The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

SMR/1842-5

International Workshop on QCD at Cosmic Energies III

28 May - 1 June, 2007

Unitarity Signatures in Cosmic Energies

U. Maor
Tel-Aviv University
Israel

Unitarity Signatures in Cosmic Energies

Uri MAOR
Tel-Aviv University

Collaboration: Gotsman, Levin (GLM)
Naftali, Kormilizin, Prygarin

Introduction:

The aim of this presentation is to search and identify unambiguos s-channel unitarity signatures in presently available high energy soft and hard diffractive processes and assess the implications of this study in Cosmic Energies.

The main experimental signature of a diffractive process is a large rapidity gap (LRG) in the $\eta-\phi$ lego plot of an hadronic final state devoid of produced hadrons.

In a QCD context a LRG signals that no colour has been exchanged between the projectiles (hadrons or partons) initiating the diffractive process - elastic or inelastic.

We shall refer to such a colourless exchange as a "Pomeron" ($\mathbb{P}$).

The $\mathbb{P}$ is presented in QCD by an exchange of a gluon ladder (2g exchange in the lowest order). It is convenient to specify the $\mathbb{P}$ in a Regge language. We make the distinction between:

i) A soft non perturbative $\mathbb{P}$ whose Regge trajectory intercept is $\alpha_s(0)-1 \simeq 0.1$. The relevant calculations are executed using a phenomenological Regge parametrization.

ii) A hard perturbative $\mathbb{P}$ whose trajectory intercep is $\alpha_H(0)-1 > 0.2$. The relevant calculations are done in pQCD.

The transition between the two sectors in DIS is smooth.

As we shall see, the experimental data enforces a severe small x reduction of the calculated diffractive channel rates. The two most dramatic manifestations of this need are:

1) A significant difference between the power like energy dependence of soft σ_{el} and the very mild energy dependence of soft σ_{diff}.
2) The hard LRG di-jets production rates are an order of magnitude smaller than the pQCD (or Regge) estimates.

Following Bjorken and GLM (1993-5) the correcting damping factor $S^2_{LRG} = \frac{\sigma_{LRG}(out)}{\sigma_{LRG}(in)}$ is called "LRG survival probability". The recent vitality of this research is associated with its relevance to the forthcoming LHC and Auger results. In particular, this approach is crucial to any realistic estimates of Higgs production via diffractive channels.

In general, the survival probability can be percieved as a product of 3 independent factors

$$S^2(s, M^2_{diff}, \Delta y) = S^2_S(s, M^2) \cdot S^2_{g\text{-}rad}(\Delta y) \cdot S^2_H(s)$$

1) S^2_S is due to the soft rescattering of the spectator hadrons or partons. It is a consequence of s-channel unitarity constraints imposed in the soft npQCD sector. Its calculation is coupled to a global fit of the soft scattering data base.

2) $S^2_{g\text{-}rad}$ is initiated by gluon radiation emitted by the partons taking part in the hard LRG diffractive sub process. This correction is strongly suppressed by the Sudakov factor which is included in the calculation of the diffractive subprocess.

3) S^2_H is a consequence of possible unitarity corrections in the hard sector initiated by high density gluons. It is commonly ignored, even though it may be relevant in DIS at small (x, Q^2).

$\Rightarrow$ Present estimates: $S^2_{LRG} = S^2_S$!

S-channel Unitarity and the Eikonal Model:

The Donnachie-Landshoff (DL) Regge parametrizat[ion] has been a bench mark of high energy phenomenology for the last 25 years. It reproduces well all two body elastic scattering with $P_L > 10$ GeV. Its specificatio[n] of the soft $\mathbb{P}$ is $\alpha_{\mathbb{P}}(t) = 1 + \epsilon + \alpha'_{\mathbb{P}} t$, where, $\epsilon = 0.081$ and $\alpha'_{\mathbb{P}} = 0.25$ GeV^{-2}. This simple parametrization is bound to violate unitarit[y] as $\sigma_{tot} \propto s^{\epsilon}$ while $\sigma_{el} \propto s^{2\epsilon}$. The intriguing question is at what energy this becomes a realistic problem? The discussion of thi[s] issue is executed in impact parameter b-spa[ce].

