000000000000000000000000000001 centimeters

to unify gravity with electromagnetism

“Klein’s paper is beautiful and impressive”

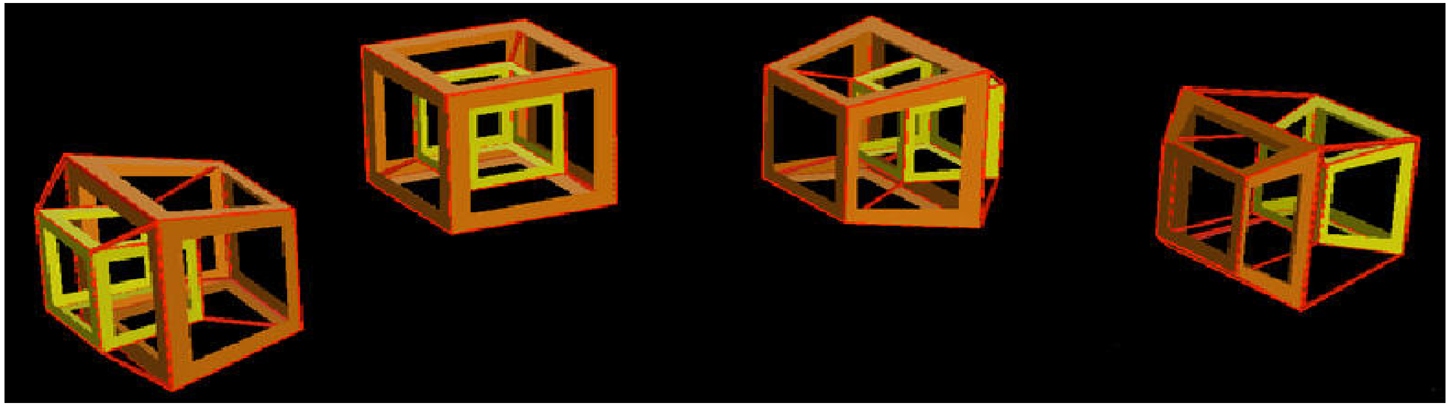
Extra dimensions can be difficult to visualize

- One picture: shadows of higher dimensional objects



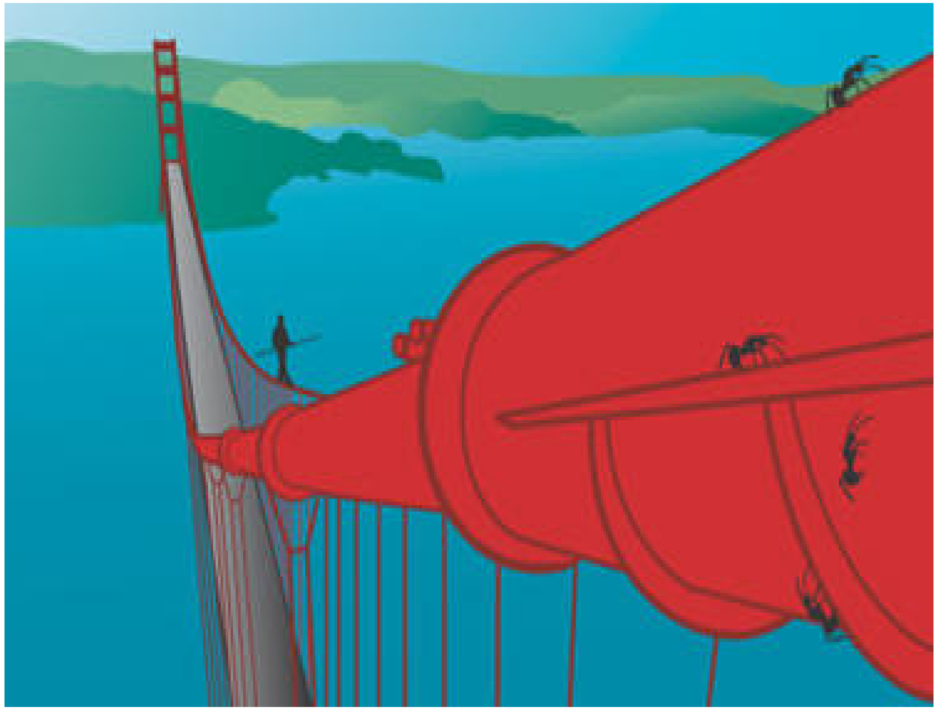
2-dimensional shadow of a rotating cube

3-dimensional shadow of a rotating hypercube



Extra dimensions can be difficult to visualize

- Another picture: extra dimensions are too small for us to observe ⇒ they are ‘curled up’ and compact

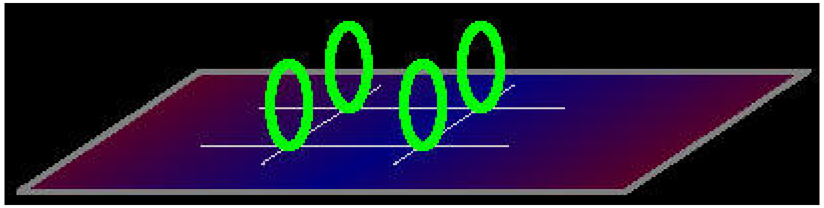


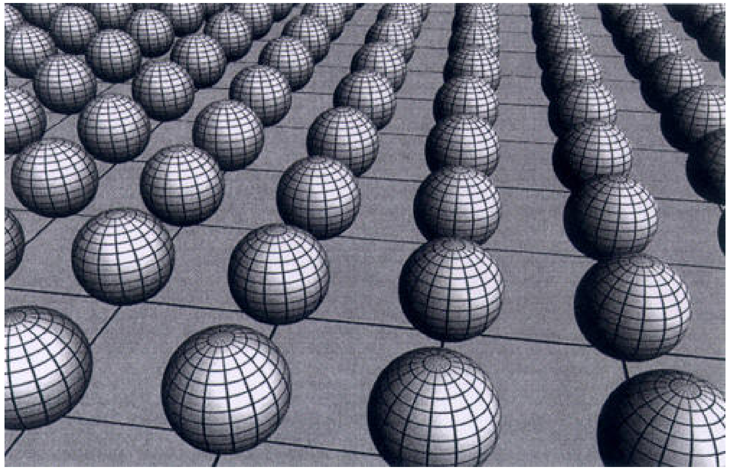
The tightrope walker only sees one dimension: back & forth.

The ants see two dimensions: back & forth and around the circle

Every point in spacetime has curled up extra dimensions associated with it

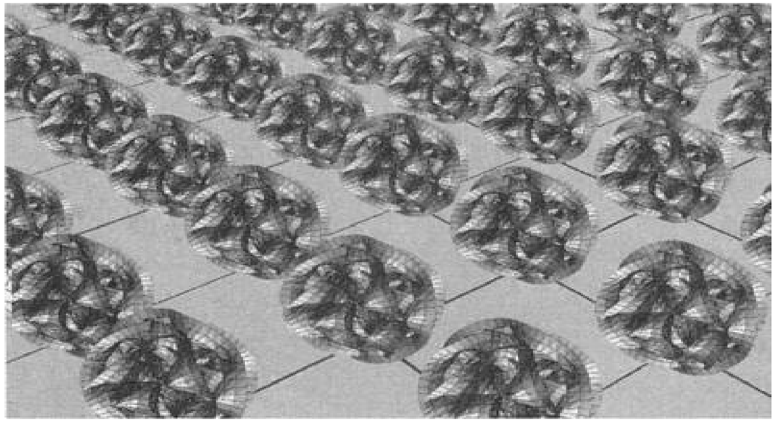
One extra dimension is a circle





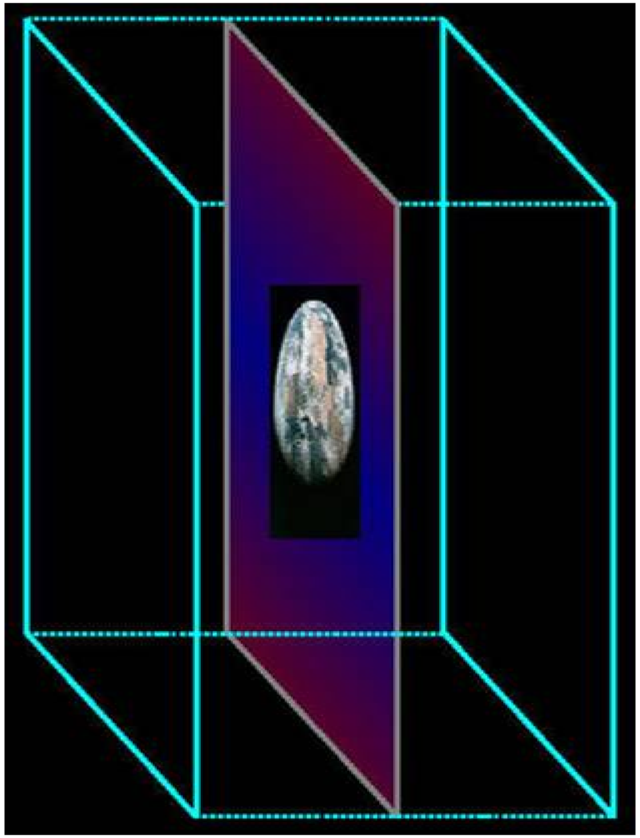
Two extra dimensions can be represented by a sphere

Six extra dimensions can be represented by a Calabi-Yau space



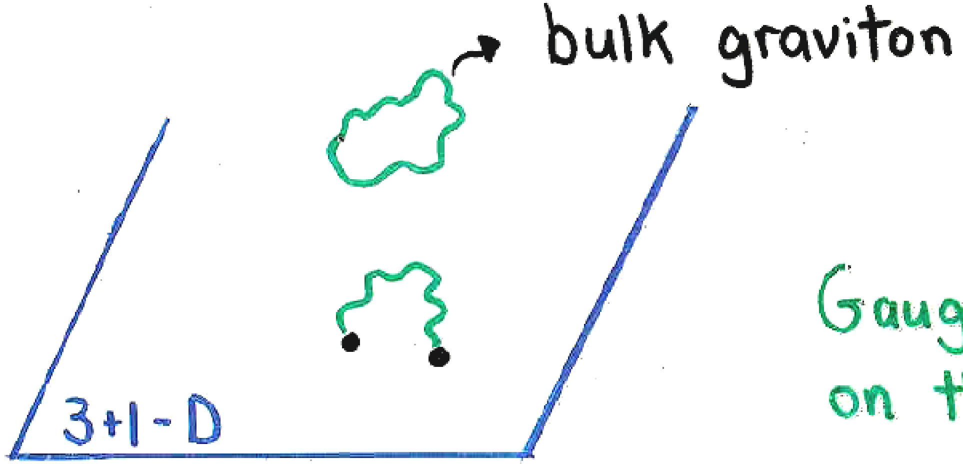
The Braneworld Scenario

- Yet another picture



- We are trapped on a 3-dimensional spatial membrane and cannot move in the extra dimensions
- Gravity spreads out and moves in the extra space
- The extra dimensions can be either very small or very large

Physics of 3-branes: string theory



Gauge theories live on the brane

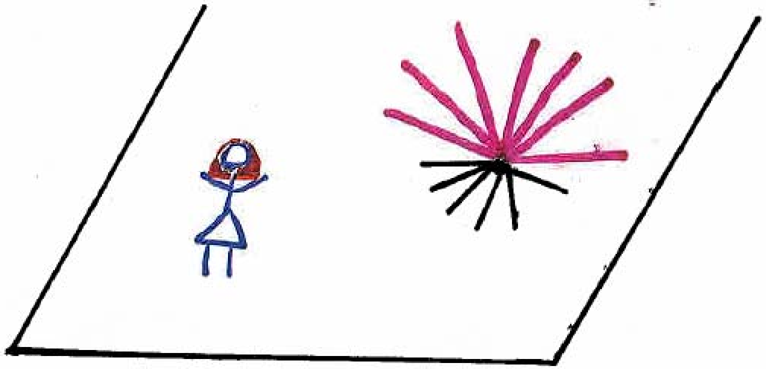
- Gauge particles live at end of strings
- Strings can pop off of brane

$\Rightarrow$ are neutral with no gauge charges

= bulk gravitons

N.B.: This is a very simplified picture

A 3-brane Universe



Standard Model
forces stuck on 3-brane

Gravitational fields
spread out over
all spacetime

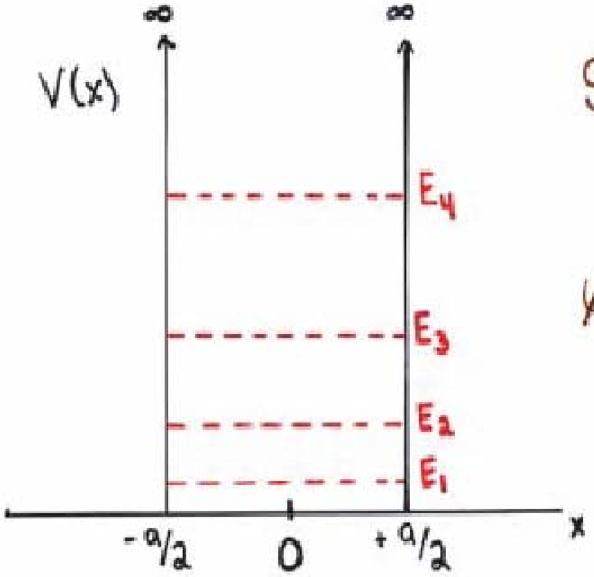
Are gravitational fields diluting too quickly?

⇒ Extra dimensions must be compactified

recover $F_{Gr} \sim 1/r^2$ on the brane

Particle in a Box

Infinite Square-Well potential



Sol'n to Schroedinger Eqn:

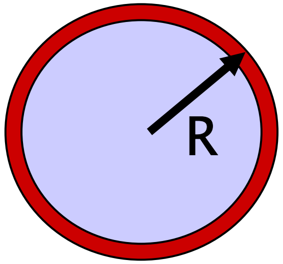
$$\psi_n(x) = \begin{cases} A_n \cos K_n x, & n=1,3,5,\ldots \\ B_n \sin K_n x, & n=2,4,6,\ldots \end{cases}$$

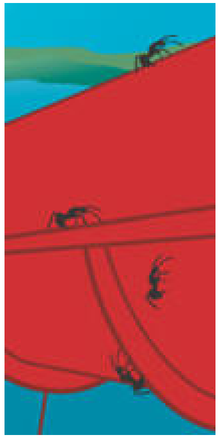
where $K_n = n\pi/a$

Momentum of the particle is <u>Quantized!</u>

$E_n \sim n^2/a^2$ (non-relativistic)

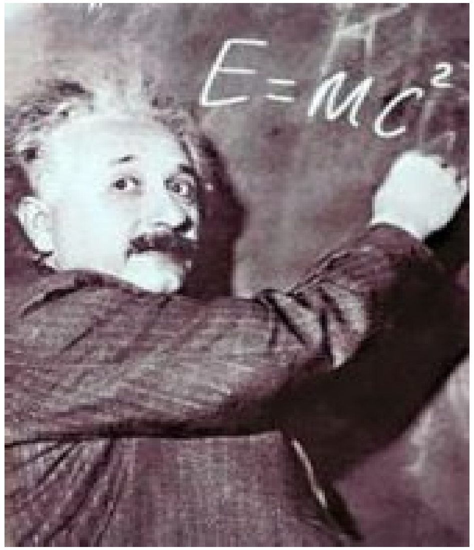
Kaluza-Klein particles





- Imagine a particle moving in a single extra dimension of size R
- It has momentum from this motion
- Quantum Mechanics says this momentum comes in steps: it has to be a multiple of 1/R
- $p_{extra} = \frac{n}{R} \qquad n = 0, 1, 2, \ldots$

Particles in extra dimensions



- This famous formula is incomplete
- For a particle in motion with momentum p in 3 spatial dimensions:

$$\boxed{E^2 = (p_x c)^2 + (p_y c)^2 + (p_z c)^2 + (mc^2)^2}$$

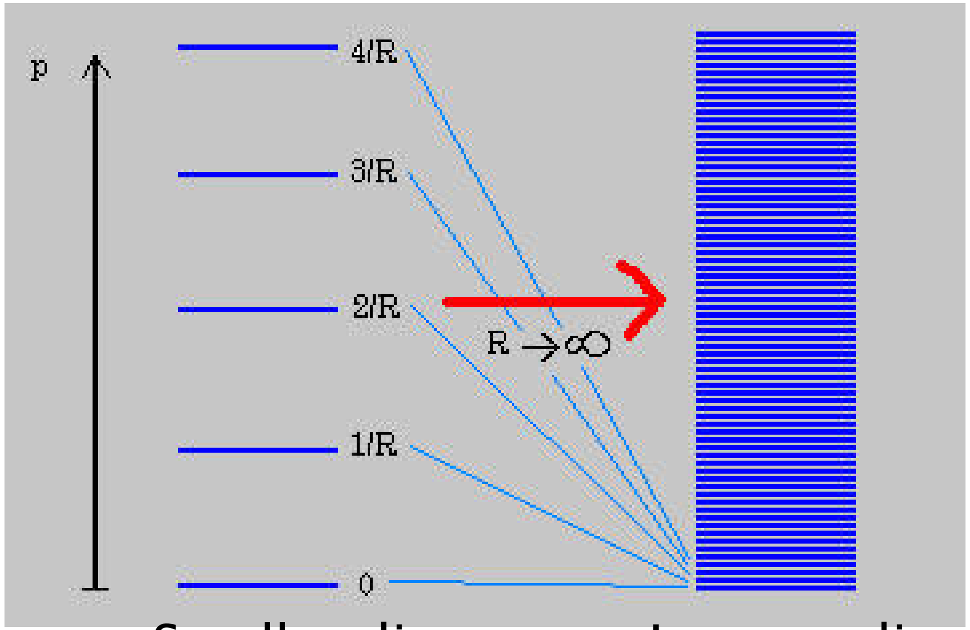
Kaluza-Klein tower of particles

$$E^2 = (p_xc)^2 + (p_yc)^2 + (p_zc)^2 + \underbrace{(p_{extra}c)^2 + (mc^2)^2}_{\text{In 4 dimensions, looks like a mass!}}$$

Recall $p_{extra} = n/R$

Tower of massive particles

Small radius gives well separated Kaluza-Klein particles



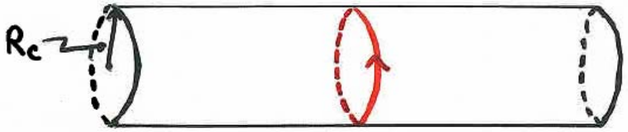
Large radius gives finely separated Kaluza-Klein particles

Small radius

Large radius

Compactification: Bulk Fields

Bulk fields expand into Kaluza-Klein towers



δ-d Kinetic motion is quantized!

$$p_\delta^2 = \frac{\vec{n}\cdot\vec{n}}{R_c^2}$$

mode numbers $\vec{n} = (n_1, n_2, \ldots n_\delta)$
label KK excitation state

Appears as tower of massive particles in 4-d

$$\Phi(x_\mu, y_i) = \sum_{\vec{n}=0}^{\infty} \phi^{(\vec{n})}(x_\mu)\, e^{i\vec{n}\cdot\vec{y}/R_c} \cdot \frac{1}{\sqrt{V_\delta}}$$

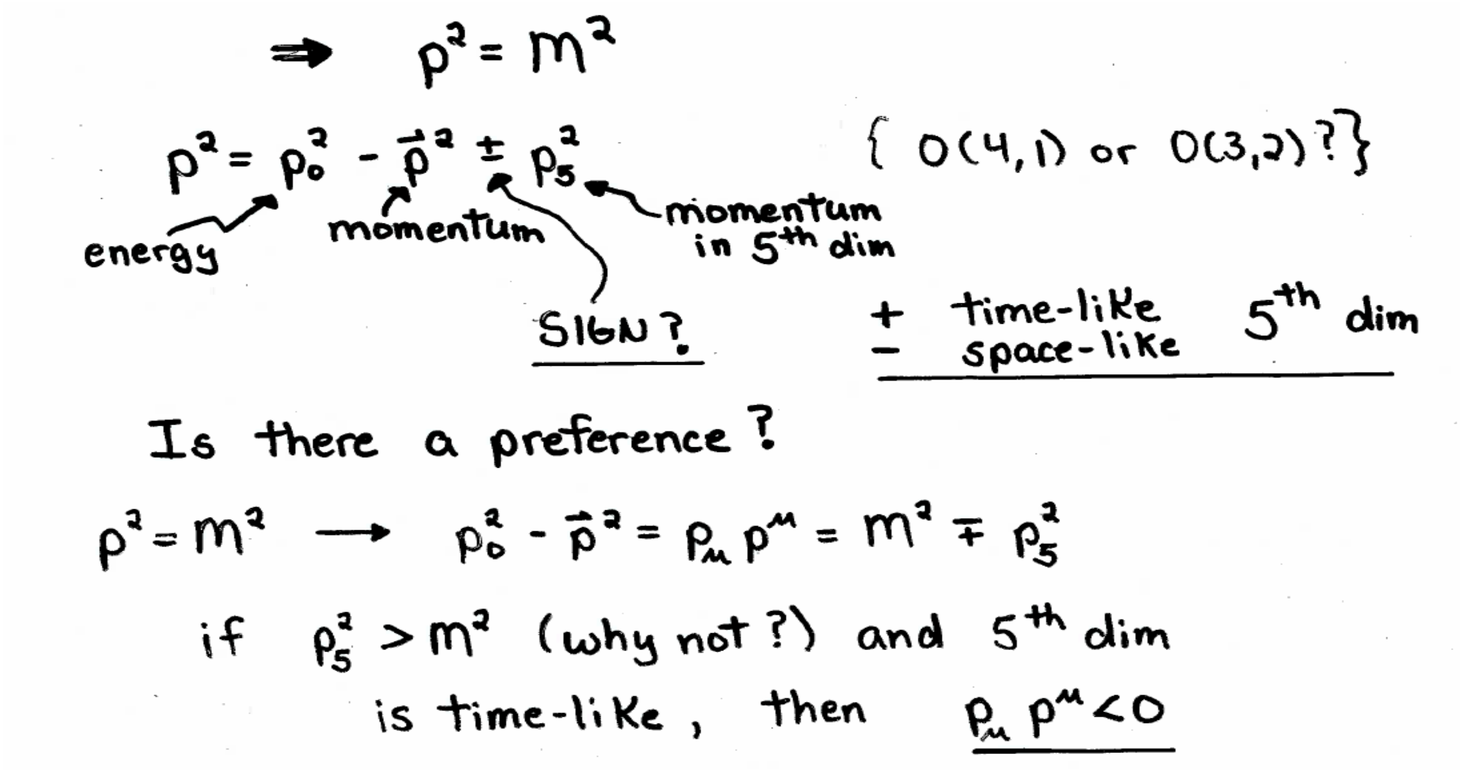
for periodic $y_i \rightarrow y_i + 2\pi R_c$ Flat space

with mass $m_{\vec{n}}^2 = \dfrac{\vec{n}\cdot\vec{n}}{R_c^2}$

K.K tower of evenly spaced states
each with identical spin + quantum numbers

Space-like vs Time-like

- Consider particle of mass M in 5D co-ords
- Assume Lorentz invariance holds in 5D



$$\Rightarrow \quad p^2 = m^2$$

$$p^2 = p_0^2 - \vec{p}^{\,2} \pm p_5^2 \qquad \{\, O(4,1) \text{ or } O(3,2)?\}$$

energy momentum momentum in 5th dim

<u>SIGN?</u>

<u>+ time-like
− space-like 5th dim</u>

Is there a preference?

$$p^2 = m^2 \longrightarrow p_0^2 - \vec{p}^{\,2} = p_\mu p^\mu = m^2 \mp p_5^2$$

if $p_5^2 > m^2$ (why not ?) and 5th dim is time-like, then $\underline{p_\mu p^\mu < 0}$

⇒<u>Tachyon</u> with possible causality problems

To avoid tachyons we generally choose extra dims to be space-like!

⇒ only ONE time dimension

Consider one extra dimension:

a simple extension of 4D $\qquad ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu - dy^2 \quad [dx_5^2]$

flat, space-like 5th d

5D Klein Gordon Eqn : (real scalar) $\qquad \underbrace{\partial_A \partial^A}_{\partial_\mu \partial^\mu - \partial_y^2} \Phi(x_\mu, y) = 0$

separate the variables : $\qquad \Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$

→ Kaluza-Klein (KK) decomposition

Klein Gordon says : $\qquad \sum_n \left(\chi_n \partial_\mu \partial^\mu \phi_n - \phi_n \partial_y^2 \chi_n\right) = 0$

if $\quad \partial_y^2 \chi_n = -m_n^2 \chi_n \qquad$ (quantized)

$$\sum_n \chi_n \underbrace{\left(\partial_\mu \partial^\mu + m_n^2\right) \phi_n}_{0} = 0$$

a set of n independent eqns

∞ set of massive scalar states

→ a KK tower

Action Approach:

$$S = \int d^4x \int_{y_1}^{y_2} dy \; \underbrace{\tfrac{1}{2}\, \partial_A \Phi\, \partial^A \Phi}_{\frac{1}{2}\partial_\mu \Phi\, \partial^\mu \Phi \;-\; \frac{1}{2}\partial_y \Phi\, \partial^y \Phi}$$

let $\Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$

$$S = \int d^4x \int_{y_1}^{y_2} dy \left\{ \tfrac{1}{2} \sum_{nm} \chi_n \chi_m \overset{\textcircled{1}}{\partial_\mu} \phi_n\, \partial^\mu \phi_m \right.$$

$$\left. - \tfrac{1}{2} \sum_{nm} \phi_n \overset{\textcircled{2}}{\phi_m}\, \partial_y \chi_n\, \partial_y \chi_m \right\}$$

To Diagonalize:

① $\int_{y_1}^{y_2} dy\; \chi_n \chi_m = \delta_{nm}$ orthonormal wavefunctions

② integrate-by-parts $\int_{y_1}^{y_2} dy\; \partial_y \chi_n\, \partial_y \chi_m$

$$= \underbrace{\chi_m \partial_y \chi_n \Big|_{y_1}^{y_2}}_{\text{Boundary conditions! } 0} - \int_{y_1}^{y_2} dy\; \chi_m \underbrace{\partial_y^2 \chi_n}_{-m_n^2 \chi_n}$$

quantized

$$S = \int d^4x \sum_n \left(\tfrac{1}{2}\, \partial_\mu \phi_n\, \partial^\mu \phi_n - \tfrac{1}{2}\, m_n^2\, \phi_n^2 \right)$$

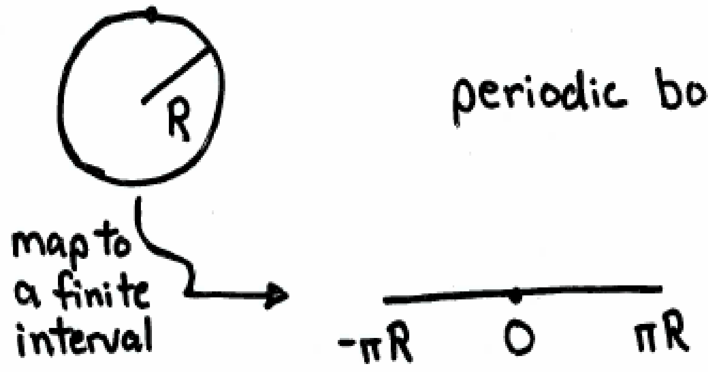
n massive scalars

$$(\partial_y^2 + m_n^2)\, \chi_n = 0 \;\Rightarrow\; \chi_n = A_n\, e^{i m_n y} + B_n\, e^{-i m_n y}$$

plane waves!

Motion in a circle: (orbital angular momentum)

$S^1 \equiv$



periodic boundary conditions

$$\chi_n(y+2\pi R) = \chi(y)$$

$$\Rightarrow \chi_n = A_n \cos {}^{ny}/_{R} + B_n \sin {}^{ny}/_{R} \quad ; \quad n=0,1,2,\ldots$$

$$\boxed{m_n = {}^{n}/_{R}}$$

Recall
$\partial_y^2 \chi_n = -m_n^2 \chi_n$

KK masses $\sim \frac{1}{\text{size}}$ of extra dimension
(for flat space)

Higher Dimensional Field Decomposition

We saw a 5D scalar → a tower of 4D scalars

What about a 5-vector A^m
or symmetric tensor h^{mN} ???

Recall:

	Lorentz (4D)	↔	Rotations (3D)
	scalar		scalar
4-vector	A^μ		$\vec{A}, \phi$
tensor	$F^{\mu\nu}$		$\vec{E}, \vec{B}$

Now

	5D	↔	4D
	scalar		$\text{scalar}(s)_n$
vector	A^M		$(A^\mu, A^5)_n$
tensor	h^{mN}		$\underbrace{(h^{\mu\nu}, h^{\mu 5}, h^{55})_n}_{\text{KK towers}}$

For δ extra dimensions:

	δD	↔	4D	$(i = 1 \ldots \delta)$
	scalar		δ scalars	
vector	A^m		$(A^\mu, A^i)_{ni}$	
tensor	h^{mN}		$(h^{\mu\nu}, h^{\mu i}, h^{ij})_{ni}$	

1 tensor

δ 4–vectors

½ δ(δ+1) scalars

- Experimental observation of KK states:

  Signals evidence of extra dimensions

- Properties of KK states:

  Determined by geometry of extra dimensions

  ⇒ Measured by experiment!

The physics of extra dimensions is the physics of the KK excitations

What are extra dimensions good for?

- Can unify the forces
- Can explain why gravity is weak
- Can break the electroweak force
- Contain Dark Matter Candidates
- Can generate neutrino masses

……

Extra dimensions can answer lots of questions!

Once observed: Things we will want to know

- How many extra dimensions are there?
- How big are they?
- What is their shape?
- What particles feel their presence?
- Do we live on a membrane?
- …
- Can we park in extra dimensions?
- When doing laundry, is that where all the socks go?

Searches for extra dimensions

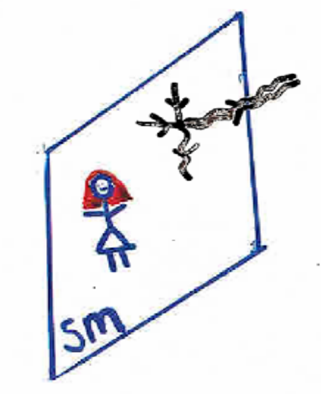
Three ways we hope to see extra dimensions:

1. Modifications of gravity at short distances
2. Effects of Kaluza-Klein particles on astrophysical/cosmological processes
3. Observation of Kaluza-Klein particles in high energy accelerators

Large Extra Dimensions

Arkani-Hamed, Dimopoulos, Dvali,
SLAC-PUB-7801

Motivation: solve the hierarchy problem by removing it!

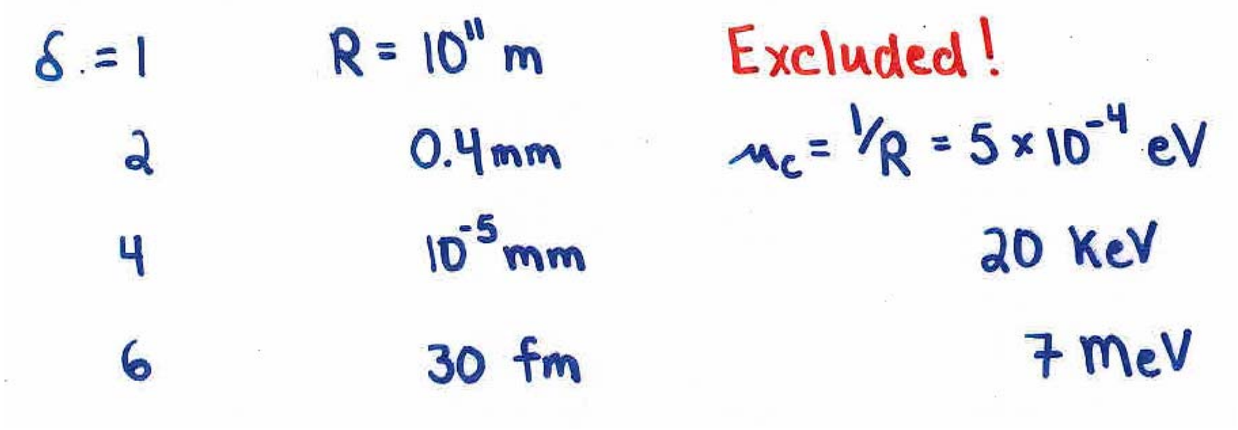


SM fields confined to 3-brane

Gravity becomes strong in the bulk

Gauss' Law: $M_{Pl}^2 = V_\delta \, M_D^{2+\delta}$, $V_\delta = R_c^{\,\delta}$

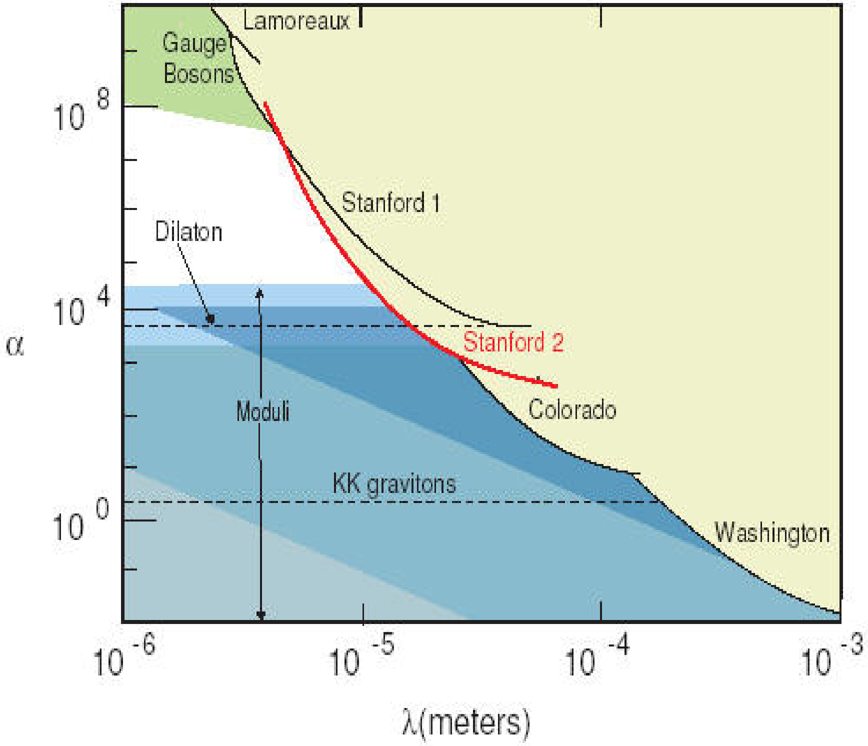
M_D = Fundamental scale in the bulk
~ TeV



$\delta = 1$	$R = 10^{11}$ m	Excluded!
2	0.4mm	$m_c = 1/R = 5 \times 10^{-4}$ eV
4	10^{-5} mm	20 KeV
6	30 fm	7 MeV

Constraints from Cavendish-type exp'ts

Parameterized as $\Delta V = - G_N m m \left\{ \alpha \, e^{-\lambda r}/r \right\}$



$$V_{gravity} \sim \frac{m_1 m_2}{M_D^{2+\delta}} \; \frac{1}{r^{\delta+1}} \qquad (r < R_c)$$

$$\sim \frac{m_1 m_2}{M_{Pl}^2} \; \frac{1}{r} \qquad (r > R_c)$$

For $\delta = 2$: $\lambda \lesssim 190\,\mu$ $[M_D \gtrsim 1.8\text{ TeV}]$

Constraints from Astrophysics/Cosmology

- Supernova Cooling

  Cullen, Perelstein
  Barger etal, Savage etal

  $NN \rightarrow NN + G_n$ can cool supernova too rapidly

- Cosmic Diffuse γ Rays

  Hannestad, Raffelt
  Hall, Smith

  $NN \rightarrow NN + G_n \rightarrow \gamma\gamma$

  $\nu\bar{\nu} \rightarrow G_n \rightarrow \gamma\gamma$

- Matter Dominated Universe

  Fairbairn

  too many KK states

- Neutron Star Heat Excess

  Hannestad, Raffelt

  $NN \rightarrow NN + G_n$

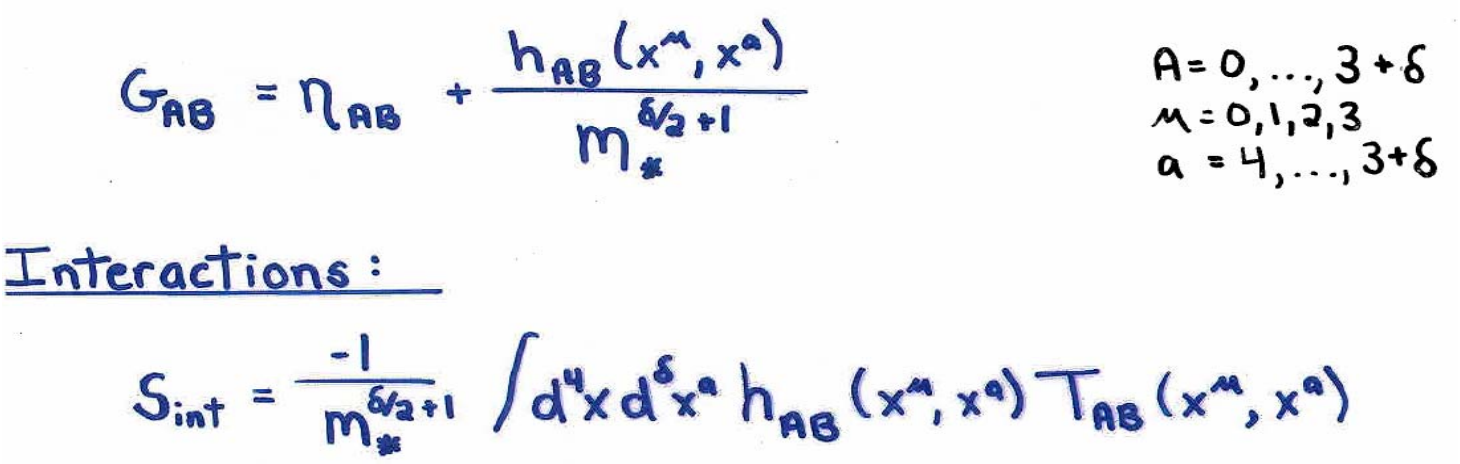
  → becomes trapped in neutron star halo and heats it

Summary of Constraints on M_D(TeV)

$\delta=$	2	3	4	5
Supernova Cooling	30	2.5		
Cosmic Diffuse γ-Rays				
SNe	80	7		
$\nu\bar{\nu}$ Annihilation	110	5		
Reheating	170	20	5	1.5
Neutron Star	450	30		
Matter Dominated Universe	85	7	1.5	
Neutron Star Heat Excess	1700	60	4	1.0

