



The Abdus Salam  
International Centre for Theoretical Physics



**310/1780-4**

**ICTP-INFN Advanced Training Course on  
FPGA and VHDL for Hardware Simulation and Synthesis  
27 November - 22 December 2006**

---

## ***DIGITAL DESIGN 4***

***Pirouz BAZARGAN SABET  
Lip 6  
University Pierre et Marie Curie (VI)  
Department ASIM  
4, place Jussieu  
75252 Paris Cedex 05  
FRANCE***

---

***These lecture notes are intended only for distribution to participants***

# Outline

## ■ Digital CMOS Design

- Boolean Algebra
- Basic Digital CMOS Gates
- **Combinational and Sequential Circuits**
- Coding - Representation of Numbers

# CMOS Circuits

How can I design a digital circuit ?

- A set of gates (cell library)

combinational gates

memory elements



# CMOS Circuits

How can I design a digital circuit ?

- Method to design combinational circuits

Karnaugh table (local optimization)

no method for global optimization

 synthesis tools



# CMOS Circuits

How can I design a digital circuit ?

- Specify the circuit

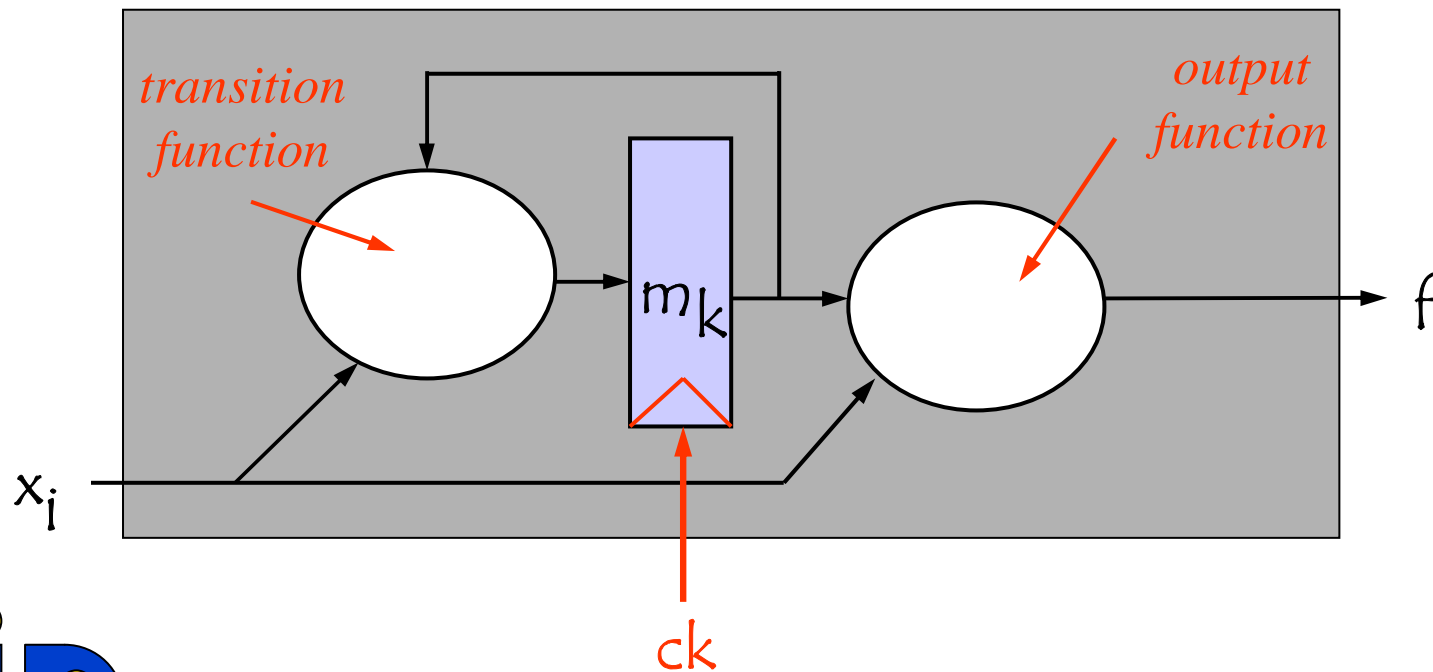
Combinational circuit : Boolean functions

Sequential circuit ?



