The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

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International Workshop on QCD at Cosmic Energies III

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Lecture Notes

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Open questions in perturbative QCD and their impact on air shower predictions

Ralph Engel *(Forschungszentrum Karlsruhe)*

Outline

Astroparticle physics and air showers

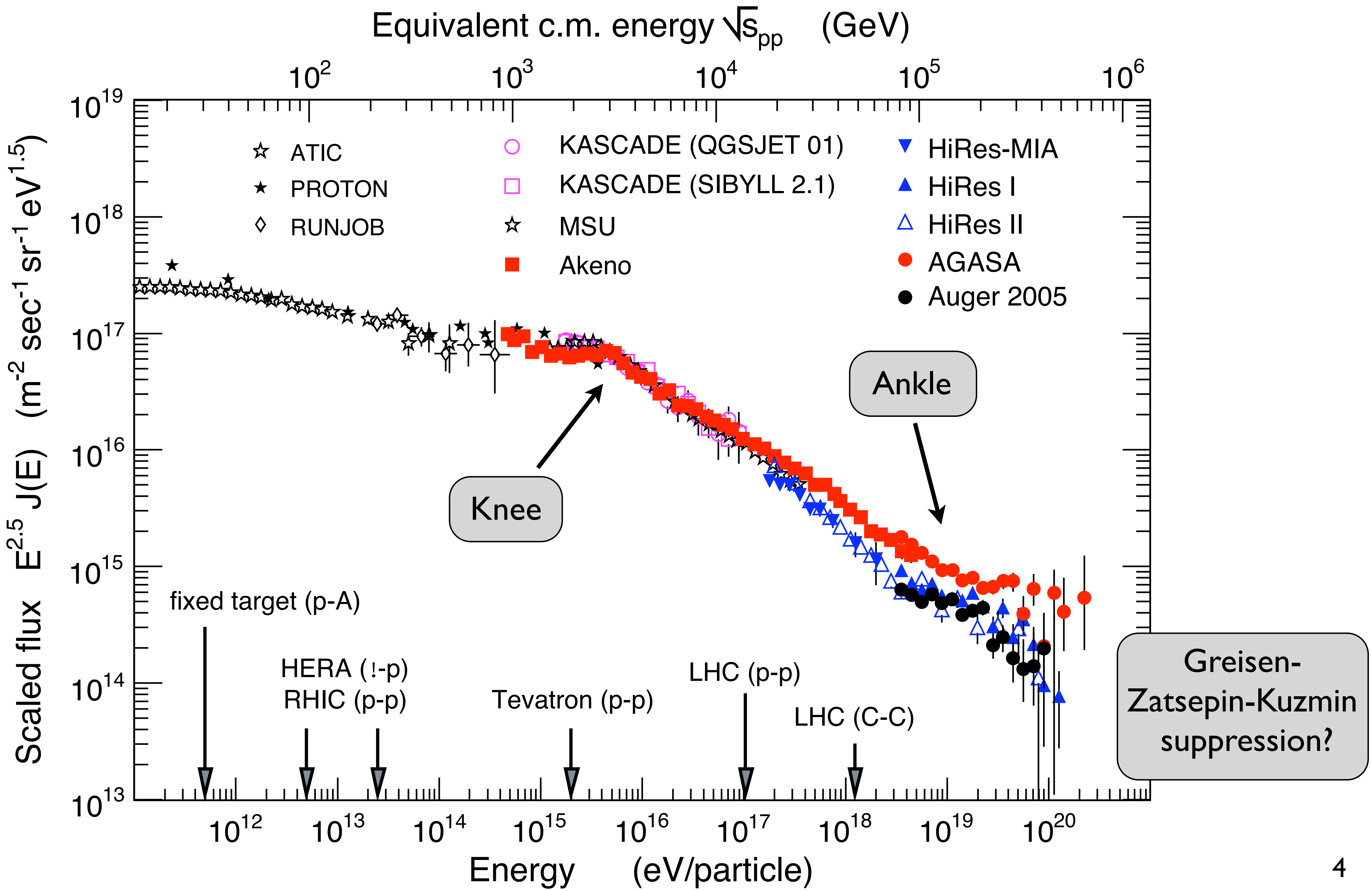
- Knee, ankle, upper end of the spectrum
- Methods of measuring composition
- Heitler-Matthews model of air showers
- Current experimental status and open problems

Perturbative QCD aspects of shower modeling

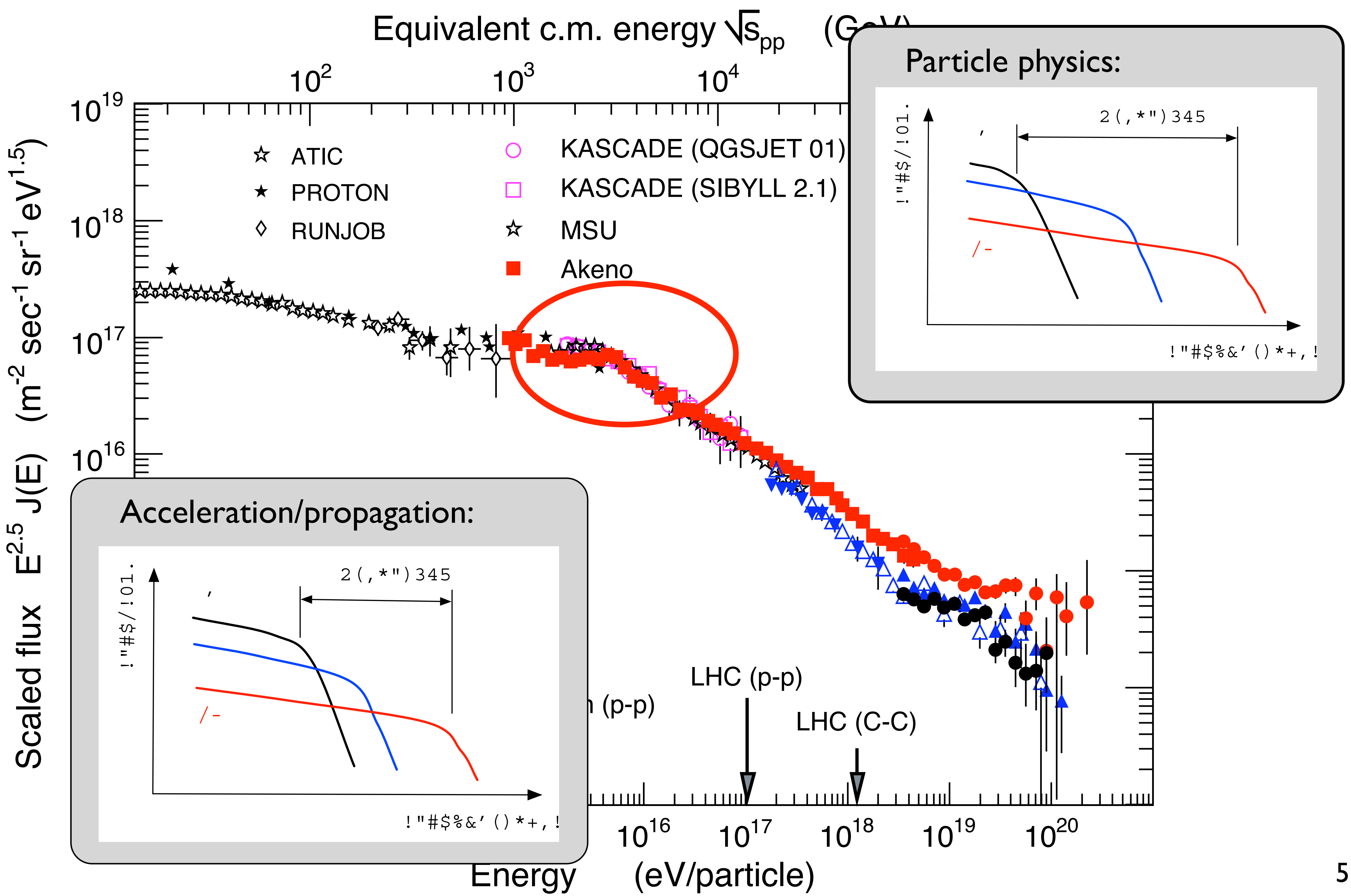
- Sensitivity of showers to hadronic interactions
- Inclusive jet cross section and low-x physics
- Impact parameter dependence and elastic/total cross section
- Multiple interaction scenarios
- Example of impact on air shower predictions

Astroparticle physics motivation

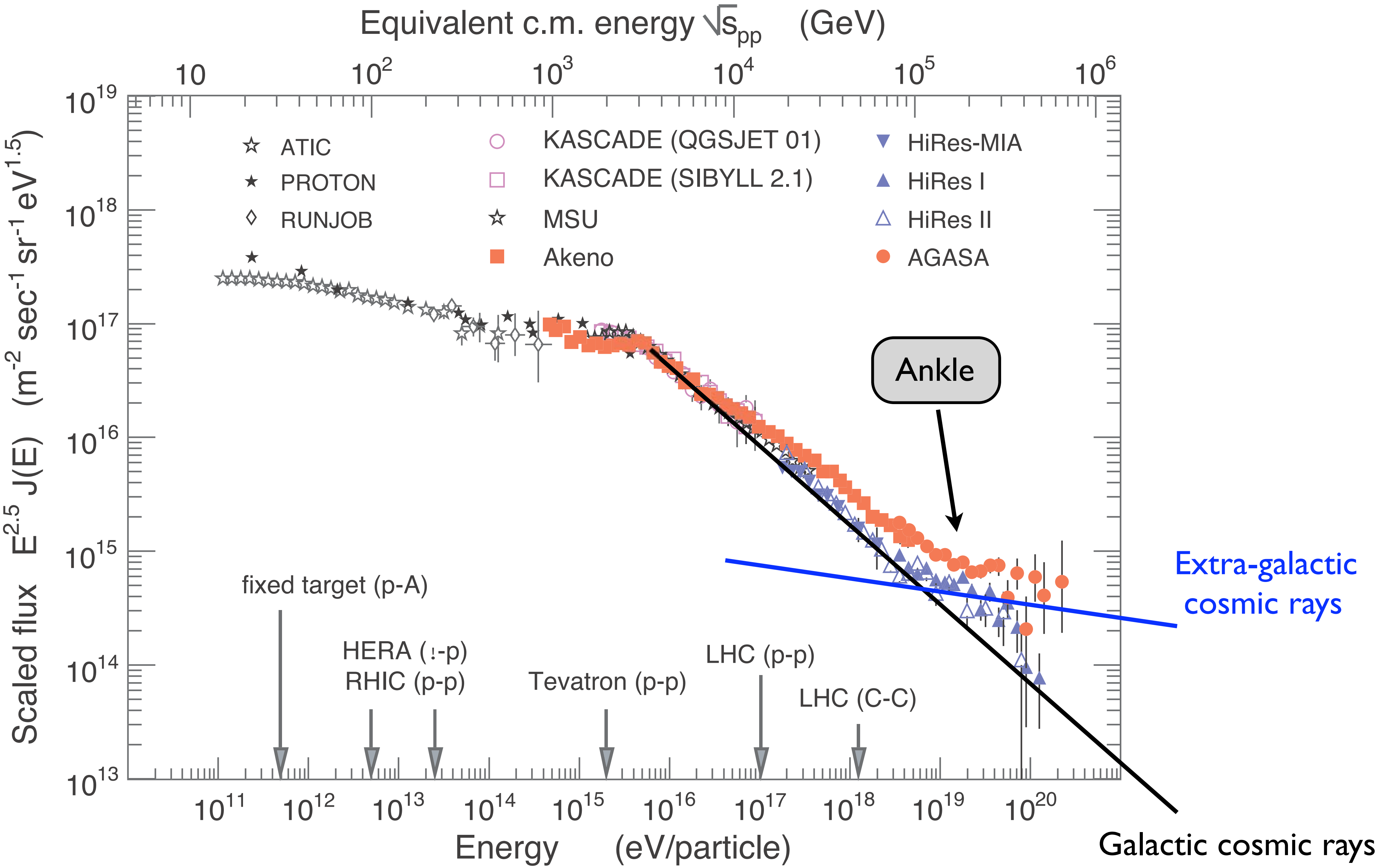
Primary cosmic ray flux



Cosmic ray composition: knee

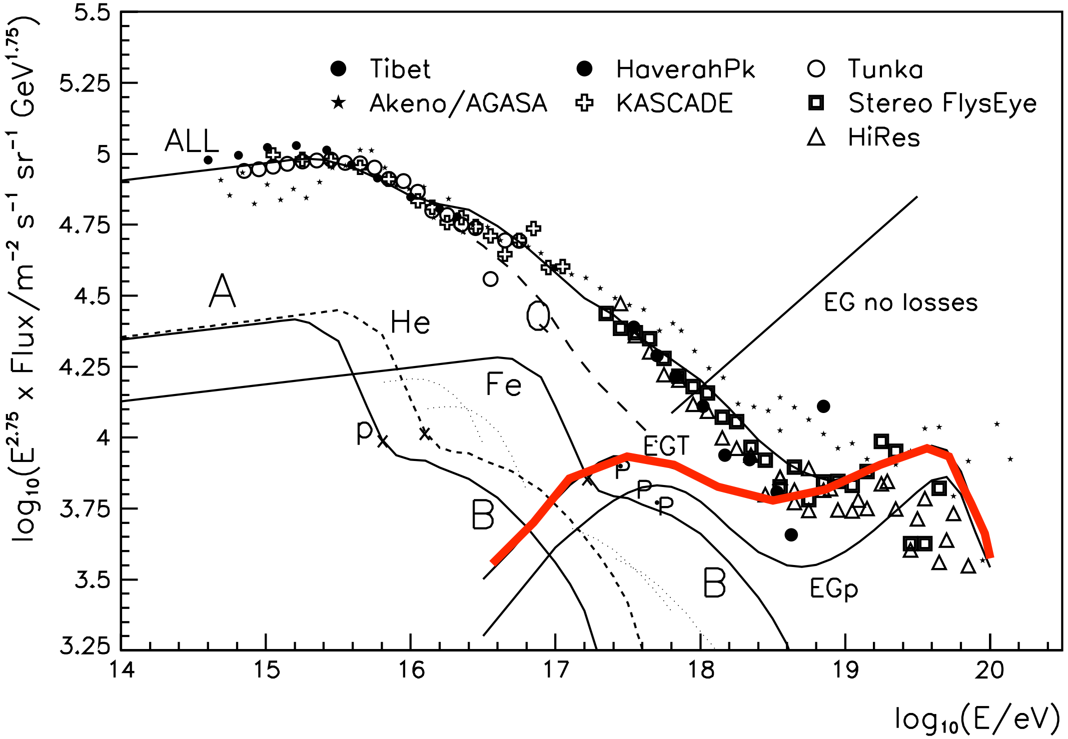


Ankle: transition region to extra-gal. cosmic rays?



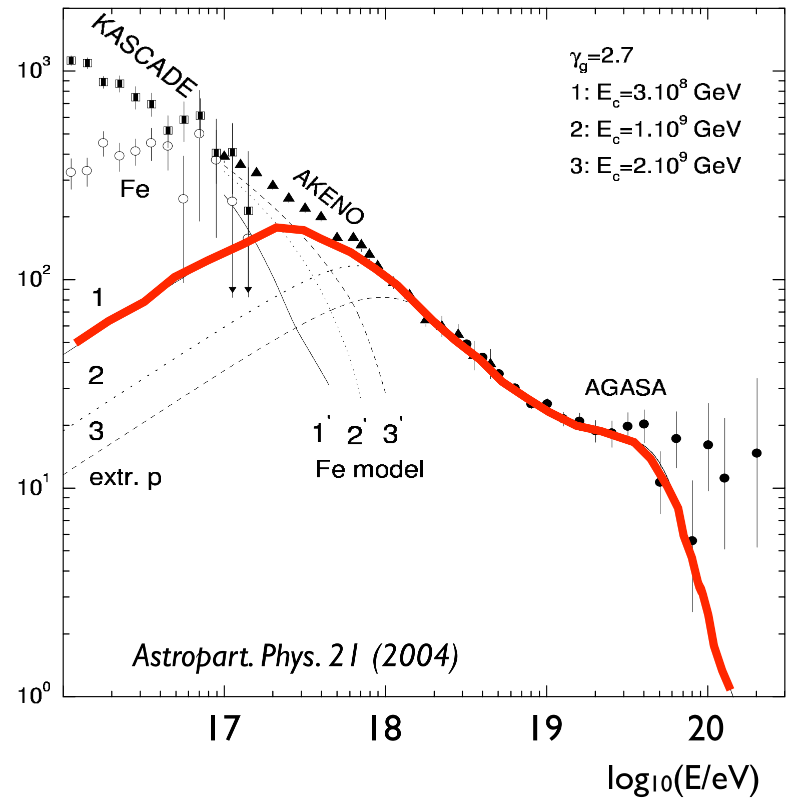
Cosmic ray composition: ankle

Hillas, J. Phys. G31 (2005)



Hillas:
- Ankle is transition galactic to extragalactic cosmic rays
- Injection spectrum *dN/dE* ~ $E^{-2.3}$

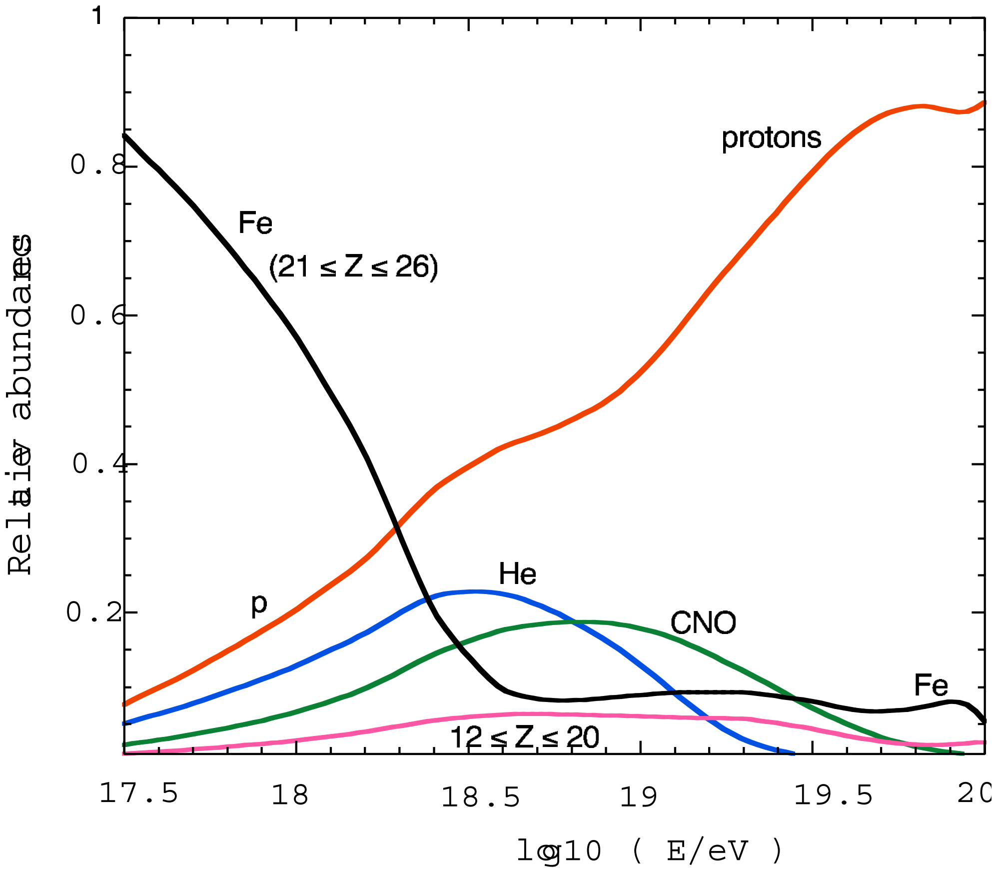
Berezinsky et al.:
- Ankle is feature due to extragalactic proton propagation
- Injection spectrum *dN/dE* ~ $E^{-2.7}$



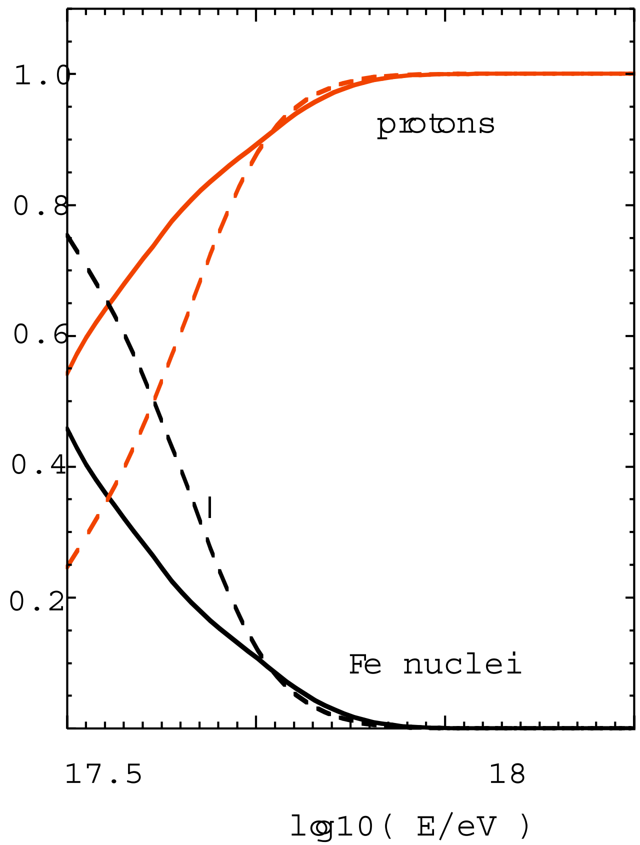
Elemental composition different

Predicted composition scenarios

Classic model of ankle



Pair-prod. model

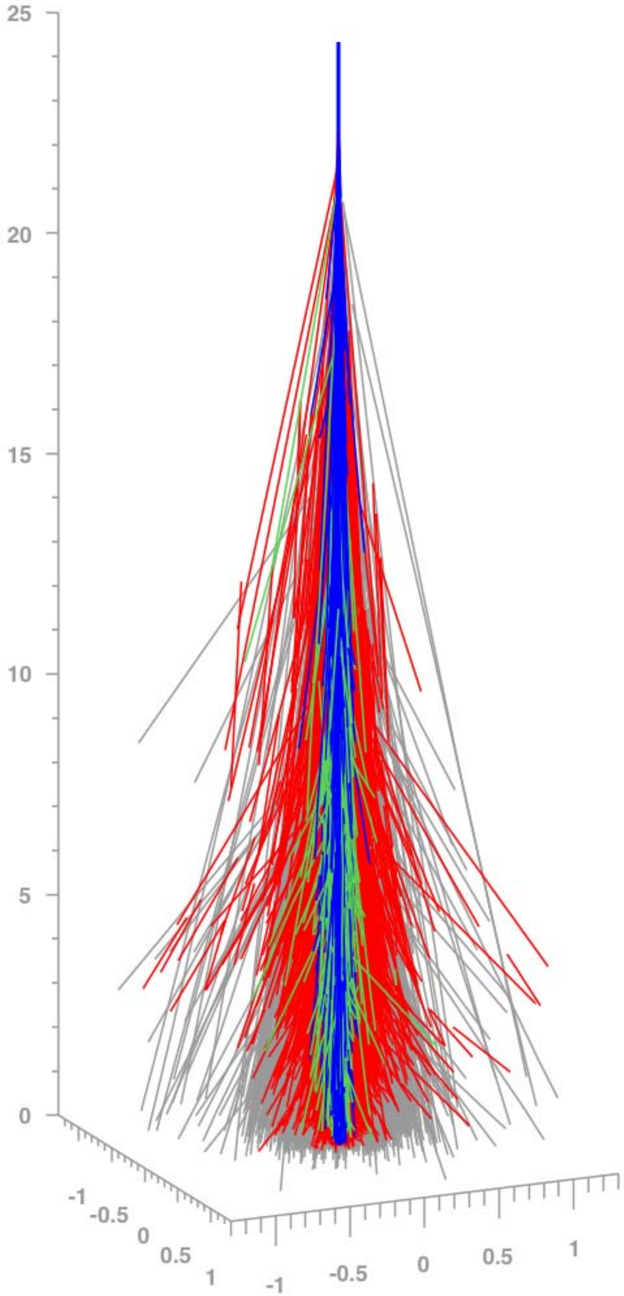


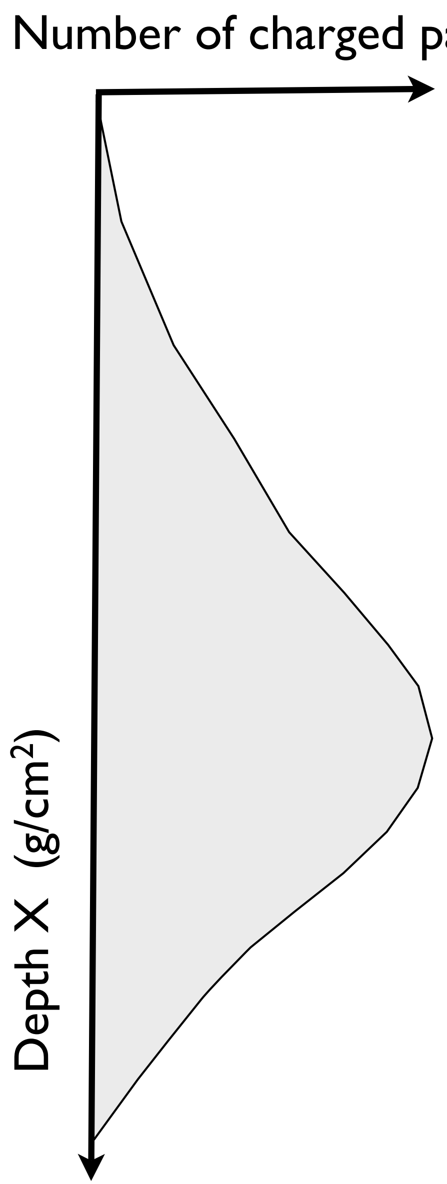
Test: measurement of composition at low energy (ankle)

