The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
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International Atomic
Energy Agency

SMR 1773 - 13

SCHOOL ON PHYSICS AT LHC: "EXPECTING LHC"
11 - 16 September 2006

Higgs bosons searches at LHC
Part IV

(Excercises)

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ICTP School: Expecting LHC
Trieste, September 11 - 16, 2006.

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EXERCISES

Exercise I: Basic calcultaional rules

1. Vertices and propagators
2. Dirac algebra
3. Cross sections and decay widths
4. Calculation of loop integrals.

Exercise II: Higgs boson production and decay mechanisms.

1. Higgs decays into fermions and gauge bosons
2. Higgs production in e^+e^- collisions
3. Higgs production in hadronic collisions
4. Higgs contributions to radiative corrections.

Exercise III: Divergences and symmetries.

1. Lagrangians and interactions
2. Electron and photon self-energies
3. Higgs boson self–energy: fermion loops
4. Higgs boson self–energy: scalar contributions.

Exercise IV: Higgs masses and couplings in the MSSM.

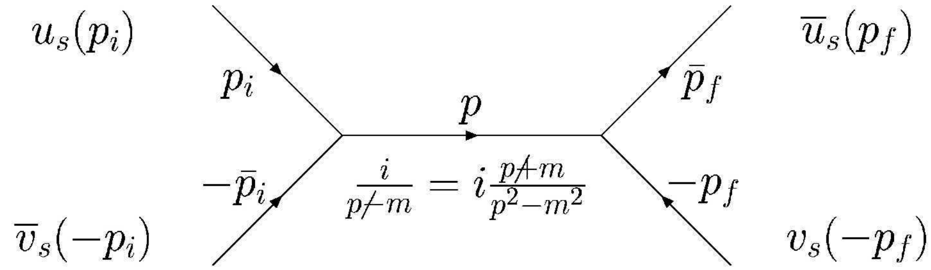
1. Derivation of the Higgs boson masses in the MSSM
2. Radiative corrections to the Higgs boson masses
3. The MSSM Higgs boson couplings

For details on phneomenology, see AD: The anatomy of EWSB hep-ph/0503172 (SM) and hep-ph/0503173 (MSSM), to appear in Physics Reports.

E1: Basic Calculational Rules

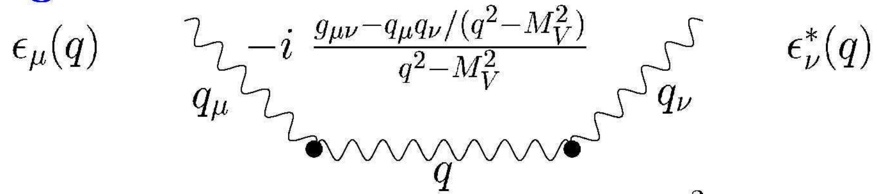
1. Vertices and propagators

Rules for fermions:



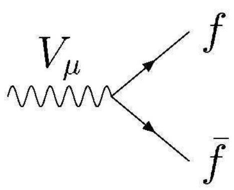
$$\Sigma_s\, u_s(p)\, \bar{u}_s(p) = \not{p} + m \quad , \quad \Sigma_s\, v_s(p)\, \bar{v}_s(p) = \not{p} - m$$

Rules for gauge bosons:



$$\Sigma_{\rm pol} = \epsilon^*_\nu \epsilon_\mu = -(g_{\mu\nu} - q_\mu q_\mu/M_V^2)$$

For the photon, discard everything which is longitudinal $(q^\mu q^\nu)$ **above. Note that the trasversality of the photon implies:** $\epsilon_\mu \cdot q^\mu = 0$.



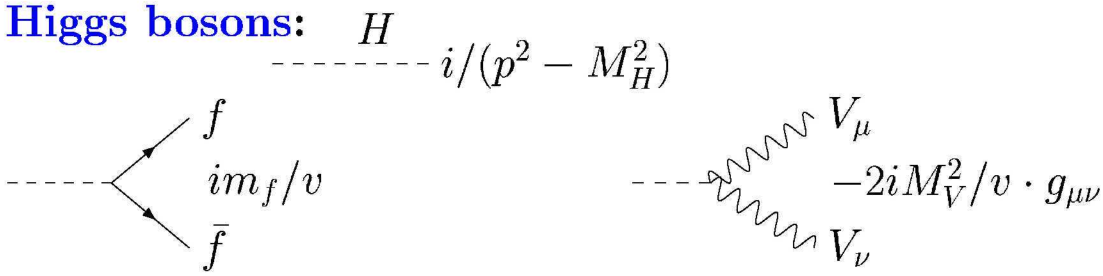
$$-ie\gamma_\mu(v_f - a_f\gamma_5)$$

$Z\ :\ v_f = (2I_f^3 - 4e_f s_W^2)/(4s_W c_W)\ ,\ a_f = 2I_f^3/(4s_W c_W)$

$W\ :\ v_f = a_f = 1/(2\sqrt{2}s_W)$

$\gamma\ :\ \ v_f = e_f\ ,\ a_f = 0$

Rules for Higgs bosons:



2. Diracology: contractions and traces of γ matrices

Basic relations:

$$\{\gamma_\mu\,,\,\gamma_\nu\} = \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu} \quad \textit{and} \quad \not{p} = p_\mu\gamma^\mu$$

$$\gamma_5 = \frac{i}{4!}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = i\gamma^0\gamma^1\gamma^2\gamma^3 \text{ and } \{\gamma_\mu, \gamma_5\} = 0$$

$$\mathrm{Tr}(\mathbf{1}) = 4\ ,\ \mathrm{Tr}(\gamma_\mu) = 0\ ,\ \mathrm{Tr}(\gamma_5) = 0$$

$$\mathrm{Tr}(A_1A_2) = \mathrm{Tr}(A_2A_1)\ ,\ \mathrm{Tr}(A_1A_2\cdots A_N) = \mathrm{Tr}(A_2\cdots A_NA_1)$$

Contractions of γ matrices

$$\begin{aligned}
\gamma_\mu\gamma^\nu &= 2g^\nu_\mu - \gamma_\mu\gamma^\nu \Rightarrow \gamma^\mu\gamma_\mu = \delta^\mu_\mu = 4 \\
\gamma^\mu\gamma_\nu\gamma_\mu &= \gamma^\mu(2g_{\mu\nu} - \gamma_\mu\gamma_\nu) = 2\gamma_\nu - 4\gamma_\nu = -2\gamma_\nu \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= (2g^{\mu\nu} - \gamma^\nu\gamma^\mu)(2g^\rho_\mu - \gamma_\mu\gamma^\rho) \\
&= 4g^{\nu\rho} - 2\gamma^\nu\gamma^\rho - 2\gamma^\nu\gamma^\rho + 4\gamma^\nu\gamma^\rho = 4g^{\nu\rho}
\end{aligned}$$

Traces of γ matrices:

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = \mathrm{Tr}(2g^{\mu\nu} - \gamma^\mu\gamma^\nu) = 2g^{\mu\nu}\mathrm{Tr}(1) - \mathrm{Tr}(\gamma^\mu\gamma^\nu) \Rightarrow \mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$$

Trace of an odd number n of γ matrices (using $\gamma_5^2 = 1$):

$$\mathrm{Tr}(\gamma^{\mu_1}\cdot\gamma^{\mu_n}) = \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5\gamma^5) = (-1)\,\mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^5\gamma^{\mu_n}\gamma^5)$$

$$= (-1)^n\,\mathrm{Tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5) = -\,\mathrm{Tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5)$$

$$\Rightarrow \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}) = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma_5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_5) = \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5) = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = \frac{1}{4}\mathrm{Tr}(\gamma^\alpha\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5) = (1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5\gamma^\alpha)$$

$$= -(1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma^\alpha\gamma_5) = -\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = 0$$

Using the same tricks as above, proof the trace of 4 γ matrices:

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4i\epsilon^{\mu\nu\sigma\rho}$$

3. Cross sections and decay widths

The differential cross section for a $2 \times n$ process $i_1 i_2 \to f_1 \cdots f_n$ is

$$d\sigma = \frac{|M(i_1 i_2 \to f_1 .. f_n)|^2}{4[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2}} \left(\Pi_n \frac{d^3 p_f}{(2\pi)^3 2 e_f} \right) (2\pi)^4 \delta^4(\Sigma p_i - \Sigma p_f)\, S$$

- **In the amplitude squared $|M|^2$, one has to average (sum) on degrees of freedom (polarisation, color) of initial (final) particles.**
- **There is a symmetry factor $S = 1/n!$ for n identical particles.**
- **The flux factor is $2(p_1 + p_2)^2 = 2s$ for $2 \to n$ process with $m_1 = m_2 = 0$. It is $2M$ for the decay of a particle with a mass M ($1 \to n$ process).**

Calculation of phase–space for a two–body process $a + b \to f_1 + f_2$:

$$\mathrm{dPS2} = \frac{1}{16\pi^2} \frac{d^3 p_1}{e_1} \frac{d^3 p_2}{e_2} \delta^4(p_a + p_b - p_1 - p_2)$$

$$\int \frac{d^3 p_2}{e_2} \delta^4(p_a + p_b - p_1 - p_2) = \frac{1}{e_2} \delta(e_a + e_b - e_1 - e_2)$$

$$\text{with :} \quad |\vec{p}_2| = |\vec{p}_a + \vec{p}_b - \vec{p}_2| \text{ and } e_2^2 = |\vec{p}_2|^2 + m_2^2$$

$$\text{and} \quad d^3 p_1 = d\Omega\, |p_1|^2\, d|p_1| \text{ with } e_1^2 = |\vec{p}_1|^2 + m_1^2$$

In the c.m. frame: $w = e_a + e_b,\ w' = e_1 + e_2 = (m_2^2 + p^2)^{\frac{1}{2}} (m_1^2 + p^2)^{\frac{1}{2}}$:

$$\frac{dw'}{dp} = p \left(\frac{1}{e_1} + \frac{1}{e_2} \right) \Rightarrow dw' = p dp \left(\frac{1}{e_1} + \frac{1}{e_2} \right) = e_1 de_1 \frac{e_1 + e_2}{e_1 e_2}$$

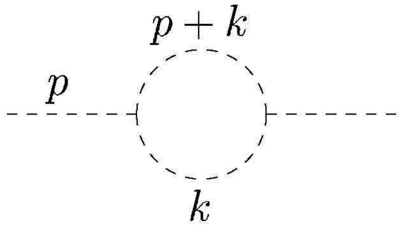
$$= \frac{d\Omega}{16\pi^2} |p| \frac{e_1 de_1}{e_1 e_2} \delta(w - w') = \frac{d\Omega}{16\pi^2} |p| \frac{dw'}{w'} \delta(w - w') \Rightarrow \frac{d\Omega}{16\pi^2} \frac{|p|}{\sqrt{s}}$$

(for the last equality, the integral over dw' has been performed). The differential cross section for a two body process is then:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2s} \times \Sigma |M(i_1 i_2 \to f_1 f_2)|^2 \times \frac{1}{16\pi^2} \left(\frac{|p|}{\sqrt{s}} \right) \times S$$

Note that $|p| = \frac{1}{2}\sqrt{s}\lambda = \frac{1}{2}\sqrt{s}[1 - m_1^2/s - m_2^2/s)^2 - 4 m_1^2 m_2^2/s^2]^{\frac{1}{2}}$.

