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Flavour Physics at the LHC
Part I

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These are preliminary lecture notes, intended only for distribution to participants.

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School on Physics at LHC: “Expecting LHC”
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(I)

Lecture I: Setting the Stage & Basics

- Introduction:
  - Why study flavour physics and CP violation?
  - The main actors of these lectures: B mesons.
  - Where to study their decays?

- CP Violation in the Standard Model:
  - Weak interactions of quarks
  - Cabibbo–Kobayashi–Maskawa matrix
  - Unitarity triangles

- Decays of B Mesons:
  - Leptonic decays
  - Semileptonic decays
  - Non-leptonic decays: $\rightarrow$ play the central rôle for CP violation

- Exploring CP Violation through Amplitude Relations

A Selection of Basic References

- Lecture Notes:
  - A. Buras.: "Flavour Physics and CP Violation",
    2004 European School on High-Energy Physics [hep-ph/0505175].
  - Y. Nir: "CP Violation: A New Era",
    3rd CERN-CLAF School of High-Energy Physics [hep-ph/0510413].
  - R.F.: "Flavour Physics and CP Violation",
    2005 European School on High-Energy Physics [hep-ph/0608010].

- Textbooks:
  - G. Branco, L. Lavoura and J. Silva: "CP Violation",
    International Series of Monographs on Physics 103, Oxford Science Publications (Clarendon Press, Oxford 1999).
  - I.I. Bigi and A.I. Sanda: "CP Violation",
    Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology (Cambridge University Press, Cambridge, 2000).

Introduction

- A flavour of "flavour physics":
  - Deals with the Fermion generations, their masses and mixing patterns:

    $\longrightarrow$ resides in the Yukawa sector of the Standard Model ...

  - Beyond the Standard Model:

    $\longrightarrow$ many new flavour phenomena!

A Key Puzzle of Flavour Physics: CP Violation

- In 1957, surprising discovery that the weak interactions are *not* invariant under parity transformations (Wu *et al.*):

$$\Rightarrow \boxed{\text{parity violation!}}$$

  - Parity transformation $\mathcal{P}$: space inversion $\vec{x} \rightarrow -\vec{x}$

- However, it was believed that the product $\mathcal{CP}$ was preserved:

  - Charge conjugation $\mathcal{C}$: particle $\rightarrow$ antiparticle

\+ e^+ e – C – e^- C e – P – e^- — e

lefthanded (×) righthanded (OK)

- In 1964, discovery of CP violation in neutral K decays (Christenson *et al.*):

$$\boxed{K_L \rightarrow \pi^+\pi^-} \qquad (\text{BR} \sim 2 \times 10^{-3})$$

- These effects are a manifestation of *indirect CP violation:*

$$\begin{array}{ccccc} (\mathcal{CP}) & (-) & (+) & & (+) \\ K_{\rm L} = & K_2 & + \;\overline{\varepsilon} K_1 & \longrightarrow & \left\{ \begin{array}{c} \pi^+\pi^- \\ \pi^0\pi^0 \end{array} \right. \end{array}$$

direct: ε' ($K_2 \to \pi\pi$)

indirect: ε ($\overline{\varepsilon} K_1 \to \pi\pi$)

$$\boxed{\varepsilon = (2.280 \pm 0.013) \times 10^{-3} \times e^{i\pi/4}}$$

- In 1999, *direct* CP violation could be established [NA48 & KTeV]:

$$\text{Re}(\varepsilon'/\varepsilon) = \left\{ \begin{array}{ll} (14.7 \pm 2.2) \times 10^{-4} & \text{[NA48 (2002)]} \\ (20.7 \pm 2.8) \times 10^{-4} & \text{[KTeV (2002)]} \end{array} \right.$$

- In 2001, discovery of CP-violating effects in B decays [BaBar & Belle], i.e. for the first time *outside* of the K system:

$$\boxed{B^0 \to J/\psi K_{\rm S}} \to \textit{mixing-induced}\ \text{CP violation!}$$

- In 2004, also observation of *direct* CP violation in $B^0 \to \pi^- K^+$ ...

Why Study Flavour Physics & CP Violation?

- New Physics (NP): $\longrightarrow$ typically new sources of flavour & CP violation
  - SUSY scenarios;
  - left–right-symmetric models;
  - scenarios with extra dimensions;
  - models with extra Z' bosons;
  - "little Higgs" scenarios ...
- ν masses: $\longrightarrow$ origin beyond the Standard Model (SM)!
  - CP violation in the neutrino sector?
  - Connection with quark-flavour physics?
- Furthermore:
  - The origin of the fermion masses, flavour mixing and their hierarchies, as well as that of CP violation lie completely in the dark:

$\longrightarrow$ involves new physics, too!

Moreover: A Link to Cosmology

- One of the key features of our Universe:

  cosmological baryon asymmetry of $\mathcal{O}(10^{-10})$

- Necessary conditions for the generation of such an asymmetry: [Sacharow '67]

  1. Deviations from thermal equilibrium.
  2. Baryon-number violation.
  3. *Elementary interactions violate CP (and C)!*

- Model calculations: $\Rightarrow$ CP violation appears too small in the SM

  $\Rightarrow$ new sources of CP violation:

  – Could be associated with very high energy scales:
    * attractive mechanism: "leptogenesis", involving new CP-violating sources in the decays of heavy Majorana neutrinos.

  – *But could also be accessible in the laboratory (see above) ...*

Challenging the Standard Model through Flavour Studies

Before searching for NP, we have first to understand the SM picture!

- Key problem for the theoretical interpretation:

  ◇ *impact of strong interactions* $\longrightarrow$ "hadronic" uncertainties!

  – Famous example: Re$(\varepsilon'/\varepsilon)$

- Prospects for the "good old" K-meson system:

  – Clean tests of the SM are offered by $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$, as their hadronic pieces can be fixed through $K \to \pi\ell\bar{\nu}$ decays!

  – These "rare" decays are *absent* at the tree level of the SM, i.e. originate there exclusively from loops, with BRs$= \mathcal{O}(10^{-10})$ $\to$ challenging ...

- The B-meson system is a *particularly promising* probe:

  – Offers various strategies: simply speaking, there are *many* B decays!

  – Search for clean SM relations that could be spoiled by NP ...

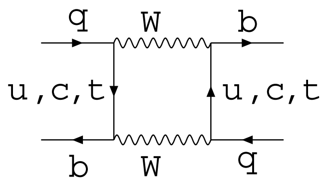
$\longrightarrow$ our focus!

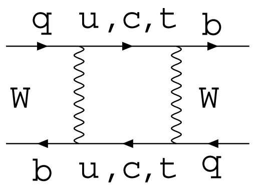
The Main Actors of these Lectures: B Mesons

- Charged B mesons: $B^+ \sim u\,\overline{b}$ $\quad B^- \sim \overline{u}\,b$
  $B_c^+ \sim c\,\overline{b}$ $\quad B_c^- \sim \overline{c}\,b$

- Neutral B mesons: $B_d^0 \sim d\,\overline{b}$ $\quad \overline{B_d^0} \sim \overline{d}\,b$
  $B_s^0 \sim s\,\overline{b}$ $\quad \overline{B_s^0} \sim \overline{s}\,b$

  – B_q^0–$\overline{B_q^0}$ mixing:





$$\Rightarrow \quad |B_q(t)\rangle = a(t)|B_q^0\rangle + b(t)|\overline{B_q^0}\rangle\ :$$

∗ Schrödinger equation $\Rightarrow$ mass eigenstates:

$$\Delta M_q \equiv M_{\rm H}^{(q)} - M_{\rm L}^{(q)}, \quad \Delta\Gamma_q \equiv \Gamma_{\rm H}^{(q)} - \Gamma_{\rm L}^{(q)}$$

∗ Decay rates: $\Gamma(\overset{(-)}{B}{}_q^0(t) \to \overset{(-)}{f})$: [→ Lecture II]

$$\cos(\Delta M_q t)\ \&\ \sin(\Delta M_q t) \quad \longrightarrow \quad \text{oscillations!}$$

Where to Study?

- B factories: asymmetric e^+e^- colliders @ $\Upsilon(4S) \to B_d^0\bar{B}_d^0,\ B_u^+B_u^-$
  - PEP-II with the *Babar* experiment (SLAC);
  - KEK-B with the *Belle* experiment (KEK):

$$\to \begin{cases} \text{could well establish CP violation in the } B \text{ system;} \\ \text{many interesting results with } \mathcal{O}(10^8)\ B\bar{B} \text{ pairs ...} \end{cases}$$

  - Discussion of a super-B factory, with increase of luminosity by $\mathcal{O}(10^2)$.

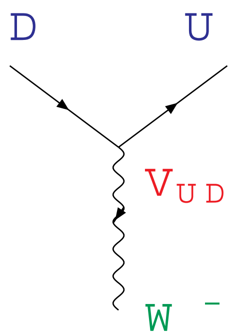
- Hadron colliders: $\to$ produce also B_s mesons, as well as B_c, Λ_b, ...
  - Tevatron: CDF and D0 have reported first B-decay results ...
  - ... to be continued at the LHC $\gtrsim 2007$:

ATLAS & CMS (can also address some B physics)
$\oplus$ *dedicated* B-decay experiment: LHCb

- Kaon physics in the LHC era:
  - Plans to measure $K^+ \to \pi^+\nu\bar{\nu}$ at the SPS (CERN);
  - $K_{\rm L} \to \pi^0\nu\bar{\nu}$ at the E391(a) experiment (KEK/J-PARC).

CP Violation
in the
Standard Model

- Weak charged-current interactions of $D \in \{d, s, b\}$, $U \in \{u, c, t\}$ quarks:



V_{UD} : Cabibbo–Kobayashi–Maskawa (CKM) matrix element

Let's have a closer look ...

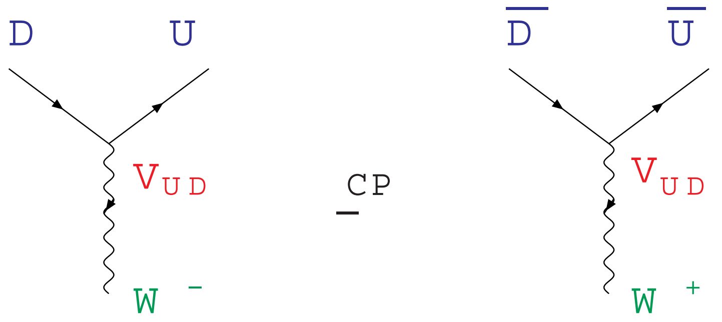
$$\mathcal{L}_{\rm int}^{\rm CC} = -\frac{g_2}{\sqrt{2}} \left(\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L}\right) \gamma^{\mu}\, \hat{V}_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix} W_{\mu}^{\dagger} + \text{h.c.}$$

- The CKM matrix connects the electroweak flavour states (d', s', b') with their mass eigenstates (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Since this transformation is unitary: $\hat{V}_{\rm CKM}^{\dagger} \cdot \hat{V}_{\rm CKM} = \hat{1} = \hat{V}_{\rm CKM} \cdot \hat{V}_{\rm CKM}^{\dagger}$

- CP-conjugate transitions:



$$V_{UD} \xrightarrow{\mathcal{CP}} V_{UD}^{*} \quad \Rightarrow$$

Are there Physical Phases in the CKM Matrix?