Low M_D disfavored for $\delta \leq 3$

Bulk Metric: Linearized Quantum Gravity



$$G_{AB} = \eta_{AB} + \frac{h_{AB}(x^\mu, x^a)}{M_*^{\delta/2+1}}$$

$A = 0, \ldots, 3 + \delta$

$\mu = 0, 1, 2, 3$

$a = 4, \ldots, 3 + \delta$

Interactions:

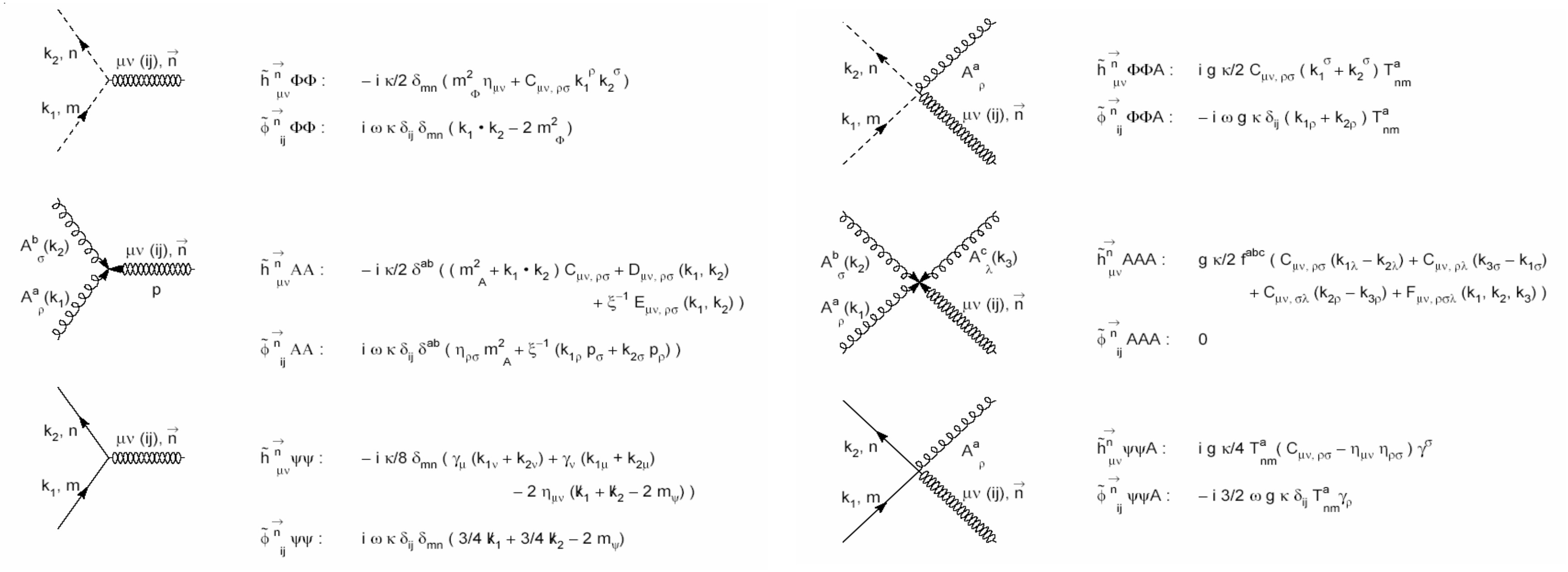
$$S_{int} = \frac{-1}{M_*^{\delta/2+1}} \int d^4x\, d^\delta x^a\, h_{AB}(x^\mu, x^a)\, T_{AB}(x^\mu, x^a)$$

- Perform Graviton KK reduction
- Expand h_{AB} into KK tower
- SM on 3-brane
  Set $T = \eta^\mu{}_A \eta^\nu{}_B \, \delta(y^a)$
- Pick a gauge
- Integrate over $d^\delta y$

$\Rightarrow$ Interactions of Graviton KK states with SM fields on 3-brane

Feyman Rules: Graviton KK Tower

Massless 0-mode + KK states have indentical coupling to matter



$\tilde{h}^{\vec{n}}_{\mu\nu}\Phi\Phi$: $-i\,\kappa/2\,\delta_{mn}\,(\,m^2_\Phi\,\eta_{\mu\nu} + C_{\mu\nu,\,\rho\sigma}\,k_1^\rho\,k_2^\sigma\,)$

$\tilde{\phi}^{\vec{n}}_{ij}\Phi\Phi$: $i\,\omega\,\kappa\,\delta_{ij}\,\delta_{mn}\,(\,k_1 \cdot k_2 - 2\,m^2_\Phi\,)$

$\tilde{h}^{\vec{n}}_{\mu\nu}AA$: $-i\,\kappa/2\,\delta^{ab}\,(\,(\,m^2_A + k_1 \cdot k_2\,)\,C_{\mu\nu,\,\rho\sigma} + D_{\mu\nu,\,\rho\sigma}\,(k_1, k_2) + \xi^{-1}\,E_{\mu\nu,\,\rho\sigma}\,(k_1, k_2)\,)$

$\tilde{\phi}^{\vec{n}}_{ij}AA$: $i\,\omega\,\kappa\,\delta_{ij}\,\delta^{ab}\,(\,\eta_{\rho\sigma}\,m^2_A + \xi^{-1}\,(k_{1\rho}\,p_\sigma + k_{2\sigma}\,p_\rho)\,)$

$\tilde{h}^{\vec{n}}_{\mu\nu}\psi\psi$: $-i\,\kappa/8\,\delta_{mn}\,(\,\gamma_\mu\,(k_{1\nu} + k_{2\nu}) + \gamma_\nu\,(k_{1\mu} + k_{2\mu}) - 2\,\eta_{\mu\nu}\,(\not{k}_1 + \not{k}_2 - 2\,m_\psi)\,)$

$\tilde{\phi}^{\vec{n}}_{ij}\psi\psi$: $i\,\omega\,\kappa\,\delta_{ij}\,\delta_{mn}\,(\,3/4\,\not{k}_1 + 3/4\,\not{k}_2 - 2\,m_\psi)$

$\tilde{h}^{\vec{n}}_{\mu\nu}\Phi\Phi A$: $i\,g\,\kappa/2\,C_{\mu\nu,\,\rho\sigma}\,(\,k_1^\sigma + k_2^\sigma\,)\,T^a_{nm}$

$\tilde{\phi}^{\vec{n}}_{ij}\Phi\Phi A$: $-i\,\omega\,g\,\kappa\,\delta_{ij}\,(\,k_{1\rho} + k_{2\rho}\,)\,T^a_{nm}$

$\tilde{h}^{\vec{n}}_{\mu\nu}AAA$: $g\,\kappa/2\,f^{abc}\,(\,C_{\mu\nu,\,\rho\sigma}\,(k_{1\lambda} - k_{2\lambda}) + C_{\mu\nu,\,\rho\lambda}\,(k_{3\sigma} - k_{1\sigma}) + C_{\mu\nu,\,\sigma\lambda}\,(k_{2\rho} - k_{3\rho}) + F_{\mu\nu,\,\rho\sigma\lambda}\,(k_1, k_2, k_3)\,)$

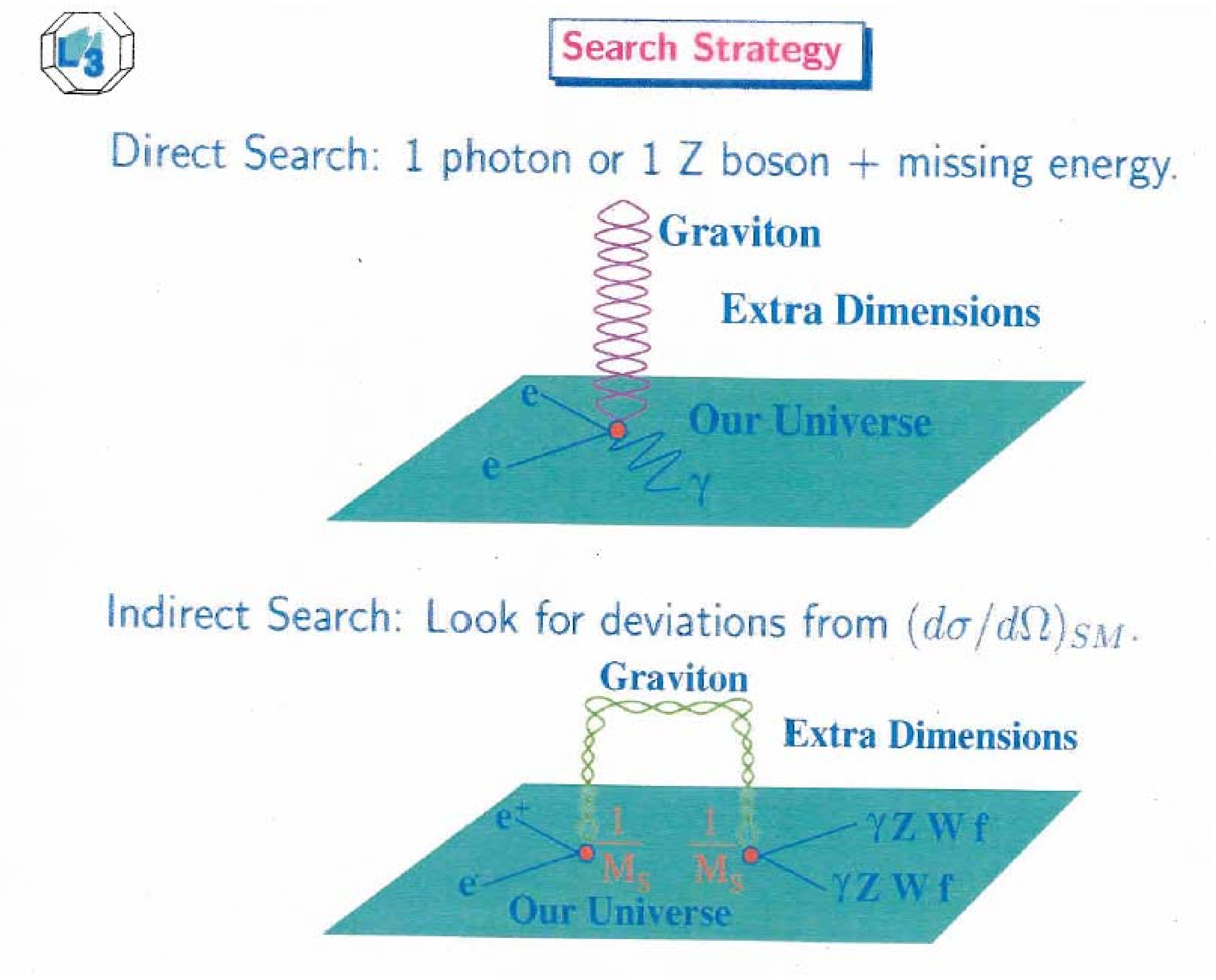
$\tilde{\phi}^{\vec{n}}_{ij}AAA$: 0

$\tilde{h}^{\vec{n}}_{\mu\nu}\psi\psi A$: $i\,g\,\kappa/4\,T^a_{nm}(\,C_{\mu\nu,\,\rho\sigma} - \eta_{\mu\nu}\,\eta_{\rho\sigma}\,)\,\gamma^\sigma$

$\tilde{\phi}^{\vec{n}}_{ij}\psi\psi A$: $-i\,3/2\,\omega\,g\,\kappa\,\delta_{ij}\,T^a_{nm}\gamma_\rho$

Han, Lykken, Zhang
Giudice, Rattazzi, Wells

Collider Tests



Search Strategy

Direct Search: 1 photon or 1 Z boson + missing energy.

Indirect Search: Look for deviations from $(d\sigma/d\Omega)_{SM}$.

Graviton Tower Exchange: $XX \to G_n \to YY$

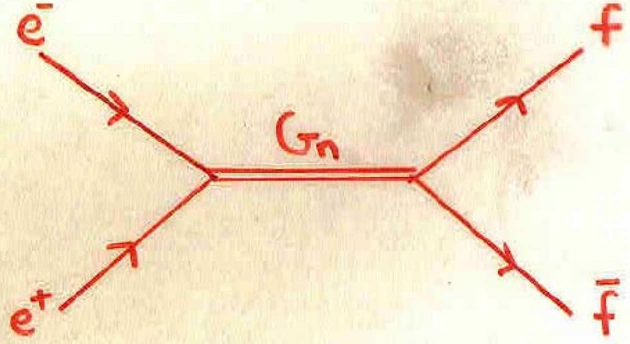
Giudice, Rattazzi, Wells
JLH

Search for 1) Deviations in SM processes
2) New processes! $(gg \to \ell\ell)$

Angular distributions reveal spin-2 exchange

Consider $e^+e^- \to f\bar{f}$

$$\mathcal{M} = \frac{1}{16\,\underline{\underline{m_{Pl}^2}}} \sum_n \frac{T^e_{\mu\nu}\, P^{\mu\nu\lambda\sigma}\, T^f_{\lambda\sigma}}{s - m_n^2 + i\epsilon}$$



G_n are densely packed!

$(\sqrt{s}\, R_c)^\delta$ states are exchanged! (~10^{30} for $\delta=2$ and $\sqrt{s} = 1$ TeV)

⇒ Approximate $\sum_{\vec{n}} \to \int dm_n^2 \; \rho(m_n)$ # of states in mass interval Δm^2

$$\rho(m_n) \sim R_c^{\delta} \, m^{\delta-2}$$

$$\sum_{\vec{n}} \frac{1}{s-m_n^2+i\epsilon} \sim R_c^{\delta} \int dm_n^2 \, \frac{m^{\delta-2}}{s-m_n^2+i\epsilon}$$

log divergent for $\delta = 2$ $\quad \sim R_c^{\delta} \Lambda^{\delta-2} \log\left(\Lambda^2/\sqrt{s}\right)$

power divergent for $\delta > 2$ $\quad \sim R_c^{\delta}$ [redacted] $\Lambda^{\delta-2}$

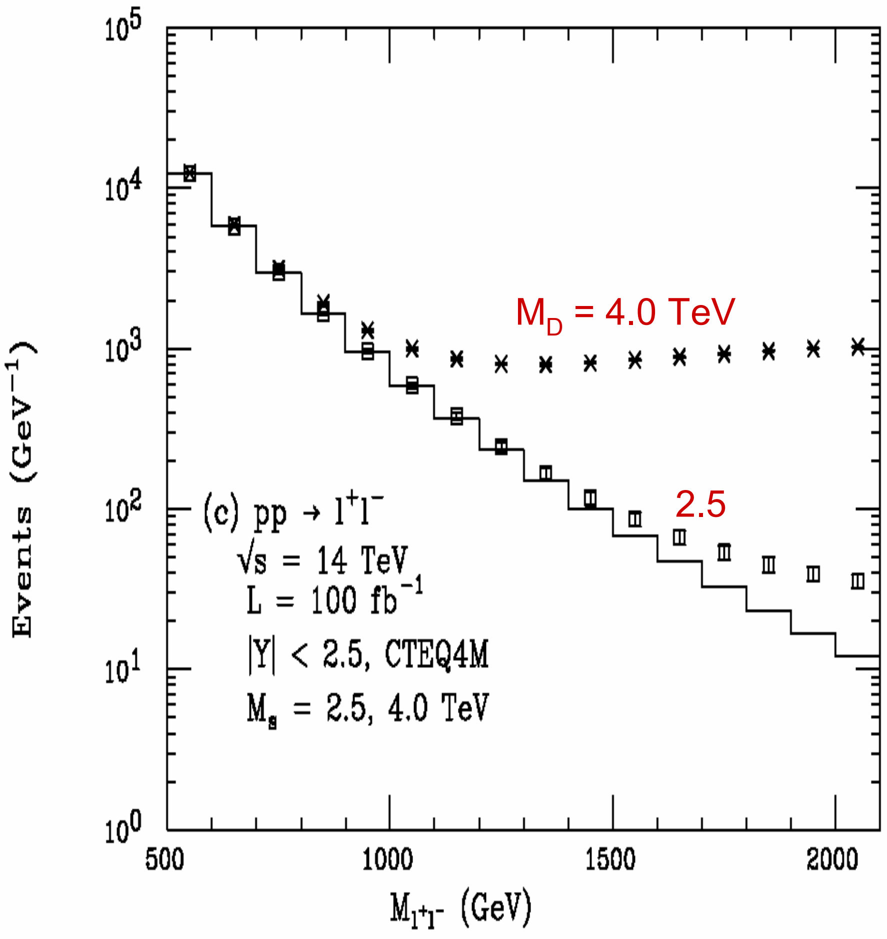
⇒ Introduce UV cut-off Λ (related to M_D in full UV theory)

Using Gauss' Law: $\quad \Gamma \sim M_{Pl}^2/\Lambda^4$

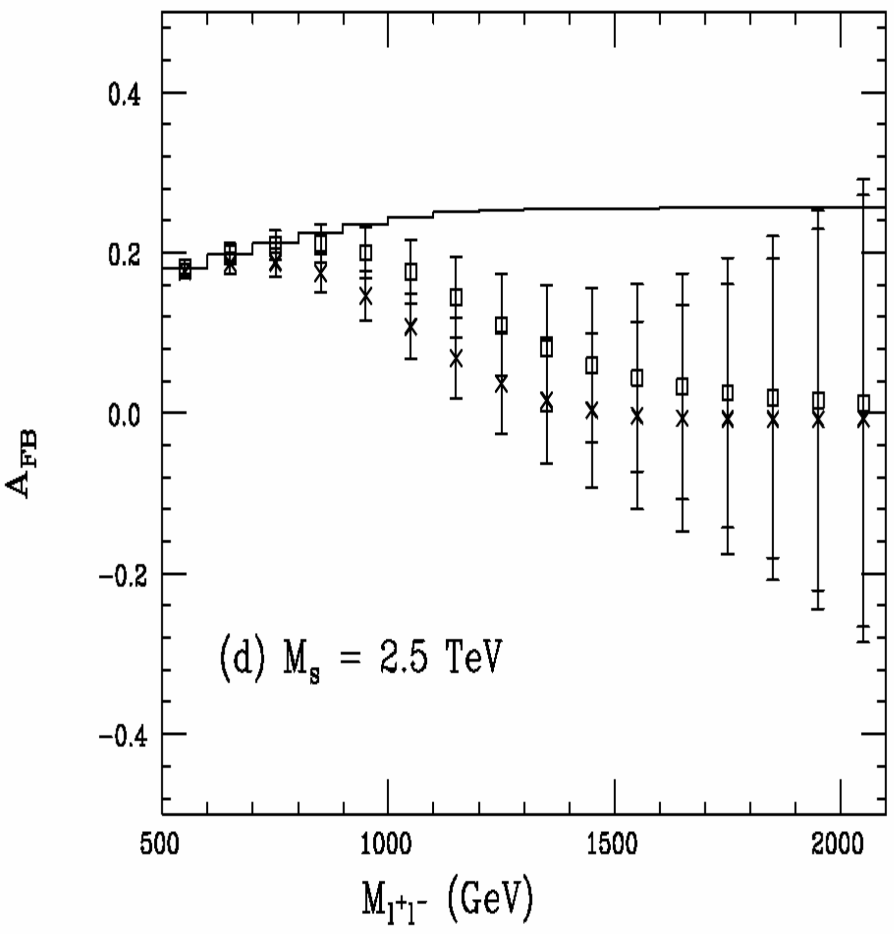
$$\frac{1}{M_{Pl}^2} \sum_{\vec{n}} \frac{1}{s-m_n^2+i\epsilon} \to \frac{1}{\Lambda^4}$$

$$\mathcal{O} = \frac{4}{\Lambda^4} T_{\mu\nu} T^{\mu\nu}$$ + corrections from higher-order terms

Drell-Yan Spectrum @ LHC



Forward-Backward Asymmetry



JLH

Graviton Exchange

Search Reach at Future Colliders

		$\sqrt{s}$	M_D (TeV)
LC:	$e^+e^- \to f\bar{f}$	500 GeV	5.0
		1 TeV	8.4
	$\gamma\gamma \to \gamma\gamma$	1 TeV	3.5
	$\gamma\gamma \to WW$		13.0
	$e\gamma \to e\gamma$		8.0
LHC:	$pp \to l^+l^-$	14 TeV	7.5
	$pp \to \gamma\gamma$		7.1

(@ design luminosity)

LHC/LC Explore the parameter space which is relevant to the hierarchy!

Limits from Virtual G_{KK} Effects

H. Zheng

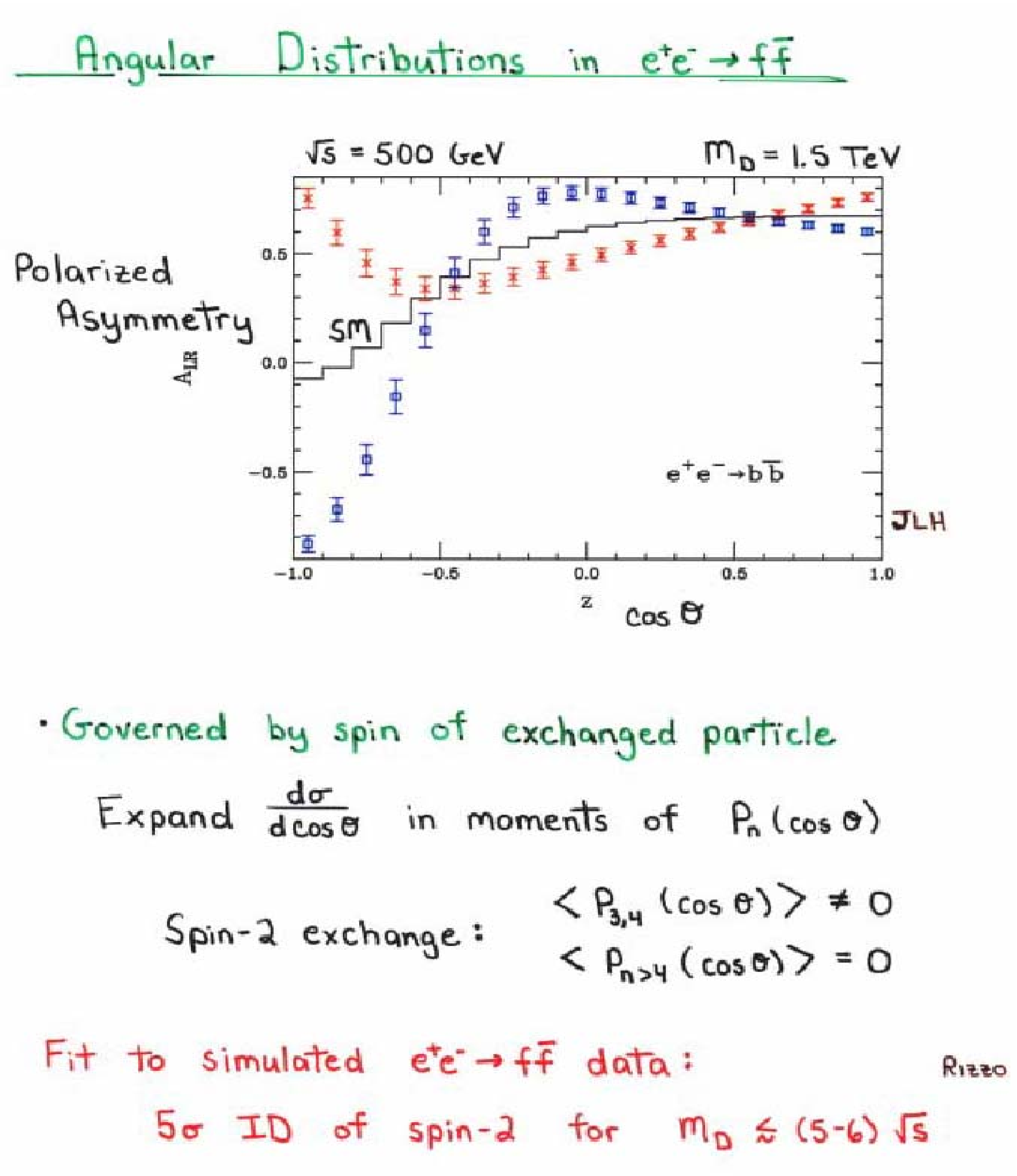
- Different notations used in different processes:
  - M_D is the fundamental mass scale – real graviton
  - M_S is the ultraviolet cutoff of the divergent sum over the KK excitations – virtual effects
- No exact relation between M_D and M_S is available
- M_D and M_S are expected to be of the same order

Hewett convention

DØ [PRL 86 (2001) 1156]: Ms (λ = +1) > 1.1 TeV; Ms (λ = -1) > 1.0 TeV

LEP Combined Results [hep-ex/0111063 v2]: Ms (λ = +1) > 1.0 TeV; Ms (λ = -1) > 1.1 TeV

CDF Preliminary [hep-ex/0111063 v2]: Ms (λ = +1) > 0.8 TeV; Ms (λ = -1) > 0.9 TeV



Angular Distributions in $e^+e^- \to f\bar{f}$

- Governed by spin of exchanged particle

Expand $\frac{d\sigma}{d\cos\theta}$ in moments of $P_n(\cos\theta)$

Spin-2 exchange: $\langle P_{3,4}(\cos\theta)\rangle \neq 0$
$\langle P_{n>4}(\cos\theta)\rangle = 0$

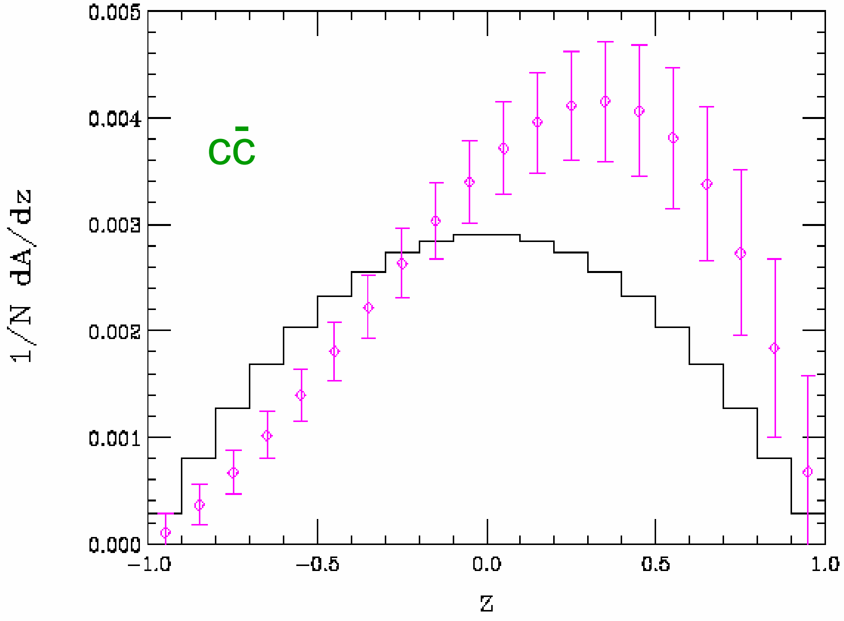
Fit to simulated $e^+e^- \to f\bar{f}$ data: Rizzo

5σ ID of spin-2 for $m_D \lesssim (5\text{-}6)\sqrt{s}$

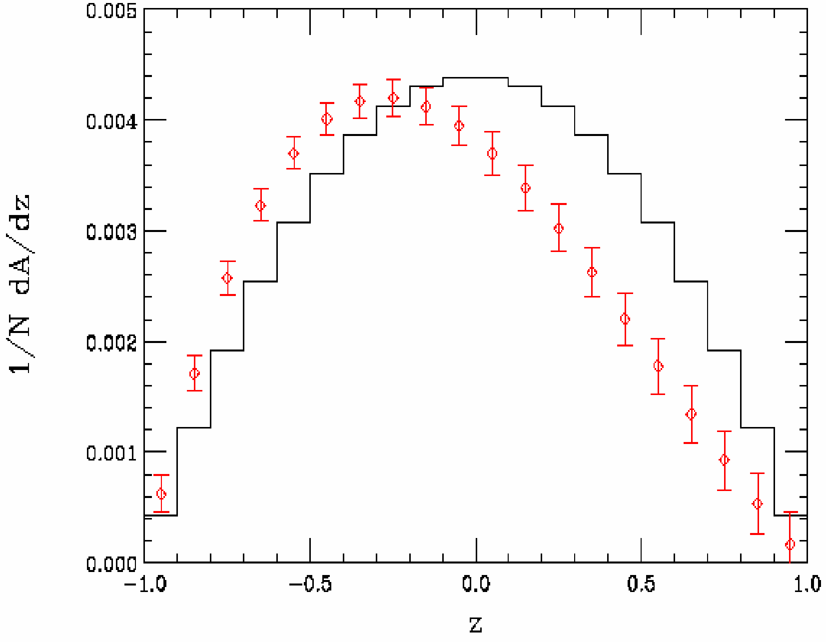
Issue: How to determine spin of exchanged particle?

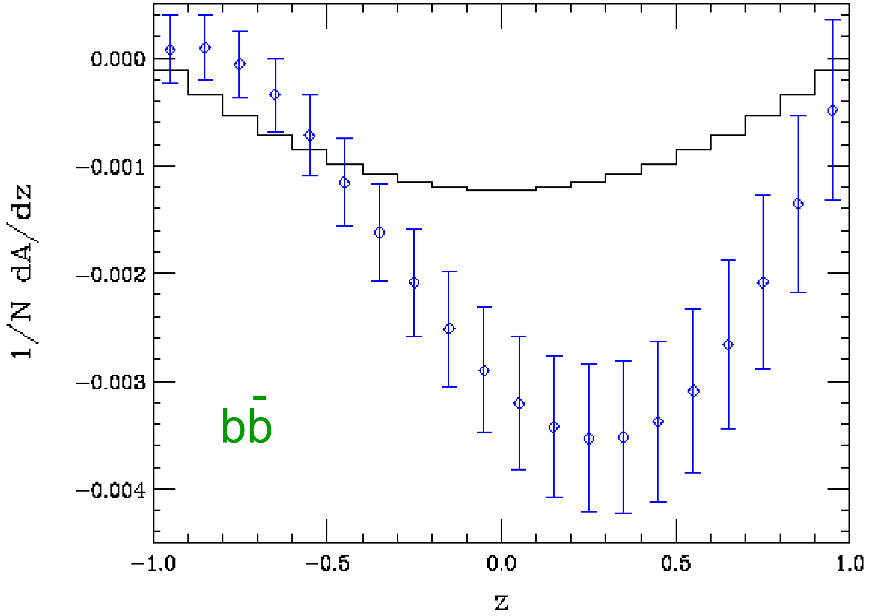
Transverse Polarization Asymmetry

Requires positron pol



$e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$

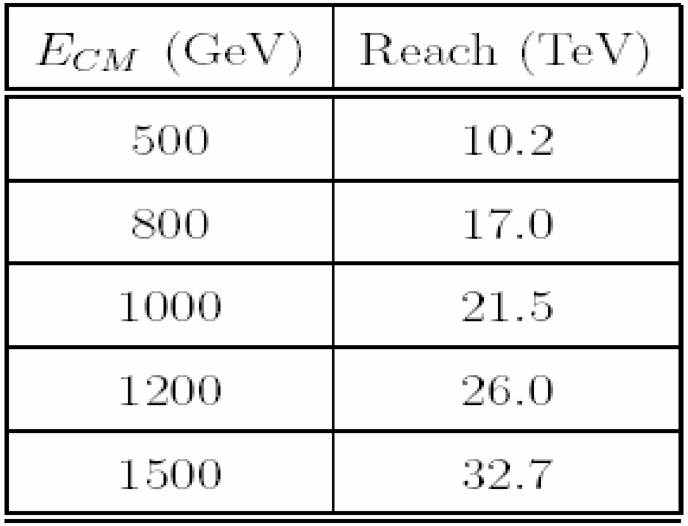




Rizzo

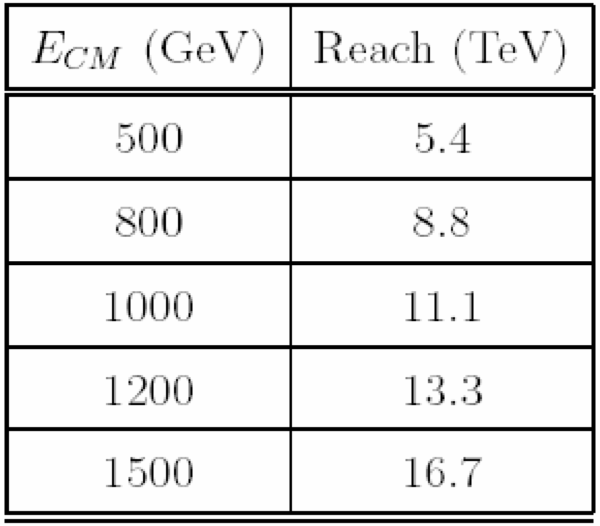
With Transverse Polarization:

Search Reach



E_{CM} (GeV)	Reach (TeV)
500	10.2
800	17.0
1000	21.5
1200	26.0
1500	32.7

ID Reach



E_{CM} (GeV)	Reach (TeV)
500	5.4
800	8.8
1000	11.1
1200	13.3
1500	16.7

Rizzo

Graviton Tower Emission

Giudice, Ratazzi,Wells
Mirabelli,Perelstein,Peskin

- $e^+e^- \to \gamma/Z + G_n$
- $q\bar{q} \to g + G_n$
- $Z \to f\bar{f} + G_n$

G_n appears as missing energy

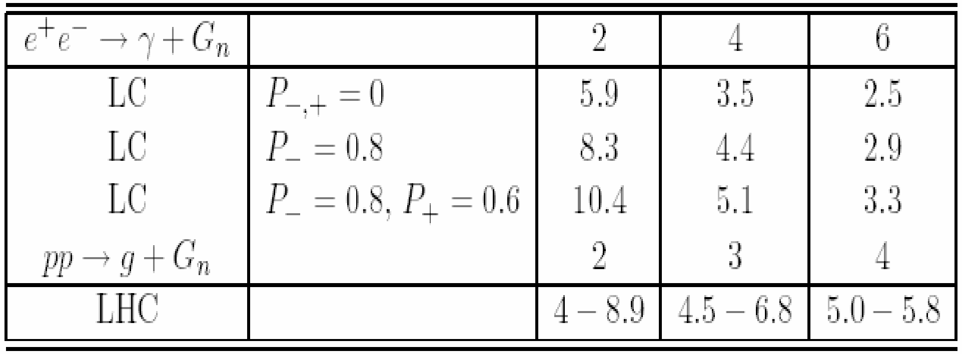
Model independent – Probes M_D directly

Sensitive to δ

Parameterized by density of states:

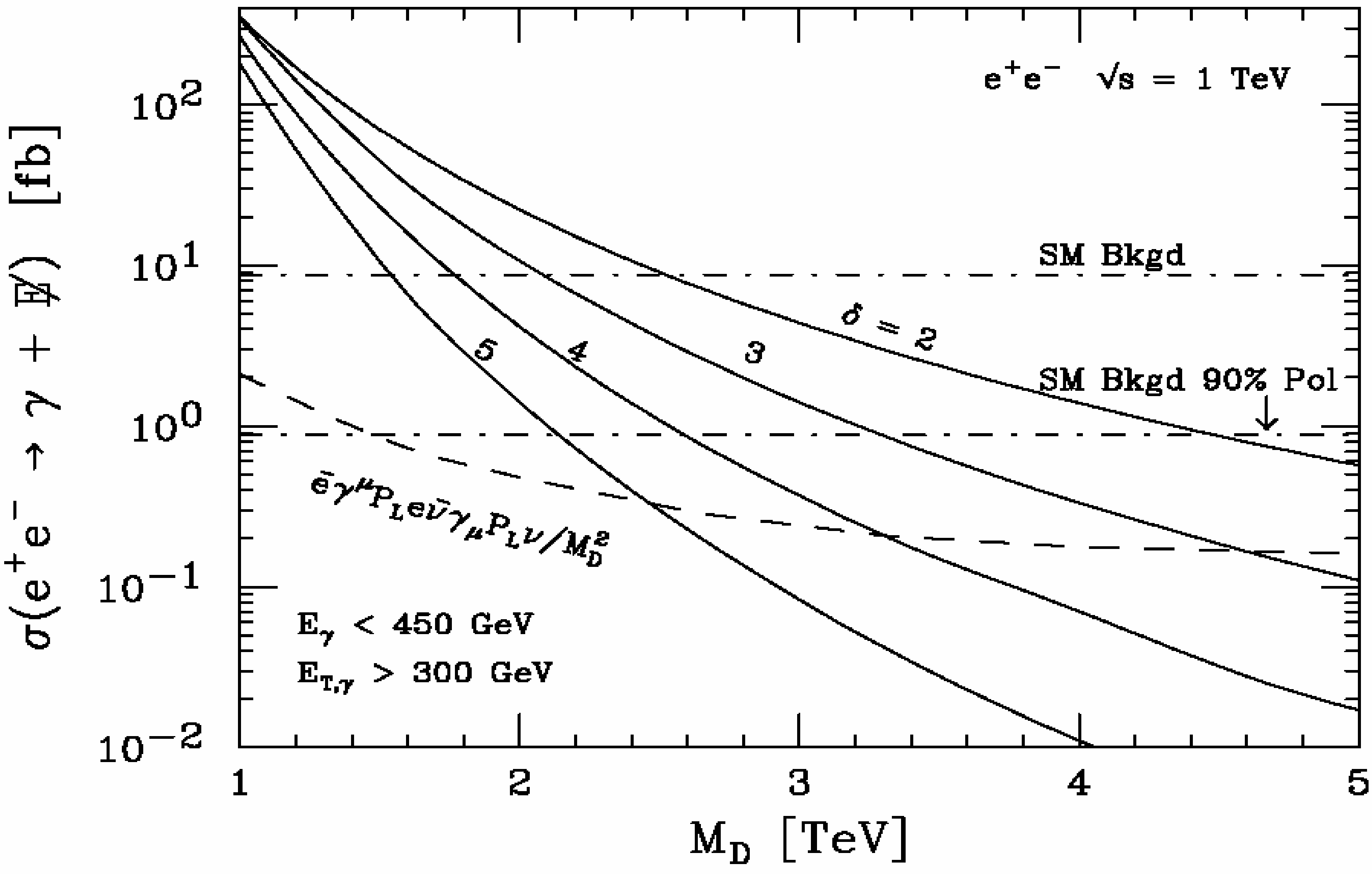
$$\sigma \sim \frac{1}{m_{Pl}^2}\,(E\,R_c)^\delta \longrightarrow \frac{1}{m_D^2}\left(\frac{E}{m_D}\right)^\delta$$

Discovery reach for M_D (TeV):



$e^+e^- \to \gamma + G_n$		2	4	6
LC	$P_{-,+} = 0$	5.9	3.5	2.5
LC	$P_- = 0.8$	8.3	4.4	2.9
LC	$P_- = 0.8,\ P_+ = 0.6$	10.4	5.1	3.3
$pp \to g + G_n$		2	3	4
LHC		$4 - 8.9$	$4.5 - 6.8$	$5.0 - 5.8$

