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SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

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Black Holes in String Theory

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LECTURE 1

Black Holes in String Theory

Gravity is in a universal and (fairly) model independent way present in string theory $\Rightarrow$ can learn lessons for realistic gravity : black holes, cosmological singularities, etc.

Good "toy model"

$$ds^2 = -\Delta_+(\rho)\Delta_-(\rho)\,dt^2 + \Delta_+(\rho)^{-1}\Delta_-(\rho)^{-1}\,d\rho^2 + \rho^2 d\Omega_2^2$$

$$\Delta_\pm(\rho) = \left(1 - \frac{r_\pm}{\rho}\right) \qquad \underline{\text{RN}} \text{ black hole.}$$

$$r_\pm = G_4\left(M \pm \sqrt{M^2 - Q^2}\right)$$

$M \to |Q|$ "extremality"

$$\to \quad ds^2_{ext} \simeq -\left(1 - \frac{r_0}{\rho}\right)^2 dt^2 + \left(1 - \frac{r_0}{\rho}\right)^{-2} d\rho^2 + \rho^2 d\Omega_2^2$$

$\rho = r_0 + r$ yields

$$\simeq \underbrace{-\frac{r^2}{r_0^2}dt^2 + \frac{r_0^2}{r^2}dr^2}_{AdS_2} + \underbrace{r_0^2 d\Omega^2}_{S^2}$$

$\to \quad ds^2_{NExt}$ with $\rho = r_+ + r^2$ yields

$$ds^2 = \underbrace{-\left(1 - \frac{r_-}{r_+}\right)\left(\frac{r^2}{r_+}\right)dt^2 + \left(1 - \frac{r_-}{r_+}\right)^{-1}\frac{r_+^2}{r^2}\,4r^2 dr^2}_{} + r_+^2 d\Omega_2^2$$

$$\bullet \; d(re^{-\eta})\,d(re^{\eta}) = dr^2 - r^2 d\eta^2 \qquad \underline{\text{Rindler}} \to \boxed{\text{Read off temperature!}}$$

issues

- information loss / Hawking radiation ($C_V < 0 \to$ need to put in a box)
- entropy = what? ($dM = \hat{k} \frac{dA}{4\pi} + \phi\, dQ$) (1st law)
- remnants? other universes?
- long lived resonances in ST? Never have been found
- locality vs non-locality?
- large redshifts etc.

+ BH's in ST vs strings

$$ds^2 = -\left(1-\left(\frac{r_H}{r}\right)^{d-3}\right) dt^2 + \left(1-\left(\frac{r_H}{r}\right)^{d-3}\right)^{-1} dr^2 + r^2 d\Omega^2$$

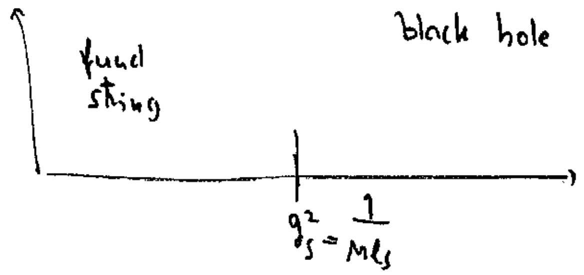
$R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \sim r_H^{-4}$ at horizon so LEEA goes bad

when $r_H \sim l_s$, $r_H^{d-3} \sim M \cdot G \sim M g_s^2 l_s^{d-2}$

Recall that $T \sim \frac{1}{r_H} \sim \frac{1}{l_s}$ and $S \sim \frac{A}{G} \sim \frac{r_H^{d-2}}{g_s^2 l_s^{d-2}}$

$\Rightarrow$ $M g_s^2 \sim \frac{1}{l_s}$ is transition point

Take a string with oscillator level N, $\hat{M}^2 \sim \frac{N}{l_s^2}$
Number of states $\sim \sqrt{c \cdot L_0} \sim \sqrt{N}$



$$S_{\text{string}} \sim \sqrt{N} \sim \hat{M} l_s \qquad S_{BH} \sim \frac{1}{g_s^2} \sim M l_s$$

Agreement if $\hat{M} = M$ (order one agreement)

Continues for systems with R-charges

Constructing BH's in ST

Dp - branes

$$\begin{cases} ds^2 = f^{-\frac{1}{2}}\left(-dt^2 + dx_{\parallel}^2\right) + f^{\frac{1}{2}} dx_{\perp}^2 \\ e^{+\Phi} = f^{(3-p)/4} \\ C_{0\cdots p} = \frac{1}{g_s}\left(1 - \frac{1}{f}\right) \\ f = 1 + c\,(g_s N_p)\left(\frac{l_s}{r}\right)^{7-p} \end{cases}$$