# CMOS Circuits

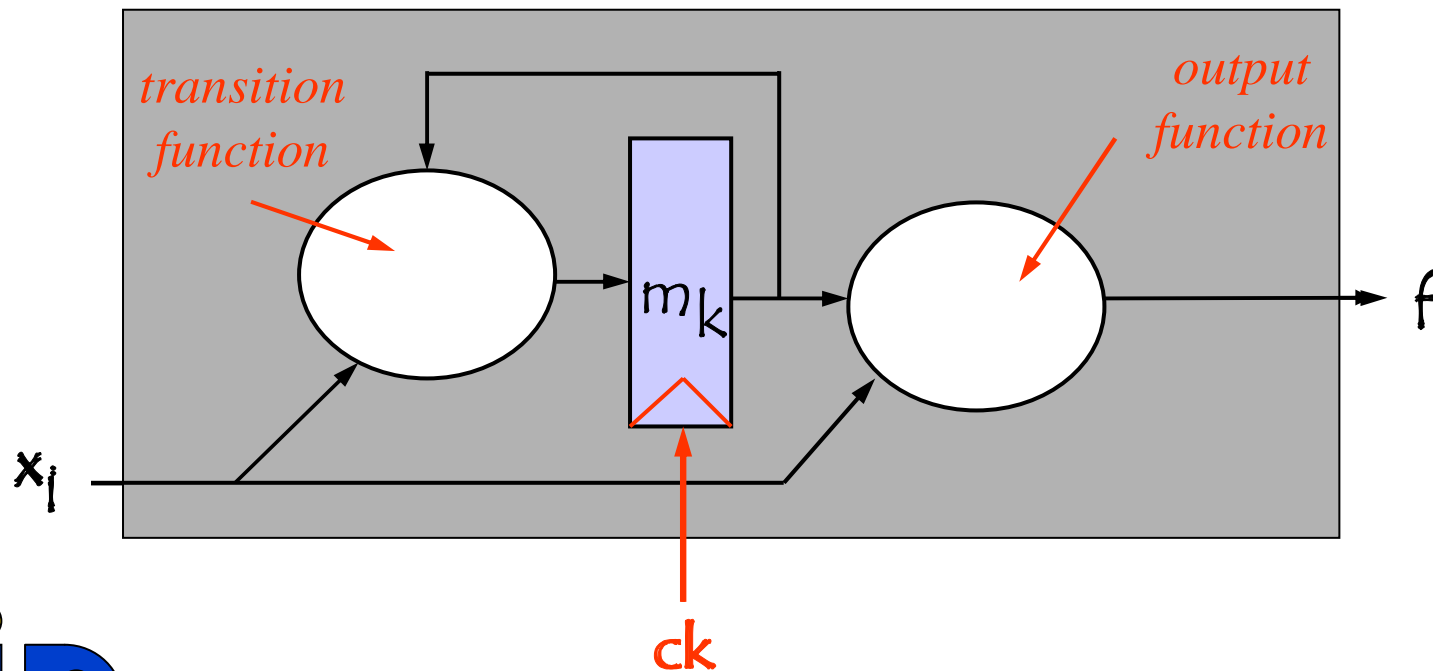
## Sequential Circuit



# CMOS Circuits

How can I design a sequential circuit ?

I need a method to go from a black box to a white box



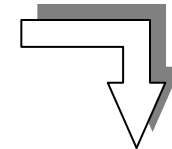
# CMOS Circuits

How can I design a sequential circuit ?

The starting point (what I know)

What the circuit is supposed to do

Capture this knowledge into a  
representation that can be  
transposed into Boolean functions



The aim

How it will do it

number of required  
memory elements





# CMOS Circuits

Representation of a sequential circuit ?