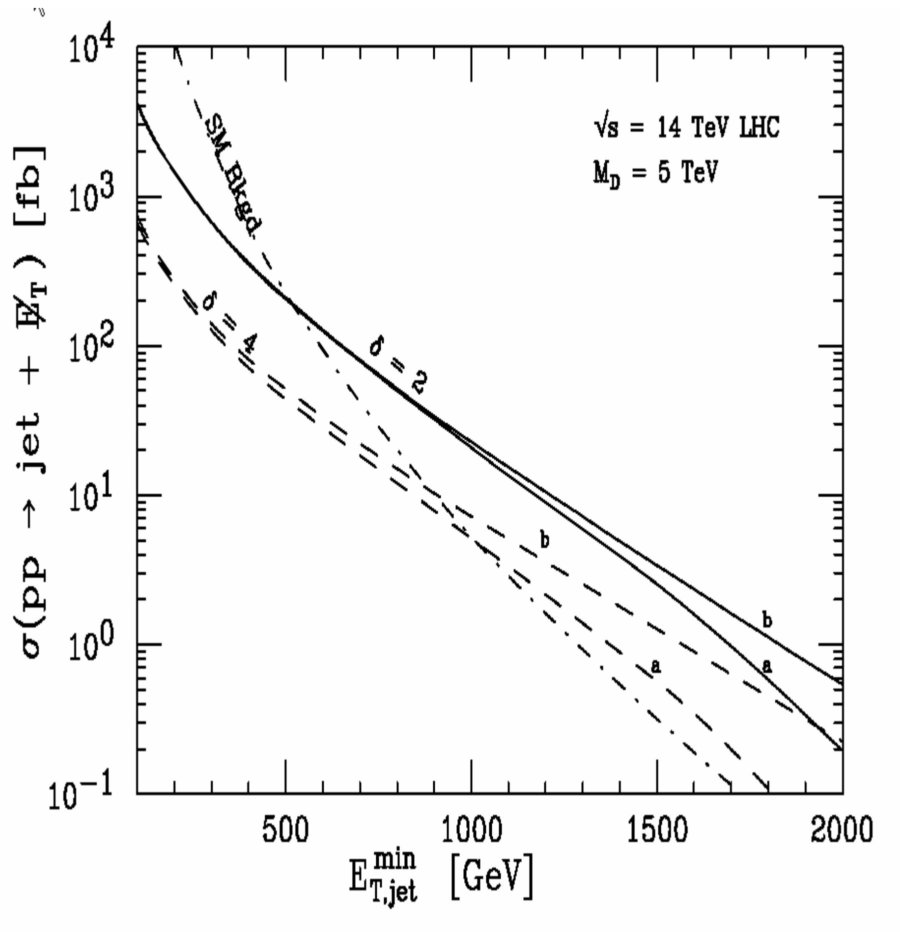
Graviton Emission

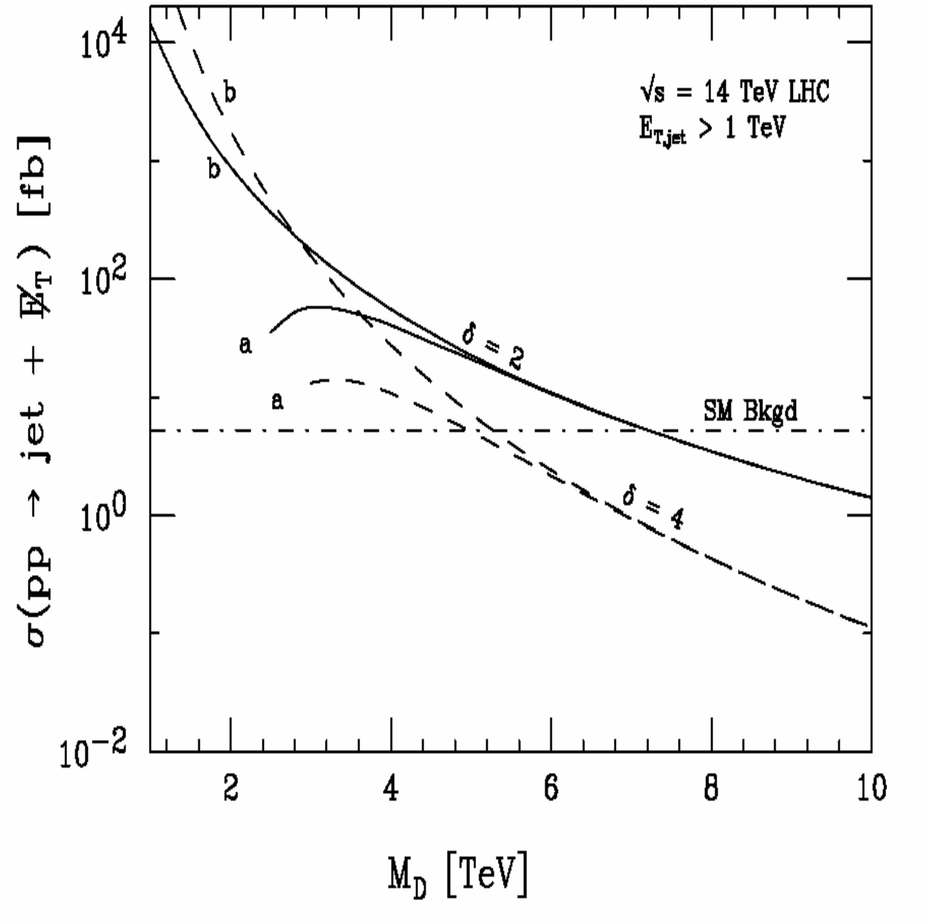


Giudice, Rattazzi, Wells

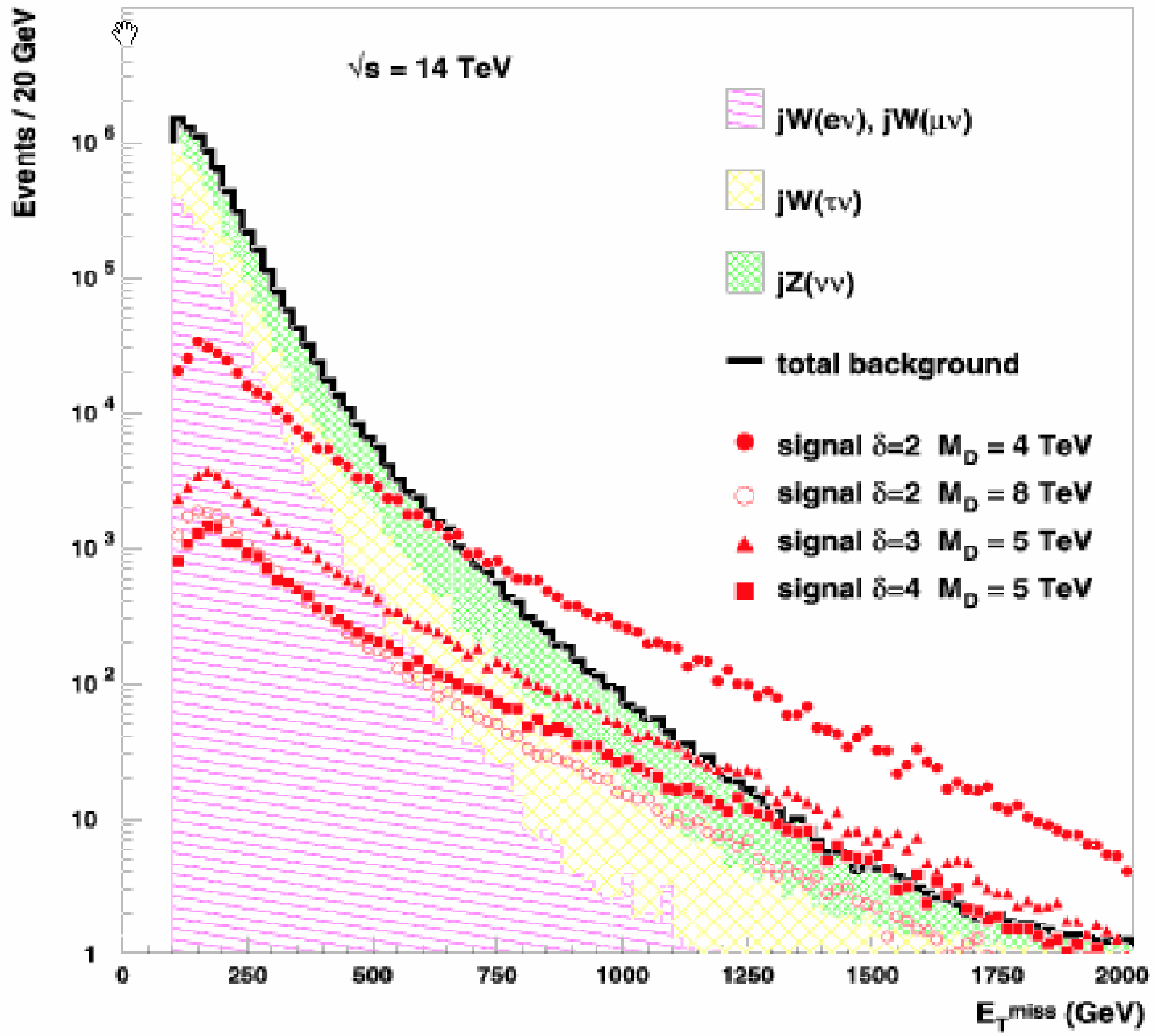
Graviton Emission @ Hadron Colliders

Giudice, Rattazzi, Wells





ATLAS Simulation

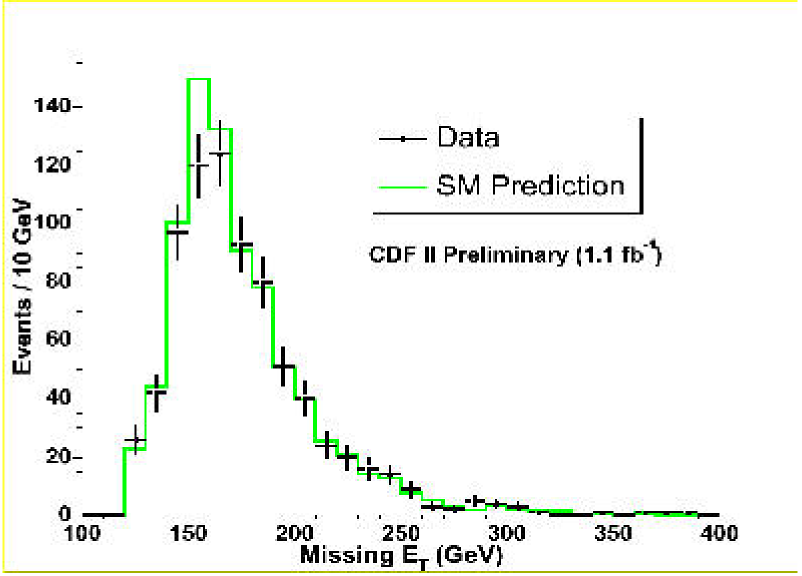


Hinchliffe,
Vacavant

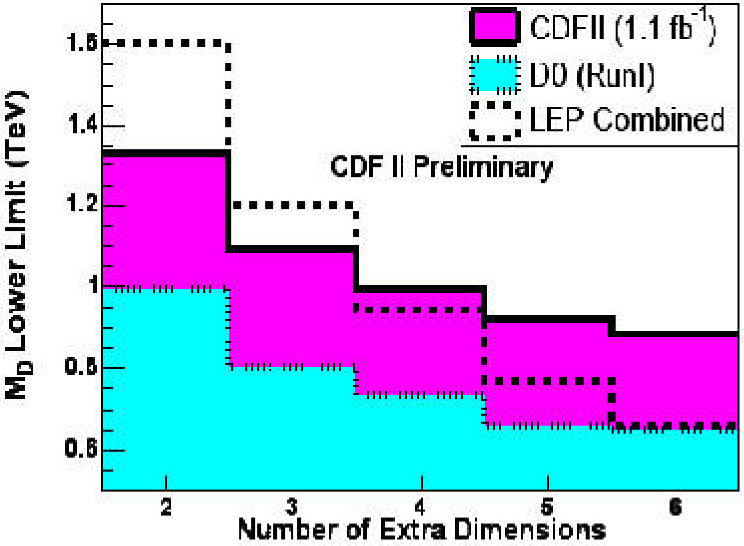
Current Bounds on Graviton Emission

Recent CDF analysis from Run II 1.1 fb^{-1} of data

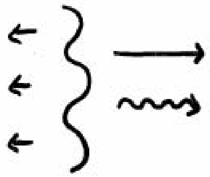
Missing Energy Distribution from monojet data



Summary of Limits



Branes gone soft: Graviton Emission $e^+e^- \to \gamma G_n$



soft brane
can recoil

Suppresses KK tower
Couplings

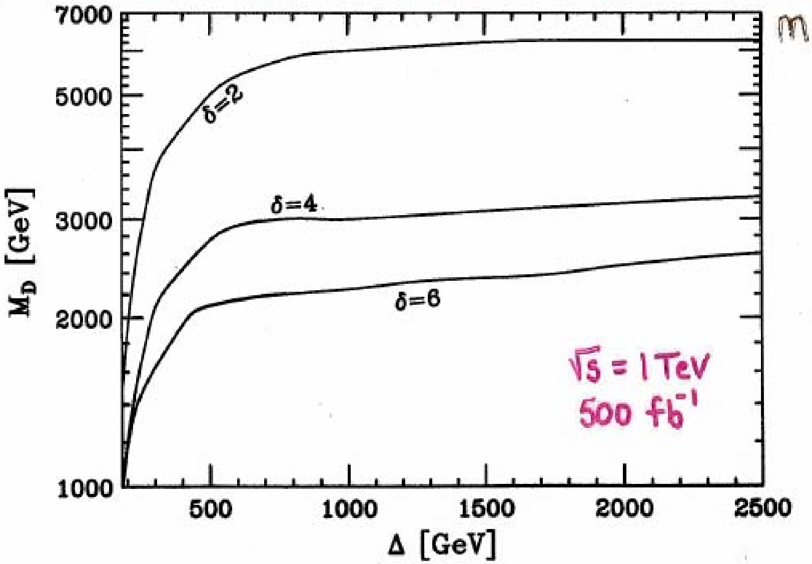
$$g_n^2 \to g_n^2 \, e^{-m_n^2/\Delta^2}$$

$\Delta \sim \sqrt{T}$ wall tension
$\lesssim m_D$

Search Reach is reduced

$$\left.\frac{d^2\sigma}{dx_\gamma \, d\cos\theta}\right|_{\rm soft} \to \left.\frac{d^2\sigma}{dx_\gamma \, d\cos\theta}\right|_{\rm stiff} e^{-s(1-x_\gamma)/\Delta^2}$$

Reach in $e^+e^- \to \gamma G_n \to \gamma \not{E}_T$

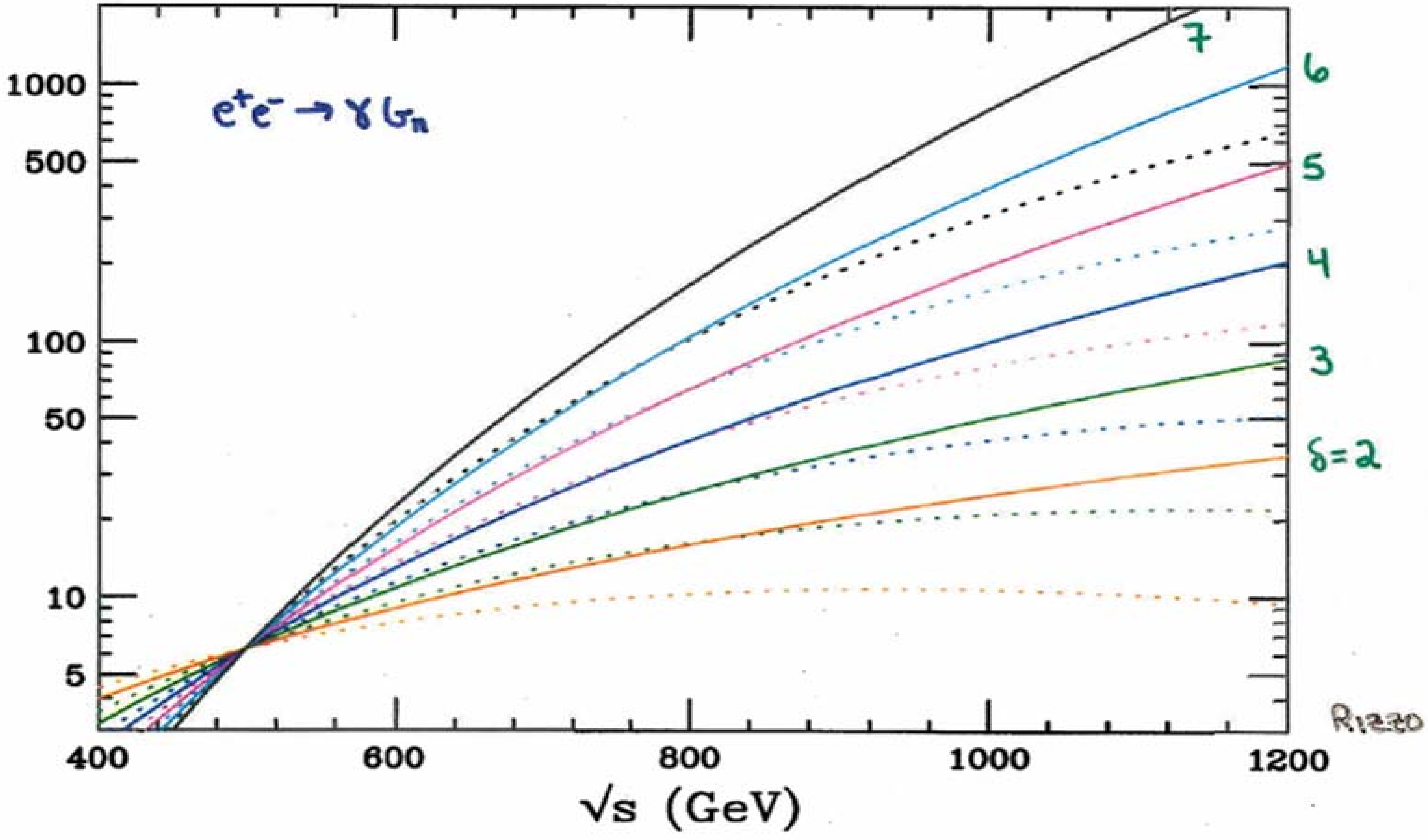


Murayama
Wells

⇐ soft brane   stiff brane ⇒

Determination of δ and M_D

solid: rigid brane
dashed: flexible brane



Normalized to $M_D = 5$ TeV, $\delta = 2$ at $\sqrt{s} = 500$ GeV

Supersymmetric Bulk

JLH, Sadri
SLAC-PUB-8782

Motivation : Embed ADD in string theory

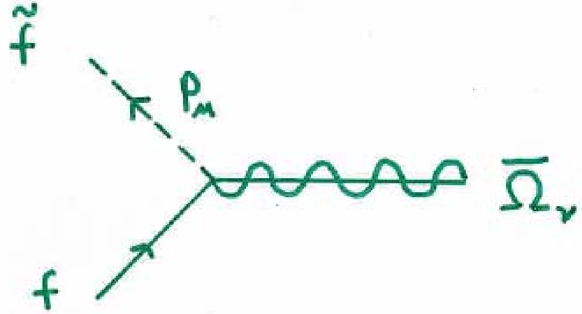
Stabilize size of dimensions

Bulk SUSY breaking

N=2 SUSY in bulk breaks to N=1 SUSY on brane

Full supermultiplet in bulk

Compactify ⇒ KK towers of gravitons AND gravitinos!



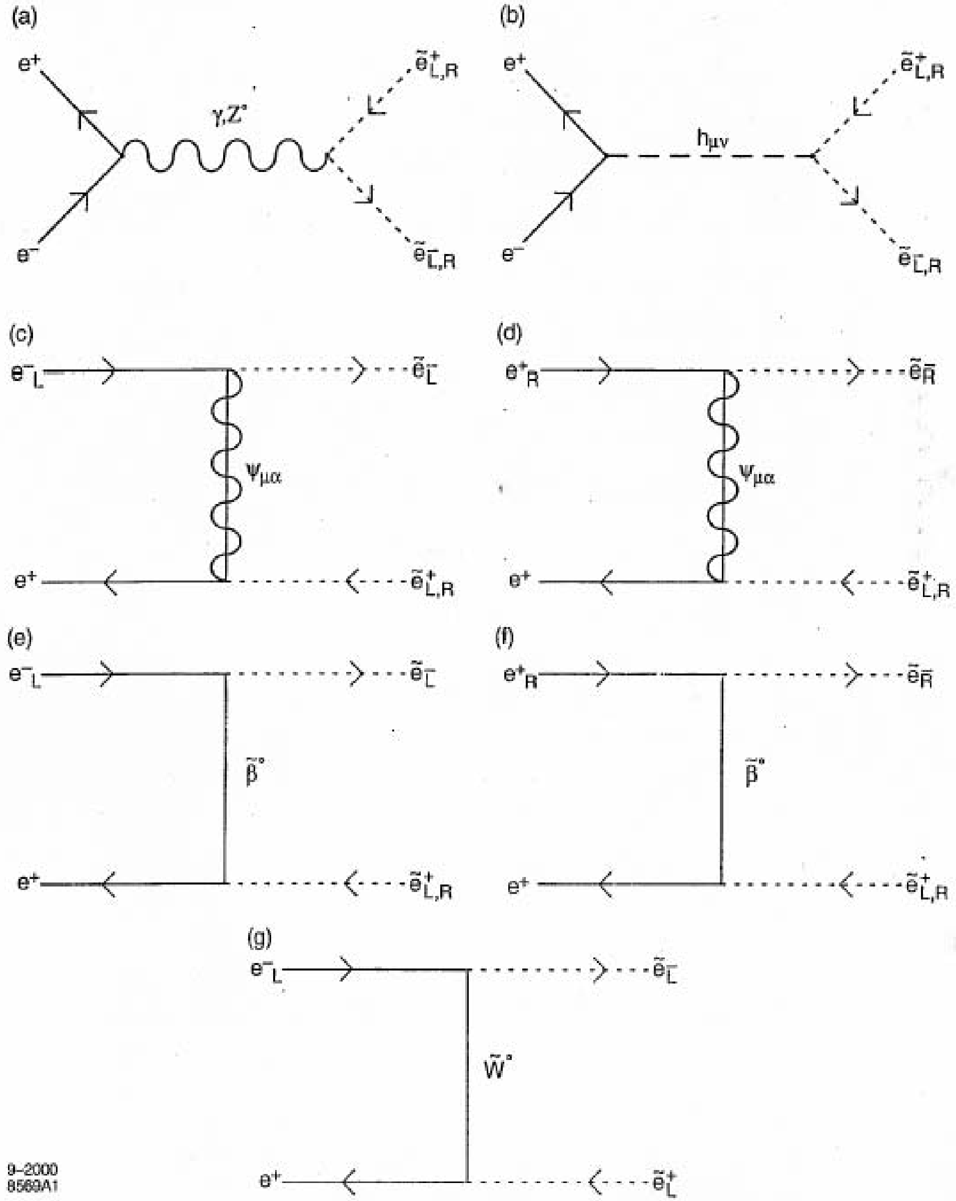
$$\frac{-i}{\sqrt{2}\,\overline{M}_{Pl}} p_\mu \gamma^\mu \gamma^\nu$$

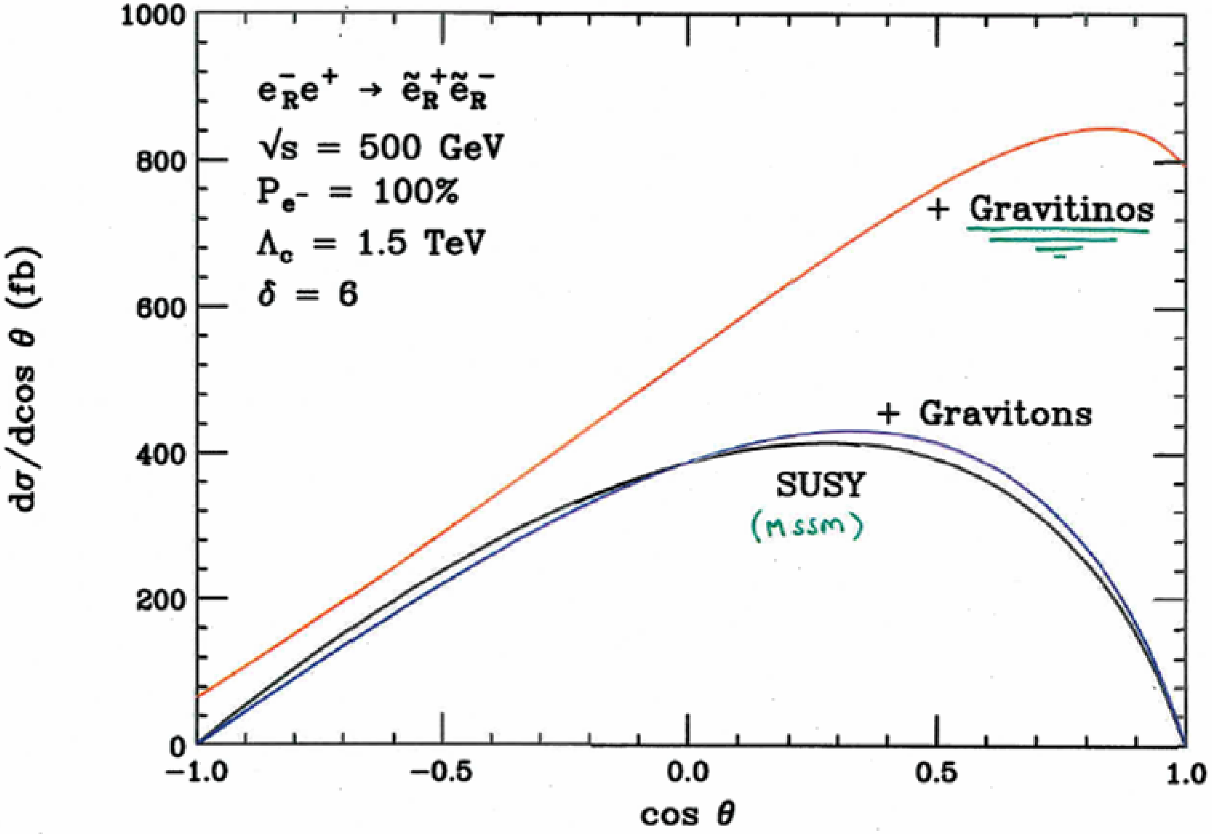
KK Gravitino Exchange : t-channel

$$\sum_{\vec{n}} P^{\vec{n},\mu\nu} \sim -i \int_{m_0^2}^{\Lambda_c^2} dm_{\vec{n}}^2 \, \rho(m_{\vec{n}}^2) \frac{|m_{\vec{n}}|^{\alpha-2}}{t - m_{\vec{n}}^2}$$

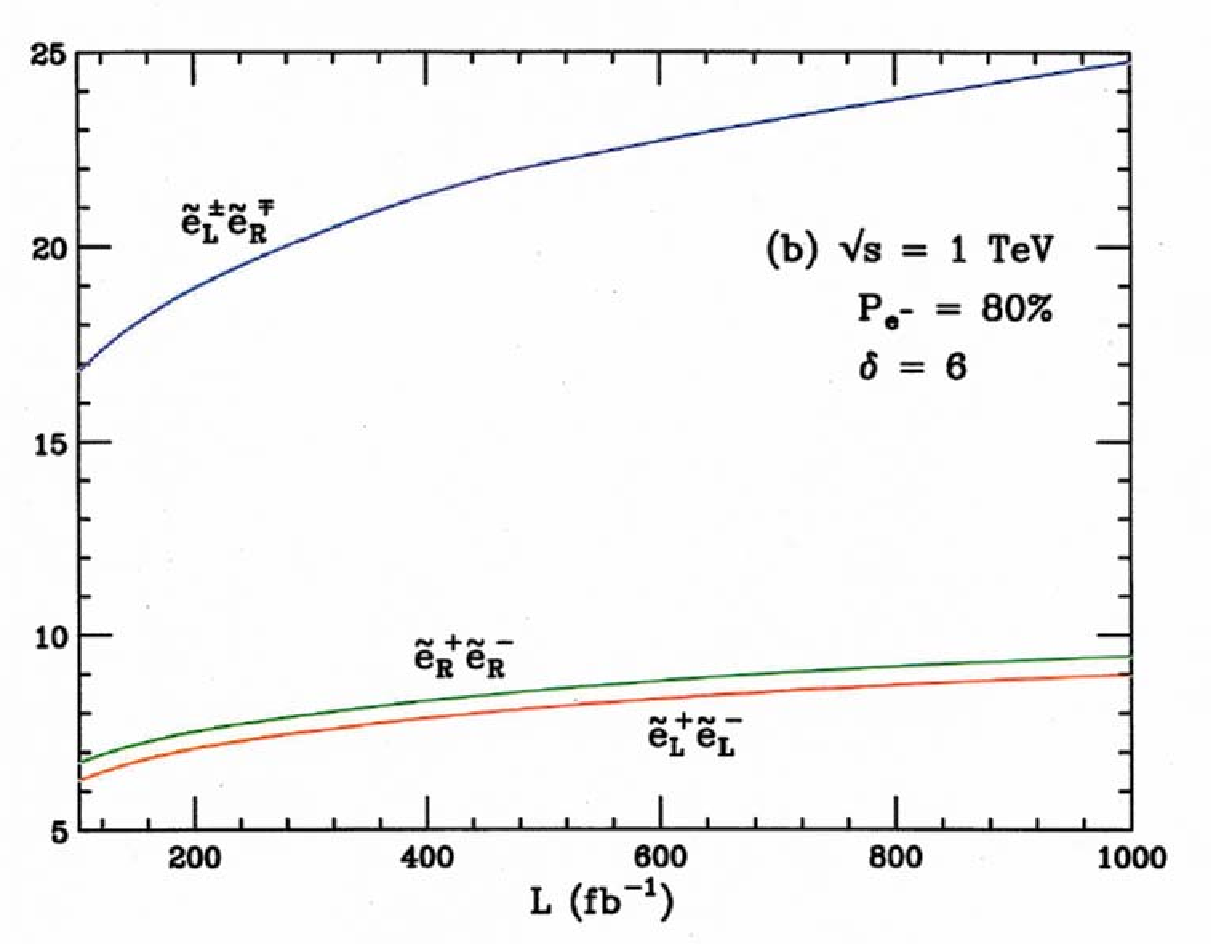
$\alpha = 0,1,2,3$

Contributions to $e^+e^- \to \tilde{e}^+_{L,R}\,\tilde{e}^-_{L,R}$




$e_R^- e^+ \to \tilde{e}_R^+ \tilde{e}_R^-$
$\sqrt{s}$ = 500 GeV
P_{e^-} = 100%
Λ_c = 1.5 TeV
δ = 6
+ Gravitinos
+ Gravitons
SUSY
(MSSM)
$d\sigma/d\cos\theta$ (fb)
cos θ
1000
800
600
400
200
0
−1.0
−0.5
0.0
0.5
1.0

Search reach for M_D

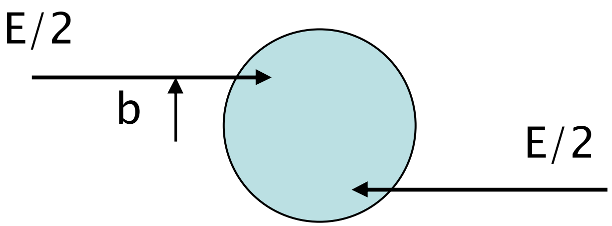


Black Hole Production @ LHC:

Dimopoulos, Landsberg
Giddings, Thomas

Black Holes produced when $\sqrt{s} > M_D$

Classical Approximation: [space curvature << E]



$b < R_s(E) \Rightarrow$ BH forms

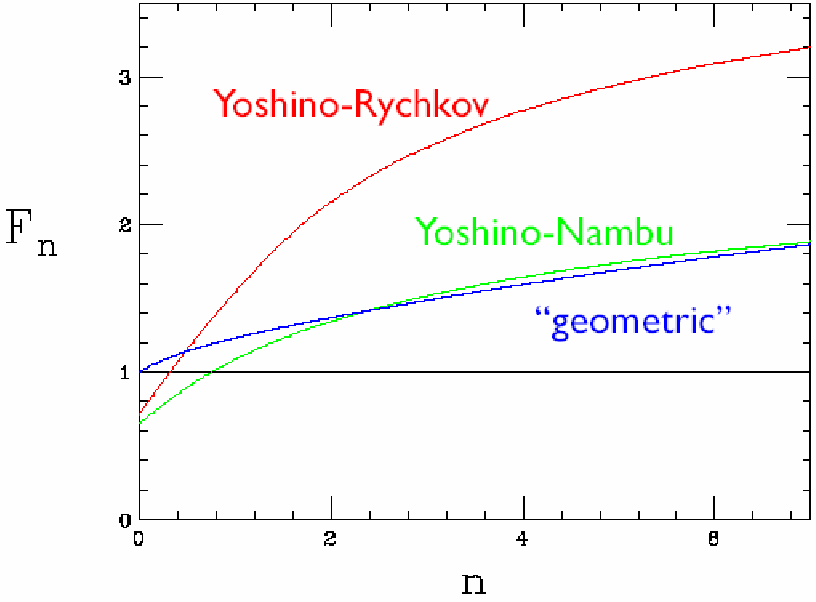
$$M_* R_s = \left[\frac{\Gamma(\frac{n+3}{2})}{(n+2)\pi^{(n+3)/2}} \frac{M_{BH}}{M_*} \right]^{1/(n+1)}$$

$M_{BH} \sim \sqrt{\hat{s}}$

Geometric Considerations:

$\sigma_{\text{Naïve}} = F_n \pi R_s^2(E)$, details show this holds up to a factor of a few

Blackhole Formation Factor



- H Yoshino & Y Nambu, gr-qc/0209003
- H Yoshino & VS Rychkov, hep-th/0503171

Potential Corrections to Classical Approx:

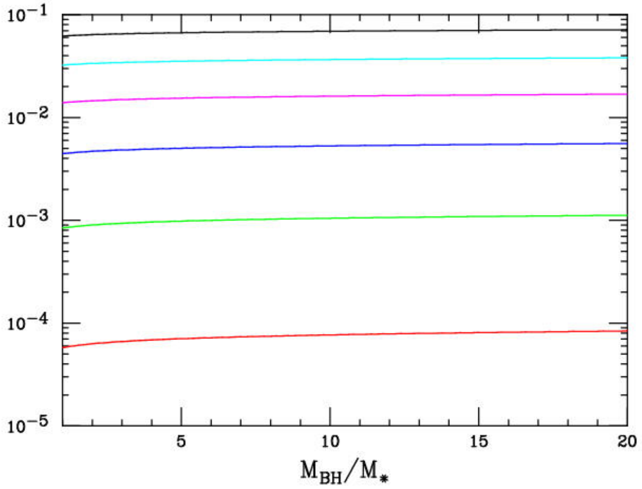
1. Distortions from

   finite R_c as $R_s \to R_c$

   Critical point for instabilities for n=5: $(R_s/R_c)^2 \sim 0.1$ @ LHC

$R_S{}^2/(2\pi R_c)^2$

n = 15 – 40



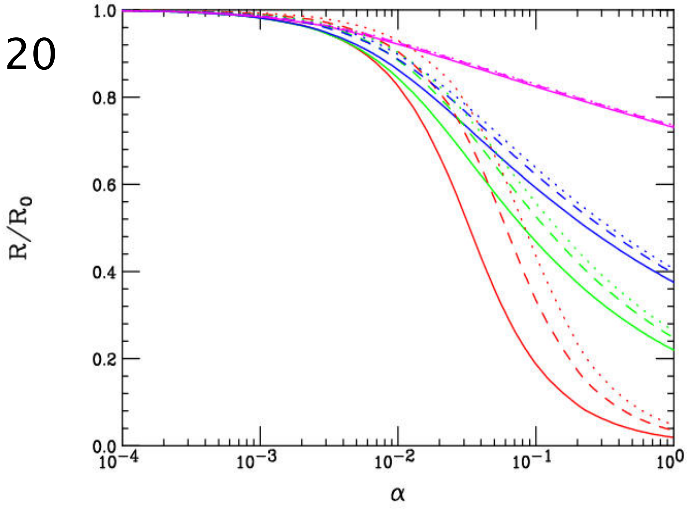
2. Quantum Gravity Effects

   Higher curvature term corrections

$$S = \frac{M_*^{D-2}}{2} \int d^D x \left(R + \frac{\alpha_1}{M_*^2} \mathcal{L}_2 + \frac{\alpha_2}{M_*^4} \mathcal{L}_3 + \ldots \right)$$

Gauss–Bonnet term

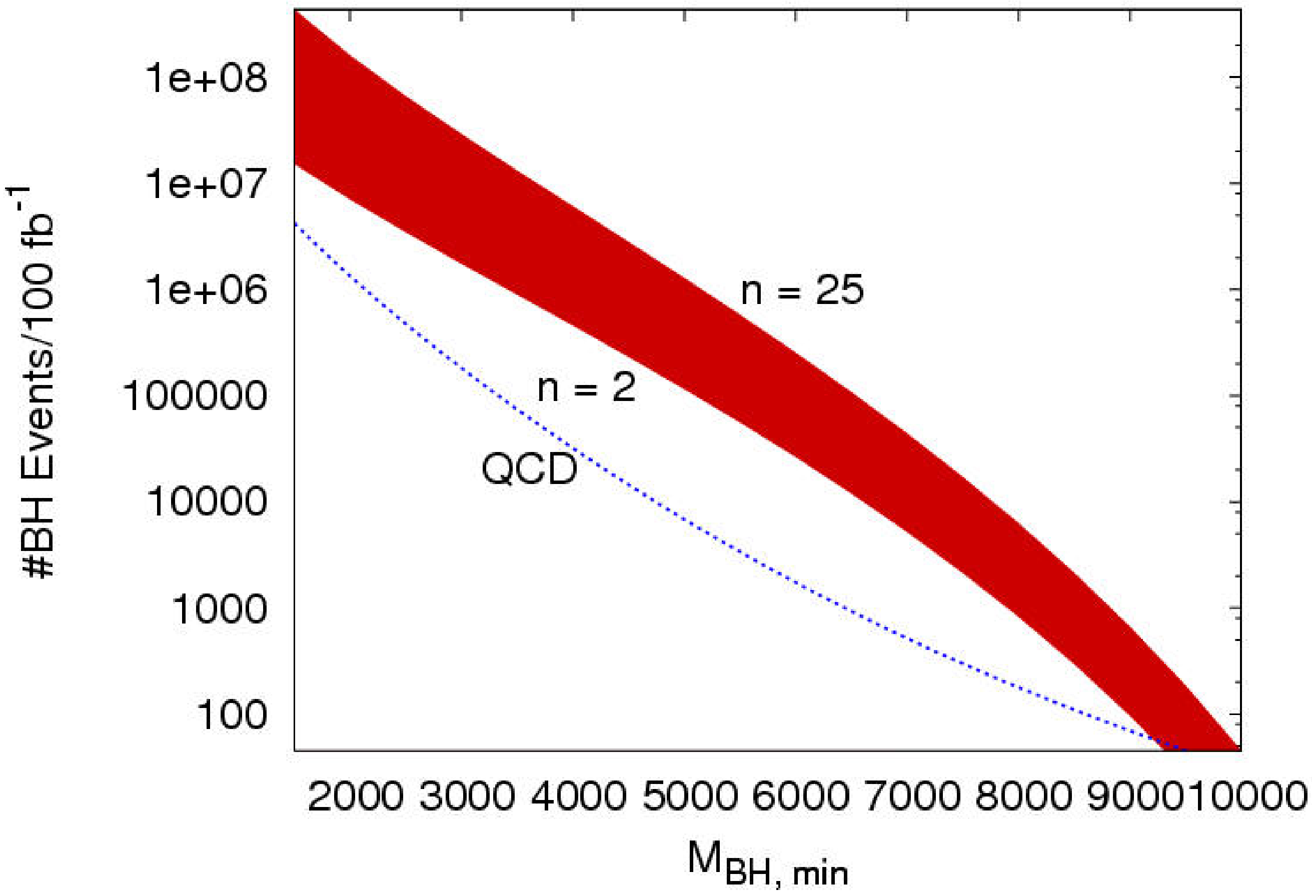
n = 2 – 20



$\alpha n^2 \leq 1$ in string models

Production rate is enormous!