NS - branes, M - branes: similar expressions

- Harmonic superposition (for BPS, <u>not</u> bound states): add arrays

- Boost / rotate as technique

- Supersymmetry important but sometimes nonextremal version also known

Also important: smearing

$$\frac{N}{V}\int_\infty \left(\left(\frac{1}{r^2+r_i^2}\right)^{\frac{7-p}{2}} dr_i = \frac{N}{V}\int \cdots = \frac{N}{V r^{7-p-\delta_i}}$$

cptification: S^1 $\quad \sum_{n\in\mathbb{Z}} \left(\frac{1}{r^2+(r_i-nR)^2}\right)^{\frac{7-p}{2}} \Rightarrow \int dv \left(\frac{1}{(r^2+v^2R^2)}\right)^{\frac{7-p}{2}}$

$$\Rightarrow \int dv \left(\frac{v'r}{R}\right)\frac{1}{r^{\frac{7-p}{2}}} = \frac{1}{R r^{\frac{6-p}{2}}}$$

0 1 2 3 4 5 6 7 8 9
x x
x x x x x x

Example D1-D5-p

$$ds^2 = f_1^{-\frac{1}{2}} f_5^{-\frac{1}{2}}(-dt^2+dx_1^2) + f_1^{\frac{1}{2}} f_5^{\frac{+1}{2}}(dx_2^2+\dots+dx_5^2)$$

$$+ f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}}(dx_6^2+\dots+dx_9^2) \qquad r^2 = x_2^2+\dots+x_5^2$$

$$e^{\Phi} = f_5^{-\frac{1}{2}} f_1^{\frac{+1}{2}} g_s \qquad f_5 = 1+\frac{g_s N_5}{r^2} \qquad f_1 = 1+\frac{g_s N_1}{V r^2}$$

Add p: $\quad -dt^2+dx_1^2 \Rightarrow (-dt^2+dx_1^2) + \frac{g_s^2 N_p}{V r^2}(dt-dx_1)^2$

Feature: supersymmetric w/ horizon (preserves 4-supersymmetries)

Compute horizon radius: $\begin{vmatrix} -1+\frac{N_p}{r^2} & -\frac{N_p}{r^2} \\ -\frac{N_p}{r^2} & 1+\frac{N_p}{r^2}\end{vmatrix}$

$$r\to 0 \qquad A \sim \left.\frac{V\cdot\left(f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}}\right)^2 \left(f_1^{\frac{1}{2}} f_5^{\frac{+1}{2}}\right)^{\frac{3}{2}} \left(f_1 f_5\right)^{-\frac{1}{4}} \cdot \sqrt{\frac{g_s^2 N_p}{V r^2}}}{g_s^2 \; f_5^{-1} f_1^{+1}} r^3 \right|_{r\to 0} \sim \sqrt{N_1 N_5 N_p}$$

More precisely: $S = 2\pi\sqrt{N_1 N_5 N_p}$: no g_s, V dependence.

* exercise: do this for D4-D4-D4-D0 on T^6

Other "famous" examples:

- M-theory with M5-M2-p on $S^1 \times CY$
- IIA with D0-D2-D4-D6 on CY
- various black holes in $AdS_5 \times S^5$
- BTZ - black hole (later)
- BMPV: spinning black holes w/ angular momentum.

LECTURE 2

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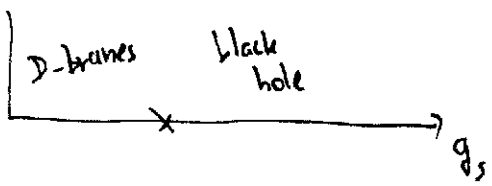


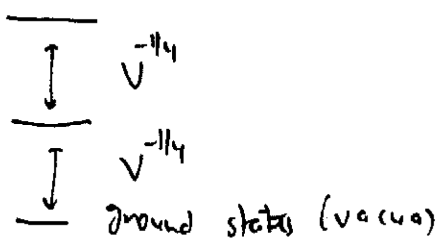
Supersymmetry, g_s-independence of the entropy, suggest that counting should be doable on the D-brane side.

$\rightarrow$ issues: does a decoupling limit exist?

What is the theory on the D1-D5-system? This is a CFT in the IR.

IR: low energies $\Leftrightarrow$ V is very small

cf. Kaluza-Klein reduction



internal momenta: throw away

2d-momenta: keep

$\Rightarrow$ Get a field theory, which is a σ-model on moduli space of classical solutions of the theory.

D1 – D5 $\Leftrightarrow$ D0 – D4

describes moduli space of N_1 instantons in $U(N_5)$ theory (ADHM equations)

thm (math): moduli space $\approx$ $(T^4)^{N_1 N_5}/S_{N_1 N_5}$ ("Hilbert Scheme")

$L_0 \leftrightarrow N_m$

$\bar{L}_0 = \frac{c}{24}$ (RR– ground state)

(Cardy $S \sim 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} \sim 2\pi\sqrt{N_1 N_5 N_m}$ ✓.