4. Calculation of loop integrals



$= -i\Pi(p^2)$

- **Measure of loop integral over internal momentum:** $\int \mathrm{d}^4k/(2\pi)^4$**.**

(For fermion loops: take trace and factor (-1) **for Fermi stats).**

$$-i\Gamma = (ig)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{i}{(p+k)^2 - m^2} \frac{i}{k^2 - m^2}$$

$$\Rightarrow \Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{(p+k)^2 - m^2} \frac{1}{k^2 - m^2}$$

- **Symmetrize the integrand using:** $1/ab = \int_0^1 \mathrm{d}x/[a + (b-a)x]^2$

$$\Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 + 2pkx + p^2x - m^2)^2}$$

- **Shift variable** $k \to k' = k + px$ **(integrand becomes** k^2 **symmetric)**

$$\Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 + p^2x(1-x) - m^2)^2}$$

- **Wick rotation** $k_0 \to ik_0$ **to go to Euclidean space** $(k^2 \to -k^2)$

$$\Gamma = -g^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 - p^2x(1-x) + m^2)^2}$$

- **Polar coordinates for d**^4k**:** $\int_{-\infty}^{+\infty} d^4k\, F(k^2) = \pi^2 \int_0^\infty dk^2\, k^2\, F(k^2)$

$$\Gamma = -\frac{g^2}{16\pi^2} \int_0^1 \mathrm{dx} \int_0^\infty y\mathrm{d}y \frac{1}{(y - p^2x(1-x) + m^2)^2}$$

- **Perform the integrals over the variables** y **and** x**:**

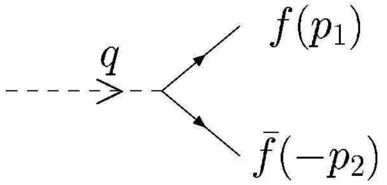
– **If integral divergent: cut-off at the energy** Λ **(**$\int_0^{\Lambda^2} \mathrm{d}k^2$**).**

– **Eventually, use the on–shell mass relation** $p^2 = m^2$**.**

E2: Higgs production and decay mechanisms

1. Higgs bosons decays

1.1 Decays into fermions: $H \to f\bar{f}$



$$-iM = \bar{u}^{s_1}(p_1)(im_f/v)v^{s_2}(-p_2)$$

$$+iM^\dagger = \bar{v}^{s_2}(-p_2)(-im_f/v)u^{s_1}(p_1)$$

$$\sum_{s_1,s_2} MM^\dagger \;=\; N_c\left(\frac{m_f}{v}\right)^2 \sum_{s_1,s_2} \bar{v}^{s_2}(-p_2)\, u^{s_1}(p_1)\bar{u}^{s_1}(p_1)v^{s_2}(-p_2)$$

with $N_c = 3(1)$ for quarks (leptons). Only one polarisation for H.

$$\begin{aligned}
(v/m_f)^2/N_c \times \Sigma|M|^2 &= \mathrm{Tr}(\not{p}_1 + m)(-\not{p}_2 - m)\\
&= \mathrm{Tr}(\gamma_\mu p_1^\mu + m)(-\gamma_\nu p_2^\nu - m)\\
&= -p_1^\mu p_2^\nu \mathrm{Tr}(\gamma_\mu\gamma_\nu) - m^2\mathrm{Tr}(1)\\
&= -4p_1.p_2 - 4m^2
\end{aligned}$$

Using $q^2 = (p_1 - p_2)^2 = 2m_f^2 - 2p_1 \cdot p_2 = M_H^2$ and defining the velocity of the final fermions $\beta_f = 2|p_f|/M_H = (1 - 4m_f^2/M_H^2)^{1/2}$

$$\Rightarrow \Sigma|M|^2 = N_c\,(m_f/v)^2\, 2(M_H^2 - 4m_f^2) = 2N_c\,(m_f^2/v^2)\, M_H^2\beta_f^2$$

The differential decay width is then simply given by:

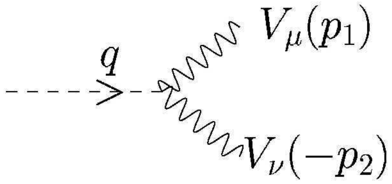
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{2M_H} \times \Sigma|M|^2 \times \frac{1}{32\pi^2} \times \frac{2|p_f|}{M_H}$$

Integrating over $\mathrm{d}\Omega = \mathrm{d}\phi\mathrm{d}\cos\theta$ (and since there is no angular dependence, $\int \mathrm{d}\Omega = 4\pi$), one obtains the partial decay width:

$$\Gamma(H \to f\bar{f}) = N_c\,\frac{m_f^2}{v^2}\,\frac{M_H}{8\pi}\,\beta_f^3$$

H decays dominantly into heaviest fermion and width $\propto M_H$.

1.2 Decays into massive gauge bosons



$$-iM = \epsilon^*_\mu(p_1)\left(-2iM_V^2/v\, g^{\mu\nu}\right)\epsilon^*_\nu(-p_2)$$

$$+iM^\dagger = \epsilon_{\mu'}(p_1)\left(-2iM_V^2/v\, g^{\mu'\nu'}\right)\epsilon_{\nu'}(-p_2)$$

$$\sum_{\text{pol}} |M|^2 = \frac{4M_V^4}{v^2}\, g^{\mu\nu}\, g^{\mu'\nu'} \sum_{\text{pol}} \epsilon^*_\mu(p_1)\epsilon_{\mu'}(p_1) \sum_{\text{pol}} \epsilon^*_\nu(-p_2)\epsilon_{\nu'}(-p_2)$$

$$\begin{aligned}(v^2/4M_V^4)\Sigma &= g^{\mu\nu}\, g^{\mu'\nu'}\,(g_{\mu\mu'} - p_{1\mu}p_{1\mu'}/M_V^2)(g_{\nu\nu'} - p_{2\nu}p_{2\nu'}/M_V^2)\\ &= (g_{\mu\mu'} - p_{1\mu}p_{1\mu'}/M_V^2)(g^{\mu\mu'} - p_2^\mu p_2^{\mu'}/M_V^2)\\ &= 4 - p_1^2/M_V^2 - p_2^2/M_V^2 + (p_1\cdot p_2)^2/M_V^4\\ &= (M_H^4/4M_V^4)\,[1 - 4M_V^2/M_H^2 + 12M_V^4/M_H^4]\end{aligned}$$

The differential decay width, $\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{2M_H} \times |M|^2 \times \frac{1}{32\pi^2}\frac{2|p_V|}{M_H} \times S$**, with** $S = \delta_V = \frac{1}{2}$ **for two identical final** Z **bosons. This finally gives** $(\int \mathrm{d}\Omega = 4\pi)$**:**

$$\Gamma(H \to VV) = \frac{\delta_V M_H^3}{16\pi v^2}\left(1 - \frac{4M_V^2}{M_H^2}\right)^{1/2}\left(1 - 4\frac{M_V^2}{M_H^2} + 12\frac{M_V^4}{M_H^4}\right)$$

The dependence on M_V **is hidden, since** $v \equiv 2M_W/g_2 = 2M_Z c_W/g_2$**. For large enough** M_H **[recall that** $H \to f\bar{f} \propto M_H$**], one has:**

$$\Gamma(H \to VV) \simeq \delta_V M_H^3/(8\pi v^2) \Rightarrow \Gamma(H \to WW) \simeq 2\Gamma(H \to ZZ)$$

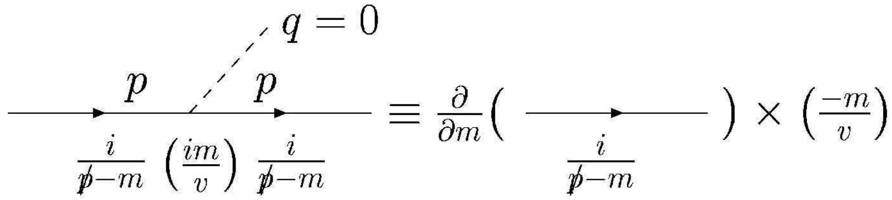
The decay widths grows like M_H^3 **i.e. is very large for** $M_H \gg M_V$**. For small** M_H**, one (two)** V **bosons can be off–shell, the width is**

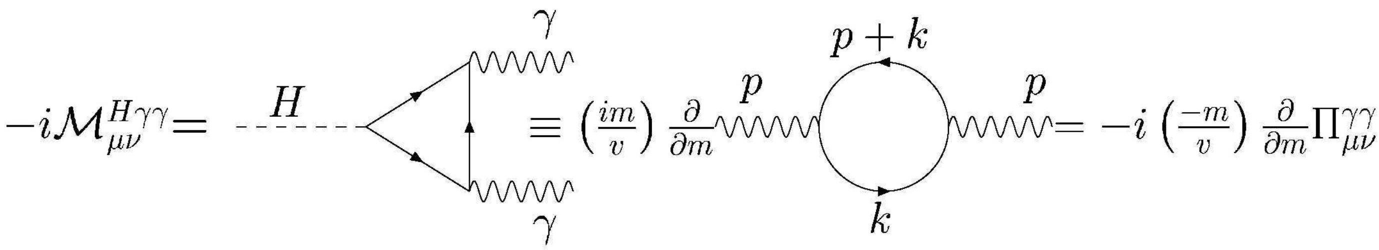
$$\Gamma = \frac{\Gamma_0}{\pi^2}\int_0^{M_H^2} \frac{\mathrm{d}q_1^2 M_V\Gamma_V}{(q_1^2 - M_V^2)^2 + M_V^2\Gamma_V^2}\int_0^{M_H^2 - q_1^2} \frac{\mathrm{d}q_2^2 M_V\Gamma_V}{(q_2^2 - M_V^2)^2 + M_V^2\Gamma_V^2}$$

$$\Gamma_0 = \frac{\delta_V M_H^3}{8\pi v^2}\,\lambda^{1/2}\left(\lambda - \frac{12q_1^2q_2^2}{M_H^4}\right)\ ,\ \lambda = \left(1 - \frac{q_1^2}{M_H^2} - \frac{q_1^2}{M_H^2}\right)^2 - \frac{4q_1^2q_2^2}{M_H^4}$$

1.3 Decays into photons and gluons: $H \to \gamma\gamma, gg$

H does not couple to massless particles at tree-level: loop induced. We have vertex diagrams with fermion (top) and W exchange for $H \to \gamma\gamma(Z\gamma)$; only top for $H \to gg$: calculation complicated. However it is simple if H momentum is small (i.e. $M_H \ll M_{\rm loop}$):