$\sigma_{\text{Naïve}} \sim n$ for large n

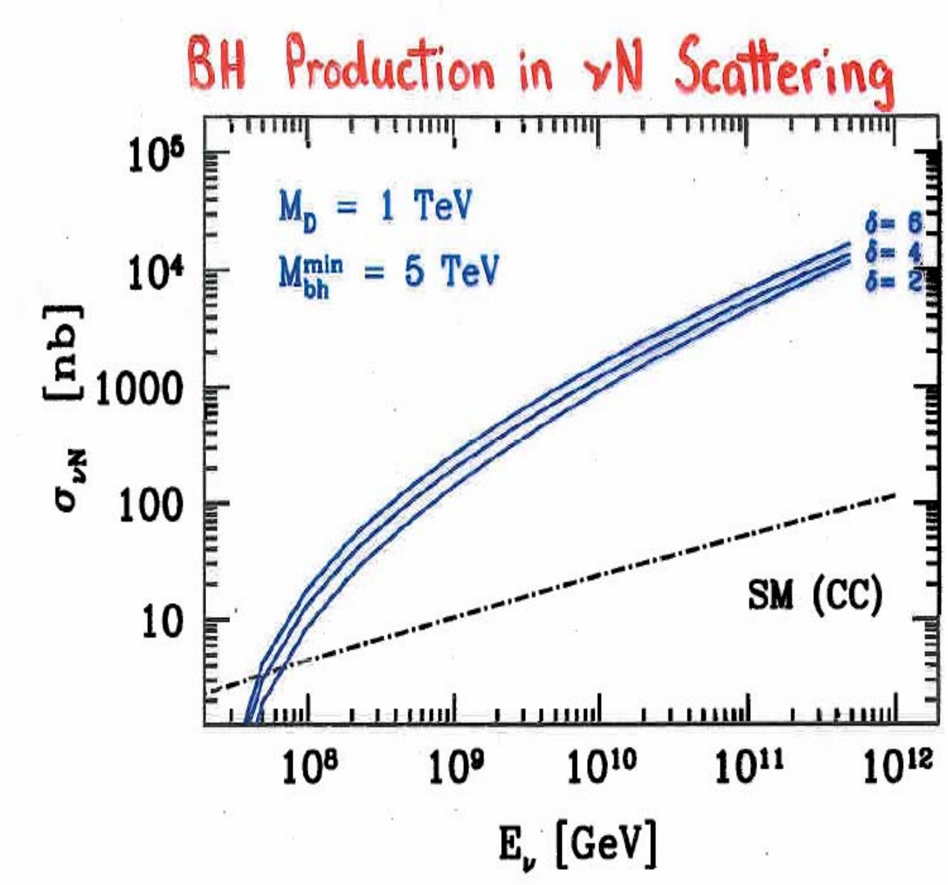


1 per sec at LHC!

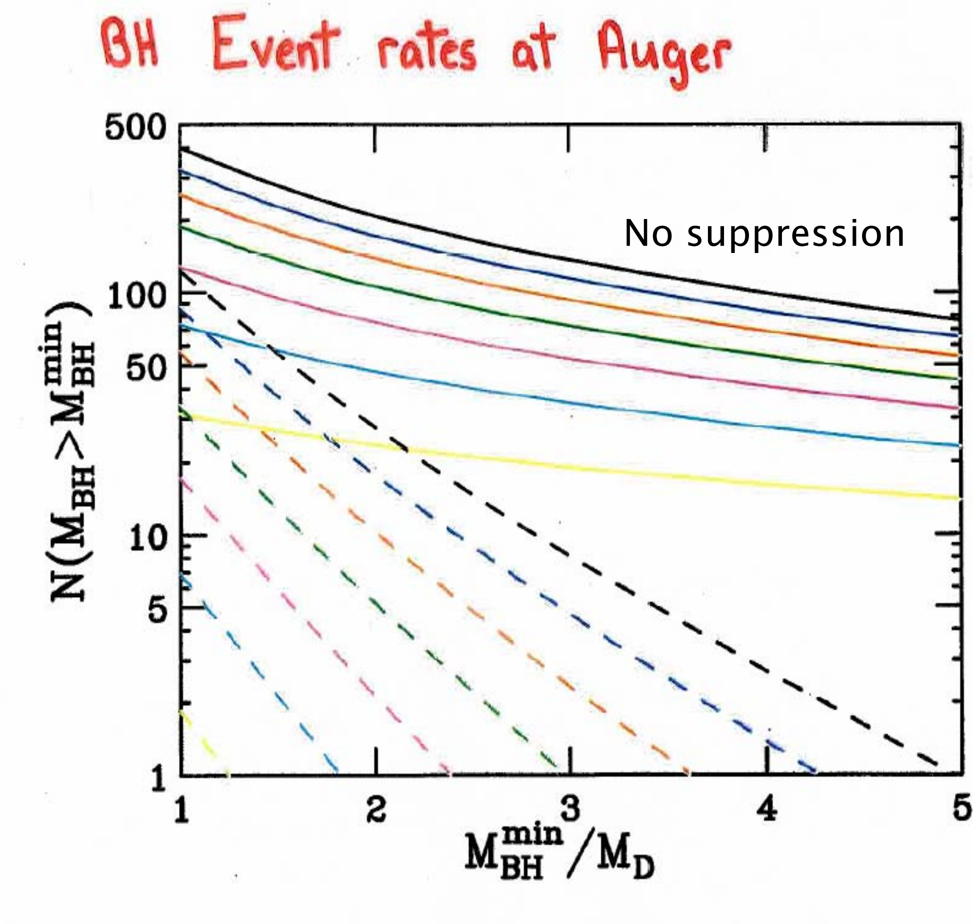
M_D = 1.5 TeV

JLH, Lillie, Rizzo

Cosmic Ray Sensitivity to Black Hole Production

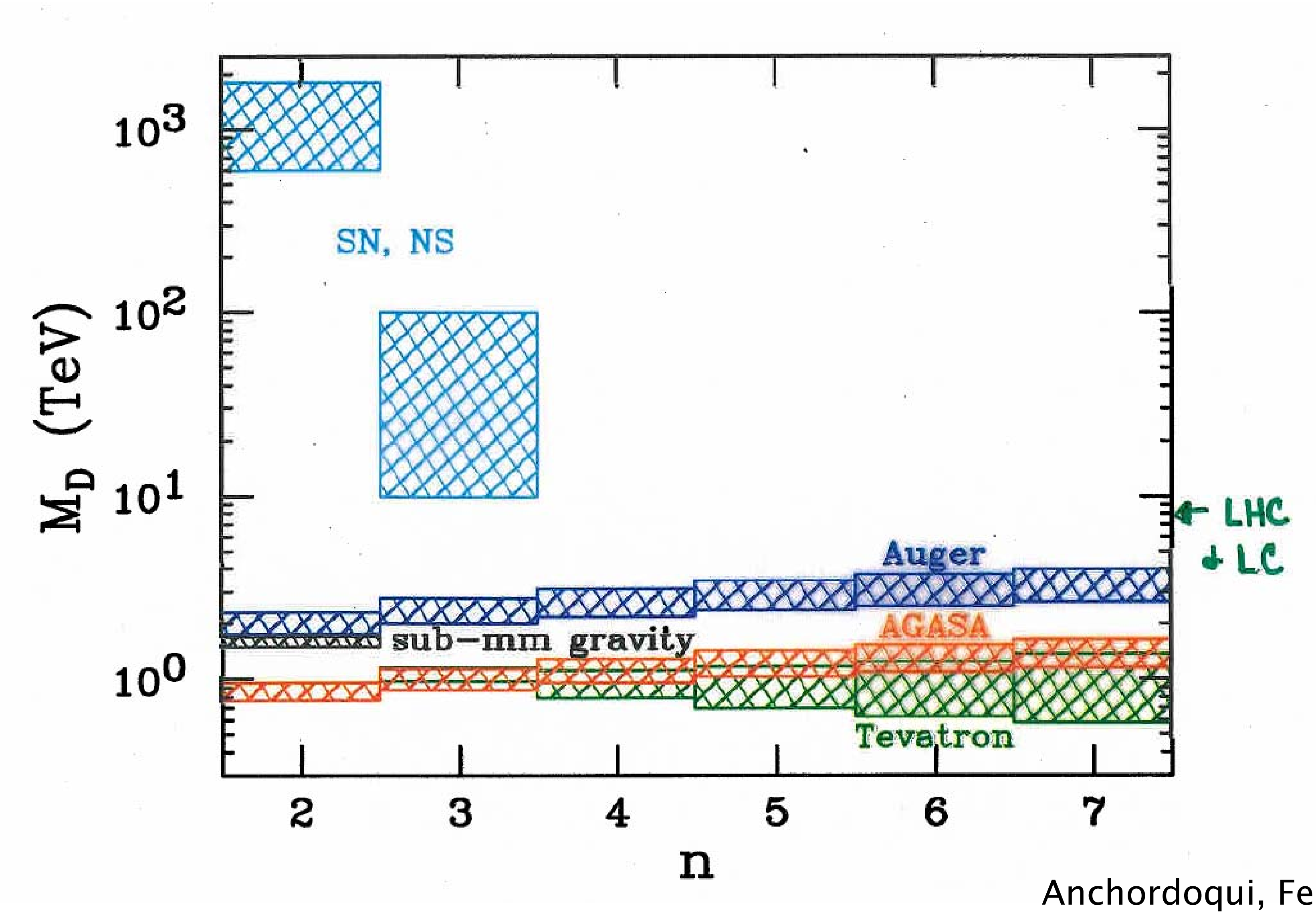


Ringwald, Tu



Anchordoqui etal

Summary of Exp’t Constraints on M_D



Anchordoqui, Feng
Goldberg, Shapere

Black Hole Decay

- Balding phase: loses ‘hair’ and multiple moments by gravitational radiation
- Spin-down phase: loses angular momentum by Hawking radiation
- Schwarzschild phase: loses mass by Hawking radiation
- Planck phase: mass & temperature reach M_D

Assume Schwarzschild phase is dominant
$\Rightarrow$ all types of SM particles emitted with Hawking spectrum

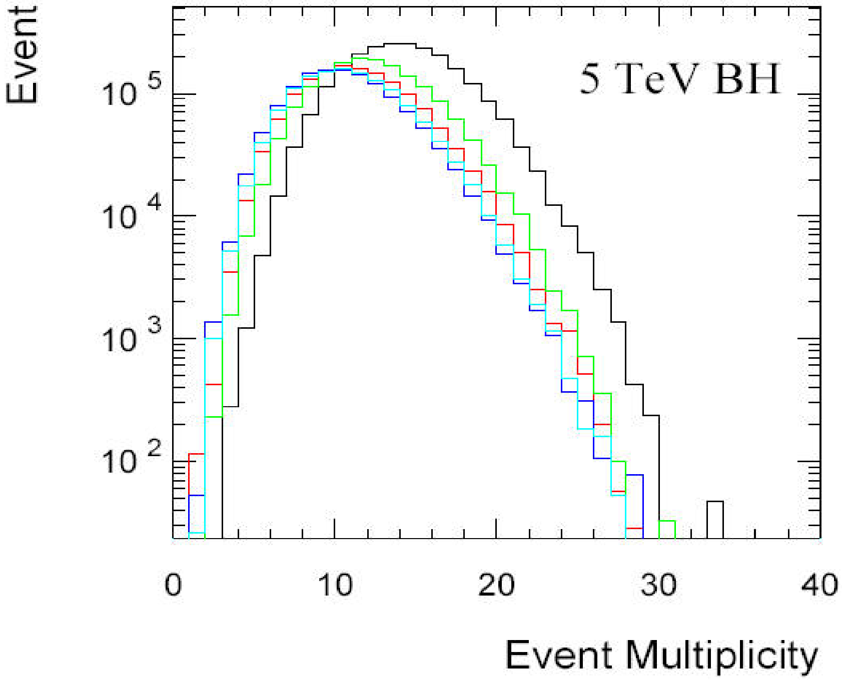
Decay Properties of Black Holes (after Balding):

Decay proceeds by thermal emission of Hawking radiation

$$T_H = \frac{(n+1)M_*}{4\pi}\left[\frac{\Gamma(\frac{n+3}{2})}{(n+2)\pi^{(n+3)/2}}\frac{M_{BH}}{M_*}\right]^{-1/(n+1)}$$

n determined to Δn = 0.75 @ 68% CL for n=2-6 from T_H and σ
This procedure doesn't work for large n

At fixed M_{BH}, higher dimensional BH's are hotter:



⟨N⟩ ~ 1/⟨T⟩

⇒ higher dimensional BH's emit fewer quanta, with each quanta having higher energy

Multiplicity for n = 2 to n = 6

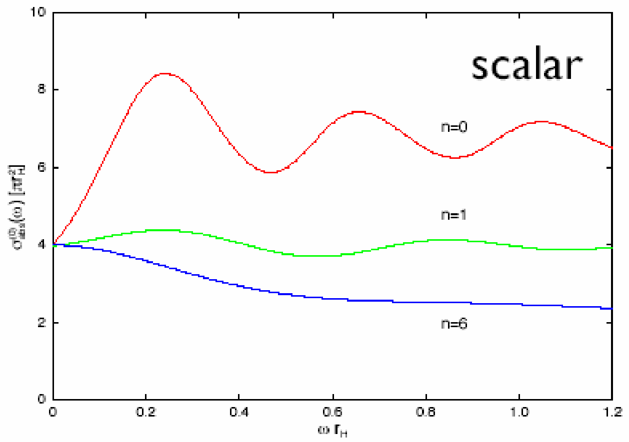
Harris etal hep-ph/0411022

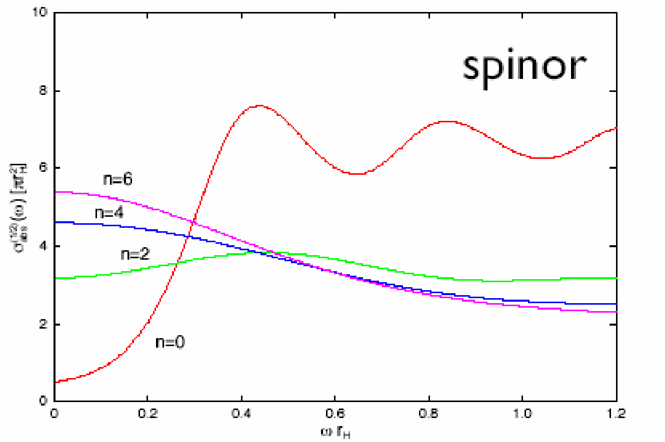
Grey–body Factors

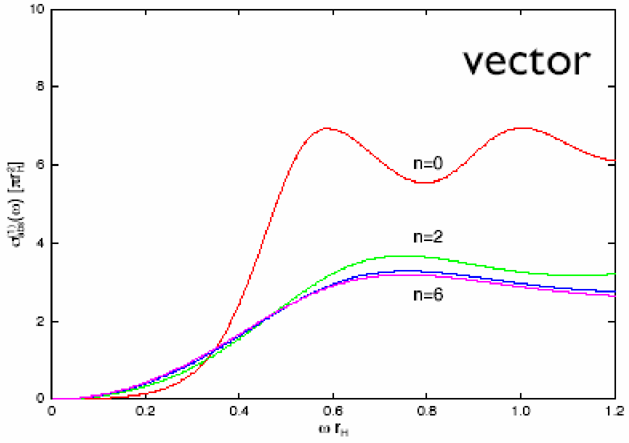
Particle multiplicity in decay:

$$\frac{dN}{dE} \propto \frac{\gamma E^2}{(e^{E/T_H} \mp 1)T_H^{n+6}}$$

γ = grey–body factor

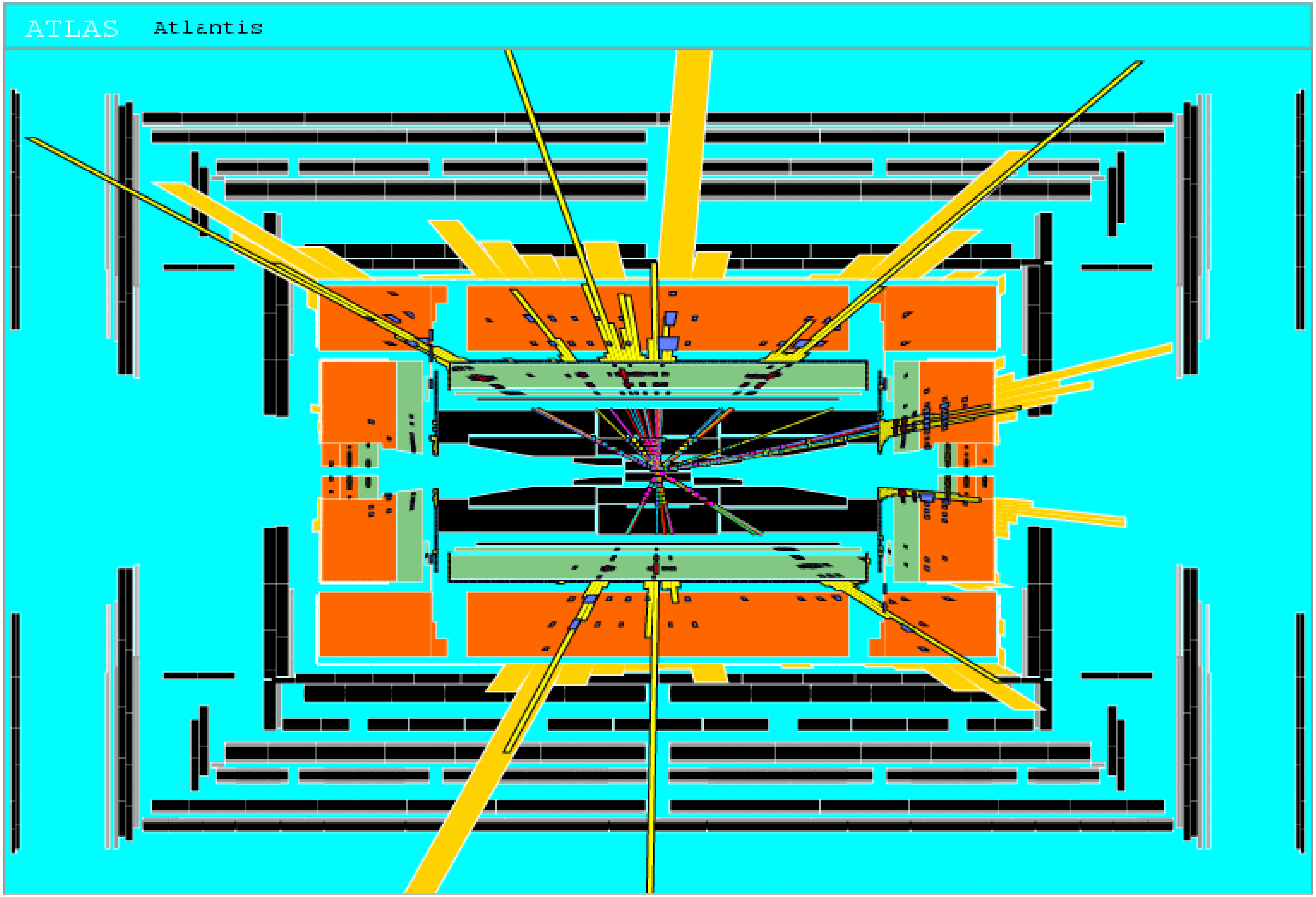






- Emission on brane only
- Low-energy vector suppression
- CM Harris, hep-ph/0502005

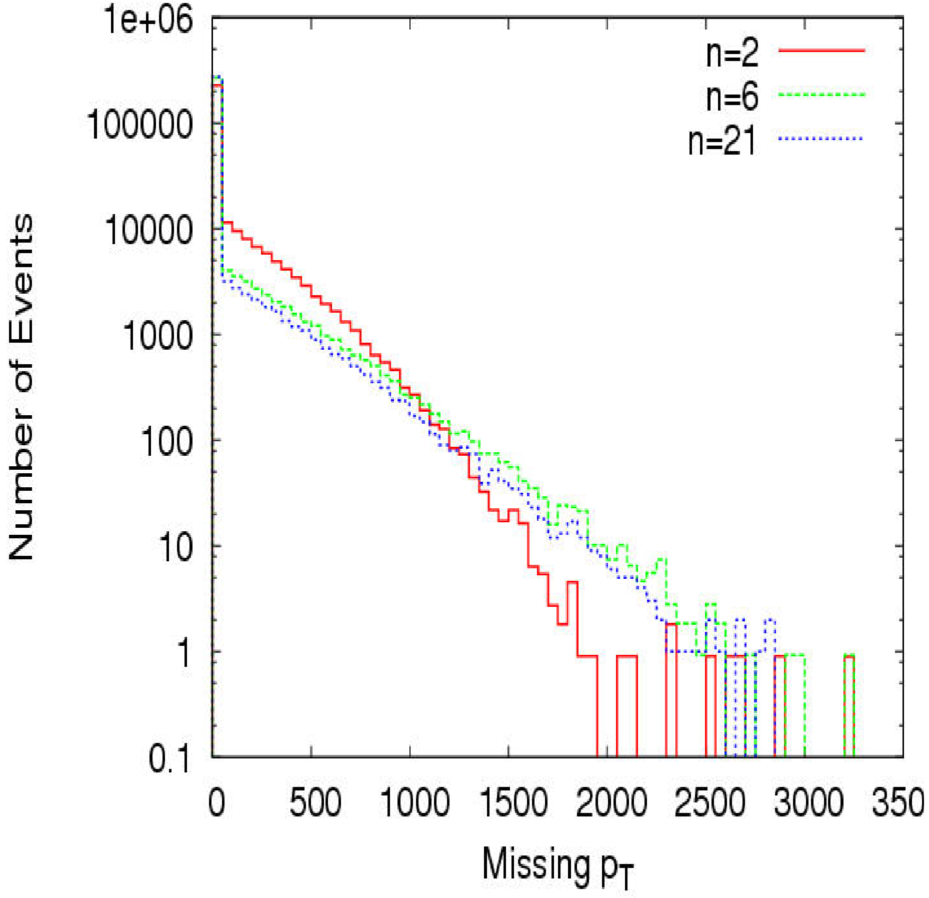
Black Hole event simulation @ LHC

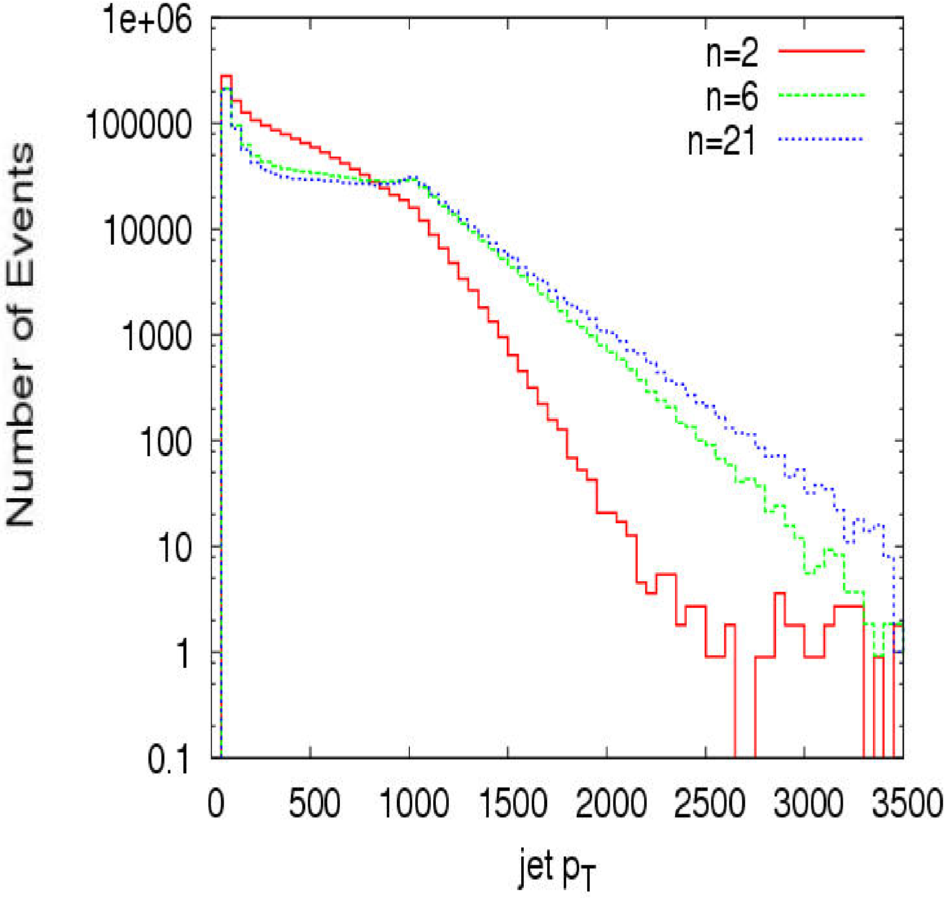


p_T distributions of Black Hole decays

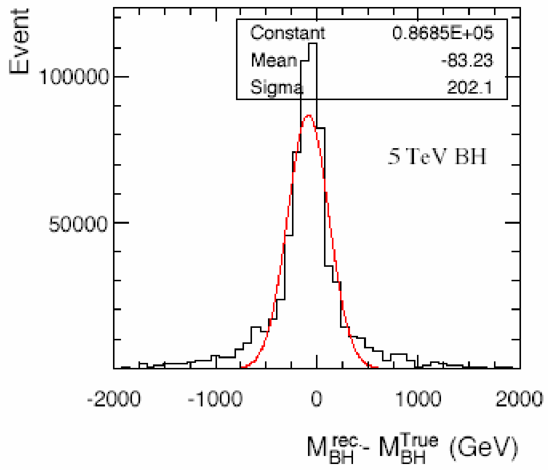
Provide good discriminating power for value of n

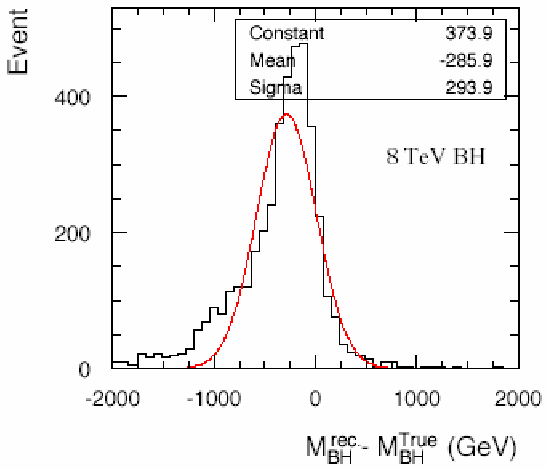
Generated using modified CHARYBDIS linked to PYTHIA
with $M_* = 1$ TeV





Measuring Black Hole Mass





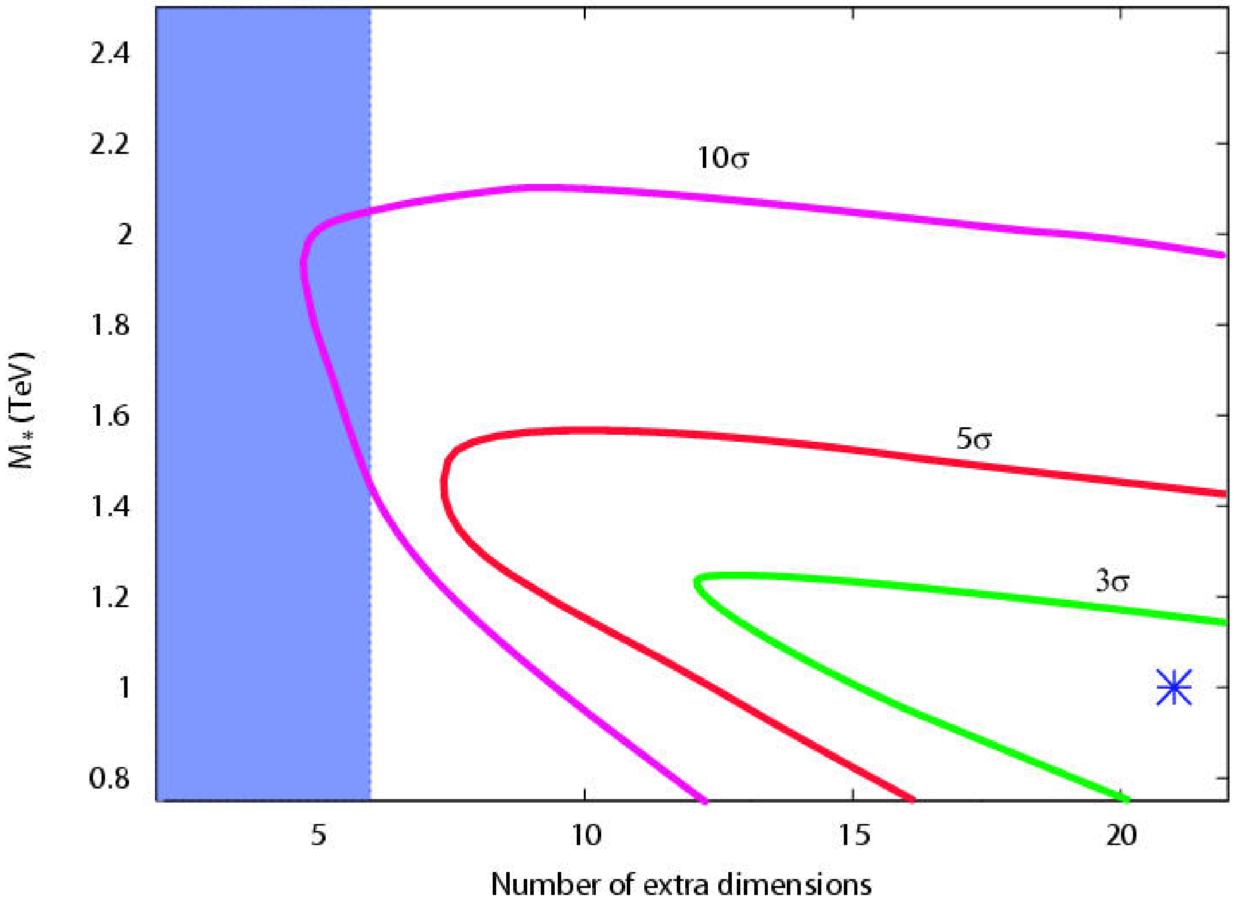
Need $\not{E}_T <$ 100 GeV for adequate resolution

➔ $\Delta M_{BH} / M_{BH} \sim 4\%$

Harris etal

Determination of Number of Large Extra Dims

Perform χ^2 fit assuming $M_* = 1$ TeV and $n = 21$
Generated 300k events (~ 10 fb^{-1})

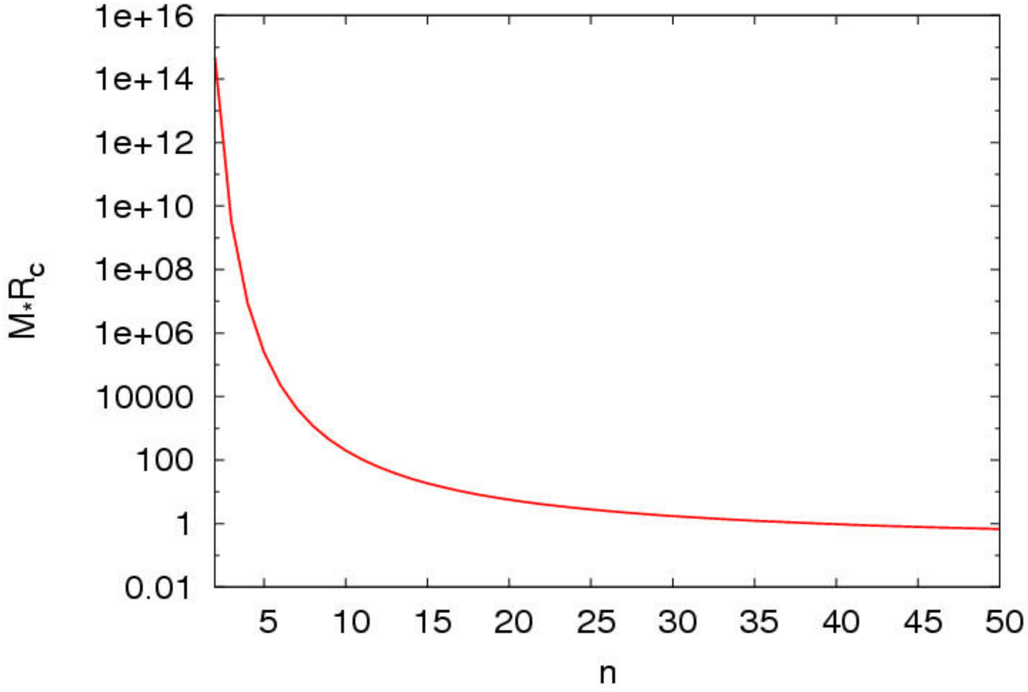


- Used p_T missing distb'n only
- Discrimination improves when jet p_T included as well
- $n < 6(7)$ excluded at 5σ for $n > 13$

Excellent resolution power for large values of n!

JLH, Lillie, Rizzo

LED: Is the hierarchy problem really solved?



$M_* R_c > 10^8$ for $n = 2–6$

Disparate values for gravity and EWK scales traded for disparate values of M_* and R_c

However,
$1 < M_* R_c < 10$ for
$n = 17 – 40$

Large n offers true solution to hierarchy!

Collider Signatures Change with large n

Graviton KK states are now ‘invisible’

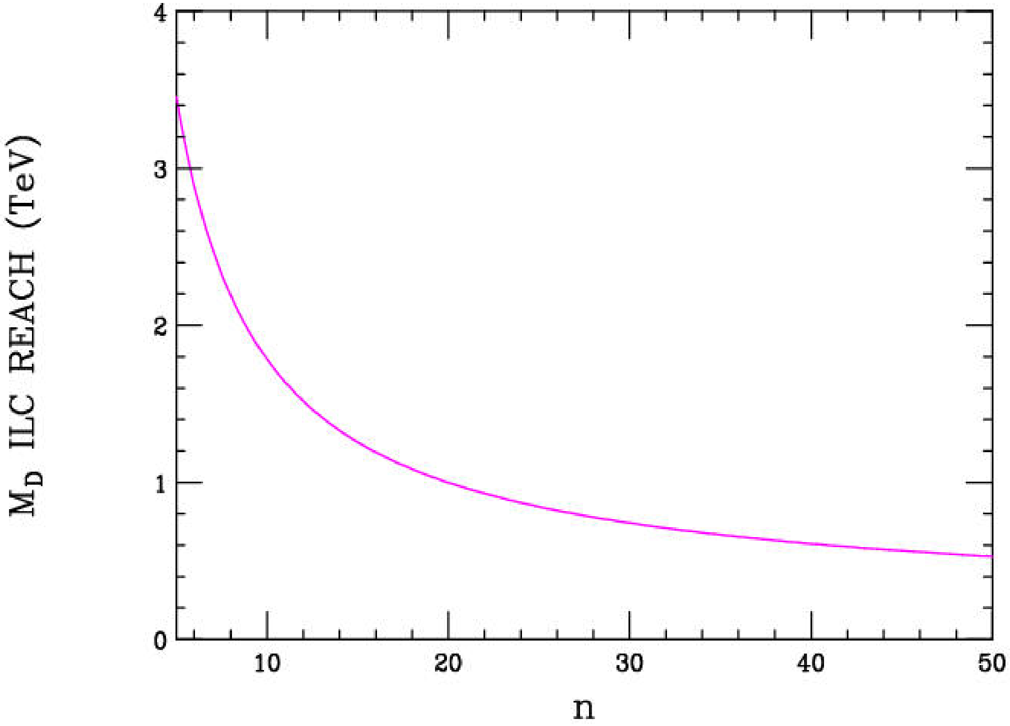
- $m_1 \sim$ TeV
- Couplings are still M_{Pl}^{-1}

Collider searches are highly degraded!

For n = 2, M_* up to 10 TeV observable at ILC, LHC

Drops to < 1 TeV for n = 20

Only viable collider signature is Black Hole production!



Questions you might ask about LED:

- Doesn’t string/M–theory fix $\delta = 6{,}7$?
- Aren’t there string–inspired models where SM gauge fields have KK excitations?
- Do all δ dimensions have to be the same size?

$m_{Pl}^2 = V_n m_*^{n+2}$ ⇒ in principle $V_n \sim R_1 R_2 \ldots R_n$

Let $R^n = R_1^p R_2^{n-p}$ with $R_1 \sim$ large

$R_2 \sim$ small $\sim 1/\text{TeV} \sim 1/m_*$

$$\Rightarrow m_{Pl}^2 = R_1^p m_*^{p-n} m_*^{n+2}$$

$$= R_1^p m_*^{p+2} \quad \text{with } 2 \leq p \leq 6$$

sm fields can propagate in small R_2^{n-p} dimensions

TeV^{-1}-size Extra Dimensions

Can arise naturally in string-inspired models

Antoniadis

The Standard Model goes into the bulk!

