The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

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ICTP-INFN Advanced Tranining Course on
FPGA and VHDL for Hardware Simulation and Synthesis
27 November - 22 December 2006

DIGITAL DESIGN 3

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Outline

- Digital CMOS Design
  - Boolean Algebra
  - Basic Digital CMOS Gates
  - Combinational and Sequential Circuits
  - Coding - Representation of Numbers

CMOS Circuits

How to implement Boolean functions in CMOS technology ?

- A complex function cannot be implemented in a single gate
- Use a network of gates

Boolean network

CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$$f = \bar{x}.y.z + x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z$$

$$f = (x+y+z) \,.\, (x+y+\bar{z}) \,.$$

$$(x+\bar{y}+z) \,.\, (\bar{x}+\bar{y}+z)$$

$$f = x.(\bar{y}+z) + \bar{x}.(y.z)$$

$$f = \bar{x}.y.z + x.\bar{y}.\bar{z} + x.z$$

$$f = x.\bar{y} + y.z$$

There is not a unique expression

CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Which gates should I use ?

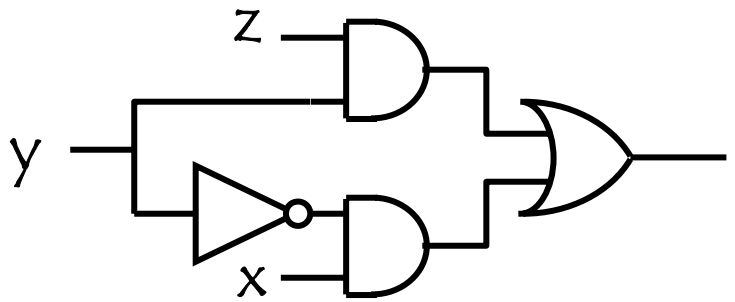
Cell library

$f = x.\overline{y} + y.z$

CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



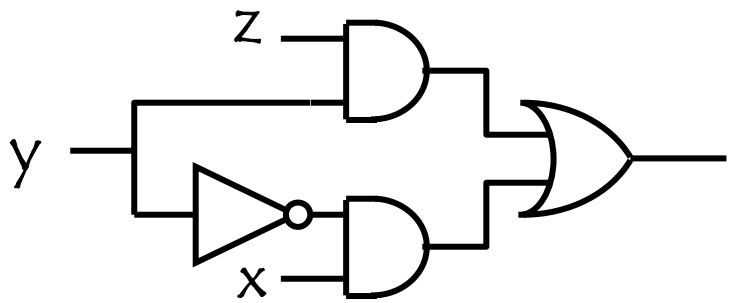
Non-inverting gates do NOT exist

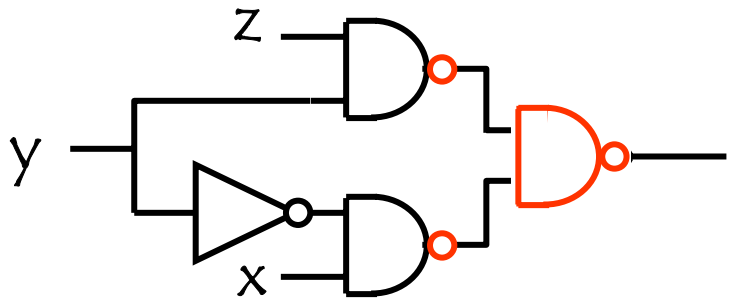
$f = x.\bar{y} + y.z$

CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1





CMOS Circuits

Example :

$$f = (\overline{x} + y) \oplus x$$

$$f = \overline{(\overline{x} + y)} \oplus \overline{x}$$

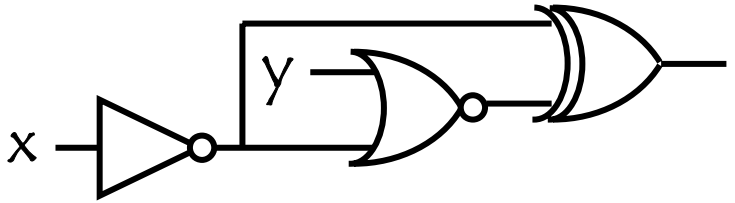
$$f = (\overline{x} + y) \cdot \overline{x} + \overline{(\overline{x} + y)} \cdot x$$

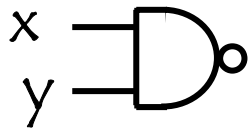
$$f = \overline{x} + y.\overline{x} + x.\overline{y}$$

$$f = \overline{x} + x.\overline{y}$$

$$f = \overline{x} + \overline{y}$$

$$f = \overline{x \cdot y}$$





CMOS Circuits

How to implement Boolean functions with a gate network ?

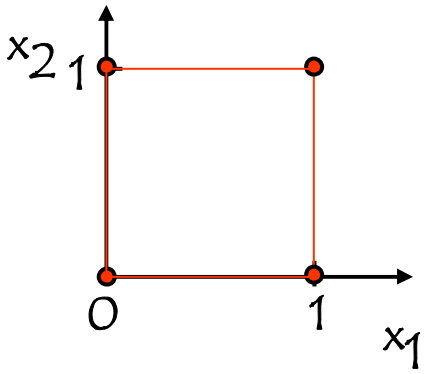
- Which expression ?

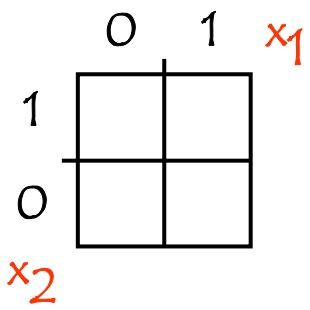
CMOS Circuits

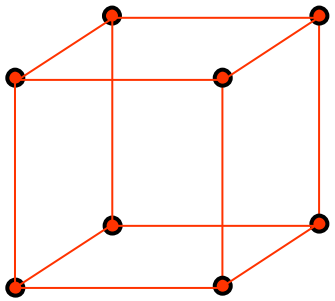
A function can be defined by its Truth table

- Karnaugh table

  Representation of the function in a space of dimension n



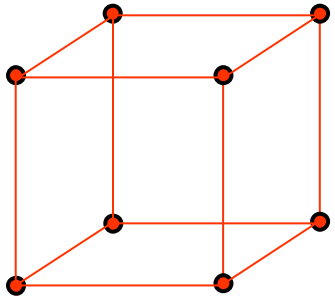


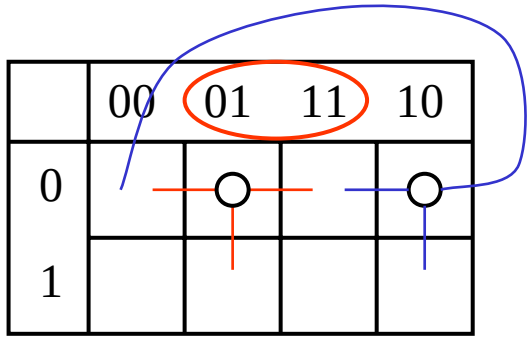


Representation of vectors' adjacency

CMOS Circuits

A Karnaugh table is represented in a flat 2-dimension table





vectors' adjacency

CMOS Circuits

Example :

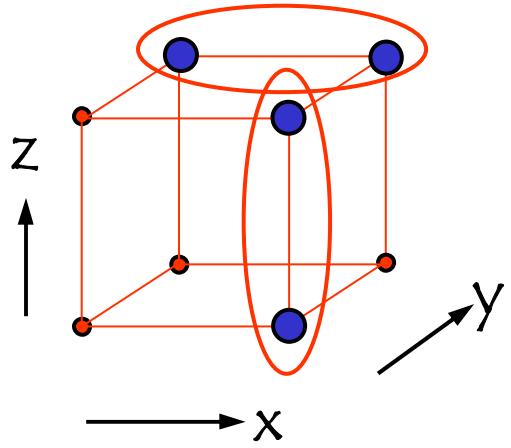
x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

	00	01	11	10	xy
0	0	0	0	1	
1	0	1	1	1	

z

CMOS Circuits

Example :