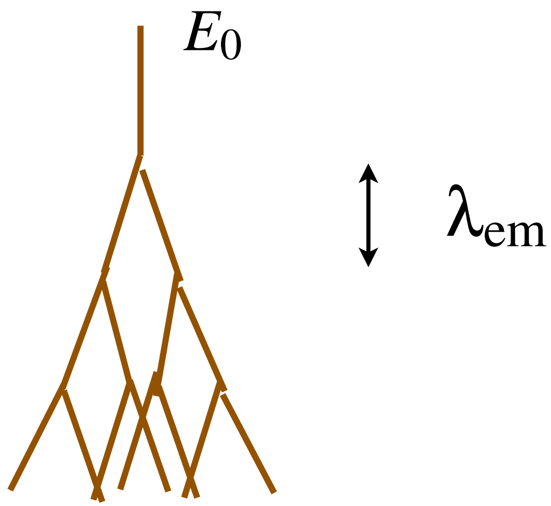
(Allard et al., 2005)

Air showers and composition sensitivity

Heitler model of em. shower



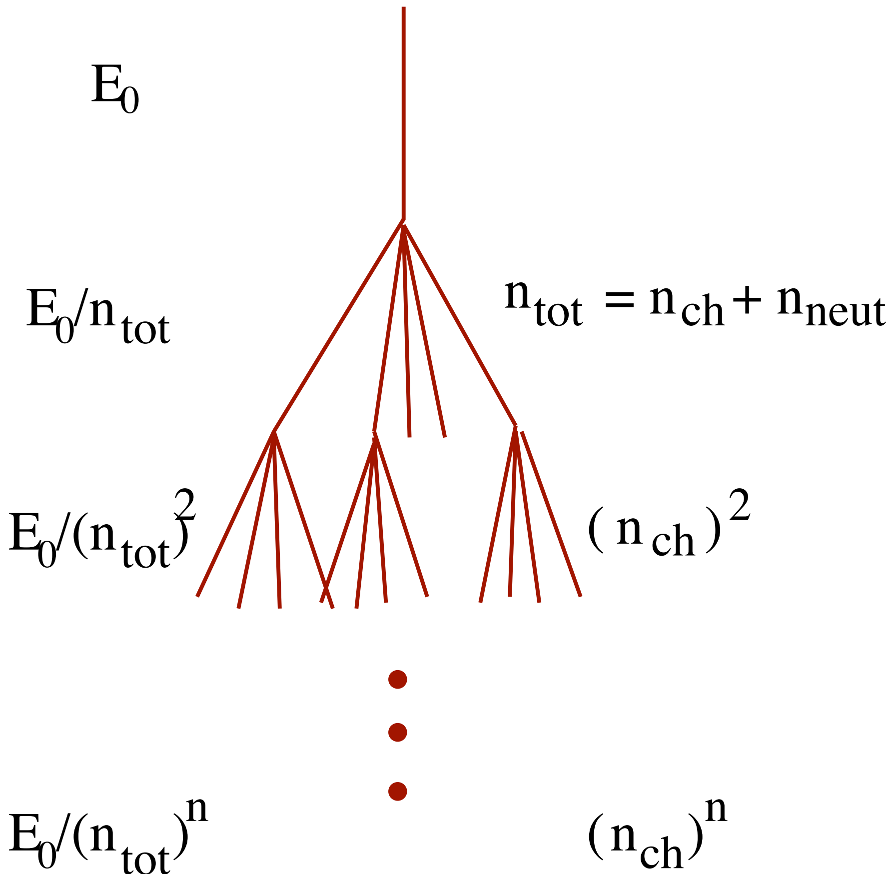




Shower maximum: $E = E_c$

$$N_{max} = E_0/E_c$$
$$X_{max} \sim \lambda_{em} \ln(E_0/E_c)$$

Heitler-Matthews model: muon production



Assumptions:

- neutral pions decay immediately
- charged pions initiate secondary cascades
- cascades stop if $E = E_{\rm dec}$

$$N_\mu = \left(\frac{E_0}{E_{\rm dec}}\right)^\alpha \qquad \alpha = \frac{\ln(n_{\rm ch})}{\ln(n_{\rm tot})} \approx 0.9$$

(Matthews, Astropart.Phys. 22(2005) 387)

Superposition model

Proton-induced shower

$$N_{\max} = E_0/E_c$$

$$X_{\max} \sim \lambda_{\text{eff}} \ln(E_0)$$

$$N_\mu = \left(\frac{E_0}{E_{\text{dec}}}\right)^\alpha \qquad \alpha \approx 0.9$$

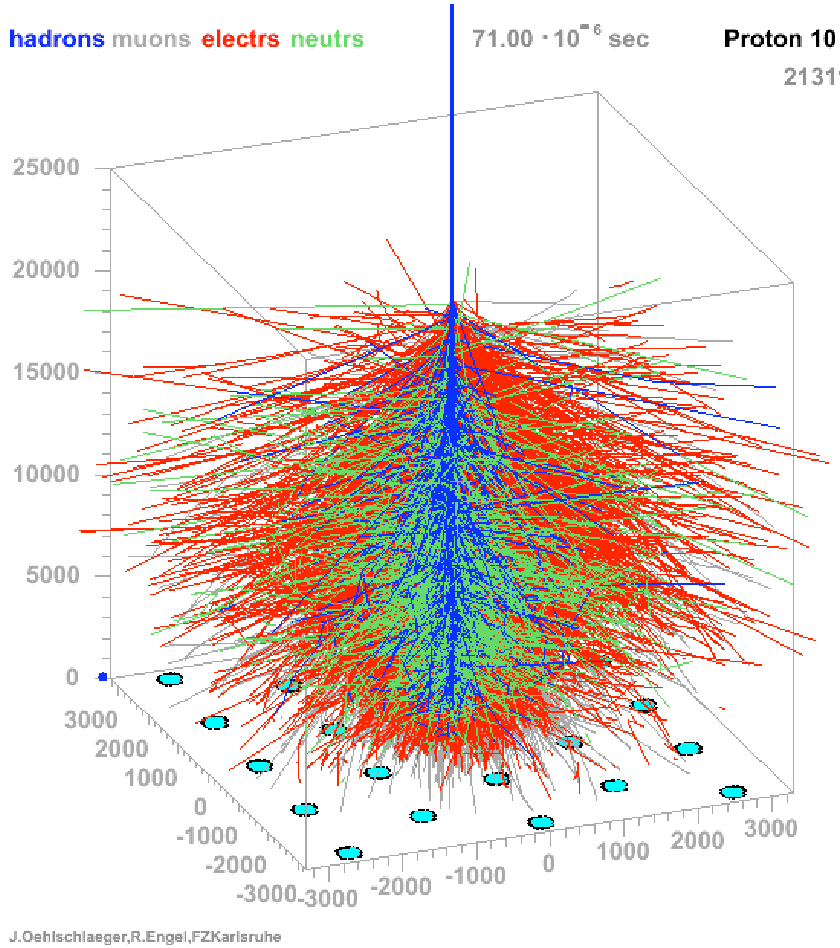
Assumption: nucleus of mass *A* and energy E_0 corresponds to *A* nucleons (protons) of energy $E_n = E_0/A$

$$N_{\max}^A = A\left(\frac{E_0}{AE_c}\right) = N_{\max}$$

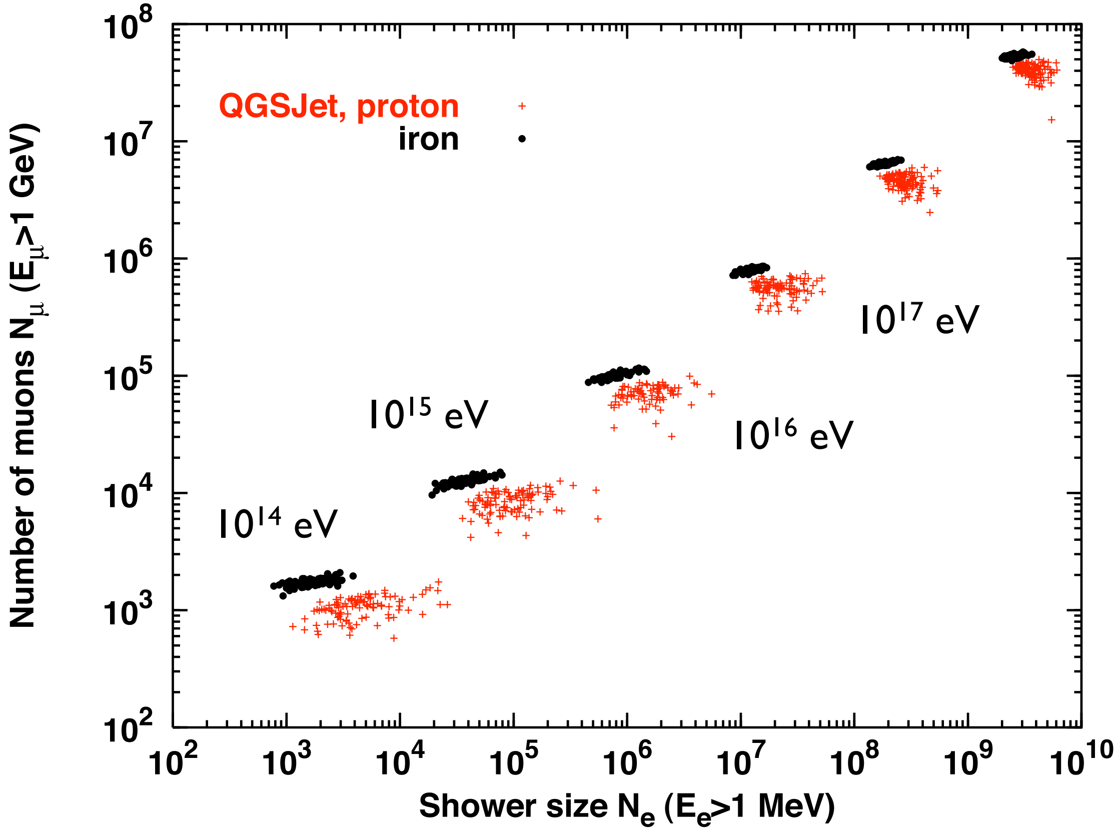
$$X_{\max}^A \sim \lambda_{\text{eff}} \ln(E_0/A)$$

$$N_\mu^A = A\left(\frac{E_0}{AE_{\text{dec}}}\right)^\alpha = A^{1-\alpha} N_\mu$$

Composition: air shower ground arrays

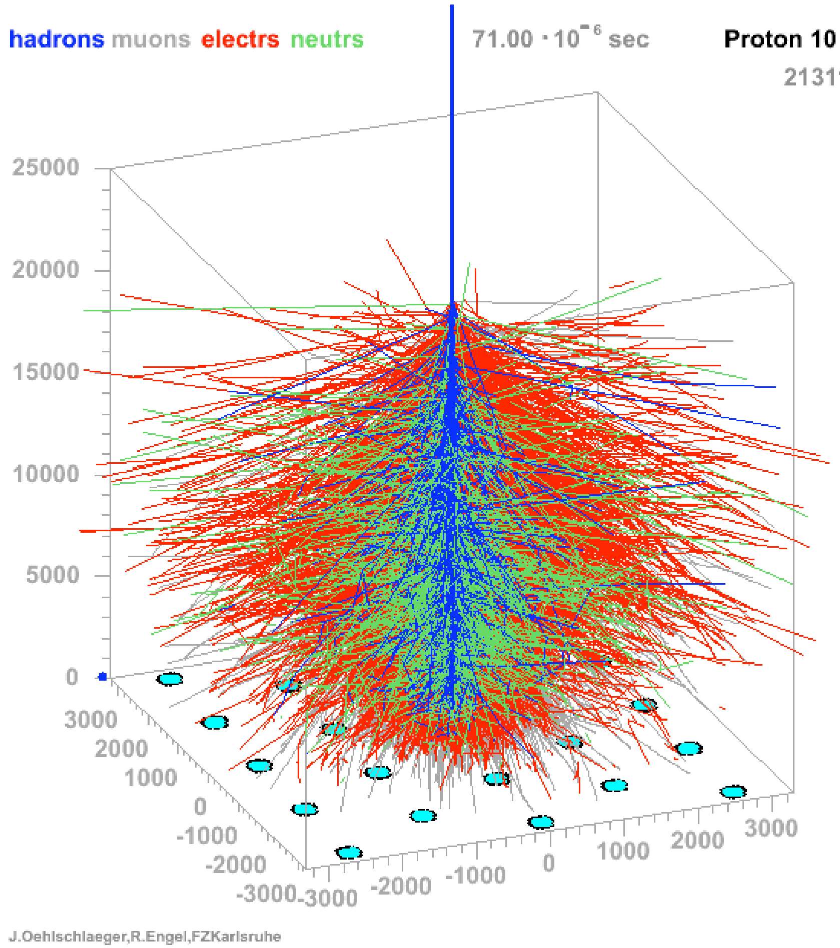


Combined energy-composition analysis

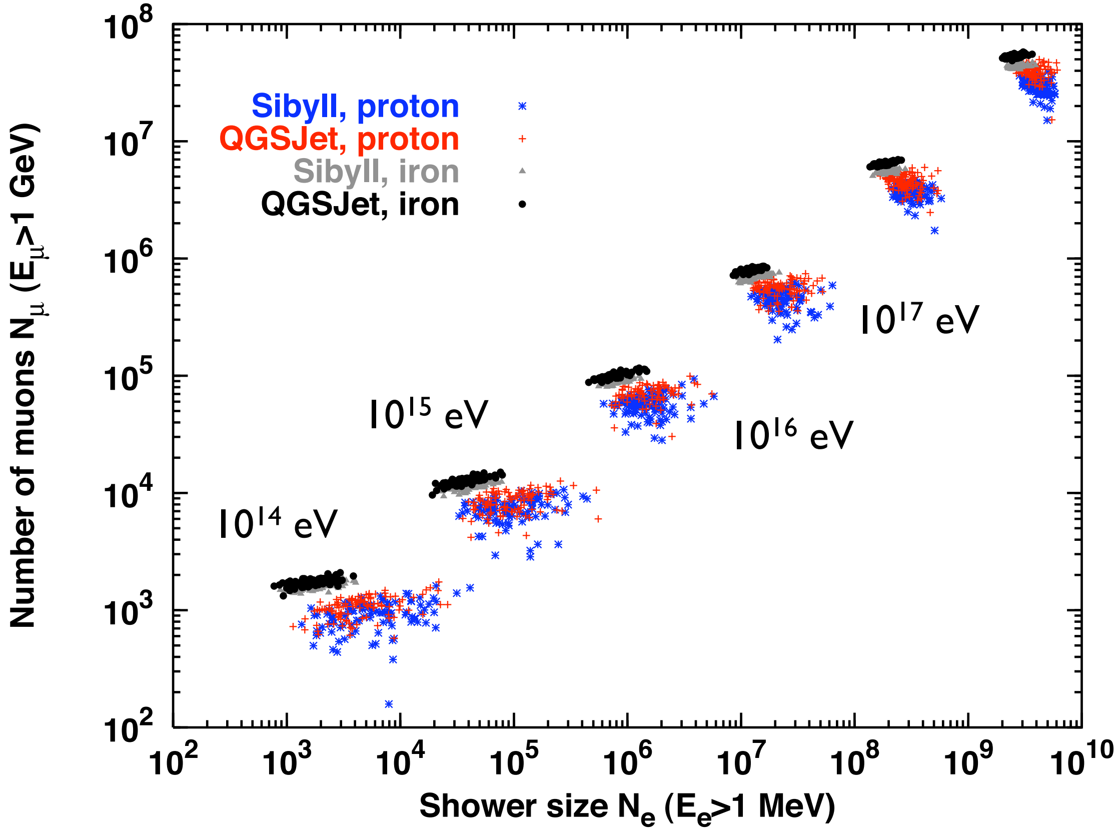


Example:
KASCADE-Grande (Karlsruhe)

Composition: air shower ground arrays

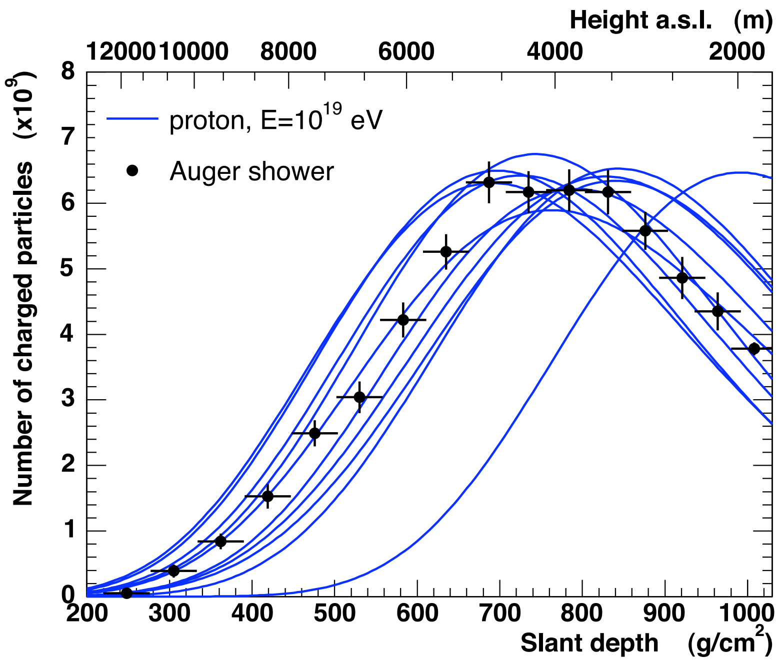


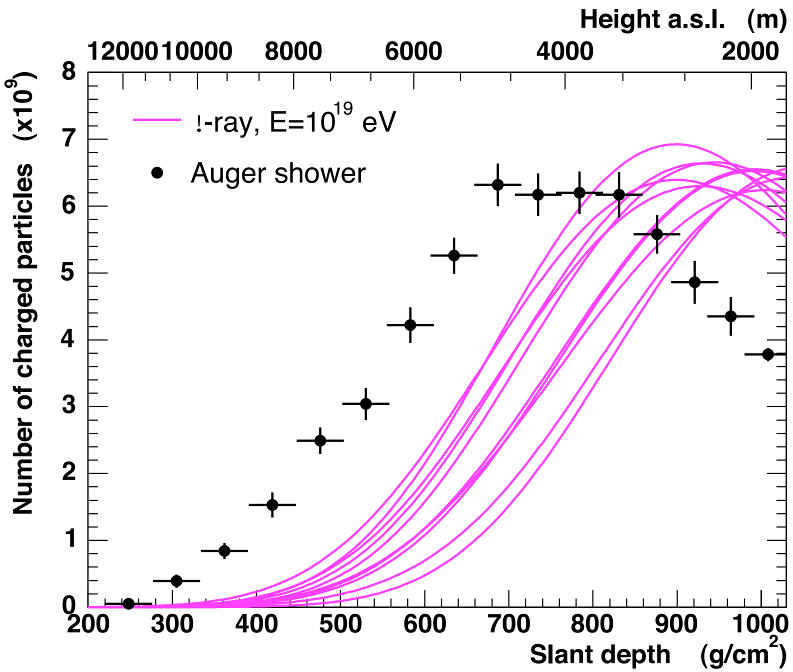
Strong model dependence !