Let's calculate the derivative of the fermionic photon self-energy:

$$-i\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = N_c \int \frac{d^4k}{(2\pi)^4}(-1)\mathrm{Tr}(-iee_f\gamma_\mu)\frac{i}{\not{k}-m}(-iee_f\gamma_\nu)\frac{i}{\not{p}+\not{k}-m}$$

$$\Pi^{\gamma\gamma}_{\mu\nu}(p) = -i\,N_c\,e^2e_f^2 \int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}\gamma_\mu(\not{k}+m)\gamma_\nu(\not{p}+\not{k}+m)}{[(p+k)^2-m^2](k^2-m^2)}$$

Using the rules for Diracology and loop integral calculations:

$$D = \int_0^1 \frac{\mathrm{d}x}{(k^2+2pkx+p^2x-m^2)^2} = \int_0^1 \frac{\mathrm{d}x}{[(k+px)^2+p^2x(1-x)-m^2]^2}$$

$$\begin{aligned} N &= \mathrm{Tr}[\gamma_\mu\,\not{k}\gamma_\nu(\not{p}+\not{k})+m^2\gamma_\mu\gamma_\nu] = k^\rho(k+p)^\sigma\mathrm{Tr}[\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\sigma+m^2\gamma_\mu\gamma_\nu] \\ &= 4[2k_\mu k_\nu+(m^2-k^2-p.k)g_{\mu\nu}] \end{aligned}$$

Shift $k \to k+px$, Wick rotation $k_0 \to ik_0$ for Euclidean space $\Rightarrow$ $k^2 \to -k^2$ and sym. integrand with $\int_{-\infty}^{+\infty} d^4k\,F(k^2) = \pi^2\int_0^\infty dy\,yF(y)$

[also use of symmetry relation $\int d^4k(k_\mu k_\nu) = \frac{1}{4}g_{\mu\nu}\int d^4k(k^2)$]:

$$\Pi_{\mu\nu}^{\gamma\gamma}(p) = -iN_c\, e_f^2\, e^2 \times 4 \times \pi^2 \times \frac{i}{16\pi^4} \times \int_0^1 \mathrm{d}x \int_0^\infty ydy \frac{[\frac{1}{2}k^2 + m^2 - x(1-x)p^2]g_{\mu\nu} + 2x(1-x)[g_{\mu\nu}p^2 - p_\mu p_\nu]}{[y + m^2 - p^2x(1-x)]^2}$$

Because of gauge invariance, photon is transverse ($\propto g_{\mu\nu}p^2 - p_\mu p_\nu$): the first term ($\propto g_{\mu\nu}$ should vanish* and we are left with:

$$\Pi_{\mu\nu}^{\gamma\gamma}(p) = \frac{N_c e_f^2 e^2}{4\pi^2}(g_{\mu\nu}p^2 - p_\mu p_\nu)\int_0^1 \mathrm{d}x \int_0^\infty ydy \frac{2x(1-x)}{[y + m^2 - p^2x(1-x)]^2}$$

We can now calculate the $H\gamma\gamma$ vertex [photons to symmetrize $\to 2$; they are on–shell and $p_{1,2} \neq p$ but $p^2 = p_1 \cdot p_2 = \frac{1}{2}M_H^2$]

$$\mathcal{M}_{\mu\nu}^{H\gamma\gamma} = -2\frac{m}{v}\frac{\partial}{\partial m}\Pi_{\mu\nu}^{\gamma\gamma}(p_1,p_2) = -\frac{4m^2}{v}\frac{\partial}{\partial m^2}\Pi_{\mu\nu}^{\gamma\gamma}(p_1,p_2)$$
$$= -\frac{2m^2}{v}\frac{N_c e_f^2 e^2}{\pi^2}(g_{\mu\nu}p_1.p_2 - p_{1\mu}p_{2\nu})\int_0^1 \mathrm{d}x \int_0^\infty \frac{-2x(1-x)ydy}{[y + m^2 - p^2x(1-x)]^3}$$

Inside the integral, we can suppose $m^2 \gg p^2(M_H^2)$ and integrate over x and y [$\int x(1-x)dx = 1/6$ and $\int y/(y+m^2)^3 dy = 1/2m^2$]

$$\mathcal{M}_{\mu\nu}^{H\gamma\gamma} = \frac{2}{3v}N_c\, e_f^2\, \frac{\alpha}{\pi}(g_{\mu\nu}p_1.p_2 - p_{1\mu}p_{2\nu})$$

Now we use the same machinery as for decays into gauge bosons:

$$|\mathcal{M}|^2 = \frac{4}{9v^2}N_c^2 e_f^4 \frac{\alpha^2}{\pi^2}\,\frac{M_H^4}{2}\,\Sigma|g^{\mu\nu}\epsilon_\mu^*(p_1)\epsilon_\nu^*(p_2)|^2 = \frac{2M_H^4}{9v^2}N_c^2 e_f^4\frac{\alpha^2}{\pi^2}$$

Integrating over phase space (with factor $\frac{1}{2}$ for identical photons):

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{9v^2}N_c^2 e_f^4 \frac{\alpha^2}{16\pi^3}$$

*This statement is not trivial to prove and we will come back to this discussion later on.

Several remarks to be made:

- **The amplitude was of course finite (no tree level contribution)!**
- **The approximation $m_f \gg M_H$ is in practice good up to $M_H \sim 2m_f$!**
- **Only tops contribute, other f have negligible Yukawa coupling.**
- **Infinitely heavy fermions do not decouple from the amplitude: a way to count the number of heavy particles coupling to the H!**
- **There are also contributions from W bosons. Also in the limit $M_H \ll M_W$ (valid for $M_H \lesssim 140$ GeV), one has:**

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{9v^2}\frac{\alpha^2}{16\pi^3}|\Sigma_f N_c e_f^2 - \frac{21}{4}|^2$$

- **The W contribution is larger (~ 4) than the t quark contribution and the interference of the two is destructive.**
- **With the same calculation, one can get the amplitude for $H \to Z\gamma$. Only difference, Zff, ZWW couplings and M_Z in phase space. Here again, the W contr. is much ($\gtrsim 10$) larger than that of top.**

- **The calculation holds also for gluons if we make the changes: $Q_e e \to g_s T_a$ which means $\alpha \to \alpha_s$ and $N_c^2 \to |\mathrm{Tr}(T_a T_a)|^2 = |\frac{1}{2}\delta_{ab}|^2 = 2$:**

$$\Gamma(H \to gg) = \frac{M_H^3}{9v^2}\frac{\alpha_s^2}{8\pi^3}$$

Decay width and branching ratios:

The total decay width of the Higgs is the sum of partial widths:

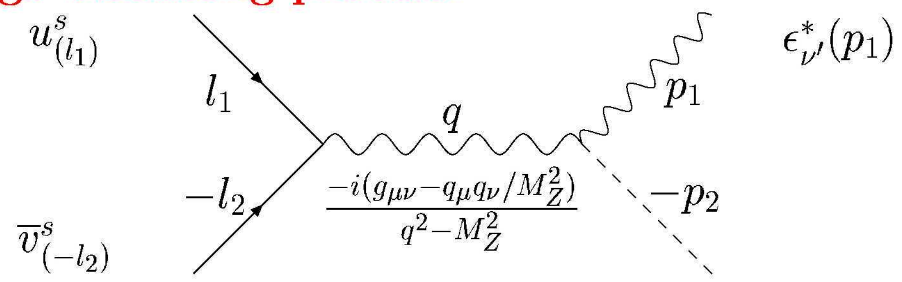
$$\Gamma_{\rm tot}(H \to \mathrm{all}) = \Sigma_f \Gamma(H \to f\bar{f}) + \Sigma_V \Gamma(H \to VV)$$

and the branching ratio for Higgs decay into a given final state is:

$$\mathrm{BR}(H \to X) = \Gamma(H \to X)/\Gamma_{\rm tot}(H \to \mathrm{all})$$

2. Higgs bosons production in e^+e^- Collisions

2.1 The Higgs–strahlung process:



$$-iM = \overline{v}^s_{(-l_2)}(-ie)\gamma_\mu(v_e - a_e\gamma_5)u^s_{(l_1)}\,\frac{-i(g^{\mu\nu} - q^\mu q^\nu/M_Z^2)}{q^2 - M_Z^2}\left(\frac{-2iM_Z^2}{v}g_{\nu\nu'}\right)\epsilon^*_{\nu'}$$

First thing to use for simplification is Dirac equation $\not{l}u(l) = m_e \sim 0$:

$$\overline{v}^s_{(-l_2)}\gamma_\mu q^\mu u^s_{(l_1)} = \overline{v}^s_{(-l_2)}[-\not{l}_2 + \not{l}_1]u^s_{(l_1)} = 2m_e \sim 0 \Rightarrow q_\mu q_\nu \to 0$$

where m_e is supposed to be much smaller than $\sqrt{s} = \sqrt{q^2}$. Then:

$$|M|^2 = \frac{4e^2M_Z^4v^{-2}}{(q^2 - M_Z^2)^2}\epsilon^\nu_{(p_1)}\epsilon^{*\mu}_{(p_1)}\overline{v}^s_{(-l_2)}\gamma_\mu(v_e - a_e\gamma_5)u^s_{(l_1)}\overline{u}^s_{(l_1)}\gamma_\nu(v_e - a_e\gamma_5)v^s_{(-l_2)}$$

Average over polarizations of $e^\pm$ and sum on those of photon:

$$\frac{1}{4}\Sigma|M|^2 = \frac{k}{4}\mathrm{Tr}\,\not{l}_1(v_e - a_e\gamma_5)\gamma_\mu(-\not{l}_2)(v_e - a_e\gamma_5)\gamma_\nu\left(-g^{\mu\nu} + \frac{p_1^\mu p_1^\nu}{M_Z^2}\right)$$

$$-\mathrm{Tr} = (v_e^2 + a_e^2)\mathrm{Tr}\,\not{l}_1\gamma_\mu\not{l}_2\gamma_\nu - 2a_ev_e\mathrm{Tr}\,\not{l}_1\gamma_\mu\not{l}_2\gamma_\nu\gamma_5$$

$$= 4(v_e^2 + a_e^2)[l_{1\mu}l_{2\nu} + l_{2\mu}l_{1\nu} - l_1.l_2g_{\mu\nu}] - 8ia_ev_e\,l_1^\alpha l_2^\beta\epsilon_{\alpha\mu\mu\beta\nu}$$

$$\frac{1}{4}\Sigma|M|^2 = k(v_e^2 + a_e^2)\left[2(l_1.l_2) - 2\frac{(l_1.p_1)(l_2.p_1)}{M_Z^2} - 4(l_1.l_2) + (l_1.l_2)\frac{p_1^2}{M_Z^2}\right]$$

$$= k(v_e^2 + a_e^2)\left[-(l_1.l_2) - 2(l_1.p_1)(l_2.p_1)/M_Z^2\right]$$

where we have used the fact that $(\epsilon_{\alpha\mu\beta\nu})\; g^{\mu\nu} - p_1^\mu p_1^\nu$ is (anti)symmetric. In the c.m. frame, one has (with $E_{1,2}^2 = M_{Z,H}^2 + |p|^2$) and $|p| = \sqrt{s}/2\lambda$):

$$l_{1,2} = \frac{\sqrt{s}}{2}(1, 0, 0, \pm 1) \text{ and } p_{1,2} = (E_{Z,H}, 0, \pm|p|\sin\theta, \pm|p|\cos\theta)$$

$$\Rightarrow k(v_e^2+a_e^2)\left[\frac{s}{2}+\frac{s(E_Z^2-|p|^2\cos^2\theta)}{2M_Z^2}\right]=k(v_e^2+a_e^2)\frac{s^2}{M_Z^2}\left[\frac{M_Z^2}{s}+\frac{\lambda^2\sin^2\theta}{8}\right]$$