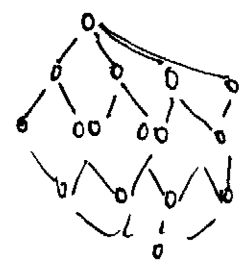
(is clearly approximate)

Various subtleties: 1) – U- duality permutes $N_1 N_5 N_m$

– Cardy is not obviously U- duality invariant?

– "fractionation" explains this

2) Should really compute an index — analogue of Witten index $(-1)^F$ — because BPS states can pair up & be lifted

T^4: short  long 

$\Rightarrow$ need $\mathrm{trace}_{\mathcal{H}}\left(F^2(-1)^F\right) \Rightarrow$ elliptic genus

3) ~~[illegible]~~

is index same as BH- horizon?

not always – unknown (cf $AdS_5 \times S^5$ examples)

4) CFT is strongly coupled

$e^{\int_{C^2} B}$ can be nonzero even when C_2 vanishes

– here $B = 0$.

"Decoupling limit"

Go back to D1–D5 system and go to regime

$$\frac{g_s N_5}{r^2} \gg 1 \qquad \frac{g_s N_1}{V r^2} \gg 1 \qquad (\text{cf Maldacena: } \alpha' \to 0 \text{ limit})$$

$$ds^2 \to \frac{r^2}{\ell^2}(-dt^2 + dx_1^2) + \frac{\ell^2}{r^2} dr^2 + \ell^2 d\Omega_3^2 + \sqrt{\frac{N_1}{V N_5}}\, ds^2_{T^4}$$

$AdS_3 \times S^3 \times T^4$

Notice that T^4 has fixed volume : attractor mechanism

Also dilaton becomes constant

Keep N_p : $\Rightarrow \left[(-dt^2 + dx_1^2) + \frac{g_s^2 N_p}{V r^2}(dt - dx_1)^2\right]$

Find AdS_3 – black hole. (BTZ)

Area is not modified.

Decoupling limit preserves all BH degrees of freedom

"proves" they can all be accounted for in strongly coupled gauge theory / CFT.

BTZ – black hole:

$$ds^2_{BTZ} = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2} dt^2 + \frac{\ell^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2$$
$$+ r^2\left(d\varphi + \frac{r_+ r_-}{\ell r^2} dt\right)^2$$

$$M = \frac{r_+^2 - r_-^2}{8\ell^2 G_3} \qquad J = \frac{r_+ r_-}{4\ell G_3}$$

$$\sim L_0 + \bar{L}_0 \qquad\qquad \sim L_0 - \bar{L}_0$$

$$S = \frac{2\pi r_+}{4 G_3} \sim 2\pi\sqrt{\frac{c}{6} L_0} + 2\pi\sqrt{\frac{c}{6}\bar{L}_0}$$

<u>Many</u> entropy explanations involve an AdS_3 decoupling limit where the BH becomes BTZ.

BTZ is a suitable quotient of $SL_2(\mathbb{R})$

The "generic" CFT so far is the M5- CFT, relevant for D0-D2-D4 system in IIA

(D6- branes: ~~kes~~ not clear if $\exists$ CFT)

M5 wrap $p^A \Sigma_A$, Σ_A: basis of four-cycles

$\rightarrow$ get string in $\mathbb{R}^5 \rightarrow AdS_3 \times S^2 \times CY$

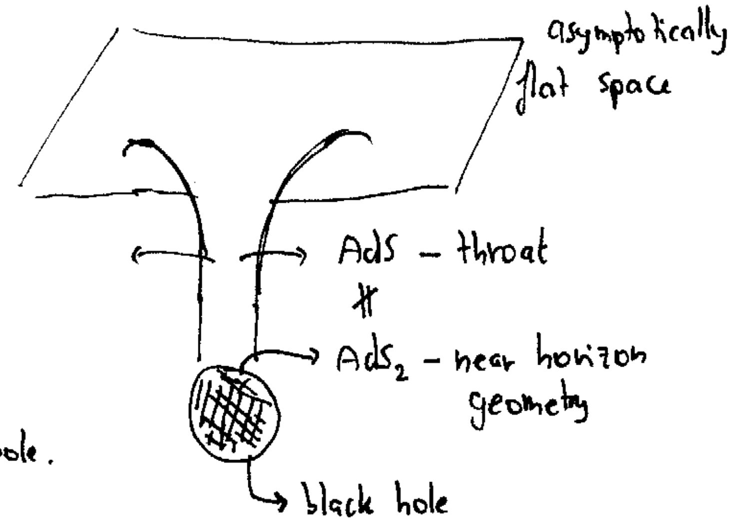
CFT is (0,4) CFT (the "MSW" theory) which describes the moduli space of deformations of the 4-cycle, not very well understood $C \sim p^A p^B p^C d_{ABC}$, $d_{ABC} \sim \int \alpha_A \wedge \alpha_B \wedge \alpha_C$

remainder?
- AdS/CFT & black holes
- Schwarzschild
- LLM
- More general solutions in d=4,5
- Mathur for $\frac{1}{2}$ BPS
- OSV, curves of marginal stability, OW?
- outlook