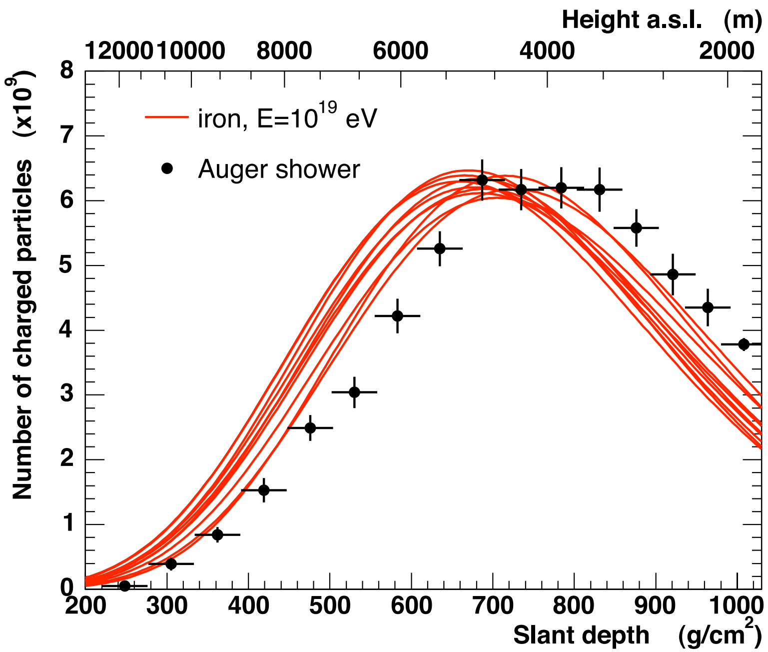
Example:
KASCADE-Grande (Karlsruhe)

Composition analysis using shower profiles



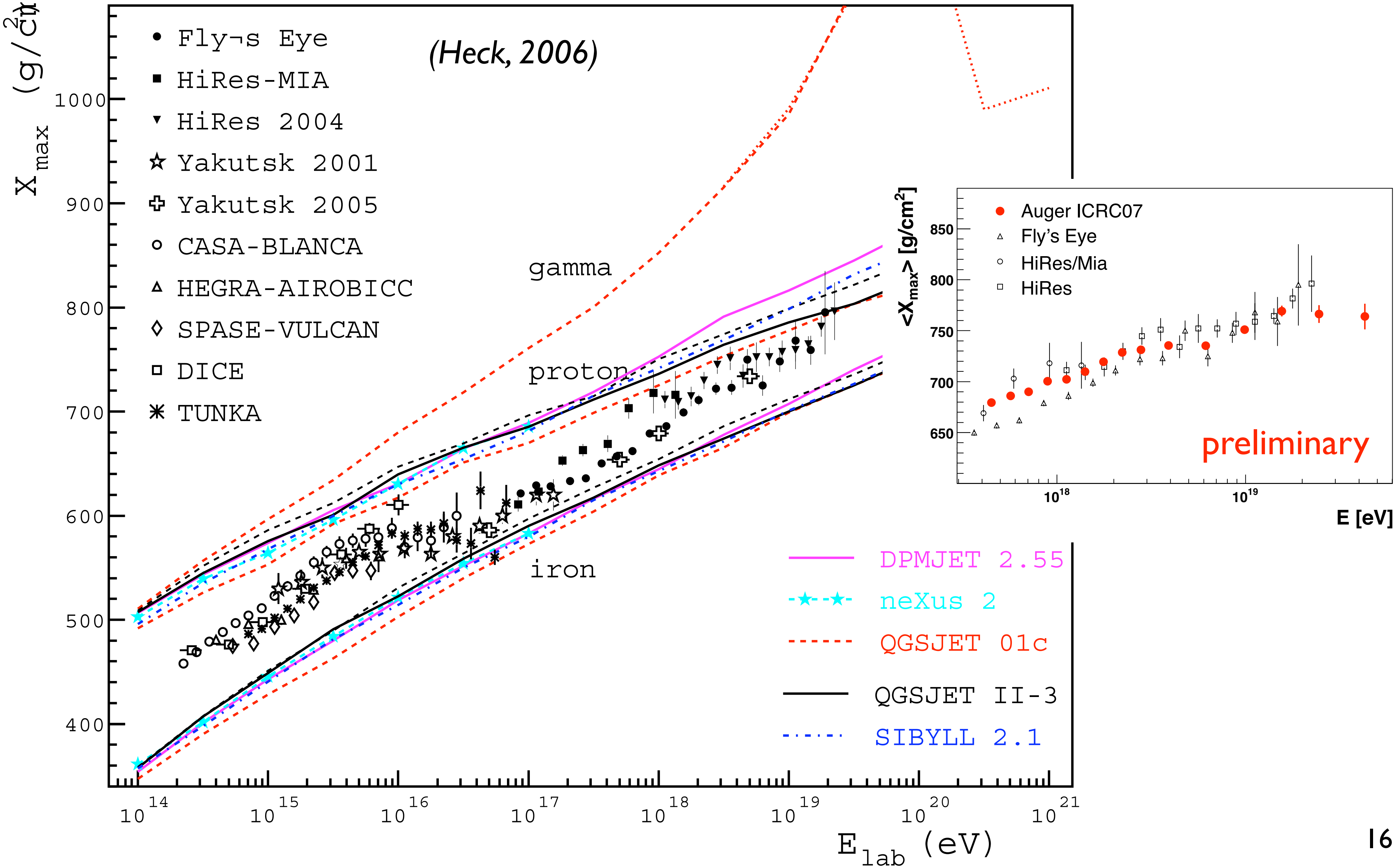


Example: event measured by Auger Collab. (ICRC 2003)



- Energy well determined
- Primary particle type: mean and fluctuations of shower depth of maximum

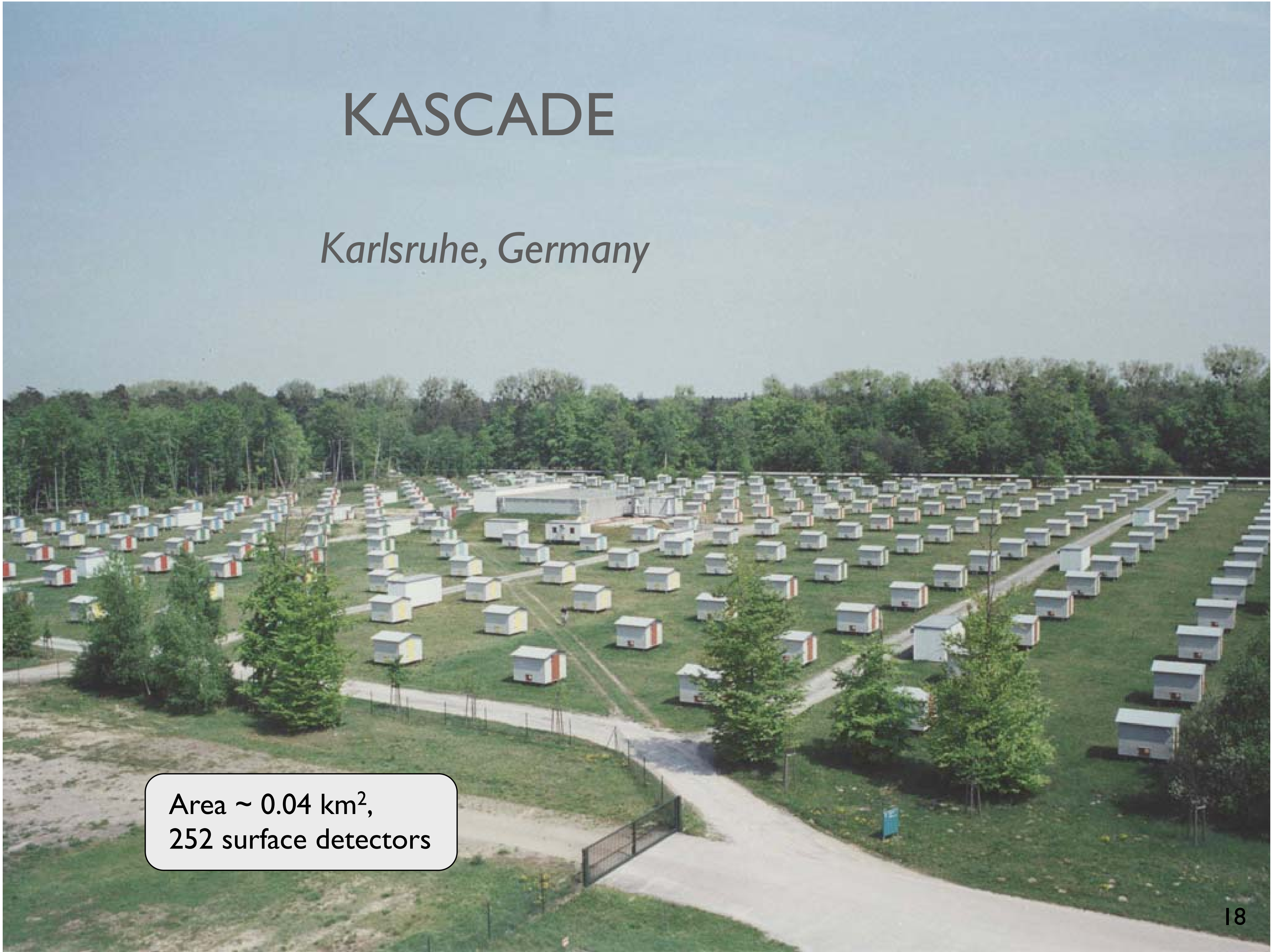
Mean shower depth of maximum



Experimental situation

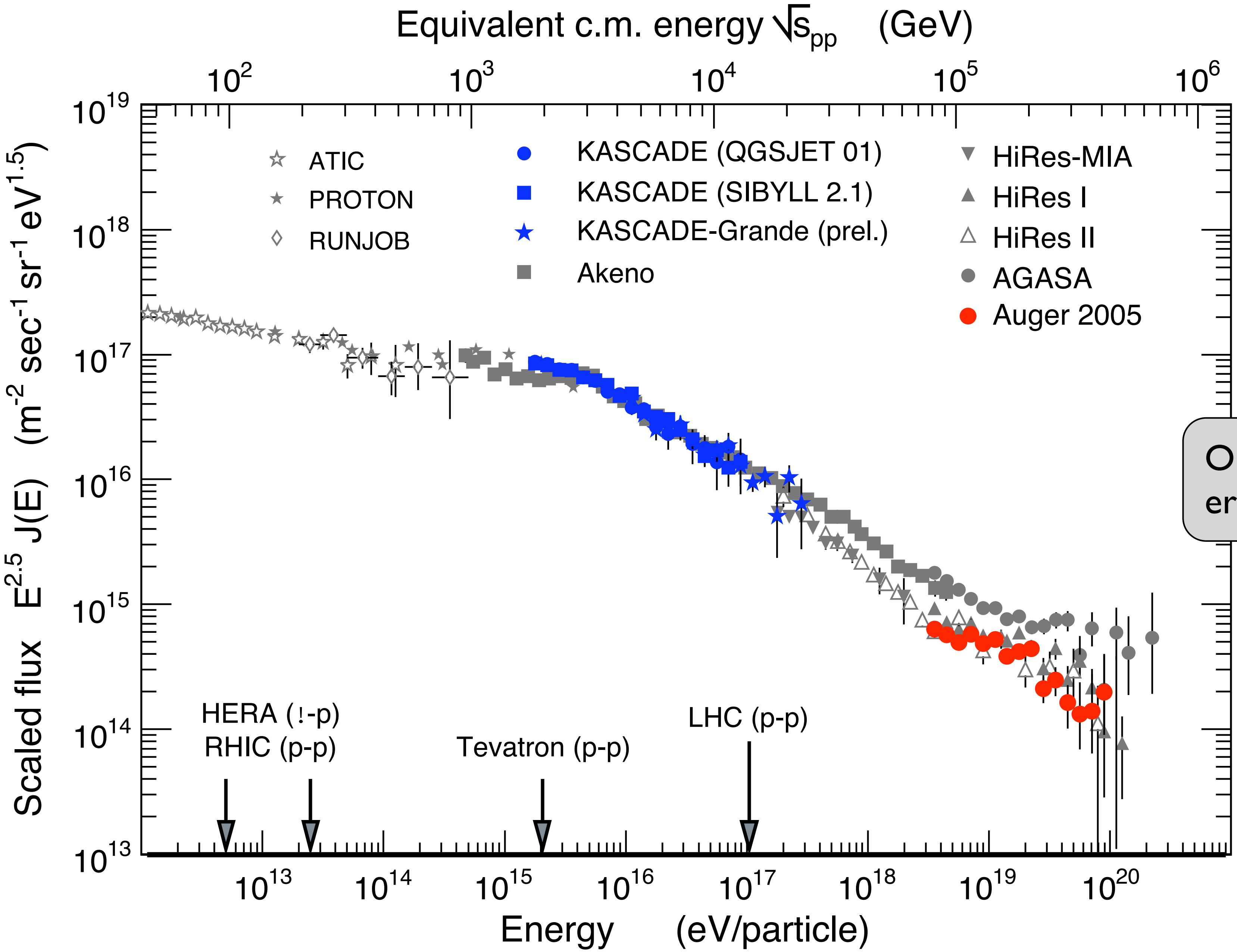
KASCADE

Karlsruhe, Germany



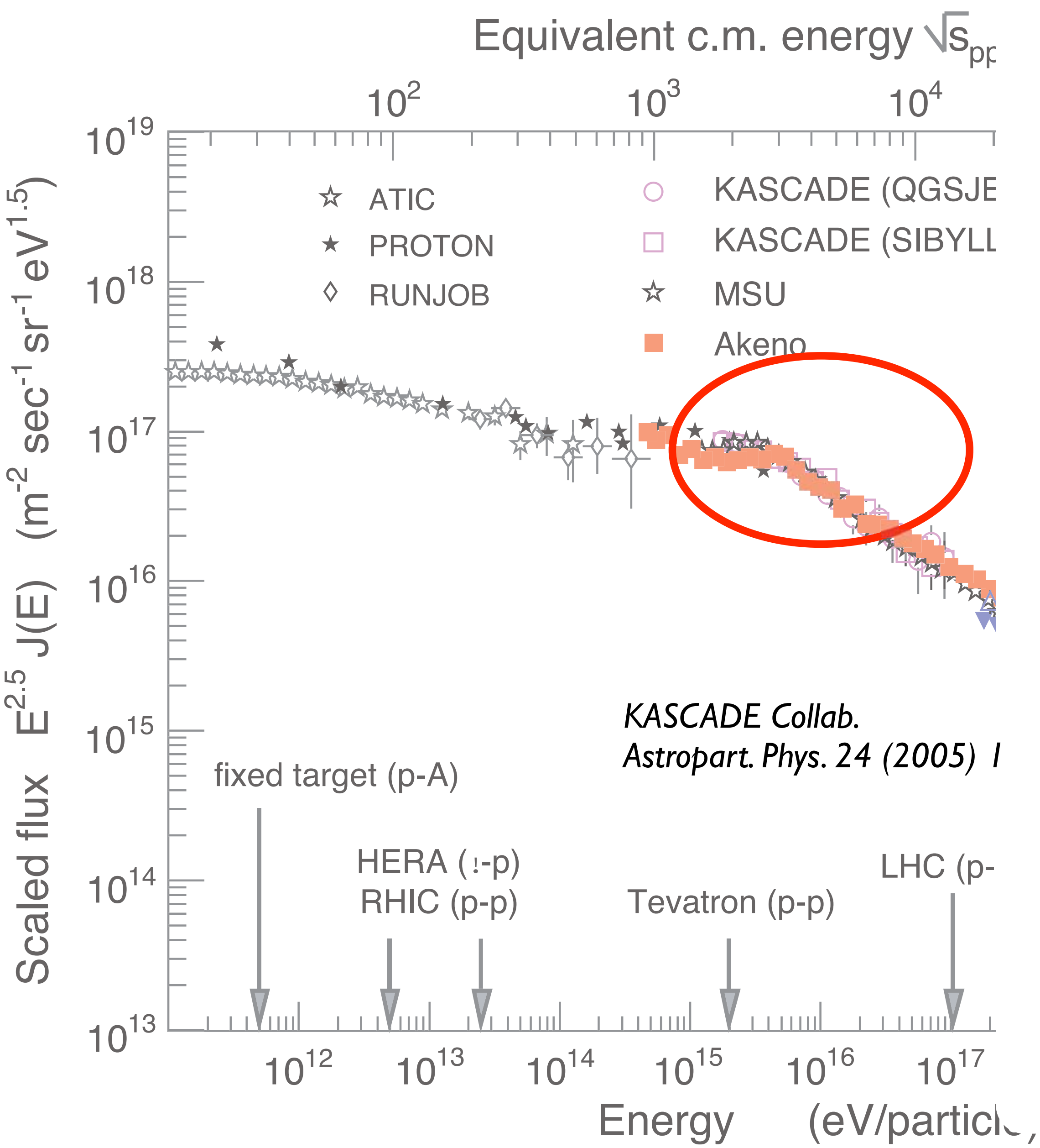
Area ~ 0.04 km^2,
252 surface detectors

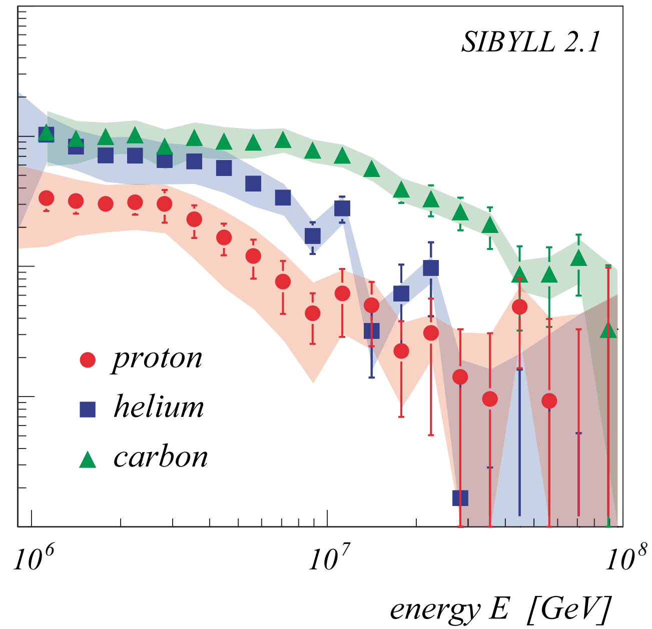
KASCADE and KASCADE-Grande

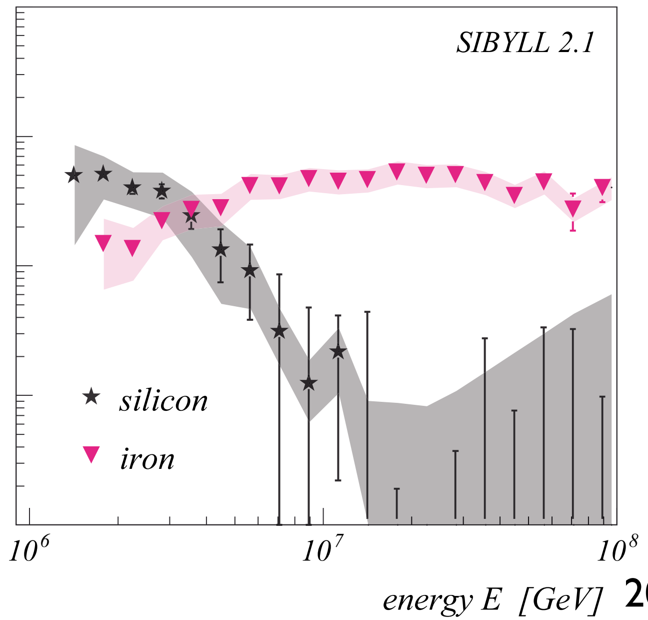


Only statistical errors shown

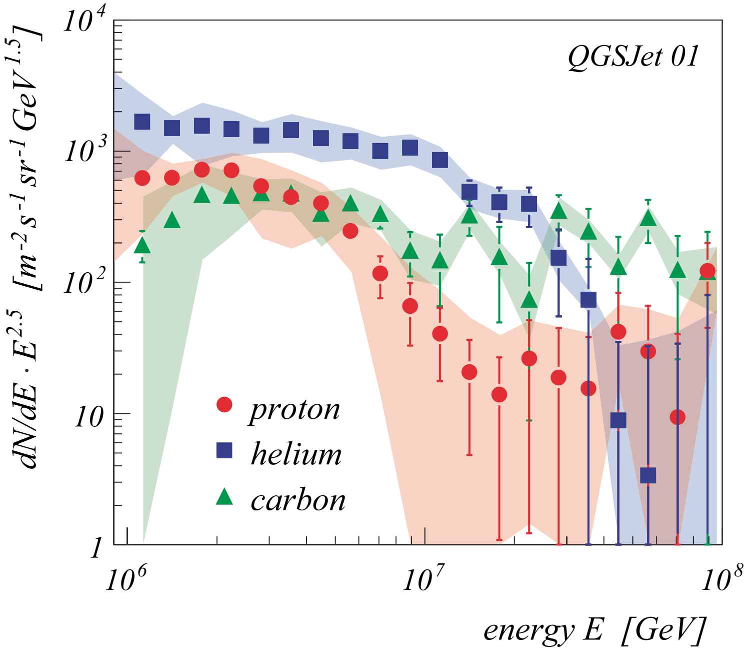
Composition in Knee region (I)

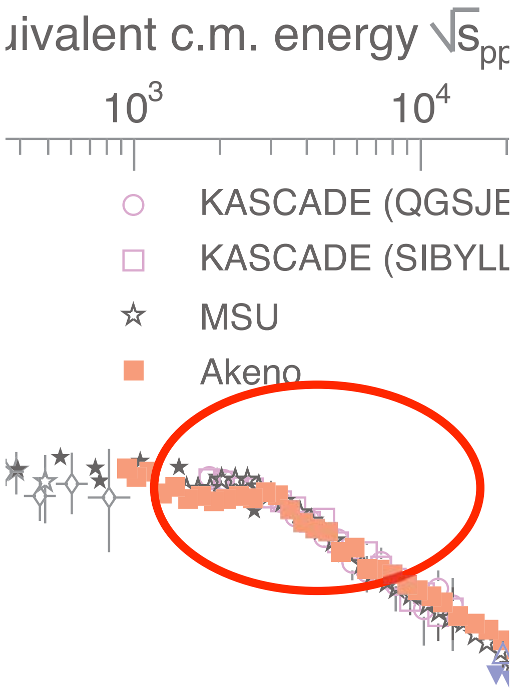


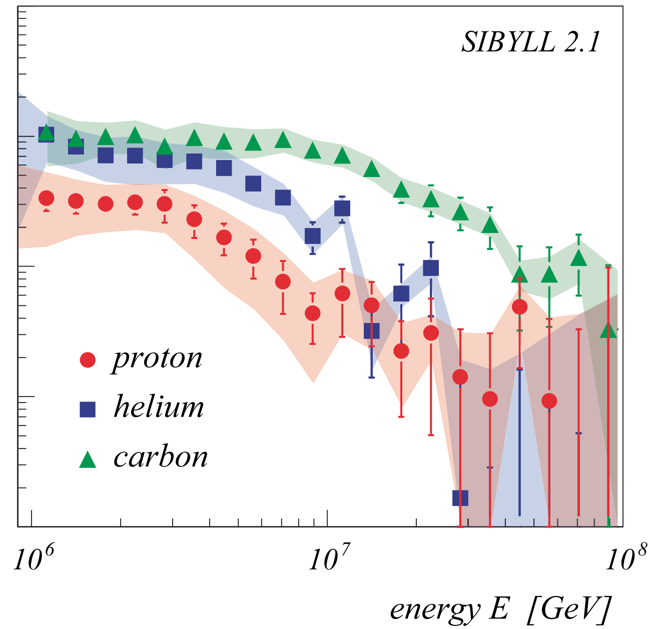


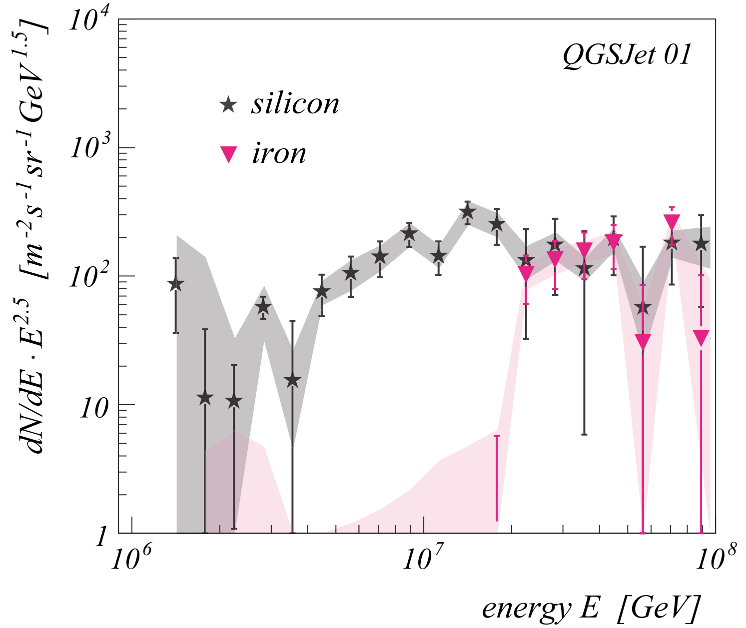


Composition in Knee region (II)