For simplicity lets write the unitarity equation for a diagonal scattering matrix, i.e. rescatterings in the initial and final state are strictly elastic.

$$2\,\mathrm{Im}\, a_{el}(s,b) = |a_{el}(s,b)|^2 + G_{in}(s,b)$$

i.e. $$\sigma_{tot}(s,b) = \sigma_{el}(s,b) + \sigma_{in}(s,b).$$

A general solution to the unitarity Eq. is: $$a_{el}(s,b) = i\left(1 - e^{-\frac{1}{2}\Omega(s,b)}\right).$$

04/07

The generality of the above holds as long as the opacity Ω is arbitrary! In this case the unitarity bound is $|a_{el}(s,b)| \leq 2$, corresponding to $\sigma_{in} = 0$, i.e. $\sigma_{tot} = \sigma_{el}$. This is an extreme option which is formally acceptable, but is not very appealing!

Assume $a_{el}(s,b)$ is imaginary $\Rightarrow$

$\Rightarrow \Omega(s,b)$ is real. This is compatible with high energy data. A small real part is easily reconstructed utilizing dispersion relations.

In this case the unitarity bound coincide with the black disc bound $|a_{el}(s,b)| \leq 1$

Accordingly, $\dfrac{\sigma_{el}(s,b)}{\sigma_{tot}(s,b)} \leq \dfrac{1}{2}$.

In the Glauber Eikonal model Ω is real and it corresponds to the zero order pre scattering correction process. In our case

$$\frac{\Omega}{2} = |a_{el}^{P}|.$$

For a real Ω, the unitarity Eq. implies that $G^{in}(s,b) = 1 - e^{-\Omega(s,b)}$. It follows that $P^{s}(s,b) = e^{-\Omega(s,b)}$ is the probability that the two initial hadrons will reach the final $\mathbb{P}$ exchange inelastic interaction intact, regardless of their rescatterings.

Consequently
$$S_D^2 = \frac{\int P^s(s,b)\, |M_D(s,b)|^2}{\int |M_D(s,b)|^2} = \frac{\sigma_D^{out}(s)}{\sigma_D^{in}(s)}.$$

D stands for LRG soft or hard diffraction.

The GLM Model:

The GLM model is an eikonal model originally concieved to explain the remarkable mild energy dependence of soft diffraction channels. The model input is a DL type $\mathbb{P}$ with $\alpha_{\mathbb{P}}(0) = 1+\Delta$, $(\Delta > \epsilon_{DL})$.

The simplicity of this toy model derives from the observation that the eikonal approximation with a central b Gaussian input (corresponding to $\frac{d\sigma_{el}}{dt} \propto e^{-B_{el}|t|}$), can be summed analytically.

This is, clearly, an over simplification, but it reproduces the forward cone data very well. None of the GLM output is particular to this model and its results have been obtained by other (more elaborate) models!

$$\Omega(s,b) = \nu(s)\, \Gamma^S(s,b)$$

$$\nu(s) = \sigma(s_0) \left(\frac{s}{s_0}\right)^{\Delta}$$

$$\Gamma^S(s) = \frac{1}{\pi R_s^2(s)}\, e^{-b^2/R_s^2(s)}$$

$$R_s^2(s) = 4R_0^2 + 4\alpha'_{P} \ln \frac{s}{s_0}$$

$$\int d^2b\, \Gamma^S(s,b) = 1 .$$

The GLM model with $\Delta = 0.1$, $\alpha'_{P} = 0.2\ GeV^{-2}$ provides an excellent reproduction of $\sigma_{tot}(s)$, $\sigma_{el}(s)$, $B_{el}(s)$. All are obtained analytically. Note that $a_{el}^{CDF}(\sqrt{s} = 1800, b=0) = 0.96 \pm 0.04$???

The formalism just presented is easily extended to diffraction (soft and hard) where the diffractive b-space amplitudes are suppressed by $\sqrt{P_S(s,b)} = e^{-\frac{\Omega}{2}}$.

$$M_D(s,b) \longrightarrow M_D(s,b)\, e^{-\frac{1}{2}\Omega(s,b)} .$$

Assuming that M_D is also Gaussian in b, its radius denoted R_D^2 we obtain

$$S_D^2(s) = \frac{a_D(s)\ \gamma\,[a_D(s), \nu(s)]}{[\nu(s)]^{a_D(s)}} \quad , \quad a_D(s) = \frac{R_S^2(s)}{R_D^2(s)},$$

$\nu = F(\sigma_{el}/\sigma_{tot})$.