The differential cross section is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta\mathrm{d}\phi}=\frac{1}{2s}\left[\frac{4e^2M_Z^4(v_e^2+a_e^2)s}{v^2(s-M_Z^2)^2M_Z^2}\left(\frac{M_Z^2}{s}+\frac{1}{8}\lambda^2\sin^2\theta\right)\right]\frac{\lambda}{32\pi^2}$$

with $\int \mathrm{d}\phi=2\pi$ **and** $\int\sin^2\theta\mathrm{d}\cos\theta=4/3$ **one gets the cross section**

$$\sigma(e^+e^-\to HZ)=\frac{\alpha M_Z^2}{12v^2}\frac{v_e^2+a_e^2}{s(1-M_Z^2/s)^2}\lambda(\lambda^2+12M_Z^2/s)$$

A few remarks:

- **The cross section drops like** $1/s$ **at high–energies (typical of an** s**–channel process). The maximum is reached at** $\sqrt{s}=M_Z+\sqrt{2}M_H$**.**
- **At the maximum LEP2 energy,** $\sqrt{s}=209$ **GeV, the cross section for** $M_H=(100)\,115$ **GeV is given by (using the fact that** $\sigma_0=4\pi\alpha^2(0)/3=86.8$ **nb with** $\alpha(0)=1/137, \alpha(s)\simeq 1/128$ **and** $\sin^2\theta_W=0.232$**):**

$$\sigma=0.42\ (0.16)\ \text{pb for}\ M_H=100\ (115)\ \text{GeV}$$

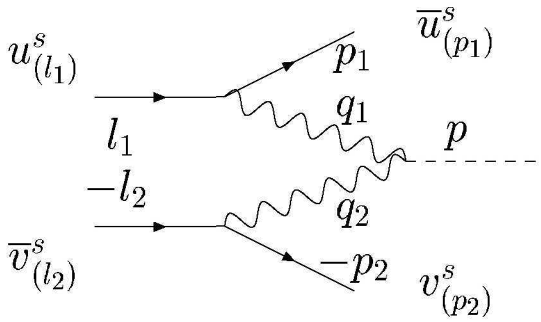
If we have an integrated luminosity of $\int\mathcal{L}\sim 100$ **pb**$^{-1}$**, this means that we have** $N=\sigma\times\int\mathcal{L}\sim 42(16)$ **Higgs boson events.**

- **Since for** $M_H\sim 100$ **GeV, BR**$(H\to b\bar{b})\sim 90\%$**, the signal is** $e^+e^-\to ZH\to Zb\bar{b}$ **and the main background is** $e^+e^-\to ZZ\to Z\bar{b}b$**.**
- **At high energies** $s\gg M_Z^2$**, one has a differential cross section**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\simeq\frac{3}{4\sigma}\sin^2\theta\ \text{with}\ \sigma\simeq\frac{\alpha M_Z^2}{12v^2}\frac{v_e^2+a_e^2}{s(1-M_Z^2/s)^2}\lambda^3$$

the behaviour in $\sin^2\theta$ **of the angular distribution and in** λ^3 **of the total cross section is typical for the production of two spin–zero particles (here, the** Z **boson is almost a Goldstone boson).**

2.2 The vector boson fusion mechanism:



$$M = \frac{i(-ie)^2(-i)^2(-2iM_V^2/v)}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)} g_{\mu'\nu'} \times \begin{array}{l} \bar{u}^s_{(p_1)}\gamma_\mu(v - a\gamma_5)u^s_{(l_1)}\left(g^{\mu\mu'} - \frac{q_1^\mu q_1^{\mu'}}{M_V^2}\right) \\ \bar{v}^s_{(-p_2)}\gamma_\nu(v - a\gamma_5)\bar{v}^s_{(l_2)}\left(g^{\nu\nu'} - \frac{q_2^\nu q_2^{\nu'}}{M_V^2}\right) \end{array}$$

Using the relations $q_1^\mu\gamma_\mu = \not{q} = \not{l}_1 - \not{p}_1 \propto m_e \sim 0$ **and** $g^{\nu\nu'}g^{\mu\mu'}g_{\mu'\nu'} = g^{\mu\nu}$**:**

$$\begin{aligned} |M|^2 &= \frac{4e^4M_V^4/v^2}{D_1^2D_2^2} \times \begin{array}{l} \bar{u}^s_{(p_1)}\gamma_\mu(v - a\gamma_5)u^s_{(l_1)}.\bar{v}^s_{(-l_2)}\gamma_\nu(v - a\gamma_5)v^s_{(-p_2)} \\ u^s_{(l_1)}\gamma^\mu(v - a\gamma_5)u^s_{(p_1)}.\bar{v}^s_{(-p_2)}\gamma^\nu(v - a\gamma_5)v^s_{(-l_2)} \end{array} \\ &= \frac{4e^4M_V^4/v^2}{D_1^2D_2^2} \times \begin{array}{l} \mathrm{Tr}\ \not{l}_1\gamma_\nu(v - a\gamma_5)\ \not{p}_1\gamma_\mu(v - a\gamma_5) \\ \mathrm{Tr}\ \not{l}_2\gamma^\nu(v - a\gamma_5)\ \not{p}_2\gamma^\mu(v - a\gamma_5) \end{array} \\ &= \frac{4e^4M_V^4/v^2}{D_1^2D_2^2} \times \begin{array}{l} (v^2 + a^2)\mathrm{Tr}\ \not{l}_1\gamma_\nu\ \not{p}_1\gamma_\mu - 2va\mathrm{Tr}\ \not{l}_1\gamma_\nu\ \not{p}_1\gamma_\mu\gamma_5 \\ (v^2 + a^2)\mathrm{Tr}\ \not{l}_2\gamma^\nu\ \not{p}_2\gamma^\mu - 2va\mathrm{Tr}\ \not{l}_2\gamma^\nu\ \not{p}_2\gamma^\mu\gamma_5 \end{array} \end{aligned}$$

Performing the trace and product using $\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu\alpha'\beta'} = \delta^\alpha_{\alpha'}\delta^\beta_{\beta'} - \delta^\alpha_{\beta'}\delta^\beta_{\alpha'}$

$$\frac{1}{4}|M|^2 = \frac{32e^4M_V^4/v^2}{D_1^2D_2^2} \times [g_S(l_1.p_2)(l_2.p_1) + g_A(l_1.l_2)(p_1.p_2)]$$

with $g_S = (v^2 + a^2)^2 + 4a^2v^2$ and $g_A = (v^2 + a^2)^2 - 4a^2v^2$

Let's write the momenta of the particles in a convenient way:

$$l_1 = (E, 0, 0, E)\ , p_1 = (\sqrt{x_1^2E^2 + p_{T1}^2}, p_{T1}\sin\theta_1, p_{T1}\cos\theta_1, x_1E)$$

$$l_2 = (E, 0, 0, -E)\ , p_2 = (\sqrt{x_2^2E^2 + p_{T2}^2}, p_{T2}\sin\theta_1, p_{T1}\cos\theta_1, -x_2E)$$

and assume high energies $s \gg M_V^2$ so that $p_{T1,T2}/E$ are rather small:

$$l_i.p_i \sim p_{Ti}^2/2x_i \ , \ \ l_1.p_2 \sim 2E^2 x_2 \ , \ \ l_2.p_1 \sim 2E^2 x_1 \ , \ \ p_1.p_2 \sim 2E^2 x_1 x_2$$

hold, together with $2l_1.l_2 = s$ and the Higgs momentum squared:

$$M_H^2 = (q - p_1 - p_2)^2 = s - 2q.p_1 - 2q.p_2 + 2p_1.p_2 = s(1-x_1)(1-x_2)$$

Using these products, one has then for the amplitude squared:

$$\begin{aligned}\frac{1}{4}|M|^2 &= \frac{32e^4 M_V^4}{v^2} \times \frac{4E^4(g_S + g_A)x_1 x_2}{(p_{T1}^2/x_1 + M_W^2)^2 (p_{T2}^2/x_2 + M_W^2)^2} \\ &= \frac{8e^4 M_V^4}{v^2} \times \frac{(g_S + g_A)s^2 x_1^3 x_2^3}{(p_{T1}^2 + x_1 M_W^2)^2 (p_{T2}^2 + x_2 M_W^2)^2}\end{aligned}$$

Let us now deal with the three body phase space:

$$\mathrm{dPS3} = \frac{1}{(2\pi)^5} \frac{\mathrm{d}^3 p_1}{2E_1} \frac{\mathrm{d}^3 p_2}{2E_2} \frac{\mathrm{d}^3 p}{2E_H} \delta^4(q - p_1 - p_2 - p)$$

Defining $\tau_H = M_H^2/s$ and using the known relation for δ functions:

$$\int \frac{\mathrm{d}^3 p}{2E_H} = \int \mathrm{d}^4 p \, \delta(p^2 - M_H^2) = \int \mathrm{d}^4 p \, \delta[s(1-x_1)(1-x_2) - s\tau_H]$$

and decomposing the momenta along the 3 directions, one obtains:

$$\mathrm{dPS3} \ = \ \frac{1}{(2\pi)^5} \frac{\mathrm{d}(x_1 E)}{2x_1 E} \mathrm{d}^2 p_{T1} \frac{\mathrm{d}(x_2 E)}{2x_2 E} \mathrm{d}^2 p_{T2} \, \delta[s(1-x_1)(1-x_2) - s\tau_H]$$

Noting that $\int \mathrm{d}p_{Ti}^2/(p_{Ti}^2 + x_i M_V^2)^2 = \pi \int_0^\infty \mathrm{d}p^2/(p^2 + x_i M_V^2)^2 = \pi^2/(x_i M_V^2)$ and using $M_W = ev/(2s_W)$, the differential cross section is given by:

$$\begin{aligned}\mathrm{d}\sigma &= \frac{1}{2s} \times \left(\frac{8e^6 M_V^4}{4M_W^2 s_W^2}\right) (g_S + g_A)s^2 x_1^3 x_2^3 \times \frac{1}{(2\pi)^5} \frac{\mathrm{d}x_1}{2x_1} \frac{\mathrm{d}x_2}{2x_2} \frac{\pi^2}{x_1 x_2 M_V^4} \delta \\ \sigma &= \frac{\alpha^3}{2M_W^2 s_W^2} (g_S + g_A) \int \mathrm{d}x_1 \int \mathrm{d}x_2 x_1 x_2 \, s\delta[s(1-x_1)(1-x_2) - s\tau_H]\end{aligned}$$