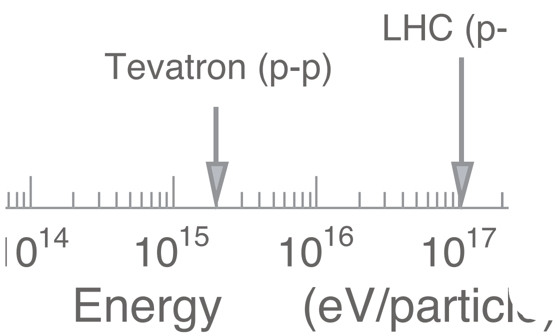


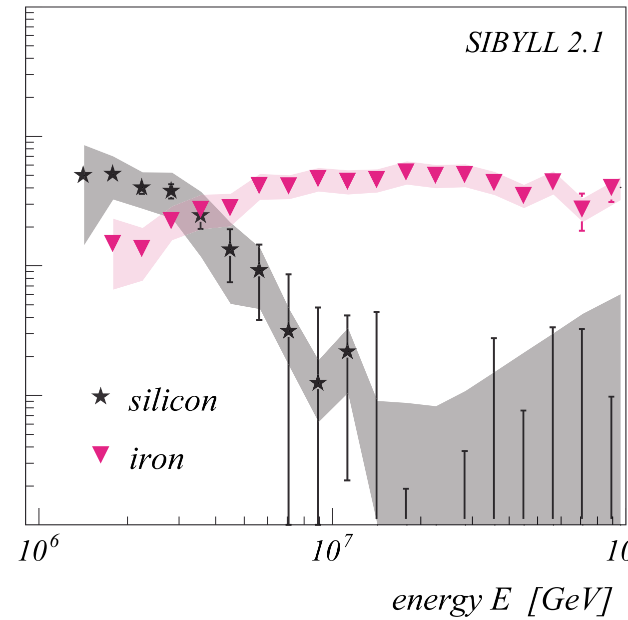






KASCADE Collab.
Astropart. Phys. 24 (2005) 1





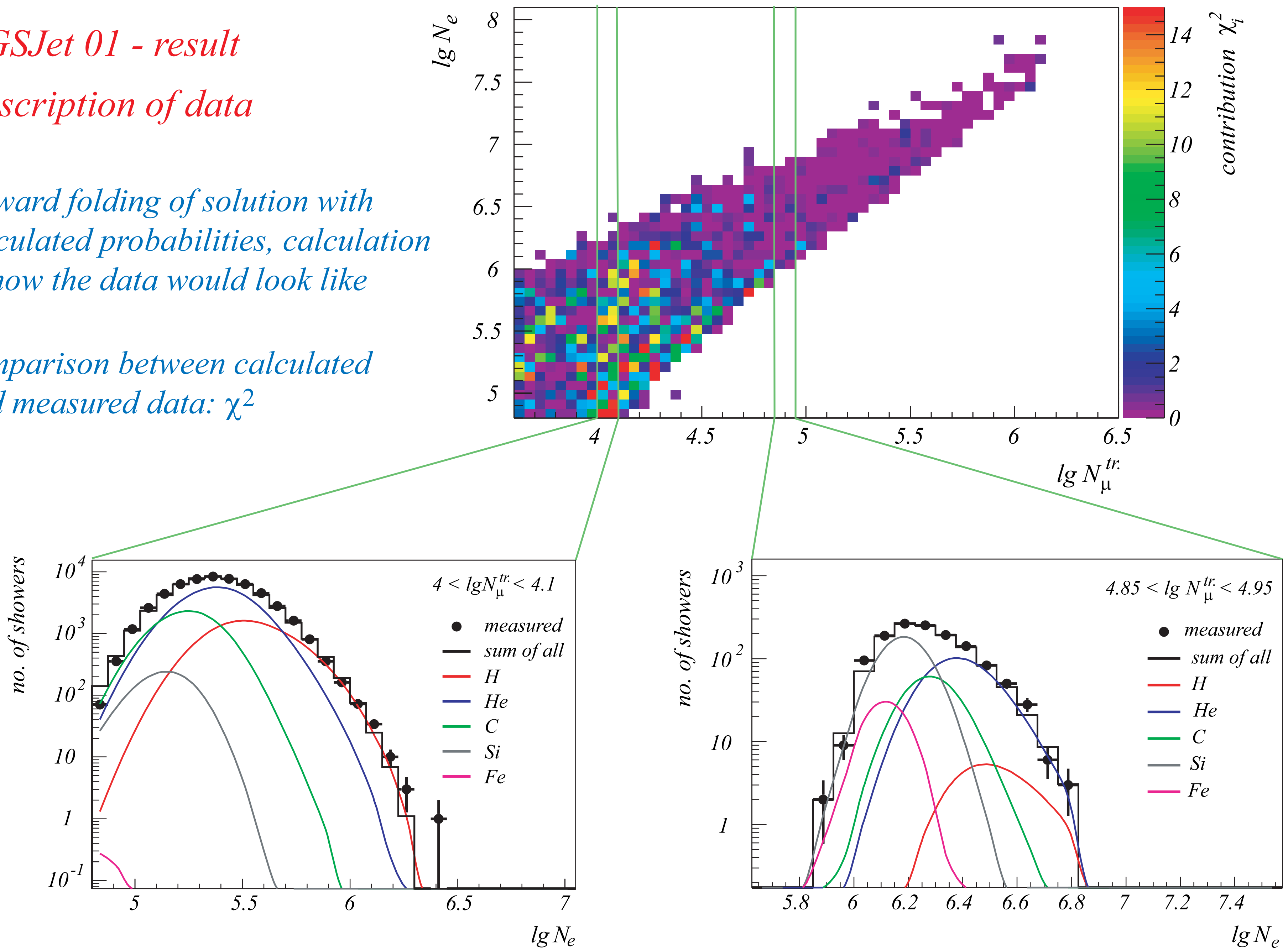
KASCADE data description: QGSJET 01

QGSJet 01 - result

Description of data

forward folding of solution with calculated probabilities, calculation of how the data would look like

comparison between calculated and measured data: χ^2



(H. Ulrich et al., C2CR 2005)

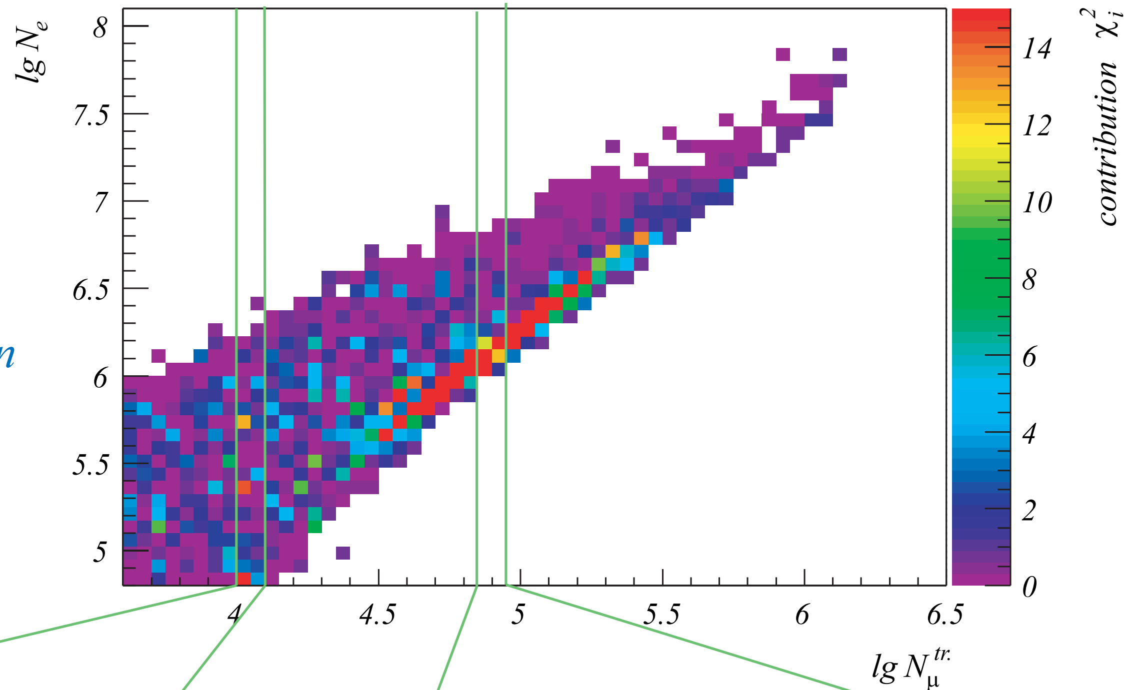
KASCADE data description: SIBYLL 2.1

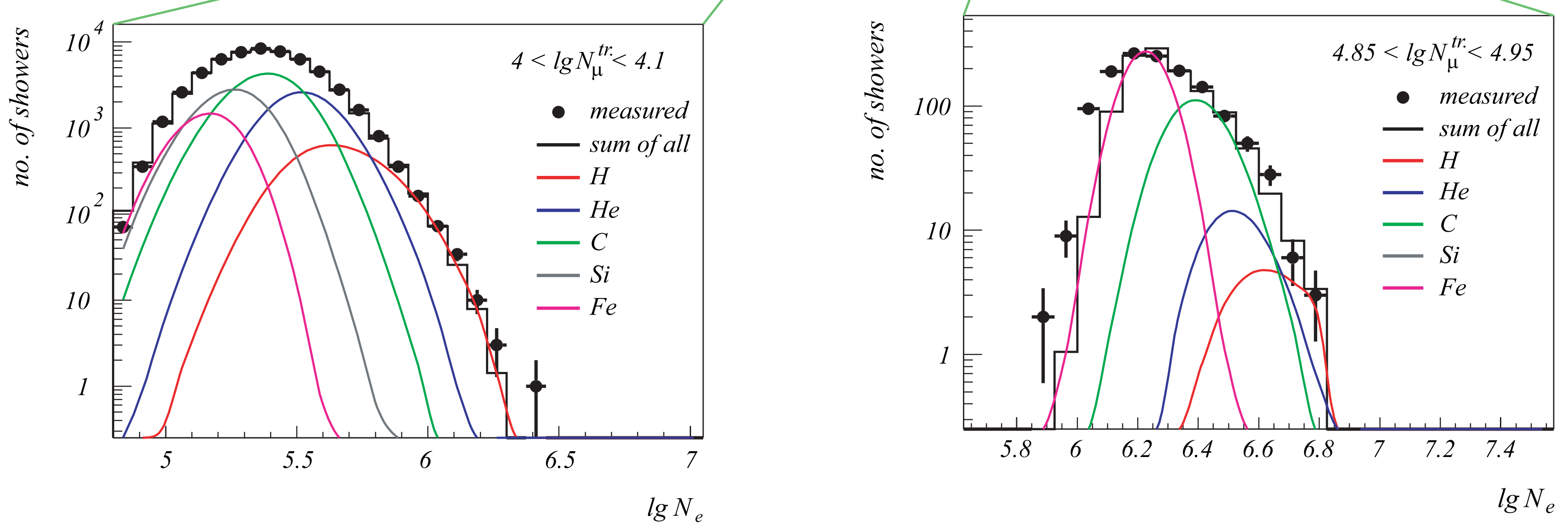
SIBYLL 2.1 - result

Description of data

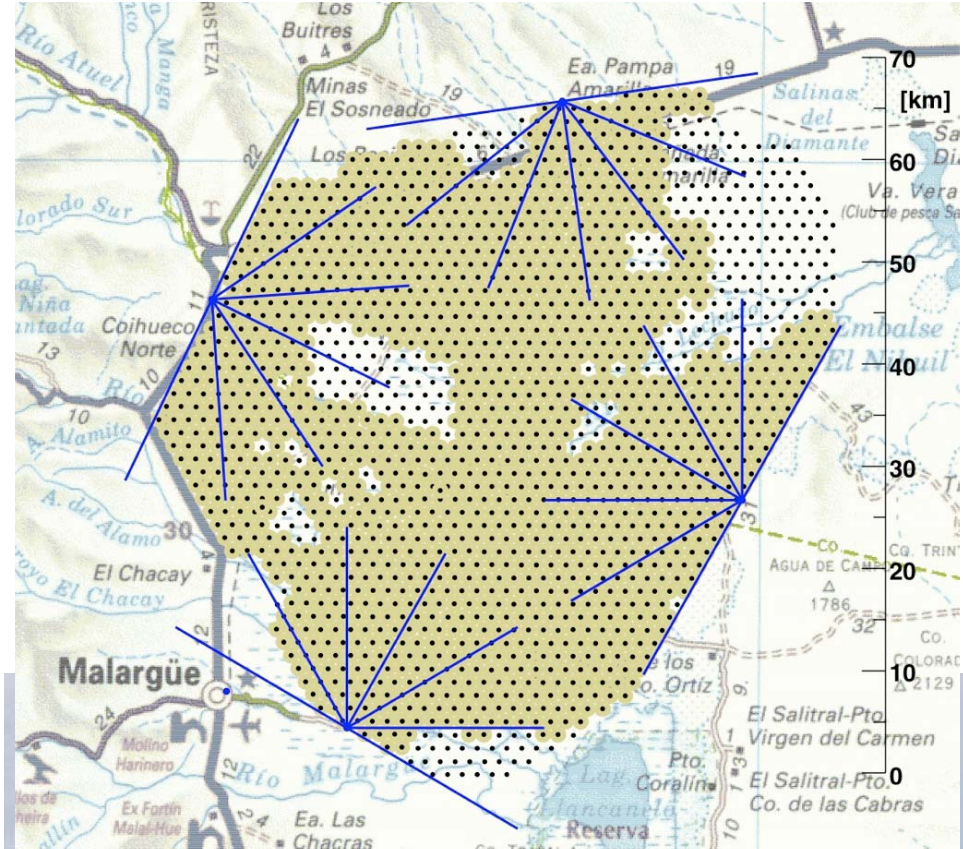
forward folding of solution with calculated probabilities, calculation of how the data would look like

comparison between calculated and measured data: χ^2





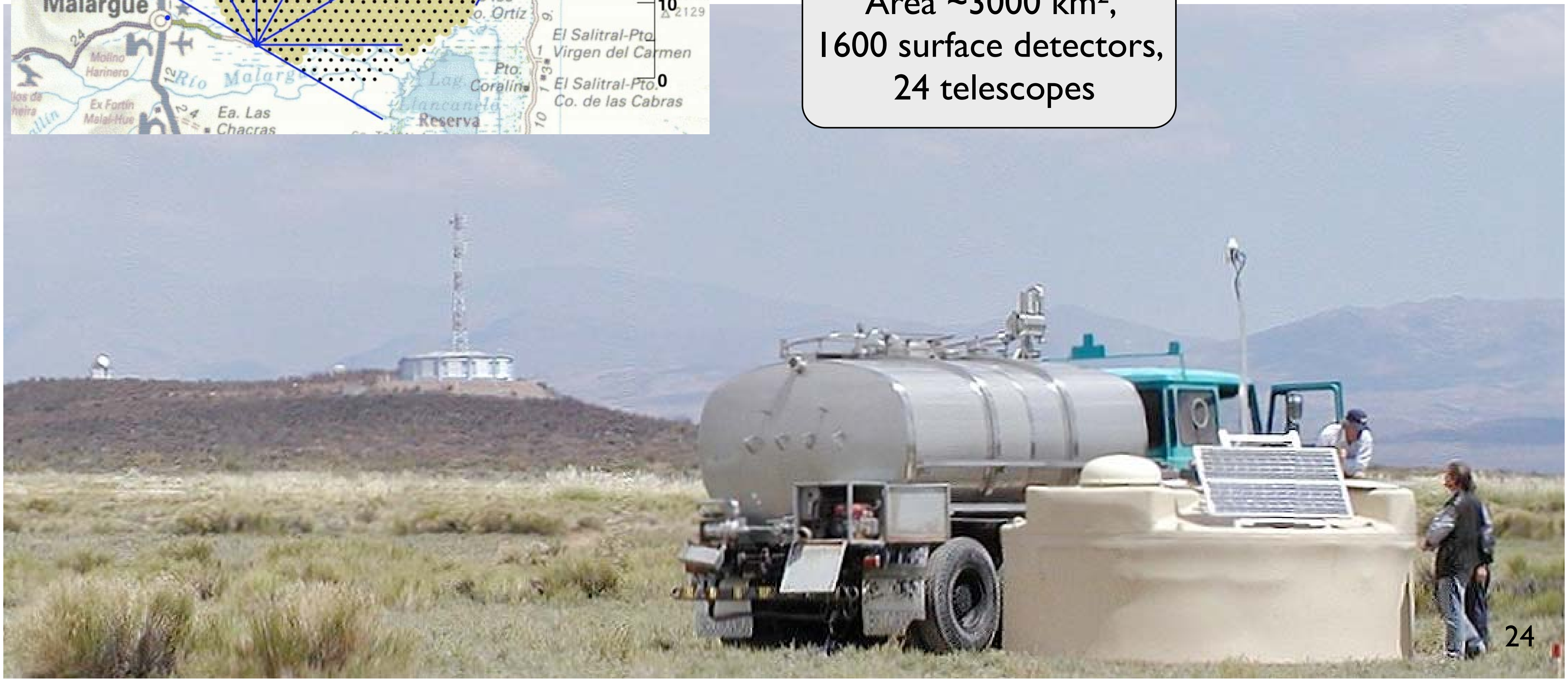
(H. Ulrich et al., C2CR 2005)