- Redefinition of the quark-field phases in $\mathcal{L}_{\text{int}}^{\text{CC}}$:

$$\left.\begin{array}{l} U \longrightarrow \exp(i\xi_U) U \\ \\ D \longrightarrow \exp(i\xi_D) D \end{array}\right\} \Rightarrow \boxed{V_{UD} \longrightarrow \exp(i\xi_U) V_{UD} \exp(-i\xi_D)}$$

- Parameters of the general $N \times N$ quark-mixing matrix (N generations):

$$\underbrace{\frac{1}{2} N(N-1)}_{\text{Euler angles}} + \underbrace{\frac{1}{2}(N-1)(N-2)}_{\text{complex phases}} = (N-1)^2$$

- Two generations: $\longrightarrow$ $\boxed{\text{Cabibbo angle } \theta_{\text{C}} \text{ (1963)}}$

$$\hat{V}_{\text{C}} = \begin{pmatrix} \cos\theta_{\text{C}} & \sin\theta_{\text{C}} \\ -\sin\theta_{\text{C}} & \cos\theta_{\text{C}} \end{pmatrix} \qquad [\sin\theta_{\text{C}} = 0.22 \text{ from } K \to \pi e \overline{\nu}_e]$$

- Three generations: $\longrightarrow$ $\boxed{\text{Kobayashi \& Maskawa (1973)}}$
  - Requires three Euler angles and one complex phase ...
  - Complex phase: allows us to accommodate CP violation in the SM!

(Exact) Parametrizations of the CKM Matrix

- "Standard" Parametrization ($\to$ PDG): [$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$]

$$\hat{V}_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- Kobayashi & Maskawa: [$c_i = \cos\theta_i$ and $s_i = \sin\theta_i$]

$$\hat{V}_{\mathrm{CKM}} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}$$

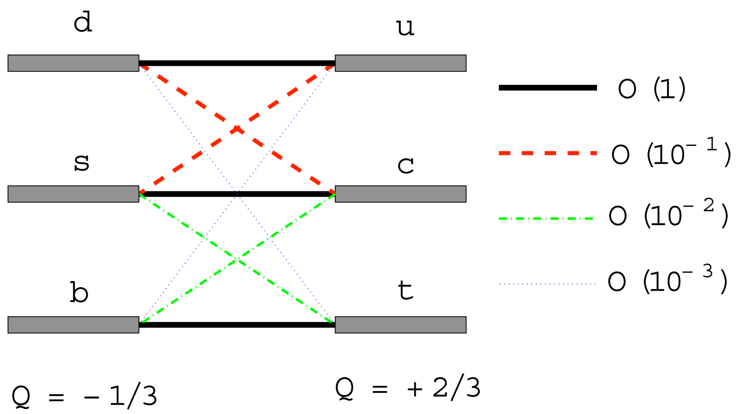
- Fritzsch & Xing: [$c_{\mathrm{u}} = \cos\theta_{\mathrm{u}}$, $s_{\mathrm{u}} = \sin\theta_{\mathrm{u}}$, etc.]

$$\hat{V}_{\mathrm{CKM}} = \begin{pmatrix} s_{\mathrm{u}}s_{\mathrm{d}}c + c_{\mathrm{u}}c_{\mathrm{d}}e^{-i\varphi} & s_{\mathrm{u}}c_{\mathrm{d}}c - c_{\mathrm{u}}s_{\mathrm{d}}e^{-i\varphi} & s_{\mathrm{u}}s \\ c_{\mathrm{u}}s_{\mathrm{d}}c - s_{\mathrm{u}}c_{\mathrm{d}}e^{-i\varphi} & c_{\mathrm{u}}c_{\mathrm{d}}c + s_{\mathrm{u}}s_{\mathrm{d}}e^{-i\varphi} & c_{\mathrm{u}}s \\ -s_{\mathrm{d}}s & -c_{\mathrm{d}}s & c \end{pmatrix}$$

Wolfenstein Parametrization

- Hierarchy of the quark transitions mediated through charged currents:

Experiment ⇒



- This hierarchy is reflected in the standard parametrization as follows:

$$s_{12} = 0.22 \quad \gg \quad s_{23} = \mathcal{O}(10^{-2}) \quad \gg \quad s_{13} = \mathcal{O}(10^{-3}) \quad \Rightarrow$$

- New parameters: $s_{12} \equiv \lambda = 0.22, \quad s_{23} \equiv A\lambda^2, \quad s_{13} e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$

- Go back to the standard parametrization and neglect all terms of $\mathcal{O}(\lambda^4)$:

$$\Rightarrow \boxed{\hat{V}_{\rm CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\,\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\,\eta) & -A\lambda^2 & 1 \end{pmatrix}}$$

[Wolfenstein (1983)]

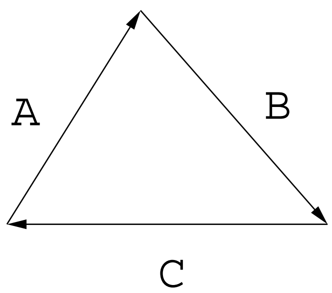
Unitarity Triangle(s) of the CKM Matrix

- Unitarity of the CKM matrix: $\boxed{\hat{V}_{\text{CKM}}^{\dagger} \cdot \hat{V}_{\text{CKM}} = \hat{1} = \hat{V}_{\text{CKM}} \cdot \hat{V}_{\text{CKM}}^{\dagger}}$ $\Rightarrow$

  – 6 normalization relations (columns and rows)

  – 6 orthogonality relations (columns and rows): $\boxed{A + B + C = 0}$

- The orthogonality relations can be represented as 6 triangles:



– unitarity triangles!

- These triangles have all the same area A_Δ, which can be interpreted as a measure of the "strength" of CP violation in the SM:

$$\boxed{2A_\Delta \equiv |J_{\text{CP}}| = \lambda^6 A^2 \eta = \mathcal{O}(10^{-5})} \rightarrow \text{"Jarlskog Parameter"}$$

- Columns:

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

$$\underbrace{V_{ud}V_{ub}^*}_{(\rho+i\eta)A\lambda^3} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{(1-\rho-i\eta)A\lambda^3} = 0$$

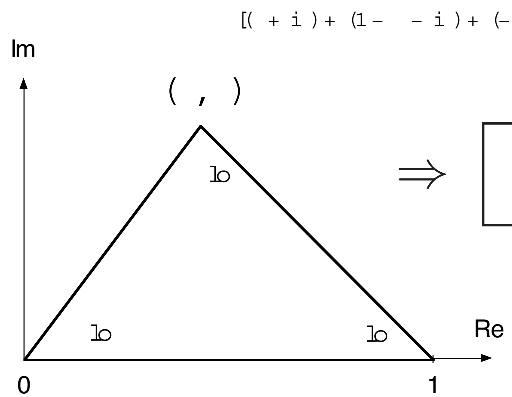
- Rows:

$$\underbrace{V_{ud}^*V_{cd}}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}^*V_{cs}}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}^*V_{cb}}_{\mathcal{O}(\lambda^5)} = 0$$

$$\underbrace{V_{cd}^*V_{td}}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}^*V_{ts}}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{cb}^*V_{tb}}_{\mathcal{O}(\lambda^2)} = 0$$

$$\underbrace{V_{ud}^*V_{td}}_{(1-\rho-i\eta)A\lambda^3} + \underbrace{V_{us}^*V_{ts}}_{-A\lambda^3} + \underbrace{V_{ub}^*V_{tb}}_{(\rho+i\eta)A\lambda^3} = 0$$

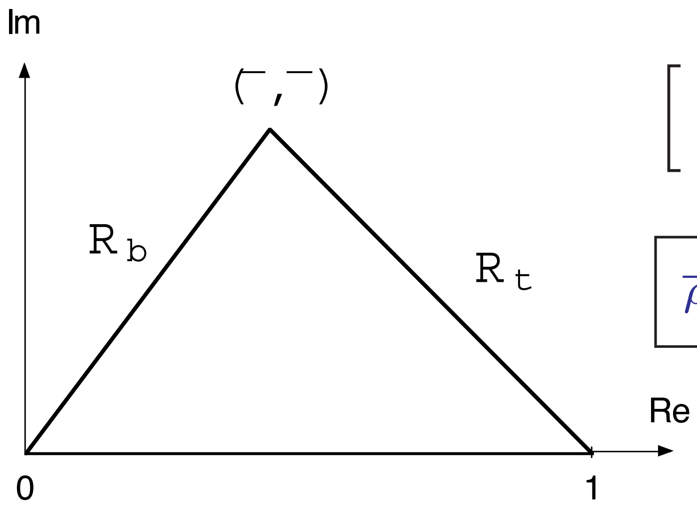
- Only in two relations, all terms are of $\mathcal{O}(\lambda^3)$, and *agree* with one another:



$\Rightarrow$ *the* unitarity triangle of the CKM matrix!