## ● State graph

**G** = (X, Y, S, T,

- ) X : set of input variables
- Y : set of output variables
- S : set of states
- T : set of transitions
- : set of output set conditions



# CMOS Circuits

○ **G** = (X, Y, S, T,  
○)

T : set of transitions

$t \in T, \quad t = (s, s', f)$

$s \in S$  : source state

$s' \in S$  : target state

$f \in \mathbf{B}_n$  : transition condition



# CMOS Circuits

○ **G** = (X, Y, S, T,  
O)

O : set of output set conditions

$o \in O, \quad o = (y, s, c)$

$y \in Y$  : output variable

$s \in S$  : state

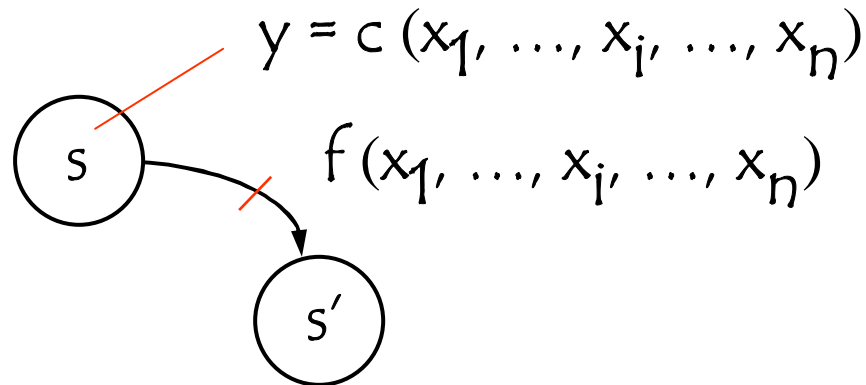
$c \in \mathbf{B}_n$  : output set condition



# CMOS Circuits

- Let consider a graph  $\mathbf{G} = (X, Y, S, T, O)$

Graphic representation of  $\mathbf{G}$



# CMOS Circuits

Representation of a sequential circuit ?

- State graph

Transpose into a graphic representation the expected behaviour of a sequential system

$$\mathbf{G} = (X, Y, S, T, O)$$



# CMOS Circuits

## Example

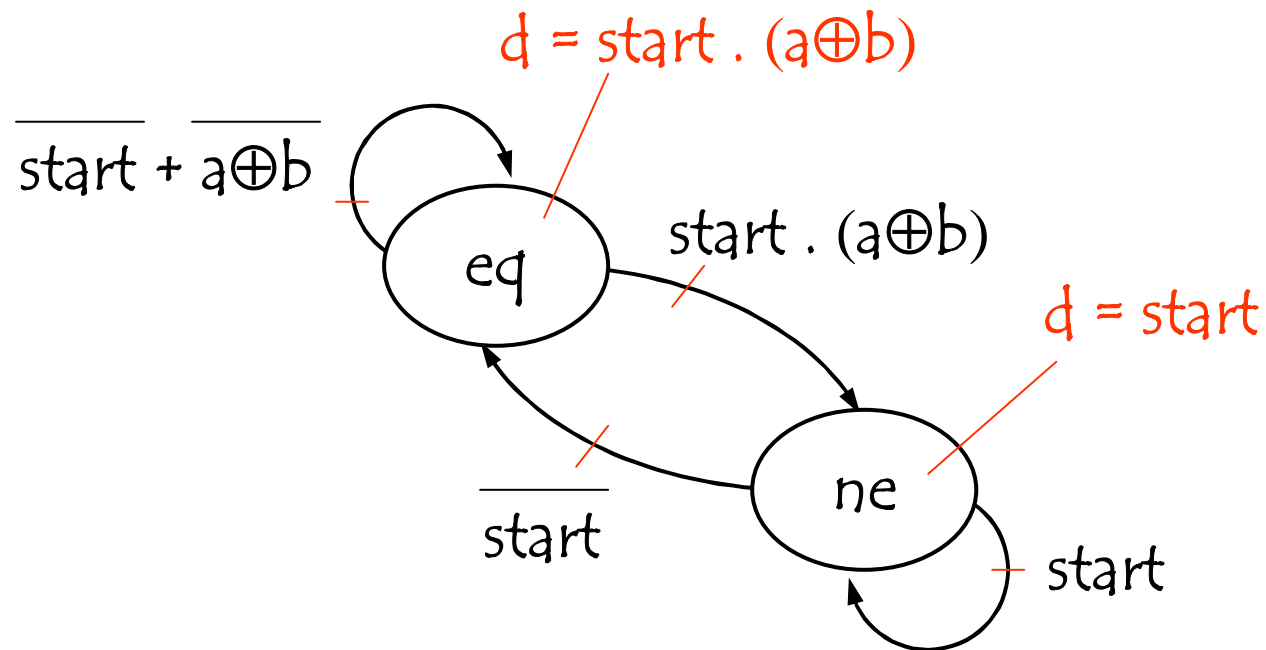
Two signals  $a$ ,  $b$  each transmitting a series of bits (1 bit at a cycle)

