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lecture notes

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The Standard Model of the Universe: from the Theory of Inflation Confronted to Observations to Today's Dark Energy

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The History of the Universe

It is a history of EXPANSION and cooling down.

EXPANSION: the space itself expands with the time.

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2 \quad , \quad a(t) =$ scale factor.

Homogeneous, isotropic and spatially flat geometry.

Cooling: temperature decreases as $1/a(t)$: $T(t) \sim 1/a(t)$.

The Universe underwent a succesion of phase transitions towards the less symmetric phases.

Wavelenghts redshift as $a(t) \quad : \quad \lambda(t) = a(t)\, \frac{\lambda(t_0)}{a(t_0)}$

Redshift $z: \quad z + 1 = \frac{a(\text{today})}{a(t)}$

The deeper you go in the past, the larger is the redshift and the smaller is $a(t)$.

Standard Cosmological Model: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant

Explains the Observations:

- 3 years WMAP data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations
- Supernova Luminosity/Distance Relations (Acceleration of the Universe expansion)
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2a}{dt^2} > 0$.
During inflation the universe expands by at least sixty efolds: $e^{60} \simeq 10^{26}$. Inflation lasts $\simeq 10^{-34}$ sec and ends by $z \sim 10^{28}$ followed by a radiation dominated era.
Energy scale when inflation starts $\sim 10^{16}$ GeV.
This energy scale **coincides** with the GUT scale ($\Longleftarrow$ CMB anisotropies).

Matter can be effectively described during inflation by an Scalar Field $\phi(t, \boldsymbol{x})$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\, a^2(t)} - V(\phi) \right]$.

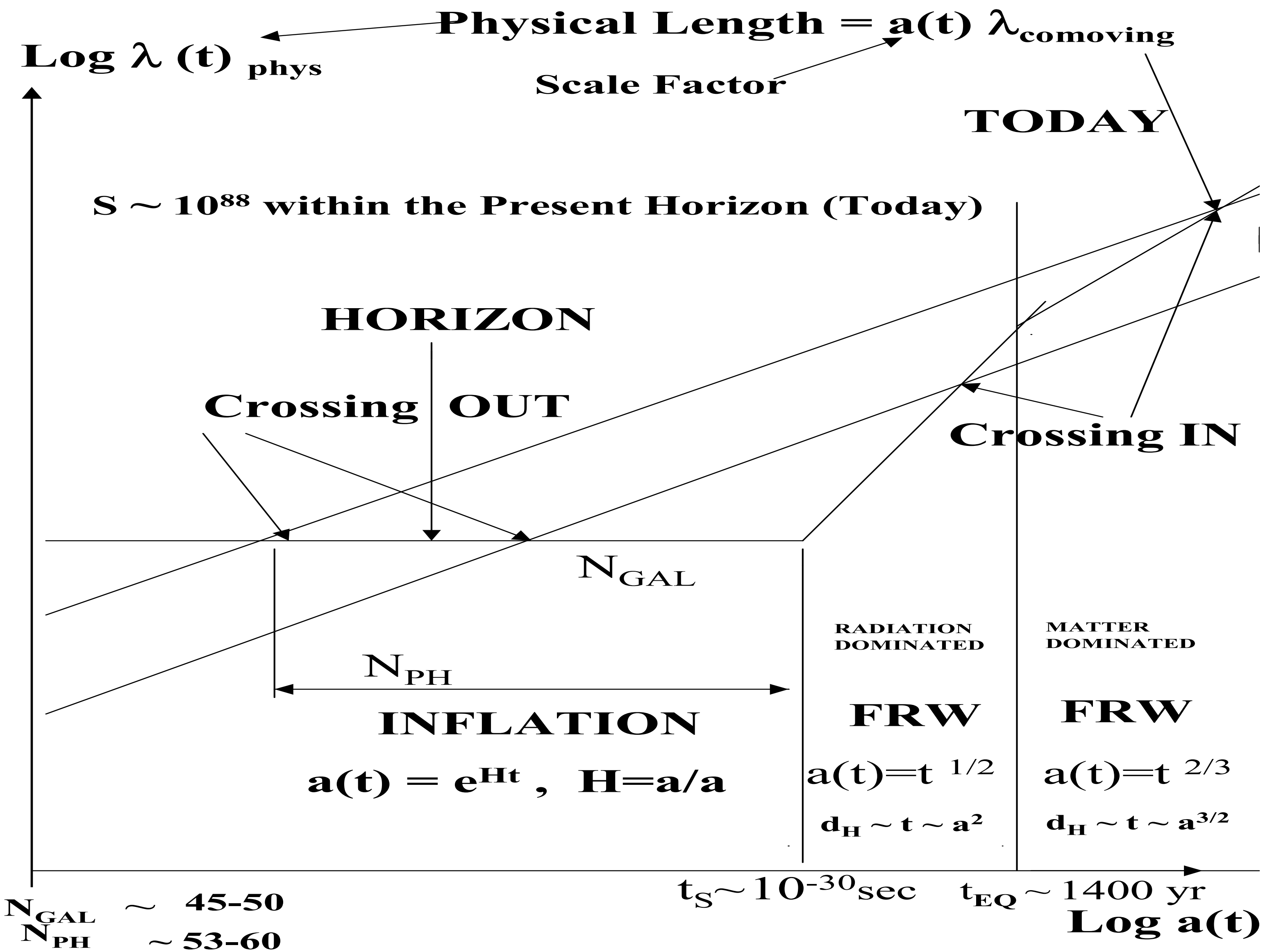
Friedmann eq.: $H^2(t) = \frac{1}{3\, M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$

Physics during Inflation

- Out of equilibrium evolution in a fastly expanding geometry. Vacuum energy DOMINATES
- Extremely dense matter at the scale of $\lesssim 10^{16}$ GeV.
- Explosive particle production due to spinodal or parametric instabilities.
- Quantum non-linear phenomena eventually shut-off the instabilities and stop inflation. Radiation dominated era follows.
- Huge redshift classicalizes the dynamics: an assembly of (superhorizon) fluctuations behave as the classical and homogeneous inflaton field.

D. Boyanovsky, H. J. de Vega, in Astrofundamental Physics, NATO ASI series vol. 562, 2000, Lectures at the Chalonge School, astro-ph/0006446.

Fluctuations Out and In the Horizon.