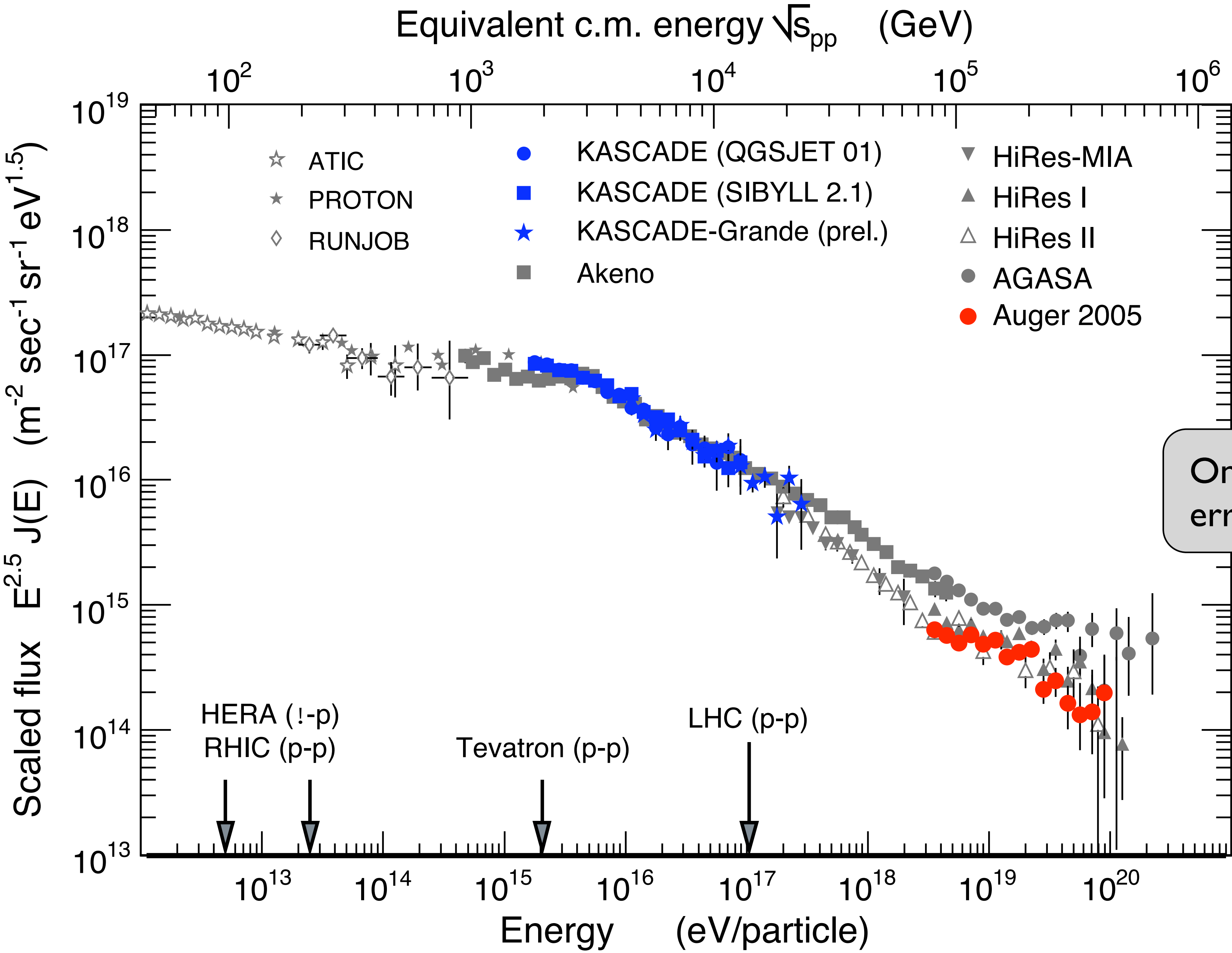
Southern Pierre Auger Observatory

Malargue, Argentina

Area ~3000 km^2,
1600 surface detectors,
24 telescopes

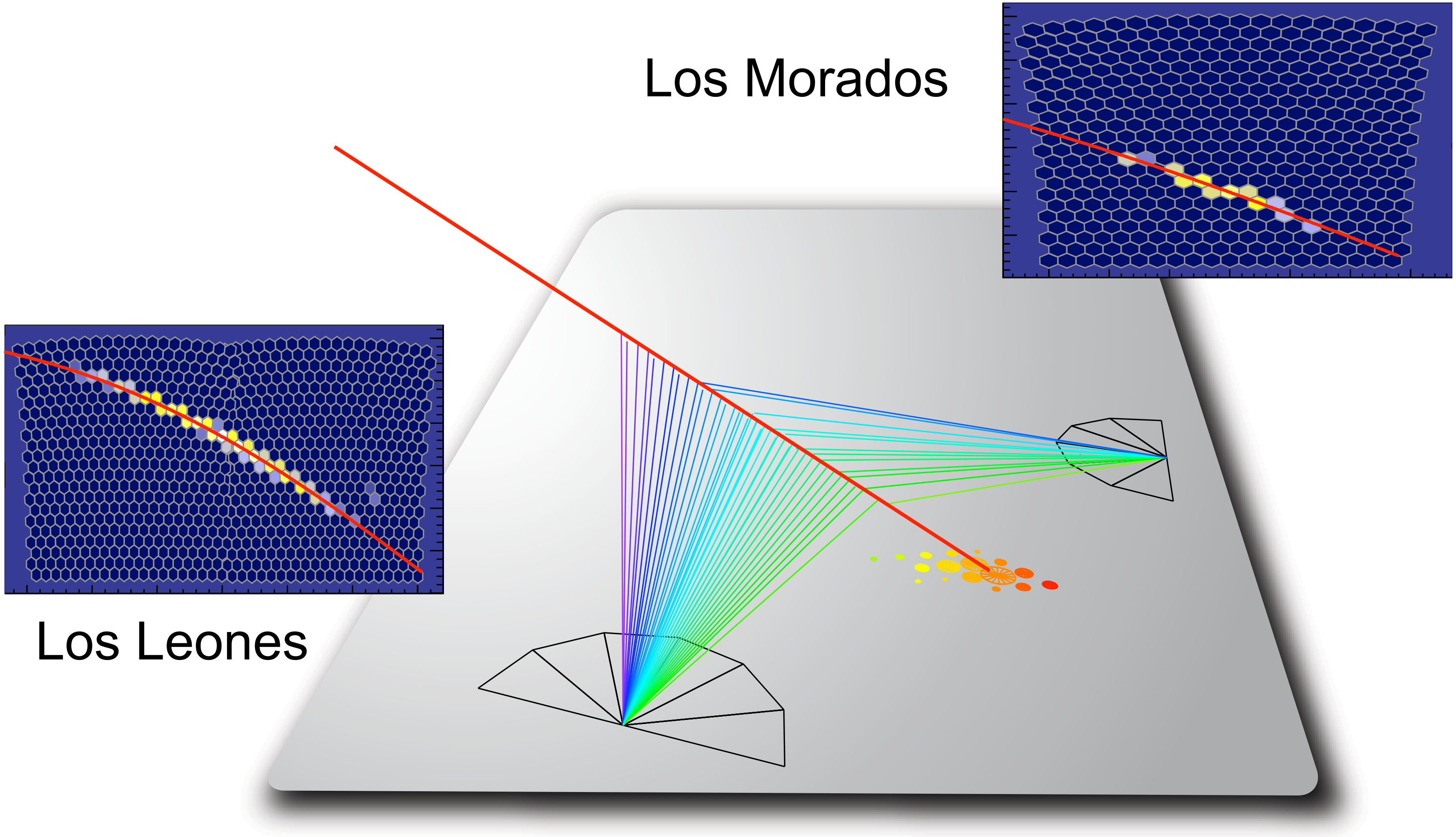


Pierre Auger Observatory



Only statistical errors shown

Auger: stereo-hybrid event

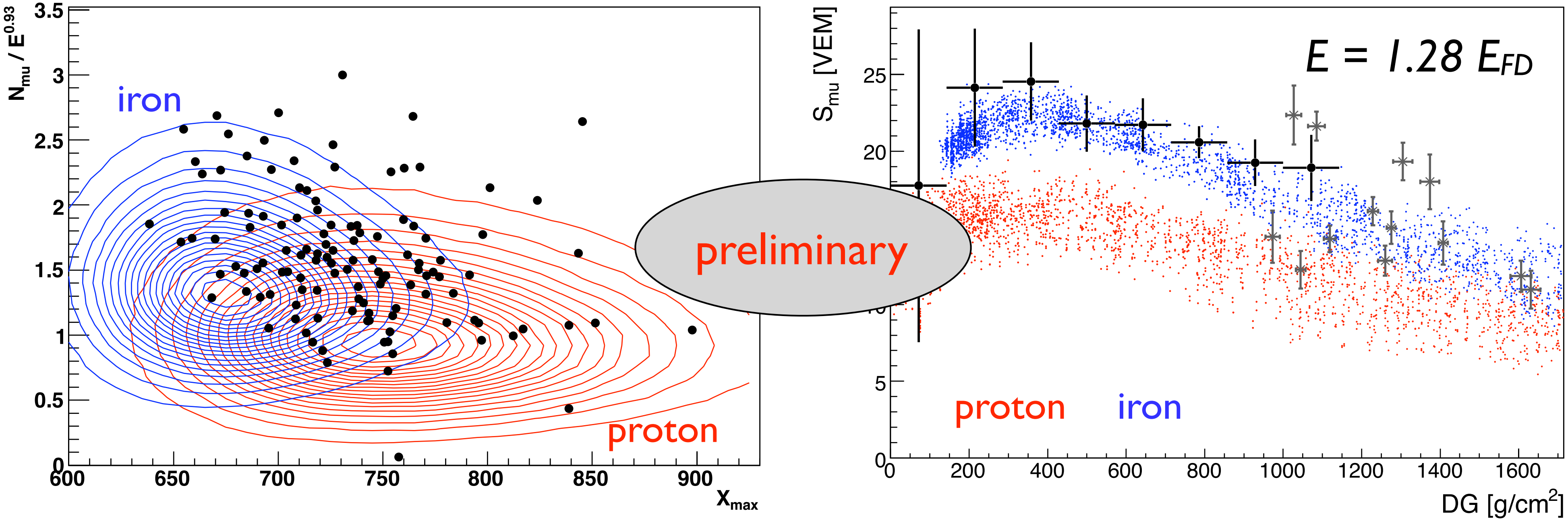


Auger ICRC 2007: golden hybrid events

Event-by-event measurement of muon signal, but lower statistics

$$S_\mu(E,\theta) = S(1000) - S_{\rm EM}(E,\theta,X_{\rm max})$$

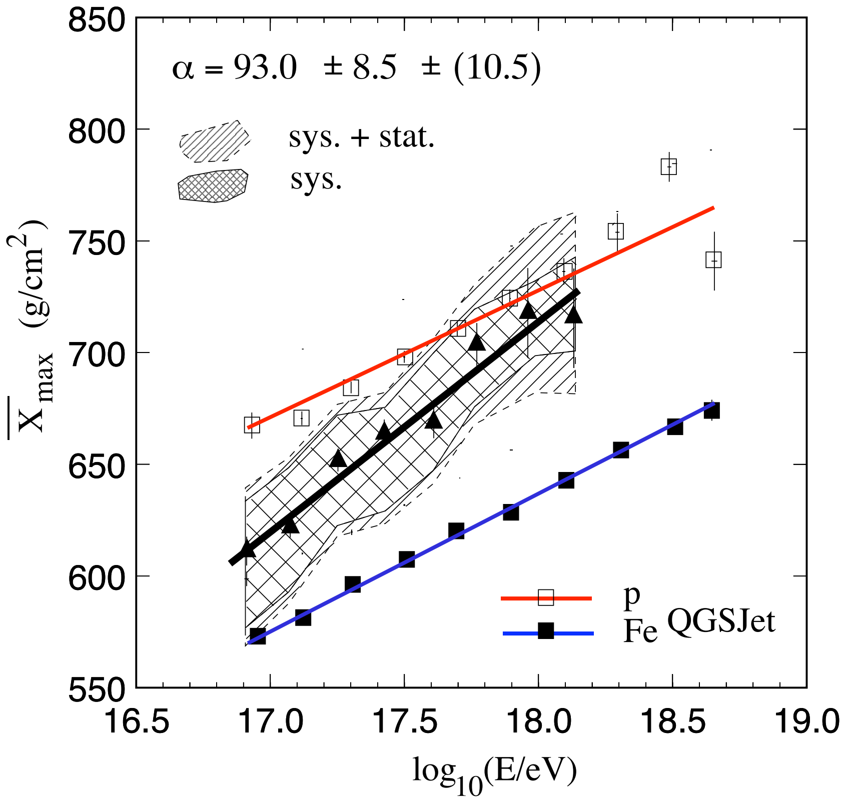
(muon density scaled to 10^{19} eV)

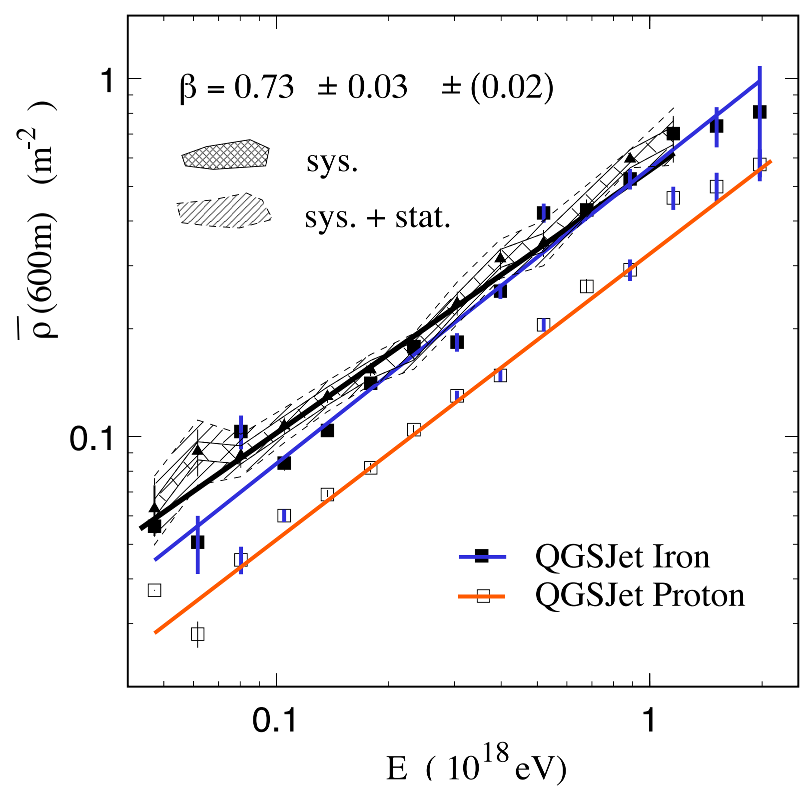


$$N_\mu^{\rm rel}\Big|_{E=1.28E_{\rm FD}} = 1.55 \pm 0.06 \qquad N_\mu^{\rm rel}\Big|_{E=E_{\rm FD}} = 1.92 \pm 0.08$$

HiRes-MIA hybrid measurement

Current models fail to describe shower data (new physics?)





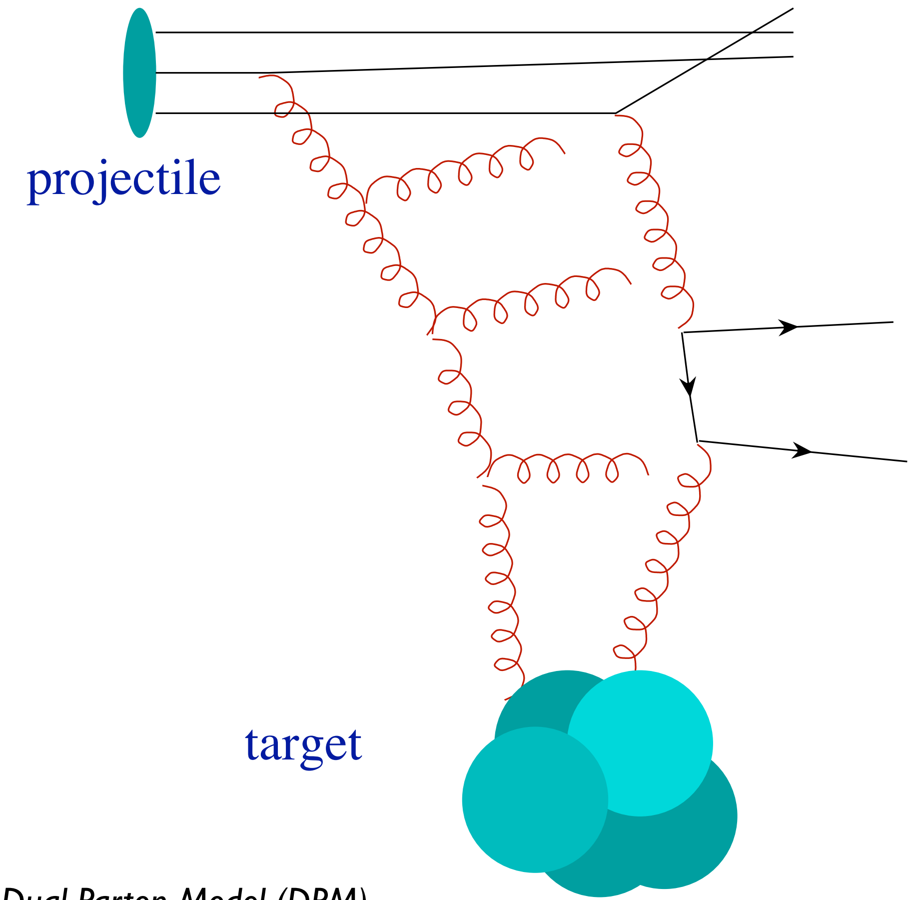
Depth of shower maximum: fast transition from heavy to light composition

Muon density measurement at 600m from shower core: very heavy primary particle expected

HiRes Fly's Eye and MIA Collabs., Phys. Rev. Lett. 84 (2000) 4276

QCD aspects of modeling hadronic interactions in air showers

Model assumptions



Dual Parton Model (DPM)
Quark-Gluon Strings Model (QGS)

- Gribov-Regge theory (pomeron)
- Minijets (cross section, multiplicity)
- Multiple interactions
- Unitarization of Born amplitudes
- Projectile / target remnants
- Glauber approximation for nuclei

- Many phenomenological model parameters

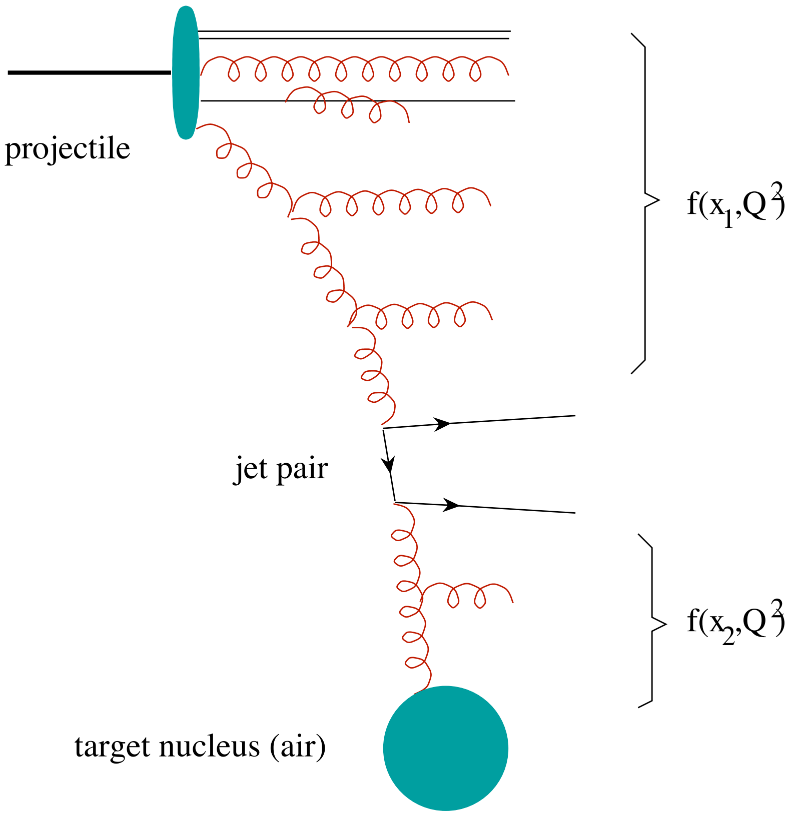
Examples:

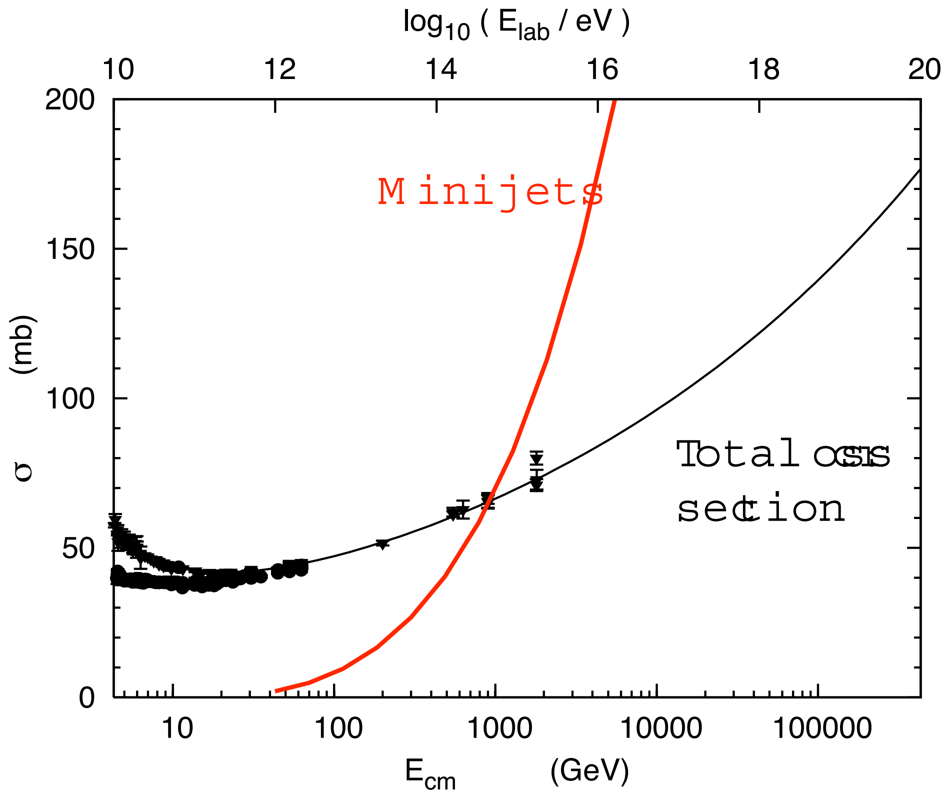
- pQCD pt cutoff (energy-dependence)
- Factorization scale / k-factor
- Energy sharing for hadron remants
- Soft multiple interactions
- Diffraction dissociation

What is the inclusive minijet cross section at ultra-high energy

- Predictions of pert. QCD valid for inclusive quantities (summed over all possible final states)
- Range in energy and transverse momentum
- DGLAP, BFKL, ..., JIMWLK evolution,
  collinear vs. kt factorization (or CGC factorization?)
- Leading vs. next-to-leading order, K-factor, factorization scale, energy conservation

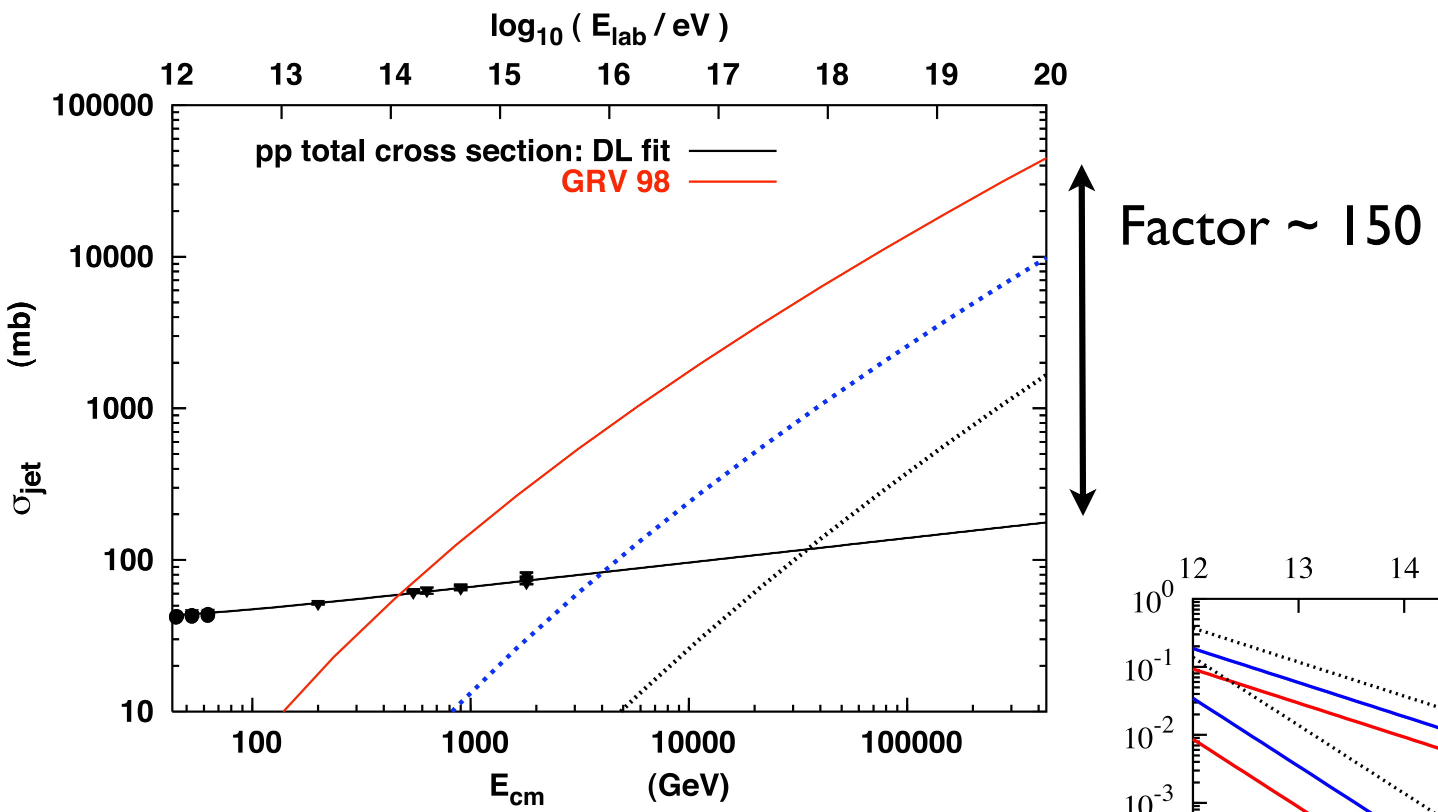
QCD parton model: minijets



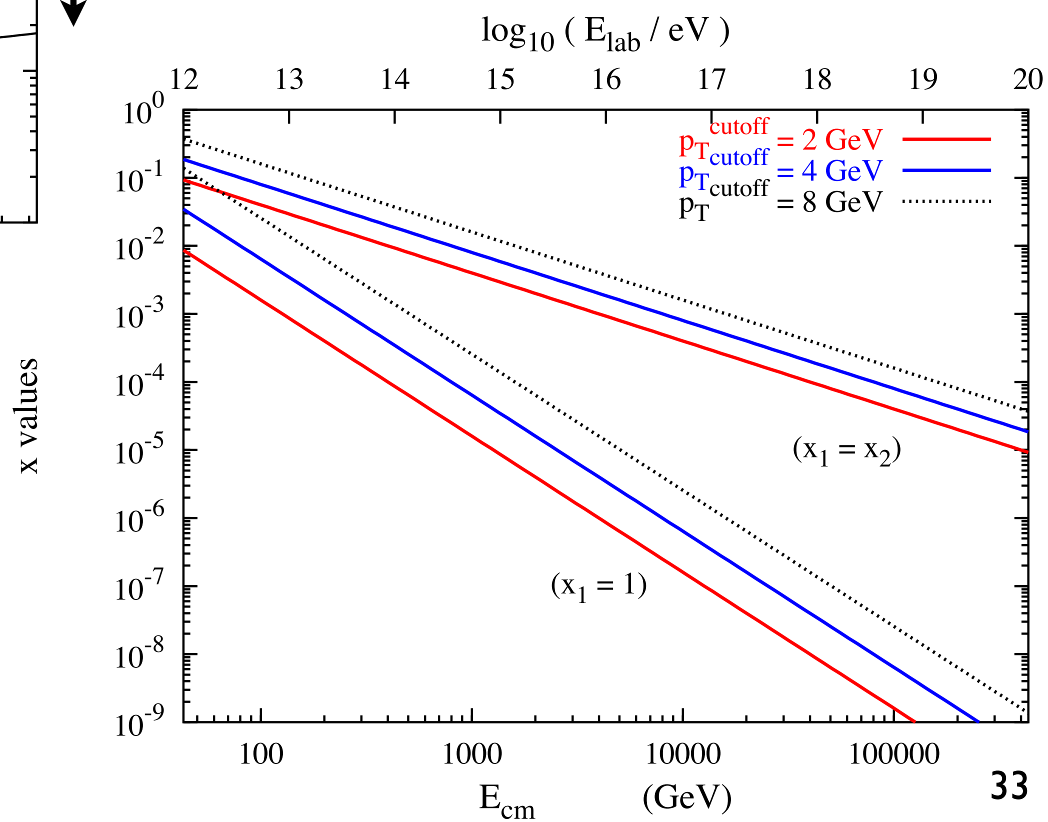


$$\sigma_{\mathrm{QCD}} = \sum_{i,j,k,l} \frac{1}{1+\delta_{kl}} \int dx_1 dx_2 \int_{p_\perp^{\mathrm{cutoff}}} d^2 p_\perp \; f_i(x_1, Q^2)\; f_j(x_2, Q^2)\; \frac{d\sigma_{i,j\to k,l}}{d^2 p_\perp}$$