Design a system that set a flag  $d$  if the values transmitted by  $a$  is different than  $b$



# CMOS Circuits

## Example



# CMOS Circuits

Representation of a sequential circuit ?

- State graph

$$\mathbf{G} = (X, Y, S, T, O)$$

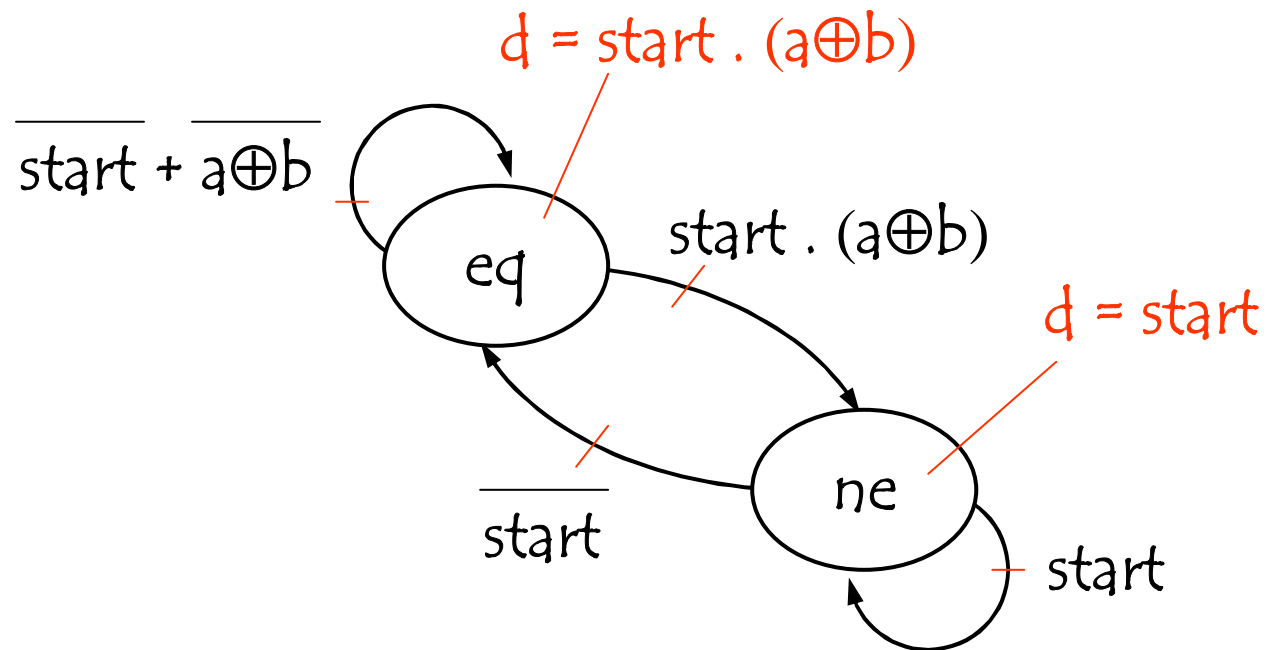
define the number of memory elements  
required to represent  $S : M$





# CMOS Circuits

## Example



2 states  $\longrightarrow$  1 memory element

# CMOS Circuits

Representation of a sequential circuit ?