$\nu(s), a_D(s)$ are obtained directly from the data.

The model reduces the calculated non corrected energy dependence of σ_{sd} but it does not reproduce the diffractive data well! We trace this defficiency to its basic input assumption that $\sigma_{diff} \ll \sigma_{el}$ (i.e. only elastic rescatterings are accounted for) which is not compatible with the data

In a two channel model we include in the rescattering chain also diffractive states. In the simplest approximation diffractively produced hadrons at a vertex are considered as a single diffractive state Ψ_D which is orthonormal to the initial proton (hadron) wave function Ψ_h:

$\langle \Psi_h | \Psi_D \rangle = 0$

The 2×2 interaction matrix T is diagonalized by two base wave functions ψ_1 and ψ_2.

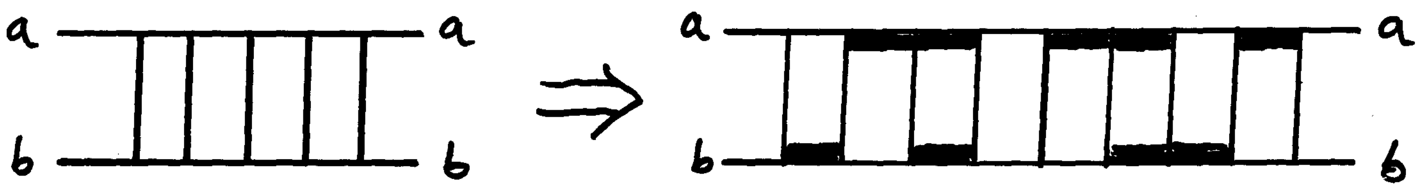
$$\Psi_h = \alpha\,\psi_1 + \beta\,\psi_2$$

$$\Psi_D = -\beta\,\psi_1 + \alpha\,\psi_2$$

$$\alpha^2 + \beta^2 = 1$$

The high energy amplitudes are

$$A_{i,j} = \langle \psi_i \psi_j \,|T|\, \psi_{i'} \psi_{j'} \rangle = A_{i,j}\, \delta_{i,i'}\, \delta_{j,j'}$$



We have 4 possible rescattering amplitudes (a,b), (a^*,b), (a,b^*), (a^*,b^*). For pp or $\bar{p}p$ $(a^*,b) = (a,b^*)$, so we have 3 amplitudes.

If we neglect double diffraction (at the cost of breaking factorization) we end with 2 amplitudes. This is a reasonable approximation if $\frac{\sigma_{dd}}{\sigma_{sd}} \ll 1$.

We identify our models by the number of their amplitudes.

Eeach of the 4 amplitudes of the two channel model keeps the eikonal structure shown for a single channel representation.

$$A_{ij}(s,b) = i\left(1 - e^{-\Omega_{ij}/2}\right)$$

$$G^{in}_{ij}(s,b) = \left(1 - e^{-\Omega_{ij}}\right)$$

$$2\,\mathrm{Im}\,A_{i,j} = |A_{ij}|^2 + G^{in}_{ij}$$

In a 3 amplitude model we have:

$$a_{el}(s,b) = i\{\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\}$$

$$a_{sd}(s,b) = i\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2-\beta^2)A_{1,2} + \beta^2 A_{2,2}\}$$

$$a_{dd}(s,b) = i\alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\}$$

If we assume that $a_{dd} \equiv 0$, we get:

$$a_{el}(s,b) = A_{1,1} - 2\beta^2(A_{1,1} - A_{1,2})$$

$$a_{sd}(s,b) = -\alpha\beta(A_{1,1} - A_{1,2})$$

In analogy with the single channel,

$$\Omega_{ij}(s,b) = \frac{g_i g_j}{\pi R^2_{i,j}}\left(\frac{s}{s_0}\right)^{\Delta} \exp\left(-\frac{b^2}{R^2_{i,j}}\right)$$

$$R^2_{i,j} = 2R^2_{0i} + 2R^2_{0j} + 4\alpha'^2_{I\!P}\ln\left(\frac{s}{s_0}\right)$$

$$R^2_{0k} = \begin{cases} R_0^2 & : k=1 \\ 0 & : k=2 \end{cases}$$

The GLM model was applied so as to fit a global data base containing all the measured data points of σ_{tot}, σ_{el}, σ_{sd}, σ_{dd} and B_{el} in the ISR – Tevatron range. We have checked 3 options:

1) A 2 amplitude model in which we assume that $\sigma_{dd} = 0$. The DD data (5 points) is ommited from the data base. The data is fitted reasonably well with $\chi^2/_{d.o.f.} = 1.5$.