Now perform the integrals using $\int \delta[f(x)] = |f'(x)|^{-1}_{x=x_0}$ **with** $f(x_0)=0$

$$\int \mathrm{d}x_1 \int \mathrm{d}x_2 \cdots = \int_0^{1-\tau_H} \mathrm{d}x_1 x_1 \left(1 - \frac{\tau_H}{1-x_1}\right) s \frac{1}{s(1-x_1)}$$
$$= \int_0^{1-\tau_H} \mathrm{d}x_1 \left[-1 + \frac{1+\tau_H}{1-x_1} + \frac{\tau_H}{(1-x_1)^2}\right] = (1+\tau_H)\log\frac{1}{\tau_H} - 2(1-\tau_H)$$

where the boundary conditions are obtained by requiring that $p_{1Z} = p_{2Z} = x_{1,2}E = 0 \Rightarrow x_1 = 0$ **and** $x_2 = 1-\tau_H/(1-x_1) = 0 \to x_1 = 1-\tau_H$**.** **Collecting all results, one obtains then the total cross section*:**

$$\sigma = \frac{\alpha^3}{2M_W^2 s_W^2}(g_S + g_A)\left[(1+\tau_H)\log\frac{1}{\tau_H} - 2(1-\tau_H)\right]$$

Let us now make a few remarks:

- **The cross section rises as** $\log(s/M_H^2)$**: small at low** $\sqrt{s}$ **and large at high** $\sqrt{s}$**. Dominant Higgs production process for** $s \gg M_H^2$**.**
- **This approximation is good only within a factor of 2 and works better at higher energies. It can be obtained in an easier way using the effective longitudinal vector boson approximation.**
- **In the case of** WW **fusion,** $g_s = 8/(2\sqrt{2})^4 = 1/8$ **and** $g_A = 0$**, one has:**

$$\sigma(e^+e^- \to H\bar{\nu}\nu) = \frac{\alpha^3}{16M_W^2 s_W^2}\left[(1+\tau_H)\log\frac{s}{M_H^2} - 2(1-\tau_H)\right]$$

- **At LEP2 energies,** $\sqrt{s} \sim 200$ **GeV, the cross section is** $\sigma \sim 5(2)\cdot 10^{-3}$ **pb for** $M_H = 100(115)$ **GeV, i.e. less than one event for** $\int \mathcal{L} = 100$ **pb**$^{-1}$**. This process is not very useful for Higgs searches at LEP2.**
- **For** ZZ **fusion with** $s_W^2 \sim 1/4$, $g_s \sim g_A \sim a_e^4 \sim 1/(16\times 9)$**: the cross section** $\sigma(e^+e^- \to e^+e^- H)$ **is** ~ 9 **times smaller than for** WW **fusion.**

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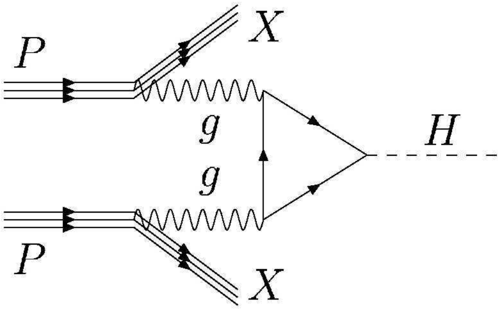
*This calculation, including details is done in: G. Altarelli, B. Mele and F. Pitolli, Nucl. Phys. B287 (1987) 205.

3. Higgs bosons production in hadronic Collisions

3.1 The gluon–gluon fusion process*



The cross section of the subprocess, $gg \to H$, is given by:

$$\mathrm{d}\hat{\sigma} = \frac{1}{2\hat{s}} \times \frac{1}{2 \cdot 8} \times \frac{1}{2 \cdot 8} |\mathcal{M}_{Hgg}|^2 \frac{\mathrm{d}^3 p_H}{(2\pi)^3 2E_H} (2\pi^4) \delta^4 (q - p_H)$$

Using the fact that $\int \mathrm{d}^3 p_H/(2E_H) = \int \mathrm{d}^4 p_H \delta(p_H^2 - M_H^2)$ and that $|M_{Hgg}|^2 = 32\pi M_H \Gamma(H \to gg)$ calculated before, one obtains for $\hat{\sigma}$:

$$\hat{\sigma} = \frac{\pi^2 M_H}{8\hat{s}} \Gamma(H \to gg)\, \delta(\hat{s} - M_H^2)$$

Convolute with gluon densities to obtain the total cross section

$$\sigma = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{\pi^2 M_H}{8\hat{s}} \Gamma(H \to gg) g(x_1) g(x_2) \delta(\hat{s} - M_H^2)$$

with $\hat{s} = s x_1 x_2$, implying $\hat{s} - M_H^2 = s(x_1 x_2 - \tau_H)$ with $\tau_H = M_H^2/s$:

$$\sigma = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{\pi^2}{8M_H} \Gamma(H \to gg)\, g(x_1) g(x_2) \delta[s(x_1 x_2 - \tau_H)]$$

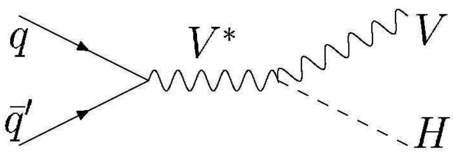
We perform the integral on x_2 $[\int \delta[f(x)] = |f'(x)|^{-1}_{x=x_0}$ with $f(x_0) = 0]$

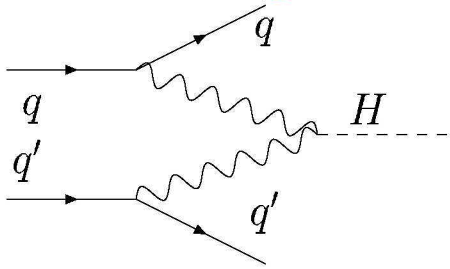
$$\sigma = \frac{\pi^2}{8M_H^3} \Gamma(H \to gg)\, \tau_H \int_{\tau_H}^1 \frac{\mathrm{d}x}{x} g(x) g(x/\tau_H) = \frac{1}{576 v^2} \frac{\alpha_s^2}{\pi} \tau_H \frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau_H}$$

where the integration bounds are $x_1^{\max} = 1, x_1^{\min} = x_1$(for $x_2 = 1) = \tau$.
At LHC, gg luminosity is large and $gg \to H$ dominant process!

*Calculation to be checked!!!

3.2 The Higgs strahlung and vector boson fusion process





The cross sections for these processes are the same as in e^+e^- collisions, provided that the following changes are performed:
– The total energy $\sqrt{s}$ is replaced by the subprocess energy $\hat{s}$.
– The average over the quark colors is made: factor $\frac{1}{3} \cdot \frac{1}{3}$.
– In the bremsstrahlung process, possibility of $q\bar{q}' \to W^* \to WH$.
– The couplings of the electrons are replaced by those of quarks:

in $q\bar{q} \to VH$: $a_e^2 + v_e^2 \to a_q^2 + v_q^2$.

in $qq \to Hqq$: $g_{S,A} \to [(v^2+a^2)(v'^2+a'^2) \pm 4(av)(a'v')$.

The cross sections for a given initial state, are given by:

$$\sigma(q\bar{q}' \to HV) = \frac{1}{9}\frac{\alpha M_V^2}{12 v^2}\frac{v_q^2 + a_q^2}{\hat{s}(1 - M_V^2/\hat{s})^2}\hat{\lambda}(\hat{\lambda}^2 + 12 M_V^2/\hat{s})$$

$$\sigma(qq \to qqH) = \frac{1}{9}\frac{\alpha^3}{2 M_W^2 s_W^2}(g_S + g_A)\left[(1+\hat{\tau}_H)\log\frac{1}{\hat{\tau}_H} - 2(1-\hat{\tau}_H)\right]$$

Summing over all possibilities for quark/antiquark initial states and folding with the proper densities, the total cross sections are:

$$\sigma[pp \to H + X] = \sum_{q,q'} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 f_q(x_1) f_{q'}(x_2)\, \hat{\sigma}[qq' \to H + X]$$

Remarks:

- **At LHC, $qq \to Hqq$ is the dominant process but not as $gg \to H$.**
- **The cross section for $q\bar{q} \to HV$ is OK for low M_H; $\sigma(HW) \sim 2\sigma(HZ)$.**
- **At Tevatron, Higgs–strahlung (esp. $q\bar{q}' \to HW$) more important.**

E3: Divergences and Symmetries

1. Lagrangians and interactions

- **Take QED Lagrangian for a fermion of charge e and mass m:**

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi + e\overline{\psi}\gamma^{\mu}A_{\mu}\psi$$

with A_μ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the electromagnetic field and tensor. U(1) gauge invariance: no $A_\mu A^\mu$ term so that photon is massless.

- **Let's add a scalar field ϕ:**

$$\mathcal{L}_\phi = |\partial_\mu \phi|^2 - m_S^2|\phi|^2 + \lambda(\phi^+\phi)^2$$

which leads to a spontaneously broken symmetry (SSB), $\langle\phi\rangle = \frac{v}{\sqrt{2}}$, and we write $\phi = (H+v)/\sqrt{2}$ with H being the physical Higgs boson.

- **And couple this field to a fermion f (à la Yukawa):**

$$\mathcal{L}_f = -\lambda_f.\overline{\psi}\psi\phi$$

After SSB, the fermion acquires a mass $m_f = \lambda_f v/\sqrt{2}$.

- **Let us introduce two scalar fields ϕ_1 and ϕ_2:**

$$\mathcal{L}_{\rm kin} = |\partial_\mu\phi_1|^2 + |\partial_\mu\phi_2|^2 - m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2$$

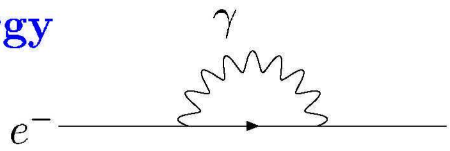
they will have a coupling to the scalar field ϕ after SSB:

$$\mathcal{L}_S = \lambda_S|\phi|^2(|\phi_1|^2 + |\phi_2|^2) + 2v\lambda_S\phi(|\phi_1|^2 + |\phi_2|^2)$$

Plus, eventually, terms in $\phi_1\phi_2$ that we take zero for simplicity.

2. Self–energy of the electron and photon

Electron self–energy



$\equiv -i\Sigma_e(\not{p})$

$$-i\Sigma_e(p) = \int \frac{d^4k}{(2\pi)^4}(-ie\gamma_\mu)\frac{i}{\not{p}+\not{k}-m}(-ie\gamma_\nu)\,\frac{-ig^{\mu\nu}}{k^2}$$

– **Perform the Dirac Algebra:** $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}$, **etc...**

– **Symmetrisation:** $1/ab = \int_0^1 dx/[a+(b-a)x]^2$**+change of variable.**

– **Switch to Euclidean space with Wick rotation** $k_0 \to ik_0, k^2 \to -k^2$.