Model building choices:

- Gauge fields in the bulk
- Higgs in the bulk or on the brane?
- Fermions:
  - Located at orbifold fixed points
  - Localized to specific points inside the bulk (Split Fermions)
  - Freely propagate inside the bulk (Universal Extra Dimensions)

Interactions

$$S = \int d^4x\, dy \, \{ -1/4\, F_{AB} F^{AB}$$

Bulk Higgs + Fermions : $+ |D_A \phi_b|^2 + i \bar{\Psi}_b \Gamma^A D_A \Psi_b$

Wall Higgs + Fermions : $+ (i \bar{\Psi}_w \Gamma^\mu D_\mu \Psi_w + |D_\mu \phi_w|^2) \delta(y)$

Localized Fermions : $+ \bar{\Psi}_L (i \Gamma^A D_A + \lambda_L \phi) \Psi_L \quad \}$

$D_A \equiv \partial_A - i g_5 T \cdot A_A \; ; \; \Gamma^\mu = \gamma^\mu \; ; \; \Gamma^5 = i \gamma_5$

KK Decomposition

$$\Phi(x_\mu, y) = \frac{\phi^0(x_\mu)}{\sqrt{2\pi R}} + \sum_{n=1}^{\infty} \frac{\phi^+(x_\mu)}{\sqrt{\pi R}} \cos\frac{ny}{R} + \frac{\phi^-(x_\mu)}{\sqrt{\pi R}} \sin\frac{ny}{R}$$

Spin Decomposition

$V_\mu^{(n)}$ acquires mass by 'eating' $V_5^{(n)}$

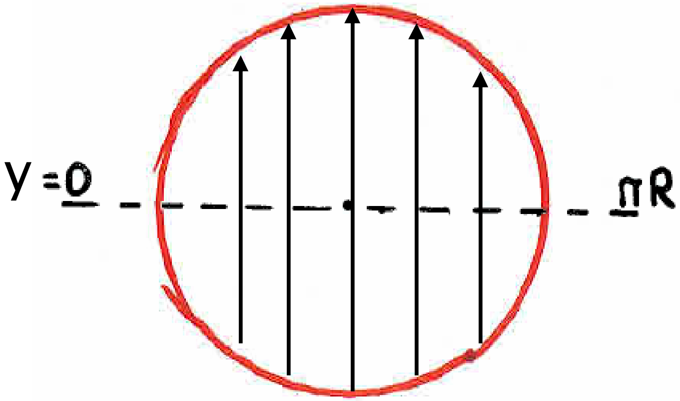
But $V_5^{(0)}$ remains!

Invoke S_1/Z_2 :

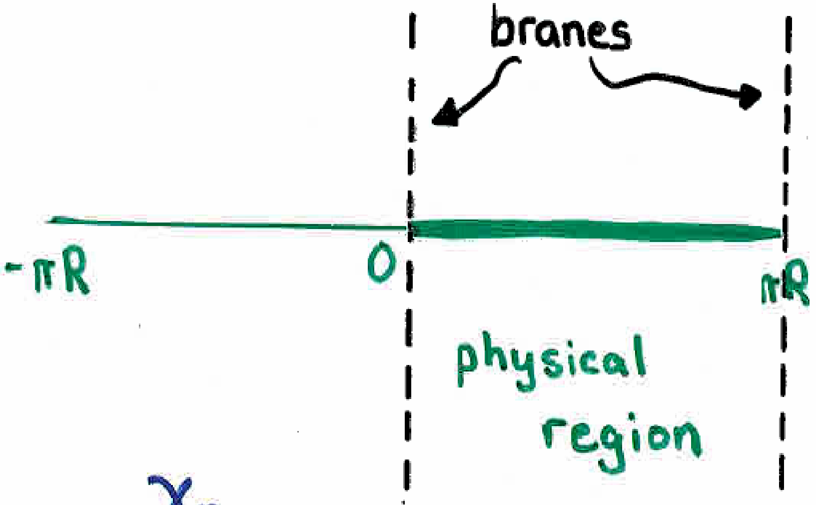
$\Rightarrow$ Choose $\left\{ \begin{matrix} V_\mu^{(n)} \\ \phi_b \end{matrix} \right.$ even; $V_5^{(n)}$ odd

Orbifolding in 1 Extra Dimension

S_1/Z_2 : $y \to -y$



Z_2 is parity on the interval



X_n

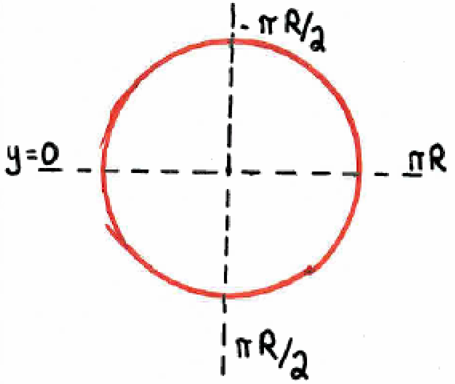
Z_2: (+) $A_n \cos \frac{ny}{R}$ even modes

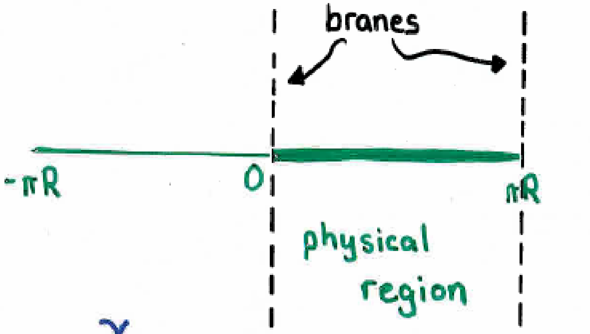
(−) $B_n \sin \frac{ny}{R}$ odd modes

Aside: Double Orbifolds!

S_1/Z_2 : $y \to -y$

Z_2 is parity on the interval





χ_n

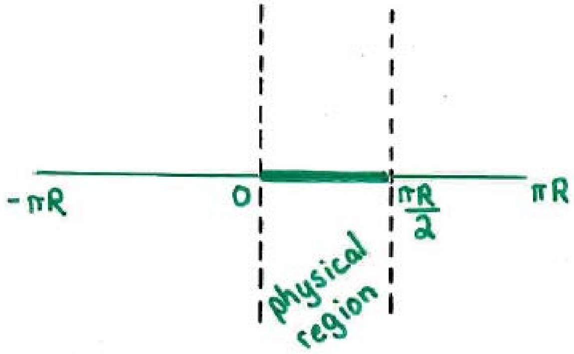
Z_2: (+) $A_n \cos \frac{ny}{R}$ even modes

(−) $B_n \sin \frac{ny}{R}$ odd modes

$S_1/(Z_2 \times Z_2')$

Z_2: $y \to -y$

Z_2': $y' \to -y'$ with $y' = y - \frac{\pi R}{2}$



(Z_2, Z_2'):

$(+,+)$ $\cos 2ny/R$

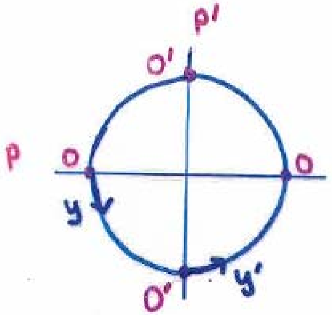
$(+,-)$ $\cos (2n+1)y/R$

$(-,+)$ $\sin (2n+1)y/R$

$(-,-)$ $\sin (2n+2)y/R$

Symmetry Breaking by Orbifolds (Boundary Conditions

Consider 5D Theory
compactified on
$S_1/(Z_2 \times Z_2')$



Physical space $y : [0, \pi R/2]$

w/ branes at orbifold fixed points

Generic 5-D bulk field : $\phi(x^\mu, -y) = P\,\phi(x^\mu, y)$

w/ 5-D symmetry $\phi(x^\mu, -y') = P'\,\phi(x^\mu, y')$

KK states	mass	
$\phi_{++} = \Sigma \frac{\phi_{++}^{(2n)}(x)}{\sqrt{2\pi R}} \cos\frac{2ny}{R}$	$2n/R$	zero modes
$\phi_{+-} = \Sigma \frac{\phi_{+-}^{(2n+1)}(x)}{\sqrt{\pi R}} \cos\frac{(2n+1)y}{R}$	$\frac{2n+1}{R}$	
$\phi_{-+} = \Sigma \frac{\phi_{-+}^{(2n+)}(x)}{\sqrt{\pi R}} \sin\frac{(2n+1)y}{R}$	$\frac{2n+1}{R}$	
$\phi_{++} = \Sigma \frac{\phi_{--}^{(2n+2)}(x)}{\sqrt{\pi R}} \sin\frac{(2n+2)y}{R}$	$\frac{2n+2}{R}$	

.g., 5D SU(5) broken by choosing — Hall, Nomura

$P_5 = (+, +, +, +, +)$, $P_5' = (-, -, -, +, +)$

KK U(1) gauge decomposition [easily generalized to non-abelian case] - Details!

$$\int d^4x\, dy \ -{}^1\!/_4\, F_{AB} F^{AB}$$

$$= \int d^4x\, dy \left\{ -{}^1\!/_4\, F_{\mu\nu} F^{\mu\nu} - {}^1\!/_4\, F_{\mu 5} F^{\mu 5} - {}^1\!/_4\, F_{5\nu} F^{5\nu} \right\}$$

$$= \int d^4x\, dy \left\{ -{}^1\!/_4\, F_{\mu\nu} F^{\mu\nu} - {}^1\!/_2\, F_{\mu 5} F^{\mu 5} \right\}$$

$$F_{\mu\nu} \equiv \partial_\mu G_\nu - \partial_\nu G_\mu$$

$$G_\mu(x,y) = \sum_n A_\mu^{(n)}(x)\, \chi^{(n)} y$$

$\chi^{(n)}(y)$ are periodic

Recall: $G_5^{(n>0)} = 0$ $\left\{ \begin{array}{c} \text{eaten by} \\ G_\mu^{(n>0)} \end{array} \right\}$ + $G_5^{(0)} = 0$ by Z_2. $\Rightarrow$ No F_{55}

This is a gauge choice!!

$$-{}^1\!/_4\, F_{\mu\nu} F^{\mu\nu} \rightarrow -{}^1\!/_4 \sum_{n,m} \left(\partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)} \right) (n \rightarrow m)\, \chi^{(n)} \chi^{(m)}$$

Given orthonormality : $\int dy\, \chi^{(n)} \chi^{(m)} = \delta_{nm}$

$$= -{}^1\!/_4 \sum_n F_{\mu\nu}^{(n)} F^{\mu\nu\,(n)}$$

$$-1/2\, F_{\mu 5} F^{\mu 5} = -1/2 \sum_{n,m} A_\mu^{(n)} A^{\mu (n)}\, \partial_y \chi^{(n)}\, \partial^y \chi^{(m)}$$

$$= +1/2 \sum A_\mu^{(n)} A^{\mu (n)}\, \partial_y \chi^{(n)}\, \partial_y \chi^{(m)}$$

$$\int dy\, \partial_y \chi^{(n)}\, \partial_y \chi^{(m)} \underset{\uparrow \text{ by parts}}{=} \int dy\, - \chi^{(m)}\, \partial_y^2\, \chi^{(n)} + \underset{=0}{\text{Boundary term}}$$

$\left[\text{either } \chi^{(n)} \text{ or } \partial_y \chi^{(n)} = 0 \text{ @ boundary} \right]$

Since $\chi^{(n)} \sim \sin ny/R,\ \cos ny/R \ + m_n = n/R$

$$\partial_y^2\, \chi^{(n)} = -m_n^2\, \chi^{(n)}$$

So, $\int dy\, m_n^2\, \chi^{(n)} \chi^{(m)} = m_n^2\, \delta_{nm}$

$$\Rightarrow -1/2\, F_{\mu 5} F^{\mu 5} = 1/2 \sum_n m_n^2\, A_\mu^{(n)} A^{\mu (n)}$$

$$S = \int d^4x \left\{ -1/4 \sum_n F_{\mu\nu}^{(n)} F^{\mu\nu (n)} + 1/2 \sum_n m_n^2\, A_\mu^{(n)} A^{\mu (n)} \right\}$$

Normalization:

$$\int_0^{2\pi R} dy\ \chi^{(n)}\chi^{(m)} = \delta_{nm}$$ normalizes χ's

For the zero-mode: $\int_0^{2\pi R} dy \cdot \text{constant} = 1$

$$\Rightarrow \text{constant} = \frac{1}{\sqrt{2\pi R}}$$

For KK-modes: $\int_0^{2\pi R}\left(\sin^2\frac{ny}{R}\ ,\ \cos^2\frac{ny}{R}\right)dy = \pi R$

$\Rightarrow$ normalized eigenfunctions are $\frac{1}{\sqrt{\pi R}}\sin\frac{ny}{R}\ ,\ \frac{1}{\sqrt{\pi R}}\cos {}^{ny}\!/\!_R$

$$\rightarrow \frac{g_5}{\sqrt{2\pi R}}V_0 + \sum_n \frac{g_5}{\sqrt{\pi R}}\cos {}^{ny}\!/\!_R\ V_{(n)}^{+}\ \Bigg|_{\substack{\text{brane}\\ y=0}}$$

$$\sim\ g_{sm}V_0 + \underset{\substack{\uparrow\\ !!!}}{\sqrt{2}}\, g_{sm}\sum_n V_{(n)}^{+}$$

TeV^{-1}-size Extra Dimensions

Can arise naturally in string-inspired models

Antoniadis

The Standard Model goes into the bulk!

Model building choices:

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- Higgs in the bulk or on the brane?
- Fermions:
  - Located at orbifold fixed points
  - Localized to specific points inside the bulk (Split Fermions)
  - Freely propagate inside the bulk (Universal Extra Dimensions)

Precision Electroweak Data (fermions @ fixed points)

Exchange of gauge KK excitations contribute to precision EW observables

Contributions include:

- Tree-level KK interactions (e.g., μ decay)
- KK – zero mode mixing (e.g., affects Z-pole observables)
- Zero mode loop corrections

KK tower exchanges induce new dim-6 operators with coefficients

$$V = \sum_n \frac{g_n^2}{g_0^2} \frac{m_W^2}{m_n^2}$$

Rizzo, Wells
Delgado, Pomerol

Perform full fit to global precision EW data set

Bound on compactification scale, $\boxed{M_c > 4.5 \text{ TeV}}$

degrades to $M_c > 2\text{-}3$ TeV for localized fermions

Searches @ Colliders (fermions @ fixed points)

- Hadron Colliders: Drell-Yan $\gamma/Z/W$ KK resonance
  dijet g KK resonance
  - $q\bar{q} \rightarrow \gamma_n/Z_n \rightarrow \ell\ell$
  - $q\bar{q} \rightarrow W_n \rightarrow \ell\nu$
  - $q\bar{q},gg \rightarrow g_n \rightarrow jj$

  Bumps!

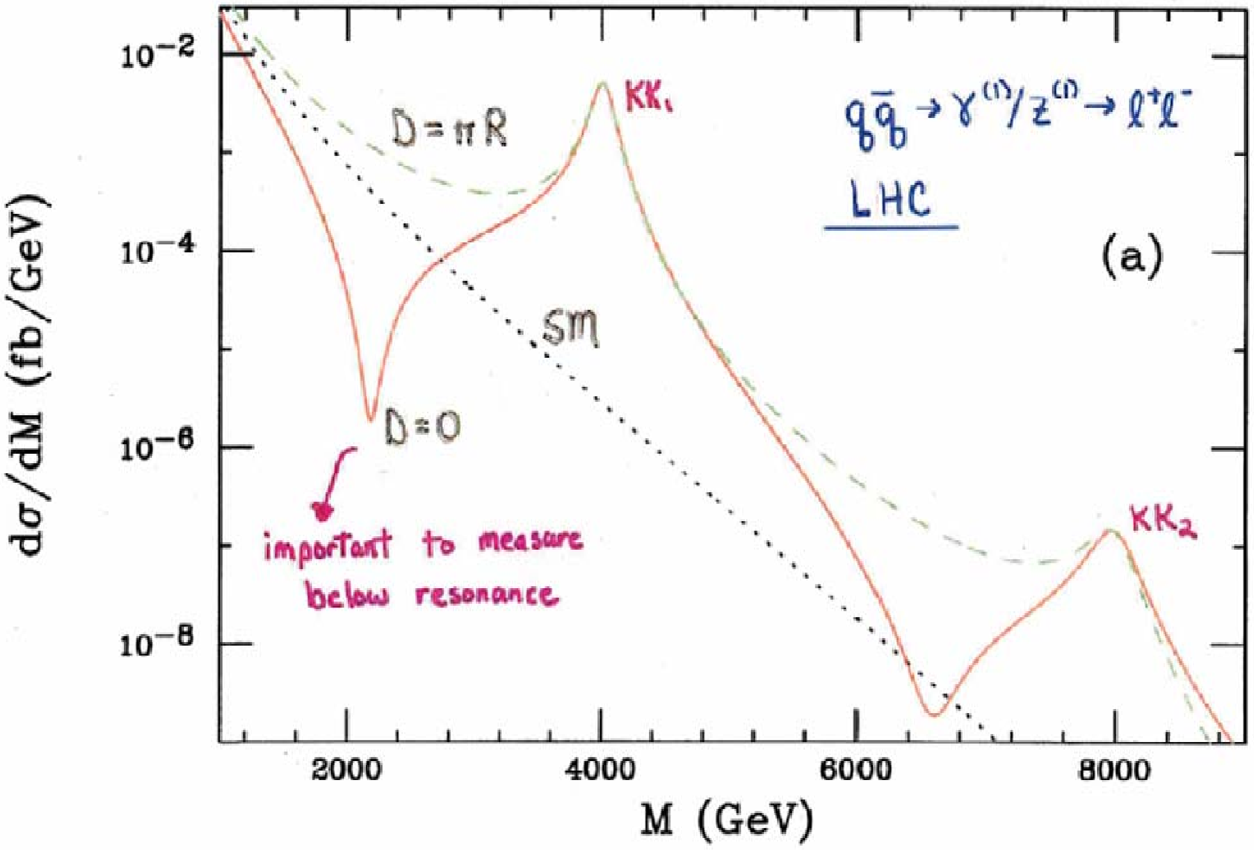
  Tevatron Run I: $M_c > 0.8$ TeV

  Run II $M_c > 1.1$ TeV

- e^+e^- Colliders: Indirect search in $e^+e^- \rightarrow \gamma_n/Z_n \rightarrow f\bar{f}$
  Observe deviation from SM
  Fit to $\sigma_{f,}$ $A_{FB}{}^f$, $A_{LR}{}^f$, $A_{pol}{}^\tau$

KK γ/Z Production @ LHC, $M_c = 4$ TeV

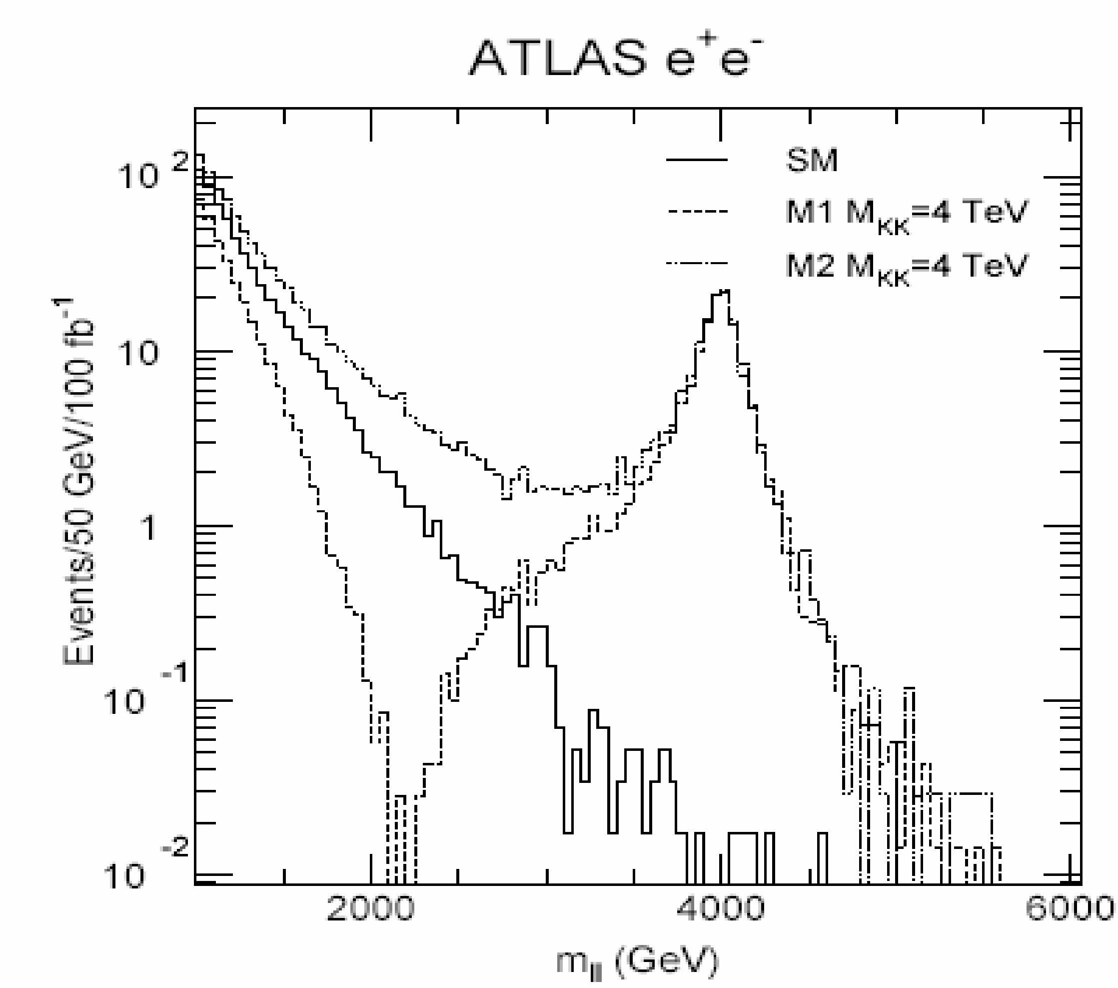
D = separation of fermions in 5th dimension



$m_2 = 2m_1$

Even spacing denotes flat space

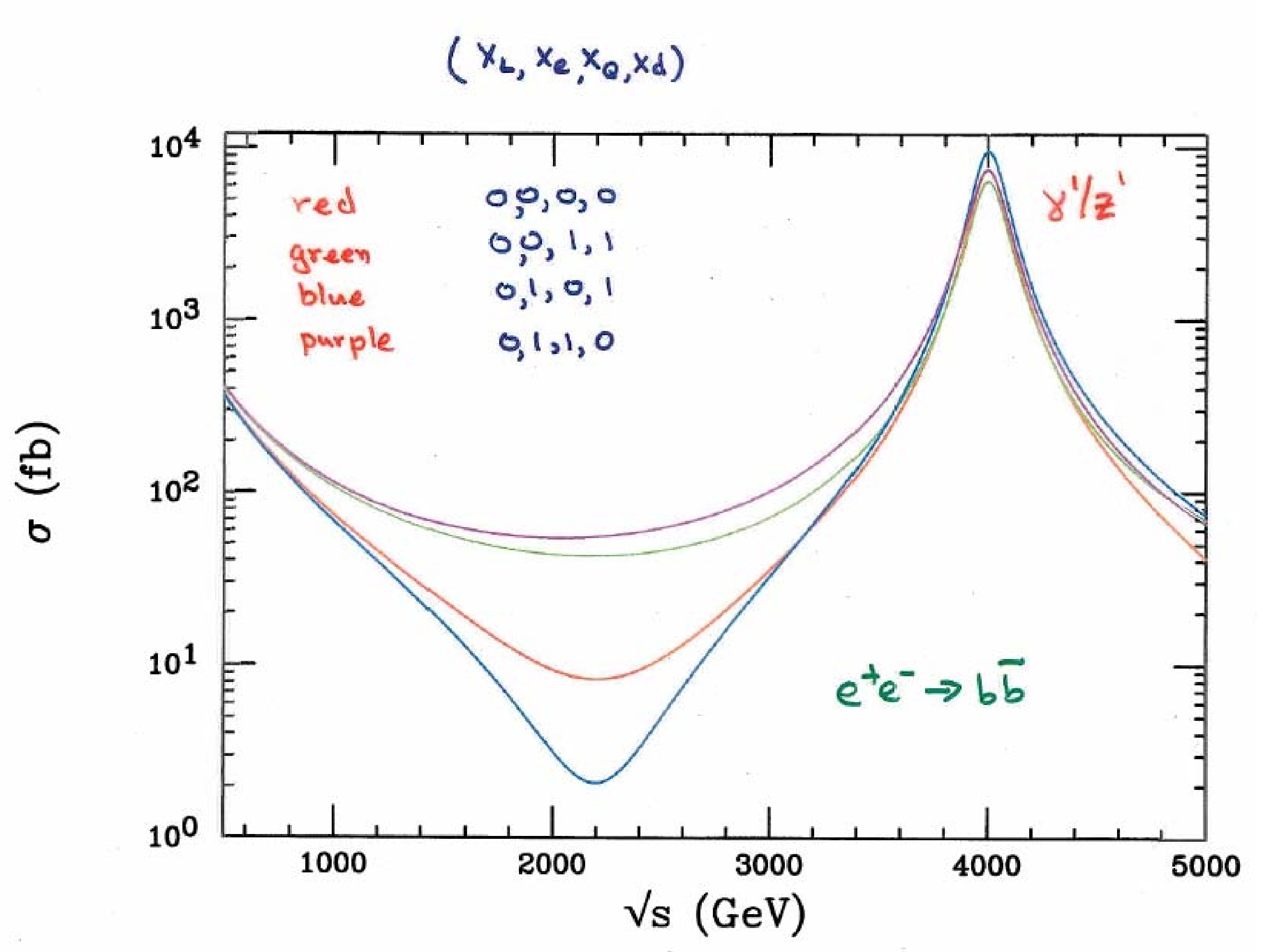
ATLAS Simulation for γ/Z KK Production



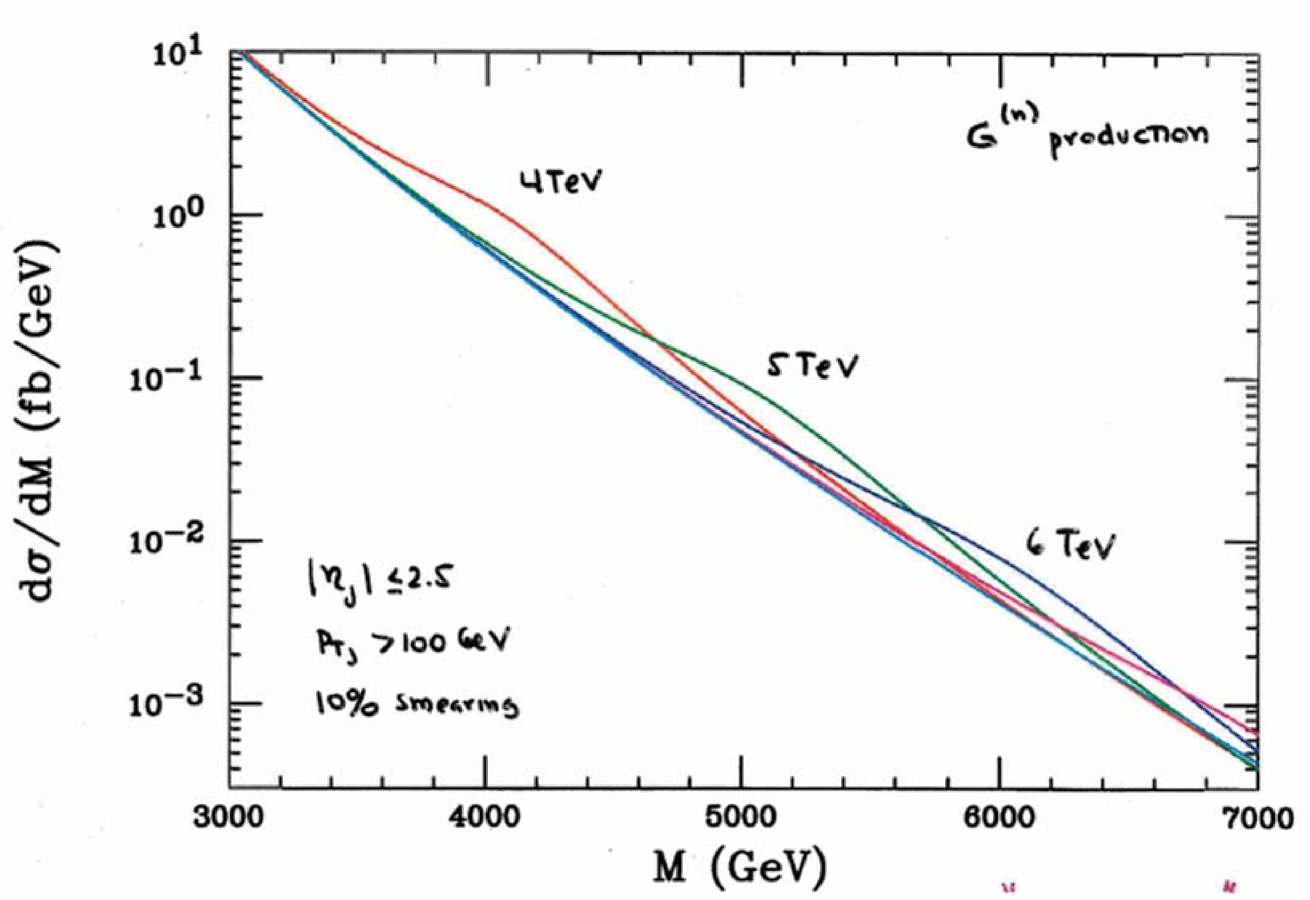
Discovery Reach: $M_c < 6.3$ TeV

Azuelos, Polesello
Les Houches 01

Further dependence on fermion location



KK gluon dijet mass bumps @ LHC



γ/Z KK Search Reach @ ILC (indirect effect)

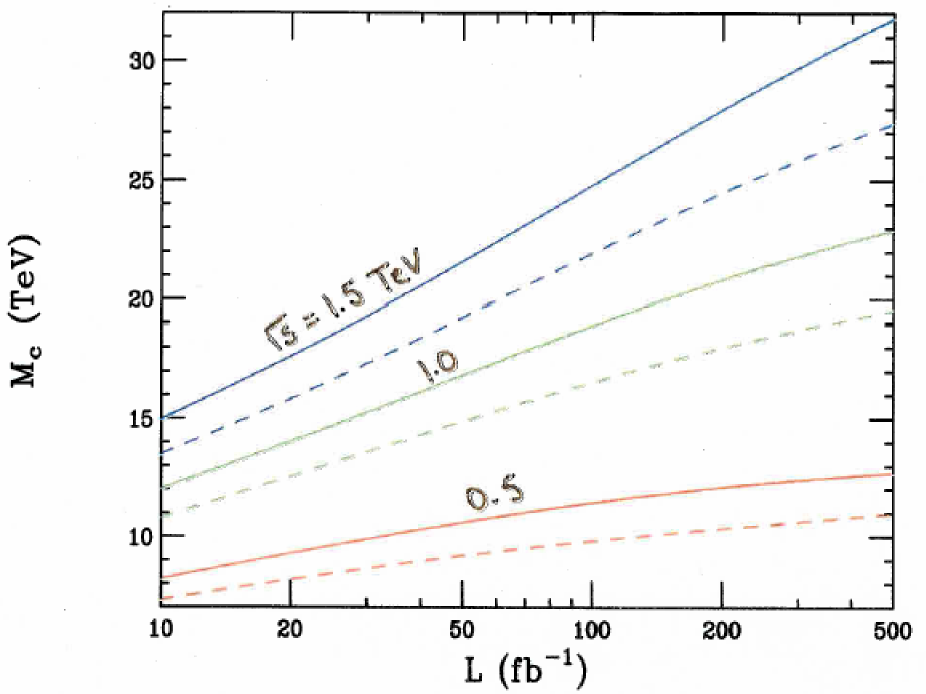
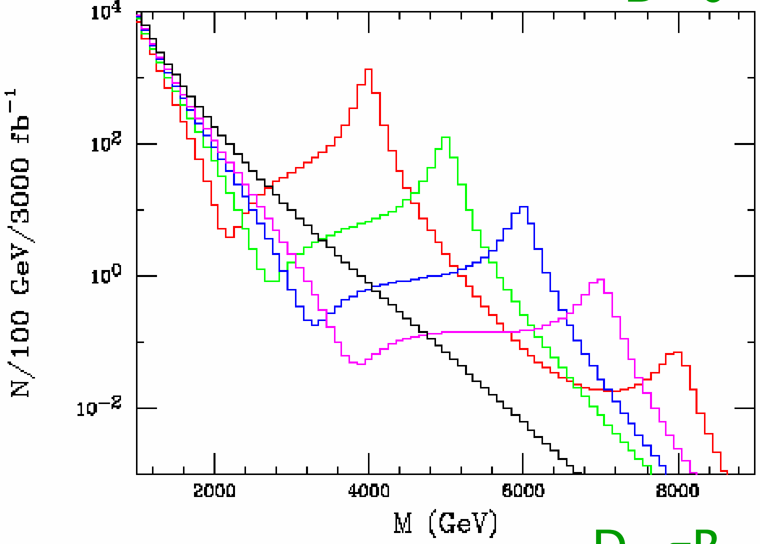


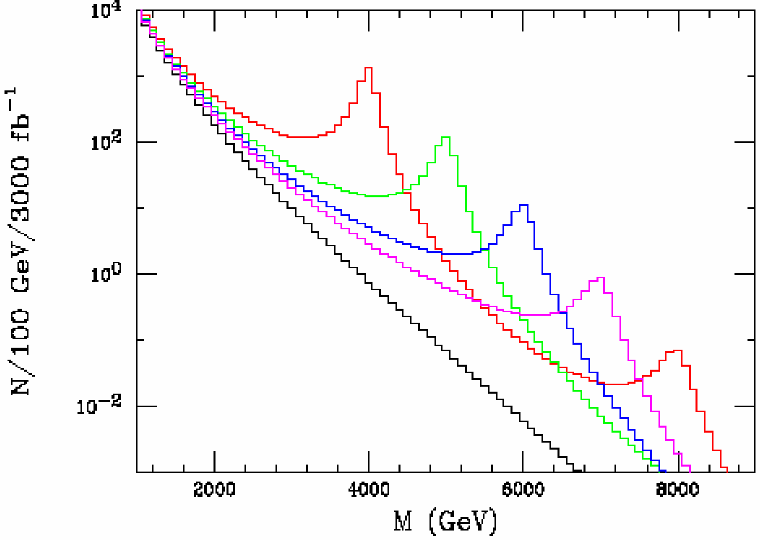
Figure 3: Search reach, M_c, for the first $Z^{(1)}/\gamma^{(1)}$ excited state as a function of the integrated luminosity assuming e^+e^- collider center of mass energies, from top to bottom, of 1.5, 1, 0.5 TeV. One extra dimension is assumed. The solid curves correspond to the case of 'conventional' couplings while the dashed curves are for the case of the AS scenario[7].

Rizzo, Wells

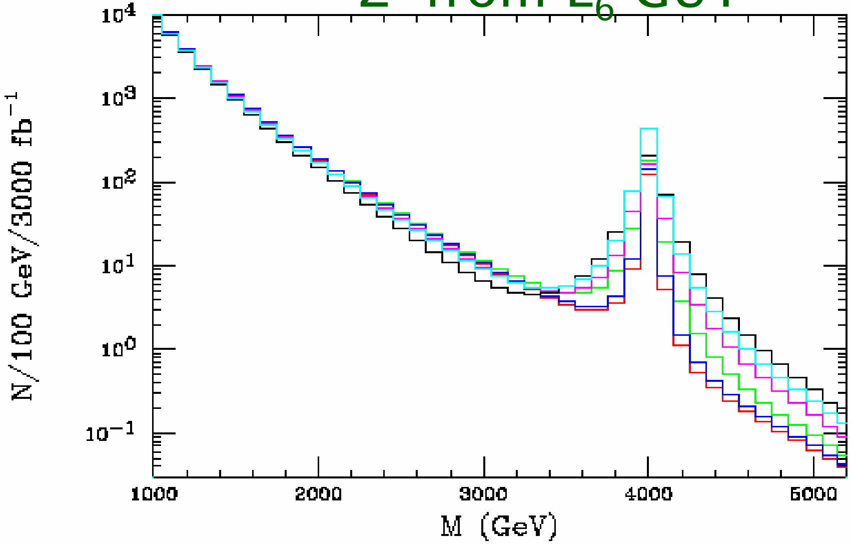
Distinguish γ/Z KK from GUT Z' Production @ LHC



D=0



D=πR



Z' from E_6 GUT

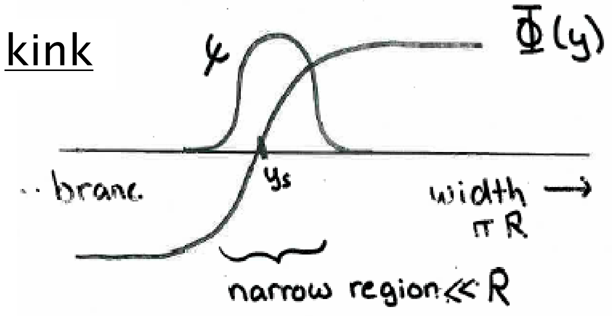
Can be difficult!
Easier @ ILC

Localized Fermions in Extra Dimension

Arkani-Hamed, Schmaltz

Spatially varying domain wall scalar

kink



- Consider a 5-d fermion ψ_s coupled to a scalar

$$m_5 \rightarrow m_5 + \lambda \Phi(y)$$

$$(m_5 + \lambda \Phi \mp \partial y)\, \chi^{(n)}_{R,L} = - m_n\, \chi^{(n)}_{L,R}$$

$\Rightarrow$ zero-mode $\quad \chi^{(0)}_{L,R} = N_{L,R} \exp\left[\pm \int dy\, [m_5 + \lambda \Phi(y)] \right]$

$\Phi(y) \sim \sigma^{-2} y \qquad = N_{L,R} \exp\{ \pm \lambda (y - y_s)^2 / 2\sigma^2 \}$

$$m_5 = \lambda/\sigma^2\, y_s$$

$\Rightarrow$ A gaussian of width $\sigma/\sqrt{\lambda}$

centered on $y_s = m_5 \sigma^2 / |\lambda|$

y_f for each fermion. Overlap of Left- & Right-handed wavefunctions give Yukawa couplings!