2) A 3 amplitude model in which coupling factorization is maintained. i.e: $g_{11}^2 = g_1 \cdot g_1$, $g_{12}^2 = g_1 \cdot g_2$ and $g_{22}^2 = g_2^2$. The model does not fit the data well with $\chi^2/_{d.o.f.} = 2.3$.

3) The same 3 amplitude model with independent couplings g_{11}^2, g_{12}^2, g_{22}^2. It fits the data well with $\chi^2/_{d.o.f.} = 1.25$.

The relatively high χ^2 obtained in the better fits (option 1 and 3) reflect the experimental high scatter of σ_{sd}.

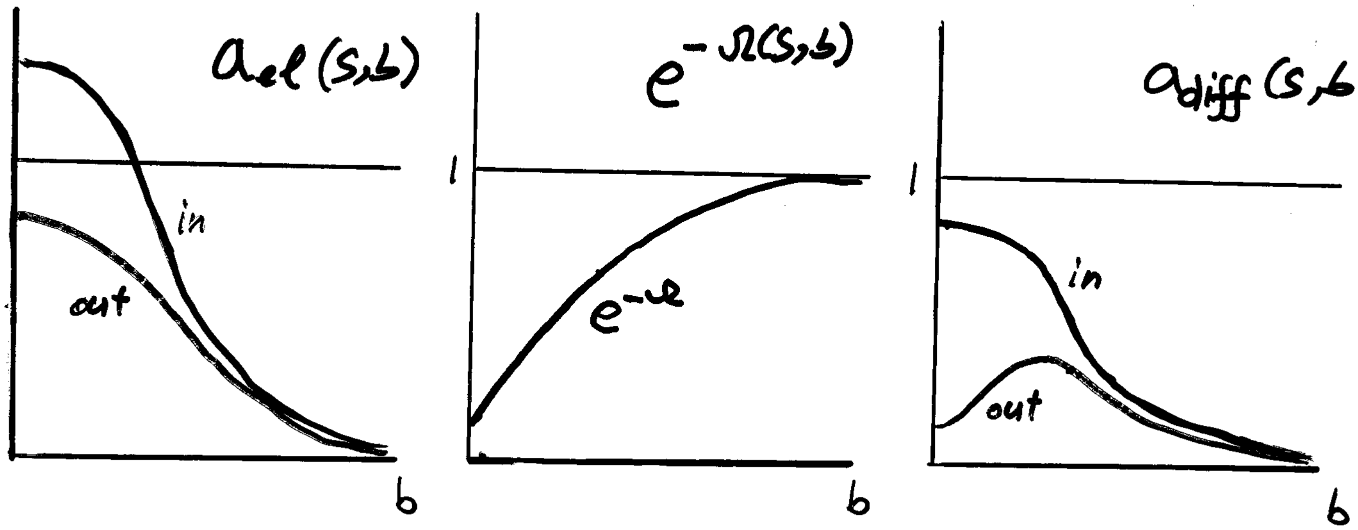
General Properties of the GLM Model:

1) The two successful versions of GLM are not compatible with factorization. Regardless of this deficiency, the model provides a parametrization which reproduces a large data base very well. As such, it is reasonable to use GLM in order to assess cross sections and survival probability at higher energies.