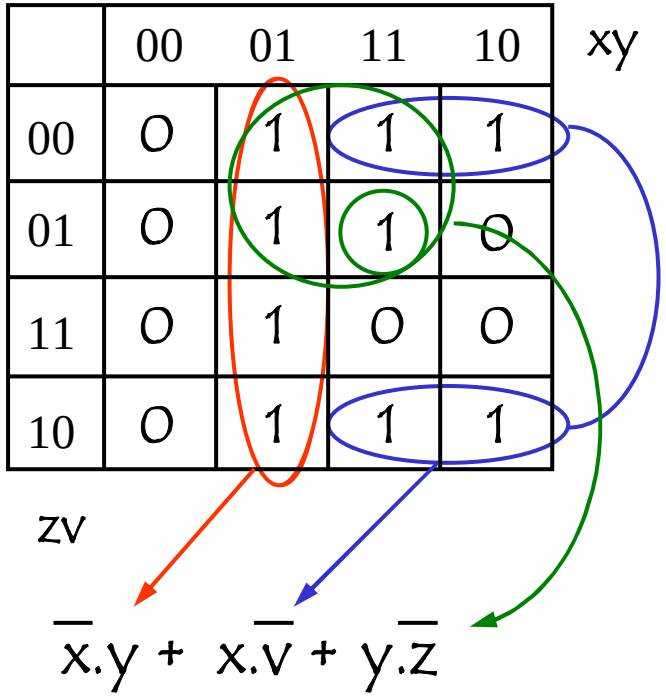


	00	01	11	10	xy
0	0	0	0	1	
1	0	1	1	1	
z					

$f = x.\overline{y} + yz$

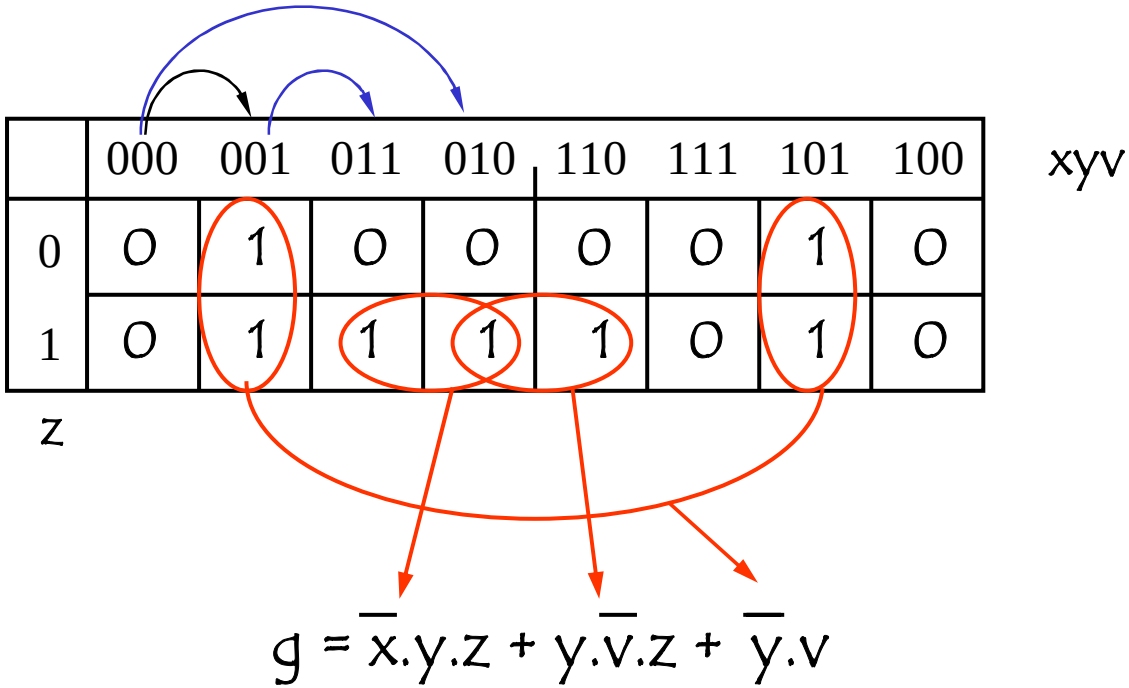
CMOS Circuits

Example :



CMOS Circuits

Example :



$$g = \overline{x}.y.z + y.\overline{v}.z + \overline{y}.v$$

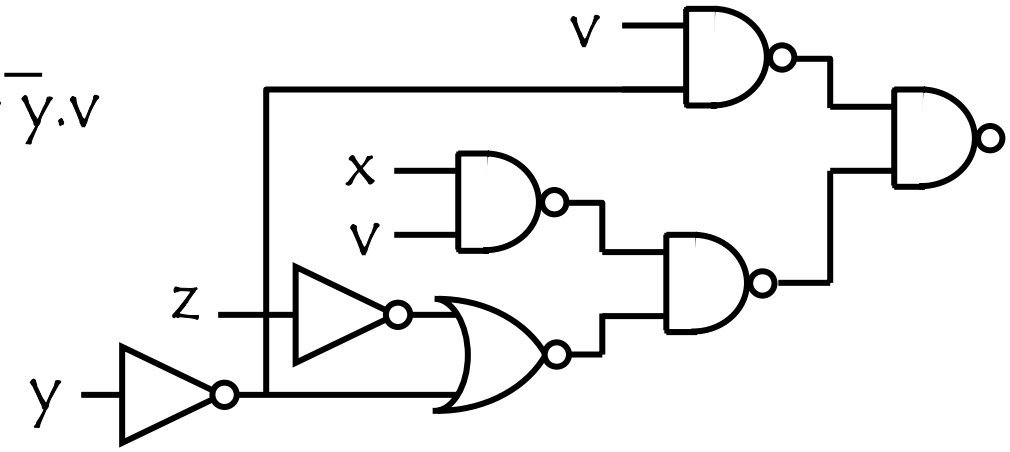
CMOS Circuits

Example :

$$g = \overline{x}.y.z + y.\overline{v}.z + \overline{y}.v$$

$$g = y.z.(\overline{x}+\overline{v}) + \overline{y}.v$$

$$g = (\overline{\overline{y}+\overline{z}}).(\overline{x.v}) + \overline{y}.v$$



CMOS Circuits

How to implement Boolean functions with a gate network ?

- Local optimization using Karnaugh tables

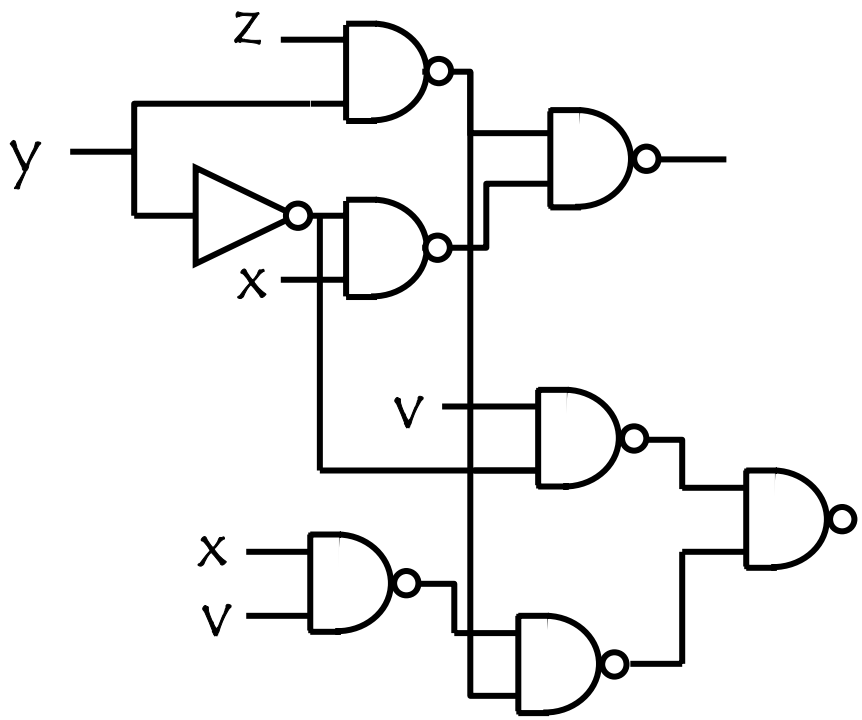
A design includes several Boolean functions

CMOS Circuits

Example :

$g = \overline{\overline{(y+z)}.\overline{(x.v)}} + \overline{y}.v$

$f = x.\overline{y} + y.z$



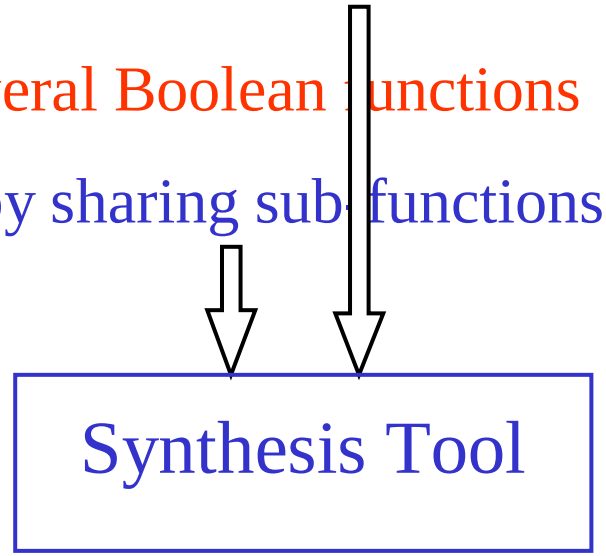
CMOS Circuits

How to implement Boolean functions with a gate network ?