LECTURE 3

AdS / CFT is relevant for BH's

Typical situation



The field theory dual to this throat-AdS should explain the entropy of the black hole.

In general non-normalizable deformations of AdS $\longleftrightarrow$ $S' = S + \int J^\alpha O_\alpha$

normalizable deformations of AdS $\longleftrightarrow$ states / density matrices ρ.

Expect that supergravity / string theory on an asymptotic AdS - space (ie normalizable deviation) computes correlation functions in a state:

$$e^{-S_{\text{bulk}}^{\text{on-shell}}} \Longleftrightarrow \mathrm{Trace}\left[\rho\, O_1 O_2 .. O_n\right] = \langle O_1 .. O_n \rangle_\rho$$

in bulk   in the field theory.

classical geometries $\longleftrightarrow$ semiclassical states
definition = what?

black hole $\longleftrightarrow$ $\rho = \sum e^{-\beta E} |E\rangle\langle E|$

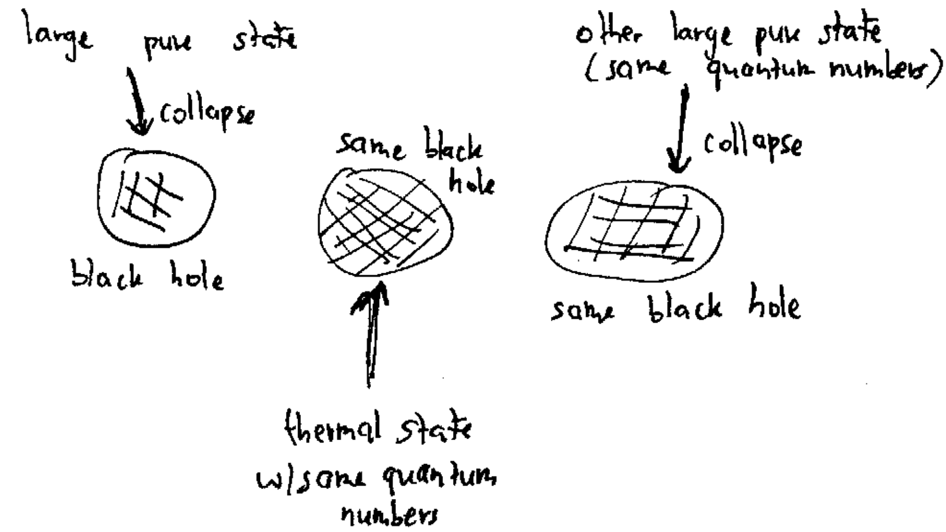
black object entropy S $\longleftrightarrow$ $\mathcal{S} = -\mathrm{Tr}(\rho \log \rho)$ definition = what?

bulk has isometry D $\longleftrightarrow$ $[\rho, \hat{D}] = 0$

ADM quantum number associated to D $\longleftrightarrow$ $\mathrm{trace}(\rho\hat{D}) = \langle \hat{D} \rangle$

more about this later....

<u>PUZZLE</u>:



$\Rightarrow$ How can different pure states & the thermal state be so similar?

See a qualitative hint of this in zero coupling $N=4$ SYM.

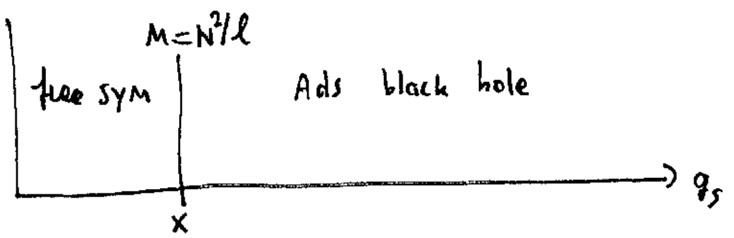
Horowitz-Polchinski: for $AdS_5 \times S^5$ Schwarzschild black holes,

$$ds^2 = -\left[1 + \frac{r^2}{\ell^2} - \frac{r_0^2}{r^2}\right] dt^2 + \left[1 + \frac{r^2}{\ell^2} - \frac{r_0^2}{r^2}\right]^{-1} dr^2 + r^2 d\Omega_3^2 + \ell^2 d\Omega_5^2 \qquad \ell = (g_s N)^{1/4}$$