The Unitarity Triangles at NLO in λ

- $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$: $\quad\Rightarrow\quad$ UT

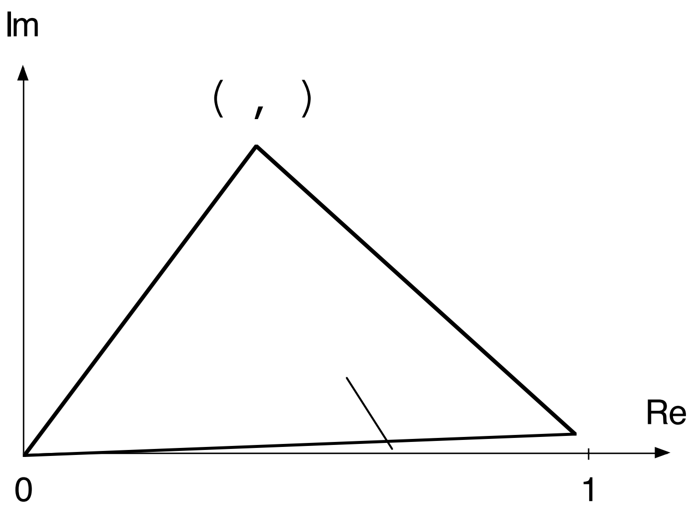


$$\left[\begin{array}{lcl} R_b & = & (1-\lambda^2/2)|V_{ub}/(\lambda V_{cb})| \\ R_t & = & |V_{td}/(\lambda V_{cb})| \end{array}\right]$$

$$\boxed{\overline{\rho} = (1-\lambda^2/2)\,\rho, \quad \overline{\eta} = (1-\lambda^2/2)\,\eta}$$

[Buras *et al.* (1994)]

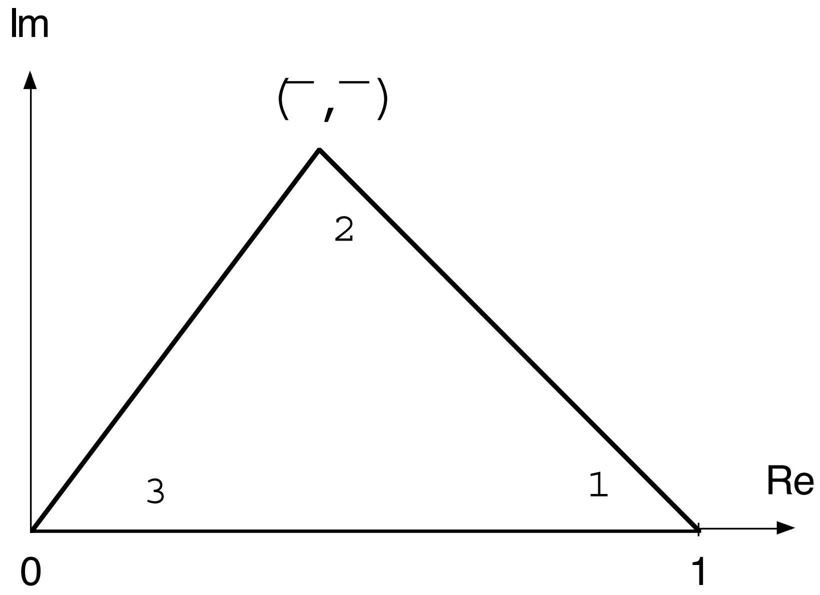
- $V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0$:



$$\boxed{\gamma = \gamma' + \delta\gamma, \quad \delta\gamma = \lambda^2\eta = \mathcal{O}(1^\circ)}$$

“Japanese” Conventions for the Angles of the UT

- Used in Asia and by the Belle Collaboration (KEK):



- Dictionary for the translation into the American/European conventions used in these lectures and by the BaBar Collaboration (SLAC):

$$\phi_1 \equiv \beta, \quad \phi_2 \equiv \alpha, \quad \phi_3 \equiv \gamma$$

Determination of the Unitarity Triangle (UT)

- Method I: *conventional ("CKM Fits") ...*
  - Semileptonic $b \to u\ell\overline{\nu}_\ell, c\ell\overline{\nu}_\ell$ decays [$\to$ UT side R_b].
  - $B^0_{d,s}$–$\overline{B^0_{d,s}}$ mixing [$\to$ UT side R_t].
  - CP violation in the kaon system, ε_K [$\to$ hyperbola].

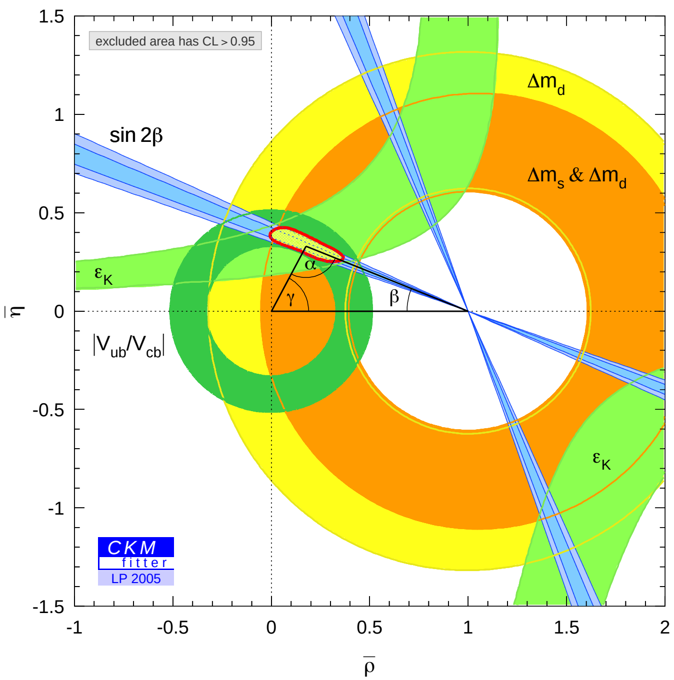
Theory	$\Rightarrow$	contours in the $\overline{\rho}$–$\overline{\eta}$ plane

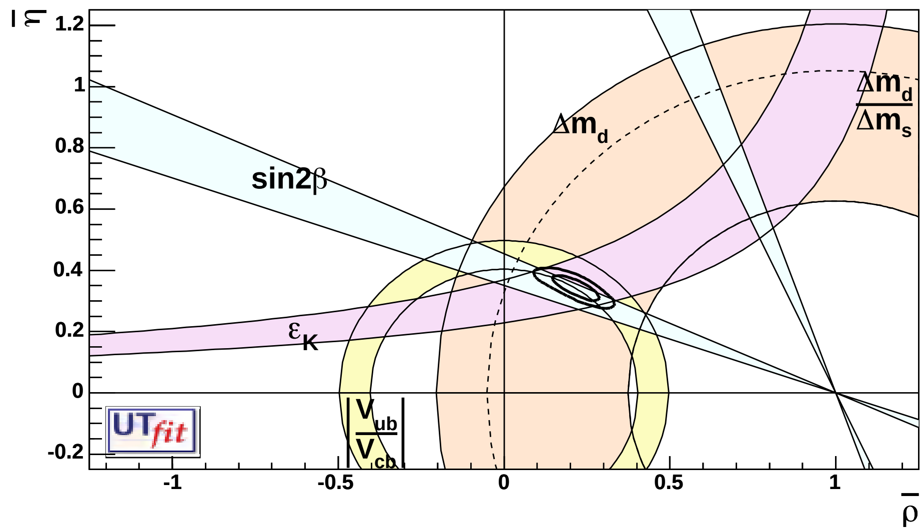
- Method II: *innovative ...*
  - CP-violating effects in B decays [$\to \sin 2\beta$, ...]

Theory	$\Rightarrow$	angles of the unitarity triangle

Status of the Unitarity Triangle

- Two competing groups: $\longrightarrow$ many plots & correlations ...
  - *CKMfitter* Collaboration [http://ckmfitter.in2p3.fr/];
  - *UTfit* Collaboration [http://www.utfit.org]:



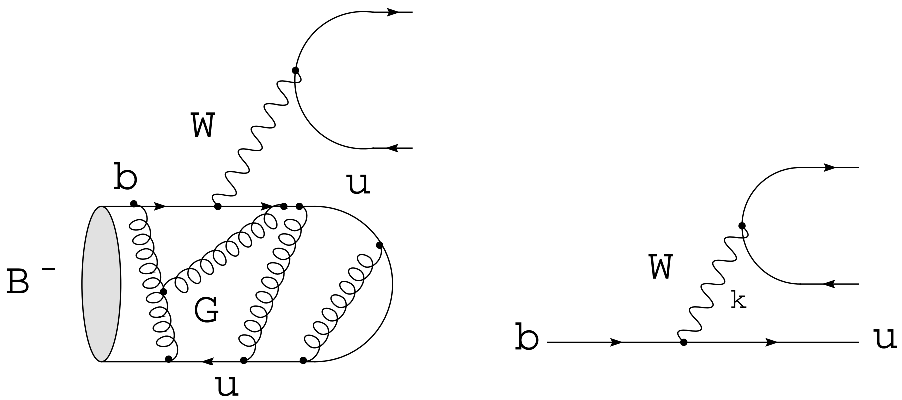


$\Rightarrow$ impressive global agreement with KM, but no longer "perfect" ...

Decays of B Mesons

Leptonic B Decays: $B^- \to \ell\overline{\nu}_\ell$

The simplest B-meson decays we can imagine:



- Calculation of the corresponding Feynman diagram:

$$T_{fi} = -\frac{g_2^2}{8} V_{ub} \underbrace{[\overline{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu]}_{\text{Dirac spinors}} \left[\frac{g_{\alpha\beta}}{k^2 - M_W^2}\right] \underbrace{\langle 0|\overline{u}\gamma^\beta(1-\gamma_5)b|B^-\rangle}_{\text{hadronic ME}}$$

- $k^2 = M_B^2 \ll M_W^2$: $\quad \Rightarrow \quad \frac{g_{\alpha\beta}}{k^2 - M_W^2} \longrightarrow -\frac{g_{\alpha\beta}}{M_W^2} \equiv -\left(\frac{8G_F}{\sqrt{2}g_2^2}\right) g_{\alpha\beta}$

$$\Rightarrow T_{fi} = \frac{G_F}{\sqrt{2}} V_{ub} [\overline{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu] \langle 0|\overline{u}\gamma_\alpha(1-\gamma_5)b|B^-\rangle$$

... the W boson is "integrated out"!

- The *whole* hadronic physics is encoded in the following quantity:

$$\langle 0|\overline{u}\gamma_\alpha(1-\gamma_5)b|B^-\rangle \;\longrightarrow\; \text{“hadronic matrix element”}$$

- Since the B^- is a pseudoscalar meson, we may write:

$$\langle 0|\overline{u}\gamma_\alpha b|B^-\rangle = 0, \quad \langle 0|\overline{u}\gamma_\alpha\gamma_5 b|B^-(q)\rangle = i f_B q_\alpha$$

f_B: B-meson decay constant $\longrightarrow$ important input!

[from lattice QCD $\longrightarrow$ see below]

- Branching ratio: $\text{BR}(B^- \to \ell\overline{\nu}_\ell) = \frac{G_\text{F}^2}{8\pi}|V_{ub}|^2 M_B m_\ell^2 \left(1-\frac{m_\ell^2}{M_B^2}\right)^2 f_B^2\tau_B$

  – Unfortunately, we encounter a tiny $|V_{ub}|$ and a helicity suppression:

    $\Rightarrow$ BRs$=\mathcal{O}(10^{-10})$ and $\mathcal{O}(10^{-7})$ for $\ell = e$ and $\ell = \mu$, respectively.

  – The helicity suppression is not effective for $\ell = \tau$, but experimentally very challenging (τ reconstruction). But nevertheless (!) ...