Cutoff dependence and x values



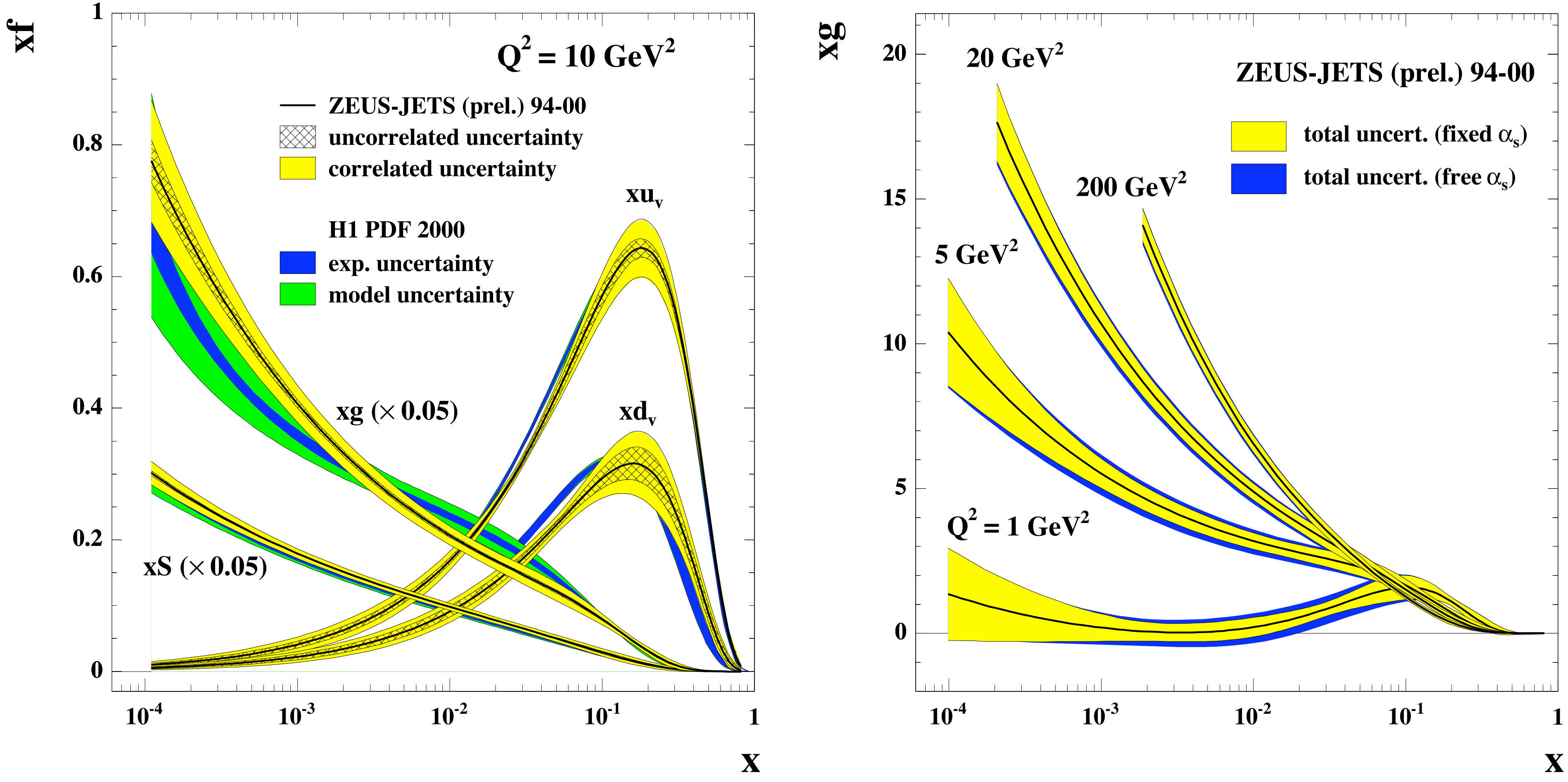
$$\hat{s} = x_1 x_2 s \geq 4p_\perp^2$$



Average number of minijet pairs

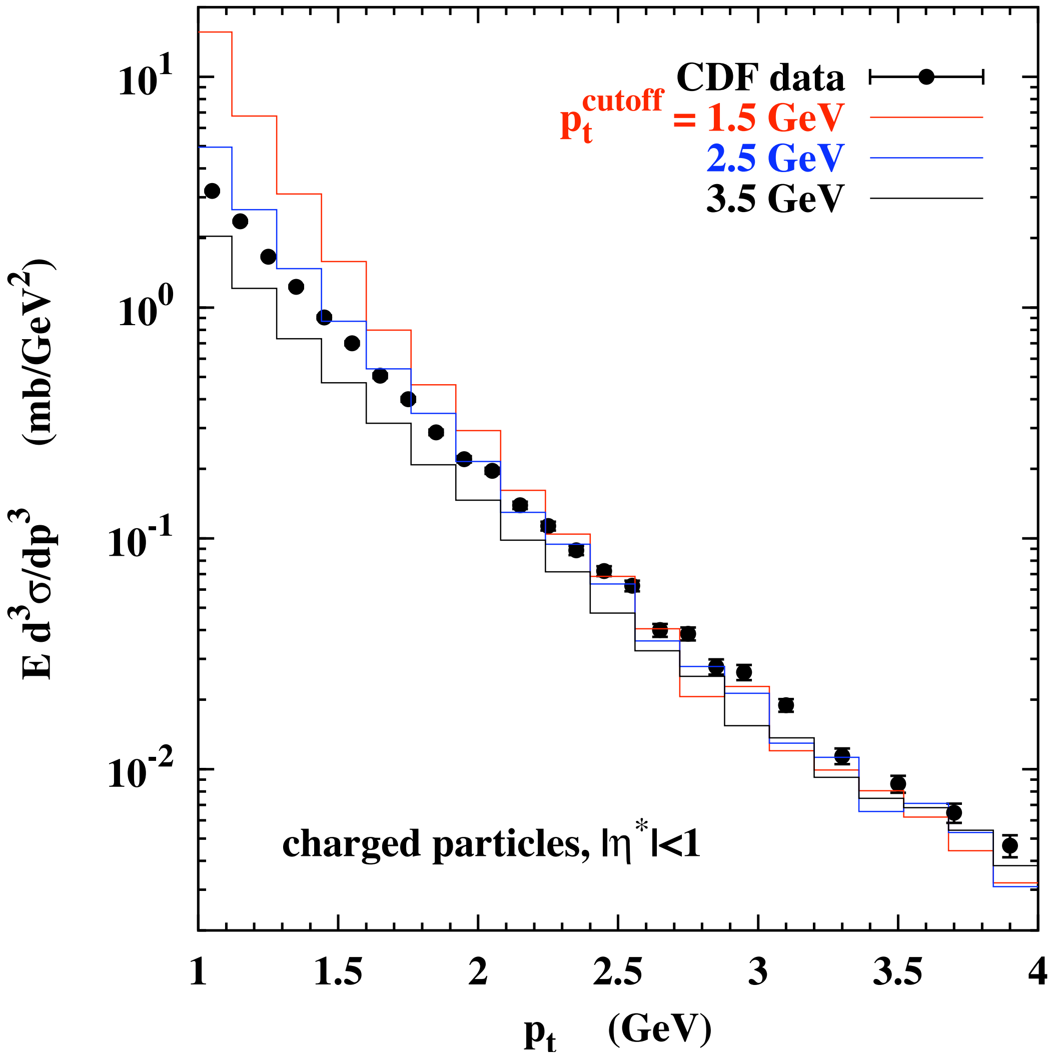
$$\langle n_{\rm jet} \rangle = \frac{\sigma_{\rm QCD}}{\sigma_{\rm ine}}$$

Parton density fits to HERA data



(HERA data, from review by Chekelian 2005)

DGLAP phenomenology very successful



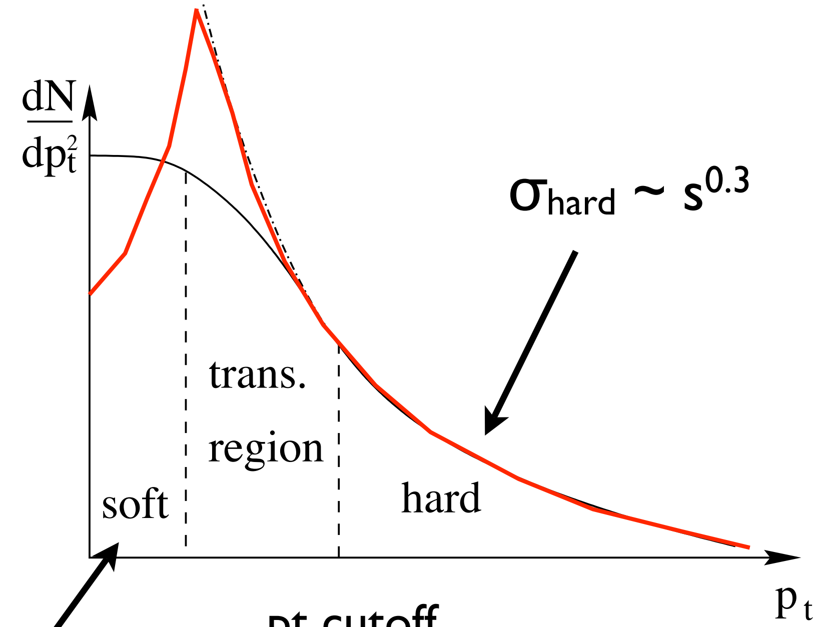
DGLAP evolution and collinear factorization:

- Calculation of inelastic graphs
- Straight-forward interpretation
- Energy conservation
- Initial- and final state radiation (parton showers)

- Not expected to be applicable at very low x !

- Optimum transverse momentum cutoff for data description varies with energy and considered process

Conceptual problem: matching soft/hard

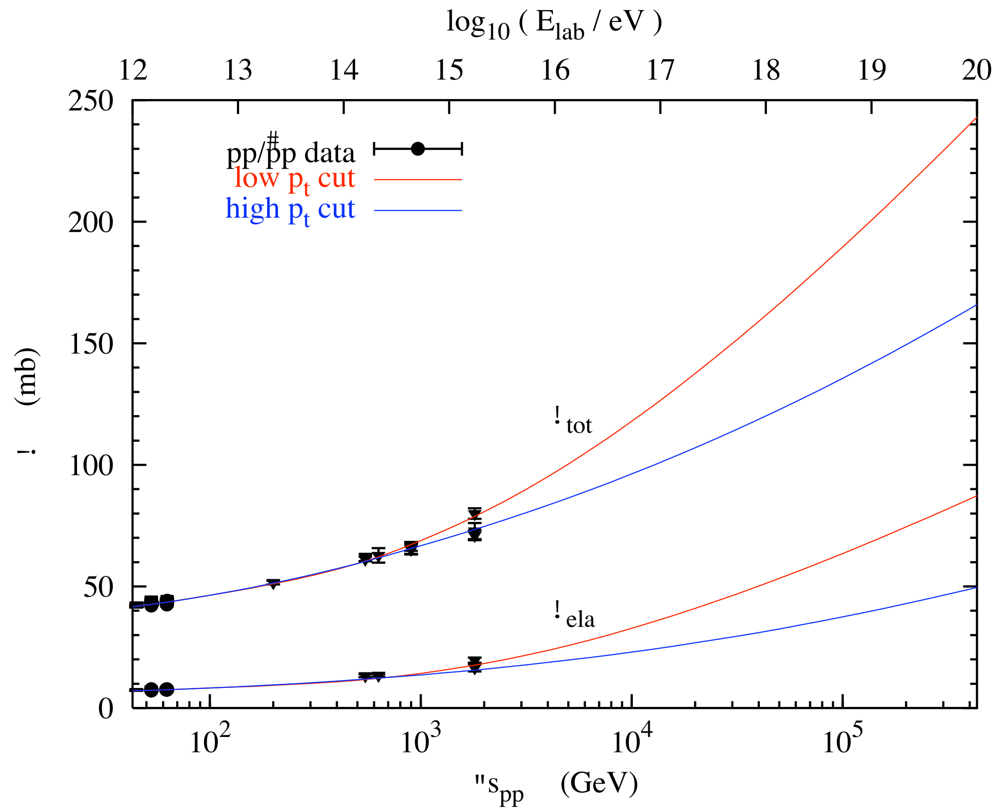


$\sigma_{hard} \sim s^{0.3}$

pt cutoff

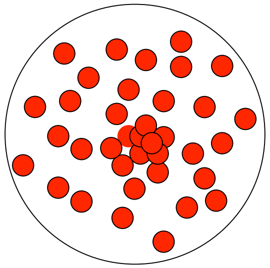
$\sigma_{soft} \sim s^{0.1}$

Example: two cross section fits
(after unitarization)

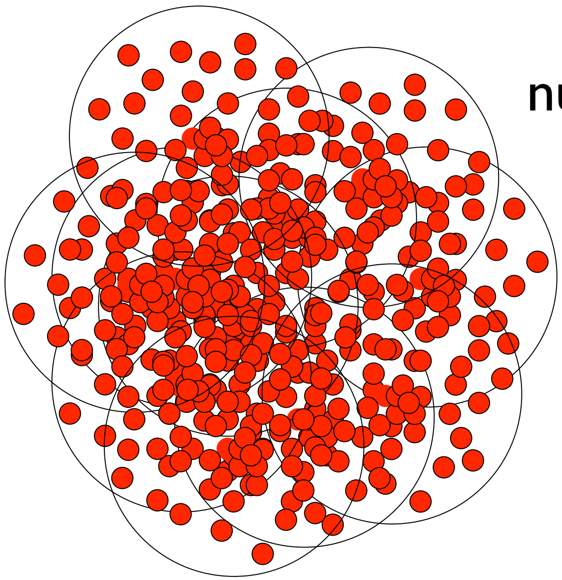


Does the transition region shift with energy?

High parton densities



nucleon



nucleus

SIBYLL: simple geometrical criterion

$$\pi R_0^2 \simeq \frac{\alpha_s(Q_s^2)}{Q_s^2} \cdot xg(x, Q_s^2)$$

$$xg(x,Q^2) \sim \exp\left[\frac{48}{11-\frac{2}{3}n_f}\ln\frac{\ln\frac{Q^2}{\Lambda^2}}{\ln\frac{Q_0^2}{\Lambda^2}}\ln\frac{1}{x}\right]^{\frac{1}{2}}$$

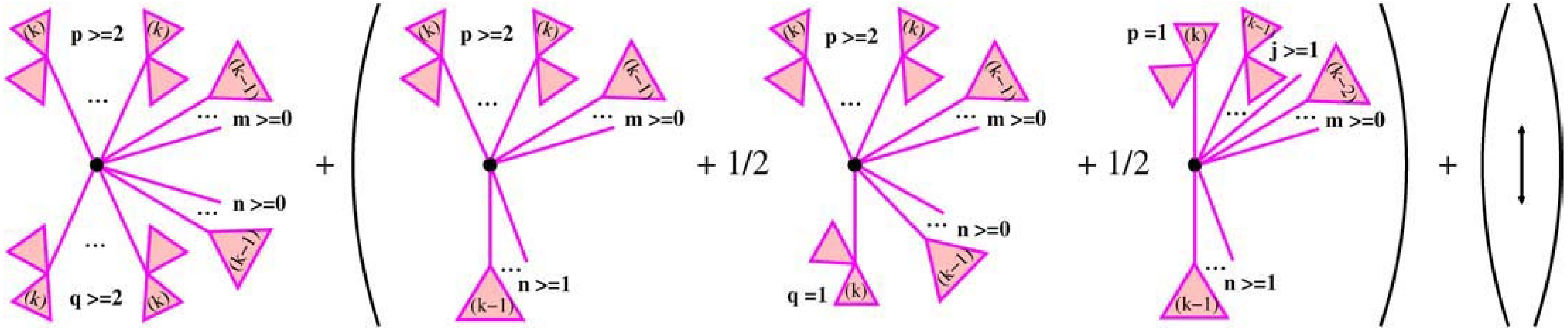
No dependence on impact parameter !

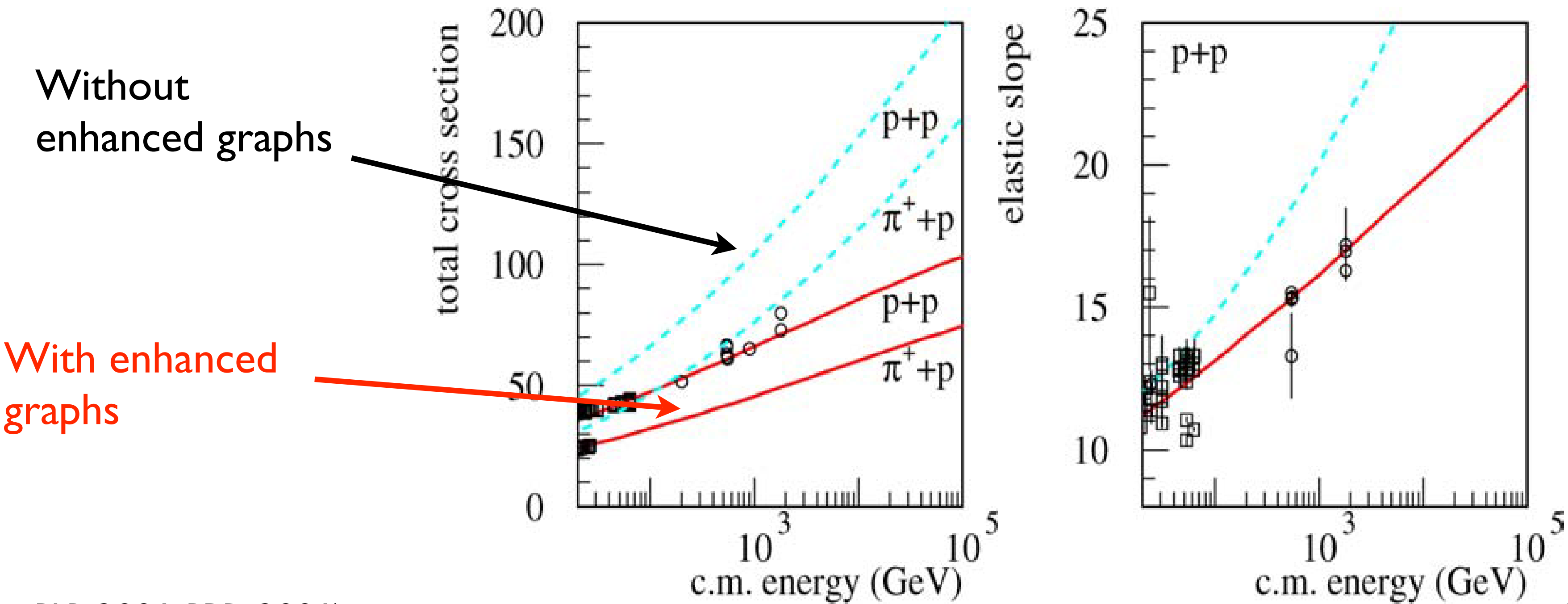
SIBYLL: $p_\perp(s) = p_\perp^0 + 0.065\text{GeV}\exp\left\{0.9\sqrt{\ln s}\right\}$

DPMJET: $p_\perp(s) = p_\perp^0 + 0.12\text{GeV}\left(\log_{10}\frac{\sqrt{s}}{50\text{GeV}}\right)^3$

QGSJET: high parton density effects

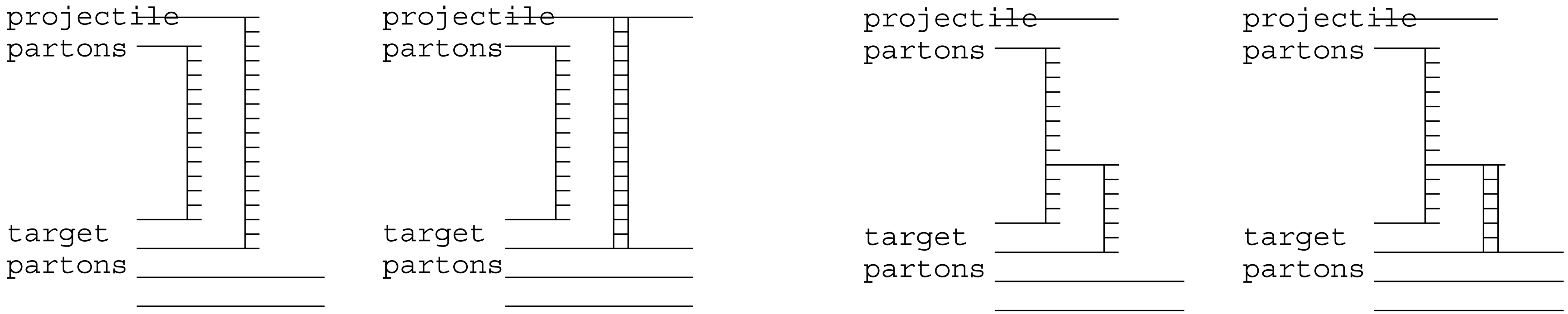
Re-summation of enhanced pomeron graphs





(Ostapchenko, PLB 2006, PRD 2006)

EPOS: high parton density effects (i)