Here matching is at $r_0 \sim \ell$, horizon is at AdS-curvature radius.

$$M G_5 = r_0^2 = \ell^2 \Rightarrow M = \frac{\ell^2}{G_5} = \frac{\ell^2 \cdot \ell^7}{g_s^2} = \frac{\ell^8}{g_s^2 \ell} = \frac{N^2}{\ell}$$

$$S = \frac{\ell^8}{G_{10}} \sim N^2$$, and notice that $\Delta = M \cdot \ell = N^2$



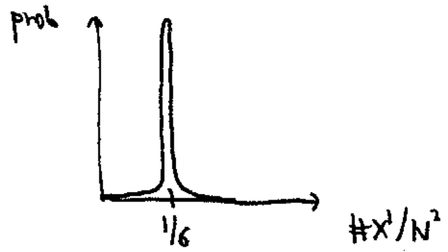
Check: in the free theory, for $\Delta = N^2$, also $S = N^2$
$\Rightarrow$ correspondence works!

Analyze typicality in free $N=4$ SYM.

Restrict attention to the six adjoint scalars X^i

Typical operator of length N^2: $\quad \mathrm{Tr}(X^1 \ldots X^4 X^6 X^5) \ldots \mathrm{Tr}(X_3^3 X_2 X_1)$

Each operator looks like a long random sequence
If one measures number of appearances of say X^1,
the probability distribution of the outcome will be

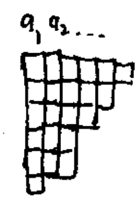


Measurement of X^i will not distinguish states from each other
Neither will $X^i X^j$ $\Longrightarrow$ almost impossible to distinguish states
$\Longrightarrow$ all states look like the ensemble average
(as is the case for a black hole)

$\Longrightarrow$ Can do better by looking at $\frac{1}{2}$-BPS states
~~Rath~~ Write $Z^1 = X^1 + iX^2 \quad Z^2 = X^3 + iX^4 \quad Z^3 = X^5 + iX^6$

$\frac{1}{2}$-BPS states $\Longleftrightarrow$ operators $\displaystyle\prod_{a_i \le N} \mathrm{tr}(Z^{a_i})$

Assume $a_1 \geq a_2 \geq a_3 \geq a_4 \ldots$
Represent the state using a Young diagram
$\#$ rows $\leq N$.



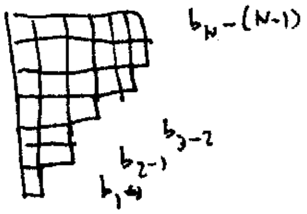
Similar representation (not identical)

N free fermions ~~&~~ in a harmonic oscillator potential

Derived from a matrix model representation of the $\frac{1}{2}$-BPS sector.

states $\quad \Psi_{-b_N-\frac{1}{2}} \Psi_{-b_{N-1}-\frac{1}{2}} \cdots \Psi_{-b_1-\frac{1}{2}} |0\rangle$

$b_i \in \mathbb{N}, \quad b_N > b_{N-1} \ldots > b_1$ (~~Fermi~~ Pauli exclusion principle)



Again, Young diagram with at most N rows.

$$\text{Energy of state} = \sum_{i=1}^{N}\left(b_i+\tfrac{1}{2}\right) = \sum_{i=1}^{N}\left(b_i-(i-1)\right) + \sum_{i=1}^{N}\left(i-\tfrac{1}{2}\right)$$

$$= \#\text{boxes} + \underbrace{\frac{N^2}{2}}_{\text{ground state energy}}$$

$$\Rightarrow \boxed{\Delta = \#\text{boxes}}$$

What is the state $\leftrightarrow$ geometry map?

Use classification of $\frac{1}{2}$-BPS solutions of supergravity due to LLM.

$$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + y e^{G} d\Omega_3^2 + y e^{-G} d\tilde{\Omega}_3^2$$

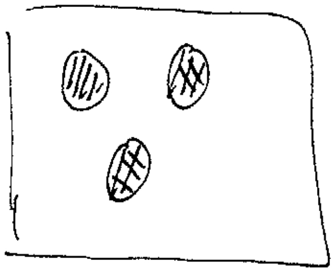
$$h^{-2} = 2y\cosh G \quad , \quad y\partial_y V_i = \varepsilon_{ij}\partial_j V \quad , \quad y(\partial_i V_j - \partial_j V_i) = \varepsilon_{ij}\partial_y z$$

$$z = \frac{1}{2}\tanh G \qquad i=1,2$$

$$z(y, x_1, x_2) = \frac{y^2}{\pi}\int dx_1' dx_2' \frac{\frac{1}{2} - u(x_1', x_2')}{[(x-x')^2 + y^2]^2}$$

Solution completely determined in terms of a single function $u(x_1, x_2)$.

smoothness $\longleftrightarrow$ $u \in \{0,1\}$ piece wise continuous



LLM- droplets: plot of $u(x_1,x_2)$

$$\int dx_1 dx_2\, u(x_1,x_2) = N$$

in suitable units.