- Local optimization using Karnaugh tables

A design includes several Boolean functions

- Global optimization by sharing sub-functions



CMOS Circuits

- Combinational logic

  The value of the ouput can be determined knwoing the value of the inputs

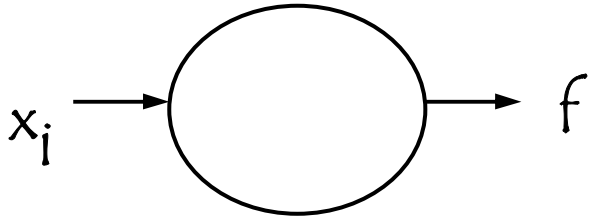
- Sequential logic

  The value of the ouput depends on the value of the inputs and the history

Notion of memory

CMOS Circuits

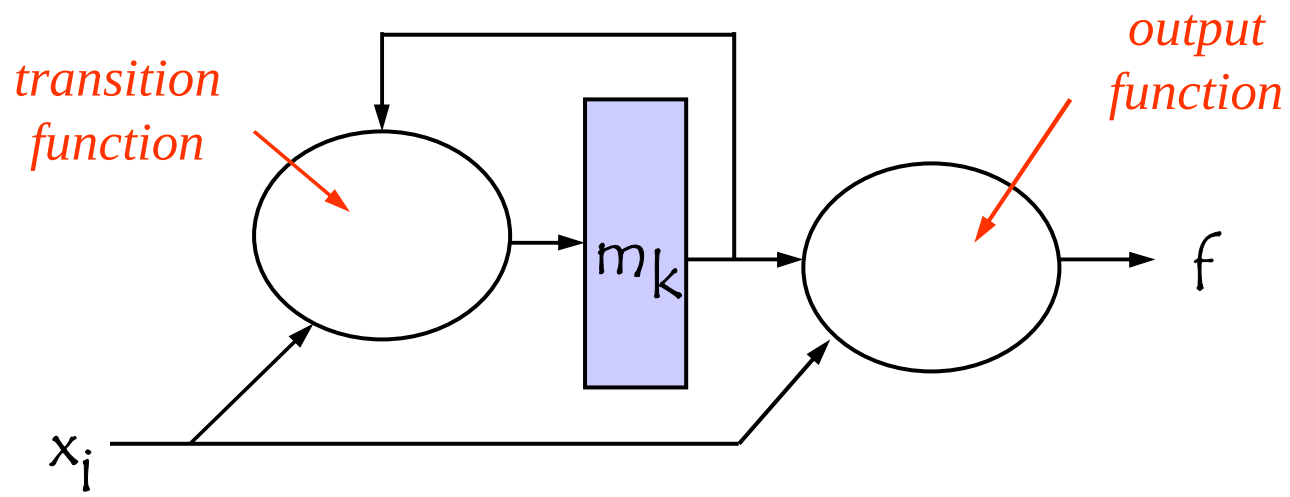
- Sequential logic



$$f(x_1, \ldots, x_i, \ldots, x_n , m_1, \ldots, m_k, \ldots, m_p)$$
$$m_k(x_1, \ldots, x_i, \ldots, x_n , m_1, \ldots, m_k, \ldots, m_p)$$

CMOS Circuits

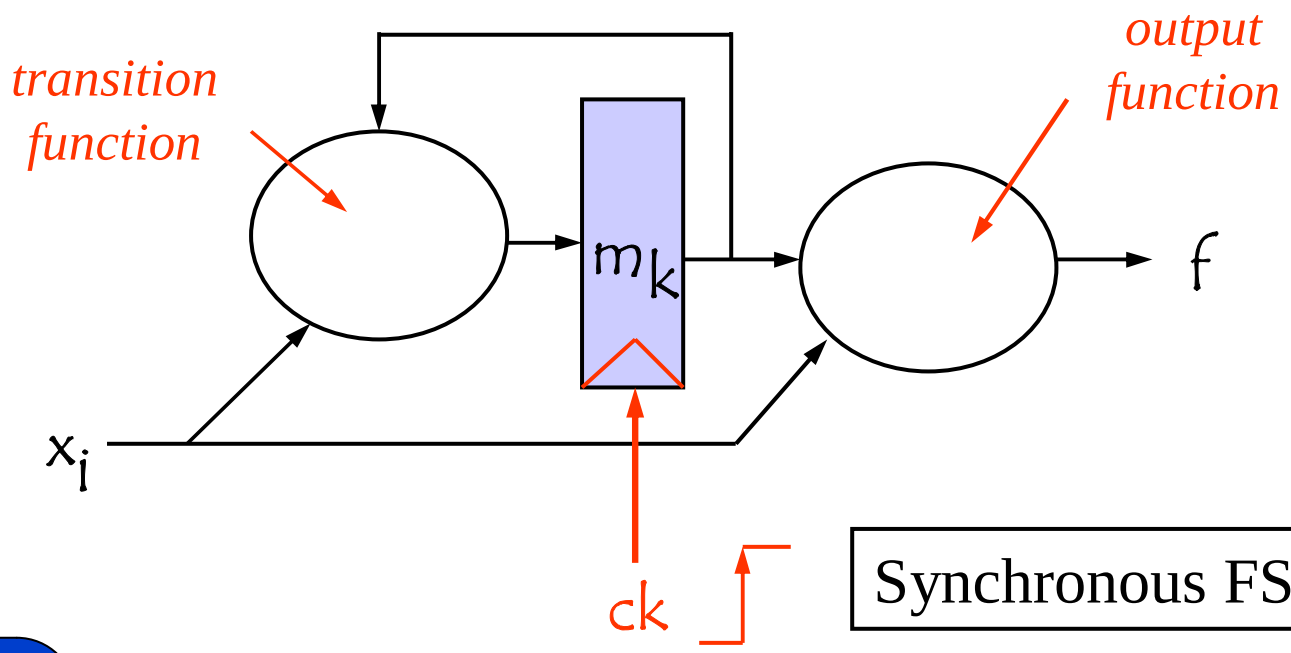
- Sequential logic



Finite State Machine (FSM)