*Scales CROSS OUT the Horizon and Later COME BACK : UNIQUE for INFLATION

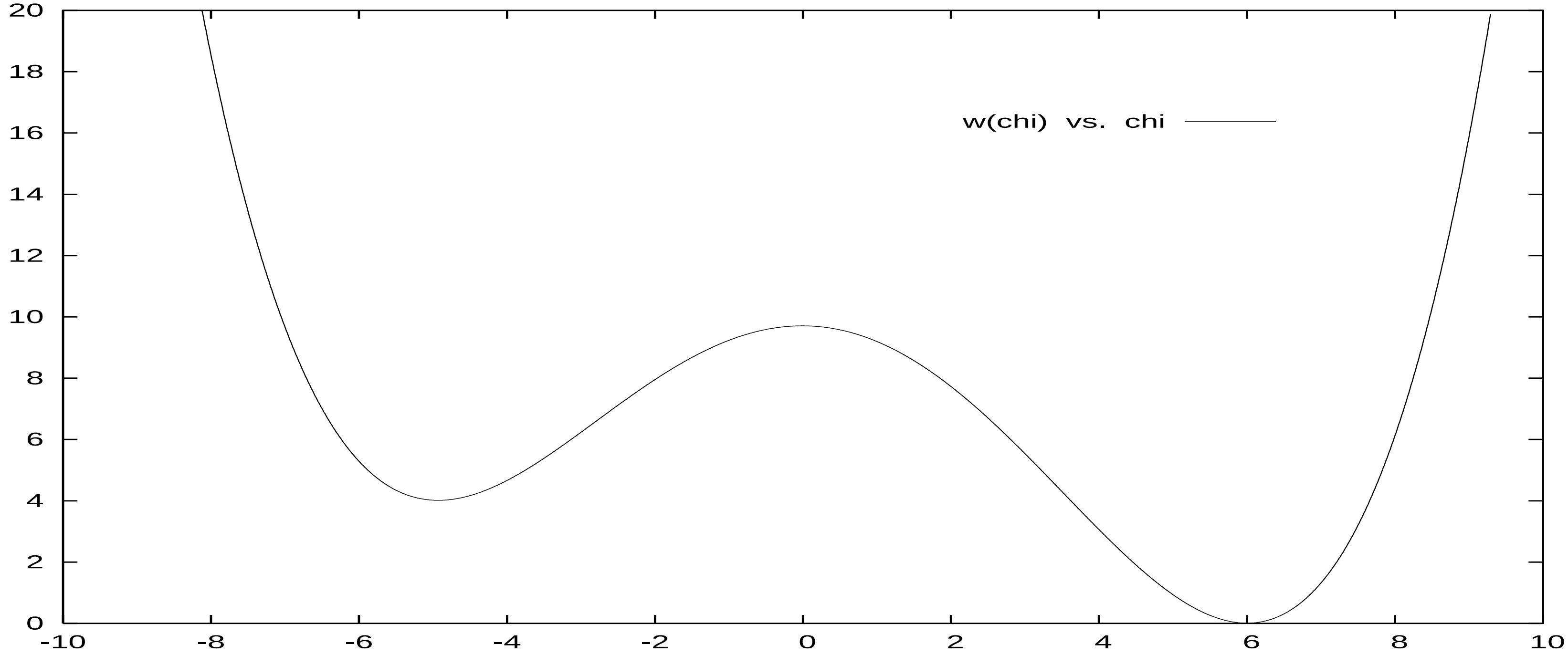
**LARGER SCALES CROSS OUT FIRST and CROSS BACK LATER

What is the Inflaton?

It is an effective field.
It can describe a fermion-antifermion pair condensate:
$\phi =< \bar{\psi}\psi >$, ψ = GUT fermion,
Such condensate can dominate the expectation value of the hamiltonian and therefore govern the cosmological expansion. [Recall that $< \psi >= 0$].
Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q \; , \; \sigma \simeq \bar{q}q$.

Slow Roll Inflaton Models



$V(\mathrm{Min}) = V'(\mathrm{Min}) = 0$: inflation ends after a finite number of efolds. Universal form of the slow-roll inflaton potential:

$$V(\phi) = N\ M^4\ w\left(\frac{\phi}{\sqrt{N}\ M_{Pl}}\right)$$

$N \sim 50$ number of efolds since horizon exit till end of inflation. $M =$ energy scale of inflation.

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N}\; M_{Pl}} \quad , \quad \tau = \frac{m\; t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m\; \sqrt{N}} \quad , \quad \left(m \equiv \frac{M^2}{M_{Pl}} \right)$$

slow inflaton, slow time, slow Hubble.
χ and $w(\chi)$ are of order one.
Evolution Equations:

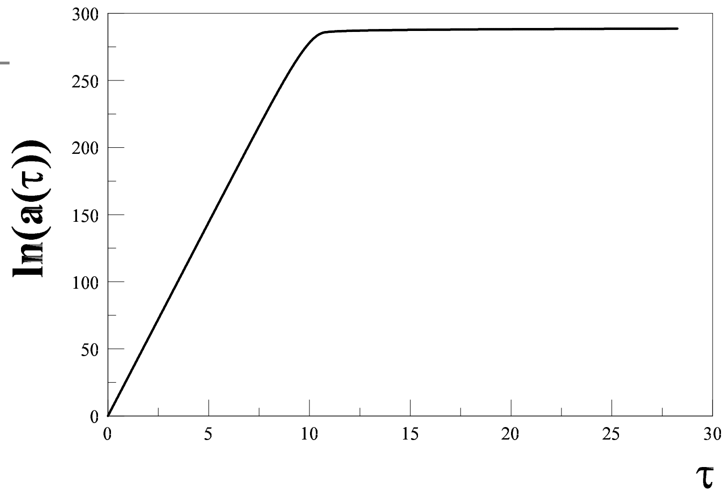
$$\mathcal{H}^2(\tau) = \frac{1}{3} \left[\frac{1}{2\; N} \left(\frac{d\chi}{d\tau} \right)^2 + w(\chi) \right] \quad ,$$

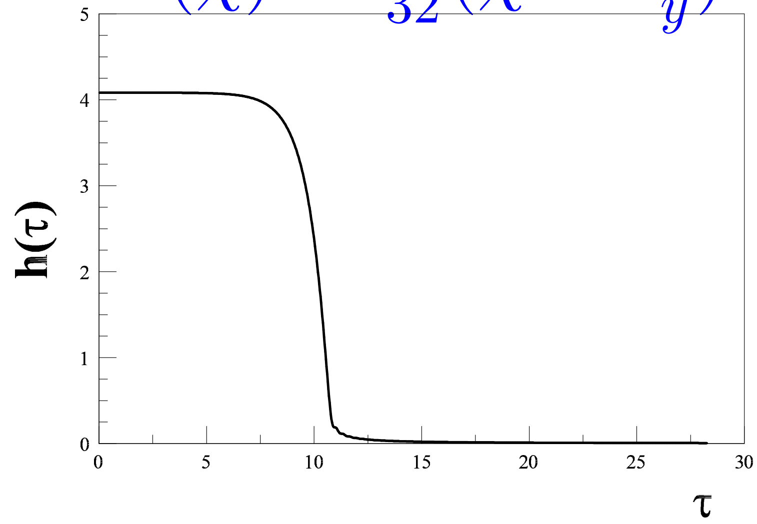
$$\frac{1}{N}\; \frac{d^2\chi}{d\tau^2} + 3\, \mathcal{H}\; \frac{d\chi}{d\tau} + w'(\chi) = 0 \quad . \tag{1}$$

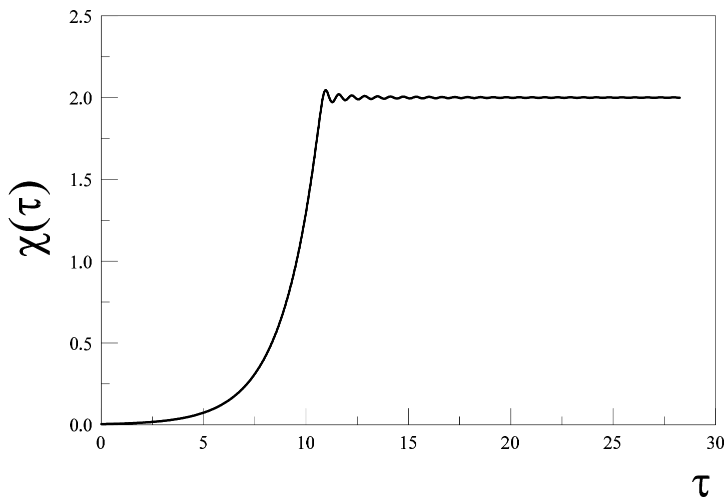
$1/N$ terms: corrections to slow-roll

Higher orders in slow-roll are obtained systematically by expanding the solutions in $1/N$.

Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$

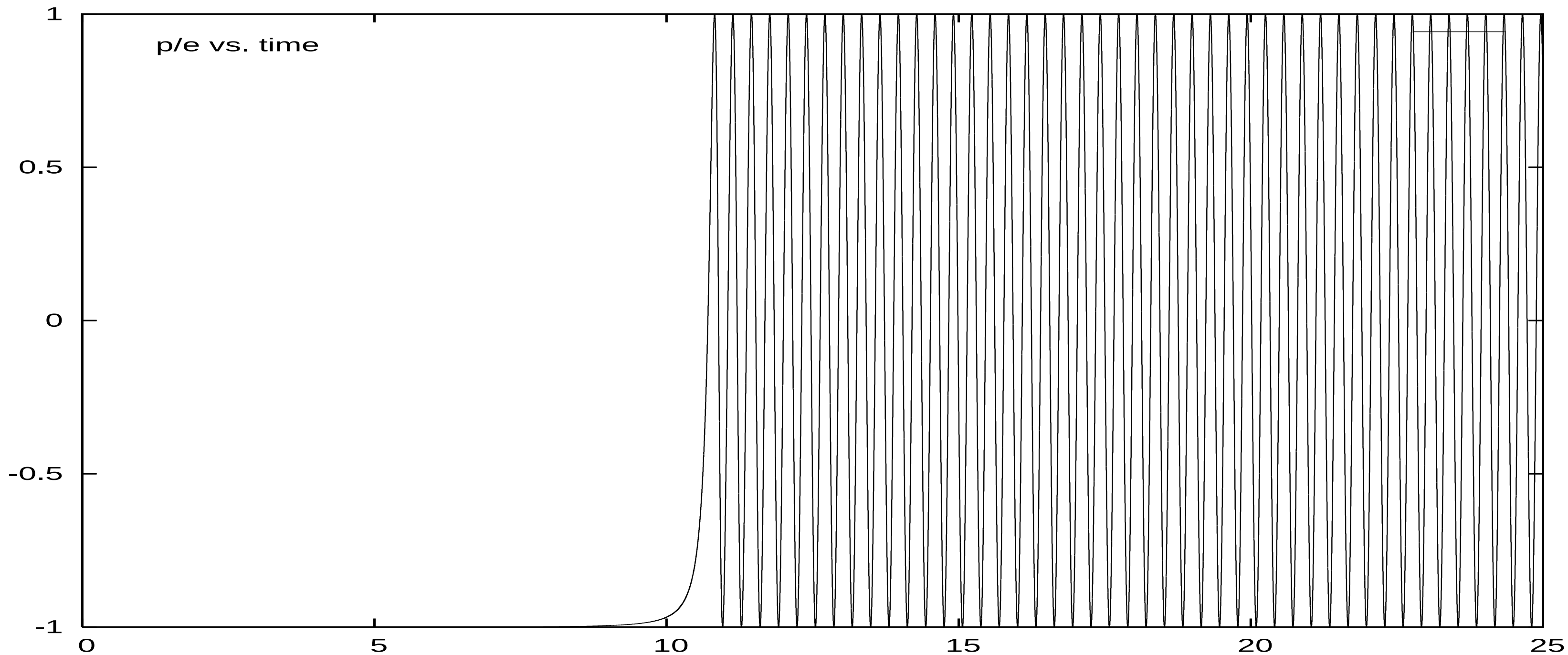






The vacuum energy transforms into particles and inflation is followed in this simplified approach by a matter dominated stage.

Equation of State: pressure/energy density



The equation of state is $p/e = -1$ during inflation.

p/e strongly oscillates between $+1$ and -1 during the matter dominated stage. We have in average $< p/e >= 0$.

We have here neglected spatial gradient terms

$$\frac{(\nabla\phi)^2}{2\ a^2(t)}$$

since $a(t)$ grows exponentially during inflation.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\ ad}^{(S)}|^2 \ k^{n_s-1}$.
[Gauge Invariant Curvature Perturbations.]
To dominant order in slow-roll:

$$|\Delta_{k\ ad}^{(S)}|^2 = \frac{N^2}{12\,\pi^2} \left(\frac{M}{M_{Pl}}\right)^4 \frac{w^3(\chi)}{w'^2(\chi)} \ .$$

Hence, for all slow-roll inflation models:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}} \left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP result $|\Delta_{k\ ad}^{(S)}| = (0.467 \pm 0.023) \times 10^{-4}$
determines the scale of inflation M

$$\left(\frac{M}{M_{Pl}}\right)^2 = 1.02 \times 10^{-5} \Longrightarrow M = 0.77 \times 10^{16} \text{ GeV}$$

The inflation energy scale turns to be the grand unification energy scale !!

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial gravitons.

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)} \quad ,$$
$$r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 \quad . \tag{2}$$

χ is the inflaton field at horizon exit.

$n_s - 1$ and r are always of order $1/N \sim 0.02$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

We choose a polynomial for $w(\chi)$. A quartic $w(\chi)$ is renormalizable. Higher order polynomials are acceptable since inflation it is an effective theory.

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3\,\chi^3 + G_4\,\chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N\,M^4\,w\left(\frac{\phi}{\sqrt{N}\,M_{Pl}}\right) = V_o \pm \frac{m^2}{2}\,\phi^2 + g\,\phi^3 + \lambda\,\phi^4\,.$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}}\left(\frac{M}{M_{Pl}}\right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N}\left(\frac{M}{M_{Pl}}\right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 50\,.$$

- Small couplings arise naturally as ratio of two energy scales: inflation and Planck.
- The inflaton is a light particle:

  $m \simeq 0.003\;M \quad , \quad m = 2.5 \times 10^{13}\mathrm{GeV}$

The number of efolds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} \, d\chi$. We choose then $N = N[\chi]$.

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

produces inflation with $0 < \sqrt{y}\,\chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

This is small field inflation.