Expect a map

fermion states $\longleftrightarrow$ droplets
CFT AdS

<u>proposal</u>: x_1, x_2 - plane ~~of~~ is the phase space of the harmonic oscillator where the fermions live

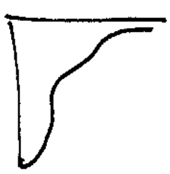
droplet = phase space density of fermion system

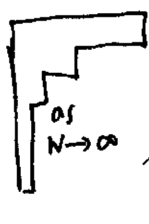
<u>recall</u> def of phase space density associated to density matrix ρ:

$$\forall A \quad : \qquad \mathrm{trace}\,(\rho.\hat{A}) = \int dp\,dq\; W_\rho(p,q)\,A(p,q)$$

There is an ordering ambiguity going from A to $\hat{A}$ $\Rightarrow$
W_ρ is ambiguous : related to higher order corrections in gravity

Ambiguities disappear in classical limit, $N \to \infty$ $\hbar N$ fixed.,
but not for all states : must approach a smooth Young diagram



role of smoothness ($\equiv$  as $N \to \infty$) in fermion system is not clear

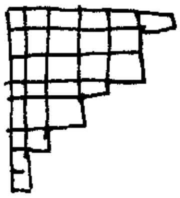
So we have precise map
states $\rightarrow$ geometries

Most states give "quantum foam" geometries w/ string scale curvature

Suitable states (semiclassical states) yield well-defined but still mildly singular spacetimes

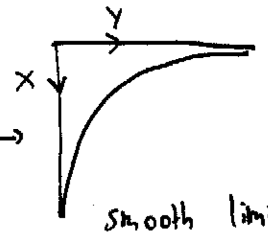
Special states yield smooth geometries

What about typicality ?



N rows
Δ boxes

$N \rightarrow \infty$



$aq^x + bq^y = 1$
for some a, b, q

smooth limit
shape w/ probability one

thm due to Vershik
basically is standard
thermodynamics

$\Rightarrow$ all states look the same & are difficult to distinguish from ensemble average

Ensemble average $\approx$ black hole
$= \sum'$ micro states
$=$ coarse grained geometry.

Unfortunately, $\frac{1}{2}$ BPS "black hole" has no macroscopic horizon $\rightarrow$ many black hole problems cannot be solved.

→ richer class of solutions exist in AdS_3

$\frac{1}{2}$-BPS : Lunin-Mathur (not complete)

$\frac{1}{4}$-BPS : Many solutions have been found but not enough to account for the entropy of the SV black hole (yet)

→ $\frac{1}{2}$ BPS-solutions (found via dualizing strings)

$$ds^2 = \frac{1}{\sqrt{f_1 f_5}}\left[-(dt+A)^2 + (dx_1+B)^2\right]$$
$$+ \sqrt{f_1 f_5}\,(dx_2^2 + \dots + dx_5^2) + \sqrt{\frac{f_1}{f_5}}\,(dx_6^2 + \dots + dx_9^2)$$

$e^{2\Phi} = f_1/f_5$, + some flux

$dB = *_4\, dA$ $\quad (*_4$ on $(x_2, x_3, x_4, x_5) \equiv \vec{x})$

$$f_5 = 1 + \frac{Q_5}{L}\int_0^L \frac{ds}{|\vec{x}-\vec{F}(s)|^2}$$

$$f_1 = 1 + \frac{Q_5}{L}\int_0^L \frac{ds\,|\vec{F}'(s)|^2}{|\vec{x}-\vec{F}(s)|^2}$$

$$A = \frac{Q_5}{L}\int_0^L \frac{F_i'(s)\,ds}{|\vec{x}-\vec{F}(s)|^2}\,, \qquad \underline{Q_1} = \frac{Q_5}{L}\int_0^L |\vec{F}'(s)|^2\, ds$$

$\vec{F}(s)$ describes a curve in $\mathbb{R}^4$. (spanned by $x_2 \dots x_5$)

$$\vec{F}(s) = \mu \sum_{k=1}^{\infty} \frac{1}{\sqrt{2k}}\left(d_k\, e^{i\frac{2\pi k}{L}s} + \bar{d}_k\, e^{-i\frac{2\pi k}{L}s}\right)$$

More general (singular) solutions: measure on the space of curves : $\mu(\vec{F}(s))$ and $f_5 = 1 + \frac{Q_5}{L}\int \mathcal{D}\vec{F}(s)\,\mu(\vec{F}(s)) \int_0^L \frac{ds}{|\vec{x}-\vec{F}(s)|^2}$

dual CFT representation?