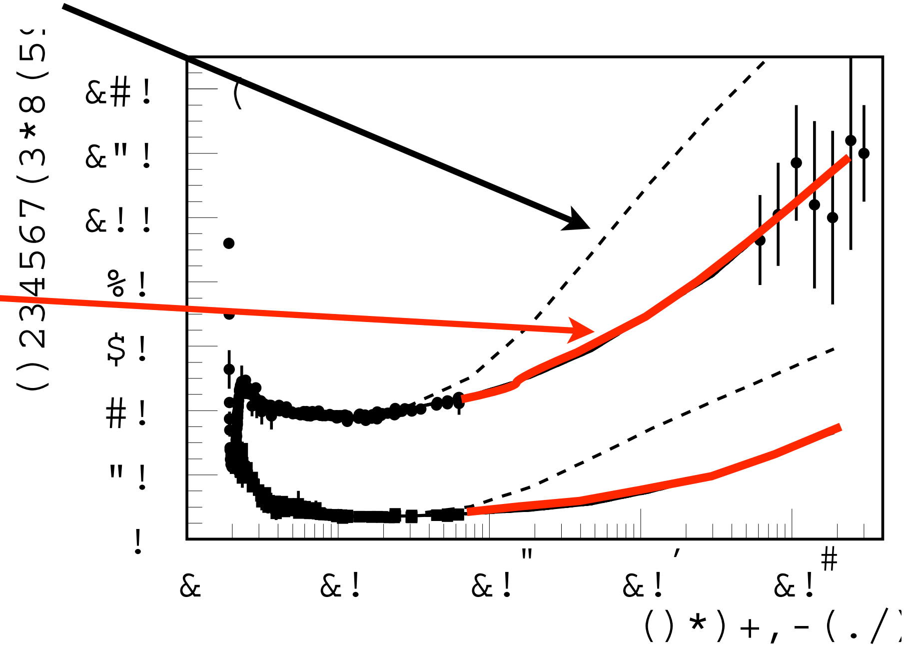


No effective coupling

$$A_{\mathrm{pom}} \sim (x_1 x_2)^{\beta}$$

With effective coupling

$$A_{\mathrm{pom}} \sim x_1^{\beta} x_2^{\beta-\varepsilon}$$



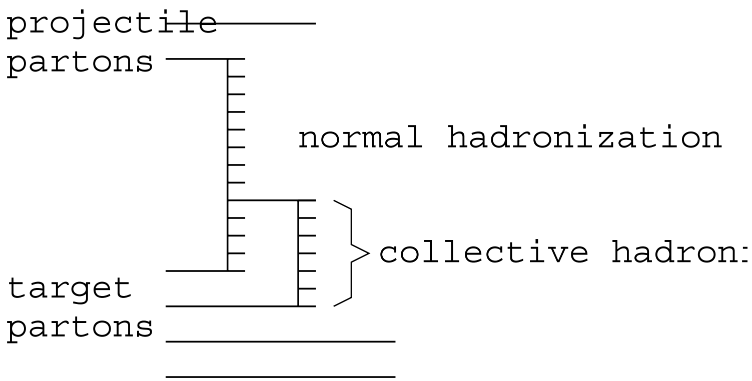
Parametrization

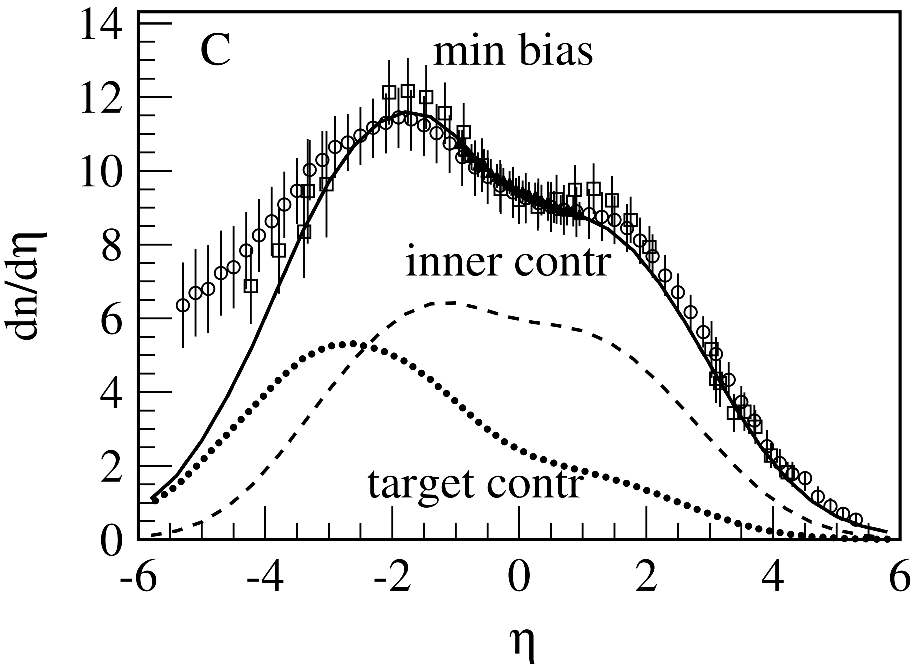
S = aS SZ ,

H = aH HZ ,

(Werner et al., PRC 2006)

EPOS: high parton density effects (ii)





Coefficient	Corresponding variable	Value
s_M	Minimum squared screening energy	$(25\,\mathrm{GeV})^2$
w_M	Defines minimum for z_0	6.000
w_Z	Global Z coefficient	0.080
w_B	Impact parameter width coefficient	1.160
a_S	Soft screening exponent	2.000
a_H	Hard screening exponent	1.000
a_T	Transverse momentum transport	0.025
a_B	Break parameter	0.070
a_D	Diquark break probability	0.110
a_S	Strange break probability	0.140
a_P	Average break transverse momentum	0.150

$$Z_T(i,j) = z_0 \exp\left(-b_{ij}^2/2b_0^2\right) + \sum_{j' \neq j}^{\text{target nucleons}} z_0 \exp\left(-b_{ij'}^2/2b_0^2\right),$$

$$b_0 = w_B \sqrt{\sigma_{inel}^{pp}/\pi}$$

$$z_0 = w_Z \log s/s_M,$$

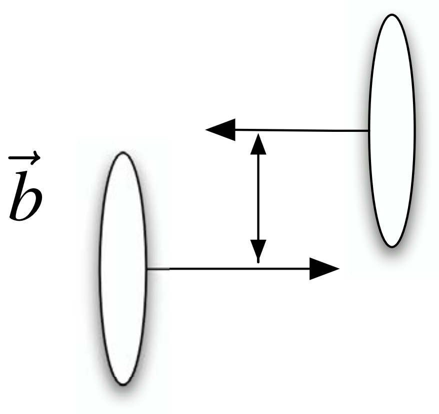
$$z_0 = w_Z \sqrt{(\log s/s_M)^2 + w_M^2},$$

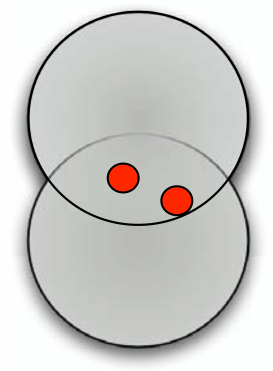
(Werner et al., PRC 2006)

Construction of exclusive final states

- Distribution of (multiple) interactions
- Parton configurations, color connection

Multiple hard interactions





Overlap function

$$\langle n(\vec{b}) \rangle = \sigma_{\mathrm{QCD}}\, A(s, \vec{b})$$

Independent interactions:
Poisson distribution

$$P_n = \frac{\langle n(\vec{b}) \rangle^n}{n!} \exp\left(-\langle n(\vec{b}) \rangle \right)$$

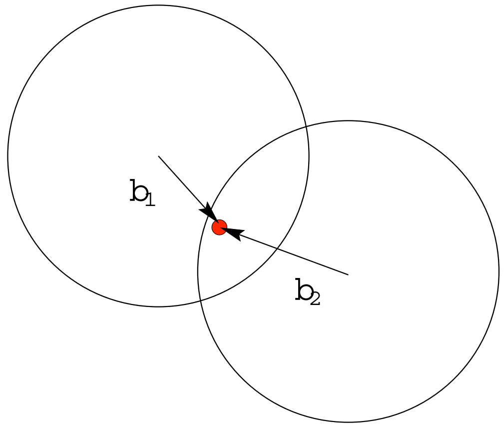
$$\sigma_{\mathrm{ine}} = \int d^2\vec{b} \sum_{n=1}^{\infty} P_n = \int d^2\vec{b} \left(1 - \exp\{ -\sigma_{\mathrm{QCD}}\, A(s, \vec{b}) \} \right)$$

Profile functions in SIBYLL

Fourier transform of em. form factor

$$F_p(q^2) = \left(1 + \frac{q_\perp^2}{\nu^2}\right)^{-2}$$

$$F_\pi(q^2) = \left(1 + \frac{q_\perp^2}{\mu^2}\right)^{-1}$$



Point-like hard interaction

$$A_{pp}^{\mathrm{hard}}(\nu_p, \vec{b}) = \int A_p(\nu_p, \vec{b}_1) A_p(\nu_p, \vec{b}_2)\ \delta^{(2)}(\vec{b}_1 - \vec{b}_2 - \vec{b})\ d^2\vec{b}_1\ d^2\vec{b}_2$$

$$A_{pp}^{\mathrm{hard}}(\nu_p, \vec{b}) = \frac{\nu_p^2}{96\pi}(\nu_p|\vec{b}|)^3 K_3(\nu_p|\vec{b}|)$$

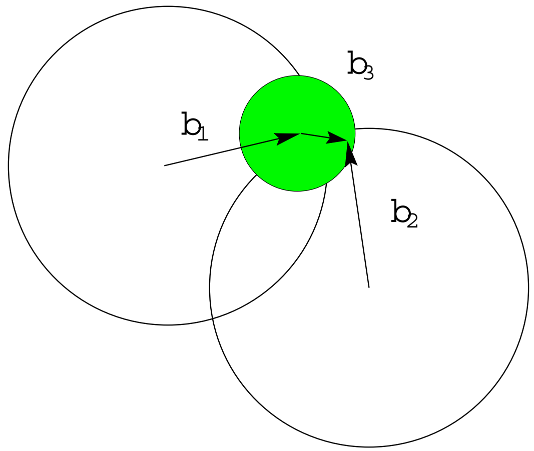
$$A_{\pi p}^{\mathrm{hard}}(\nu, \mu, \vec{b}) = \frac{1}{4\pi}\frac{\nu^2\mu^2}{\mu^2 - \nu^2}\left((\nu|\vec{b}|)K_1(\nu|\vec{b}|) - \frac{2\nu^2}{\mu^2 - \nu^2}\left[K_0(\nu|\vec{b}|) - K_0(\mu|\vec{b}|)\right]\right)$$

Profile functions in QGSJET and EPOS

Fourier transform of exponential

$$A_{\rm soft}(s,\vec{b}) \quad = \quad \frac{1}{4\pi B_s(s)} \exp\left\{-\frac{\vec{b}^2}{4B_s(s)}\right\}$$

$$B_s(s) = B_0 + \alpha'_{\rm pom}(0)\ln\left(\frac{s}{s_0}\right)$$



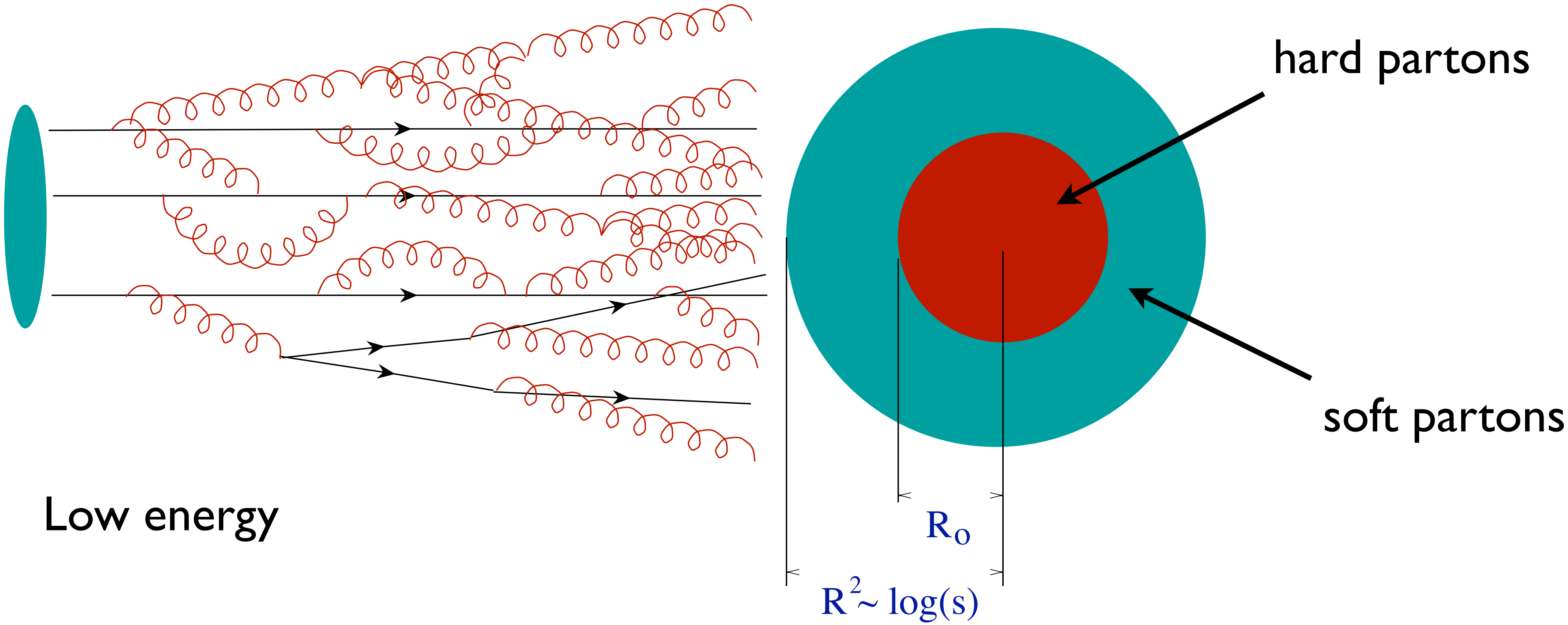
$$A_{pp}^{\rm pom}(x_1,x_2,s,\vec{b}) = \int A_{\rm soft}(s_1,\vec{b}_1)\, A_{\rm soft}(s_2,\vec{b}_2)\, A_{\rm hard}(x_1x_2s,\vec{b}_3)\, \delta^{(2)}(\vec{b}_1-\vec{b}_2+\vec{b}_3-\vec{b})\, d^2\vec{b}_1\, d^2\vec{b}_2\, d^2\vec{b}_3$$

$$A_{pp}^{\rm pom}(x_1,x_2,s,\vec{b}) = \frac{1}{4\pi B_{\rm eff}} \exp\left\{-\frac{\vec{b}^2}{4B_{\rm eff}}\right\}$$

Transverse size depends on kinematics

$$B_{\rm eff} = B_s(s_1) + B_s(s_2) + B_h(x_1x_2s)$$

Impact parameter diffusion



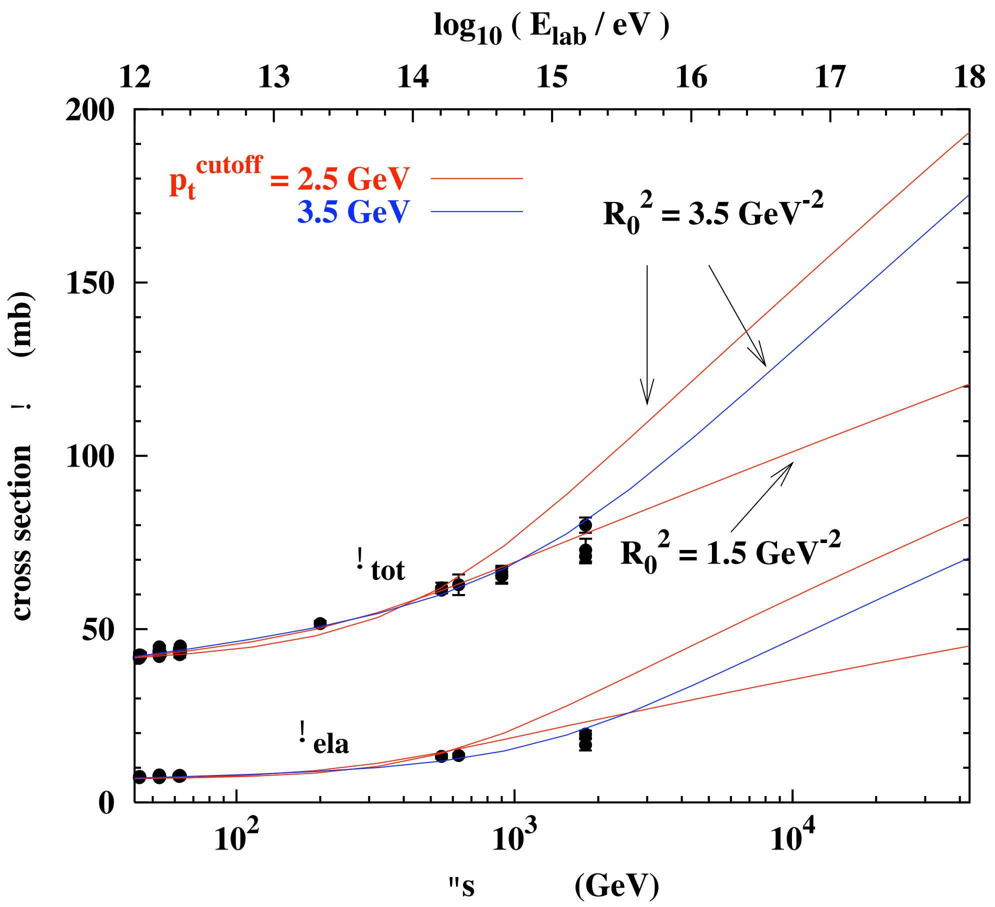
High energy

$$\Delta\vec{b}\cdot\Delta\vec{p}_{\perp} \sim 1$$

$$A_{\text{soft}}(s,\vec{b}) = \frac{1}{4\pi R^2(s)}\exp\left\{-\frac{\vec{b}^2}{4R^2(s)}\right\}$$

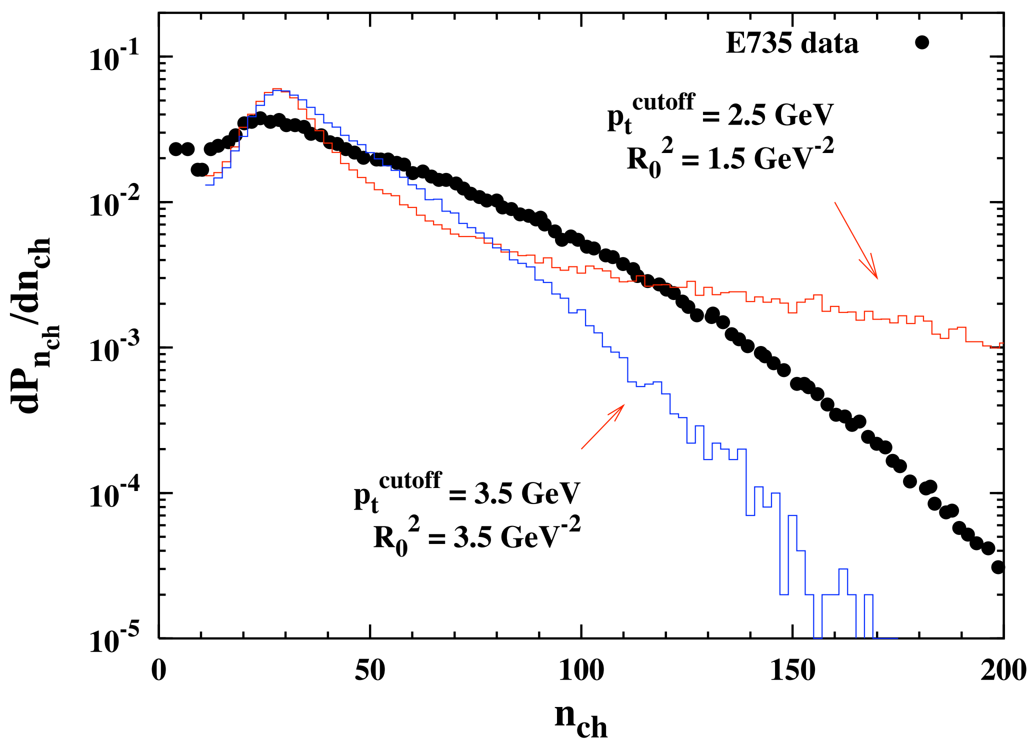
$$R^2(s) = R_0^2 + \alpha' \ln s$$

Correlation of hard cross section and impact parameter profile



Cross section fits with energy-independent Gaussian profiles for hard interaction

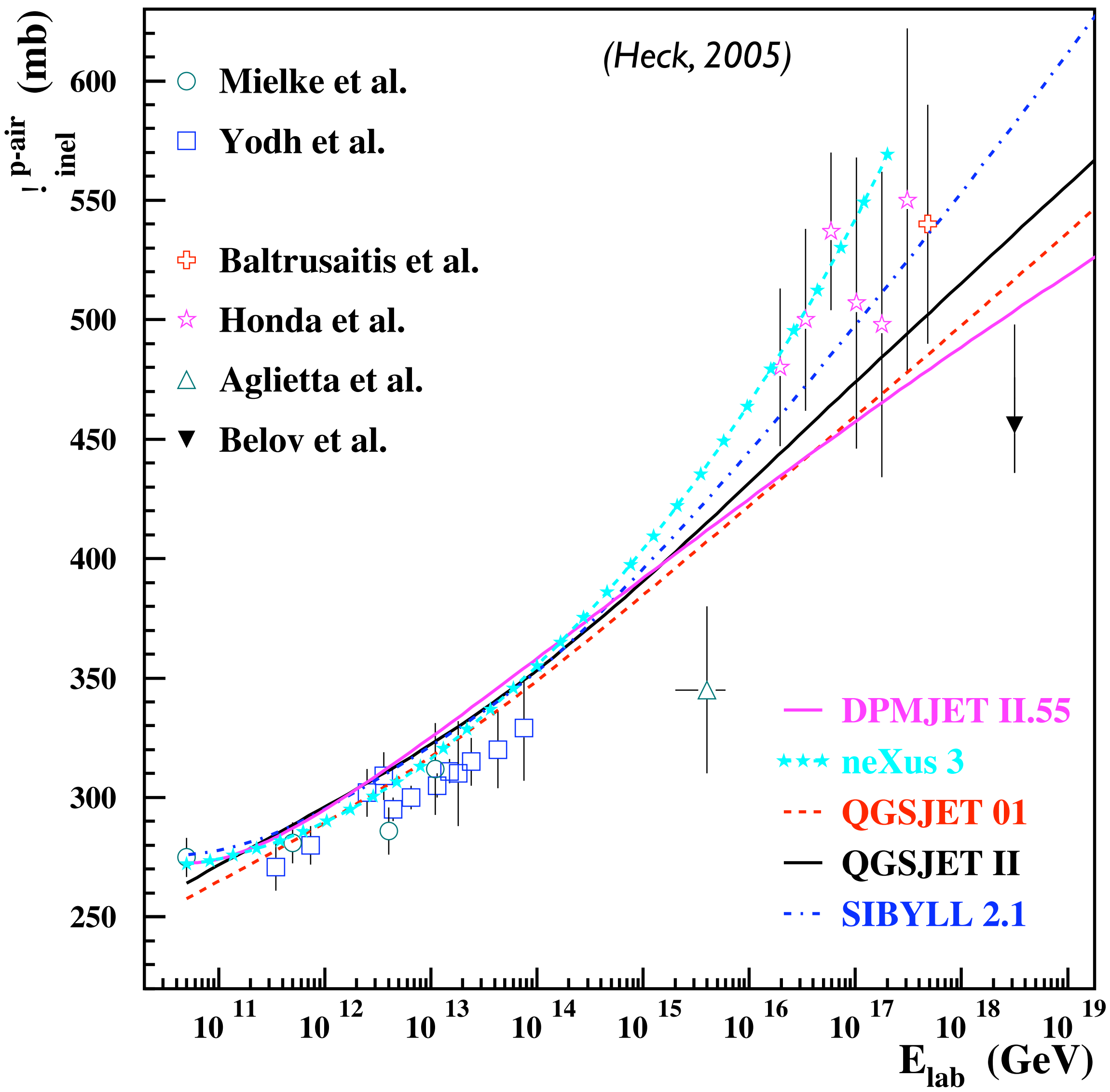
Charged particle multiplicity distribution