From the above integral: $\quad y = z - 1 - \log z$

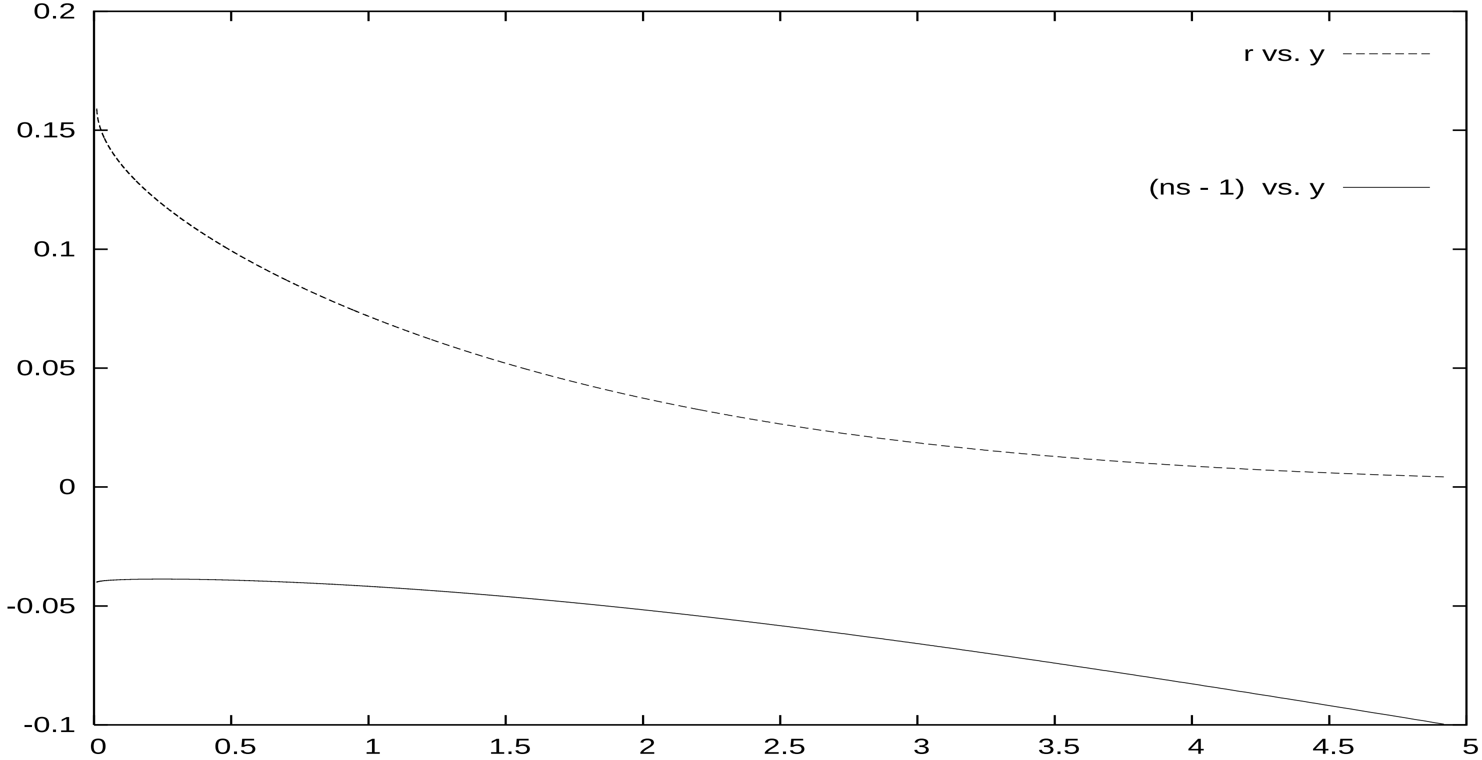
where $z \equiv y\,\chi^2/8$ and we have $0 < y < \infty$ for $1 > z > 0$.

Spectral index n_s and the ratio r as functions of y:

$n_s = 1 - \frac{y}{N} \frac{3\,z+1}{(z-1)^2} \quad , \quad r = \frac{16\,y}{N} \frac{z}{(z-1)^2}$

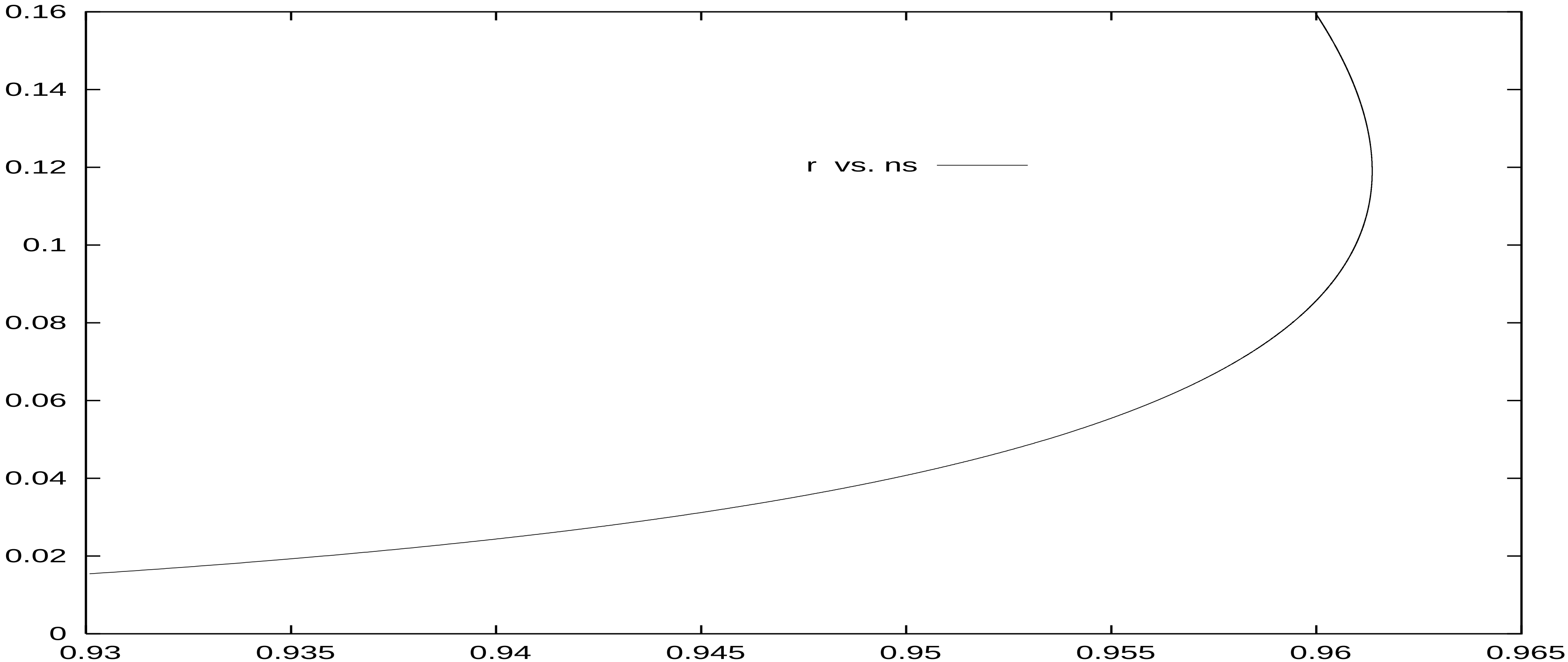
Binomial New Inflation: (y = coupling).

r decreases monotonically with y :
(strong coupling) $0 < r < \frac{8}{N} = 0.16$ (zero coupling).



n_s first grows with y, reaches a maximum value $n_{s,maximum} = 0.96139\ldots$ at $y = 0.2387\ldots$ and then n_s decreases monotonically with y.

Binomial New Inflation



$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a double valued function of n_s.

Trinomial Inflationary Models

- Trinomial Chaotic inflation:

$$w(\chi) = \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 \; .$$

- Trinomial New inflation:

$$w(\chi) = -\frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 + \frac{2}{y}\,F(h) \; .$$

h = asymmetry parameter. $w(\min) = w'(\min) = 0$,

$y =$ quartic coupling, $F(h) = \frac{8}{3}\,h^4 + 4\,h^2 + 1 + \frac{8}{3}\,|h|\,(h^2+1)^{\frac{3}{2}}$.

H. J. de Vega, N. G. Sanchez, Single Field Inflation models allowed and ruled out by the three years WMAP data. Phys. Rev. D 74, 063519 (2006), astro-ph/0604136.