– **Integrate over momentum (symmetric integrand and regul.):**

$$\int_{-\infty}^{+\infty} d^4k\, F(k^2) = \pi^2 \int_0^{\Lambda^2} dk^2\, k^2\, F(k^2)$$

Definition of the correction to the mass: $\delta m_e = \Sigma_e(\not{p})|_{p=m}$**:**

$$\begin{aligned}\Rightarrow \delta m_e &= \frac{m_e e^2}{8\pi^2}\int_0^1 dx(1+x)\int_0^{\Lambda^2} dyy[y+m_e^2x^2]^{-2}\\ &= \frac{m_e e^2}{8\pi}\left[\frac{3}{2}\log\frac{\Lambda^2}{m_e^2}\right]+\cdots = \frac{3\alpha}{4\pi}m_e\log\frac{\Lambda^2}{m_e^2}+\cdots\end{aligned}$$

UV divergence (at large k^2**) logarithmic.**

In principle $\Lambda = \infty \Rightarrow$ **renormalization:** $m_e^{\rm phys} = m_e^{\rm nu} + \delta m_e$.

But QED valid up to GUT (M_P**) scale, i.e.** $\Lambda = M_{\rm GUT}(M_P)$.

Correction logarithmic AND proportional to m_e**, therefore small:**

$\delta m_e \sim 0.2 m_e$ **for** $\Lambda \sim M_P \sim 10^{19}$ **GeV.**

More fundamental: correction small due to chiral symmetry:

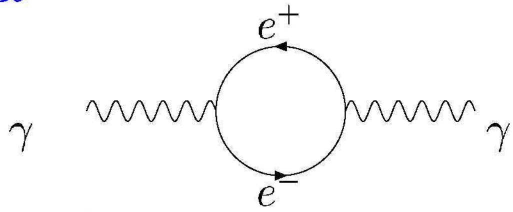
if $m_e = 0$, $\mathcal{L}_{\rm QED}$ **is invariant under the chiral transformation:**

$\psi_L \to e^{i\theta_L}\psi_L$ **and** $\psi_R \to e^{i\theta_R}\psi_R$ **with** $\psi_{L,R} = 1/2(1 \mp \gamma_5)\psi$.

But m_e **breaks the chiral symmetry** $\to$ **correction** $\propto$ **to the mass.**

$\Rightarrow$ **Symmetry** $\equiv$ **protection for the mass.**

Photon self–energy



$\equiv -i\Pi^{\gamma\gamma}_{\mu\nu}(p^2)$

$$-i\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = \int \frac{d^4k}{(2\pi)^4}(-1)\mathrm{Tr}(-ie\gamma_\mu)\frac{i}{\not k - m}(-ie\gamma_\nu)\frac{i}{\not p + \not k - m}$$

We already made the calculation and we reached the level where:

$$\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = -ie^2 \times 4 \times 2\pi^2 \times \frac{i}{16\pi^4} \times \int_0^1 \mathrm{d}x \int_0^\infty y dy \frac{[\frac{1}{2}y^2 + m^2 - x(1-x)p^2]g_{\mu\nu} + 2x(1-x)[g_{\mu\nu}p^2 - p_\mu p_\nu]}{[y + m^2 - p^2x(1-x)]^2}$$

using usual tricks and a cut–off Λ for integral on k^2, one gets

$$\delta m_\gamma = \frac{1}{4}g^{\mu\nu}\Pi^{\gamma\gamma}_{\mu\nu}(0) = \frac{e^2}{16\pi^2}\int_0^1 \mathrm{d}x \int_0^{\Lambda^2} dy \frac{y^2 + 2m^2 y}{(y+m^2)^2} \sim \frac{\alpha}{4\pi}\Lambda^2 \quad !!!$$

But we must have $m_\gamma \equiv 0$ at all orders because of gauge invariance.
Problem: the cut–off Λ^2 violates the QED gauge invariance.

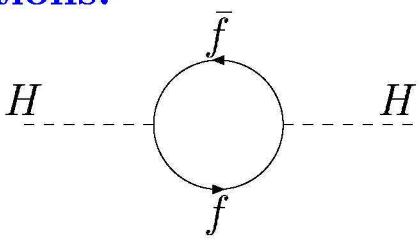
Solution: dimensional regularization, preserves gauge invariance!
We work in a space–time of $n = 4 - \epsilon$ dimensions:
– **Internal momentum in n dim: $\int d^n k/(2\pi)^n$ etc...**
– **Dirac algebra in n dim: $\mathrm{Tr}(I) = n, \gamma_\mu\gamma^\mu = nI, g^\mu_\mu = n$, etc..**
– **UV divergence: poles in $1/(n-4) = 1/\epsilon$ with $\epsilon \to 0$.**

In this case, one would have for the integral:
$A(m^2) = \int [k^2 - m^2]^{-1} \sim am^2/\epsilon + \cdots \Rightarrow m^2\partial A/\partial 2m^2 \sim am^2/\epsilon$.
With this regularization, $\delta m_\gamma = 0$ at all orders: $\Rightarrow$ massless photon.

Another example of a protection for a mass by a symmetry...

3. Higgs boson self–energy

Fermionic contributions:



$$-i\Sigma_H(p^2) \;=\; N_f \int \frac{d^4k}{(2\pi)^4}(-1)\mathrm{Tr}\,(-\frac{i\lambda_f}{\sqrt{2}})\frac{i}{\not k - m}(-\frac{i\lambda_f}{\sqrt{2}})\frac{i}{\not p + \not k - m}$$

Usual calculation. Simpler: $p^2 = M_H^2 = 0$ (fermion heavy compared to M_H). Using a cut–off Λ for the integral on k^2, one obtains:

$$\Sigma_H(p^2=0) \;=\; 4N_f(\frac{\lambda_f}{\sqrt{2}})^2\frac{1}{16\pi^2}\int_0^1 dx \int_0^{\Lambda^2} dy\, \frac{y(-y+m_f^2)}{(y+m_f^2)^2}$$

After the trivial integral on x and the one on y, one gets:

$$\Delta M_H^2 = \;=\; N_f \frac{\lambda_f^2}{8\pi^2}\left[-\Lambda^2 + 6m_f^2\log\frac{\Lambda}{m_f} - 2m_f^2\right] + \mathcal{O}(1/\Lambda^2)$$

We have thus a quadratic divergence, $\delta M_H^2 \sim \Lambda^2$.

Divergence is independent of M_H, and does not disappear if M_H=0: The choice M_H=0 does not increase the symmetry of $\mathcal{L}_{\rm SM}$.

Here, the cut–off does not break any symmetry and the problem is not solved with dimensional regularization (though we have only poles in $1/\epsilon$ and the quadratic divergence is not apparent).

If we fix the cut–off Λ to $M_{\rm GUT}$ or M_P: $\Rightarrow M_H \sim 10^{14}$ to 10^{17} GeV! The Higgs boson mass prefers to be close to the very high scale:

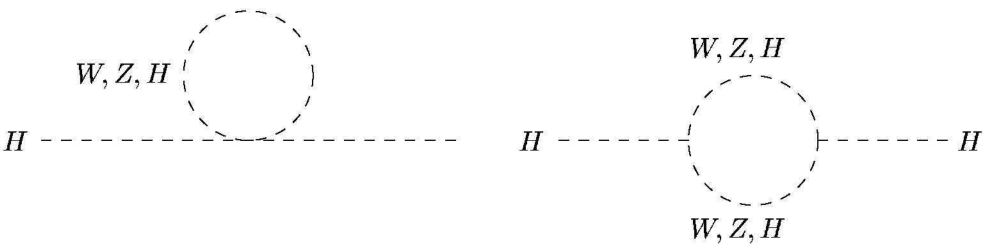
This is the hierarchy problem.

But we want a light Higgs ($\lesssim 1$ TeV) for unitarity etc... reasons
We need thus to make: $M_H^2|^{\rm Physical} = M_H^2|^0 + \Delta M_H^2 + \text{countreterm}$
And adjust this counterterm with a precision of 10^{30} **(30 digits)!**

This is the naturalness problem.

In a complete theory, no problem formally: we adjust the bare M_H **and the counterterm which are infinite, to have the physical finite mass. This is the case of the log divergence of** m_e **in QED.**
However, we want to give a physical meaning to the cut–off Λ **and the logarithmic and quadratic divergences are of different nature.**

In the Standard Model:
besides the fermions, there are also contributions to M_H **from the massive gauge bosons and from the Higgs boson itself:**



Total contributions of fermions and bosons in the SM at one–loop:

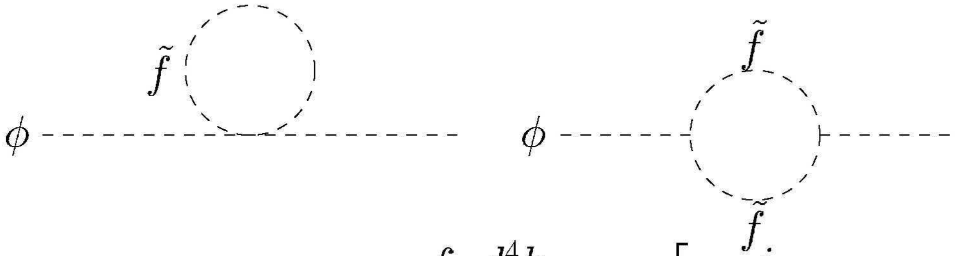
$$\Delta M_H^2 \propto [3(M_W^2 + M_Z^2 + M_H^2)/4 - \sum m_f^2](\Lambda^2/M_W^2)$$

We can adjust the unknown M_H **so that the quadratic divergence disappears (would be a prediction for Higgs mass,** $M_H \sim 200$ **GeV).**

However: does not work at two–loop level or at higher orders....