- State graph

$$\mathbf{G} = (X, Y, S, T, O)$$

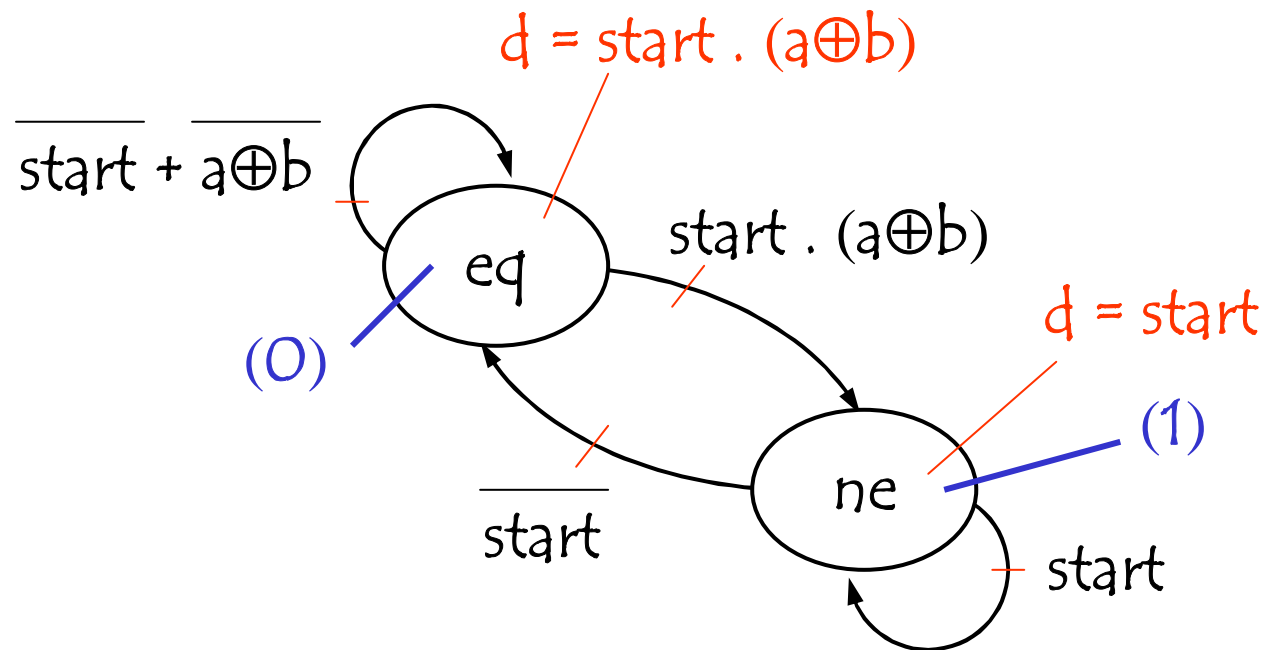
define the number of memory elements required to represent  $S : M$

represent each state  $s$  by a vector of  $M$



# CMOS Circuits

## Example



2 states  $\longrightarrow$  1 memory element

# CMOS Circuits

Representation of a sequential circuit ?