$\frac{1}{2}$-BPS states in CFT on $(M_4^N)/S_N$

→ given by $H^*((M_4)^N/S_N)$

thm $\sum p^N \dim H^*((M_4)^N/S_N) = \prod_{k=1}^{\infty} \frac{(1+p^k)^{b_1+b_3}}{(1-p^k)^{1+b_2+b_4}}$

where $b_i = \dim H^i(M_4)$

⇒ looks like b_1+b_3 fermions & $1+b_2+b_4$ bosons

[to see this for $M_4 = K3$ can use type II – heterotic duality; BPS states of D-branes map to perturbative heterotic states; in heterotic on T^4 there are 20 left-moving scalars and 4 right-moving ones and these are the above 24 bosons ($1+b_2+b_4 = 24$ for K3)]

⇒ $\frac{1}{2}$-BPS states in the CFT $\xleftrightarrow{1-1}$ Hilbert space obtained by quantizing phase space of classical solutions of gravity.

– density matrices ⟷ phase space measure on the space of loops → gravitational solution

– coherent states ⟷ classical (smooth) geometries

– ensemble average ⟷ $M=0$ BTZ.

Many of the results for $N=4$ SYM vs LLM carry over.
Almost all states look identical etc.

<u>Somewhat richer set of possibilities:</u>

Solutions correspond to four bosons : b_1, b_2, a^+, a^-
that carry a $U(1)$ quantum number : $0 \quad 0 \quad +1 \quad -1$

① Small rotating black hole :

$$\rho \sim e^{-\beta L_0 - \mu J} \qquad L_0 = \sum_{k>0} k (b_1)_{-k} (b_1)_k + \text{other bosons}$$

$$J = \sum_{k>0} a^+_{-k} a^+_k - \sum_{k>0} a^-_{-k} a^-_k$$

typical state : $\underbrace{(\text{thermal piece})}_{N-J} . (a^+_{-1})^J |0\rangle$

↳ "Bose-Einstein condensate"

$$S \sim \sqrt{N-J} \qquad (N = Q_1 Q_5)$$

② Small black ring

$$S \sim \sqrt{N - JD} \qquad D = \text{"dipole moment"}$$

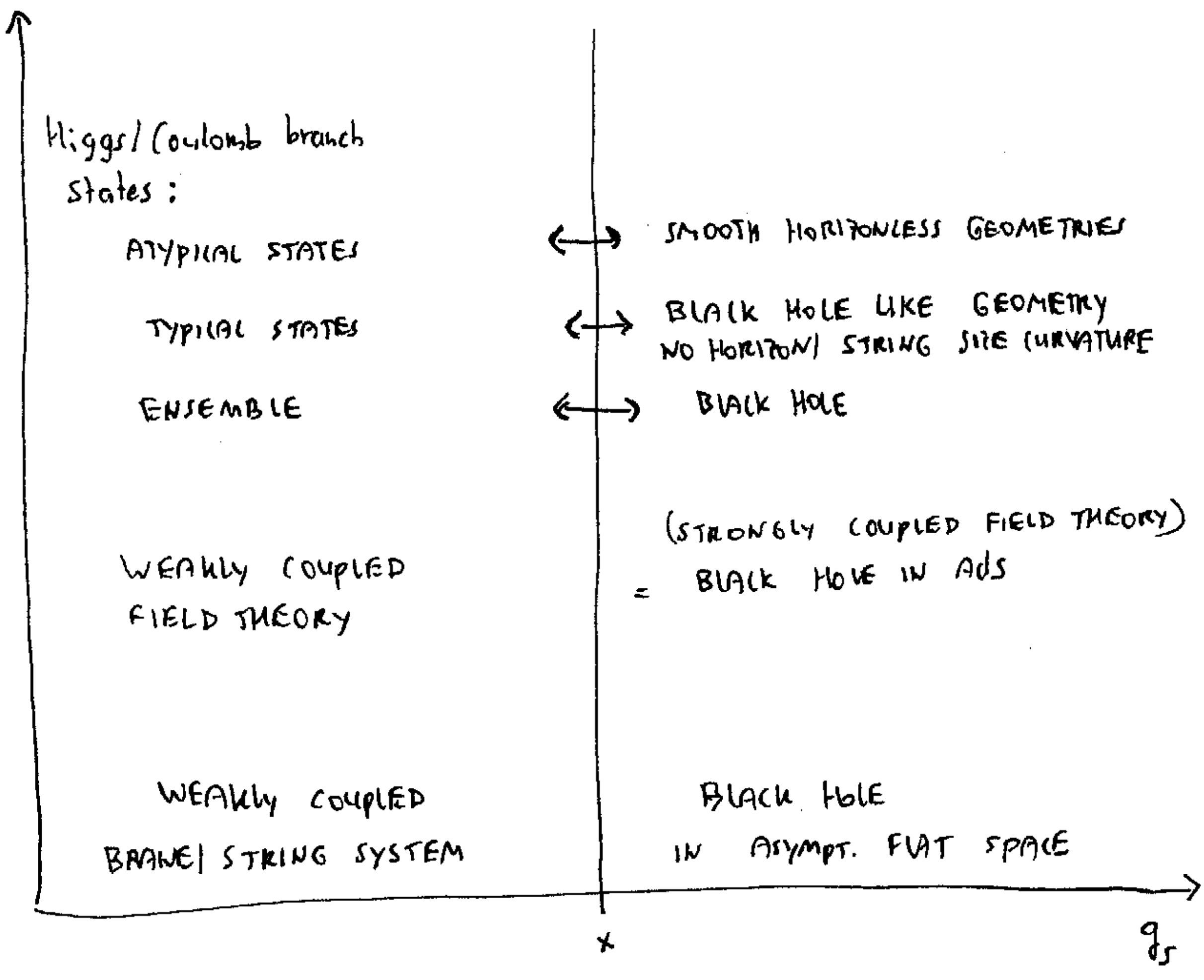
typical state $\underbrace{(\text{thermal piece})}_{N-JD} . (a^+_{-D})^J |0\rangle$