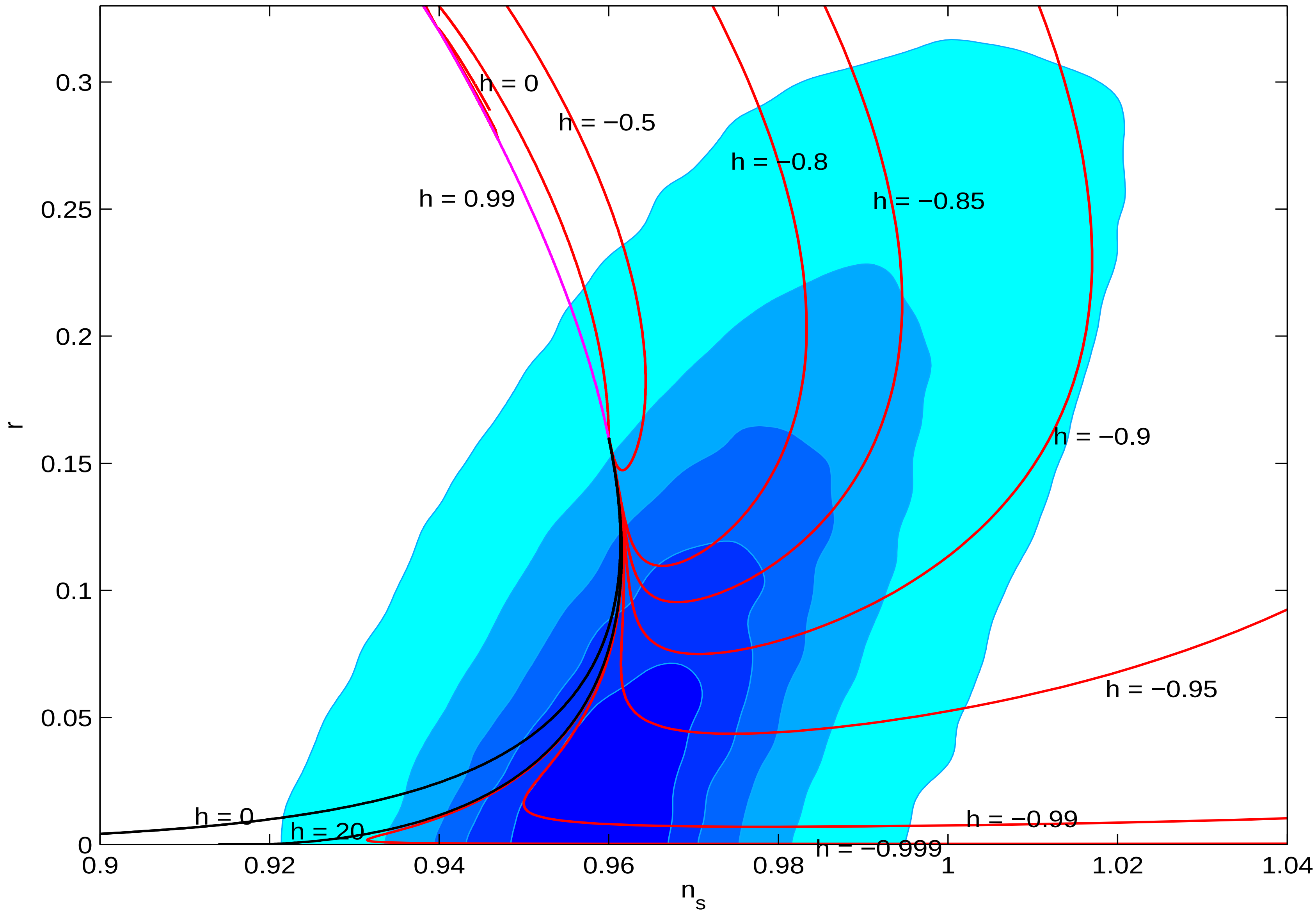
Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

We found n_s and r and the couplings y and h by MCMC.
NEW: We imposed as a hard constraint that r and n_s are given by the trinomial potential.
Our analysis differs in this crucial aspect from previous MCMC studies of the WMAP data.

The color–filled areas correspond to $12\%,\ 27\%,\ 45\%,\ 68\%$ and 95% confidence levels according to the WMAP3 and Sloan data.
The color of the areas goes from the darker to the lighter for increasing CL.

MCMC Results for Trinomial New Inflation.



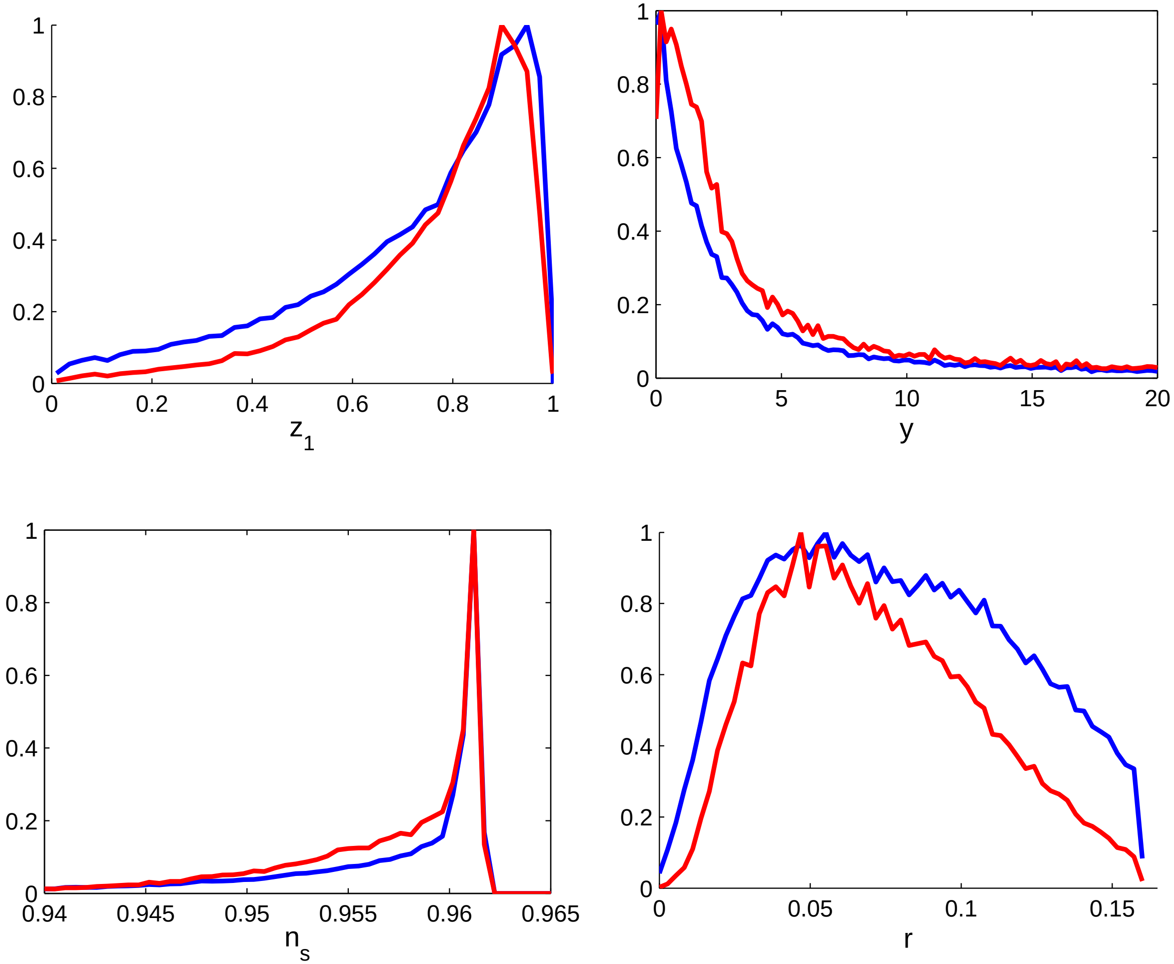
MCMC Results for Trinomial New Inflation.