- State graph

$$\mathbf{G} = (X, Y, S, T,$$

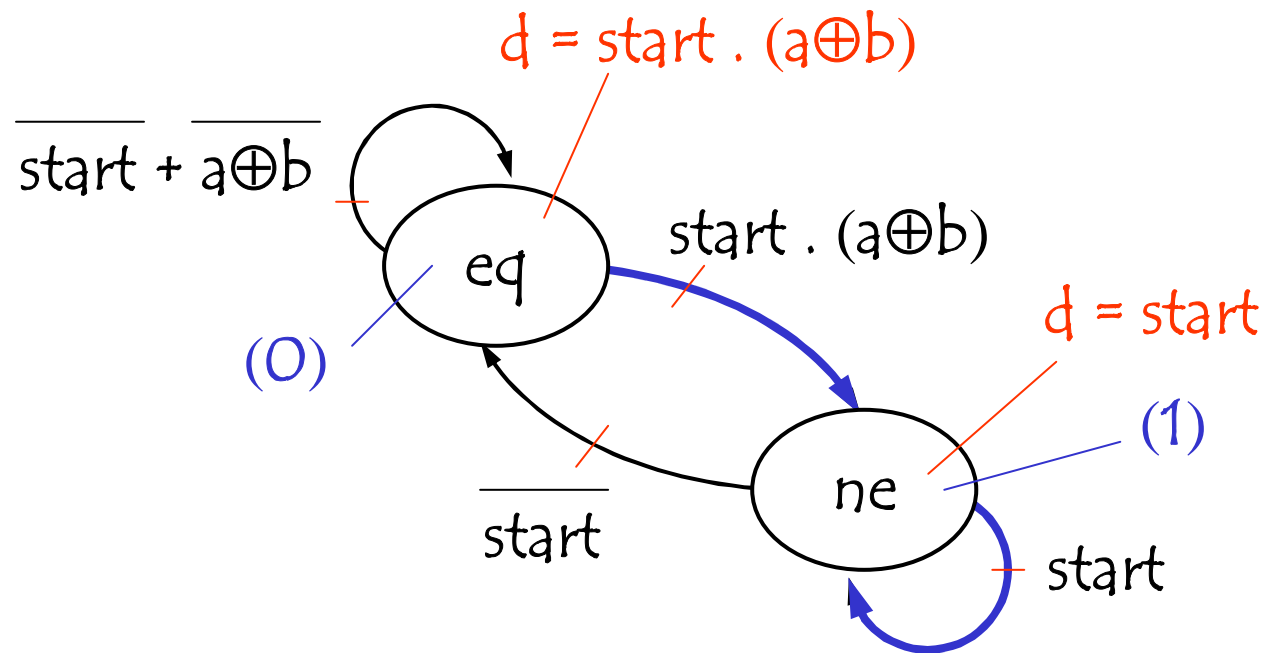
define the <sup>O</sup>*transition function* :  $m_k$

$m_k$  = sum of the Boolean function of the transitions that have as target a state where  $m_k=1$



# CMOS Circuits

## Example



$$m_1 = \overline{m_1} \cdot start \cdot (a \oplus b) + m_1 \cdot start$$

# CMOS Circuits

Representation of a sequential circuit ?

- State graph

$$\mathbf{G} = (X, Y, S, T,$$

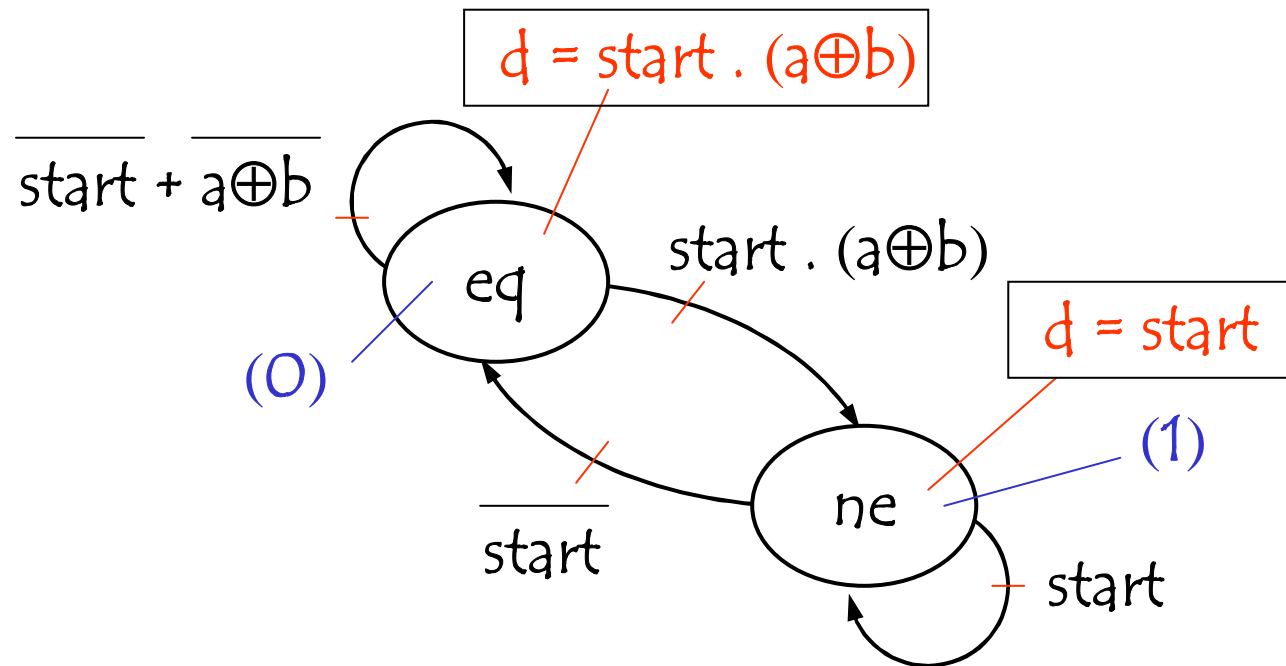
define the <sup>O</sup>output function :  $y_i$