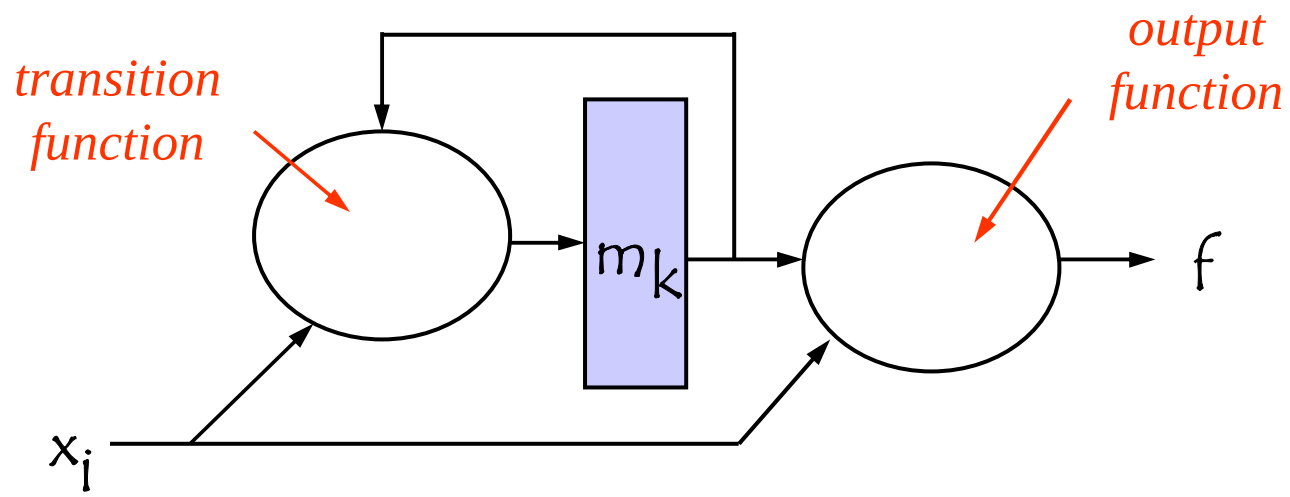
CMOS Circuits

- Finite State Machine



CMOS Circuits

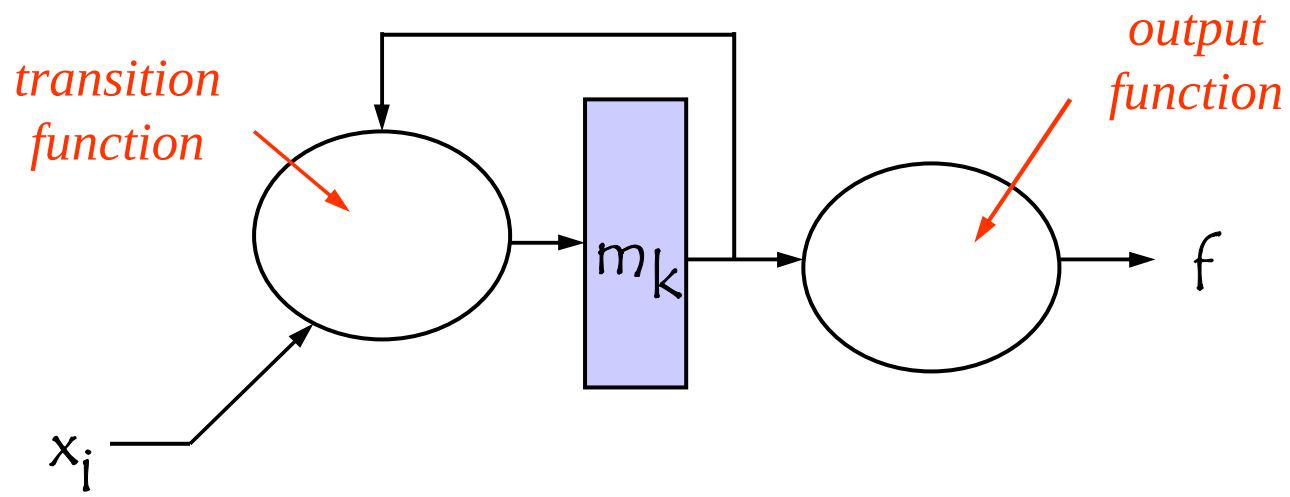
- Finite State Machine



Mealy FSM

CMOS Circuits

- Finite State Machine



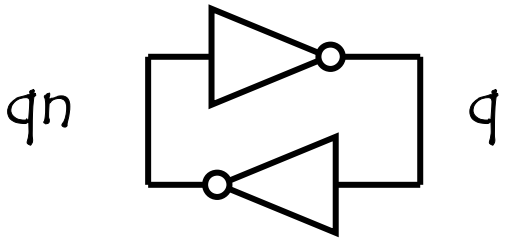
Moore FSM

CMOS Circuits

Memory :

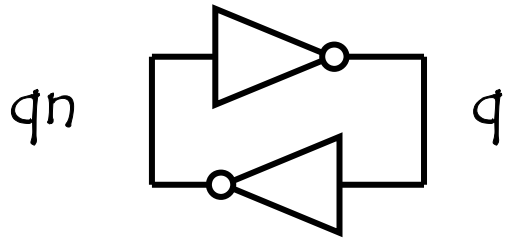
Hold a data (0 or 1)

Write a data (0 or 1)



CMOS Circuits

Memory :



$q = \overline{qn}$ $\quad qn = \overline{q}$

$f = q = \overline{\overline{q}}$ $\quad f(q) = q$

$$\frac{\partial f^{+}}{\partial q} = f_{q1} \cdot \overline{f_{q0}} = 1$$

$$\frac{\partial f^{-}}{\partial q} = \overline{f_{q1}} \cdot f_{q0} = 0$$

q depends always on itself in a positive way

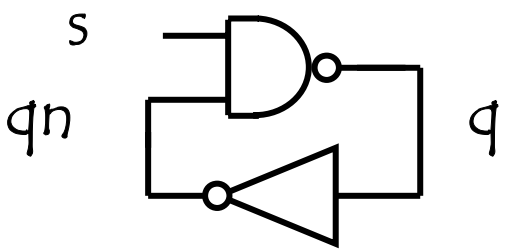
this dependency cannot be disabled

CMOS Circuits

Memory :

Hold a data (0 or 1)

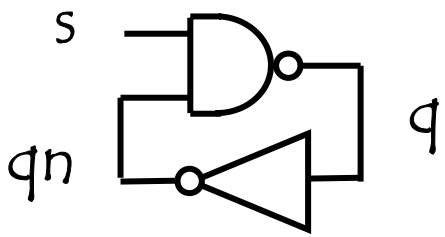
Write a data (0 or 1)



s	q	qn
0	1	0
1	q	qn

CMOS Circuits

Memory :



$q = \overline{s.qn}$ $\quad qn = \overline{q}$

$f = q = \overline{s \,.\, (\overline{q})}$

$f = \overline{s} + q$ $\quad f = q + \overline{q} \,.\, \overline{s}$

$$\frac{\partial f^{+}}{\partial q} = 1 \,.\, s = s \qquad \frac{\partial f^{-}}{\partial q} = \overline{s} \,.\, 0 = 0$$

if s = 1 q is a positive function of itself

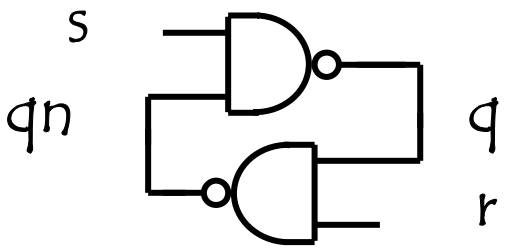
if s = 0 q is a combinational function

CMOS Circuits

Memory :

Hold a data (0 or 1)

Write a data (0 or 1)

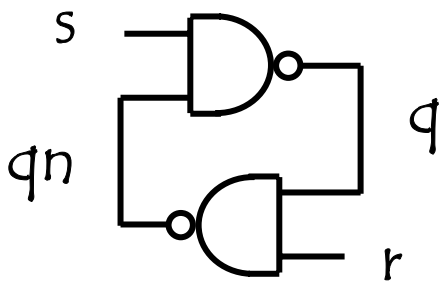


s	r	q	qn
0	1	1	0
1	0	0	1
1	1	q	qn
0	0	1	1

RS flip flop

CMOS Circuits

Memory :



$$q = \overline{s.qn} \qquad qn = \overline{r.q}$$

$$f = q = \overline{s \,.\, (\overline{r \,.\, q})}$$

$$f = \overline{s} + (r \,.\, q)$$

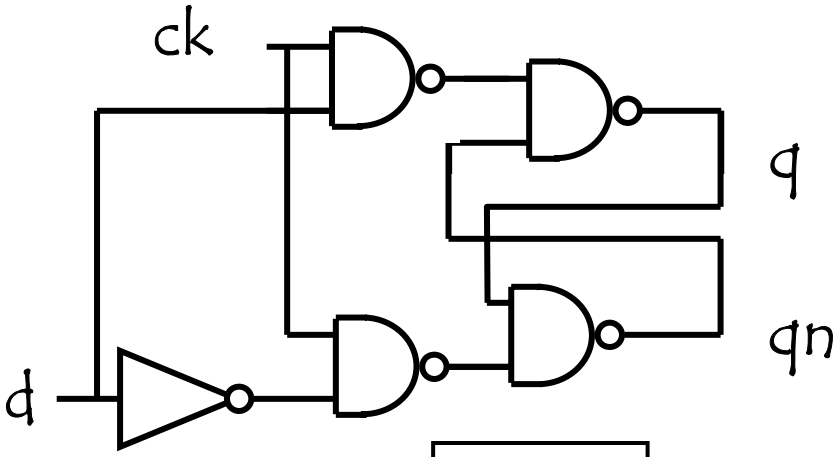
$$f = q \,.\, (r+\overline{s}) + \overline{q} \,.\, \overline{s}$$

$$\frac{\partial f^{+}}{\partial q} = s \,.\, (\overline{s} + r) = s.r \qquad \frac{\partial f^{-}}{\partial q} = \overline{s} \,.\, \overline{(\overline{s} + r)} = 0$$

CMOS Circuits

Synchronous Memory :