Bounds: $r > 0.016 \; (95\% \; \mathrm{CL}) \quad , \quad r > 0.049 \; (68\% \; \mathrm{CL})$
Most probable values: $n_s \simeq 0.958 \quad , \quad r \simeq 0.054$.
The most probable trinomial potential for new inflation is symmetric and has a moderate nonlinearity with the quartic coupling $y \simeq 1.2\ldots$ and $h \simeq 0$.
The $\chi \to -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$.

$$w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$$

C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, astro-ph/0703417.

Probability Distributions. Trinomial New Inflation.

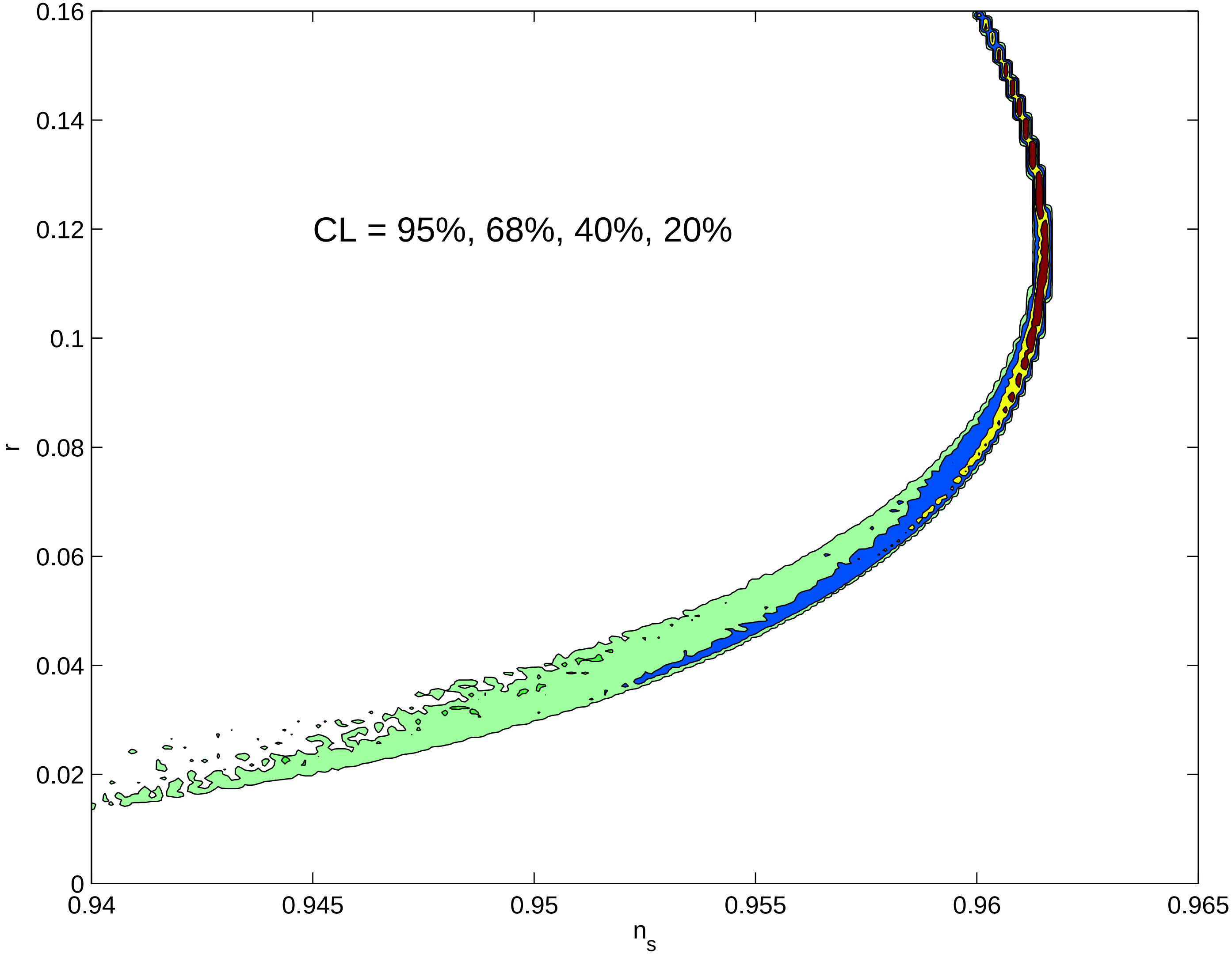


CL: $r > 0.016$ (95%) , $r > 0.049$ (68%)