Proton Decay

Induced by short distance physics above m_*

Local QQQL interaction:

$$S \sim \int d^4x\, dy \, \frac{(Q^T C_5 L)^\dagger (U^{c\,T} C_5 D^c)}{m_*^3}$$

$$C_5 \equiv \gamma^0 \gamma^2 \gamma^5$$

Corresponding 4-d operator:

5d fields → 0-mode fields

Calculate wavefunction overlap in y

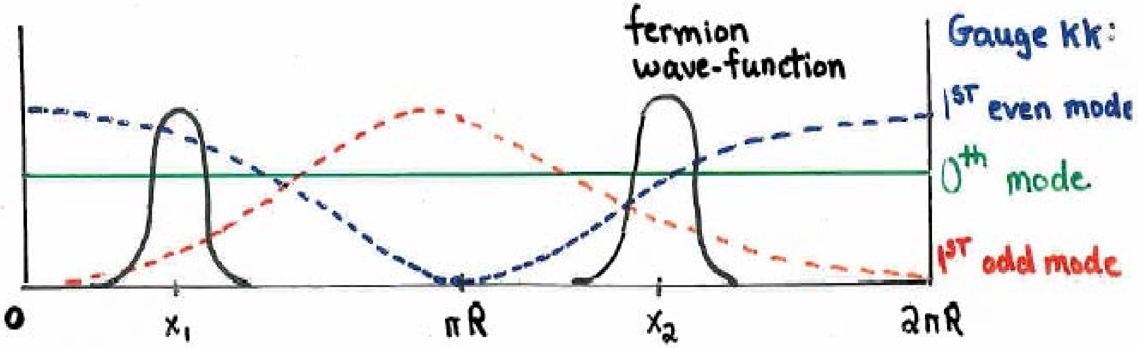
$$S \sim \int d^4x \;\delta\; \frac{(q\ell)^\dagger (u^c d^c)}{m_*^2}$$

$$\delta \sim \int dy \left[e^{-\lambda y^2/2\sigma^2} \right]^3 e^{-\lambda (y-r)^2/2\sigma^2}$$

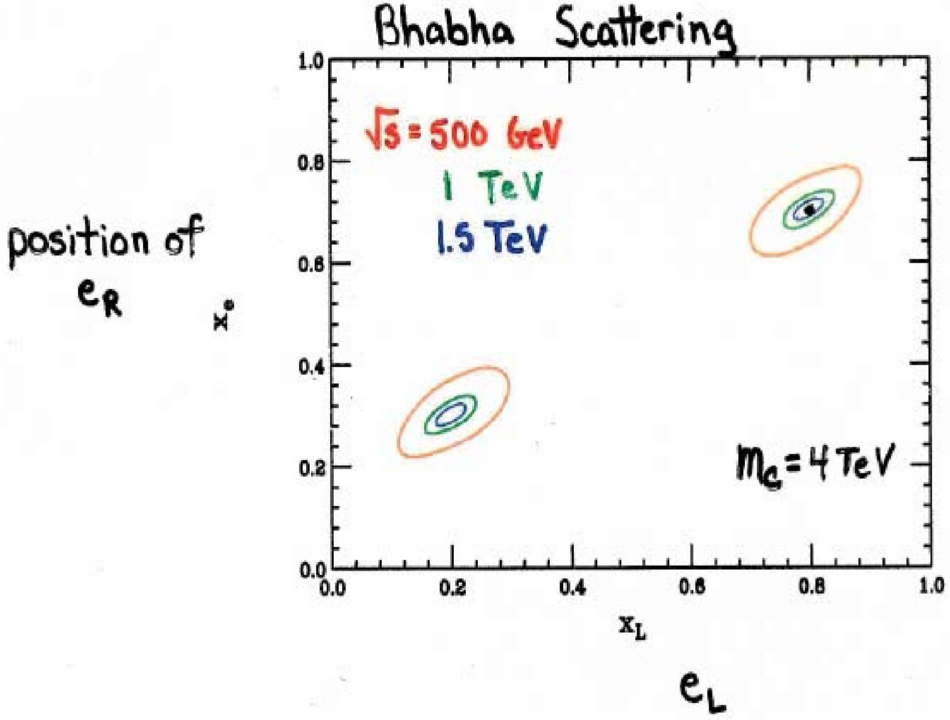
$$\sim e^{-\overset{\sigma^2}{3/4}\lambda r^2} \sim 10^{-33} \quad \text{if} \quad \underbrace{\frac{\sqrt{\lambda}}{2\sigma}}_{\text{½-width of Gaussian}} \underbrace{r}_{\text{separation distance between } q+\ell} = 10$$

⇒ 4-d coupling is exponentially supressed!

Fermions can be localized at different points in a thick brane



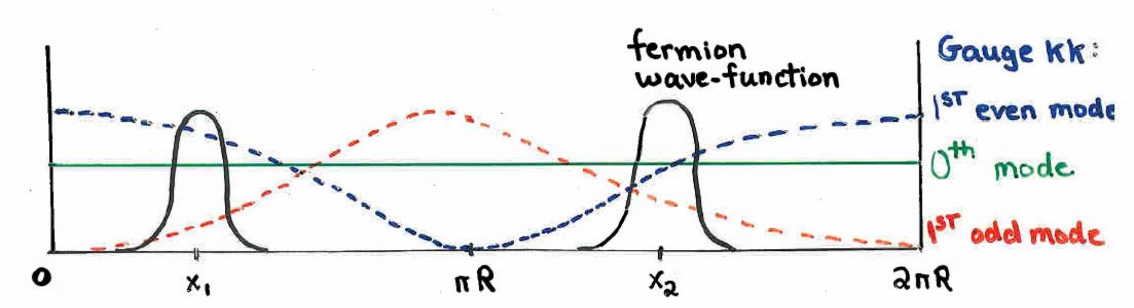
Gauge KK couplings probe relative fermion locations!



S_1/Z_2

Rizzo

Exponential Fall-off of Scattering Cross Sections



If collision energy is high enough, the two interacting partons will probe separation distance between them!

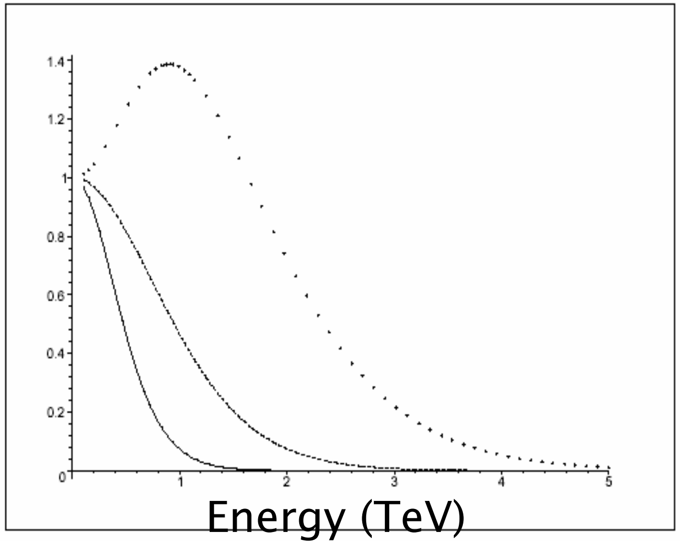
Exponential fall-off of cross section for fermion pair production

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2}{s^2}\left[\left(1+\frac{1}{16\sin^4\theta_w}\right)\frac{u^2(P_0(s)+P_0(t))^2}{\cos^4\theta_w} + \frac{t^2P_d^2(s)+s^2P_d^2(t)}{2\cos^4\theta_w}\right]$$

$$P_d(t) \simeq -\frac{\pi R}{\sqrt{-t}}\, e^{-\sqrt{-t}\,d}$$

Arkani-Hamed, Grossman, Schmaltz

σ/σ_{SM} for μ pair production



Universal Extra Dimensions

Appelquist, Cheng, Dobrescu

- All SM fields in TeV^{-1}, 5d, S^1/Z_2 bulk
- No branes! ⇒ translational invariance is preserved
  ⇒ tree–level conservation of p_5
- KK number conserved at tree–level
- broken at higher order by boundary terms
- KK parity conserved to all orders, $(-1)^n$

Consequences:

1. KK excitations only produced in pairs
   Relaxation of collider & precision EW constraints
   $R_c^{-1} \geq 300$ GeV!
2. Lightest KK particle is stable (LKP) and is Dark Matter candidate
3. Boundary terms separate masses and give SUSY–like spectrum

Universal Extra Dimensions: Bosonic SUSY

Phenomenology looks like Supersymmetry:

Heavier particles cascade down to LKP

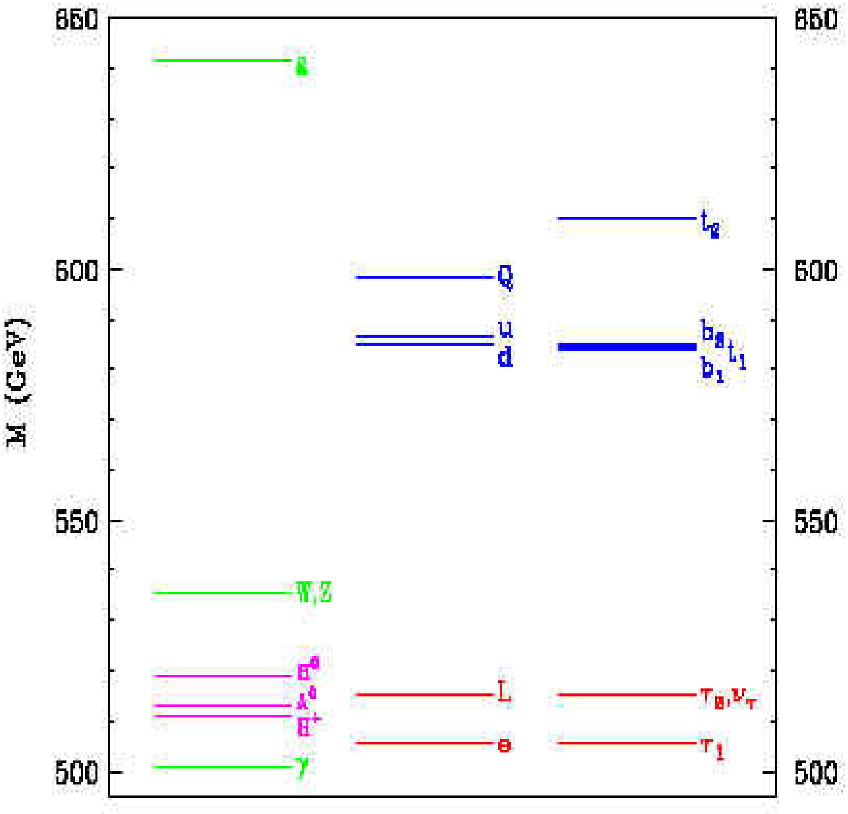
LKP: Photon KK state

appears as missing E_T

SUSY-like Spectroscopy

Confusion with SUSY if discovered @ LHC !

Spectrum looks like SUSY !



Chang, Matchev,Schmaltz

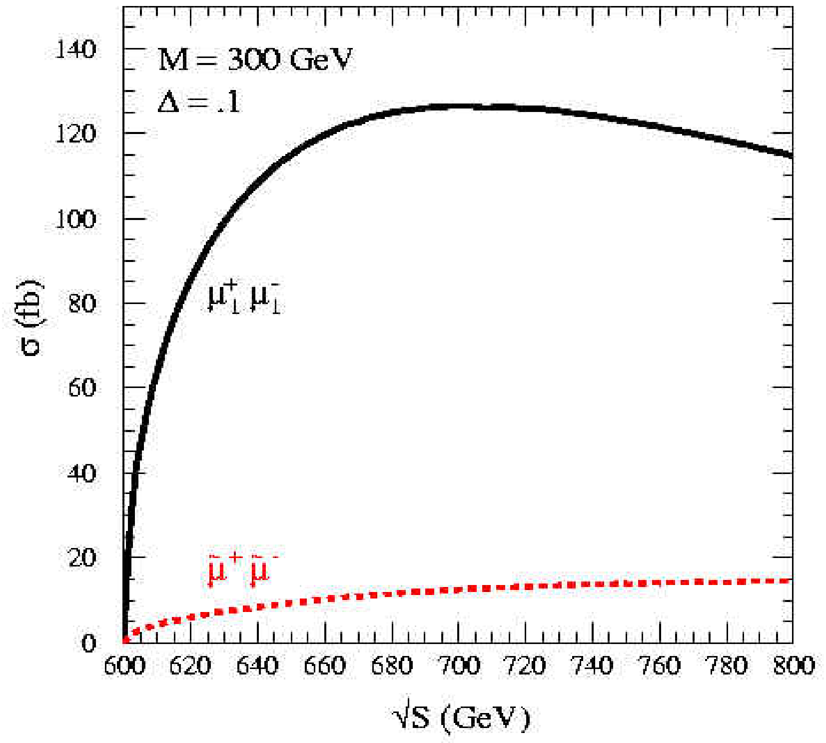
How to distinguish SUSY from UED I:

Observe KK states in e^+e^- annihilation

Measure their spin via:

- Threshold production, s−wave vs p−wave
- Distribution of decay products

- However, could require CLIC energies...



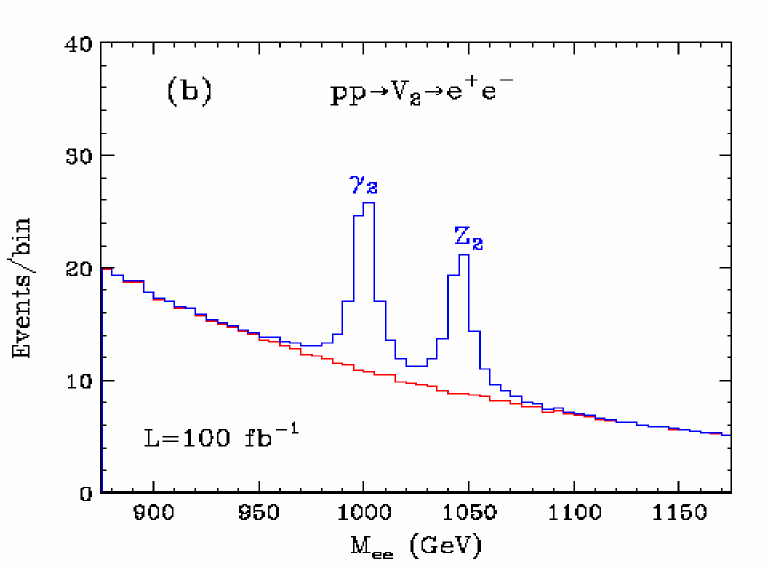
JLH, Rizzo, Tait
Datta, Kong, Matchev

How to distinguish SUSY from UED II:

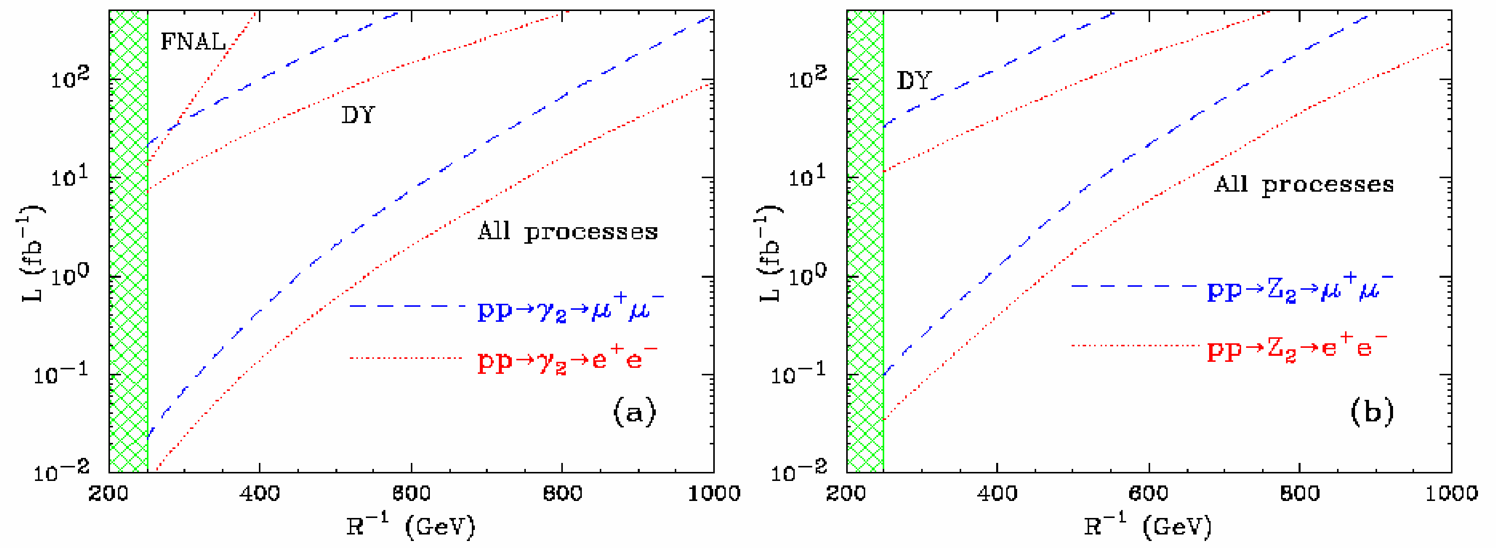
Datta, Kong, Matchev

Observe higher level (n = 2) KK states:

- Pair production of $q_2 q_2$, $q_2 g_2$, $V_2 V_2$
- Single production of V_2 via (1) small KK number breaking couplings and (2) from cascade decays of q_2



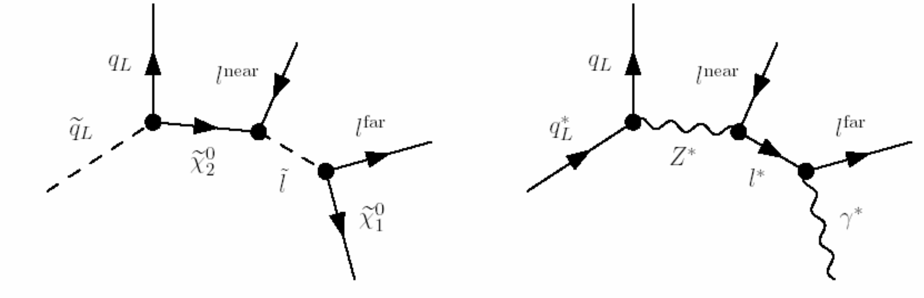
Discovery reach @ LHC



How to distinguish SUSY from UED III:

Measure the spins of the KK states @ LHC – Difficult!

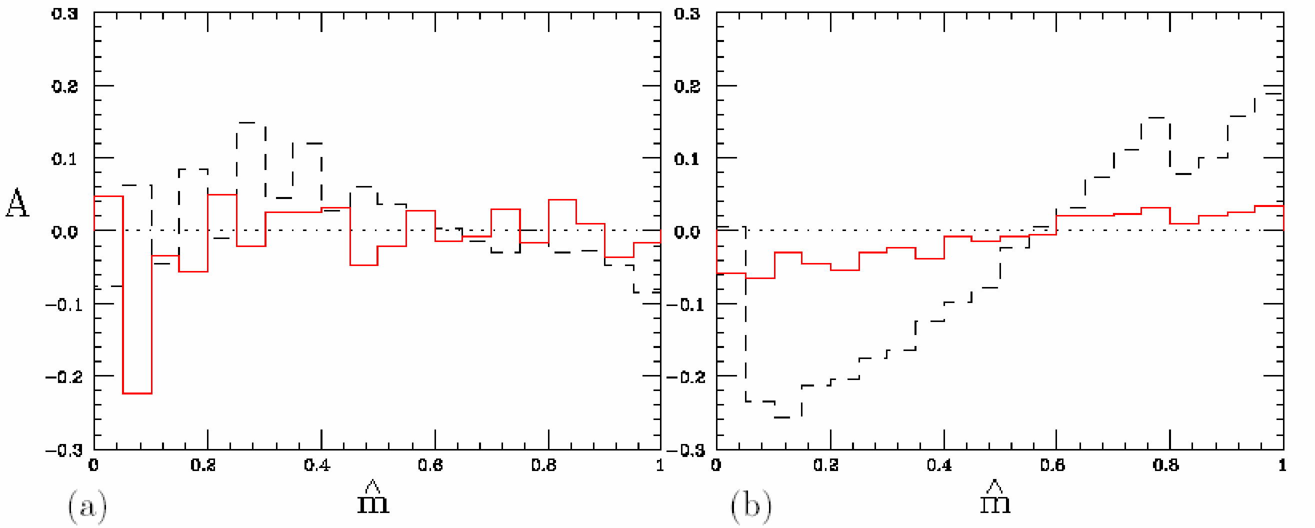
Decay chains in SUSY and UED:



Form charge asymmetry:

$$A = \frac{\mathrm{d}P/\mathrm{d}m_{jl^+} - \mathrm{d}P/\mathrm{d}m_{jl^-}}{\mathrm{d}P/\mathrm{d}m_{jl^+} + \mathrm{d}P/\mathrm{d}m_{jl^-}}$$

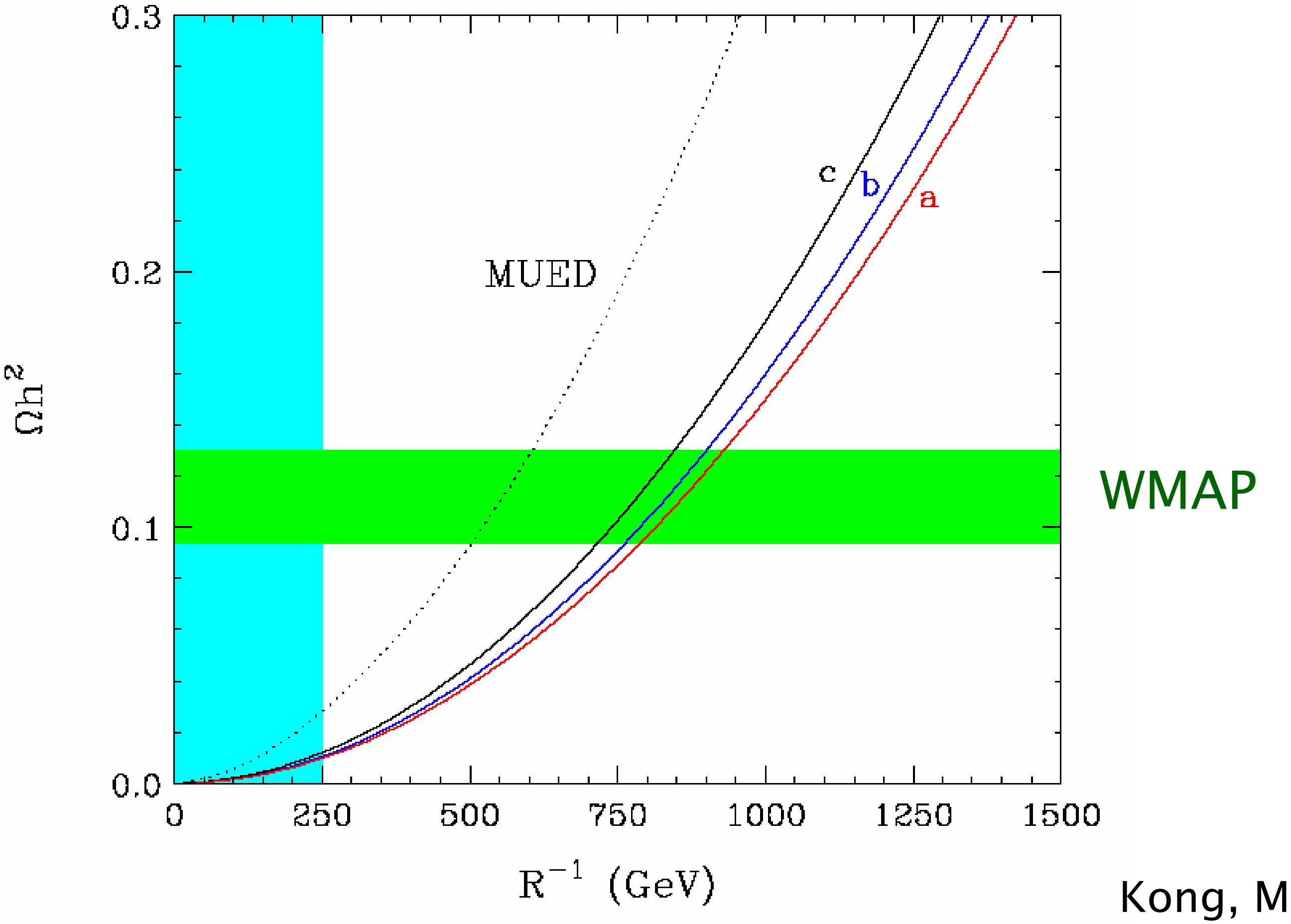
Smillie, Webber



Works for some, but not all, regions of parameter space

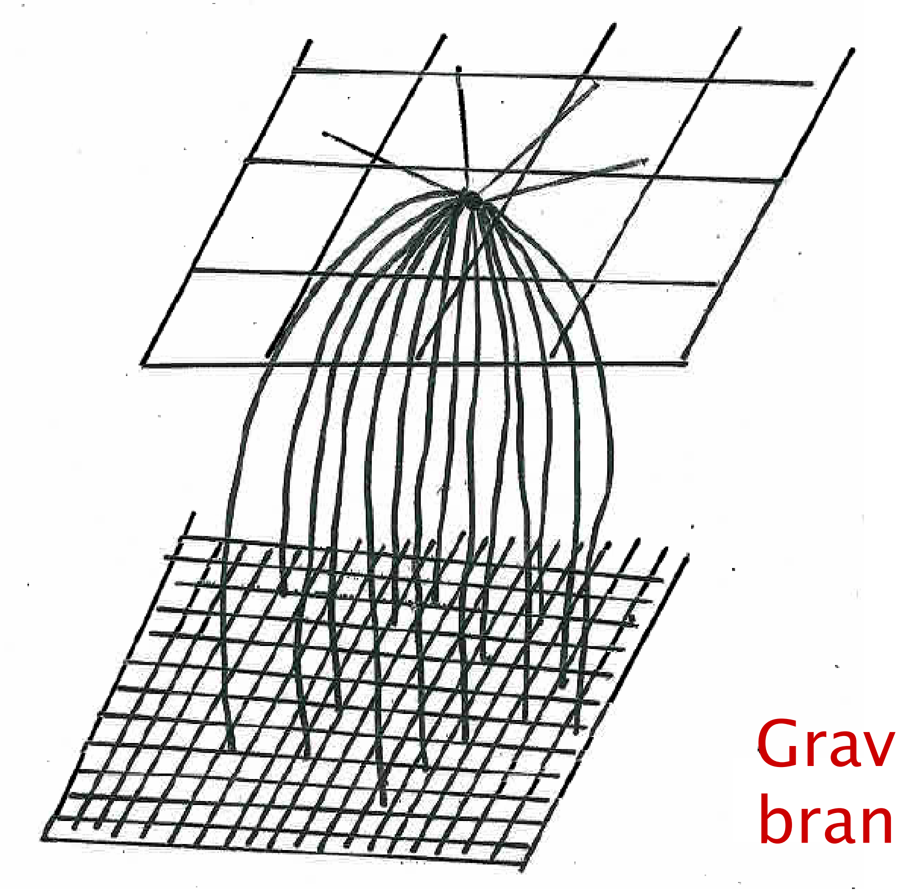
UED Dark Matter Candidate: γ_1

Calculate relic density from γ_1 annihilation and co-annihiliation



Kong, Matchev
Tait, Servant

Non-Factorizable Curved Geometry: Warped Space



Area of each grid is equal

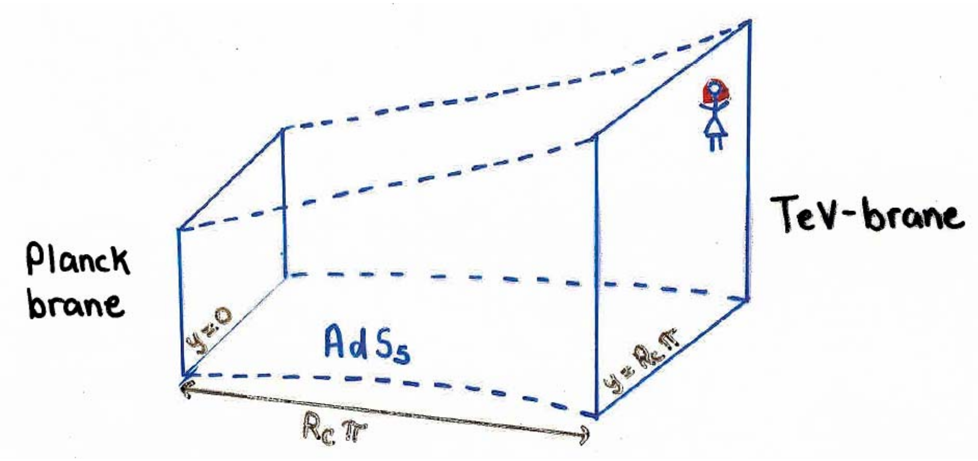
Field lines spread out faster with more volume

⇒ Drop to bottom brane

Gravity appears weak on top brane!

Localized Gravity: Warped Extra Dimensions

Randall, Sundrum



Bulk = Slice of AdS_5

$\Lambda_5 = -24M_5^3k^2$
k = curvature scale

5-D non-factorizable geometry:

$$ds^2 = \underbrace{e^{-2ky}}_{\text{Warp factor}}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$M_5 \sim M_{Pl} \sim k$ ⇒ no additional hierarchies!

Physical scales on SM 3-brane:

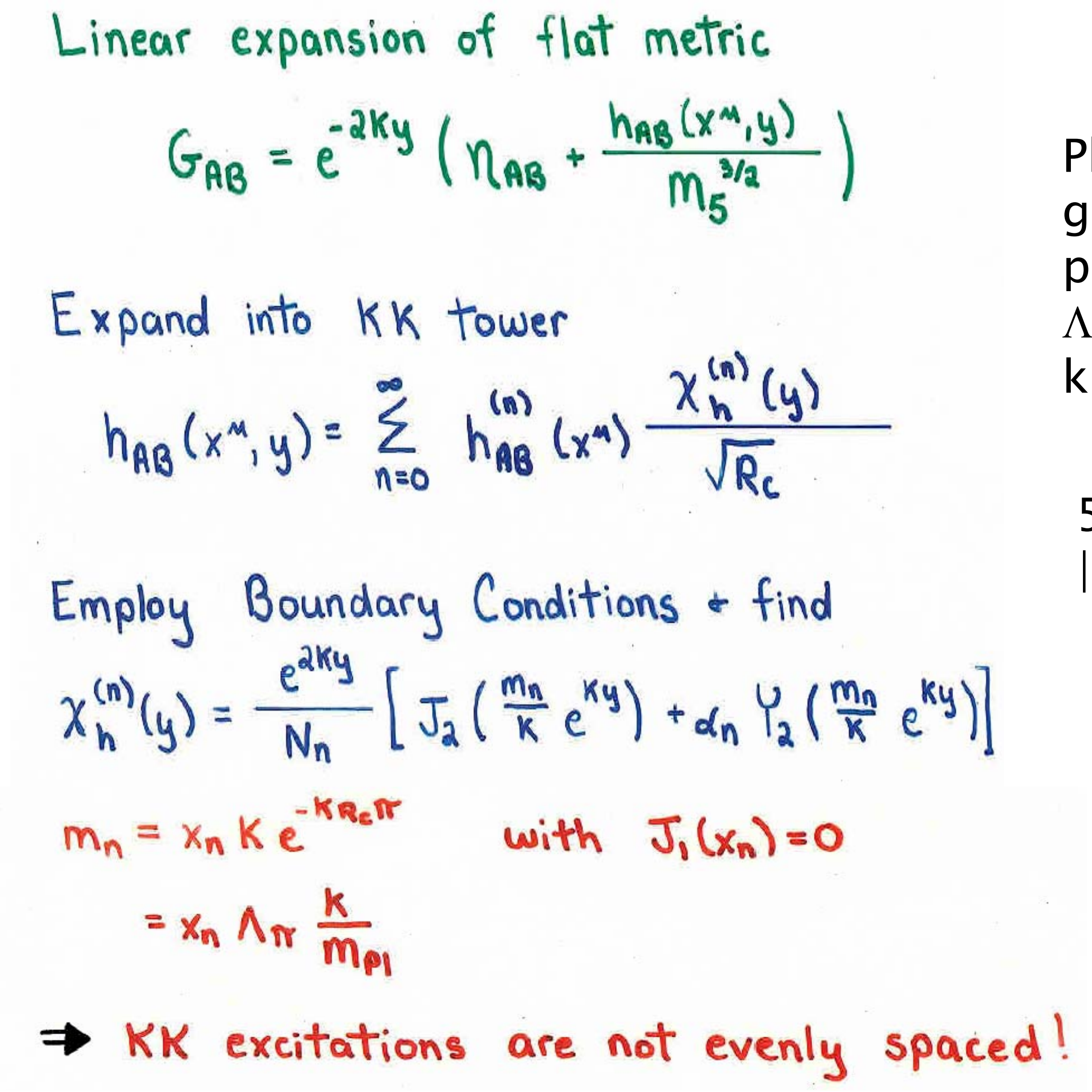
$$\Lambda_\pi = e^{-kR_c\pi}M_{Pl}$$
$$\simeq \text{TeV} \quad \text{if} \quad kR_c \sim 11$$

Naturally stablized via Goldberger-Wise

Hierarchy is generated by exponential!

4-d Effective Theory

Davoudiasl, JLH, Rizzo



Linear expansion of flat metric

$$G_{AB} = e^{-2ky}\left(\eta_{AB} + \frac{h_{AB}(x^\mu,y)}{M_5^{3/2}}\right)$$

Expand into KK tower

$$h_{AB}(x^\mu,y) = \sum_{n=0}^{\infty} h_{AB}^{(n)}(x^\mu)\,\frac{\chi_h^{(n)}(y)}{\sqrt{R_c}}$$

Employ Boundary Conditions + find

$$\chi_h^{(n)}(y) = \frac{e^{2ky}}{N_n}\left[J_2\left(\frac{m_n}{k}e^{ky}\right) + \alpha_n Y_2\left(\frac{m_n}{k}e^{ky}\right)\right]$$

$$m_n = x_n k e^{-kR_c\pi} \qquad \text{with} \quad J_1(x_n) = 0$$

$$= x_n \Lambda_\pi \frac{k}{M_{Pl}}$$

⇒ KK excitations are not evenly spaced!

Phenomenology governed by two parameters:

$\Lambda_\pi/m_1 \sim$ TeV

$k/M_{Pl} \lesssim 0.1$

5-d curvature:

$|R_5| = 20k^2 < M_5^2$

Interactions

$$\mathcal{L} \sim \frac{-1}{M_5^{3/2}} T^{\alpha\beta}(x)\, h_{\alpha\beta}(x, \phi = \pi)$$

$$= \frac{-1}{\overline{M}_{Pl}} T^{\alpha\beta}(x)\, h^{(0)}_{\alpha\beta}(x) - \frac{1}{\Lambda_\pi} T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}(x)$$

zero-mode decouples

TeV-suppressed ⇒ can be directly produced!

Recall $\Lambda_\pi = M_{Pl} e^{k\pi r} \sim \text{TeV}$

4-d Effective Theory

Action: $S = \frac{m_5^3}{4} \int d^4x \int dy \sqrt{-G}\; R^{(5)}$

Linear expansion of flat metric: $G_{AB} = e^{-2ky}\left(\eta_{AB} + \frac{h_{AB}(x^\mu,y)}{m_5^{3/2}}\right)$

Gauge: $\partial^\alpha h_{\alpha\beta} = h^\alpha{}_\alpha = 0$

Expand into KK tower: $h_{AB}(x^\mu,y) = \sum_n h^{(n)}_{AB}(x^\mu)\, \frac{\chi^{(n)}(y)}{\sqrt{R_c}}$

Can derive Eq of motion for $h^{(n)}_{AB}$: $(\eta^{AB}\partial_A\partial_B - m_n^2)\, h^{(n)}_{\mu\nu}(x^\mu) = 0$

Putting this ~~that~~ into action yields:

$$-\frac{1}{R_c^2}\frac{d}{d\theta}\left(e^{-4kR_c\theta}\frac{d\chi^{(n)}}{d\theta}\right) = m_n^2\, e^{-2kR_c\theta}\,\chi^{(n)}$$

- 2^{ND} order Bessel's Eqn!