$y_i =$  sum of the output conditions concerning  $y_i$



# CMOS Circuits

## Example



$$d = \overline{m_1}.start.(a \oplus b) + m_1.start$$

# CMOS Circuits

Representation of a sequential circuit ?

- State graph

$\mathbf{G} = (X, Y, S, T,$   
implement<sup>o</sup> the Boolean functions :  $m_k, y_j$





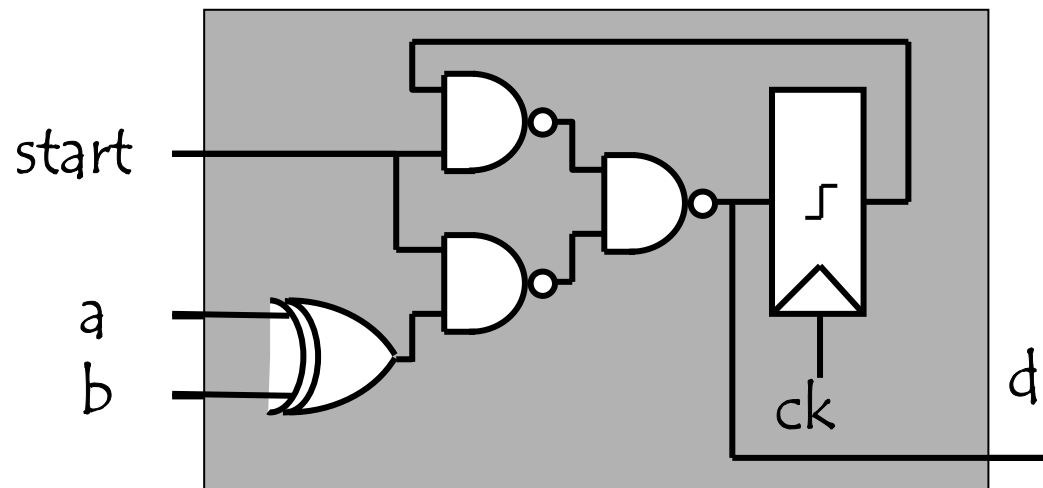
# CMOS Circuits

## Example

$$m_1 = \overline{m_1} \cdot \text{start} \cdot (a \oplus b) + m_1 \cdot \text{start}$$

$$d = \overline{m_1} \cdot \text{start} \cdot (a \oplus b) + m_1 \cdot \text{start}$$

$$m_1 = \text{start} \cdot (a \oplus b) + \text{start} \cdot m_1$$



# CMOS Circuits

How can I design a sequential circuit (summary) ?

○ **G** = (X, Y, S, T,

○)

- graphic representation the behaviour
- define the number of memory elements required to represent  $S : M$
- **represent each state by a vector of  $M$**
- define the *transition function*
- define the *output function*
- implement the Boolean functions