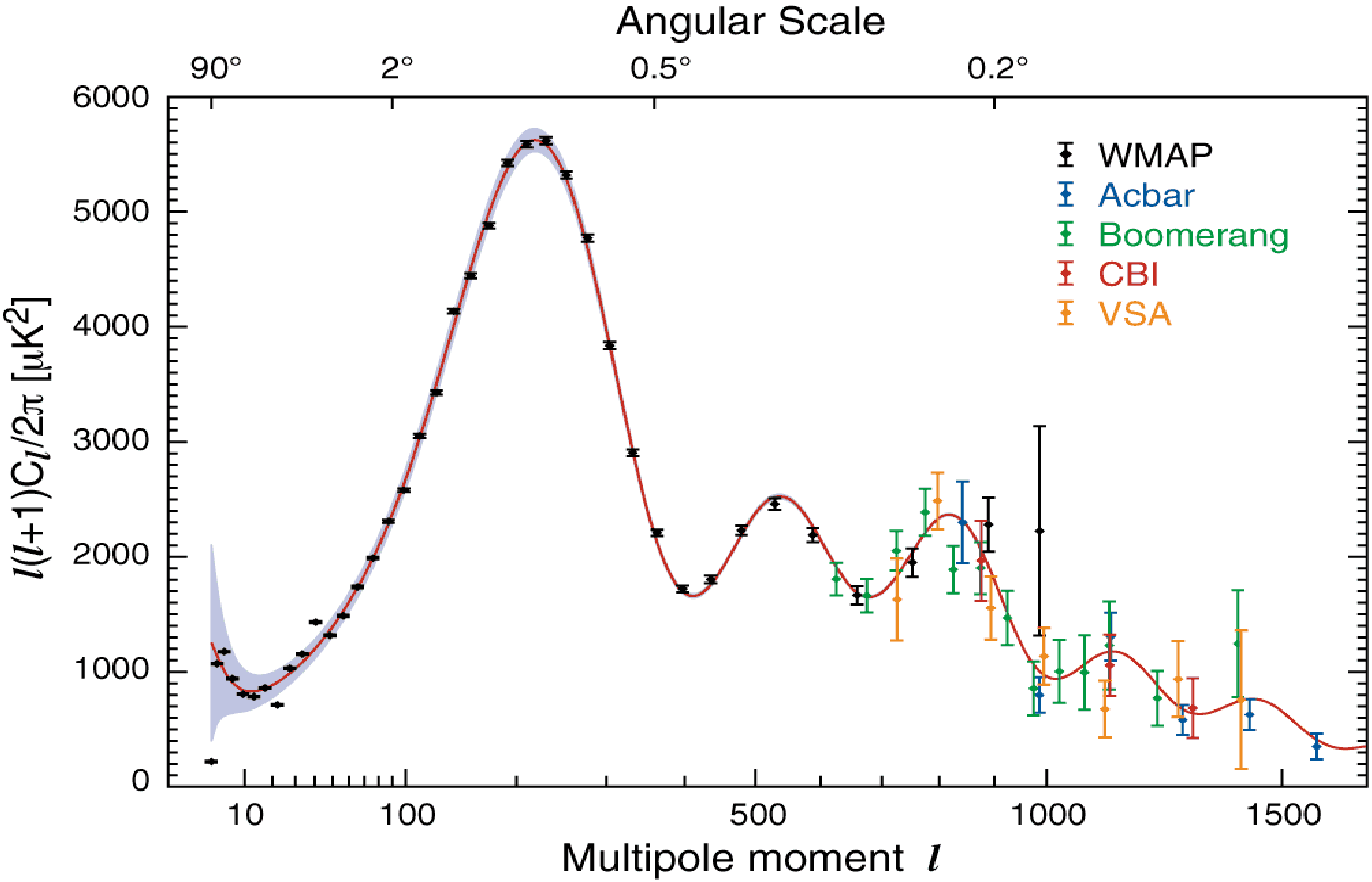
ML: $r = 0.054$, ML: $y = 1.2$,

$$z_1 = 1 - \frac{y}{8\left(|h|+\sqrt{h^2+1}\right)}\,\chi^2 \,.$$

r vs. n_s data within the Trinomial New Inflation Region.



WMAP 3 years data set plus other CMB data



Theory and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

Quadrupole Suppression and Fast Roll

Slow-roll inflation is generically preceded by a fast-roll stage where $\dot{\phi}^2 \sim V(\phi)$. Fast-Roll typically lasts 1 efold.

The slow-roll regime is an attractor with a large basin of attraction.

If the quadrupole modes ($\sim$ Hubble radius today) exited the horizon 1.5 efolds after the beginning of fast roll, then the quadrupole modes get suppresed $\sim 20\%$ in agreement with the observations.

$\Longrightarrow N_{total\ efolds} \simeq 60 + 1.5$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
CMB quadrupole suppression: I. Initial conditions of inflationary perturbations, II. The early fast roll stage. Phys. Rev. **D74**, 123006 and123007 (2006).

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they all meet at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the see-saw mechanism: $m_\nu \sim \frac{M^2_{\rm Fermi}}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least three independent ways.

Moreover, moduli potentials: $V_{moduli} = M^4_{\rm SUSY} \; v\left(\frac{\phi}{M_{Pl}}\right)$

ressemble inflation potentials provided $M_{\rm SUSY} \sim 10^{16}$ GeV.
First observation of SUSY in nature??

De Sitter Geometry and Scale Invariance

The De Sitter metric is scale invariant:

$$ds^2 = \frac{1}{(H\,\eta)^2}\left[(d\eta)^2 - (d\vec{x})^2\right] \; .$$

$\eta =$ conformal time.

But inflation only lasts for N efolds !

Corrections to scale invariance:

$|n_s - 1|$ as well as the ratio r are of order $\sim 1/N$

$n_s = 1$ and $r = 0$ correspond to a critical point.

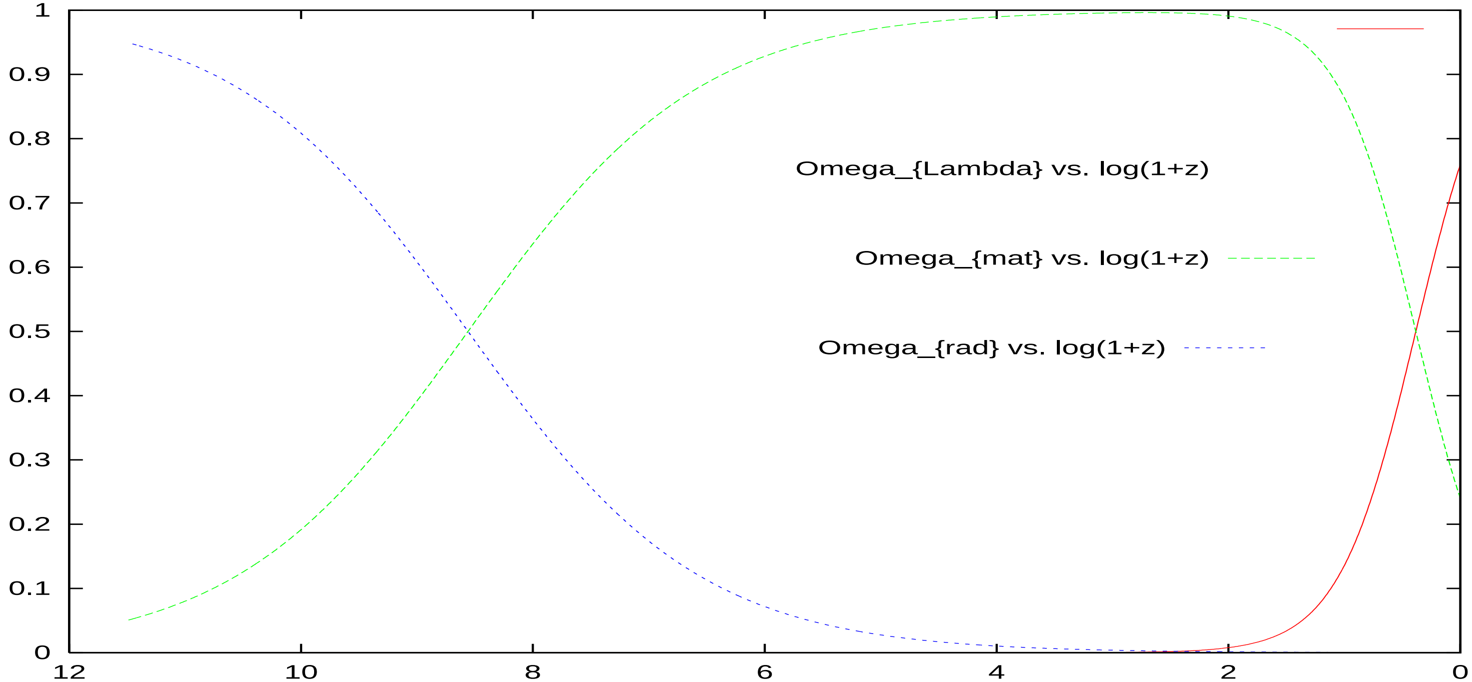
It is a gaussian fixed point around which the inflation model hovers in the renormalization group (RG) sense with an almost scale invariant spectrum during the slow roll stage.

The quartic coupling:

$$\lambda = \frac{G_4}{N}\left(\frac{M}{M_{Pl}}\right)^4 \quad , \qquad N = \log\frac{a(\text{inflation end})}{a(\text{horizon exit})}$$

runs like in four dimensional RG in flat euclidean space.

The Universe is made of radiation, matter and dark energy



Electro-Weak phase transition: $z \sim 10^{15}$, $T_{\rm EW} \sim 100$ GeV.

QCD phase transition (conf.): $z \sim 10^{12}$, $T_{\rm QCD} \sim 170$ MeV.

BBN: $z \sim 10^{9}$, $\quad \ln(1+z) \sim 21$, $\quad T \simeq 0.1$ MeV.

Rad-Mat equality: $z \simeq 3050$, $\ln(1+z) \simeq 8$, $T \simeq 0.7$ eV.

CMB last scattering: $z \simeq 1100$, $\ln(1+z) \simeq 7$, $T \simeq 0.25$ eV.

Mat-DE equality: $z \simeq 0.47$, $\ln(1+z) \simeq 0.38$, $T \simeq 0.345$ meV.

Today: $z = 0$, $\ln(1+z) = 0$, $T = 2.725$K $= 0.2348$ meV.

