The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

ICTP/SISSA
Joint Colloquium in Mathematics

Announcement

Tuesday, 24 April 2007, at 15.05 hrs.

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Kahane polynomials and their derandomization
(Joint work with J. Bourgain)

Abstract:

In 1957 Erdös studied trigonometric polynomials with all coefficients of absolute value 1 (called unimodular polynomials) and was led to the conjecture that the the maximum modulus of a unimodular polynomial of degree n is at least $(1+c)\sqrt{n}$ for some positive absolute constant c. This conjecture was disproved by Littlewood in 1966 and, on the basis of numerical evidence, Littlewood conjectured that there are unimodular polynomials that deviate by $o(\sqrt{n})$ from its mean-square value $\sqrt{(n+1)}$. The existence of such polynomials, of any given degree, was proved by Kahane in 1980 using probabilistic methods, obtaining a remainder term of $O(n^{(1/2-1/17)}\sqrt{(\log n)})$.

In this lecture a new construction will be given of these polynomials with the improved remainder term $O(n^{(1/2-1/9+epsilon)})$, first using probabilistic methods, and then with an explicit construction. The explicit construction makes use of Deligne's Riemann hypothesis for L-functions over varieties in positive characteristic associated to mixed exponential sums in arbitrarily many variables, as well as of sieves and recent results about gaps between squarefree numbers.

Venue: ICTP Main Lecture Hall, Main Building, entrance level