Evidence for $B^- \to \tau^- \bar{\nu}_\tau$ at Belle in Spring '06

- Branching ratio for this purely leptonic decay:
  - *New* value @ ICHEP '06 (preliminary):

$$\boxed{\mathrm{BR}(B^- \to \tau^- \bar{\nu}_\tau) = \left(1.79^{+0.56+0.39}_{-0.49-0.46}\right) \times 10^{-4}}$$

  - Previous value (affected by some coding error):

$$\mathrm{BR}(B^- \to \tau^- \bar{\nu}_\tau) = \left(1.06^{+0.34+0.18}_{-0.28-0.16}\right) \times 10^{-4}$$

- The SM expression for this BR and the measured values of G_F, M_B, m_τ and the B-meson lifetime allow the extraction of the following product:

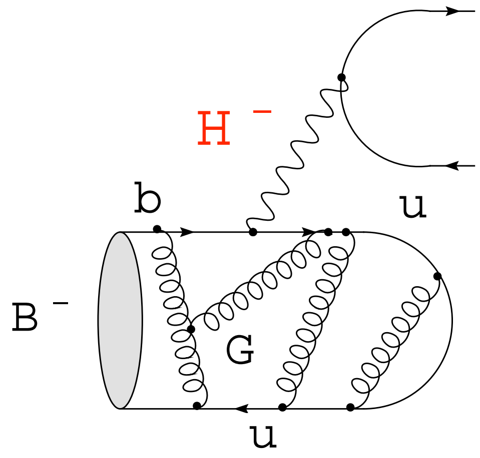
$$\boxed{f_B |V_{ub}| = \left(10.1^{+1.6+1.1}_{-1.4-1.3}\right) \times 10^{-4}\,\text{GeV}.}$$

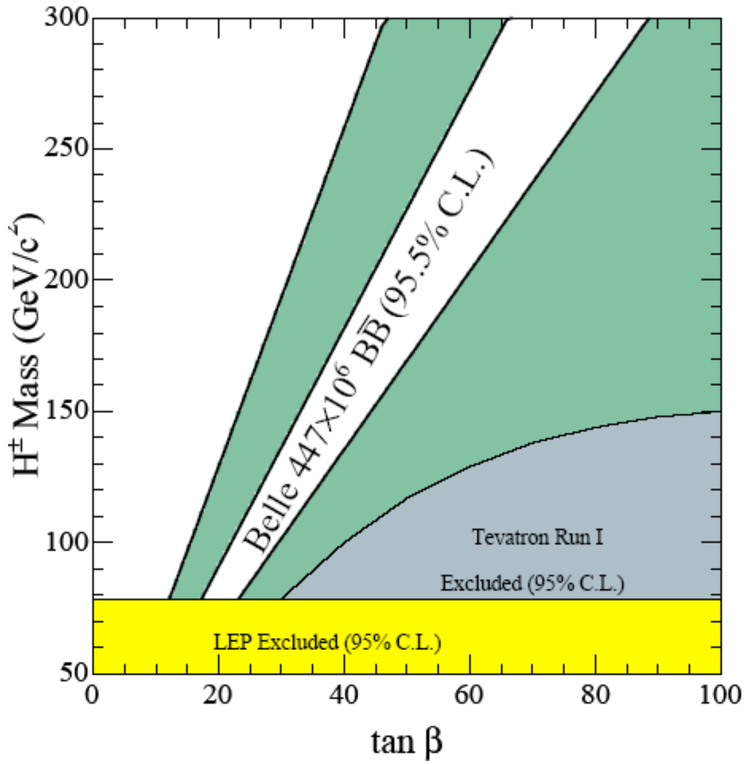
- This quantity is very interesting, as knowledge of $|V_{ub}|$ allows us to extract f_B, thereby providing tests of non-perturbative calculations.

[K. Ikado *et al.* (Belle), hep-ex/0604018; T. Browder's talk @ ICHEP '06]

- When leaving the SM, $B^- \to \tau^- \bar{\nu}_\tau$ is sensitive to effects form charged Higges (e.g. 2HDM), which can be constrained through f_B and $|V_{ub}|$:
  - SM prediction is simply modified by the following factor:[1] [Hou ('92)]

$$r_H = \left[1 - \left(\frac{M_B}{M_H}\tan\beta\right)^2\right]^2 \xrightarrow{\text{data}} r_H = 1.13 \pm 0.51$$





[Tom Browder (Belle Collaboration), talk at the ICHEP '06 conference]

[1] Note that $\tan\beta = v_2/v_1$ is defined through the ratio of vev's and does not involve the UT angle β.

Moreover: Leptonic D Decays

- Consider the decay $D^+ \to \mu^+\nu$: $\to$ analogous to $B^- \to \mu^-\overline{\nu}$, but ...
  - Governed by the CKM factor $|V_{cd}| = |V_{us}| + \mathcal{O}(\lambda^5) = \lambda[1 + \mathcal{O}(\lambda^4)]$, whereas $B^- \to \mu^-\overline{\nu}$ involves $|V_{ub}| = \lambda^3 R_b$:

    $\Rightarrow$ we win $\mathcal{O}(\lambda^4)$ in the rate $\Rightarrow$ $D^+ \to \mu^+\nu$ accessible at CLEO-c!
  - Determine the decay constant f_{D^+}:

    $\Rightarrow$ interesting testing ground for lattice calculations!

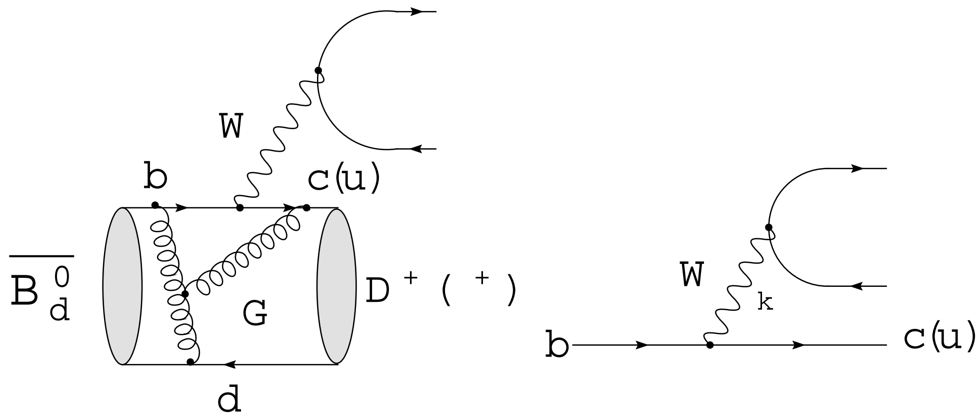
- Lattice QCD: [recent overview: talk by Christine Davies @ EPS-HEP '05]
  - Numerical evaluation of QCD path intergral using a space-time lattice!
  - Important recent progress:

    The "quenched" approximation, which had to be applied for many many years and ingnores quark loops, is no longer required!

- Show-down at the Lepton–Photon '05 and EPS-HEP '05 conferences:
  - Lattice [FNAL/MILC/HPQCD]: $f_{D^+} = (201 \pm 3 \pm 17)$MeV
  - CLEO-c result: $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$MeV $\to$ stay tuned!

Semileptonic B Decays

More complicated than the leptonic case ...



- Calculation of the corresponding diagram ($b \to u$ transition is analogous):

$$T_{fi} = -\frac{g_2^2}{8} V_{cb} \underbrace{\left[\overline{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu\right]}_{\text{Dirac spinors}} \left[\frac{g_{\alpha\beta}}{k^2 - M_W^2}\right] \underbrace{\langle D^+|\overline{c}\gamma^\beta(1-\gamma_5)b|\overline{B^0_d}\rangle}_{\text{hadronic ME}}$$

- $k^2 \sim M_B^2 \ll M_W^2$: $$\frac{g_{\alpha\beta}}{k^2 - M_W^2} \longrightarrow -\frac{g_{\alpha\beta}}{M_W^2} \equiv -\left(\frac{8G_\text{F}}{\sqrt{2}g_2^2}\right) g_{\alpha\beta},$$

$$\Rightarrow T_{fi} = \frac{G_\text{F}}{\sqrt{2}} V_{cb} \left[\overline{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu\right] \langle D^+|\overline{c}\gamma_\alpha(1-\gamma_5)b|\overline{B^0_d}\rangle$$

... the W boson is "integrated out"!

- The *whole* hadronic physics is encoded in the following quantity:

$$\langle D^+|\bar{c}\gamma_\alpha(1-\gamma_5)b|\overline{B^0_d}\rangle \;\longrightarrow\; \text{“hadronic matrix element”}$$

- Since the $\overline{B^0_d}$ and D^+ are pseudoscalar mesons, we may write:

$$\langle D^+(k)|\bar{c}\gamma_\alpha\gamma_5 b|\overline{B^0_d}(p)\rangle = 0$$

$$\langle D^+(k)|\bar{c}\gamma_\alpha b|\overline{B^0_d}(p)\rangle =$$
$$= F_1(q^2)\left[(p+k)_\alpha - \left(\frac{M_B^2 - M_D^2}{q^2}\right)q_\alpha\right] + F_0(q^2)\left(\frac{M_B^2 - M_D^2}{q^2}\right)q_\alpha$$

$F_{1,0}(q^2)$: $B \to D$ form factors $[q \equiv p - k]$

$\Rightarrow$ *two* hadronic parameters involved (instead of f_B)!

[from lattice, QCD sum rules, ...]

… nevertheless: extractions of $|V_{cb}|$ & $|V_{ub}|$!

$m_b \gg \Lambda_{\rm QCD}$ → Heavy Quark Effective Theory and Heavy Quark Expansions

Since no time for details in these lectures, just results[2] …

- The leptonic and hadronic moments in inclusive $b \to c\ell\bar{\nu}$ processes yield a value of $|V_{cb}|$, which agrees also with analyses of exclusive decays:

$$|V_{cb}| = (42.0 \pm 0.7) \times 10^{-3}.$$

[Gambino and Uraltsev (2004); Buchmüller and Flächer (2006)]

- $|V_{ub}|$ is more challenging (experimentally and theoretically); there is a $1\,\sigma$ discrepancy between the values from inclusive and exclusive transitions:

$$|V_{ub}|_{\rm incl} = (4.4 \pm 0.3) \times 10^{-3}, \quad |V_{ub}|_{\rm excl} = (3.8 \pm 0.6) \times 10^{-3};$$

… has to be settled!