$$\rho \sim e^{-\beta L_0 - \mu J - \nu D} \qquad D = \sum_{k>0} \frac{1}{k} (a^+_{-k} a^+_k) + \dots$$

D = "dipole operator"
Is not a conserved charge
Not clear how to extend to the interacting theory
(Similar to giant graviton number operator in $N=4$ SYM)

Many more complicated black objects exist in AdS_3
- concentric black rings
- black hole bound states
- etc

It is an open problem to understand & classify all these from the CFT point of view.
They somehow all combine to build up the CFT partition function, & connect it to OSV etc.
Many solutions can be constructed using classification of supersymmetric solutions of 5d supergravity



- does not mean that we cannot find a large number of supergravity solutions, perhaps even to explain a fraction of the entropy.

Comments:
- do not know whether states are dual to geometries are Σ(geometries) — single geometry seems reasonable
- need that sizes of bound states <u>grow</u> as g_s is increased. Amazingly, this indeed happens
- geometries start to deviate from black hole background at location of horizon? Not completely clear (does <u>not</u> seem to work for BTZ)
- how can there be a large fluctuation in the metric at the horizon if large black holes have such a good semiclassical description?
- what happens if you fall in? What are eigenstates of operators that the observer uses to measure stuff? Does the wave function collapse? (would be weird — Brownian motion through geometries? cf gas..)
- probably, as you fall in, you notice nothing, but then your wavefunction starts to decohere and to thermalize, you lose consciousness and become part of the black hole wave function.

☒

Some references (incomplete! ignoring 99.9% of the important papers about black holes)

* black hole review: e.g. A. Peet, TASI lectures on Black holes in String Theory, hep-th/0008241

* AdS/CFT including AdS_3/CFT_2

  : O. Aharony, S. Gubser, J. Maldacena, H. Ooguri, Y. Oz hep-th/9905111

* Review of the D1-D5-P system: J. David, G. Mandal and S. Wadia hep-th/0203048

* Correspondence Principle: - Horowitz, Polchinski, hep-th/9612146

* Map between ~~BPS~~ states/geometry & typicality of states in N=4 SYM:

  - V. Balasubramanian, J. de Boer, V. Jejjala, J. Simon hep-th/0508023

* Map between $\frac{1}{2}$-BPS states & geometries, dipole operator for AdS_3/CFT_2:

  - F. Alday, J. de Boer, I. Messamah, hep-th/0511246, hep-th/0607222

* A more recent AdS_3/CFT_2 review:

  - P. Kraus / hep-th/0609074

Detailed discussions of the microstate $\leftrightarrow$ geometry map
& associated issues, see e.g.

— S. Mathur, hep-th/0502050
— I. Bena, N. Warner, hep-th/0701216

The last paper also discusses many 5d supergravity solutions.
More about 4d/5d solutions, black rings, etc can e.g. be
found in

— R. Emparan, H. Reall, hep-th/0608012
— F. Denef, G. Moore, hep-th/0702146

Obviously, there are ∞-many papers with improvements,
more details, more discussions, etc., but this should
help you get going.