Orthonormality $\int_{-\pi}^{\pi} d\theta\, e^{-2kR_c\theta}\,\chi^{(m)}\chi^{(n)} = \delta_{mn}$

Soln: $\chi^{(n)}(y) = \frac{e^{2ky}}{N_n}\left[J_2\left(\frac{m_n}{k}e^{ky}\right) + \alpha_n Y_2\left(\frac{m_n}{k}e^{ky}\right)\right]$

normalization

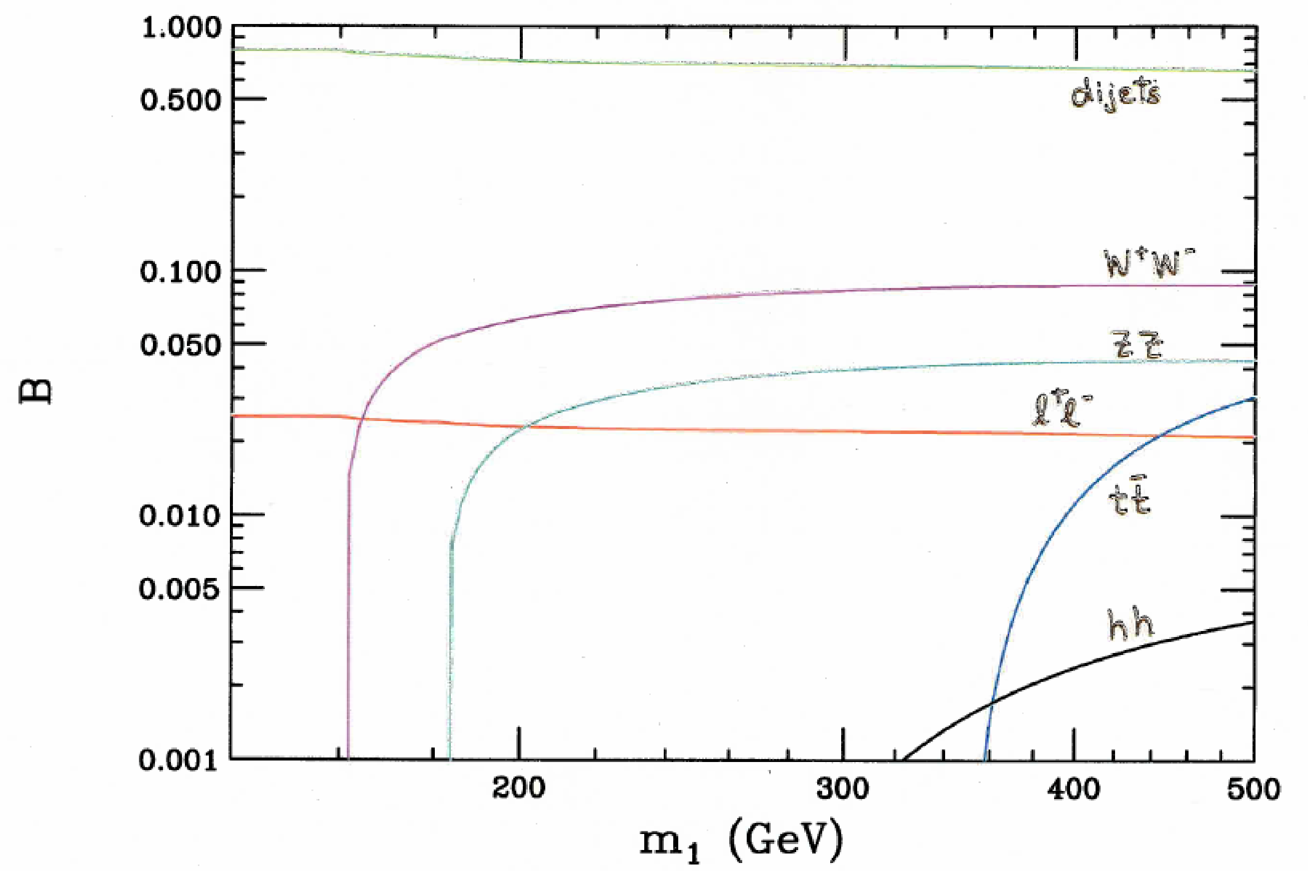
Require 1^{ST} derivative of $\chi^{(n)}$ be continuous @ orbifold fixed points

$\Rightarrow \quad \alpha_n \sim x_n^2\, e^{-2k\pi R_c} \rightarrow$ <u>SMALL</u>

Find $m_n = x_n\, k\, e^{-k\pi R_c}$

$= x_n \Lambda_\pi\, k/m_{Pl}$

Graviton Branching Fractions

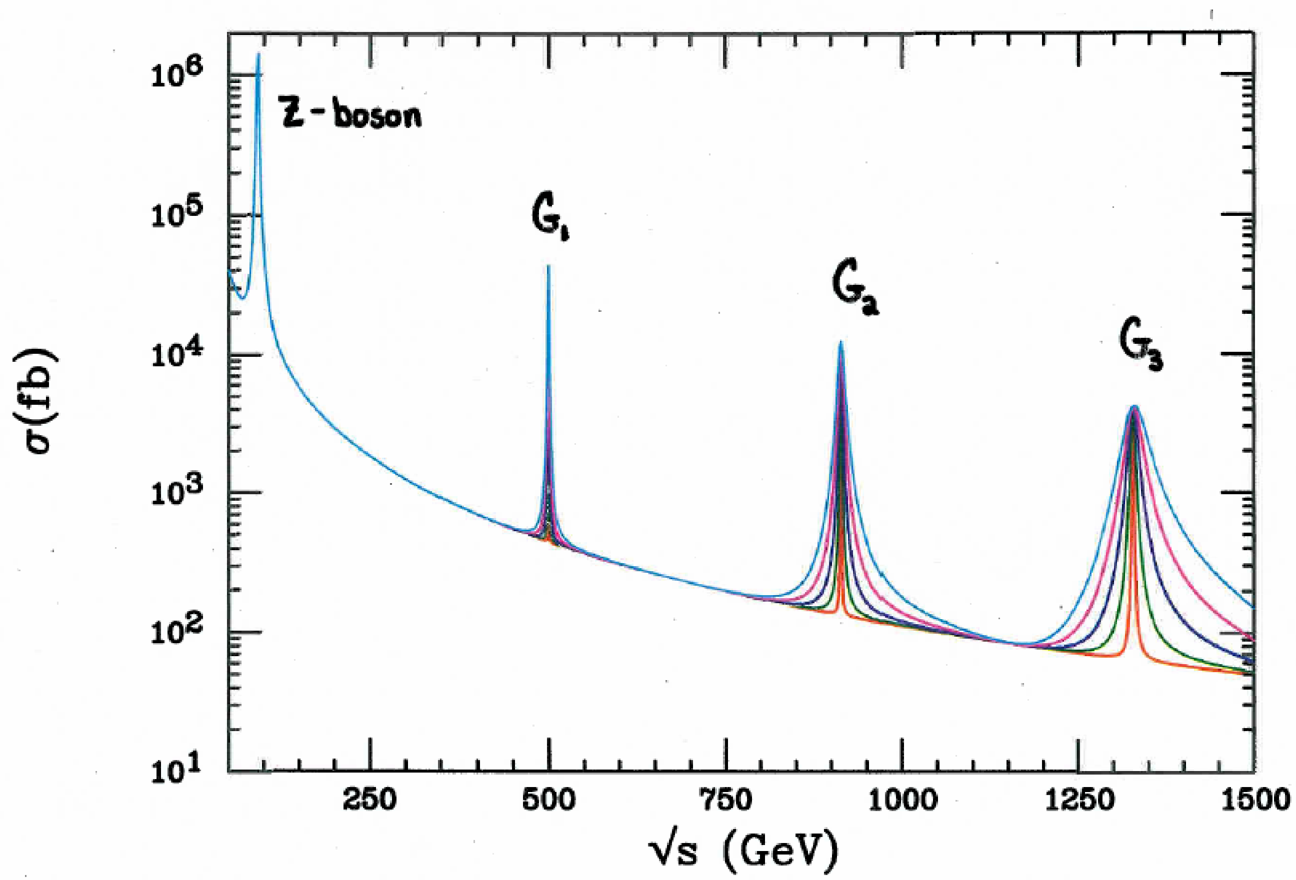


$B_{\gamma\gamma} = 2B_{\ell\ell}$

Randall–Sundrum Graviton KK Spectrum

Davoudiasl, JLH, Rizzo

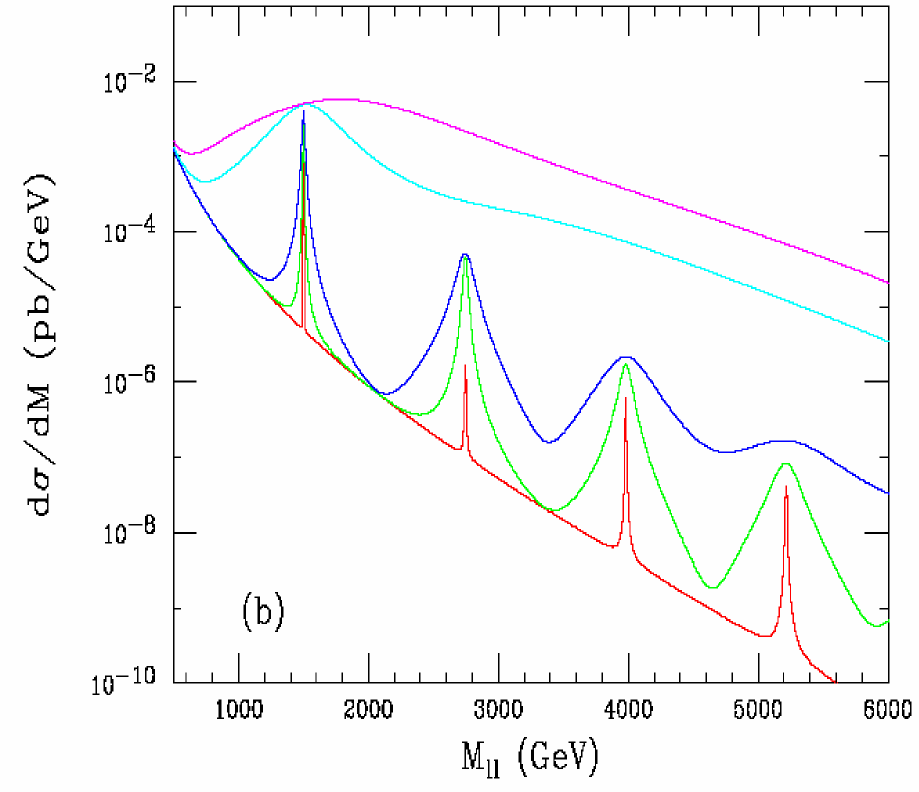
$e^+e^- \to \mu^+\mu^-$

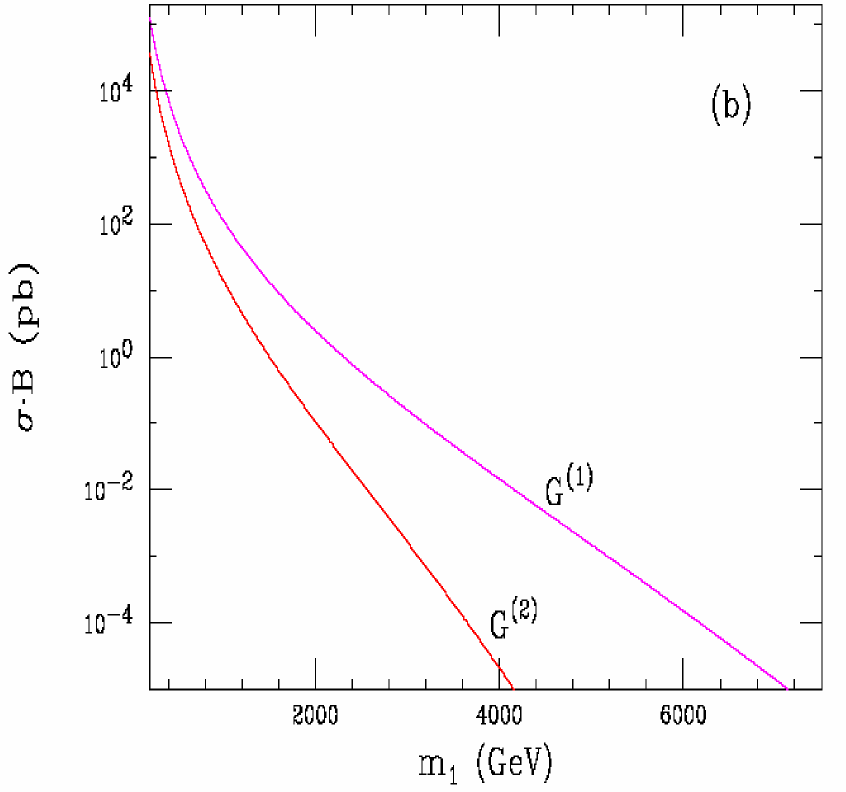


Different curves for k/M_{Pl} = 0.01 – 0.1

Unequal spacing signals curved space

Graviton KK Production @ LHC

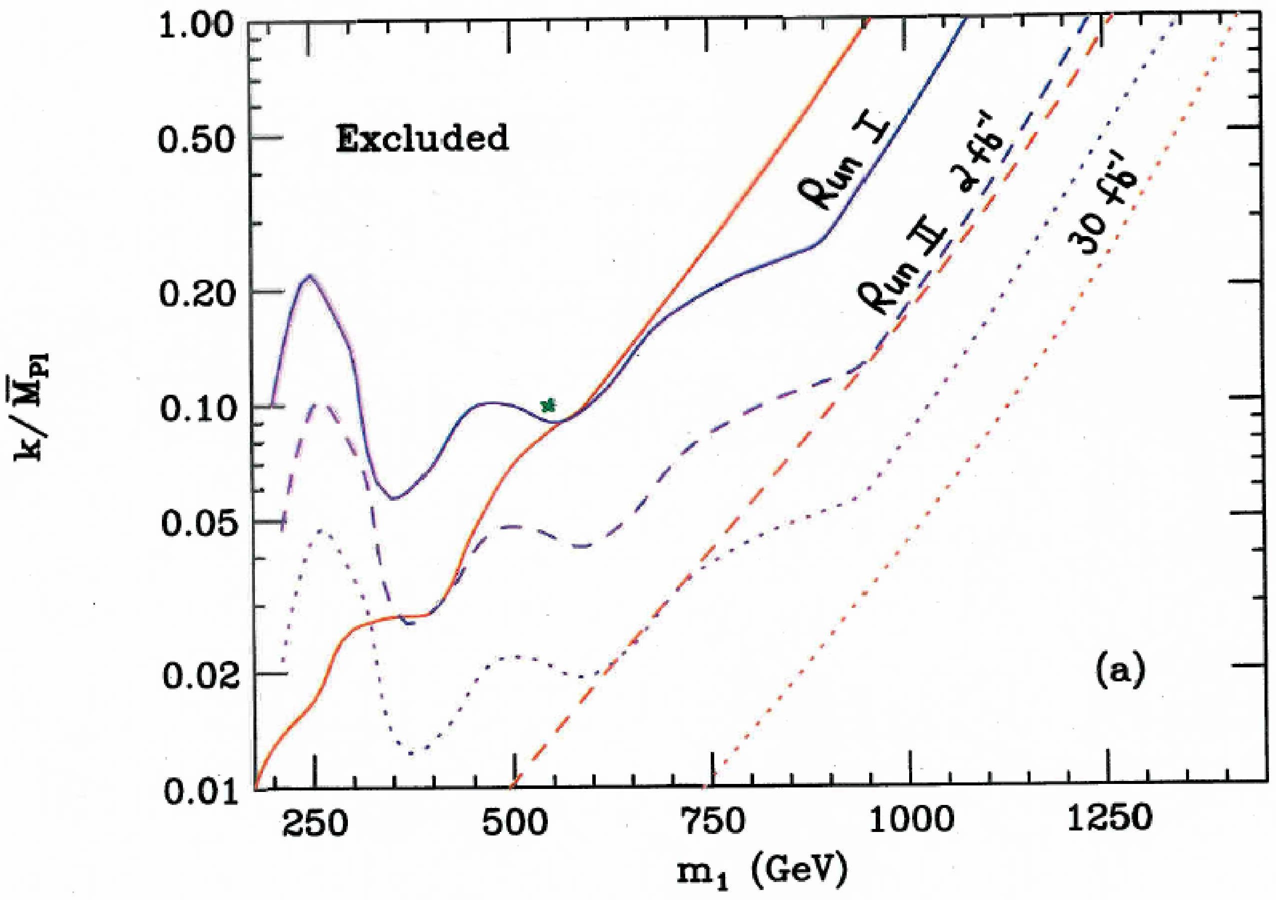




Different curves for $k/M_{Pl} = 0.01 - 1.0$

Davoudiasl, JLH, Rizzo

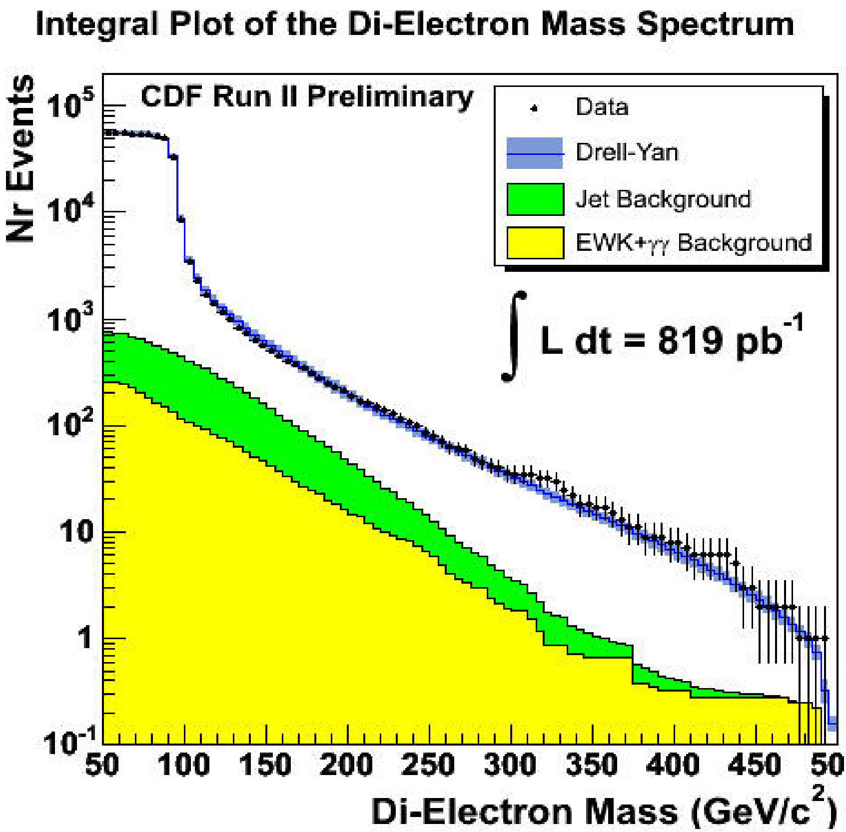
Tevatron Bump Search: Drell-Yan & Dijets

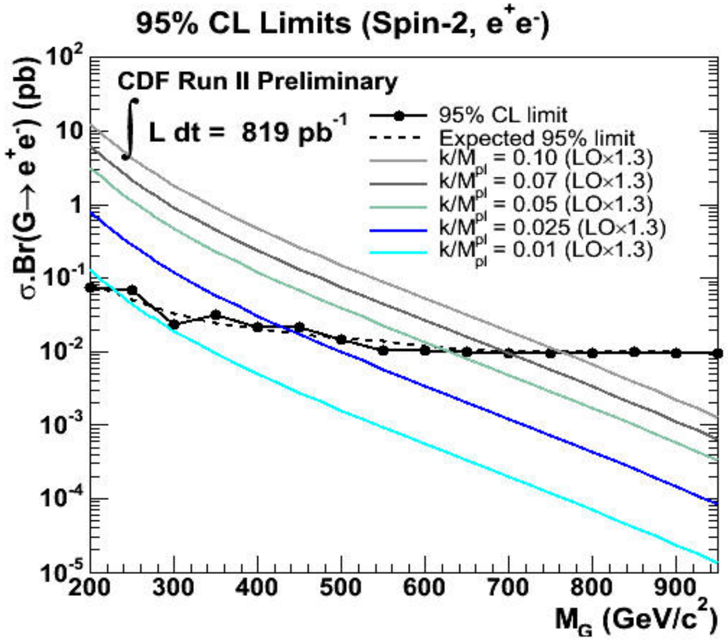


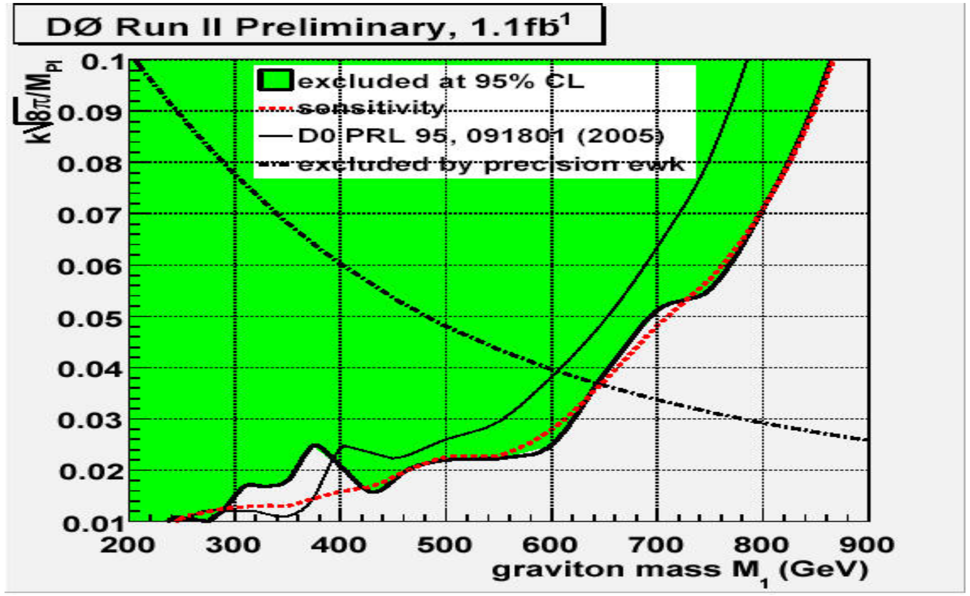
Davoudiasl, JLH, Rizzo

Tevatron limits on RS Gravitons

CDF Drell-Yan spectrum

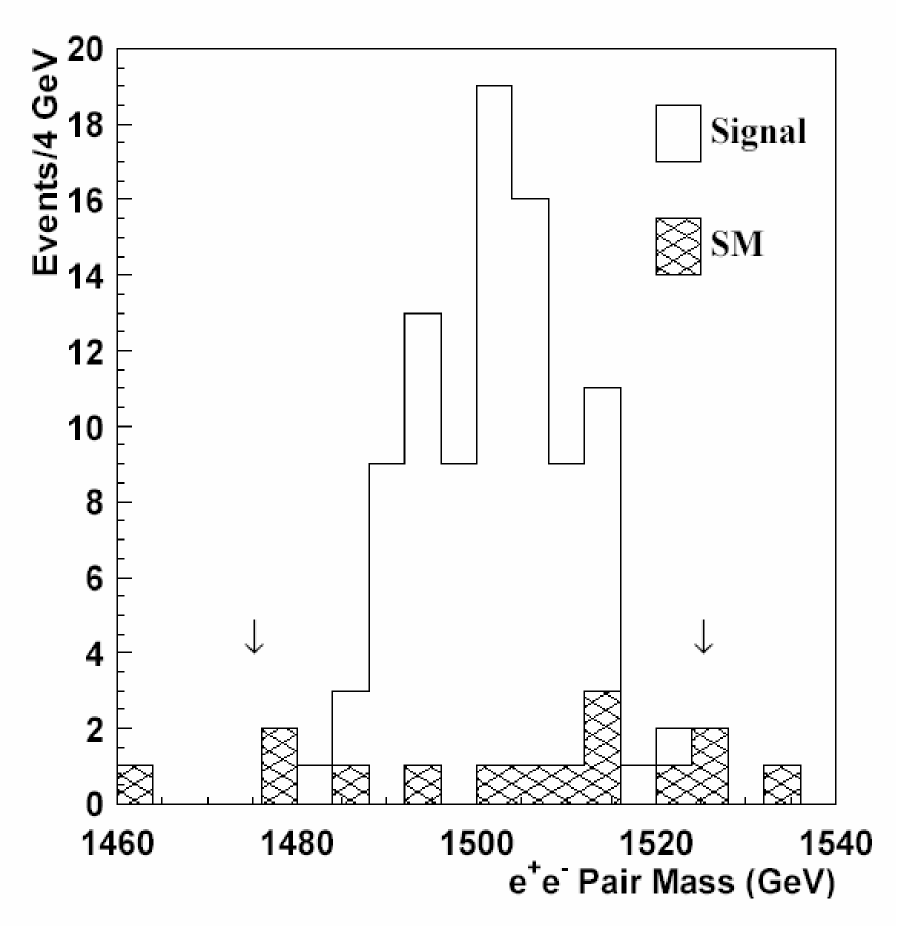






Graviton KK Search @ LHC: Issue = Narrow Width

ATLAS Simulation



Search Reach

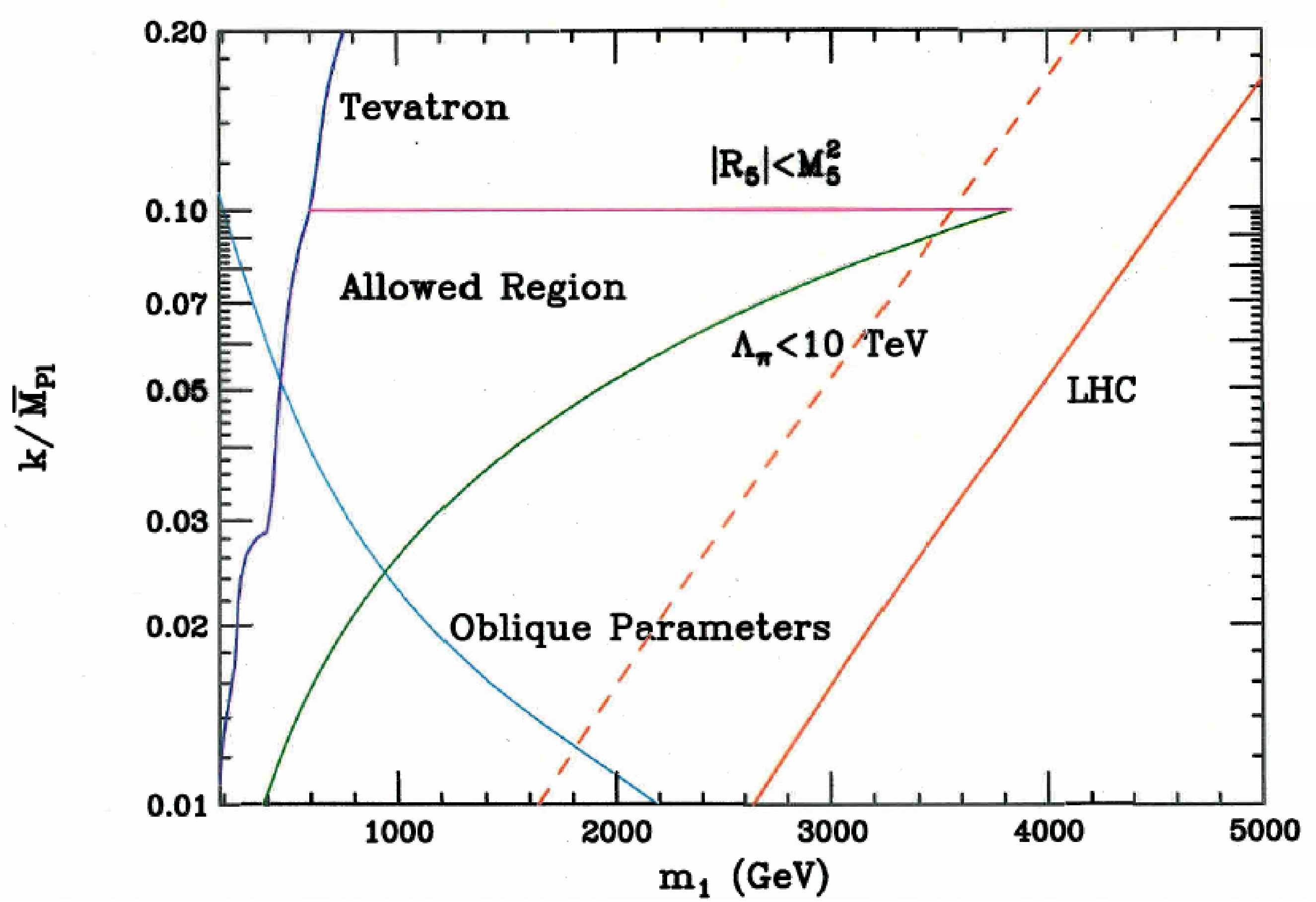
$m_1 > 1830$ GeV

for $k/M_{Pl} = 0.01$

With 100 fb^{-1}

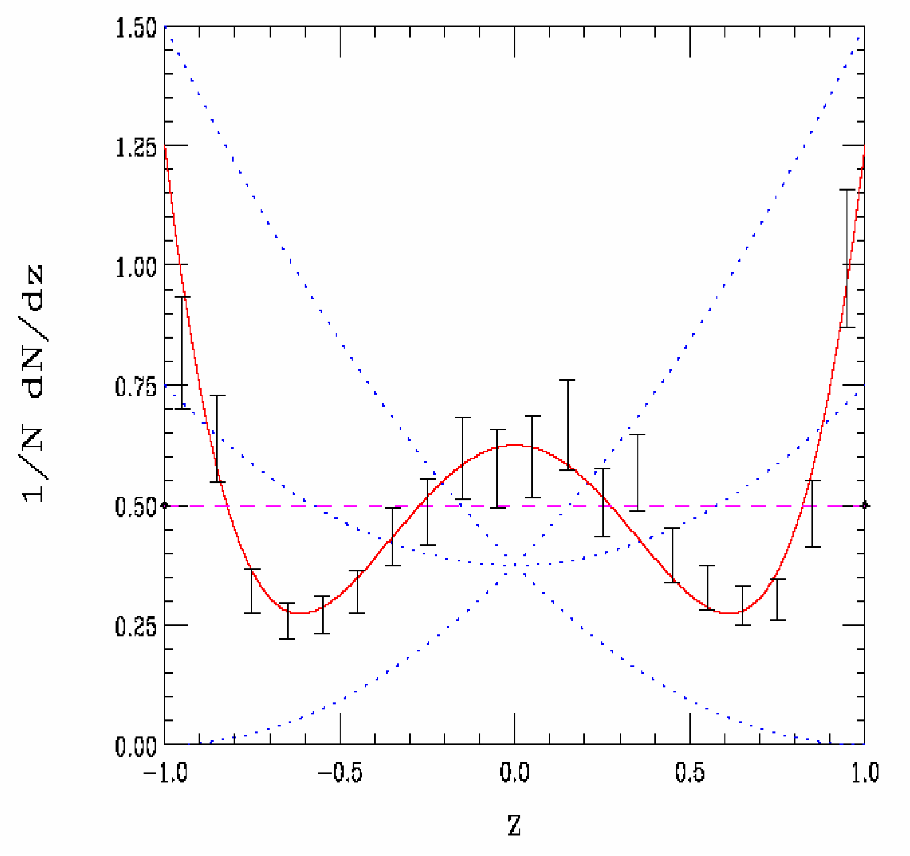
Allanach etal

Summary of Theory & Experimental Constraints

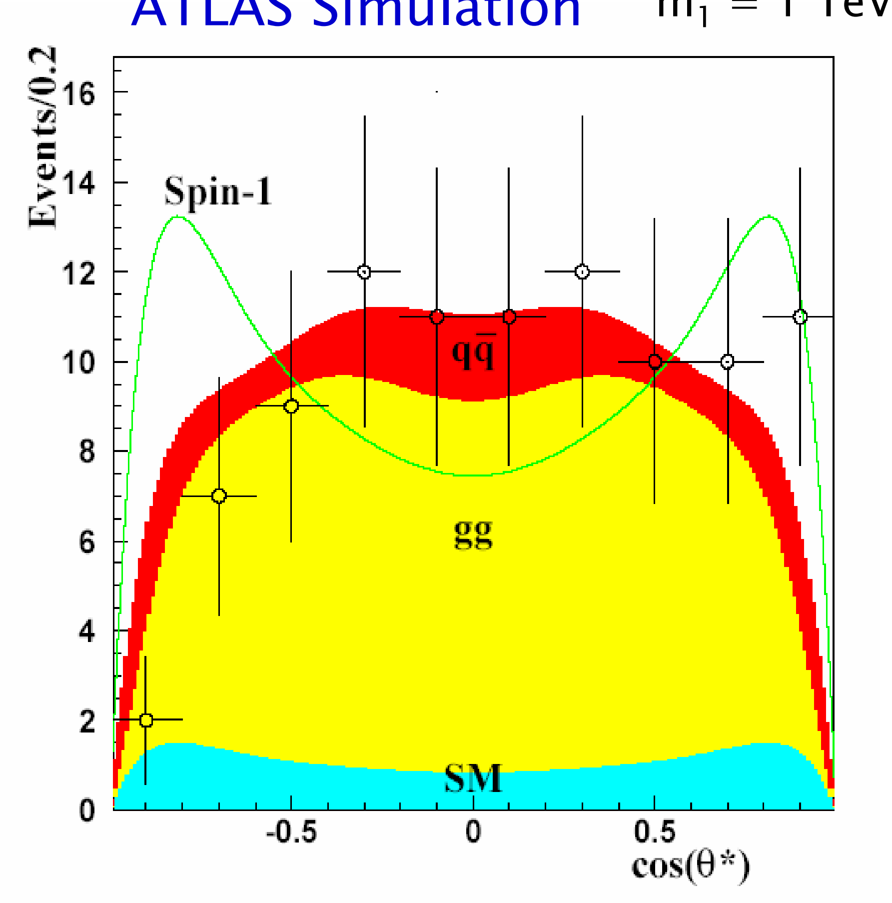


LHC can cover entire allowed parameter space!!

Spin-2 Determination



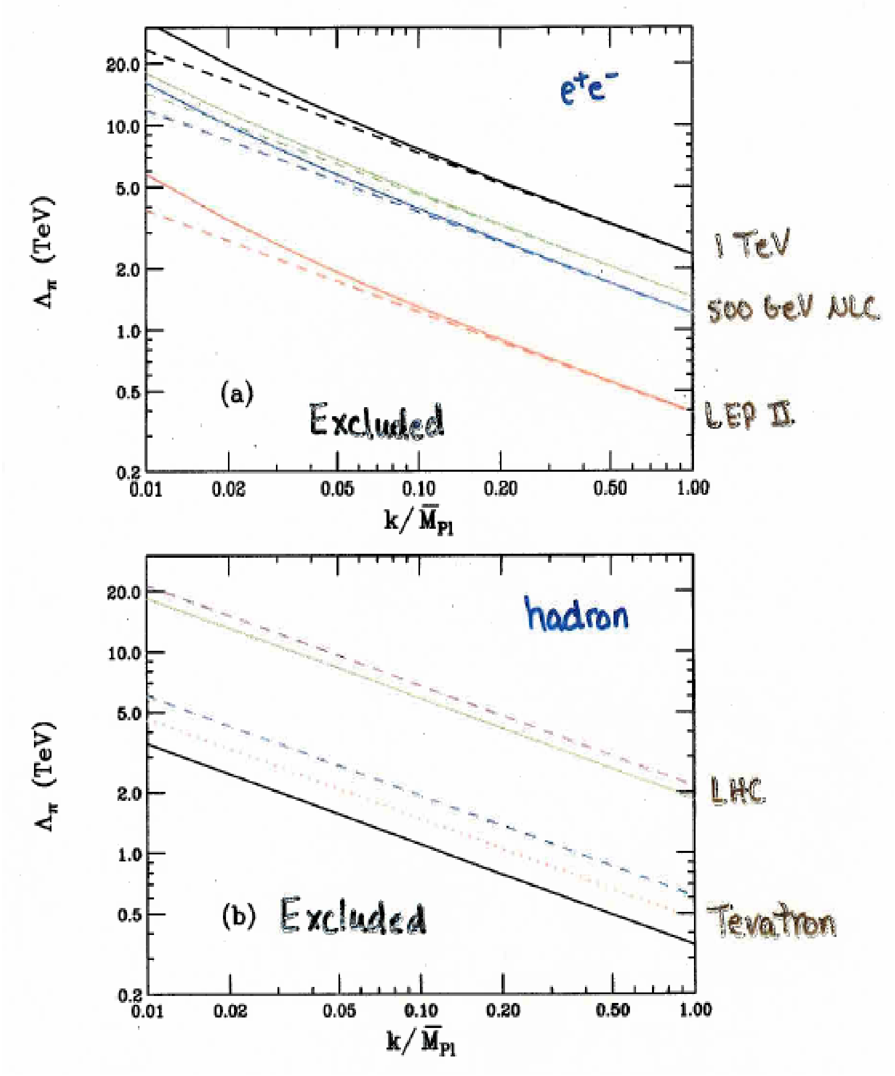
ATLAS Simulation m_1 = 1 TeV



Sit on resonance and measure angular distribution of lepton pair

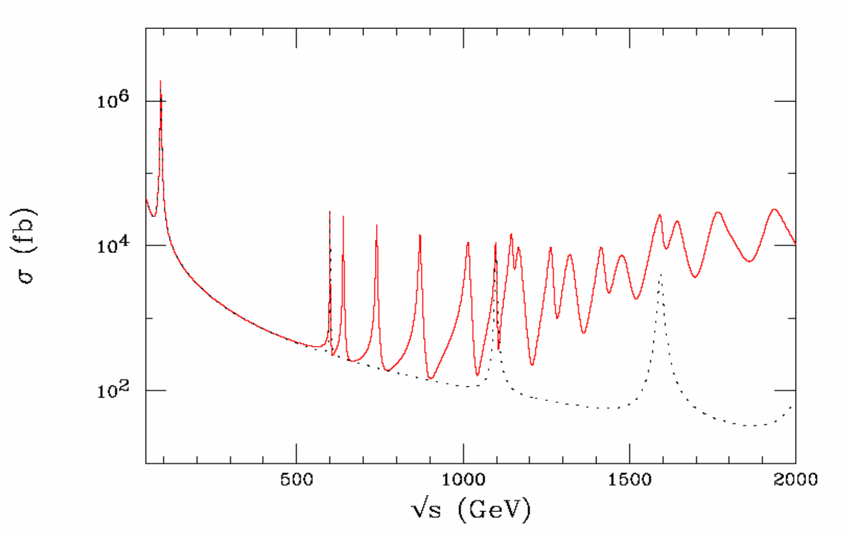
Allanach etal

Bounds from Contact Interaction Searches

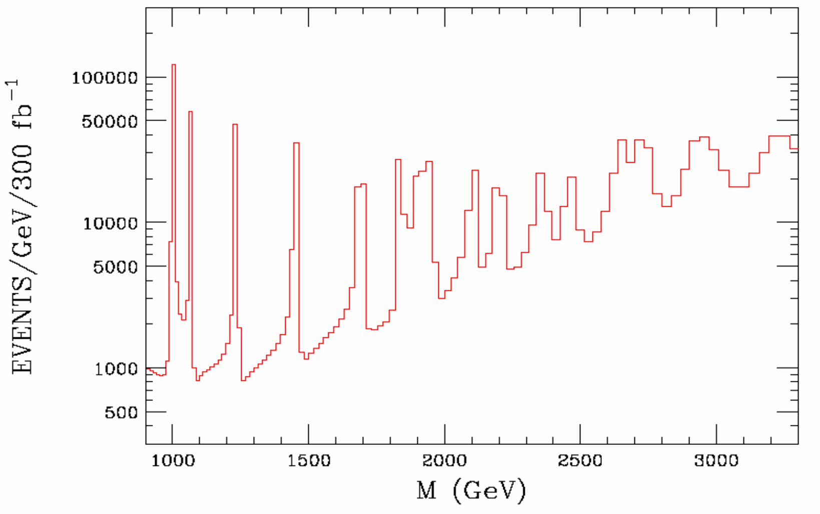


Extend Manifold: AdS_5 x S^δ

$e^+e^- \rightarrow \mu^+\mu^-$



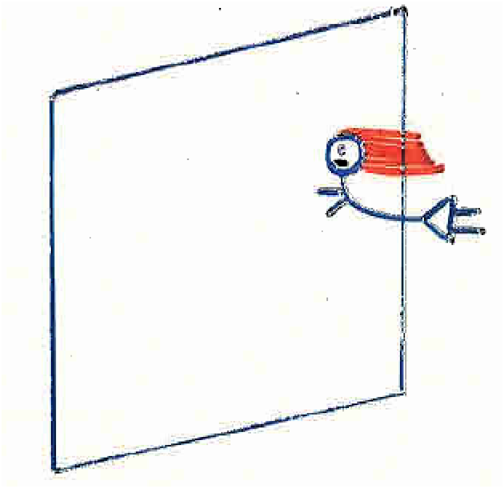
Drell-Yan



Gives a forest of KK graviton resonances!

Davoudiasl, JLH, Rizzo

Peeling the Standard Model off the Brane



- Model building scenarios require SM bulk fields
  - Gauge coupling unification
  - Supersymmetry breaking
  - ν mass generation
  - Fermion mass hierarchy

SM gauge fields alone in the bulk violate custodial symmetry!

Gauge boson KK towers have coupling $g_{KK} = 8.4 g_{SM}$!!

Precision EW Data Constrains: $m_1^A > 25$ TeV $\Rightarrow \Lambda_\pi > 100$ TeV!