- $|V_{cb}|$ and $|V_{ub}|$ with $\lambda = 0.225 \pm 0.001$ yield the following numbers for R_b:

$$R_b^{\rm incl} = 0.45 \pm 0.03\,, \qquad R_b^{\rm excl} = 0.39 \pm 0.06.$$

[2] See the "Heavy Flavour Averaging Group": http://www.slac.stanford.edu/xorg/hfag/

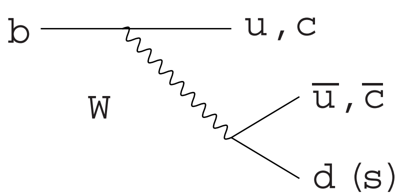
Key Rôle for CP Violation:

Non-Leptonic B Decays

$\longrightarrow$ only quarks in the final states!

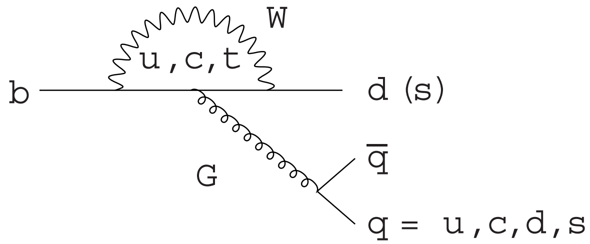
Topologies & Classification

- Tree diagrams:

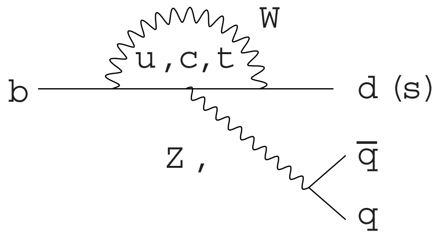


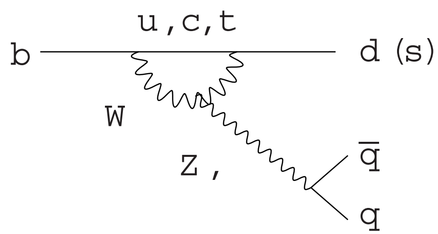
- Penguin diagrams:

  ◇ QCD penguins:



  ◇ Electroweak (EW) penguins:





- Classification (depends on the flavour content of the final state):

  – Only tree diagrams.

  – Tree and penguin diagrams.

  – Only penguin diagrams.

Low-Energy Effective Hamiltonians

- Operator product expansion (OPE): $\Rightarrow$

$$\boxed{\langle f|\mathcal{H}_{\text{eff}}|i\rangle = \frac{G_{\rm F}}{\sqrt{2}} V_{\rm CKM} \sum_k C_k(\mu)\, \langle f|Q_k(\mu)|i\rangle}$$

[$G_{\rm F}$: Fermi constant, $V_{\rm CKM}$: CKM factor, μ: renormalization scale]

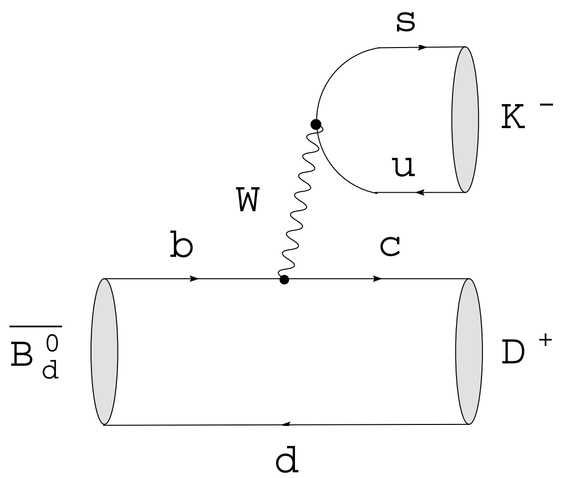
- The operator product expansion allows a separation of the short-distance from the long-distance contributions:
  - *Perturbative* Wilson coefficients $C_k(\mu) \to$ short-distance physics.
  - *Non-perturbative* hadronic MEs $\langle f|Q_k(\mu)|i\rangle \to$ long-distance physics.

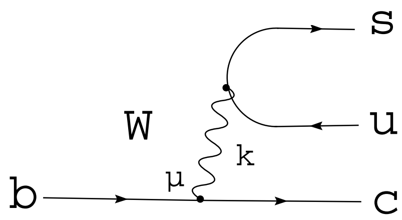
- The Q_k are local operators, which are generated through the electroweak interactions and QCD, and govern "effectively" the considered decay.

- The Wilson coefficients $C_k(\mu)$ describe the scale-dependent "couplings" of the interaction vertices that are associated with the operators Q_k.

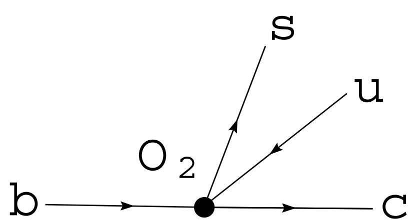
- Illustration through an example: $\boxed{\overline{B^0_d} \to D^+ K^-}$ ... pure tree decay:

  – $b \to c\bar{u}s$:





  – "Integrate out" the W boson: $\to$ contract the W propagator ...

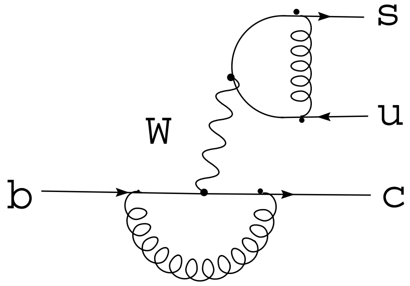


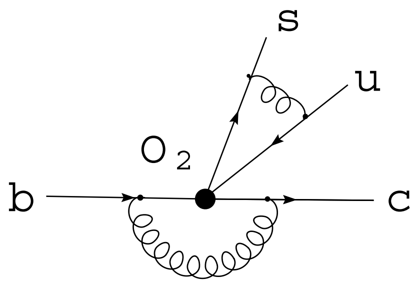
$$\frac{g_{\nu\mu}}{k^2 - M_W^2} \xrightarrow{k^2 \ll M_W^2} -\frac{g_{\nu\mu}}{M_W^2} \equiv -\left(\frac{8G_{\rm F}}{\sqrt{2}g_2^2}\right) g_{\nu\mu}$$

$$\Rightarrow \mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{cb} \underbrace{\left[\bar{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha\right]\left[\bar{c}_\beta \gamma^\mu (1-\gamma_5) b_\beta\right]}_{\text{"current–current" operator } O_2} \equiv \frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{cb} C_2 O_2$$

– Impact of QCD, i.e. exchange of gluons:

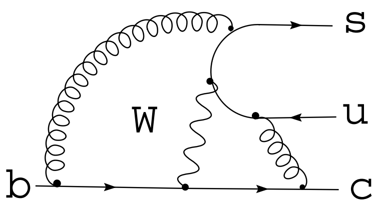
∗ *Factorizable* QCD corrections:

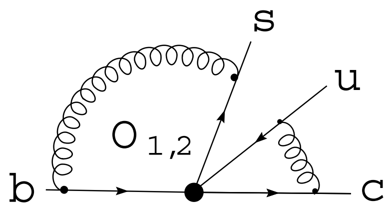




$\longrightarrow$ C_2 acquires a renormalization-scale dependence, i.e. $C_2(\mu) \neq 1$

∗ *Non-factorizable* QCD corrections:





$\longrightarrow$ generation of a second current–current operator:

$$O_1 \equiv [\overline{s}_\alpha \gamma_\mu (1-\gamma_5) u_\beta]\,[\overline{c}_\beta \gamma^\mu (1-\gamma_5) b_\alpha]$$

$\longrightarrow$ *operator mixing* through QCD effects!

- The results for the $C_k(\mu)$ contain $\log(\mu/M_W)$ terms, which become large for renormalization scales μ in the GeV regime:

$\longrightarrow$ what shall we do?

- Use *renormalization-group improved perturbation theory*:

  – The fact that the transition matrix element $\langle f|\mathcal{H}_{\text{eff}}|i\rangle$ *cannot* depend on the renormalization scale μ implies a renormalization-group equation.

  – Its solution can be written as follows:

$$\boxed{\vec{C}(\mu) = \hat{U}(\mu, M_W) \cdot \vec{C}(M_W)} \qquad (1)$$

  – The initial conditions $\vec{C}(M_W)$ describe the *short-distance* physics at the high-energy scales, and are related to the "Inami–Lim functions".

  – The following terms can be systematically summed up through (1):

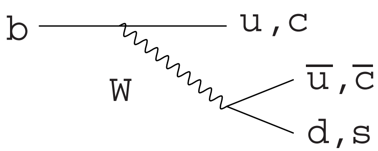
$$\underbrace{\alpha_s^n \left[\log\left(\frac{\mu}{M_W}\right)\right]^n}_{\text{(LO)}}, \quad \underbrace{\alpha_s^n \left[\log\left(\frac{\mu}{M_W}\right)\right]^{n-1}}_{\text{(NLO)}}, \quad \ldots$$

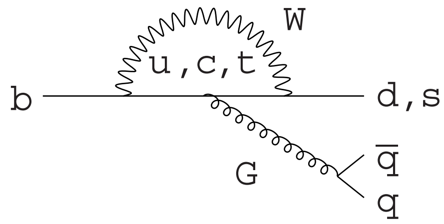
- Low-energy effective Hamiltonians provide a nice tool to deal with weak B- and K-meson decays, as well as with B^0–$\bar{B}^0$ and K^0–$\bar{K}^0$ mixing.

Let Penguins Enter the Stage ...

- Particularly interesting decay class: $|\Delta B| = 1,\ \Delta C = \Delta U = 0$

- $\Delta C = \Delta U = 0 \quad \Rightarrow$ **tree and penguin processes may contribute:**





$$\underbrace{V_{ur}^*V_{ub} + V_{cr}^*V_{cb} + V_{tr}^*V_{tb} = 0}_{\text{CKM unitarity } (r \in \{d,s\})} \quad \Rightarrow$$

only *two* weak amplitudes!