Summary: the problem of the quadratic divergences to M_H **is there.**
There is no symmetry which protects M_H **in the SM.**

The contributions of (2) scalars



$$\Delta M_H^2 = \Sigma_H(p^2) = (i)N_S \int \frac{d^4k}{(2\pi)^4}(i\lambda_S)\left[\frac{i}{k^2-m_1^2} + \frac{i}{k^2-m_2^2}\right]$$
$$+N_S(i\lambda_S v)^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2-m_1^2}\frac{i}{(k+p)^2-m_1^2} + m_1 \leftrightarrow m_2\right]$$

And here, really simple calculation... We get after integration:

$$\begin{aligned}\Delta M_H^2 &= \frac{\lambda_S N_S}{16\pi^2}\Big[-2\Lambda^2 + 2m_1^2 \log\big(\frac{\Lambda}{m_1}\big) + 2m_2^2\log\big(\frac{\Lambda}{m_2}\big)\Big] \\ &- \frac{\lambda_S^2 v^2 N_S}{16\pi^2}\Big[-2 + 2\log\big(\frac{\Lambda}{m_1}\big) + 2\log\big(\frac{\Lambda}{m_2}\big)\Big] + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)\end{aligned}$$

Again, quadratic divergences. But let us now assume:

- **Scalar couplings related to fermion couplings:** $\lambda_f^2 = -\lambda_S$ **(!).**
- **Multiplicative factors are the same:** $N_S = N_f$ **(nb: 2 scalars!).**
- **To simplify, the scalars have the same mass:** $m_1 = m_2 = m_S$**.**

Let us now add the fermionic and scalar contributions:

$$\Delta M_H^2|^{\rm tot} = \frac{\lambda_f^2 N_f}{4\pi^2}\Big[(m_f^2 - m_S^2)\log\big(\frac{\Lambda}{m_S}\big) + 3m_f^2\log\big(\frac{m_S}{m_f}\big)\Big]$$

The quadratic divergences have disappeared in the sum!! Logarithmic divergence still there, but even with $\Lambda = M_P$**, contribution small. It disappears also if in addition we have** $m_S = m_f$**!**

⇒ Symmetry fermions–scalars → no divergence in Λ^2
"Supersymmetry" no divergences at all: M_H **is protected!**

E4: Higgs masses and couplings in the MSSM

1. The MSSM Higgs boson masses

To obtain the physical Higgs fields and their masses from V_H,

$$V_H = \overline{m}_1^2(|H_0^1|^2 + |H_1^+|^2) + \overline{m}_2^2(|H_2^0|^2 + |H_2^-|^2) - \overline{m}_3^2(H_1^+ H_2^- - H_1^0 H_2^0 + \text{hc})$$
$$+\frac{g_2^2+g_1^2}{8}(|H_1^0|^2 + |H_1^+|^2 - |H_2^0|^2 - |H_2^-|^2)^2 + \frac{g_2^2}{2}|H_1^{+*}H_1^0 + H_2^{0*}H_2^-|^2$$

develop $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ into real (corresponding to CP–even and charged Higgses) and imaginary (CP–odd Higgs and Goldstones) parts and diagonalize the mass matrices:

$$\mathcal{M}_{ij}^2 = \frac{1}{2}\partial^2 V_H/\partial H_i \partial H_j|_{\langle \text{Re}(H_{1,2}^0)\rangle = v_{1,2}, \langle \text{Im}(H_{1,2}^0)\rangle = 0, \langle H_{1,2}^\pm \rangle = 0}$$

To obtain the masses and mixing angles, two useful relations are:

$$\text{Tr}(\mathcal{M}^2) = M_1^2 + M_2^2 \quad , \quad \text{Det}(\mathcal{M}^2) = M_1^2 M_2^2$$
$$\sin 2\theta = \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}} \ , \ \cos 2\theta = \frac{\mathcal{M}_{11}-\mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}$$

where M_1 and M_2 are the physical masses and θ the mixing angle.

The procedure in the case of the CP–even Higgs bosons:

The neutral part of the scalar potential is (drop subscripts ...):

$$V_H = \overline{m}_1^2|H_1|^2 + \overline{m}_2^2|H_2|^2 + \overline{m}_3^2(H_1H_2 + \text{hc}) + (M_Z^2/4v^2)(|H_1|^2 - |H_2|^2)^2$$

First perform the first derivative of the scalar potential:

$$\begin{aligned}
\partial V_H/\partial H_1^0 &= 2\bar{m}_1^2 H_1 + 2\bar{m}_3^2 H_2 + M_Z^2/v^2 \ H_1(H_1^2 - H_2^2) \\
\partial V_H/\partial H_2^0 &= 2\bar{m}_2^2 H_2 + 2\bar{m}_3^2 H_1 + M_Z^2/v^2 \ H_2(H_2^2 - H_1^2)
\end{aligned}$$

At the minimum, $\partial V_H/\partial H_{1,2}=0$, leading to the two relations:

$$\bar{m}_1^2=-\bar{m}_3^2\tan\beta-\frac{1}{2}M_Z^2\cos(2\beta)\ ,\ \bar{m}_2^2=-\bar{m}_3^2\text{cot}\beta+\frac{1}{2}M_Z^2\cos(2\beta)$$

Then make the second derivative with respect to H_1 and H_2:

$$\begin{aligned}
\frac{\partial^2 V_H}{\partial H_1^0\partial H_1^0} &= 2\bar{m}_1^2+\frac{M_Z^2}{v^2}(H_1^2-H_2^2+2H_1H_1)=2\bar{m}_1^2+M_Z^2(3c_\beta^2-s_\beta^2)\\
\frac{\partial^2 V_H}{\partial H_2^0\partial H_2^0} &= 2\bar{m}_2^2+\frac{M_Z^2}{v^2}(H_2^2-H_1^2+2H_2H_2)=2\bar{m}_2^2+M_Z^2(3s_\beta^2-c_\beta^2)\\
\frac{\partial^2 V_H}{\partial H_1^0\partial H_2^0} &= 2\bar{m}_3^2+\frac{M_Z^2}{v^2}(-2H_1H_2)=2\bar{m}_3^2-M_Z^2\sin 2\beta
\end{aligned}$$

Using the previous relations for $\bar{m}_1$ and $\bar{m}_2$ in terms of $\bar{m}_3$ and M_Z, we then obtain the mass matrix for the CP even Higgs bosons:

$$\mathcal{M}_R^2=\begin{bmatrix}-\bar{m}_3^2\tan\beta+M_Z^2\cos^2\beta & \bar{m}_3^2-M_Z^2\sin\beta\cos\beta\\ \bar{m}_3^2M_Z^2\sin\beta\cos\beta & -\bar{m}_3^2\text{cot}\beta+M_Z^2\sin^2\beta\end{bmatrix}$$

In the case of the CP–odd Higgs boson: use the same expressions for $\partial^2V/\partial H_i^0\partial H_j^0$ as above but set the fields to zero at the minimum:

$$\mathcal{M}_I^2=\begin{bmatrix}-\bar{m}_3^2\tan\beta & \bar{m}_3^2\\ \bar{m}_3^2 & -\bar{m}_3^2\text{cot}\beta\end{bmatrix}$$

Since $\text{Det}\mathcal{M}_I^2=0$, one eigenvalue is zero (the Goldstone) while the other one corresponds to the mass of the pseudoscalar Higgs:

$$M_A^2=-\bar{m}_3^2(\tan\beta+\text{cot}\beta)=-2\bar{m}_3^2/\sin 2\beta$$

The mixing angle θ is, in fact, just the angle β:

$$\begin{aligned}
\sin 2\theta &= 2\left[\left(\frac{s_\beta}{c_\beta}-\frac{c_\beta}{s_\beta}\right)^2+4\right]^{-\frac{1}{2}}=2\left[\frac{c_{2\beta}^2}{s_{2\beta}^2/4}+4\right]^{-\frac{1}{2}}=s_{2\beta}\\
\cos 2\theta &= \left(-\frac{s_\beta}{c_\beta}+\frac{c_\beta}{s_\beta}\right)\left[\cdots\cdots\right]^{-1/2}=\left(\frac{c_{2\beta}}{s_{2\beta}}\right)\left[s_{2\beta}\right]=c_{2\beta}
\end{aligned}$$

In the case of the charged Higgs boson, same excercice:

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix}$$

with a massless charged Goldstone and a charged Higgs with mass:

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

Back to the CP–even Higgsses: **inject expression of M_A^2 into $\mathcal{M}_R^2$:**

$$\mathcal{M}_R^2 = \begin{bmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2+M_Z^2)\sin\beta\cos\beta \\ -(M_A^2+M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{bmatrix}$$

Calculating determinant and trace, one obtains for the masses:

$$\mathrm{Det}\mathcal{M}_R^2 = (M_A^2 s^2 + M_Z^2 c^2)(M_A^2 c^2 + M_Z^2 s^2) - (M_A^2 + M_Z^2) s^2 c^2 = M_A^2 M_Z^2 c_{2\beta}^2 \equiv M_h^2 M_H^2$$
$$\mathrm{Tr}\mathcal{M}_R^2 = M_A^2 s^2 + M_Z^2 c^2 + M_A^2 c^2 + M_Z^2 s^2 = M_A^2 + M_Z^2 \equiv M_h^2 + M_H^2$$

To obtain the CP–even Higgs masses, solve the equation:

$$M_h^2(M_A^2 + M_Z^2 - M_h^2) = M_A^2 M_Z^2 c_{2\beta}^2 \Rightarrow M_h^4 - M_h^2(M_A^2 + M_Z^2) + M_A^2 M_Z^2 c_{2\beta}^2 = 0$$

with discriminant $\Delta = (M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta$, the two solutions are: $M_{h,H}^2 = \frac{1}{2}(M_A^2 + M_Z^2 \mp \sqrt{\Delta})$ giving (h is the lightest Higgs):

$$M_{h,H}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right]$$

The mixing angle α which rotates the fields is ($-\frac{\pi}{2} \leq \alpha \leq 0$)

$$\tan 2\alpha = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}} = \frac{-(M_A^2 + M_Z^2)\sin 2\beta}{(M_Z^2 - M_A^2)\cos 2\beta} = \tan 2\beta \, \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

We see that we have an important constraint on the lightest h boson mass in the MSSM:

$$M_h \leq \min(M_A, M_Z) \cdot |\cos 2\beta| \leq M_Z$$

besides some other (alos important) relations:

$$M_H > \max(M_A, M_Z) \quad \text{and} \quad M_{H^\pm} > M_W$$

If we send M_A to infinity, we will have for the Higgs masses and α:

$$M_h \sim M_Z|\cos 2\beta| \ , \ M_H \sim M_{H^\pm} \sim M_A \ , \qquad \alpha \sim \frac{\pi}{2} - \beta$$

This is the decoupling regime: all Higgses are heavy except for h.

The h boson is lighter than M_Z and should have been seen at LEP2 (we have $\sqrt{s}_{\rm LEP2} \sim 200$ GeV $> M_h + M_Z \sim 180$ GeV).

So what happened in this case? Maybe the MSSM is ruled out?

No! This relation holds only at first order (tree–level): there are strong couplings involved, in particular the htt and $h\tilde{t}\tilde{t}$ couplings.

$\Rightarrow$ Calculation of radiative corrections to M_h necessary.

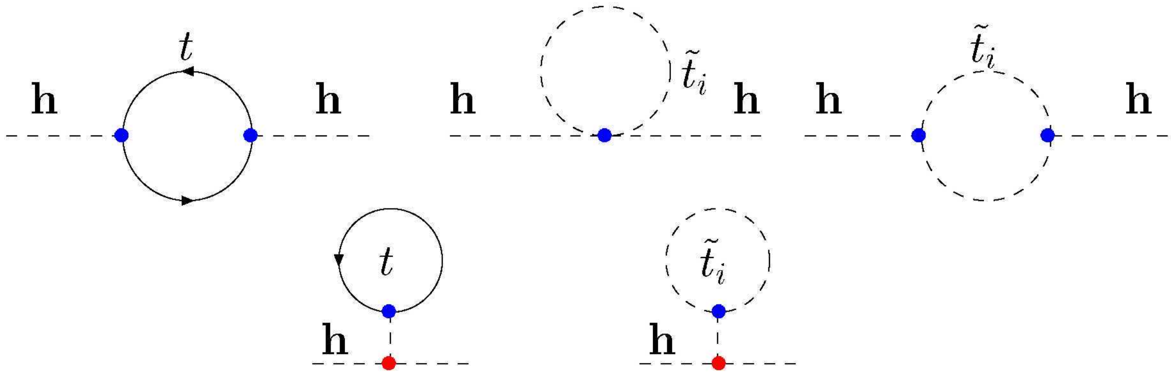
We have to include the important corrections due to top (s)quarks.