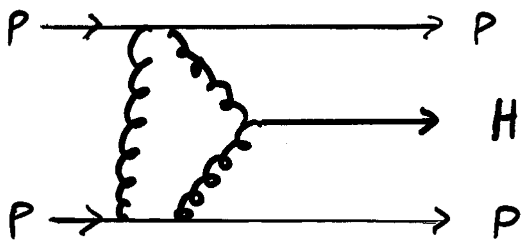
2) GLM is compatible with s-channel unitarity. Accordingly $|a_{el}(s,b)| \le 1$. In 1 channel $\frac{\sigma_{el}}{\sigma_{tot}} \le \frac{1}{2}$. The two channel GLM model is compatible with the Pumplin bound: $\frac{\sigma_{el}(s,b)+\sigma_{diff}(s,b)}{\sigma_{tot}(s,b)} \le \frac{1}{2}$. In GLM notation: $\frac{|a_{el}(s,b)|^2+|a_{sd}(s,b)|^2+|a_{dd}(s,b)|^2}{2a_{el}(s,b)} \le \frac{1}{2}$.

3) Given a non screened input diffraction amplitude (soft or hard), GLM provides a procedure to calculate the screened (unitarity compatible) output amplitude and its survival probability.

4) In the GLM model the b-dependence of a diffractive amplitude is fundamentally different from the b-dependence of the elastic amplitude due to the extra $e^{-\Omega(s,b)}$ factor in σ_{diff}.



5) Exclusive Higgs production in central diffraction $p+p \to p + LRG + H + LRG + p$



is a promising LHC channel with a high discovery potential due to its 2 LRG signature + a very good signal/background ratio. Our calculated survival probability is reduced with the opening of more screening options. Our results:

$S_H^2(14{,}000) = 3.6\%,\ 2.7\%,\ 0.7\%$ for 1,2,3 amplitudes

A general remark: All models atempting to fit the soft scattering data bas in the ISR-Tevatron range have to include also a Regge (non Pomeron) secondary component. This is crucial in the ISR-SPS range. As it stands, the data in the SPS-Tevatron range is not suffecient to constrain the $\mathbb{P}$ parameters. Regardless, the GLM over all fit is very stable!

GLM Predictions Above the LHC:

GLM provide cross sections and elastic slope predictions above the LHC up to GZK and Planck. To the best of my knowledge this is the only set of predictions based on the complete existing soft scattering data base in which s-unitarity and Pumplin bounds are in built.

Obviously, we are interested to learn on the onsetting of the unitarity black disc.

$\sqrt{s}$ TeV	$\sigma_{tot}(DL)$ mb	σ_{tot} mb	σ_{el} mb	σ_{sd} mb	σ_{dd} mb	σ_{el}/σ_{tot}	$\frac{\sigma_{el}+\sigma_{diff}}{\sigma_{tot}}$
1.8	73.0	78.0	16.3	9.6	3.8	0.21	0.38
14	101.5	110.5	25.3	11.6	4.9	0.23	0.38
30	115.0	124.8	29.7	12.2	5.3	0.24	0.38
60	128.6	139.0	34.3	12.7	5.7	0.25	0.38
120	143.9	154.0	39.6	13.2	6.1	0.26	0.38
250	162.0	172.0	45.9	13.6	6.6	0.27	0.38
500	181.2	190.0	52.7	14.0	7.0	0.28	0.39
GZK → 1000	202.7	209.0	60.2	14.3	7.4	0.29	0.39
10^{11}	3980	912.0	413.7	9.0	6.0	0.45	0.47
$1.22 \cdot 10^{16}$ Planck	31200	1970.0	985.0	0	0	0.50	0.50

We provide 4 pieces of information obtained from our input:

1) Predicted values of $\frac{\sigma_{el}}{\sigma_{tot}}$ and $\frac{\sigma_{el}+\sigma_{diff}}{\sigma_{tot}}$. These numbers indicate the critical energy at which we have a complete black b-amplitude. This is attained only at, or close, to the Planck mass.

2) We are interested to assess when does the inner core of the proton reach the unitarity black bound and what is the radius of this core. Following are some representative values of $A_{el}(s, b=0)$ and the radius of the black core when applicable.

$\sqrt{s}$ TeV	$A_{el}(b=0)$	width of black disc	$\frac{\sigma_{el}}{\sigma_{tot}}$
1.8	0.62	—	12.13
14	0.71	—	15.26
500	0.85	—	20.50
1000	0.93	—	20.70
10^5	1.00	0.5 f	
10^{11}	1.00	3.0 f	
planck 10^{16}	1.00	4.5 f	

3) Obviously, the GLM model suggests a very slow blackening rate. This is very different from the philosophy advanced by Frankfurt, Strikman et.al. who suggests a much faster approach to the black bound. Their argument is that $A_{el}(s, b=0)$ can be estimated through $A_{el}(s, b=0) = \frac{\sigma_{tot}(s)}{4\pi B_{el}(s)}$.