- Integrate out the W boson and the top quark ($\rightarrow$ penguins):

$$\mathcal{H}_{\text{eff}} = \frac{G_{\text{F}}}{\sqrt{2}}\left[\sum_{j=u,c} V_{jr}^* V_{jb}\left\{\underbrace{\sum_{k=1}^{2} C_k(\mu)\, Q_k^{jr}}_{\text{current–current}} + \underbrace{\sum_{k=3}^{10} C_k(\mu)\, Q_k^{r}}_{\text{penguins}}\right\}\right] + \text{h.c.}$$

- Four-quark operators Q_k^{jr} ($j \in \{u, c\}$, $r \in \{d, s\}$):

  - Current–current operators (tree-like processes):

$$\begin{aligned} Q_1^{jr} &= (\bar{r}_\alpha j_\beta)_{\text{V–A}}(\bar{j}_\beta b_\alpha)_{\text{V–A}} \\ Q_2^{jr} &= (\bar{r}_\alpha j_\alpha)_{\text{V–A}}(\bar{j}_\beta b_\beta)_{\text{V–A}} \end{aligned}$$

  - QCD penguin operators:

$$\begin{aligned} Q_3^{r} &= (\bar{r}_\alpha b_\alpha)_{\text{V–A}} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V–A}} \\ Q_4^{r} &= (\bar{r}_\alpha b_\beta)_{\text{V–A}} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V–A}} \\ Q_5^{r} &= (\bar{r}_\alpha b_\alpha)_{\text{V–A}} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_6^{r} &= (\bar{r}_\alpha b_\beta)_{\text{V–A}} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}} \end{aligned}$$

  - EW penguin operators:

$$\begin{aligned} Q_7^{r} &= \tfrac{3}{2}(\bar{r}_\alpha b_\alpha)_{\text{V–A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_8^{r} &= \tfrac{3}{2}(\bar{r}_\alpha b_\beta)_{\text{V–A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}} \\ Q_9^{r} &= \tfrac{3}{2}(\bar{r}_\alpha b_\alpha)_{\text{V–A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V–A}} \\ Q_{10}^{r} &= \tfrac{3}{2}(\bar{r}_\alpha b_\beta)_{\text{V–A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V–A}} \end{aligned}$$

[Here α, β are $SU(3)_{\text{C}}$ indices, V$\pm$A refers to $\gamma_\mu(1 \pm \gamma_5)$, $q' \in \{u, d, c, s, b\}$ runs over the active quark flavours at $\mu = \mathcal{O}(m_b)$, and the $e_{q'}$ are the electrical charges]

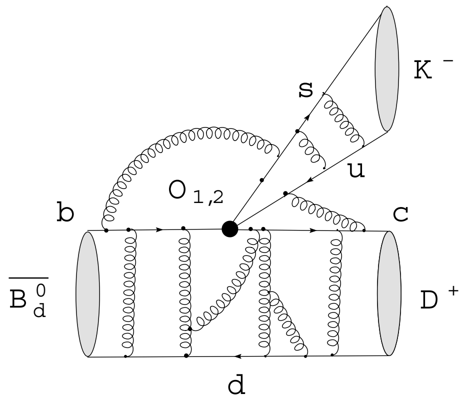
- The Wilson coefficients at $\mu = m_b$ for different renormalization schemes:

	$\Lambda^{(5)}_{\overline{\rm MS}} = 160{\rm MeV}$			$\Lambda^{(5)}_{\overline{\rm MS}} = 225{\rm MeV}$			$\Lambda^{(5)}_{\overline{\rm MS}} = 290{\rm MeV}$		
Scheme	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
C_1	-0.283	-0.171	-0.209	-0.308	-0.185	-0.228	-0.331	-0.198	-0.245
C_2	1.131	1.075	1.095	1.144	1.082	1.105	1.156	1.089	1.114
C_3	0.013	0.013	0.012	0.014	0.014	0.013	0.016	0.016	0.014
C_4	-0.028	-0.033	-0.027	-0.030	-0.035	-0.029	-0.032	-0.038	-0.032
C_5	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.010
C_6	-0.035	-0.037	-0.030	-0.038	-0.041	-0.033	-0.041	-0.045	-0.036
C_7/α	0.043	-0.003	0.006	0.045	-0.002	0.005	0.047	-0.002	0.005
C_8/α	0.043	0.049	0.055	0.048	0.054	0.060	0.053	0.059	0.065
C_9/α	-1.268	-1.283	-1.273	-1.280	-1.292	-1.283	-1.290	-1.300	-1.293
C_{10}/α	0.302	0.243	0.245	0.328	0.263	0.266	0.352	0.281	0.284

[Detailed discussion: A.J. Buras, hep-ph/9806471]

Factorization of the Hadronic Matrix Elements

- The problem:



- Transition amplitude:[3]

$$\left[a_2 \equiv C_1 + \frac{C_2}{N_{\rm C}}\right]$$

$$\langle K^- D^+|\mathcal{H}_{\rm eff}|\overline{B^0_d}\rangle = \frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{cb} \Big[\overbrace{\left(\frac{C_1}{N_{\rm C}} + C_2\right)}^{\equiv a_1} \langle K^- D^+|(\overline{s}_\alpha u_\alpha)_{\text{V-A}}(\overline{c}_\beta b_\beta)_{\text{V-A}}|\overline{B^0_d}\rangle$$

$$+2\, C_1 \langle K^- D^+|(\overline{s}_\alpha\, T^a_{\alpha\beta}\, u_\beta)_{\text{V-A}}(\overline{c}_\gamma\, T^a_{\gamma\delta}\, b_\delta)_{\text{V-A}}|\overline{B^0_d}\rangle \Big]$$

- "Factorization" of the hadronic matrix elements:

$$\langle K^- D^+|(\overline{s}_\alpha u_\alpha)_{\text{V-A}}(\overline{c}_\beta b_\beta)_{\text{V-A}}|\overline{B^0_d}\rangle\Big|_{\rm fact}$$

$$= \langle K^-|\,[\overline{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha]\,|0\rangle \langle D^+|\,[\overline{c}_\beta \gamma^\mu (1-\gamma_5) b_\beta]\,|\overline{B^0_d}\rangle$$

$$\propto f_K[\to \text{“decay constant”}] \times F_{BD}[\to \text{“form factor”}]$$

$$\langle K^- D^+|(\overline{s}_\alpha\, T^a_{\alpha\beta}\, u_\beta)_{\text{V-A}}(\overline{c}_\gamma\, T^a_{\gamma\delta}\, b_\delta)_{\text{V-A}}|\overline{B^0_d}\rangle\Big|_{\rm fact} = 0$$

[3] Here we use the well-known $SU(N_{\rm C})$ colour-algebra relation $T^a_{\alpha\beta} T^a_{\gamma\delta} = (\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta}/N_{\rm C})/2$.

- Long history of factorization:

  Schwinger (1964); Farikov & Stech (1978); Cabibbo & Maiani (1978); Bjorken (1989); Dugan & Grinstein (1991); Politzer & Wise (1991); ...

- Factorization in weak decays in the large-N_C limit:

  Buras, Gérard & Rückl (1986); Buras and Gérard (1988).

- Interesting recent developments: $\rightarrow$ important target $B \rightarrow \pi\pi, \pi K$

  - QCD Factorization (QCDF):

    Beneke, Buchalla, Neubert & Sachrajda (1999–2001); ...

  - Perturbative Hard-Scattering (PQCD) Approach:

    Li & Yu ('95); Cheng, Li & Yang ('99); Keum, Li & Sanda ('00); ...

  - Soft Collinear Effective Theory (SCET):

    Bauer, Pirjol & Stewart (2001); Bauer, Grinstein, Pirjol & Stewart (2003); ...

  - QCD light-cone sum-rule methods:

    Khodjamirian (2001); Khodjamirian, Mannel & Melic (2003); ...

Data indicate large non-factorizable corrections $\Rightarrow$ remain a theoretical challenge ...

[Buras *et al.*; Ali *et al.*; Bauer *et al.*; Chiang *et al.*; ...]

Towards Studies of

CP Violation in the

B-Meson System

Amplitude Structure of Non-Leptonic B Decays

- Low-energy effective Hamiltonian $\mathcal{H}_{\text{eff}}$ (see above): $\Rightarrow$

$$A(\overline{B} \to \overline{f}) = \langle \overline{f} | \mathcal{H}_{\text{eff}} | \overline{B} \rangle$$

$$= \frac{G_{\text{F}}}{\sqrt{2}} \left[\sum_{j=u,c} V_{jr}^* V_{jb} \left\{ \sum_{k=1}^{2} C_k(\mu) \langle \overline{f} | Q_k^{jr}(\mu) | \overline{B} \rangle + \sum_{k=3}^{10} C_k(\mu) \langle \overline{f} | Q_k^{r}(\mu) | \overline{B} \rangle \right\} \right]$$

- On the other hand, we may write $A(B \to f)$ as follows:

$$A(B \to f) = \langle f | \mathcal{H}_{\text{eff}}^{\dagger} | B \rangle$$

$$= \frac{G_{\text{F}}}{\sqrt{2}} \left[\sum_{j=u,c} V_{jr} V_{jb}^* \left\{ \sum_{k=1}^{2} C_k(\mu) \langle f | Q_k^{jr\dagger}(\mu) | B \rangle + \sum_{k=3}^{10} C_k(\mu) \langle f | Q_k^{r\dagger}(\mu) | B \rangle \right\} \right]$$

- CP trafos: $\boxed{(\mathcal{CP}) Q_k^{jr\dagger} (\mathcal{CP})^{\dagger} = Q_k^{jr}, \quad (\mathcal{CP}) | M \rangle = e^{i\phi_{\text{CP}}(M)} | \overline{M} \rangle}$ $\Rightarrow$

$$A(B \to f) = e^{i[\phi_{\text{CP}}(B) - \phi_{\text{CP}}(f)]}$$

$$\times \frac{G_{\text{F}}}{\sqrt{2}} \left[\sum_{j=u,c} V_{jr} V_{jb}^* \left\{ \sum_{k=1}^{2} C_k(\mu) \langle \overline{f} | Q_k^{jr}(\mu) | \overline{B} \rangle + \sum_{k=3}^{10} C_k(\mu) \langle \overline{f} | Q_k^{r}(\mu) | \overline{B} \rangle \right\} \right]$$

- Consequently, the decay amplitudes can be written in the following form:

$$A(\overline{B} \to \overline{f}) = e^{+i\varphi_1}|A_1|e^{i\delta_1} + e^{+i\varphi_2}|A_2|e^{i\delta_2}$$

$$A(B \to f) = e^{i[\phi_{\mathrm{CP}}(B)-\phi_{\mathrm{CP}}(f)]}\left[e^{-i\varphi_1}|A_1|e^{i\delta_1} + e^{-i\varphi_2}|A_2|e^{i\delta_2}\right]$$

– CP-violating weak phases $\varphi_{1,2}$ originate from the CKM factors $V_{jr}^* V_{jb}$.

– CP-conserving "strong" amplitudes $|A_{1,2}|e^{i\delta_{1,2}}$ involve the hadronic matrix elements of the four-quark operators:

$$|A_j|e^{i\delta_j} = \sum_k \underbrace{C_k(\mu)}_{\text{pert. QCD}} \times \underbrace{\langle \overline{f}|Q_k^j(\mu)|\overline{B}\rangle}_{\text{"unknown"}}$$

$\Rightarrow$ encode the *hadron dynamics* of the considered decay!