Write a data d when the clock ck = 1



d latch

s	r	q	qn
0	1	1	0
1	0	0	1
1	1	q	qn
0	0	1	1

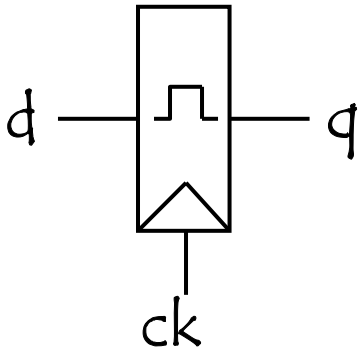
if ck.d = 1 s = 0

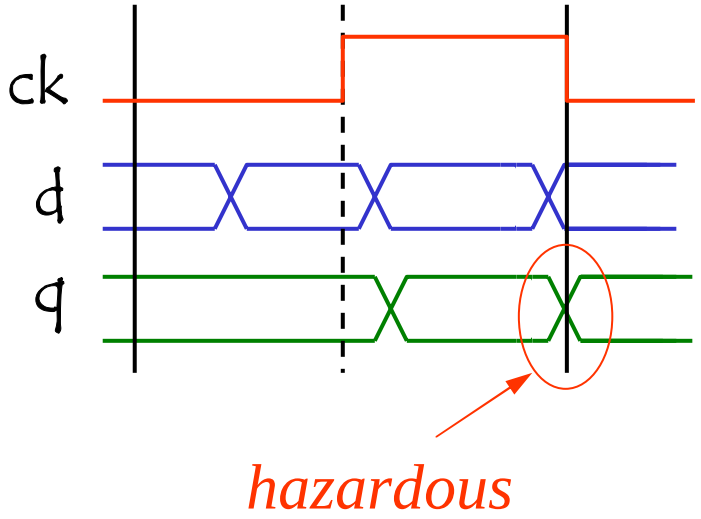
if ck.$\overline{d}$ = 1 r = 0

CMOS Circuits

Synchronous Memory :

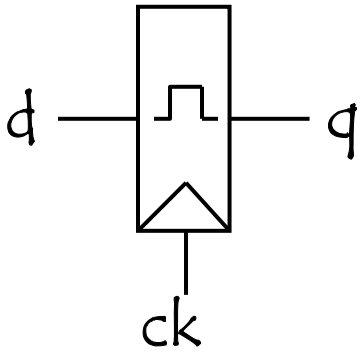
Write a data d when the clock ck = 1

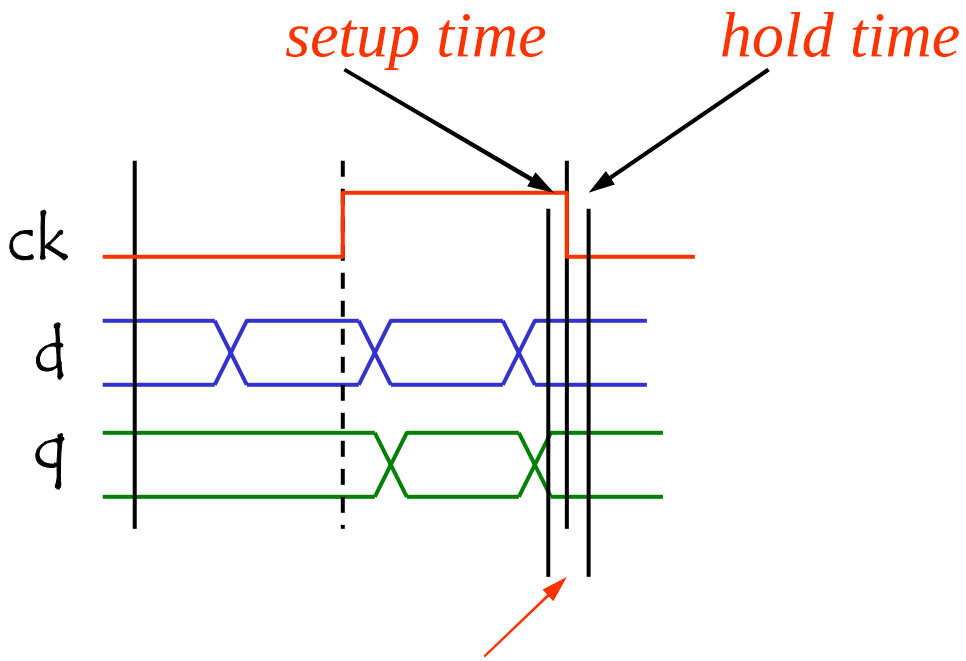




CMOS Circuits

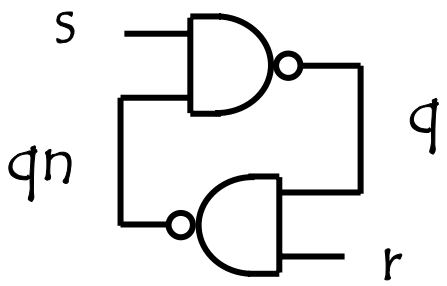
Synchronous Memory :

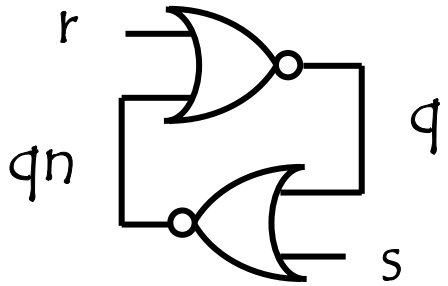




CMOS Circuits

Memory :





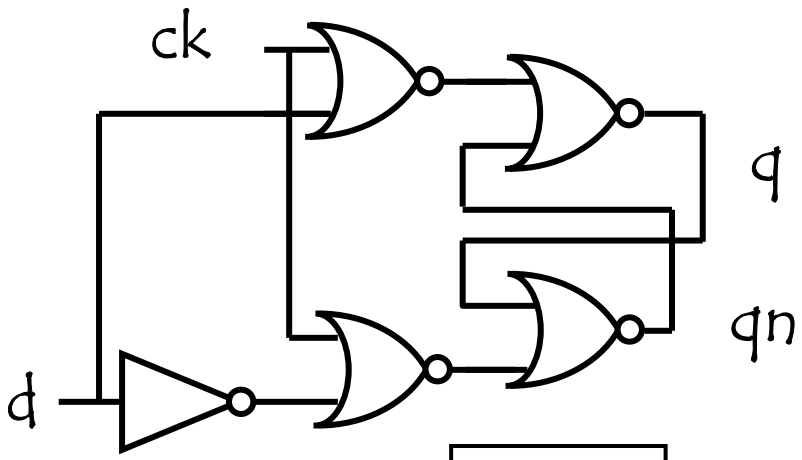
s	r	q	qn
0	1	0	1
1	0	1	0
1	1	0	0
0	0	q	qn

RS flip flop

CMOS Circuits

Synchronous Memory :

Write a data d when the clock ck = 0



d latch

s	r	q	qn
0	1	0	1
1	0	1	0
1	1	0	0
0	0	q	qn

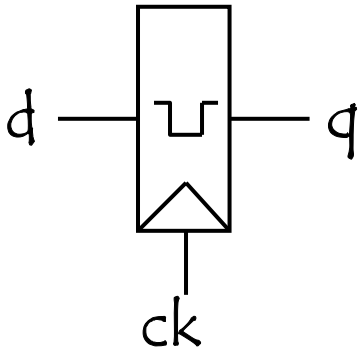
if $\overline{ck}.d = 1$ s = 1

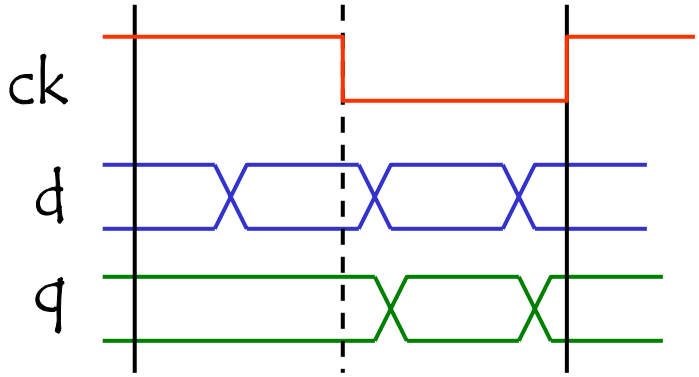
if $\overline{ck}.\overline{d} = 1$ r = 1

CMOS Circuits

Synchronous Memory :