This approximation is based on a b-Gaussian profile identical to GLM. Checking the numbers, one obtains that $A_{el}(b=0) \simeq 1.0$ at or close to LHC.

GLM prediction is that this takes place at $\sqrt{s} \simeq 10^5$ TeV !!!

Following are some critical comments:

1) The Gaussian b-profile approximation holds only in the forward $|t|$ cone, where $|t| < 0.2-0.4$ GeV (depending on s). The $b=0$ value of $A_{el}(s, b=0)$ gets its main contribution from high $|t|$, where the Gaussian description is not valid!

2) In the above approximation the unitarity bound implies that $\frac{\sigma_{tot}}{B_{el}} \leq 4\pi = 12.57$.

More over, $\frac{\sigma_{tot}}{B_{el}}$ has to remain constant with s above the LHC energy where this bound was reached. This is not reasonable!

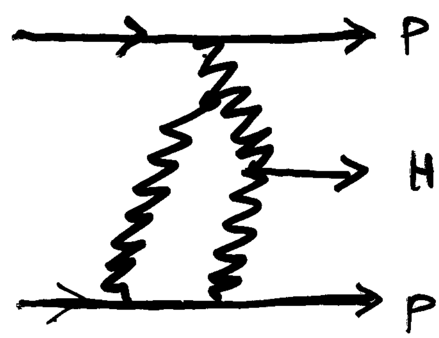
3) Regardless of the above, I like the basic idea advanced by FK asociating the hard and soft interaction to the proton core and periphery. But, this has to be based on a careful pp data analysis!

Eikonalization in the hard sector:

We wish to examine if eikonalization is a viable procedure with which we can assess the role of unitarity in a strictly hard scattering.

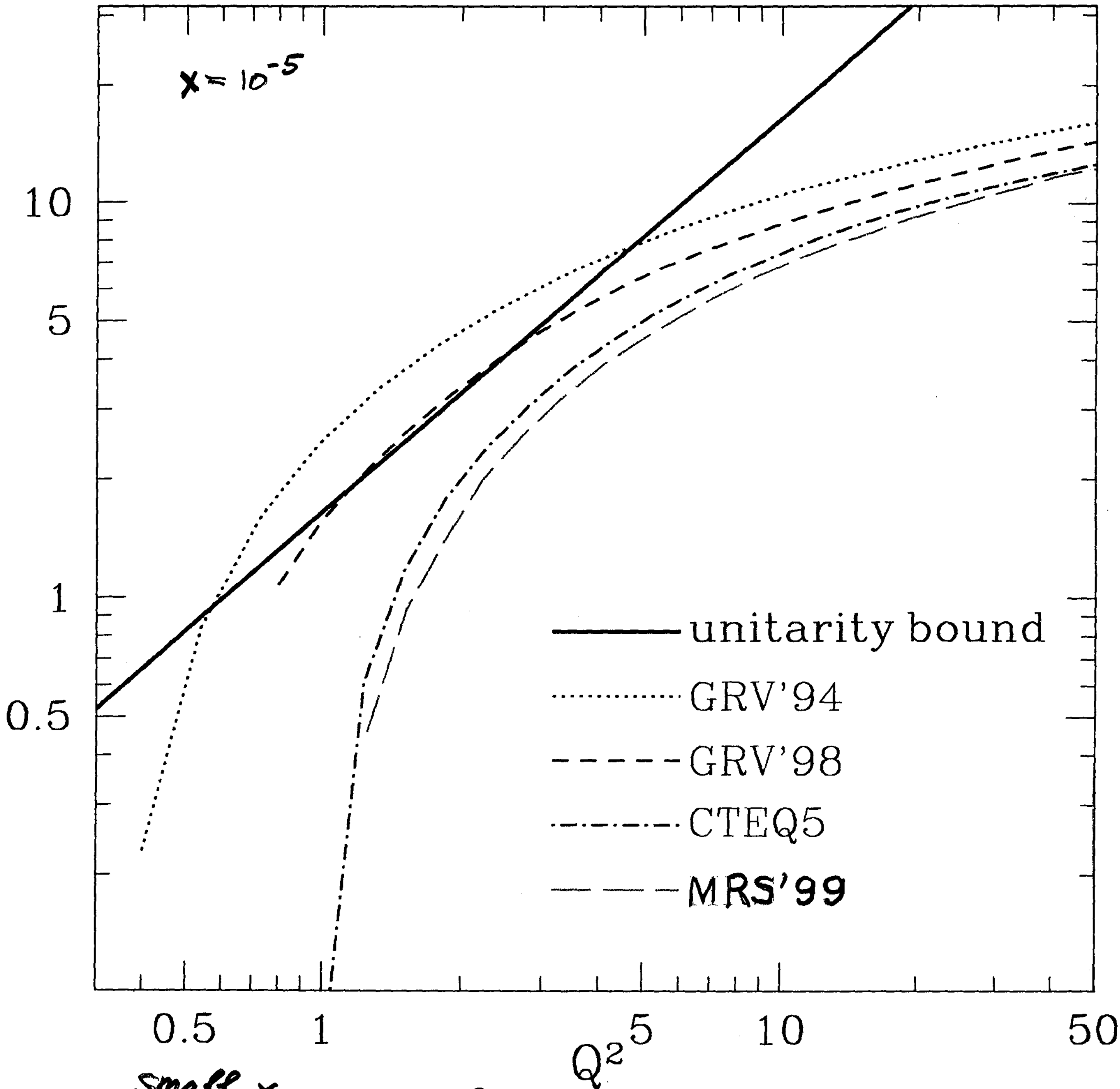
Note that the suppression due to gluon radiation from the partons participating in the hard subprocess is already included in the hard calculation through the Sudakov factor.