– The *convention-dependent* phase factor $e^{i[\phi_{\mathrm{CP}}(B)-\phi_{\mathrm{CP}}(f)]}$ has to *cancel* in all physical observables, in particular in the CP asymmetries!

Direct CP Violation

- The most straightforward CP asymmetry ("direct" CP violation):[4]

$$\mathcal{A}_{\rm CP} \equiv \frac{\Gamma(B\to f)-\Gamma(\overline{B}\to\overline{f})}{\Gamma(B\to f)+\Gamma(\overline{B}\to\overline{f})} = \frac{|A(B\to f)|^2-|A(\overline{B}\to\overline{f})|^2}{|A(B\to f)|^2+|A(\overline{B}\to\overline{f})|^2}$$

$$= \frac{2|A_1||A_2|\sin(\delta_1-\delta_2)\sin(\varphi_1-\varphi_2)}{|A_1|^2+2|A_1||A_2|\cos(\delta_1-\delta_2)\cos(\varphi_1-\varphi_2)+|A_2|^2}$$

- Provided the two amplitudes satisfy the following requirements:

  i) non-trivial CP-conserving strong phase difference $\delta_1-\delta_2$;

  ii) non-trivial CP-violating weak phase difference $\varphi_1-\varphi_2$:

$\Rightarrow$ **CP violation originates through interference effects!**

- Goal: extraction of $\varphi_1-\varphi_2$ ($\to$ UT angle) from the measured $\mathcal{A}_{\rm CP}$!
- Problem: uncertainties related to the strong amplitudes $|A_{1,2}|e^{i\delta_{1,2}}$ ...

[4] B-meson counterpart of ε'/ε; could be established through $B_d^0\to\pi^-K^+$ and CP conjugate in '04.

Two Main Strategies

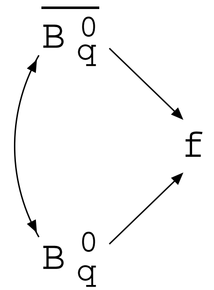
- Amplitude relations allow us in fortunate cases to eliminate the hadronic matrix elements ($\rightarrow$ typically strategies to determine γ):

  - *Exact* relations: pure "tree" decays (e.g. $B \rightarrow KD$). [$\rightarrow$ below].

  - *Approximate* relations, which follow from the *flavour symmetries* of strong interactions, i.e. $SU(2)$ isospin or $SU(3)_{\mathrm{F}}$:

$$B \rightarrow \pi\pi,\ B \rightarrow \pi K,\ B_{(s)} \rightarrow KK.$$

- Decays of neutral B_d and B_s mesons: [$\rightarrow$ Lecture II]



Interference effects through B^0_q–$\overline{B^0_q}$ mixing

  - "Mixing-induced" CP violation!

  - If one CKM amplitude dominates (e.g. $B_d \rightarrow \psi K_{\mathrm{S}}$):

$$\Rightarrow \text{ hadronic matrix elements cancel!}$$

  - Otherwise, we have to rely again on amplitude relations ...

Exploring CP Violation

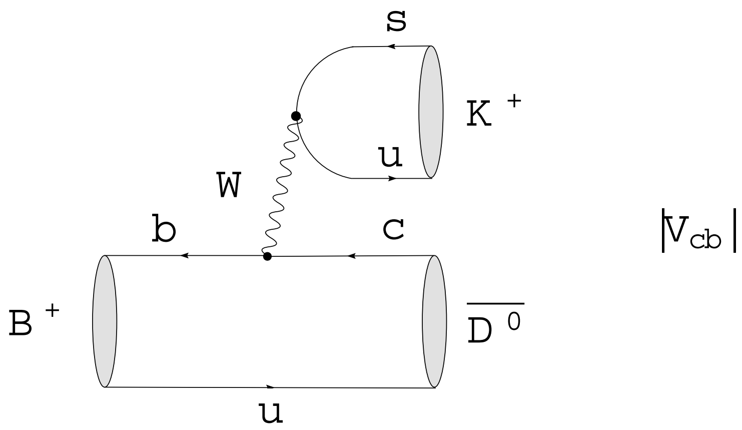
through

Amplitude Relations:

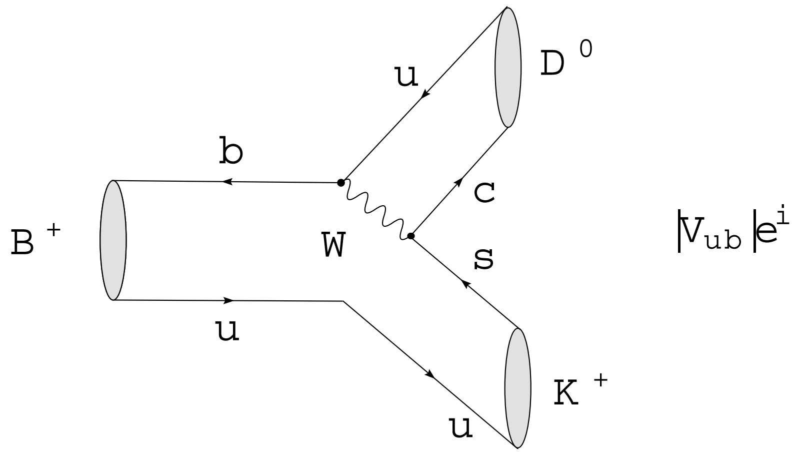
$$B^{\pm} \to K^{\pm} D,\ B_c^{\pm} \to D_s^{\pm} D$$

The Classical Triangle Approach [Gronau & Wyler ('91)]

- $B^+ \to K^+\overline{D^0}$: $\longrightarrow$ *"colour-allowed"* decay

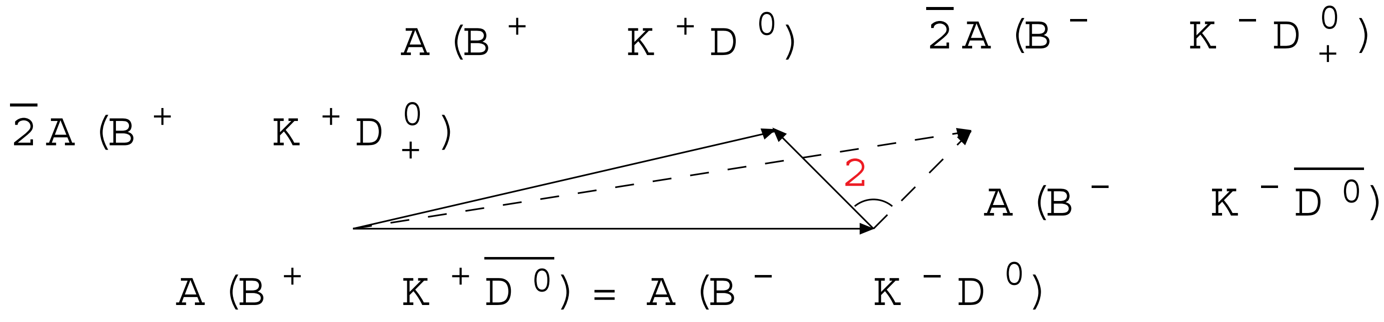


- $B^+ \to K^+D^0$: $\longrightarrow$ *"colour-suppressed"* decay



- $B^+ \to K^+D^0_+$: $\longrightarrow$ $\underbrace{\text{CP eigenstate } D^0_+}_{|D^0_+\rangle=\frac{1}{\sqrt{2}}[|\overline{D^0}\rangle+|D^0\rangle]}$ $\Rightarrow$ **interference effects!**

- We then arrive at the following amplitude triangles:



$\Rightarrow$ theoretically *clean* determination of γ!

- The triangles are unfortunately very squashed:

$$\left|\frac{A(B^+ \to K^+ D^0)}{A(B^+ \to K^+ \overline{D^0})}\right| \approx \frac{1}{\lambda}\frac{|V_{ub}|}{|V_{cb}|} \times \frac{a_2}{a_1} \approx 0.4 \times 0.3 = \mathcal{O}(0.1)$$

$\Rightarrow$ approach is very difficult in practice!

- nother – more subtle – problem: BR$(B^+ \to K^+ D^0)$

  - $D^0 \to K^- \ell^+ \nu_\ell$ would be ideal to measure this tiny branching ratio:

    however, huge background from semi-leptonic B decays $\Rightarrow$

  - Have to use Cabibbo-allowed hadronic $D^0 \to f_{\rm NE}$ decays:

$$B^+ \to K^+ D^0 \, [\to f_{\rm NE} = \pi^+ K^-, \rho^+ K^-, ...].$$

  - Unfortunately, another decay path into the *same* final state through

$$B^+ \to K^+ \overline{D^0} \, [\to f_{\rm NE}],$$

    where BR$(B^+ \to K^+ \overline{D^0})$ is $\mathcal{O}(10^2)$ *larger* than BR$(B^+ \to K^+ D^0)$, while $\overline{D^0} \to f_{\rm NE}$ is DCS, i.e. $\mathcal{O}(10^{-2})$ *smaller* than $D^0 \to f_{\rm NE}$:

    $\Rightarrow$ interference effects of $\mathcal{O}(1)$!

- But: If two different final states are measured, γ can be extracted!

  [Atwood, Dunietz & Soni (1997)]

Several Variants in the Literature ...

- The angle γ can be determined in a variety of ways through CP violation in pure tree decays of the kind $B \to D^{(*)}K^{(*)}$:
  - Important example: Dalitz plot analyses with $D \to K_S\pi^+\pi^-$.

- Combination of all methods for the current B-factory data:

$$\gamma|_{D^{(*)}K^{(*)}} = \begin{cases} (62^{+35}_{-25})^\circ & \text{(CKMfitter)} \\ (65 \pm 20)^\circ & \text{(UTfit)} \end{cases}$$

  - Here a second solution $180^\circ + \gamma|_{D^{(*)}K^{(*)}}$ was discarded, as it is disfavoured by the fits of the UT and further $B_d \to D^\pm\pi^\mp$ data.
  - This result is also consistent with $\gamma|_{\pi^+\pi^-,\pi^-K^+} = (74 \pm 6)^\circ$ following from *flavour-symmetry* relations between $B_d^0 \to \pi^+\pi^-, \pi^-K^+$ decays.