Write a data d when the clock ck = 0

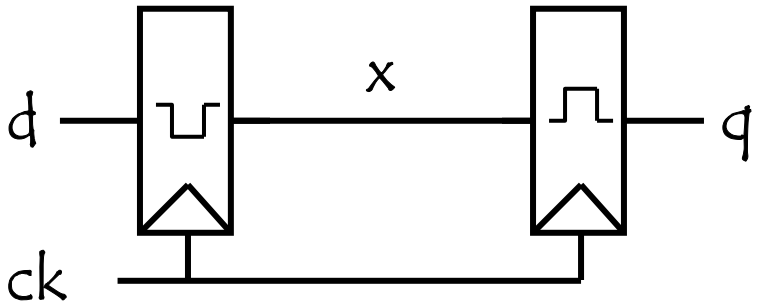


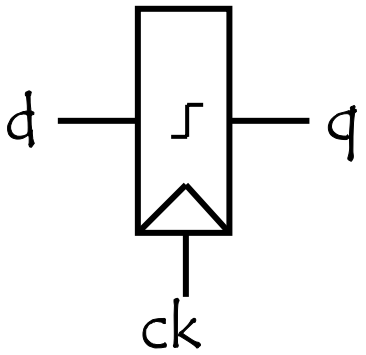


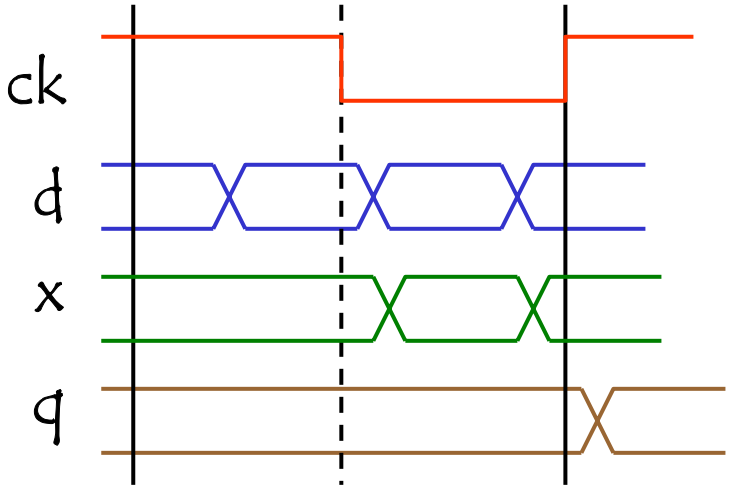
CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck



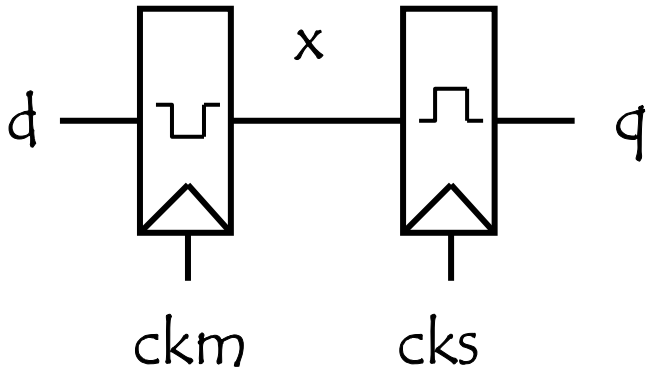


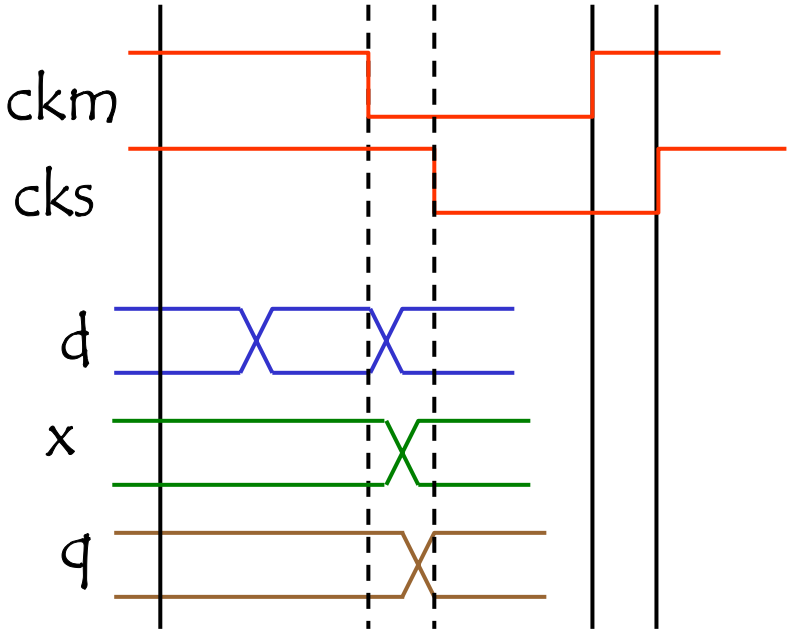


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck

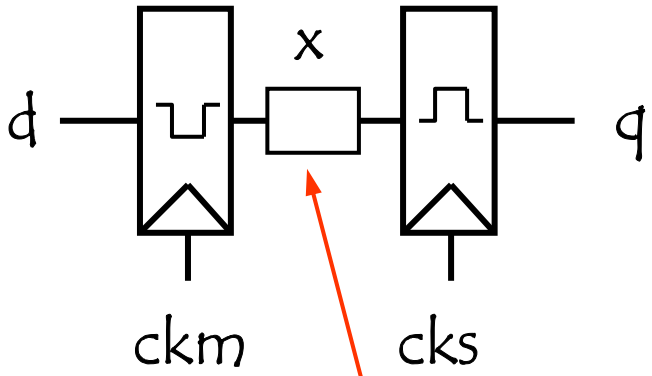


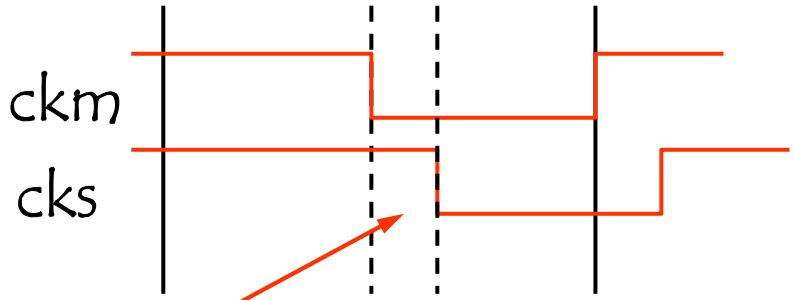


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck



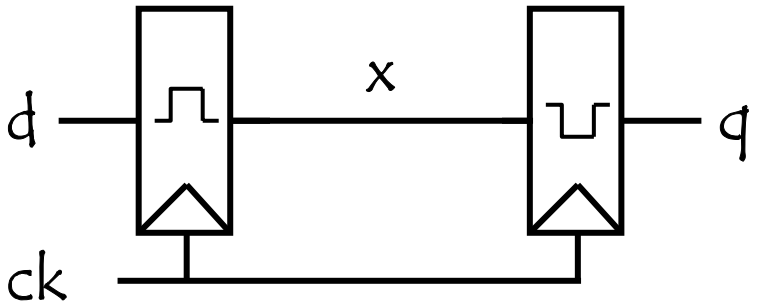


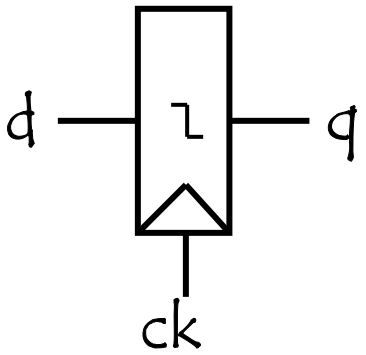
delay < skew

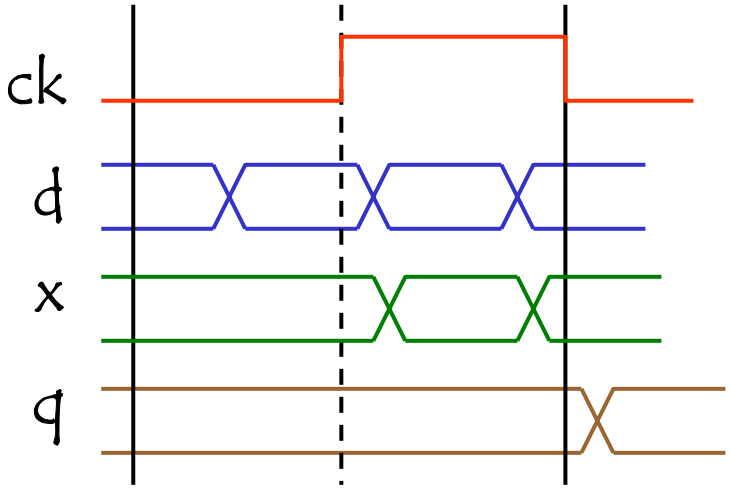
CMOS Circuits

Synchronous Memory :

