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**Alpha Spectrometry:
Exercise 2**

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Alpha Spectrometry: Exercise 2

Task

The task is to calculate the activity concentration (including uncertainty) of ^{210}Po in a sample, calculated to the date of chemical separation of Po from the sample matrix.

Basic data (note: ks = kiloseconds)

Item	Symbol	Quantity	Unit
Tracer		Po-209	
Tracer calibration date		5 July 2005	
Po separation date		5 July 2007	
Count date		9 Aug 2007	
Count time	t_C	212.048	ks
Tracer mass	m_T	9.322	g
Tracer activity concentration on calibration date	$C_{T,T}$	7.8578 ± 0.0880	mBq/g
Sample mass	m_s	2.365	g
Counts Po-209 region	n_{209}	3251	Counts
Counts Po-210 region	n_{210}	4989	Counts
Background + blank Po-209 region	$G_{209,B}$	0.033 ± 0.012	Counts/ks
Background + blank Po-210 region	$G_{210,B}$	0.083 ± 0.019	Counts/ks
Detection efficiency	ϵ	0.2157	

Notes

Number of days from calibration date to count date = $t_T = 765$ days

Number of days from separation date to count date = $t_S = 35$ days

λ_T = decay probability constant for $^{209}\text{Po} = 1.8606 \times 10^{-5} \text{ days}^{-1}$

λ_P = decay probability constant for $^{210}\text{Po} = 5.0092 \times 10^{-3} \text{ days}^{-1}$

For simplicity, the following are ignored in this exercise:

- Uncertainty on the Po-209 and Po-210 decay probability constants
- Detection efficiency and gravimetric weighing uncertainties
- Tailing of Po-210 peak into Po-209 peak area
- Emission probabilities of Po-209 and Po-210 (i.e. assume both Po-209 peaks are included in the counts measured), and decay of Po-209 and Po-210 over the course of the count time

Step 1: Calculate the relevant tracer activity to the count date

(a) Adjust tracer activity concentration to the count date

$$C_{T,C} = C_{T,T} \times e^{-\lambda_T t_r} =$$

(b) Calculate activity Po-209 added (adjusted to count date)

$$A_{T,C} = m_T \times C_{T,C} =$$

Step 2: Calculate count rates for Po-209 and Po-210 peaks

(a) Calculate gross count rates

$$G_{209} = \frac{n_{209}}{t_C} =$$

$$G_{210} = \frac{n_{210}}{t_C} =$$

(b) Calculate net count rates (i.e. minus background+blank)

$$N_{209} = G_{209} - G_{209,B} =$$

$$N_{210} = G_{210} - G_{210,B} =$$

Step 3: Calculate activity concentration of Po-210 in sample

(a) Basic calculation

$$C_{P,C} = \frac{N_{210}}{N_{209}} \times \frac{A_{T,C}}{m_s} =$$

(b) Adjust result to Po separation date

$$C_{P,S} = \frac{C_{P,C}}{e^{-\lambda_P t_S}} =$$

Step 4: Calculate Po tracer recovery (optional)

$$R_T = \frac{N_{209}}{\varepsilon \times A_{T,C}} \times 100 =$$

Step 1: Calculate the relevant tracer activity to the count date

(a) Adjust tracer activity concentration to the count date

$$\begin{aligned}C_{T,C} &= C_{T,I} \times e^{-\lambda_T t_r} \\ &= (7.8578 \pm 0.088) \times \exp(-1.8606 \times 10^{-5} \times 765) \\ &= (7.7467 \pm 0.0868) \text{mBq/g}\end{aligned}$$

(b) Calculate activity Po-209 added (adjusted to count date)

$$\begin{aligned}A_{T,C} &= m_T \times C_{T,C} \\ &= 9.322 \times (7.7467 \pm 0.0868) \\ &= (72.215 \pm 0.809) \text{mBq}\end{aligned}$$

Step 2: Calculate count rates for Po-209 and Po-210 peaks

(a) Calculate gross count rates

$$\begin{aligned}G_{209} &= \frac{n_{209}}{t_c} \\ &= \frac{3251 \pm \sqrt{3251}}{212.048} \\ &= (15.331 \pm 0.269)\text{counts/ks}\end{aligned}$$

$$\begin{aligned}G_{210} &= \frac{n_{210}}{t_c} \\ &= \frac{4989 \pm \sqrt{4989}}{212.048} \\ &= (23.528 \pm 0.333)\text{counts/ks}\end{aligned}$$

(b) Calculate net count rates (i.e. minus background+blank)

$$\begin{aligned}N_{209} &= G_{209} - G_{209,B} \\ &= (15.331 \pm 0.269) - (0.033 \pm 0.012) \\ &= 15.298 \pm \sqrt{0.269^2 + 0.012^2} \\ &= (15.298 \pm 0.269)\text{counts/ks}\end{aligned}$$

$$\begin{aligned}N_{210} &= G_{210} - G_{210,B} \\ &= (23.528 \pm 0.333) - (0.083 \pm 0.019) \\ &= 23.445 \pm \sqrt{0.333^2 + 0.019^2} \\ &= (23.445 \pm 0.334)\text{counts/ks}\end{aligned}$$

Step 3: Calculate activity concentration of Po-210 in sample

(a) Basic calculation

$$\begin{aligned} C_{P,C} &= \frac{N_{210}}{N_{209}} \times \frac{A_{T,C}}{m_s} \\ &= \frac{23.445}{15.298} \times \frac{72.215}{2.365} = 46.795 \text{mBq/g} = 46.795 \text{Bq/kg} \end{aligned}$$

$$\begin{aligned} \mu(C_{P,C}) &= C_{P,C} \times \sqrt{\left(\frac{\mu(N_{210})}{N_{210}}\right)^2 + \left(\frac{\mu(N_{209})}{N_{209}}\right)^2 + \left(\frac{\mu(A_{T,C})}{A_{T,C}}\right)^2} \\ &= 46.795 \times \sqrt{\left(\frac{0.334}{23.445}\right)^2 + \left(\frac{0.269}{15.298}\right)^2 + \left(\frac{0.809}{72.215}\right)^2} = 1.181 \text{Bq/kg} \end{aligned}$$

Therefore: $C_{P,C} = 46.795 \pm 1.181 \text{ Bq/kg}$

(b) Adjust result to Po separation date

$$\begin{aligned} C_{P,S} &= \frac{C_{P,C}}{e^{-\lambda_P t_S}} \\ &= \frac{46.795 \pm 1.181}{\exp(-5.0092 \times 10^{-3} \times 35)} = 55.76 \pm 1.41 \text{Bq/kg} \end{aligned}$$

Step 4: Calculate Po tracer recovery (optional)

$$\begin{aligned} R_T &= \frac{N_{209}}{\varepsilon \times A_{T,C}} \times 100 \\ &= \frac{15.298}{0.2157 \times 72.215} \times 100 = 98.2\% \end{aligned}$$

$$\begin{aligned} \mu(R_T) &= R_T \times \sqrt{\left(\frac{\mu(A_{T,C})}{A_{T,C}}\right)^2 + \left(\frac{\mu(N_{209})}{N_{209}}\right)^2} \\ &= 98.2 \times \sqrt{\left(\frac{0.809}{72.215}\right)^2 + \left(\frac{0.269}{15.298}\right)^2} = 2.0\% \end{aligned}$$

Therefore: $R_T = 98.2 \pm 2.0 \%$