- Expected uncertainty that may be achieved @ LHCb by ~ 2010:

$$\Delta\gamma|_{D^{(*)}K^{(*)}} = \pm 5^\circ.$$

[Detailed overview: WG5 Report of CKM 2005, L. Cavotto *et al.*, hep-ph/0603019]

There is another species

of charged B mesons:

B_c System

Preliminaries

- Discovered by CDF through $B_c^+ \to J/\psi\, \ell^+ \nu_\ell$: [CDF, *PRL* **81** (1998) 2432]

$$
\begin{aligned}
M_{B_c} &= (6.40 \pm 0.39 \pm 0.13)\text{GeV} \\
\tau_{B_c} &= (0.46^{+0.18}_{-0.16} \pm 0.03)\text{ps.}
\end{aligned}
$$

- D0 observed $B_c^+ \to J/\psi\, \mu^+ X$: [D0 Note 4539-CONF (Aug 2004)]

$$
\begin{aligned}
M_{B_c} &= (5.95^{+0.14}_{-0.13} \pm 0.34)\text{GeV} \\
\tau_{B_c} &= (0.448^{+0.123}_{-0.096} \pm 0.121)\text{ps.}
\end{aligned}
$$

- Also evidence for $B_c^+ \to J/\psi\pi^+$ at CDF: [hep-ex/0505076; Note 8004 ('06)]

$$
\Rightarrow \quad M_{B_c} = (6.2765 \pm 0.0040 \pm 0.0027)\text{GeV.}
$$

- Run-II of the Tevatron will provide further insights into B_c physics, and a huge number of B_c mesons will be produced at LHCb:

Can insights into CP violation be obtained from B_c-meson decays?

The Triangle Approach in the B_c-Meson System

- B_c counterparts of $B_u^\pm \to K^\pm D$:

$B_c^\pm \to D_s^\pm D$: $\quad \Rightarrow \quad$ also a determination of γ!

[M. Masetti (1992)]

- For the extraction of γ, the following amplitude relations are used:

$$\sqrt{2}A(B_c^+ \to D_s^+ D_+^0) = A(B_c^+ \to D_s^+ D^0) + A(B_c^+ \to D_s^+ \overline{D^0})$$

$$\sqrt{2}A(B_c^- \to D_s^- D_+^0) = A(B_c^- \to D_s^- \overline{D^0}) + A(B_c^- \to D_s^- D^0)$$

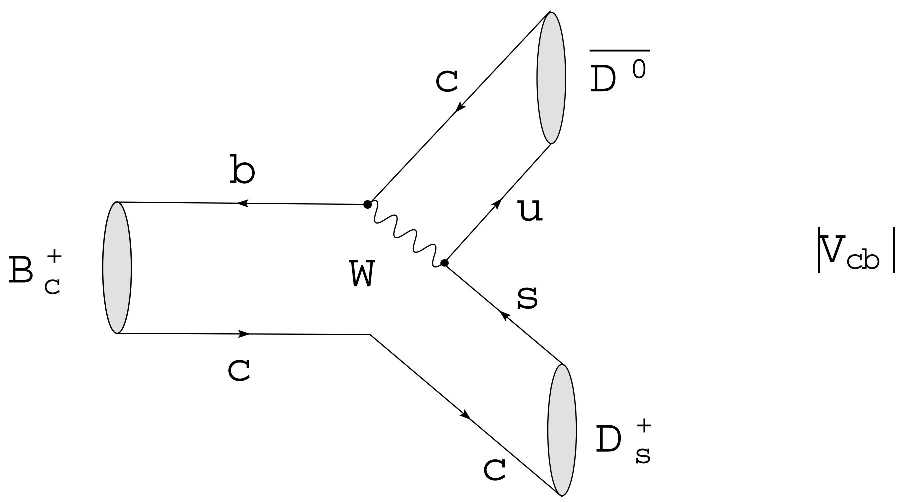
$$A(B_c^+ \to D_s^+ \overline{D^0}) = A(B_c^- \to D_s^- D^0)$$

$$A(B_c^+ \to D_s^+ D^0) = A(B_c^- \to D_s^- \overline{D^0}) \times e^{2i\gamma}$$

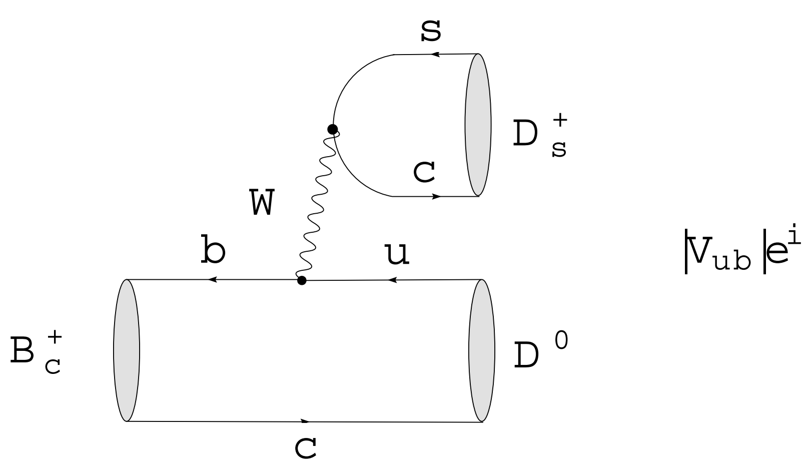
- The situation appears completely analogous to the $B^\pm \to K^\pm D$ case ...

- However, there is an important difference: Feynman diagrams $\longrightarrow$

  - $B_c^+ \to D_s^+ \overline{D^0}$: $\longrightarrow$ *"colour-suppressed"* decay



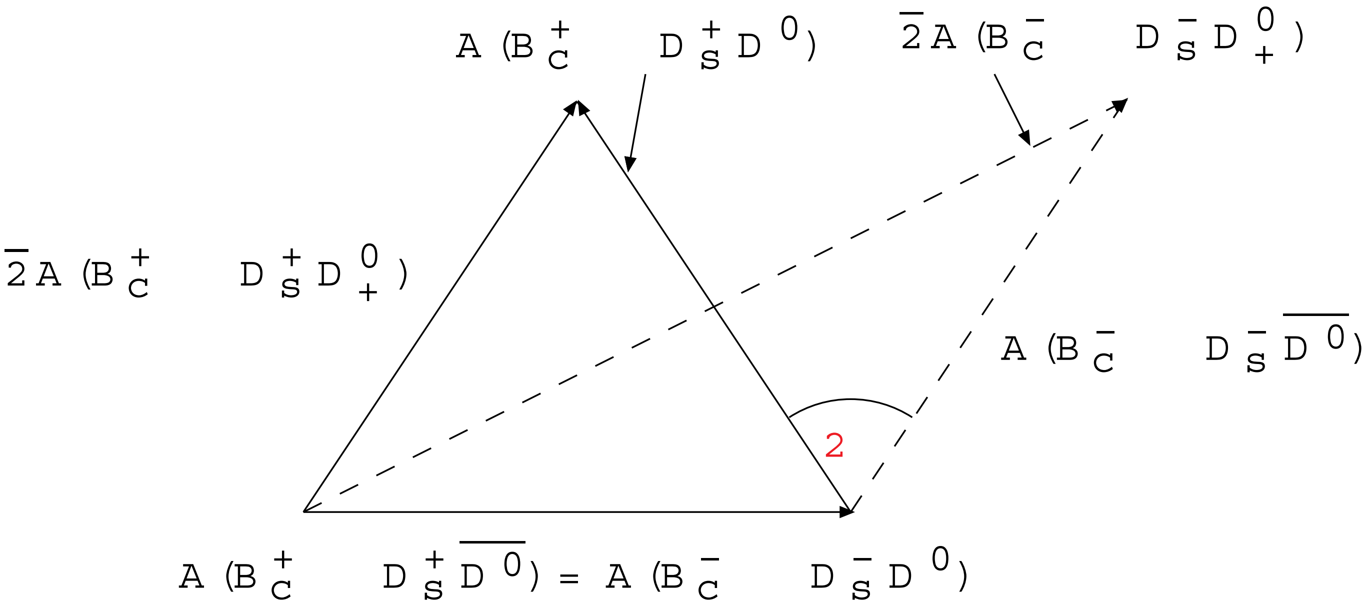
  - $B_c^+ \to D_s^+ D^0$: $\longrightarrow$ *"colour-allowed"* decay



– Consequently, in the $B_c^\pm \to D_s^\pm D$ system, the amplitude associated with the *small* CKM element V_{ub} is *not* colour-suppressed, whereas the larger CKM element V_{cb} enters with a colour-suppression factor:

$$\Rightarrow \quad \left|\frac{A(B_c^+ \to D_s^+ D^0)}{A(B_c^+ \to D_s^+ \overline{D^0})}\right| \approx \frac{1}{\lambda}\frac{|V_{ub}|}{|V_{cb}|} \times \frac{a_1}{a_2} \approx 0.4 \times 3 = \mathcal{O}(1)$$

$\Rightarrow$ *non-squashed* triangles, i.e. *ideal* theoretical realization:



But experimentally still challenging ...

[R.F. & Wyler (2000)]

Other Interesting Aspects of B_c Mesons

- Lowest lying bound state of two heavy quarks, $\bar{b}$ and c: $\Rightarrow$
  - QCD dynamics of the B_c^+ mesons is similar to quarkonium systems, such as $\bar{b}b$ and $\bar{c}c$, which are approximately non-relativistic.
  - Important difference: B_c contains open flavour

$\Rightarrow$ **stable under strong interactions!**

- Quarkonium-like B_c mesons provide an important laboratory to explore the interplay of strong and weak interactions:
  - Heavy-Quark Expansions (HQE)
  - Non-Relativistic QCD (NRQCD)
  - Factorization, ...

Can be tested in a setting complementary to weak hadron decays!

$\longrightarrow$ B_c lifetime & inclusive decays, leptonic and semileptonic decays, ...

[For more details, see *B Decays at the LHC*, hep-ph/0003238]

The Major Lessons of Lecture I

- Weak interactions of quarks and CP violation in the SM:
  - Complex phase in CKM matrix ($N \geq 3$): $\rightarrow$ CP violation!
  - Key target: $\rightarrow$ UT of the CKM matrix $\rightarrow$ "overconstrain" ...

The B-meson system is a central aspect of this game ...

- First contact with B decays: $\rightarrow$ strong interaction effects enter ...
  - Leptonic decays: $\rightarrow$ simplest transitions: "decay constant" f_B
  - Semileptonic decays: $\rightarrow$ more complicated, but $\rightarrow$ $|V_{cb}|$ & $|V_{ub}|$.
  - Non-leptonic decays: $\rightarrow$ key rôle for the exploration of CP violation:
    - $*$ Central Problem: hadronic uncertainties $\rightarrow$ get rid of them ...

... possible through amplitude relations or ...

- Neutral B decays: particularly promising (also @ LHC) $\rightarrow$ Lecture II