The QCD Phase Transition

Hubble scale then: $1/H \sim 10^{-5}$ sec
$\gg t_{QCD} \sim 1/[\alpha_s^2 \, T_{\rm QCD}] \sim 10^{-23}$ sec $\Longrightarrow$ Very fast!.
Hence, the transition happens in thermal equilibrium.

Hubble radius then $c/H \sim 10$ km. $\Longrightarrow$ 1 pc today.

Probably a first order transition: Bubbles of the confined phase appear in the quark-gluon plasma $\Longrightarrow$ hadronization.

Supercooling very short $\sim 10^{-3} \; 1/H$.
Bubble separation $1\,\mathrm{cm} \sim 10^{-6} \; c/H$.
Bubbles grow slowly and eventually fill the whole space.
Latent heat of the transition reheats the universe.

BBN happens at $200\,\mathrm{secs} \gg 10^{-5}$ sec: signatures ERASED.

D. Boyanovsky, H. J. de Vega, D. J. Schwarz, Ann. Rev. Nucl. Part. Sci. 56, 441(2006), hep-ph/0602002.

Little Bang vs. Big Bang

Similarities: baryon free, entropy is dominated by radiation, longitudinal expansion in RHIC similar to the Hubble expansion.

Differences: Cosmology:

Local Thermal Equilibrium: $1/H \sim 10^{-5}\,\mathrm{s} \gg t_{QCD} \sim 10^{-23}$ s,
Starting energy density $\gg$ QCD phase transition energy density ~ 1 GeV/fm^3 $\Longrightarrow$
weakly interacting QGP **was initially** present due to asymptotic freedom.

URHIC: expansion time scale $\sim 10^{-22}\,\mathrm{sec} \sim 10\, t_{QCD} \Longrightarrow$
non-equilibrium effects can be relevant.
Strongly interacting *liquid* initially present (‘color glass condensate’).

Primordial Magnetic Fields

Astrophysical observations show the presence of large scale magnetic fields $\sim \mu G$ correlated on scales up to ~ 1 Mpc (cluster of galaxies).

Origin?: Dynamo mechanisms amplify seed magnetic fields. Typical growth rates $\Gamma \sim$ Gyr^{-1} over time scales $\sim 10 - 12$ Gyr

Origin of Seeds?: Inflation and/or phase transitions.

If the electroweak and/or the chiral phase transitions occurred out of equilibrium they can be a significant source of primordial magnetic fields

Cosmic magnetic fields may be one of the few observational relics of primordial phase transitions.

D. Boyanovsky, H. J. de Vega, M. Simionato 'Large scale magnetogenesis from a non-equilibrium phase transition in the radiation dominated era', Phys.Rev.D67,123505(2003).

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark!
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \, \rho_c = (2.39 \text{ meV})^4 \, , \; 1 \text{ meV} = 10^{-3} \text{ eV}$.
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest neutrino ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions, familons, majorons .....
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological analogue to the Casimir effect in Minkowski with non-trivial boundaries.

Summary and Conclusions

- Inflation can be formulated as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV.
- Effective theory does work because: $H \ll M \ll M_{Pl}$. Inflaton mass small: $m \sim H/\sqrt{N}$. Infrared regime!
- The slow-roll approximation is a $1/N$ expansion ($N \sim 50$).
- MCMC analysis of WMAP+LSS data plus the Trinomial Inflation potential indicates a spontaneously symmetry breaking potential (new inflation): $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$.
- Lower Bounds: $r > 0.016 \; (95\% \; \text{CL}) \; , \; r > 0.049 \; (68\% \; \text{CL})$. The most probable values are $n_s \simeq 0.958 \; , \; r \simeq 0.054$ with a quartic coupling $y \simeq 1.2$.

Summary and Conclusions 2

Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-8}$.
Same order of magnitude as graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

Galaxy formation. Gigantic black-holes ($M \sim 10^9 \; M_\odot$) as galaxy nuclei, early star formation...

Nature of Dark Energy?

Nature of Dark Matter? 83% of the matter in the universe.

[DM = Stable particles beyond the standard model].

Light (> 1 meV) or Heavy (< 100 GeV) DM ??

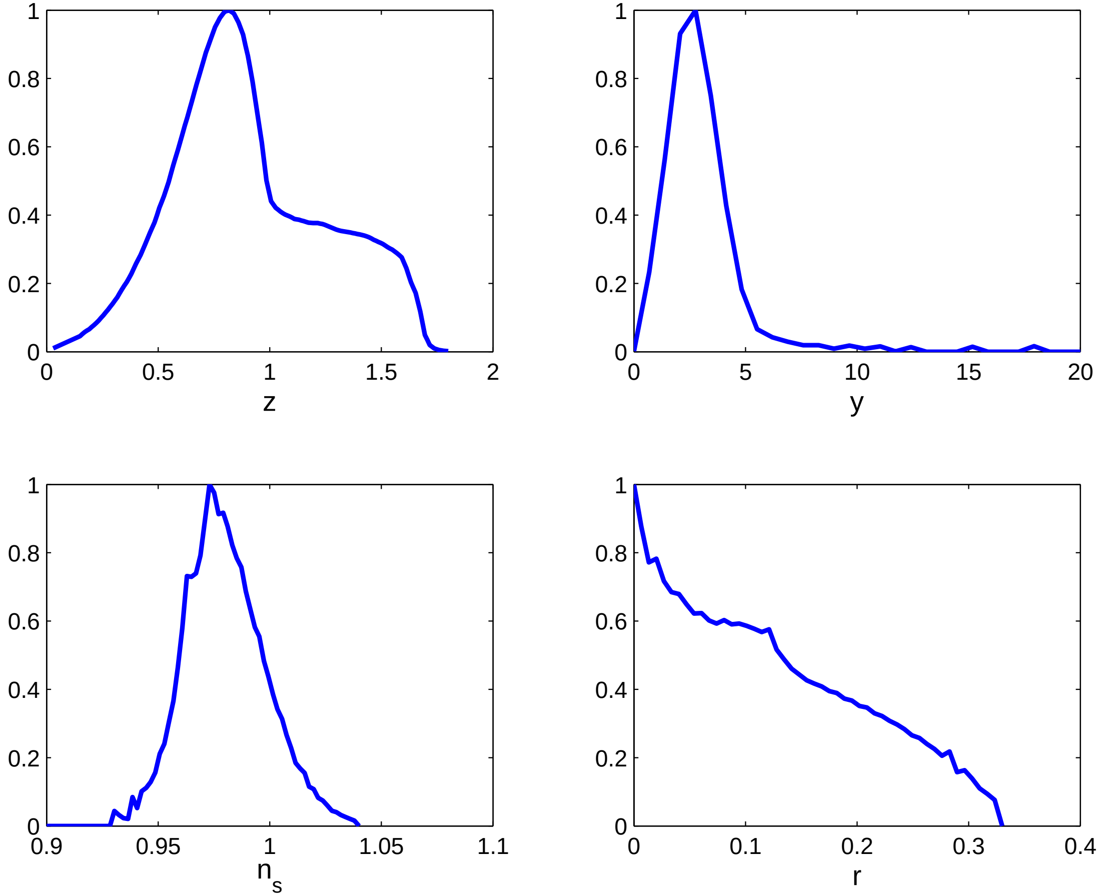
We need to learn the physics of light particles (< 1 MeV).

Neutrinos, sterile neutrinos? axions?

Some unknown light particle??

THANK YOU VERY MUCH

FOR YOUR ATTENTION!!



ML: $y = 4.2$, MV: $y = 3.1$