Example: proton-air cross section

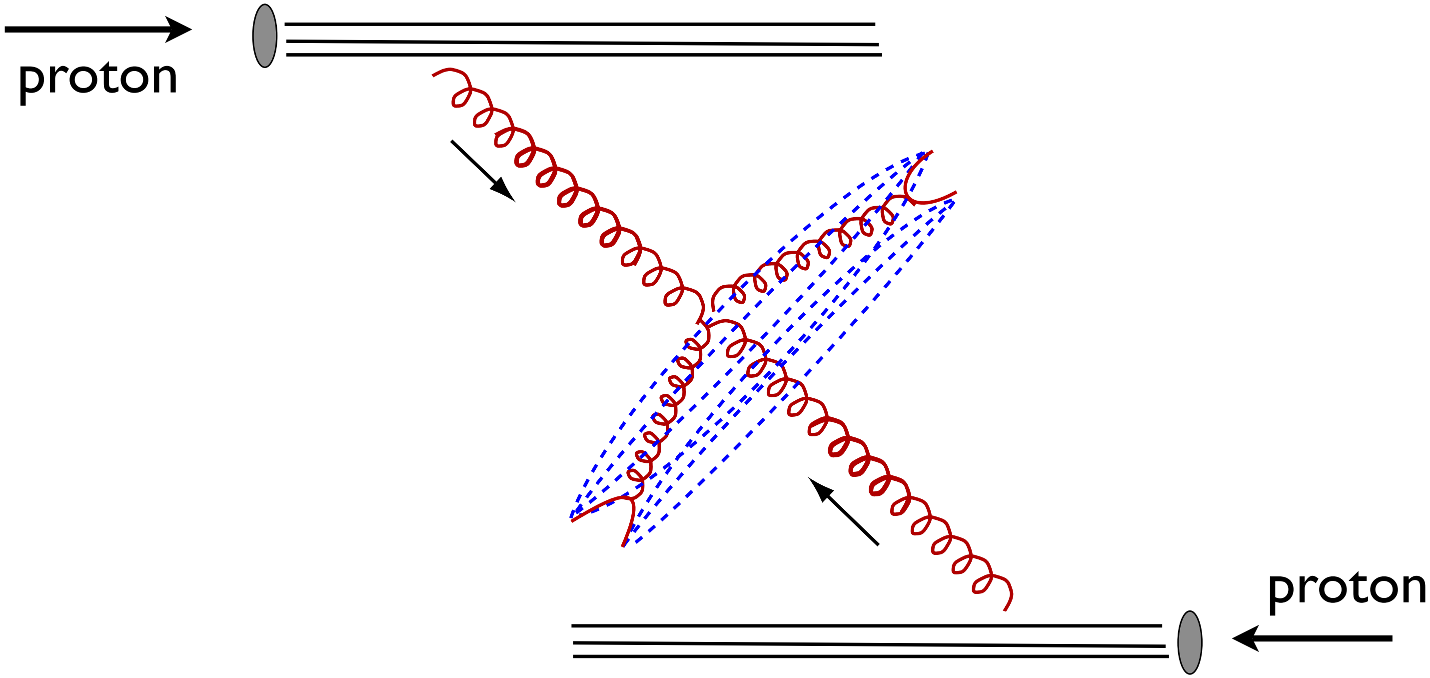
DPMJET: moderate cross section increase (Gaussian profile, energy-dep. cutoff)

SIBYLL: fast increase of cross section with energy (form factor profile)



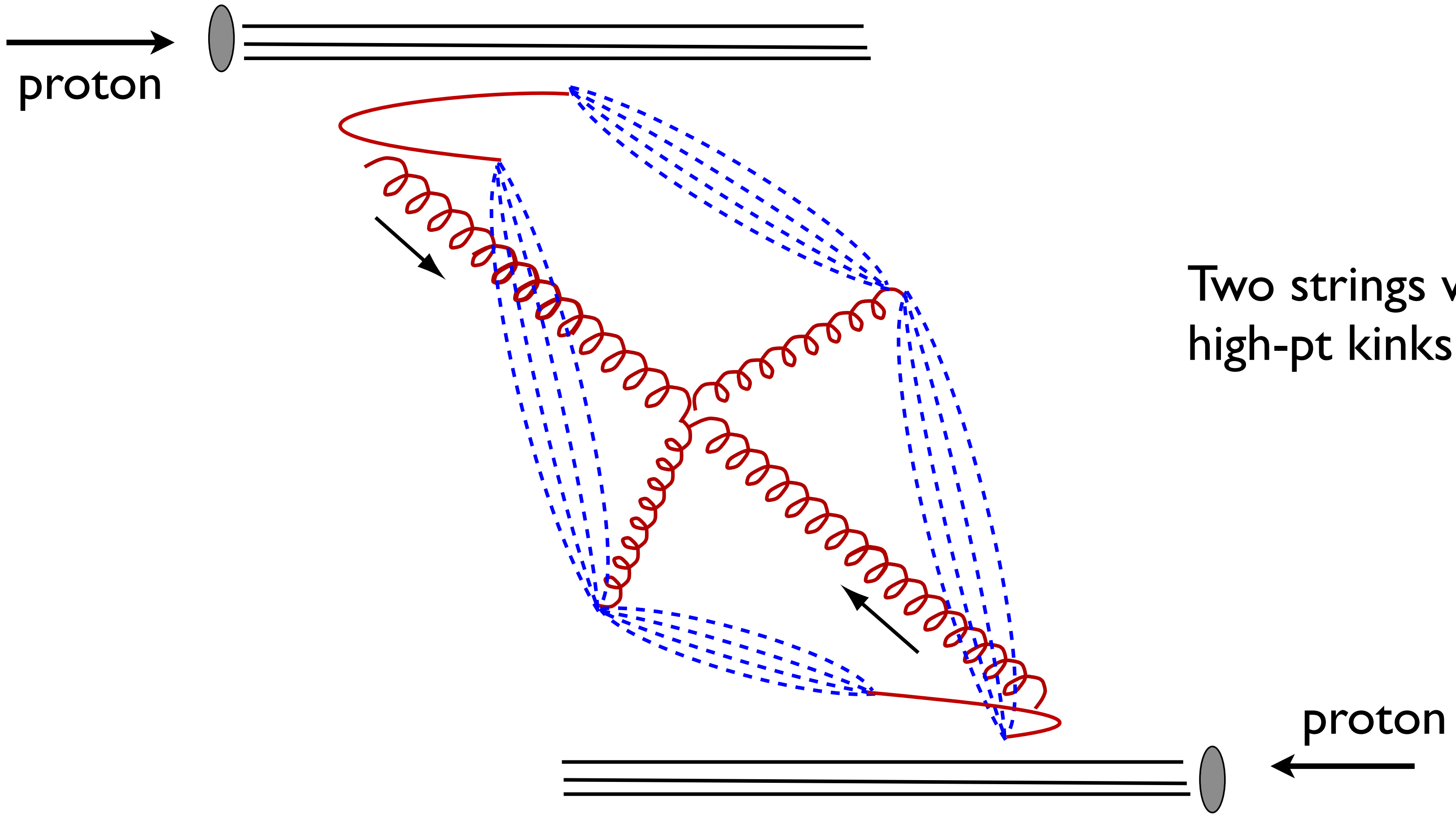
Details: PRD66 (2002) 033011

Two-gluon scattering: SIBYLL & DPMJET II



Kinematics etc. given by parton densities and perturbative QCD
Two strings stretched between quark pairs from gluon fragmentation

Two-gluon scattering: QGSJET

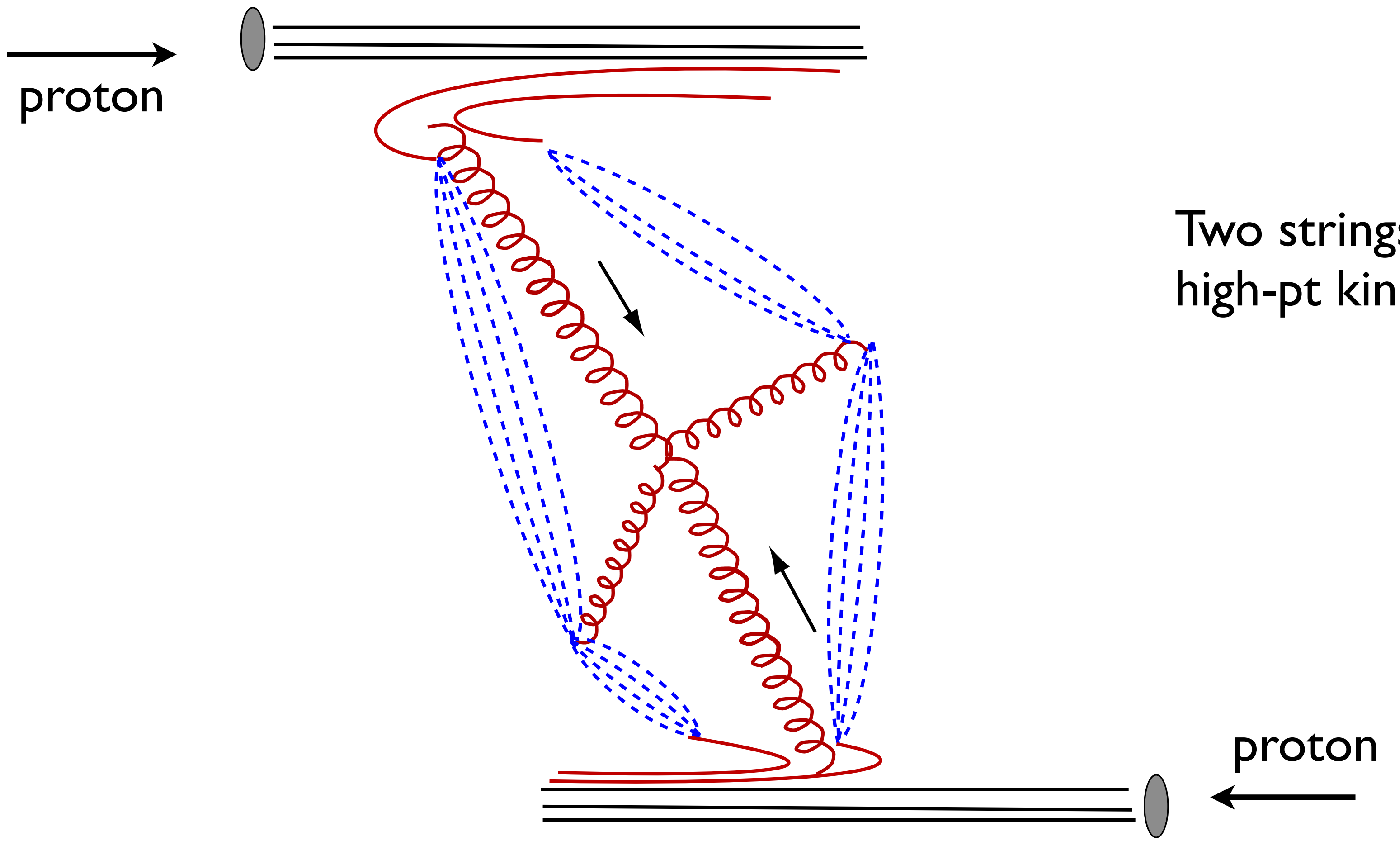


Two strings with high-pt kinks

Sea quark pairs form end of strings, generated from model distribution

$$\frac{dP}{dx} \sim \frac{1}{\sqrt{x}}$$

Two-gluon scattering: EPOS



Two strings with high-pt kinks

Independent sea quarks form string ends for color neutral building block

Sensitivity to physics of first interaction

Muon production:

$$N_\mu = \left(\frac{E_0}{E_{\rm dec}}\right)^\alpha \qquad \alpha = \frac{\ln(n_{\rm ch})}{\ln(n_{\rm tot})} \approx 0.9$$

$$N_\mu = n_{\rm ch}^{(\rm first)} \left(\frac{E_0}{n_{\rm tot}^{(\rm first)} E_{\rm dec}}\right)^\alpha = k^{1-\alpha} \left(\frac{E_0}{E_{\rm dec}}\right)^\alpha$$

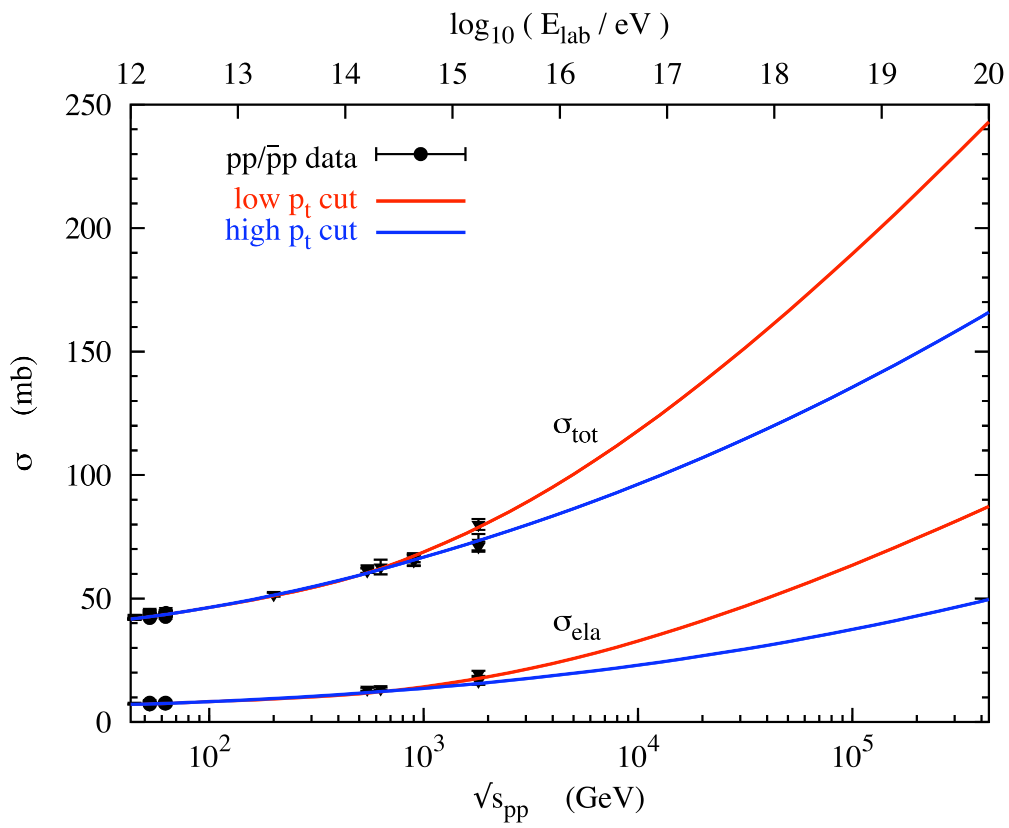
Multiplicity increase by
factor of 2: 5 -7% more muons,
factor of 10: 25% more muons

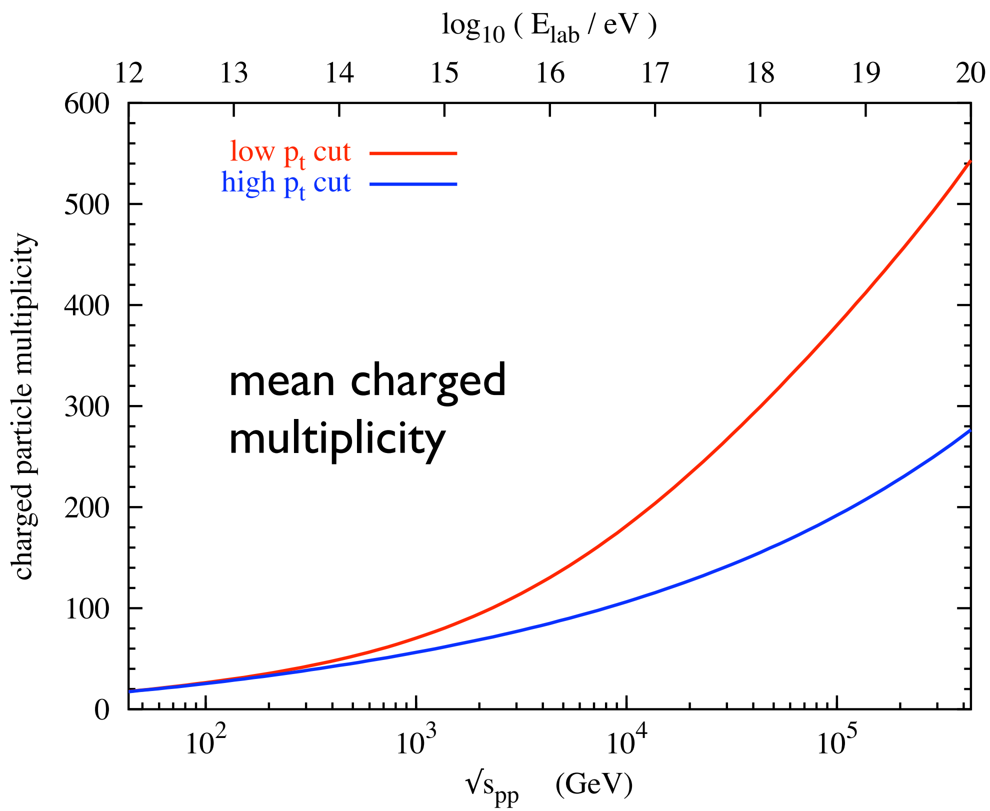
Electromagnetic component: much higher sensitivity

Importance of hard cross section

Fit of SIBYLL 2.1 with different energy dependence of transverse momentum cutoff

$$p_\perp(s) = p_\perp^0 + 0.065\text{GeV} \exp\left\{0.9\sqrt{\ln\, s}\right\}$$

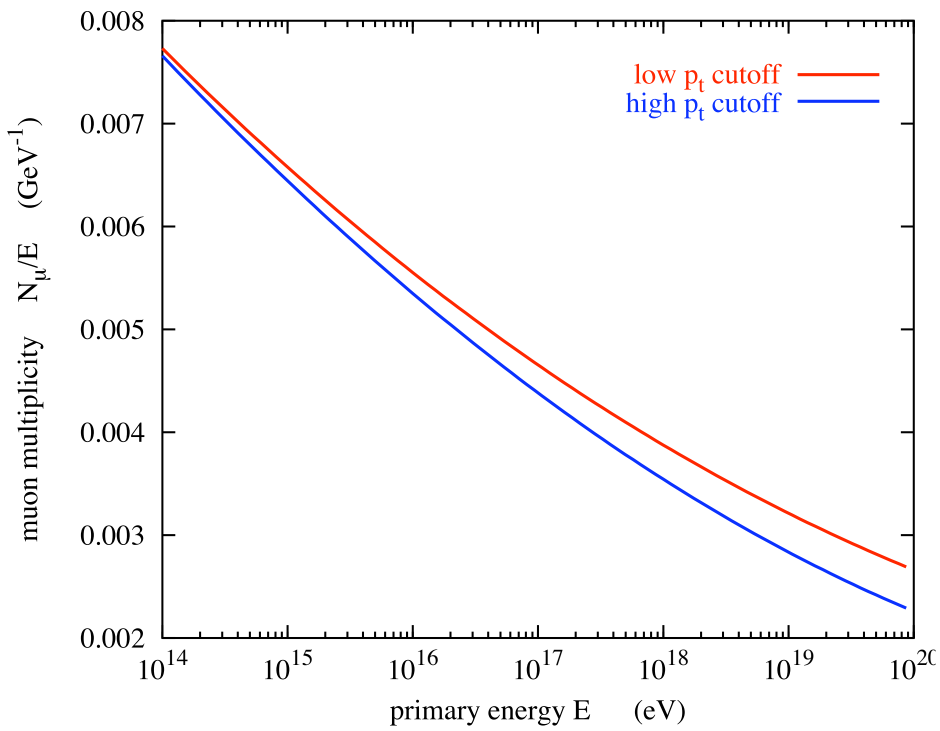




(see also talk by H.-J. Drescher)

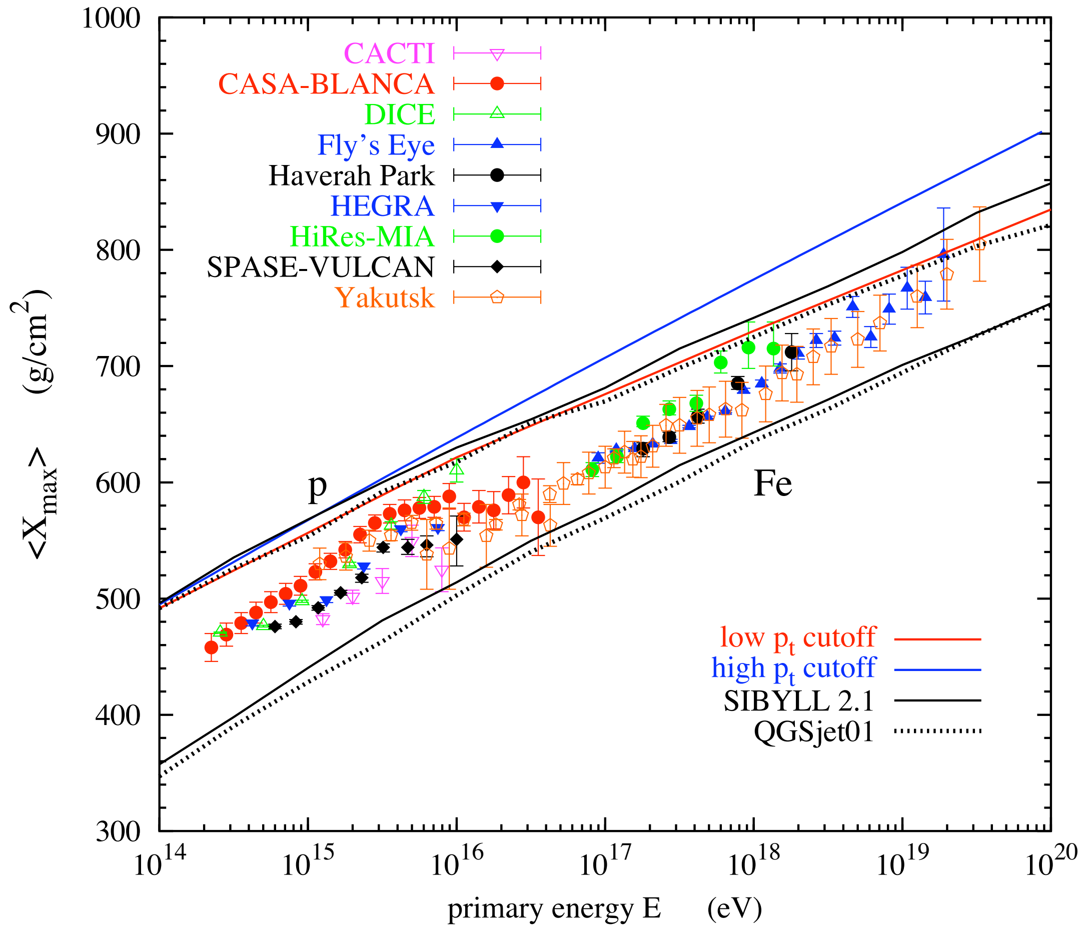
Air shower predictions

Muon multiplicity



Change: <15%

Shower depth of maximum



Change: up to 65 g/cm^2

Conclusions

- Discovery age of astroparticle physics
- Strong dependence of composition analysis on air shower modeling
- Data quality very high: constraints on interaction models
  - *hybrid measurements*
  - *distribution edges*
  - *energy regions with almost mono-elemental composition*
- Many open questions in pert. QCD lead to considerable uncertainty of model extrapolation to high energy
  - *inclusive cross section (evolution equations, saturation, factorization, ...)*
  - *multiple interaction (profile function, unitarization, correlations, ...)*
  - *combination with non-perturbative concepts (strings, regge theory, ...)*