Write a data d on the falling edge of the clock ck



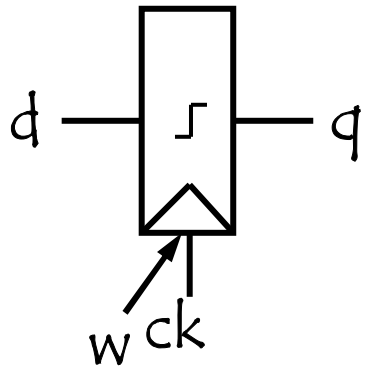


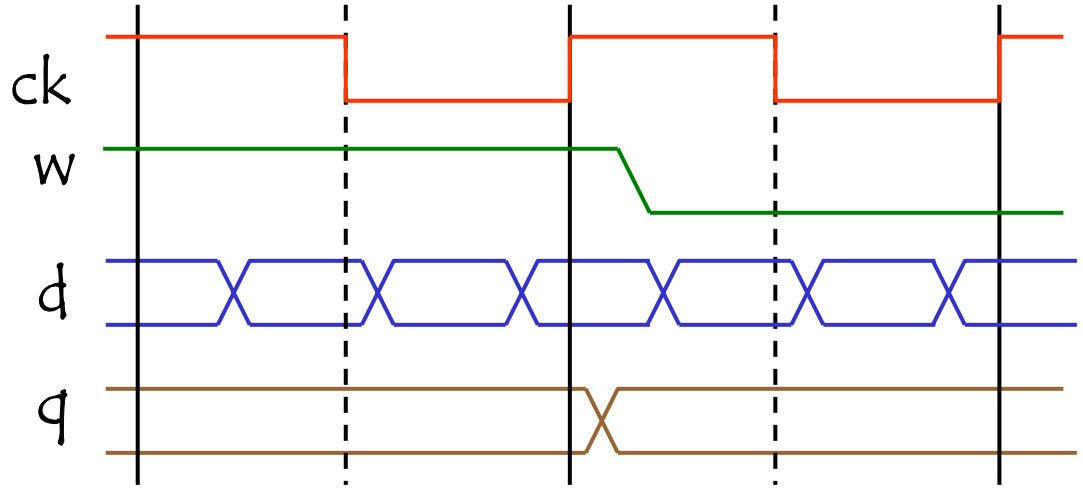


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck
when a condition is true (write enable)

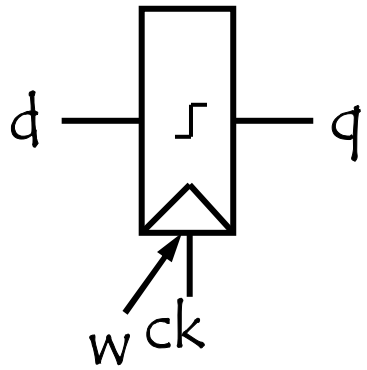


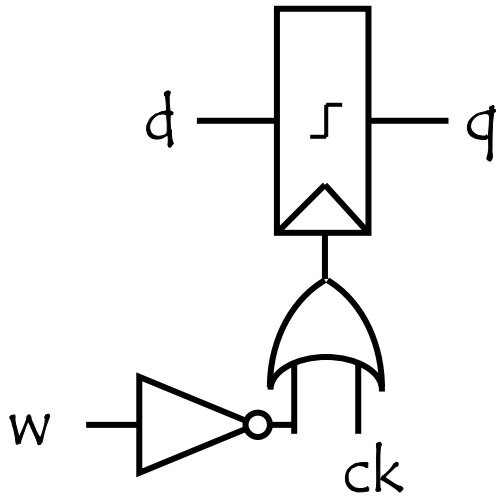


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck
when a condition is true (write enable)

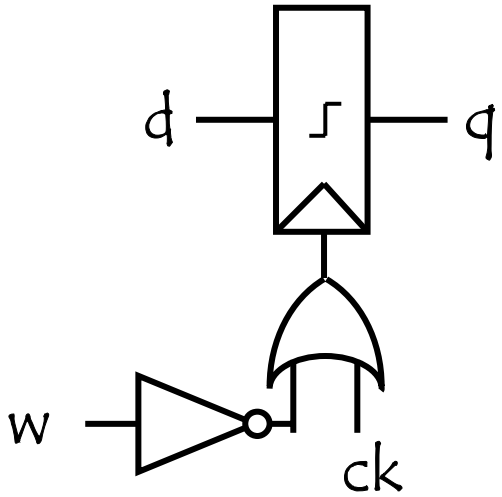


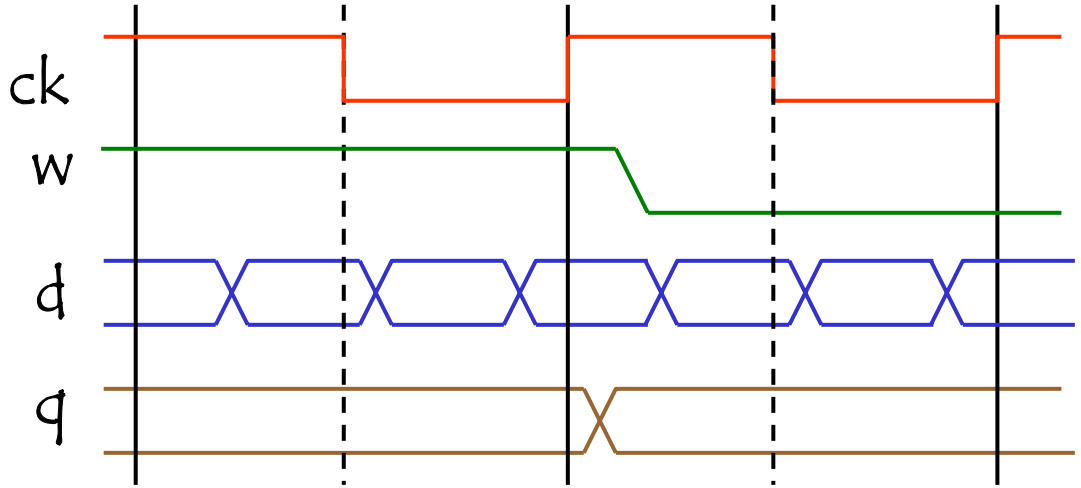


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck
when a condition is true (write enable)

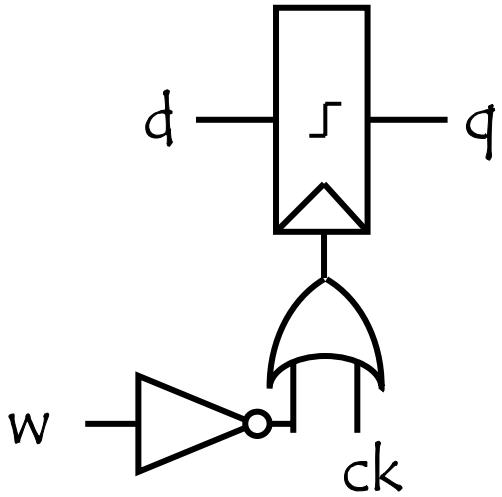


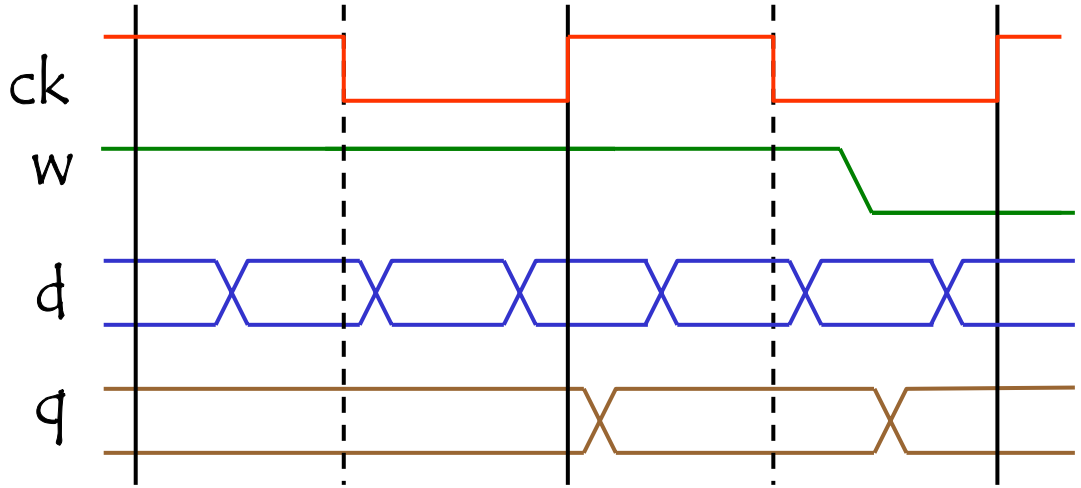


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck
when a condition is true (write enable)