2. Calculation of radiative corrections to M_h

Let us do the calculation, but with some simplifications:

- take the (decoupling) limit $M_A \to 0$ and use $\tan\beta \gg 1$ ($M_h^{\rm max}$)
- assume no stop mixing and same masses, $m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_{\tilde{t}}$
- simple couplings: $h\bar{t}t \sim h\tilde{t}\tilde{t} \sim \lambda_t$, $hh\tilde{t}^*\tilde{t} \sim \lambda_t^2$ with $\lambda_t = \sqrt{2}m_t/v$
- work in the limit $M_h \ll m_t, m_{\tilde{t}}$.

In addition to two–point functions including fermion/scalar loops, we have also tadpole contributions (counterterm corrections):



- **The calculation is almost already done: for two–point function:**

$$\Delta M_h^2|_2 \; = \; \frac{3\lambda_t^2}{4\pi^2}\left[(m_t^2 - m_{\tilde{t}}^2)\log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) + 3m_t^2\log\left(\frac{m_{\tilde{t}}}{m_t}\right)\right]$$

- **For the tadpole contributions, the calculation is very simple:**

$$\begin{aligned}\Delta M_h^2|_1 \; &= \; iN_f\left(\frac{-iM_H^2}{v}\right)\frac{i}{-M_h^2}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)(-4mi)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2-m_f^2} \\ &+ \; iN_S\left(\frac{-iM_H^2}{v}\right)\frac{i}{-M_h^2}(iv\lambda_S)i\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2-m_S^2} \\ &= \; \frac{4N_f m_f\lambda_f}{\sqrt{2}v16\pi^2}\int_0^{\Lambda^2}dy\frac{y}{y+m_f^2} + \frac{N_S\lambda_S}{16\pi^2}\int_0^{\Lambda^2}dy\frac{y}{y+m_S^2}\end{aligned}$$

Using λ_S=-λ_f^2=-$2m_f^2/v$ **and** $\int ... = \Lambda^2 - m^2\log(\Lambda^2/m^2)$**, one obtains**

$$\Delta M_h^2|_1 \; = \; -\frac{3\lambda_t^2}{4\pi^2 v^2}\left[m_{\tilde{t}}^2\log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) - m_t^2\log\left(\frac{\Lambda}{m_{\tilde{t}}}\right)\right]$$

- **The total correction to the** h **boson mass is then given by:**

$$\Delta M_h^2 \; = \; \frac{3m_t^4}{2\pi^2 v^2}\log\frac{m_{\tilde{t}}}{m_t} = \frac{3g^2}{2\pi^2}\frac{m_t^4}{M_W^2}\log\frac{m_{\tilde{t}}^2}{m_t^2}$$

Its depends on m_t^4 **and** $\log(m_{\tilde{t}}^2/m_t^2)$**, and is large:** $M_h^{\max} \to M_Z + 40$ **GeV! This explains why the** h **boson has not been seen at LEP2.**

3. The Higgs boson couplings

Can be derived by looking at the relevant parts of the Lagrangian. Here will discuss briefly couplings to Higgs bosons, massive gauge bosons and fermions.

Trilinear and Quartic scalar couplings

The trilinear (3 fields) et quartic (4 fields) couplings among Higgs bosons can be obtained from the scalar potential V_H by making:

$$\lambda^2_{ijk} = \frac{\partial^3 V_H}{\partial H_i \partial H_j \partial H_k}\Big|_{\langle H_1^0\rangle = v_1, \langle H_2^0\rangle = v_2, \langle H_{1,2}^\pm\rangle = 0}$$

$$\lambda^2_{ijkl} = \frac{\partial^4 V_H}{\partial H_i \partial H_j \partial H_k \partial H_l}\Big|_{\langle H_1^0\rangle = v_1, \langle H_2^0\rangle = v_2, \langle H_{1,2}^\pm\rangle = 0}$$

with the H_i expressed in terms of the fields $h, H, A, H^\pm$ and $G^0, G^\pm$ with rotations of angles β et α. Examples (unit: $\lambda_0 = -iM_Z^2/v$):

$$\lambda_{hhh} = 3\cos 2\alpha \sin(\beta+\alpha) + \text{rad. corr.}$$

$$\lambda_{hhhh} = 3\cos^2\alpha / M_Z^2 + \text{rad. corr.} \qquad (\text{in units of } \lambda_0^2)$$

In the decoupling limit, $M_A \gg M_Z$ we have $\alpha \to \beta - \pi/2$:

$$\lambda_{hhh} \to 3\cos^2(2\beta) = 3M_h^2/M_Z^2 = \lambda^3|_{\text{MS}}$$

$$\lambda_{hhhh} \to 3\cos^2(2\beta)/M_Z^2 = 3M_h^2/M_Z^4 = \lambda^4|_{\text{MS}}$$

⇒ In the decoupling limit, $M_A \gg M_Z$, the Higgs potential of the MSSM becomes like the one of the SM: only one light Higgs with a mass $M_h \lesssim 130$ GeV and with standard interactions. All other Higgsses are heavy and decouple (but self–couplings are non–zero).

Couplings to gauge bosons:

Higgs couplings to massive gauge bosons are obtained from the kinetic terms of H_1 et H_2 in the $\mathrm{SU(2)_L \times U(1)_Y}$ Lagrangian:

$$\mathcal{L}_{\text{kin.}} = (D^\mu H_1)^\dagger (D_\mu H_1) + (D^\mu H_2)^\dagger (D_\mu H_2)$$

Develop D_μ and make the usual transformations on the fields:

$$D_\mu = i\partial_\mu - g\frac{\vec{\tau}_a}{2}W_\mu^a - g'\frac{Y_{H_i}}{2}B_\mu$$

$$W_{1,2,3}, B \to W^\pm, Z, \gamma \quad ; \quad H_{1,2} \to h, H, A, H^\pm, G^0, G^\pm \text{ via rotations } \beta, \alpha$$

$$\begin{aligned} g_{h_iVV} &\equiv \text{coefficients de}\, h_i V_\mu V_\mu \quad (g_{\mu\nu}) \\ g_{h_ih_jV} &\equiv \text{coefficients de}\, h_i h_j V_\mu \quad (\partial_\mu \to p_\mu) \\ g_{h_ih_jVV} &\equiv \text{coefficients de}\, h_i h_j V_\mu V_\mu \quad (g_{\mu\nu}) \end{aligned}$$

Some very important couplings for Higgs phenomenology:

$$\begin{aligned} Z^\mu Z^\nu h &: \frac{igM_Z}{\cos\theta_W}\sin(\beta-\alpha)g^{\mu\nu} \quad , \quad Z^\mu Z^\nu H : \frac{igM_Z}{\cos\theta_W}\cos(\beta-\alpha)g^{\mu\nu} \\ W^\mu W^\nu h &: igM_W\sin(\beta-\alpha)g^{\mu\nu} \quad , \quad W^\mu W^\nu H : igM_W\cos(\beta-\alpha)g^{\mu\nu} \\ Z^\mu hA &: \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu \quad , \quad Z^\mu HA : -\frac{g\sin(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu \end{aligned}$$

– **γ massless: no coupling with the neutral Higgsses at tree–level.**
– **CP invariance: no ZZA and Zhh, ZHh, ZHH couplings e.g.**
– **Couplings of h and H complementary: $g_{hZZ}^2 + g_{HZZ}^2 = g_{\rm MS}^2$!**
– **Decoupling limit ($M_A \to \infty, \alpha \to \beta - \frac{\pi}{2}$): $\sin(\beta-\alpha) \to 1, \cos(\beta-\alpha) \to 0$:**

$$\Rightarrow g_{hVV} = g_{H_{\rm MS}VV} \;, g_{HVV} = 0 \; (= g_{AVV}) \qquad .$$

Yukawa couplings to fermions:

The Higgs couplings to fermions come from Superpotential W:

$$W = \sum_{\text{i,j=gen}} Y^u_{ij}\, \hat{u}^i_R \hat{H}_2.\hat{Q}^j + Y^d_{ij}\, \hat{d}^i_R \hat{H}_1.\hat{Q}^j + Y^l_{ij}\, \hat{l}^i_R \hat{H}_1.\hat{L}^j + \mu \hat{H}_1.\hat{H}_2$$

with $\mathcal{L}_{\text{Yuk}} = -\frac{1}{2}\sum_{ij}\left[\overline{\psi}_{iL}\frac{\partial^2 W}{\partial z_i \partial z_j}\psi_j + \text{h.c.}\right]$ **evaluated in terms of** H_1, H_2. **Take bilinears out, digagonal** Y **matrices** Y **with relations to masses, and expressings** H_1, H_2 **in terms of the physical fields, we get:**

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} &= -\frac{g m_u}{2M_W \sin\beta}\left[\bar{u}u(H\sin\alpha + h\cos\alpha) - i\bar{u}\gamma_5 u\, A\cos\beta\right] \\ &\quad -\frac{g m_d}{2M_W \cos\beta}\left[\bar{d}d(H\cos\alpha - h\sin\alpha) - i\bar{d}\gamma_5 d\, A\sin\beta\right] \\ &\quad +\frac{g}{2\sqrt{2}M_W}\left\{H^+\bar{u}[m_d\tan\beta(1+\gamma_5) + \frac{m_u}{\tan\beta}(1+\gamma_5)]d + \text{hc}\right\}\end{aligned}$$

Couplings in termsof those of H_{SM} **[factor** $-(i)gm_f/2M_W = -im_f/v$**] and their values in the decoupling limit** $[\cos\alpha \to \sin\beta, \sin\alpha \to -\cos\beta]$**:**

f	g_{ffh}	g_{ffH}	g_{ffA}
u	$\cos\alpha/\sin\beta \to 1$	$\sin\alpha/\sin\beta \to -\tan\beta$	$\cot\beta$
d	$-\sin\alpha/\cos\beta \to 1$	$\cos\alpha/\cos\beta \to \tan\beta$	$\tan\beta$

– **The couplings of** $H^\pm$ **have the same intensity as those of** A.
– **For** $\tan\beta > 1$: **Cplgs to** d **enhanced, cplgs to** u **suppressed.**
– **For** $\tan\beta \gg 1$: **couplings to** b **quarks** b $(m_b \tan\beta)$ **very strong.**
– **For** $M_A \gg M_Z$: h **couples like the SM Higgs boson and** H **like** A.