Davoudiasl, JLH, Rizzo
Pomarol

Derivation of Bulk Gauge KK Spectrum

1) Gauge bosons in the bulk

(DHR
Pomarol)

$$S_A = -1/4 \int d^5x \sqrt{-G}\, G^{MK} G^{NL} F_{KL} F_{MN}$$

KK expand: $A_\mu(x,\phi) = \sum_n A_\mu^{(n)}(x) \frac{\chi_A^{(n)}(\phi)}{\sqrt{r_c}}$

integrate over ϕ + impose orthonormality

$$\Rightarrow \chi_A^{(n)} = \frac{e^\sigma}{N_n^A}\left[J_1(z_n^A) + \alpha_n^A Y_1(z_n^A)\right] \quad ; \quad z_n^A = \frac{m_n^A}{k} e^\sigma$$

$x_n^A = z_n^A(\phi = \pi)$ given by

$$J_1(x_n^A) + x_n^A J_1'(x_n^A) + \alpha_n^A \left[Y_1(x_n^A) + x_n^A Y_1'(x_n^A)\right] = 0$$

$$\mathcal{L}_{ffA} \sim g \bar{\psi} \gamma^\mu \psi \left[A_\mu^{(0)}(x) + \sqrt{2\pi k r_c} \sum_{n=1}^{\infty} A_\mu^{(n)}(x)\right]$$

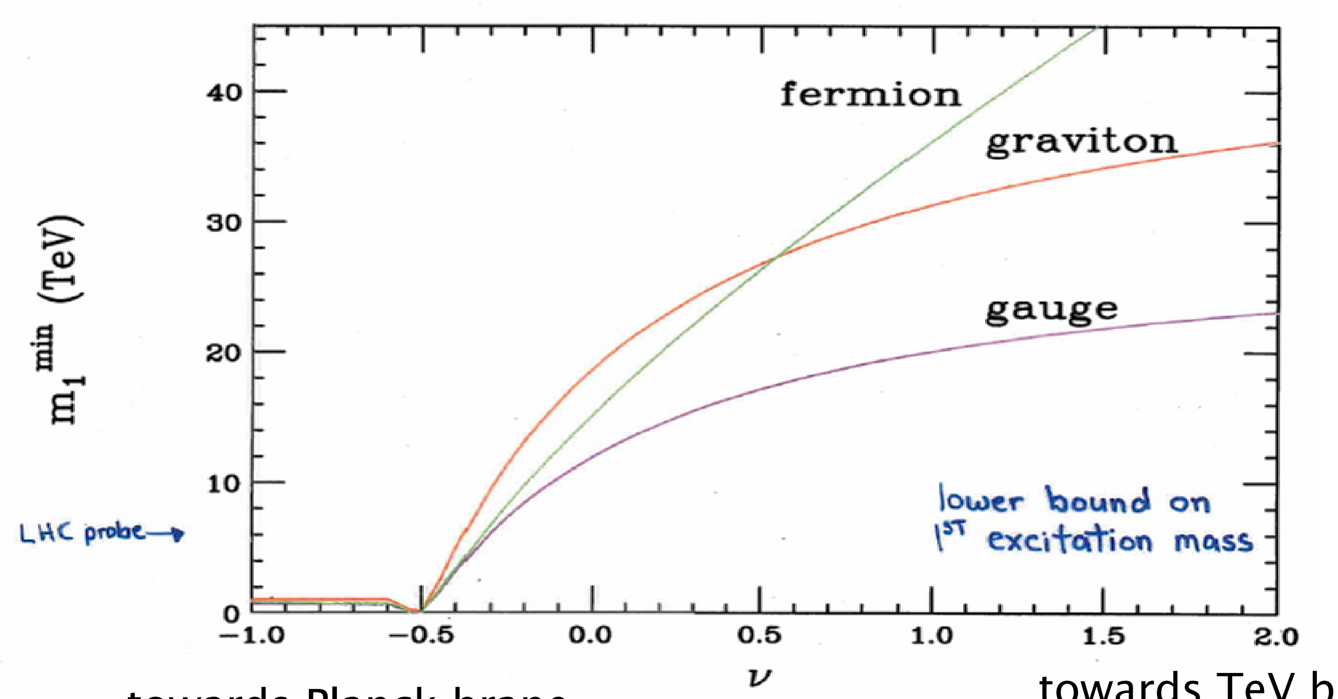
$$\Lambda_\pi = e^{-kr_c\pi}\, \overline{M}_{Pl} \sim \text{TeV} \quad \Rightarrow \quad kr_c \sim 11\text{-}12$$

$$g^{(n)} = \sqrt{2\pi k r_c}\, g^{(0)} \sim (8\text{-}9)\, g^{(0)} \;!$$

Fix 1: Add Fermions in the Bulk

- Introduces new parameter, related to fermion Yukawa
  - $m_f^{bulk} = \nu k$, with $\nu \sim O(1)$
- Zero-mode fermions couple weaker to gauge KK states than brane fermions

Precision EW Constraints



towards Planck brane

towards TeV brane

Notes: (RS) Fermions in the bulk

5-d Action:

$$S_f = \int d^4x \int d\phi \sqrt{G} \left[V_n^M \left(\frac{i}{2} \overline{\Psi}\, \gamma^n\, \partial_M \Psi + h.c. \right) - sgn(\phi) m \overline{\Psi}\Psi \right]$$

Required by Z_2 symmetry

$$\Psi_{L,R}(x,\phi) = \sum^{\infty} \psi_{L,R}^{(n)}(x)\, \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}}\, \hat{f}_{L,R}^{(n)}(\phi)$$

$$\int_{-\pi}^{\pi} d\phi\ e^{\sigma}\, \hat{f}_L^{(m)*}\, \hat{f}_L^{(n)} = \int_{-\pi}^{\pi} d\phi\ e^{\sigma}\, \hat{f}_R^{(m)*}\, \hat{f}_R^{(n)} = \delta^{mn}$$

choose $\hat{f}_L^{(n)}$ to be Z_2-even and $\hat{f}_R^{(n)}$ to be Z_2-odd

$$\hat{f}_{L,R}^{(n)}(\phi) = \frac{e^{\sigma/2}}{N_n^{L,R}} \left[J_{\frac{1}{2}\mp\nu}(z_n^{L,R}) + \beta_n^{L,R}\, Y_{\frac{1}{2}\mp\nu}(z_n^{L,R}) \right]$$

$$\hat{f}_L^{(0)} = \frac{e^{\nu\sigma}}{N_0^L}$$

Doublet		Singlet	
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T_L^{(2)}$	$T_R^{(2)}$	$t_L^{(2)}$	$t_R^{(2)}$
$T_L^{(1)}$	$T_R^{(1)}$	$t_L^{(1)}$	$t_R^{(1)}$
$T_L^{(0)}$	X	X	$t_R^{(0)}$

Hmwk: Repeat this derivation for flat space

Fix 2: Enlarge EW gauge group to $SU(2)_L \times SU(2)_R$

Agashe, Sundrum etal

Brane Kinetic Terms

- Originally introduced to allow infinite 5th dim to recover 4-d behavior at short distances

Dvali, Gabadadze, Porrati, Shifman

- Generated at loop-order from brane quantum effects from presence of $[S^1/Z_2]$ orbifold and/or matter fields on the brane

- Required as brane counter terms for bulk quantum effects

Georgi, Grant, Hailu

⇒ Brane Kinetic terms are naturally present!

Size is determined by the full UV theory – as of yet unknown.

$$S_{Gravity} = \frac{m_5^3}{4} \int d^4x \int r_c \, d\phi \, \sqrt{-g} \, \Big\{ R^{(5)} \longrightarrow \text{bulk piece}$$

$$+ \left[g_0 \, \delta(\phi) + g_\pi \, \delta(\phi - \pi) \right] R^{(4)} + \cdots \Big\}$$

induced brane kinetic terms

Higgsless EWSB

Csaki, Grojean,Murayama, Pilo, Terning

What good is a Higgs anyway??

- Generates W,Z Masses
- Generates fermion Masses
- Unitarizes scattering amplitudes $(W_L W_L \rightarrow W_L W_L)$

Do we really need a Higgs?

And get everything we know right….

Our laboratory: Standard Model in 1 extra warped dimension

$\Rightarrow$ Minimal Particle Content!

Generating Masses

Consider a massless 5-d field

$$\partial^2\phi = (\partial_\mu\partial^\mu - \partial_5\partial^5)\, \phi = 0$$

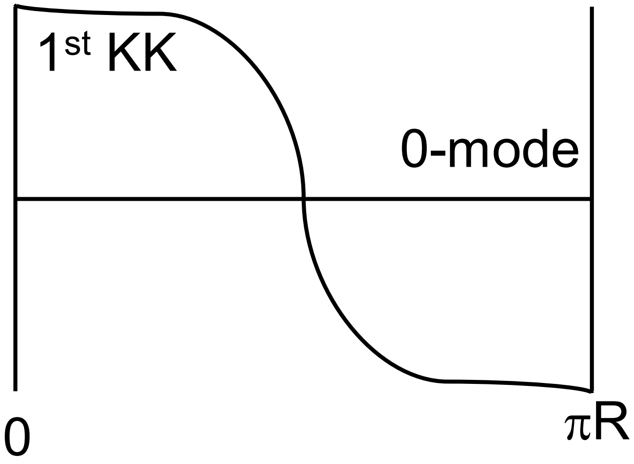
looks like $(\partial_\mu\partial^\mu - m_n^2)\, \phi = 0$ in 4-d (KK tower)

The curvature of the 5-d wavefunction $\phi(y)$ is related to its mass in 4-d

Toy Example: Flat space with U(1) gauge field in bulk with S^1/Z_2 Orbifold

$A_\mu(y) \sim \cos(ny/R)$

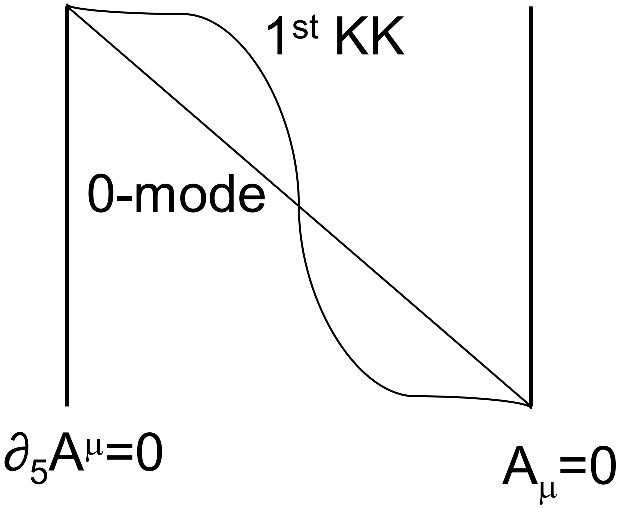
$A_5(y) \sim \sin(ny/R)$



0-mode is flat & y independent

$\Rightarrow m_0 = 0$

If **The Same** boundary conditions are applied at both boundaries, 0-mode is massless and U(1) remains unbroken



Orbifold Boundary Conditions:

$$\partial_5 A\mu = 0$$
$$A_5 = 0$$

A^μ cannot be flat with these boundary conditions!

$$A(y) \sim \Sigma_n\ a_n \cos(m_n y) + b_n \sin(m_n y)$$

$$\partial_5 A(y) \sim m_n \Sigma_n\ (-a_n \sin(m_n y) + b_n \cos(m_n y)$$

BC's: $A(y=0) = 0 \quad \Rightarrow a_n = 0$

$\partial_5 A(y=\pi R) = 0 \quad \Rightarrow \cos(m_n \pi R) = 0$

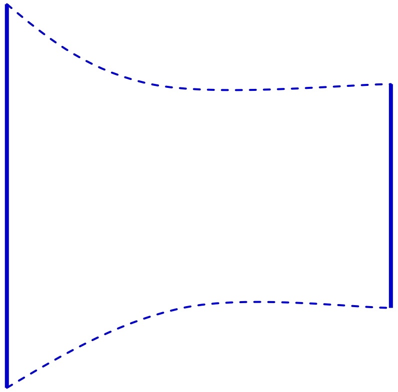
$$\boxed{m_n = (n + ½)/R}$$

The zero mode is massive!
A_5 acts as a Goldstone
U(1) is broken

Realistic Framework:

Agashe etal hep-ph/0308036
Csaki etal hep-ph/0308038

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in 5-d Warped bulk



Planck brane

TeV-brane

BC's restricted by variation of the action at boundary

$SU(2)_L \times SU(2)_R \rightarrow SU(2)_D$

SU(2) Custodial Symmetry is preserved!

$SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$

$W_R^{\pm}$, Z_R get Planck scale masses

$W^{\pm}$, Z get TeV scale masses
γ left massless!

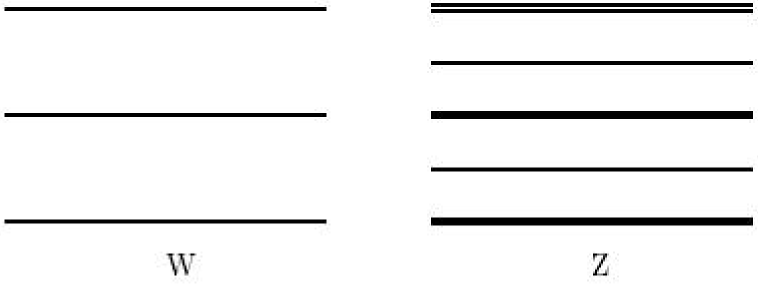
Parameters: $\kappa = g_{5R}/g_{5L}$ (restricted range)
$\delta_{L,Y,B,D}$ brane kinetic terms
g_{5L} fixed by G_F, g_{5B}/g_{5L} fixed by M_Z

Gauge KK Spectrum

$\psi_n \sim z[a_n J_1(m_n z) + b_n Y_1(m_n z)]$, $z = e^{ky}/k$

Masses are fixed by model parameters

Schematic KK Spectra



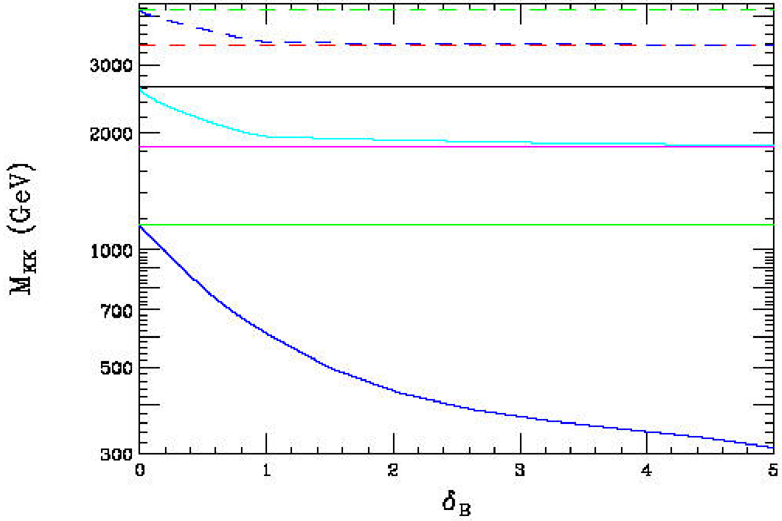
Every other neutral gauge KK level is degenerate!

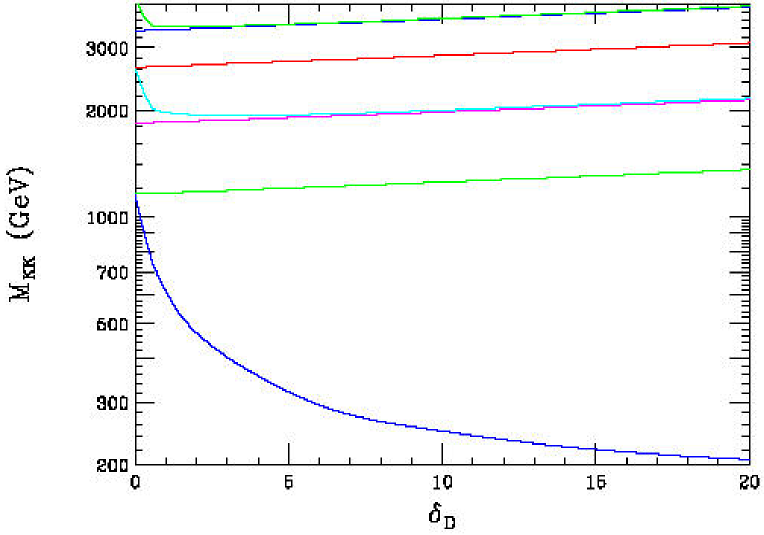
Brane terms split this degeneracy
And give lighter KK states

Davoudiasl, JLH, Lillie, Rizzo
hep-ph/0312193,0403300

Effects of Brane terms

$\kappa = 1$

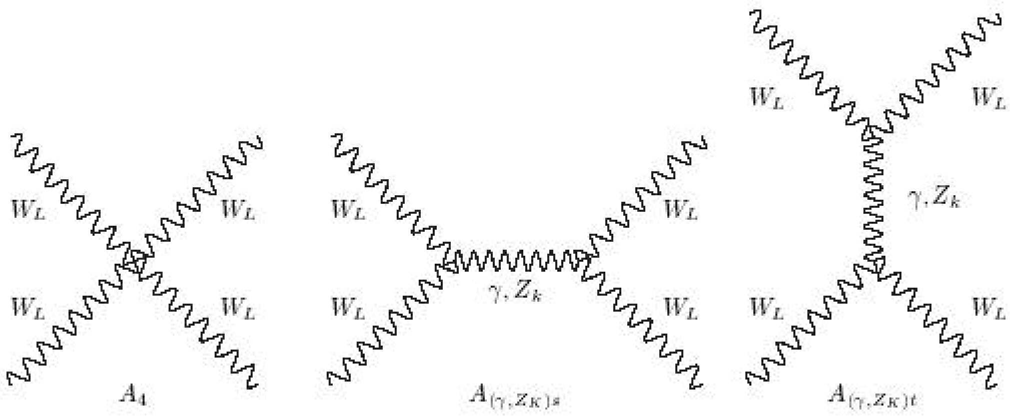




Unitarity in Gauge Boson Scattering

- SM without Higgs violates perturbative unitarity in $W_L W_L \rightarrow W_L W_L$ at $\sqrt{s} \sim 1.7$ TeV
- Higgs restores unitarity if $m_H <$ TeV
  What do we do without a Higgs??

Exchange gauge
KK towers:



Conditions on KK masses & couplings:

$(g_{1111})^2 = \Sigma_k \, (g_{11k})^2$

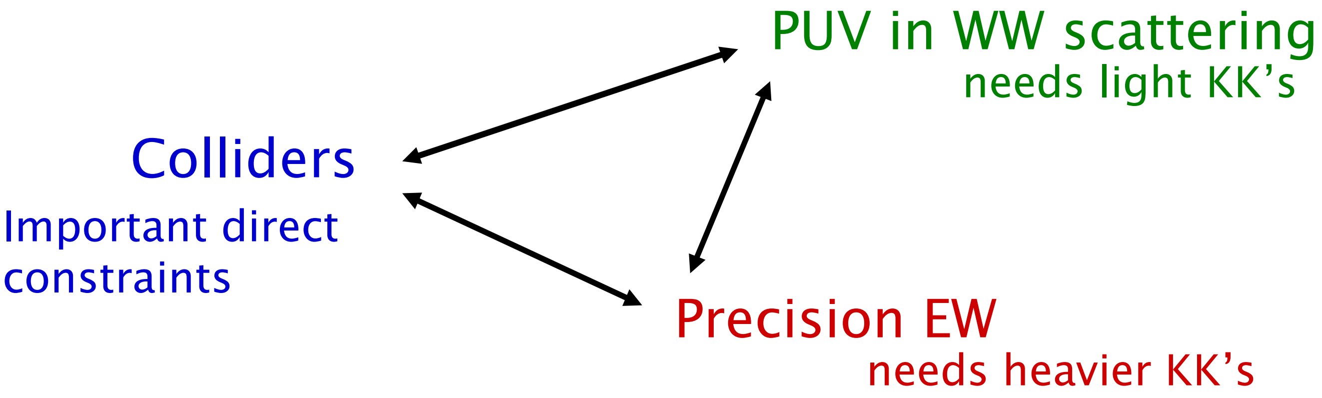
Csaki etal, hep-ph/0305237

$4(g_{1111})^2 \, M_1{}^2 = \Sigma_k \, (g_{11k})^2 \, M_k{}^2$

Necessary, but not sufficient, to guarantee perturbative unitarity!

What are the preferred gauge KK masses?

Tension Headache:



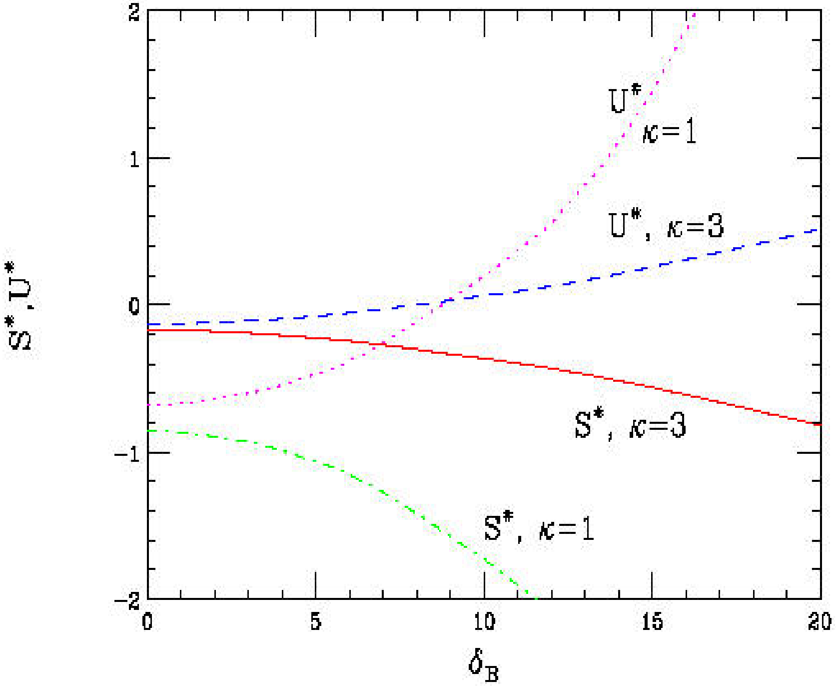
PUV in WW scattering
needs light KK's

Colliders
Important direct
constraints

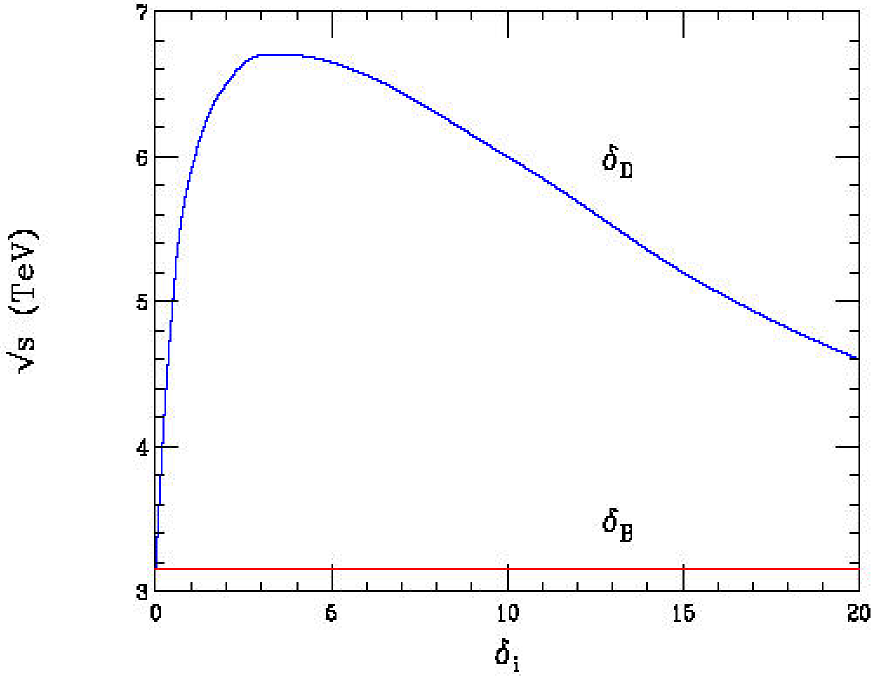
Precision EW
needs heavier KK's

Is there a consistent region of parameter space?

Precision EW pseudo-oblique parameters



Scale of unitarity violation in W_L scattering

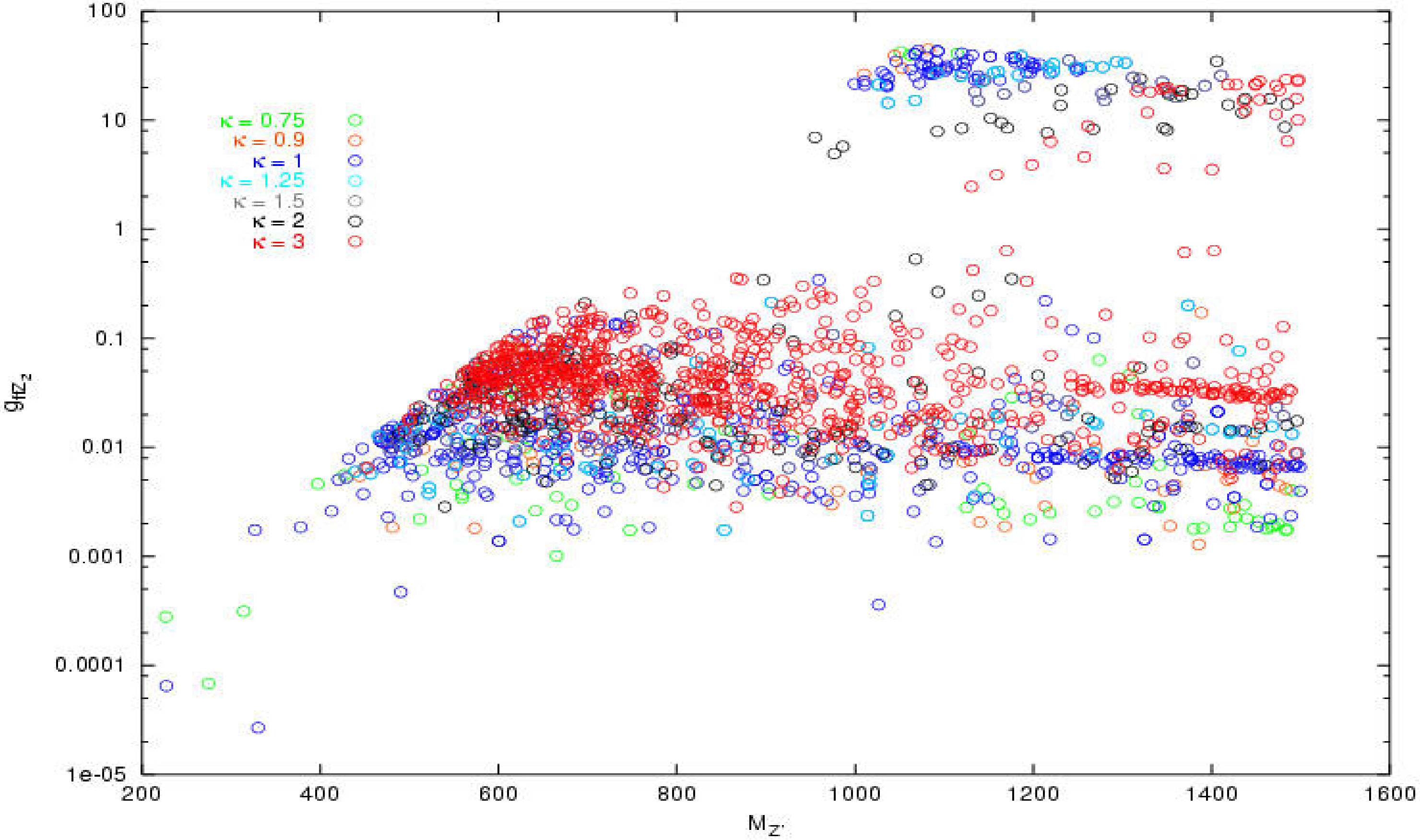


Davoudiasl, JLH, Lillie, Rizzo
hep-ph/0312193,0403300

Monte Carlo Exploration of Parameter space

Over 3M points scanned

Points which pass all constraints
except PUV: None passed PUV!



Prefers light Z’ with small couplings
Perfect for the Tevatron RunII and LC!!

JLH, Lillie, Rizzo
hep-ph/0407059

Little Higgs: The Basics

- The Higgs becomes a component of a larger multiplet of scalars, Σ
- Σ transforms non-linearly under a new global symmetry

- New global symmetry undergoes SSB
  - $\Rightarrow$ leaves Higgs as goldstone
- Part of global symmetry is gauged
  - $\Rightarrow$ Higgs is pseudo-goldstone

- Careful gauging removes Higgs 1-loop divergences

$$\delta\, m_h{}^2 \sim \frac{\Lambda^2}{(16\pi^2)^2} \;, \quad \Lambda > 10\ \mathrm{TeV}, \quad @\ 2\text{-loops!}$$

Minimal Model: The Littlest Higgs

Arkani-Hamed,
Cohen, Katz, Nelson

$\Lambda > 10$ TeV: non-linear σ model is strongly-coupled

Λ 10 TeV:

- Global Symmetry: SU(5) → SO(5) via SSB with $\langle\Sigma_0\rangle$
  $\Sigma(x) = e^{2i\Pi/f}\,\Sigma_0, \quad \Pi = \Sigma_a\, \pi^a(x) X^a \quad \Rightarrow$ 14 Goldstone Bosons
  $f \sim \Lambda/4\pi =$ G.B. decay constant ~ TeV
- Gauged Symmetry: $G_1 \times G_2$
  $[SU(2) \times U(1)]^2 \rightarrow SU(2)_L \times U(1)_Y$ via SSB with $\langle\Sigma_0\rangle$
  $W_H^{\pm}, Z_H, B_H$ acquire mass ~ f
  $W^{\pm}, W_3^0, B^0$ remain massless

14 Goldstone Bosons $\Rightarrow$ 4 eaten under SSB
complex triplet φ
complex doublet h
} massless at tree-level

φ Acquires mass at 1-lopp via gauge interactions ~ f

h acquires mass at 2-loops ~ $f/4\pi$

3-Scale Model

$\Lambda > 10$ TeV: New Strong Dynamics

? UV Completion ?

Global Symmetry

$f \sim \Lambda/4\pi \sim$ TeV: Symmetires Broken

Pseudo-Goldstone Scalars

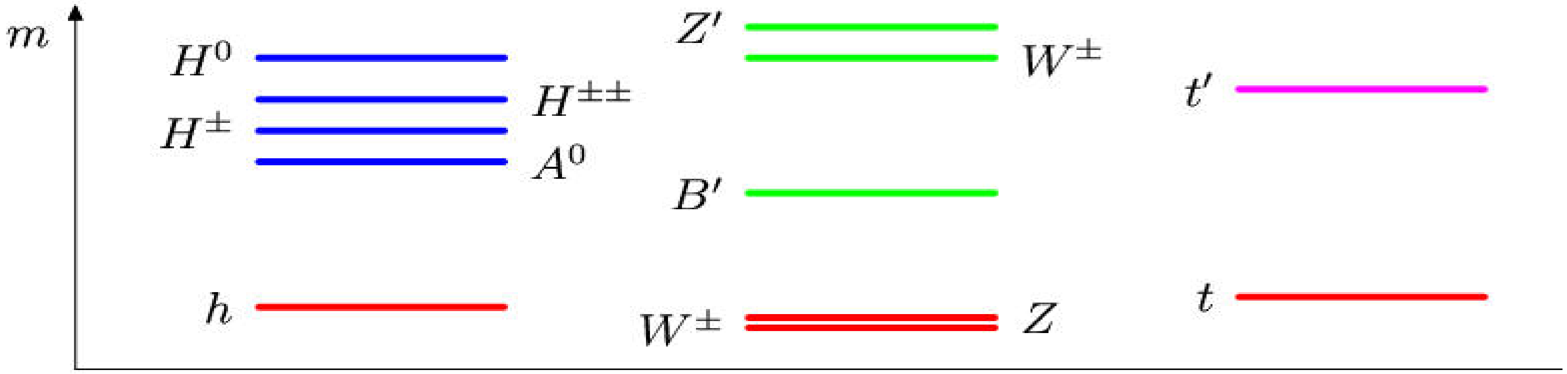
New Gauge Fields

New Fermions

$v \sim f/4\pi \sim 100$ GeV: Light Higgs

SM vector bosons & fermions

Sample Spectrum

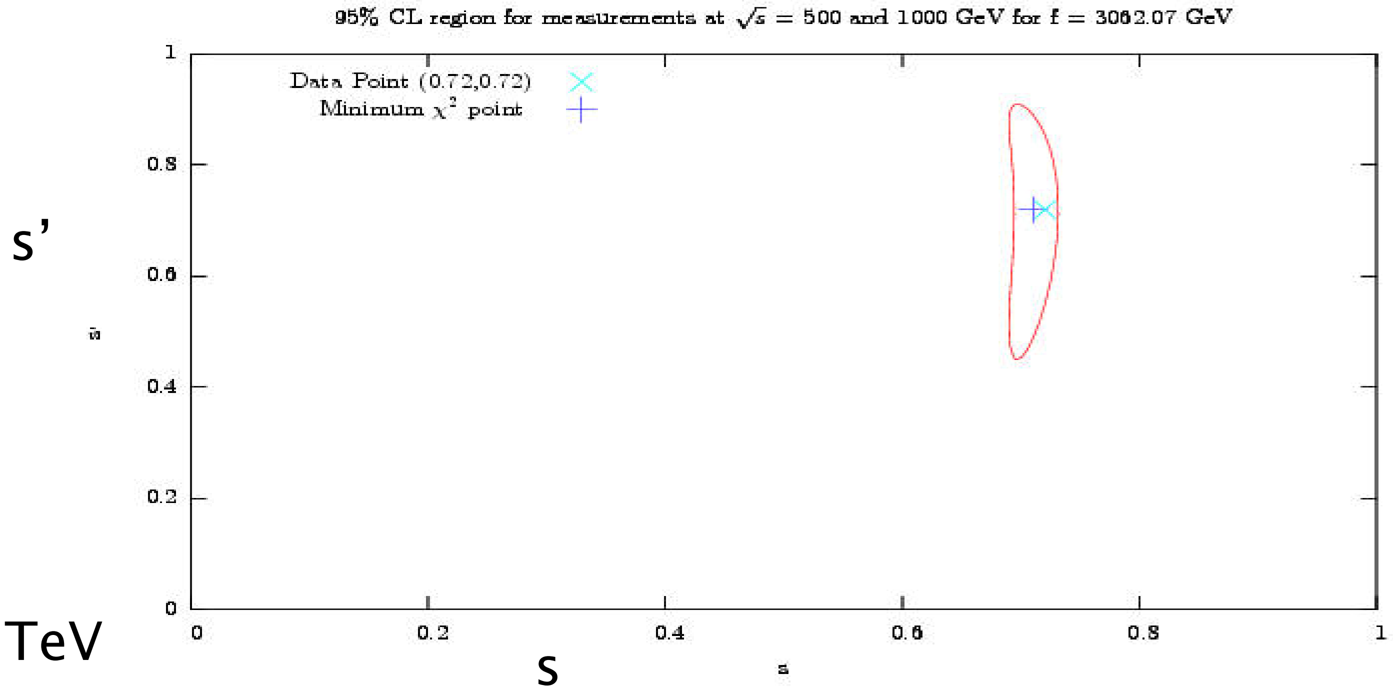


Hallmark of Little Higgs: Determine Couplings of new fields to the Higgs

Perelstein, Peskin, Pierce

- TTh coupling measured at LHC
- $Z_H Zh$ coupling measured at LC in $e^+e^- \rightarrow Zh$

Z_H observed at LHC $\Rightarrow$ mass is known
Couplings depend on 2 parameters: s and s'
Perform a 2-parameter fit



Conley, JLH, Le

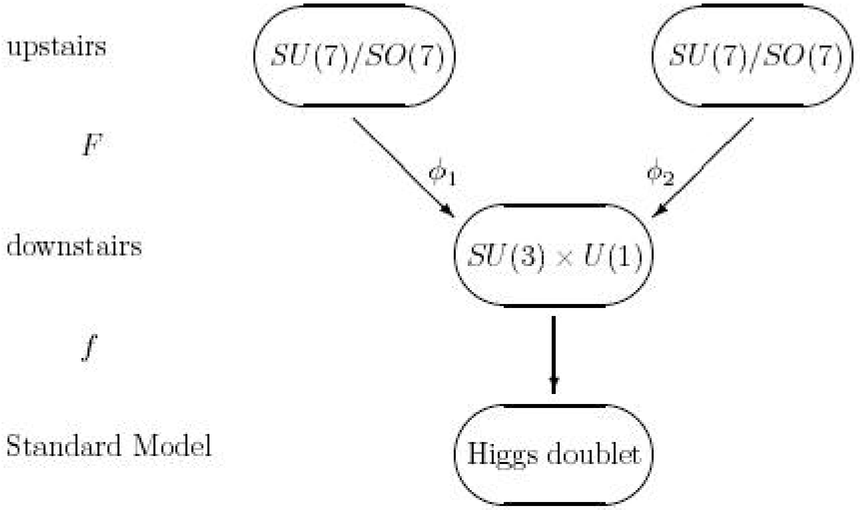
$M_{Z_H} = 2$ TeV

A UV Completion

Kaplan, Schmaltz, Skiba

Keep stacking Little Higgs Theories

- Upstairs Little Higgs: Strongly coupled @ Λ ~ 100 TeV
  non-linear σ model
  symmetry breaks @ F ~ 10 TeV
- Downstairs Little Higgs: Weakly coupled @ Λ ~ 10 TeV
  linear σ model
  symmetry breaks @ f ~ 1 TeV



Summary of Extra Dimensions

- Many models of extra dimensions exist!
- Extra dimensions were founded to resolve the hierarchy, but now stand on their own for answering many open questions of the Standard Model
- Extra dimensions which resolve the gauge hierarchy are testable at the LHC/ILC. These models can be proved or disproved regarding their relevance to the hierarchy
- If discovered, collider measurements can reveal many properties of extra dimensions
- If discovered, our view of the universe will be forever changed.

Summary of Physics Beyond the Standard Model

- There are many ideas for scenarios with new physics! Most of our thinking has been guided by the hierarchy problem
- They must obey the symmetries of the SM
- They are testable at the LHC
- We are as ready for the LHC as we will ever be
- The most likely scenario to be discovered at the LHC is the one we haven't thought of yet.

Exciting times are about to begin.
Be prepared for the unexpected!!

Fine-tuning does occur in nature

2001 solar eclipse as viewed from Africa

Most Likely Scenario @ LHC:

H. Murayama