KMR (hep-ph/0602247) recently showed that the rescattering depicted in the diagram results in a very small correction.



One may ask, nevertheless, what is the consequence of $xG(x,Q^2)$ getting too close to the black unitarity bound. If so, the SC due to the percolation of a $q\bar{q}$ dipole can be calculated in the eikonal model. This is simply demonstrated in a dipole LLA DGLAP.

Ayala, Gay-Ducati, Levin: $\frac{\partial^2 xG(x,Q^2)}{\partial y\, \partial \ln Q^2} < \frac{2}{\pi} R_H^2 Q^2$

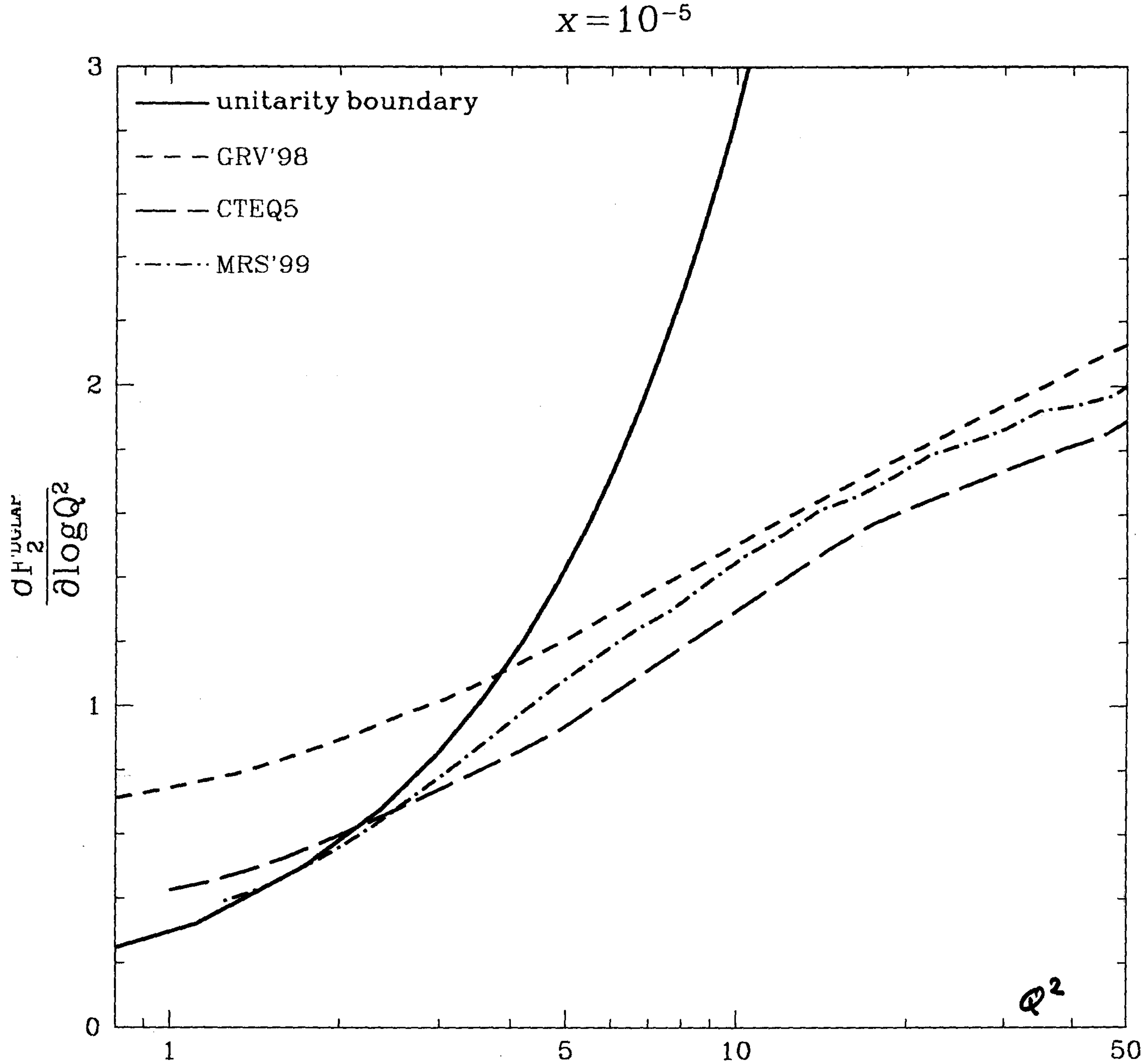


small x
LLA DGLAP: $\frac{\partial^2 xG(x,Q^2)}{\partial y\, \partial \ln Q^2} = \frac{N_c}{\pi} \alpha_s(Q^2)\, xG(x,Q^2)$

$xG(x,Q^2) < \frac{2}{\pi N_c \alpha_s(Q^2)} R_H^2 Q^2$

$R_H^2 \simeq 4\ \mathrm{GeV}^{-2}$ obtained from HERA $\gamma p \to J/\psi\, p$

In a correlated small x bound:



$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \frac{\partial xG(x,Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2}\, xG(x,Q^2)$$

$$\frac{\partial xG(x,Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2}\int d^2b_\perp \, \mathrm{Im}\, a_{el}^H(x, r_\perp^2)$$

Setting $a_{el}^H \equiv 1$ provides the bound which was tested by H1 and Zeus. $Q^2 = \frac{4}{r_\perp^2}$

Hard eikonalization was applied by GLM to reproduce the HERA experimental data of $\frac{\partial F_2}{\partial \ln Q^2}$ and $\gamma p \to J/\psi\, p$.

Despite its phenomenological success the GLM program was unable to suggest definite unitarity signatures derived from the small (x, Q^2) data.

The trigger for our analysis was the poor reproduction of the small (x, Q^2) $\frac{\partial F_2}{\partial \ln Q^2}$ data by the early p.d.f. editions.

The overestimation was corrected by our eikonalization, but then similar proper results were obtained by newer p.d.f. editions. Similar problems were detected at even smaller (x, Q^2) values. The new data was fitted by our model and then reproduced soon after by, yet, newer p.d.f. editions. etc. In 2001 we got tired and retired from this game.

Saturated (or high density) p.d.f.'s reproduce the data (explicitly $\frac{\partial F_2}{\partial \ln Q^2}$ and $\gamma p \to J/\psi\, p$) but the problem is that these p.d.f.'s offer an excellent postdiction while their predictive power, at exceedingly small (x, Q^2), may be limitted.

Phenomenological models in the spirit of Golec-Biernat, Wüsthoff, Bartels, Kowalski, ... are doing well but their translation to p-p scattering is not simple. As far as I know there is no decisive pQCD unitarity signature observed in p-p scattering as yet!