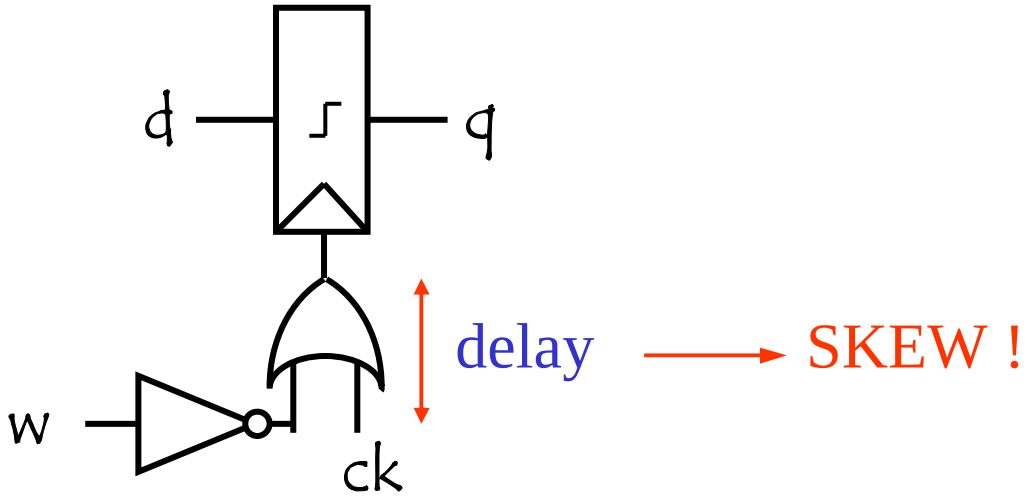




CMOS Circuits

Synchronous Memory :

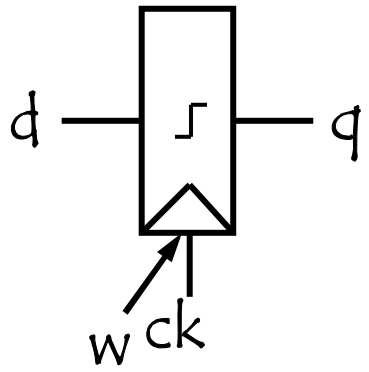
Write a data d on the rising edge of the clock ck
when a condition is true (write enable)

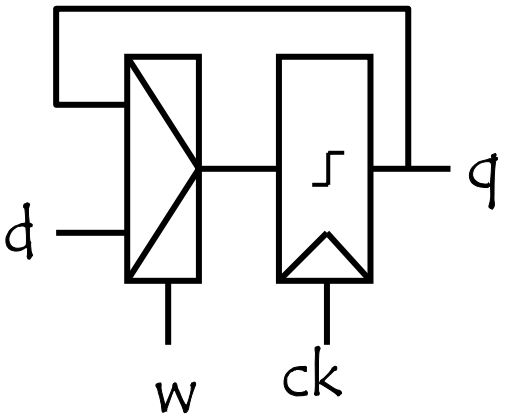


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck
when a condition is true (write enable)

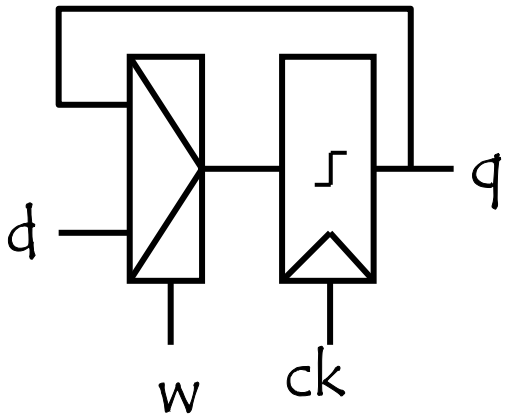




CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck
when a condition is true (write enable)



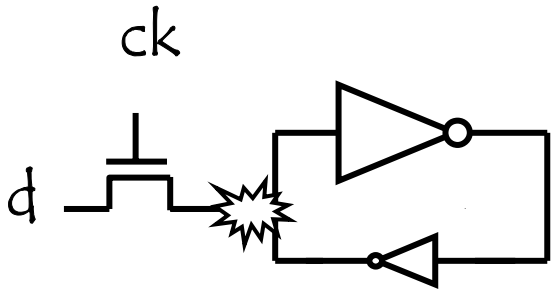
44 tr per register !!

CMOS Circuits

Memory :

Hold a data (0 or 1)

Write a data (0 or 1)

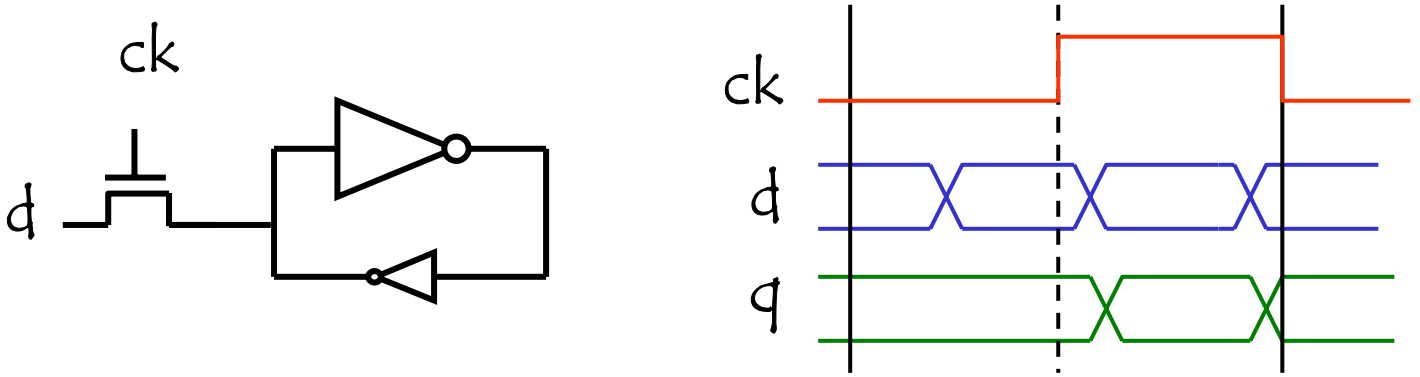


short circuit !

CMOS Circuits

Synchronous Memory :

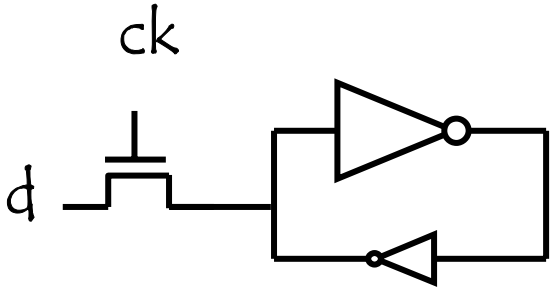
Write a data d when the clock ck = 1

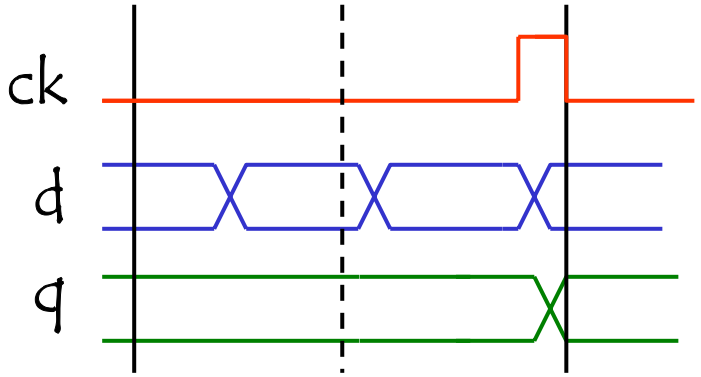


CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck





CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck