The Abdus Salam
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Educational, Scientific and
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IAEA
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International Workshop on QCD at Cosmic Energies III

28 May - 1 June, 2007

MC Generator of High Energy QCD Particle Multi-production

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MC generator of high energy QCD particle multi-production

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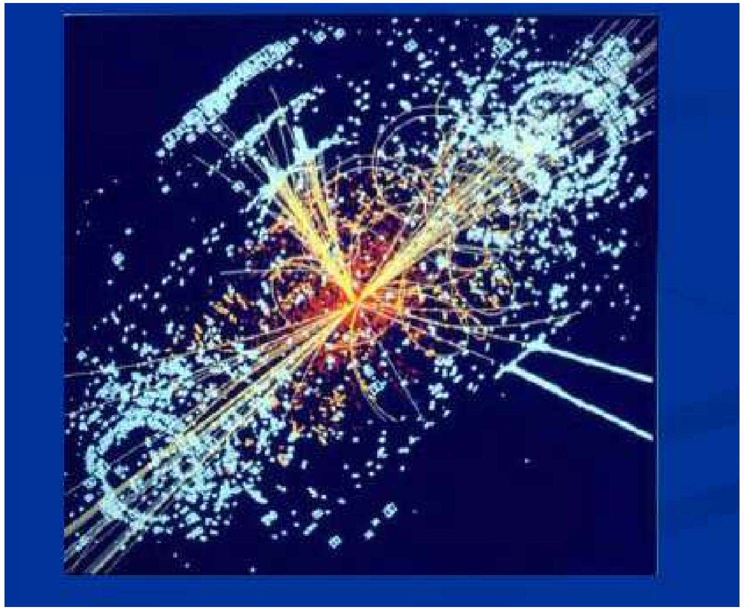
Monte Carlo jet emission as a QCD $Summa$.

Here a pedagogical tour



High Energy, high E_T event with intense hadron emission:

Need QCD radiation structure

High E_T (short distance) and inclusive level (IR and collinear safe):

quantitative QCD predictions are possible

Major instruments:

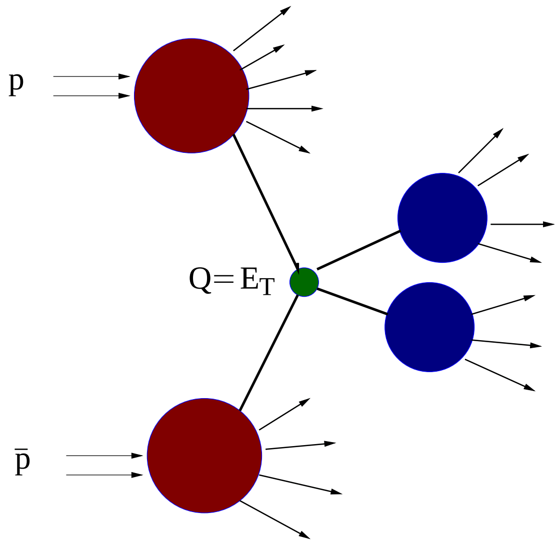
-asymptotic freedom $\beta_0 = 11 - \frac{2}{3} n_f > 0$

-factorization of collinear and IR singularities

-resummation of collinear and IR log-enhanced contributions

IR and collinear factorization/resummation results:

- evolution equation of single inclusive hadron: DGLAP 1977
- jet calculus and branching process: Konishi Ukawa Veneziano 1978
- Monte Carlo programs: Odorico, Fox+Wolfram 1980
- parton branching and QCD coherence: Mueller, Fadin,.... 1981
- Monte Carlo program with QCD coherence: Webber+GM 1984



MC the (hard) QCD $Summa$:

Elementary hard distribution

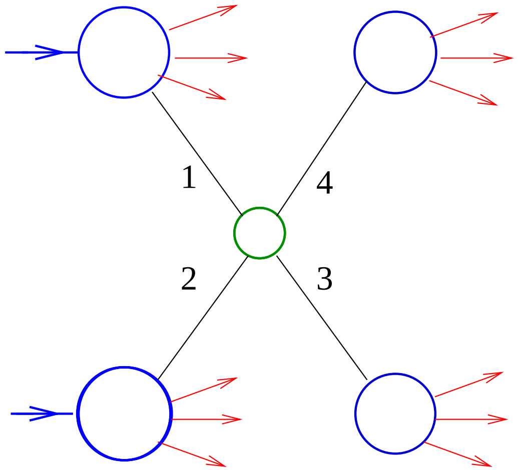
Structure function

Fragmentation function

The path into the Monte Carlo generator

- first stage: primary partons from hard matrix elements
- second stage: initial state bremsstrahlung emission (DGLAP)
- third stage: final state branching emission
- forth stage: hadronization (preconfinement)
- improvements: hard matrix el. (NLO); branching (large angle soft emission)

First stage: Generate $p_1\, p_2\, p_3\, p_4$ of primary partons



$$d\sigma_{P_1P_2}(E_t) = \int dx_1 dx_2 F_{P_1}^{p_1}(x_1, Q)\, F_{P_2}^{p_2}(x_2, Q) \times \frac{d\hat{\sigma}_{p_1p_2\to p_3p_4}(Q)}{dp_3 dp_4}$$

$$Q \sim E_T\,, \qquad p_1 = x_1 P_1\,, \quad p_2 = x_2 P_2$$

Improving hard process $d\hat{\sigma}_{p_1p_2\to p_3p_4}$ is a major effort now:

- get a more reliable control of primary hadrons
- build the library of important SM and BSM processes

Need of LO, NLO, NNLO

- high accuracy needed for LHC discoveries
- new processes for LHC: The LHC “priority” wishlist

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

What is needed for a NLO calculation

- compute n-point Born amplitude: $M_n^{(0)}$
- compute $n+1$-point Born amplitude: $M_{n+1}^{(0)}$
- compute one-loop corrections to n-point Born amplitude: $M_n^{(1)}$
- sum and check that all divergences are canceled at the (proper) inclusive level

NLO calculation strategies

- semi-numerical methods
- analytical methods
  - unitarity
  - reduction to elements (box + triangle + bubble + constant)
  - recurrence relations
  - supersimmetry: $\mathrm{QCD}=\{\mathrm{SYM},\mathcal{N}=4\}-4\{\mathrm{SYM},\mathcal{N}=1\}+\{\mathrm{SYM},\mathcal{N}=0\}$

Feynman diagrams have unexpected simple structures

$$gg \to gg$$

[illegible]

where x, y, X, Y, $\tilde{X}$ and $\tilde{Y}$ are defined in eqs. (3.17) and (3.18).

For $-+++$, the functions are

[illegible]

+ another 15 pages

High order QCD amplitudes as curves in twistor space

1986 Park+Taylor: the many page long 4-gluon amplitude (MHV and tree level) can be reduced to a small single line formula

General multi-gluon amplitude:

$$\mathcal{M}_n = g_s^{n-2} \sum_{\text{perm}} \text{Tr}(\, t^{a_1} t^{a_2} \cdots t^{a_n}\,) \cdot M(p_1\lambda_1; p_2\lambda_2; \cdots p_n\lambda_n)$$

$M(\lambda_1\lambda_2 \cdots \lambda_n)$ colour ordered amplitude

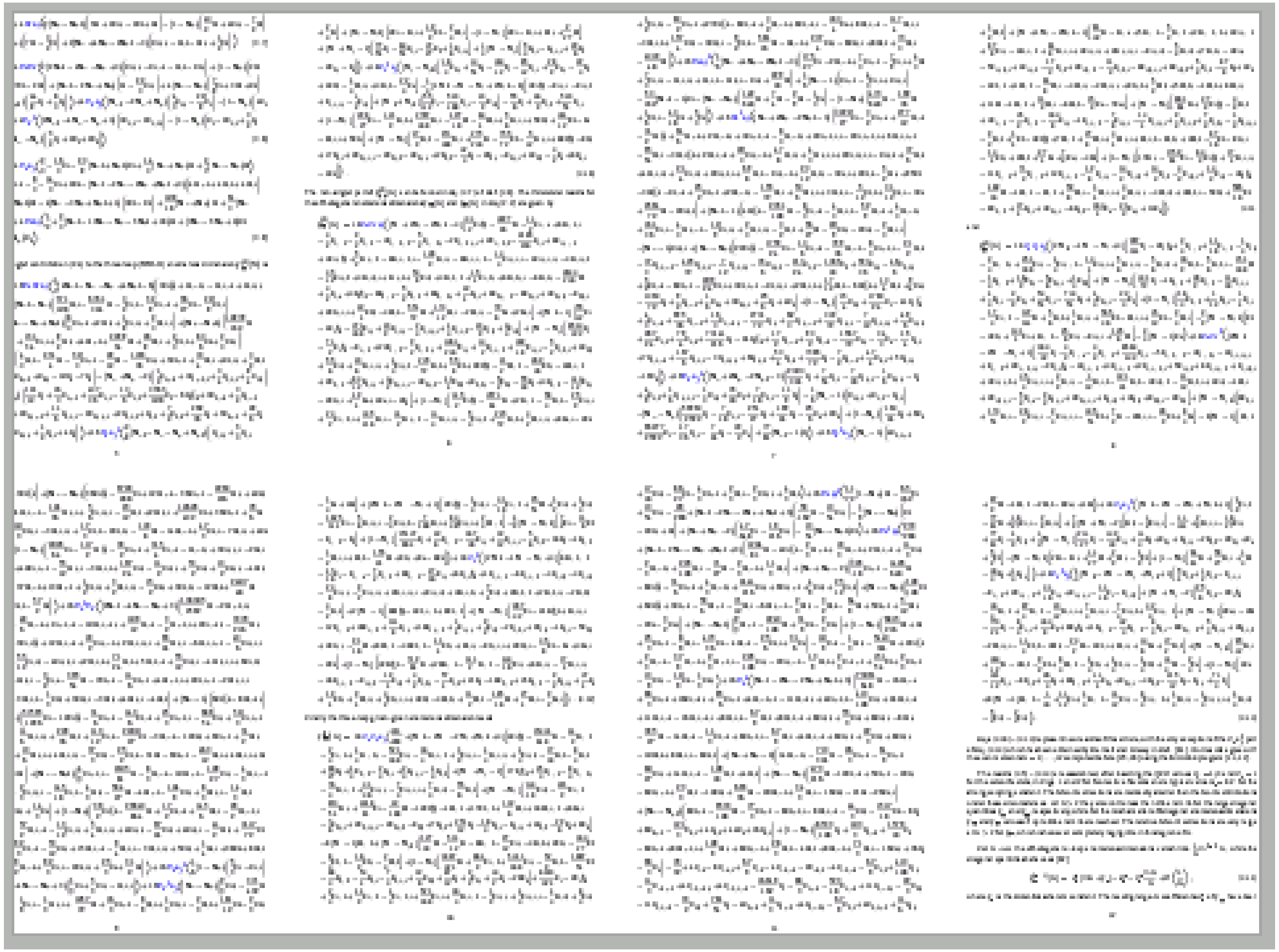
MHV configuration $\lambda_i = \lambda_j = -1, \;\; \lambda_k = +1$

$$M_{\text{tree}}^{\text{MHV}}(+-++-+++) = \frac{\langle p_i p_j \rangle^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \cdots \langle p_n p_1 \rangle},$$

$$\langle p_i p_j \rangle = \sqrt{2 p_i p_j}\, e^{i\phi_{ij}}$$

Space-like singlet anomalous dimension at three loops

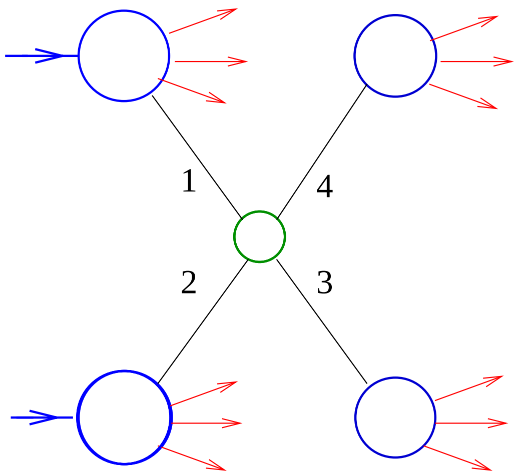
Vogt Moch Vermaseren, Nucl. Phys. B691(2004) 129; B688(2004)101



For *expected* simple structure here see Yuri's talk

Second stage:

Generate initial state bremsstrahlung radiation $k_1\, k_2 \cdots k_n$



Use DGLAP evolution equation

$$P_1 \to p_1 + k_1\, k_2 \cdots, \qquad p_1 = x_1 P_1$$

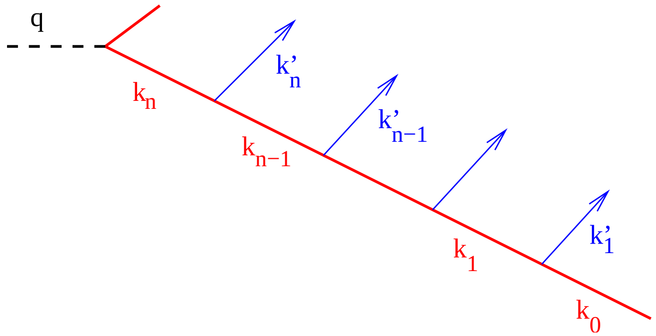
$$\partial_{Q^2} F(x, Q^2) = \int_0^1 \frac{dz}{z} P(z, \alpha_s)\, F\left(\frac{x}{z}, Q^2\right)$$

Collinear cutoff Q_0 required

Subtleties:

- evolution variable: k_t (coherence)
- splitting function: $P(z, \alpha_s)$ (known at three-loop VVM 2004)
- running coupling argument: k_t, and physical scheme

Space-like evolution variable (coherence)



$$\frac{-k_i^2}{k_{i,+}} = \frac{-k_{i-1}^2}{k_{i-1,+}} + \frac{{k'}_i^2}{k'_{i,+}} + \frac{k_{i,+}k'_{i,+}}{k_{i-1,+}} \left(\frac{\vec{k}_{it}}{k_{i,+}} - \frac{\vec{k}'_{it}}{k'_{i,+}} \right)$$

mass singularities factorize

$$-\,k_{i-1}^2 \ll -k_i^2\, z_i^{-1}\,, \qquad \frac{k_{i,+}}{k_{i-1,+}} \equiv z_i$$

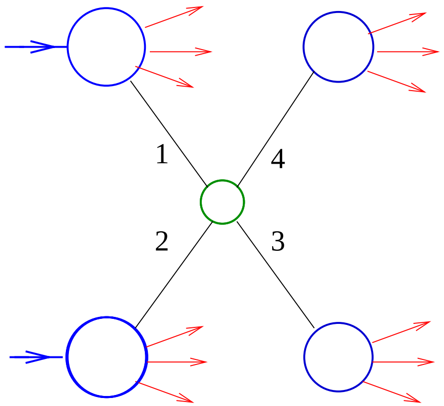
Actually, cancellation (coherence) leading to k_t-ordering

pre-QCD observation by Gribov 1960's

$$-k_i^2 < -k_{i-1}^2 < -k_i^2\, z_i^{-1} \quad \Rightarrow \quad -k_{i-1}^2 < -k_i^2 \quad \simeq \quad k_{t,\,i-1}^2 < k_{t\,,i}^2$$

Yuri's talk for connection with crossing symmetric case (fragmentation function)

Third stage: Final state emission

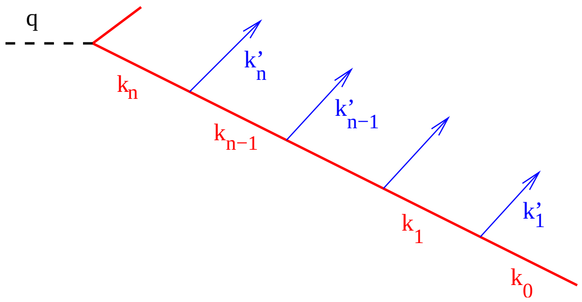


Radiation off the *primary partons* given by:

$p_3,\ p_4 +$ bremsstrahlung: $k_1 + \cdots$ and $k'_1 + \cdots$

$P_1 \to p_1 + k_1 \cdots$ and $P_2 \to p_2 + k'_1 \cdots$

Consider first the fragmentation function of a primary parton



Mass-singularity phase space + coherence

$\Rightarrow$ angular-ordering: $Q = \theta\, E$

$$Q^2 \partial_{Q^2}\, D(x, Q) = \int_0^1 \frac{dz}{z}\, P(z, \alpha_s)\, D\left(\frac{x}{z}, zQ\right)$$

Generalization to parton branching $a \to b_1 b_2$ Bassetto, Ciafaloni+GM 1983

$$Q^2 \partial_{Q^2}\, G_a(Q) = \tfrac{1}{2} \int_\epsilon^{1-\epsilon} \frac{dz}{z} P_a^{b_1 b_2}(z, \alpha_s)\ \{G_{b_1}(zQ) G_{b_2}((1-z)Q) - G_a(Q)\}$$

HERWIG 1983

$p\bar{p} \to W^+ + X\,, \quad W^+ \to t\bar{b}:$ a MC event

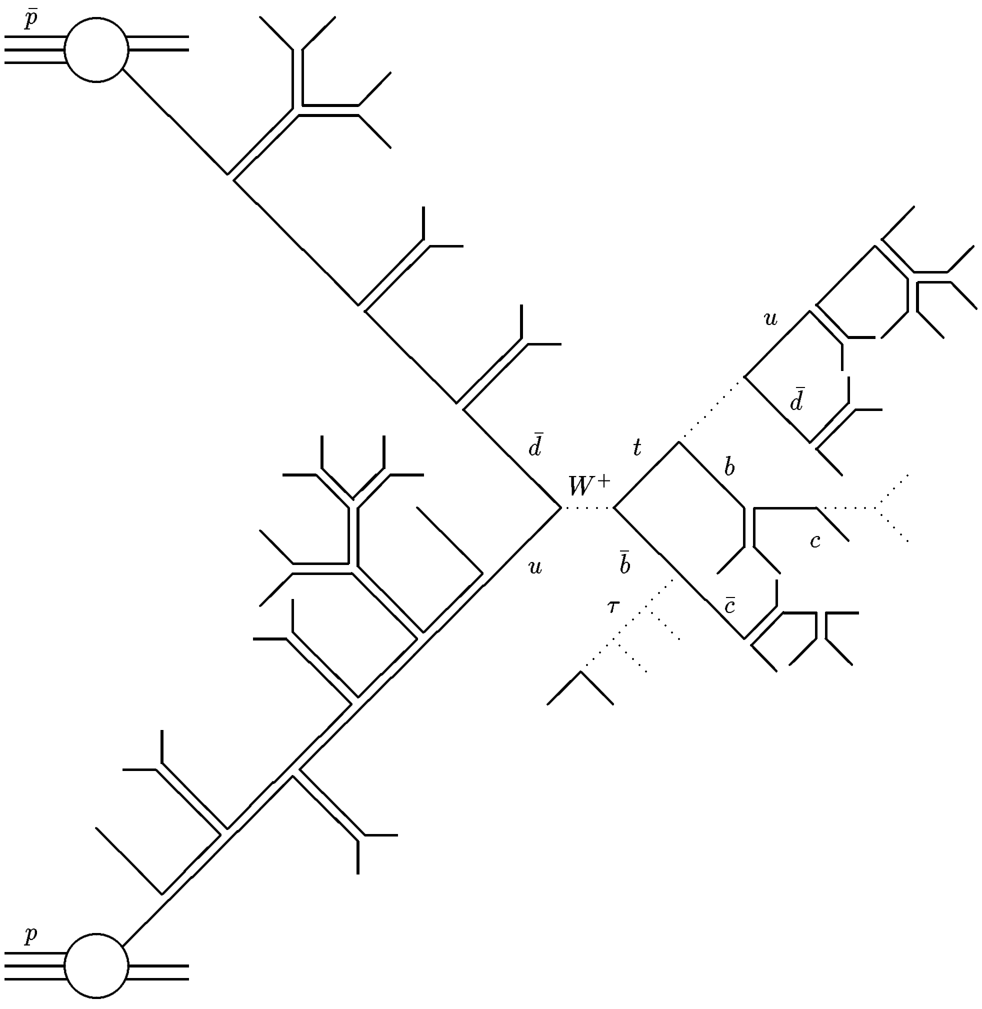


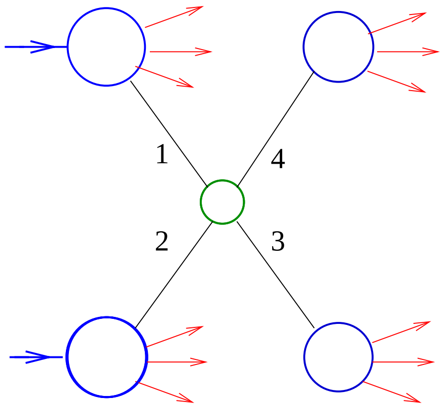
Figure 1: Colour structure of a $\bar{p}p \to W^+ + X$, $W^+ \to t\bar{b}$ event.

G^{ab}_μ: gluon with two indexes

$SU(N_c) \to U(N_c)$

As a consequence of factorization, one can construct a single Monte Carlo program, such as

Forth stage: hadronization



QCD radiation of partons

how to go to hadrons?

beyond perturbative QCD

Preconfinement (D.Amati, G.Veneziano, PL83(79)87) :

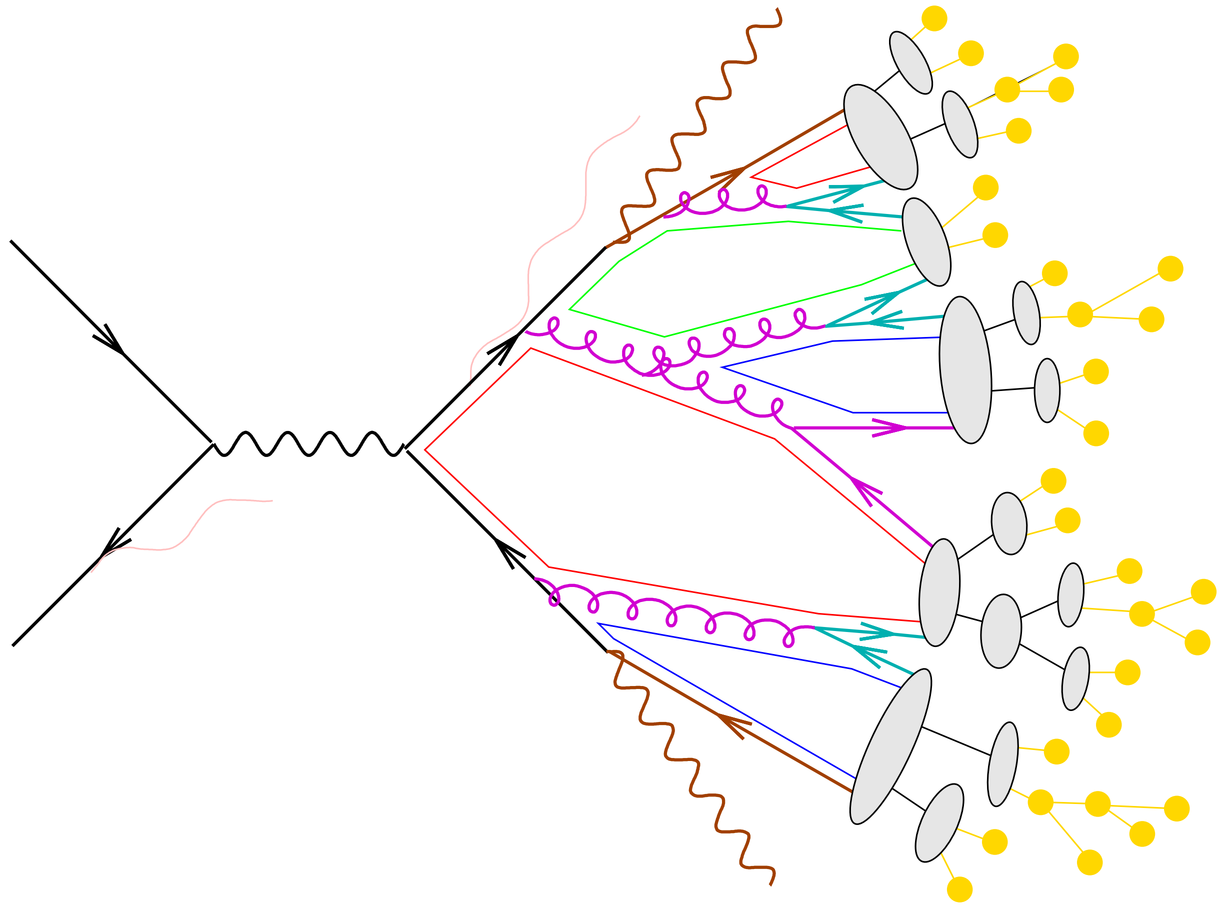
- colour connection $\longrightarrow$ Sudakov suppression in mass distribution

  $\longrightarrow$ colour connected partons at small mass

- colour connected $\longrightarrow$ small mass colour singlet $\longrightarrow$ model $\longrightarrow$ hadrons

- results not too sensitive to hadronization model

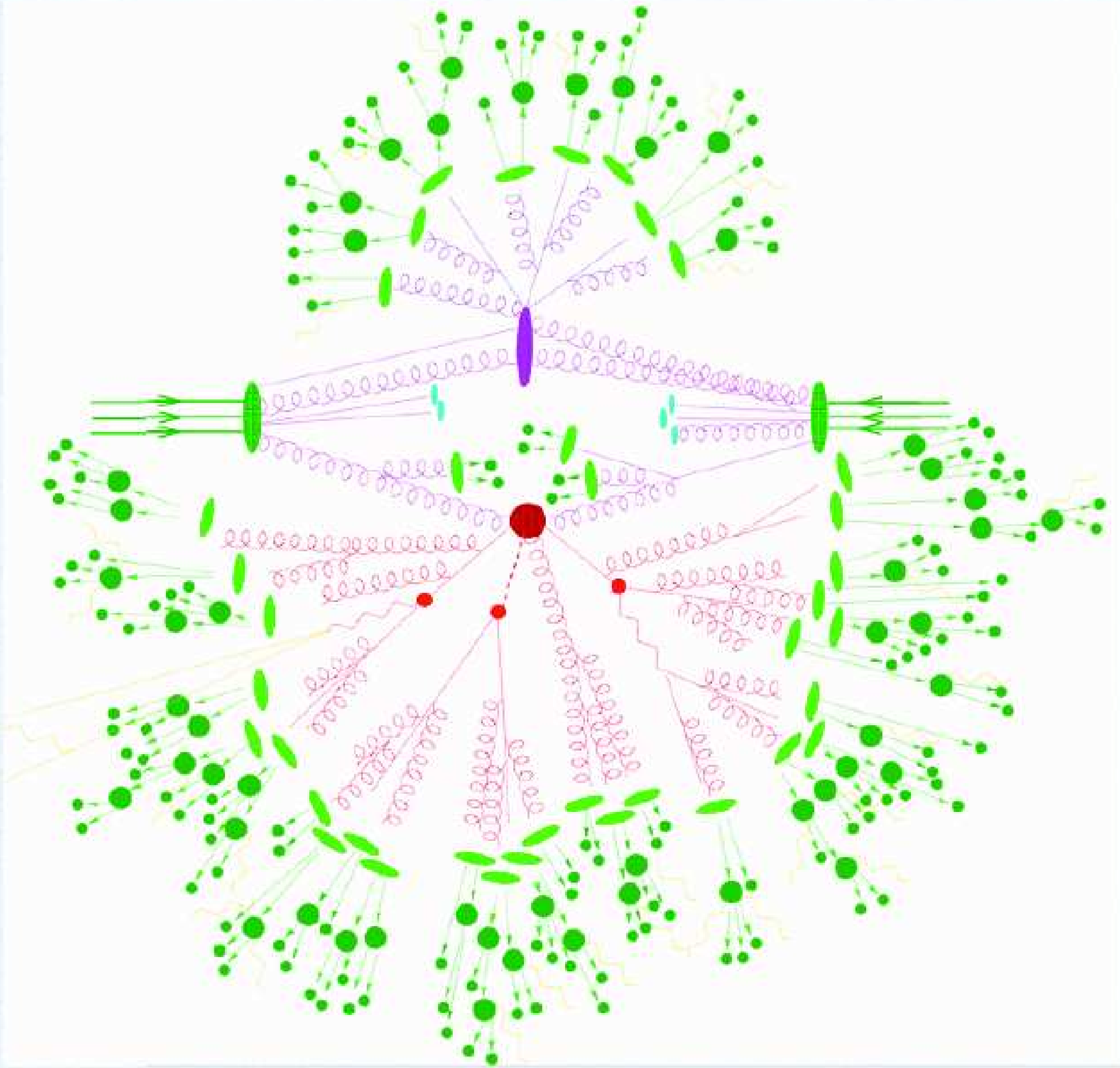
Cluster Hadronization Model



- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. $t \longrightarrow bW$
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster $\longrightarrow$ hadrons
- hadronic decays

QCD Monte Carlo simulation:	e^+e^-annihilation lepton-hadron DIS hadron-hadron (largeE_T)

- Herwig (B.Webber&GM, 1984): e^+e^-, ℓ—h and h-h
- Pythia (T.Sjostrand 1987): e^+e^-, ℓ—h and h-h
- Ariadna (L. Lonnblad 1992): e^+e^- mostly
- others for specific processes



Improvements

- library of hard processes: Heavy quarks, Electroweak, Higgs, Sypersymmetry...
- matching with exact fix order results: LO, NLO, NNLO....
- include QCD results in high energy scattering (BFKL, CCFM, ...) H.Jung 1992
- include low p_t radiation: underlying event
- include large angle soft emission

Include large angle soft emission

Generating functional (couting emitted partons)

$$G_{ab}[E,u] = \sum_n \int \prod_i (\bar{\alpha}_s dq_i\, u_i)\; M^2_{ab}(q_1 \cdots q_n)\,, \qquad \bar{\alpha}_s = \frac{N_c\,\alpha_s}{\pi}$$

Multi-soft gluon distribution (planar limit): Bassetto, Ciafaloni&GM 1981

$$M^2_{ab}(q_1 \cdots q_n) = w_{ab}(q_\ell) \cdot M^2_{a\ell}(q_1 \cdots q_{\ell-1}) \cdot M^2_{\ell b}(q_{\ell+1} \cdots q_n)\,, \qquad w_{ab}(q) = \frac{(ab)}{(aq)(qb)}$$

Give evolution equation (virtual corrections included) Banfi, Smye&GM 2002

$$E\partial_E G_{ab}[E,u] = \int \frac{d\Omega_q}{4\pi}\,\bar{\alpha}_s w_{ab}(q)\Big\{G_{aq}[E,u] \cdot G_{qb}[E,u] - G_{ab}[E,u]\Big\}$$

To solve by Monte Carlo introduce Sudakov form factor (with a cutoff Q_0)

$$\ln S_{ab}(E) = -\int_{Q_0}^{E} \frac{d\omega_q}{\omega_q} \int \frac{d\Omega_q}{4\pi}\,\bar{\alpha}_s w_{ab}(q) \cdot \theta(q_{tab} - Q_0)$$

"Solution" of the evolution equation as a dipole branching process $(ab) \longrightarrow (aq)(qb)$

$$G_{ab}[E,u] = S_{ab}(E,Q_0) + \int d\mathcal{P}_{ab}(q)\; G_{aq}[\omega_q,u] \cdot G_{qb}[\omega_q,u]$$

$$d\mathcal{P}_{ab}(q) = \left\{ \frac{d\omega_q}{\omega_q} \frac{S_{ab}(E)}{S_{ab}(\omega_q)} \right\} \left\{ \frac{d\Omega_q}{4\pi} \bar{\alpha}_s w_{ab}(q) \right\} \cdot \theta(q_{tab} - Q_0)$$

$$r_{ab}(E, \omega_q) = \frac{S_{ab}(E)}{S_{ab}(\omega_q)} \qquad \int dr_{ab}(E, \omega_q) = 1 - S_{ab}(E)$$

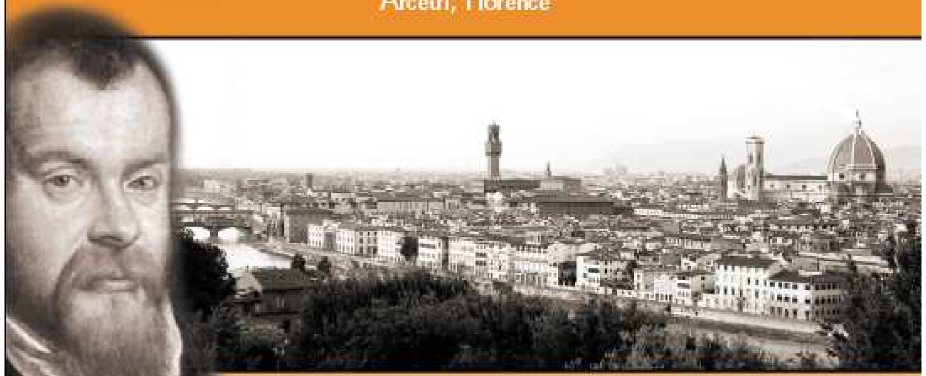
$$dR_{ab}(\Omega_q) = \mathcal{N}_{ab} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s w_{ab}(q) \qquad \int dR_{ab}(\Omega_q) = 1$$

1. ab-dipole emits? Take a random number $0 < \rho < 1$:
   $\rho < S_{ab}(E)$ no emission
   $\rho > S_{ab}(E)$ emission with energy ω_q with $\rho \cdot S_{ab}(\omega_q) = S_{ab}(E)$
2. obtain Ω_q by sampling $dR_{ab}(\Omega_q)$
3. repeat procedure for each new generated dipole till no dipole emits within the resolution Q_0.

Strong energy ordering: $\omega_n \ll \omega_{n-1} \cdots \ll \omega_1 \ll E_a, E_b$

How to implement energy conservation and the full splitting function?

The Galileo Galilei Institute for Theoretical Physics
Arcetri, Florence



Advancing Collider Physics: from Twistors to Monte Carlos

Aug 27th - Oct 26th, 2007

High energy physics is poised to enter a new era of discovery with the start of LHC operations in 2007. The LHC will play a key role in exploring spontaneous symmetry breaking in the Standard Model and in discovering new physics phenomena beyond the Standard Model (BSM).

The goal of this Workshop is to develop theoretical tools needed to test the Standard Model with unprecedented accuracy, analyse spontaneous symmetry breaking and uncover evidence for BSM physics at present and future high-energy colliders.

Another important aspect of the Workshop will be to support the transfer of knowledge between different communities - new formal developments (like twistors) will be explored and their applications to QCD and electroweak physics will be promoted and studied.

The main topics of the program include:

1. Higgs Production
2. QCD and Higher-order Computations
3. Electroweak Physics
4. Computer Algebra
5. New Formal Developments

Organizers: **Andreas Brandhuber** (Queen Mary University, London), **Vittorio Del Duca** (INFN Torino), **Nigel Glover** (IPPP Durham), **David Kosower** (CEA Saclay), **Giampiero Passarino** (University of Torino), **William Spence** (Queen Mary University, London), **Gabriele Travaglini** (Queen Mary University, London), **Dieter Zeppenfeld** (University of Karlsruhe).

GGI: http://www.fi.infn.it/GGI/

Final consideration

- MC (for QCD jets) needs hard scale $Q \gg \Lambda_{QCD}$
- MC as "summa" of most hard QCD studies
- MC not a solution of $\mathcal{L}_{\rm QCD}$, but universal features in e^+e^-, DIS and hh-collisions
- MC enormous impact on experimental analysis and planing of future beyond SM investigations
- MC continuously improved to include new theoretical results
- MC future (not planed) developments
  - $1/N_c^2 = 1/9$ corrections: beyond parton model?
  - include single collinear and singe infrared logarithms
  - include unbiased high energy scattering (BFKL dynamics)
- large-angle-soft-emission: amazing connection to BFKL dynamics A.Mueller&GM,PhysLettB(2003)