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MC Generator of High Energy QCD Particle Multi-production

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MC generator of high energy QCD particle multi-production

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Monte Carlo jet emission as a QCD Summa.

Here a pedagogical tour



High Energy, high E_T event with intense hadron emission:

Need QCD radiation structure

High E_T (short distance) and inclusive level (IR and collinear safe): quantitative QCD predictions are possible

Major instruments:

-asymptotic freedom $\beta_0 = 11 - \frac{2}{3}n_f > 0$

-factorization of collinear and IR singularities

-resummation of collinear and IR log-enhanced contributions

IR and collinear factorization/resummation results:

- evolution equation of single inclusive hadron: DGLAP 1977
- jet calculus and branching process: Konishi Ukawa Veneziano 1978
- Monte Carlo programs: Odorico, Fox+Wolfram 1980
- parton branching and QCD coherence: Mueller, Fadin,.... 1981
- Monte Carlo program with QCD coherence: Webber+GM 1984



MC the (hard) QCD Summa:

Elementary hard distribution

Structure function

Fragmentation function

The path into the Monte Carlo generator

- first stage: primary partons from hard matrix elements
- second stage: initial state bremsstrahlung emission (DGLAP)
- third stage: final state branching emission
- forth stage: hadronization (preconfinement)
- improvements: hard matrix el. (NLO); branching (large angle soft emission)

First stage: Generate $p_1 p_2 p_3 p_4$ of primary partons



$$d\sigma_{P_1P_2}(E_t) = \int dx_1 dx_2 F_{P_1}^{p_1}(x_1, Q) F_{P_2}^{p_2}(x_2, Q)$$
$$\times \frac{d\hat{\sigma}_{p_1p_2 \to p_3p_4}(Q)}{dp_3 dp_4}$$
$$Q \sim E_T, \qquad p_1 = x_1 P_1, \quad p_2 = x_2 P_2$$

Improving hard process $d\hat{\sigma}_{p_1p_2 \rightarrow p_3p_4}$ is a major effort now:

- get a more reliable control of primary hadrons
- build the library of important SM and BSM processes

Need of LO, NLO, NNLO

• high accuracy needed for LHC discoveries

• new processes for LHC: The LHC "priority" wishlist

process $(V \in \{Z, W, \gamma\})$	relevant for
1. $pp \rightarrow VV$ jet	$t\bar{t}H$, new physics
2. $pp \rightarrow t\bar{t}b\bar{b}$	$t\bar{t}H$
3. $pp \rightarrow t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \rightarrow VVb\bar{b}$	VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics
5. $pp \rightarrow VV + 2$ jets	VBF $\rightarrow H \rightarrow VV$
6. $pp \rightarrow V + 3$ jets	various new physics signatures
7. $pp \rightarrow VVV$	SUSY trilepton

What is needed for a NLO calculation

- compute *n*-point Born amplitude: $M_n^{(0)}$
- compute n+1-point Born amplitude: $M_{n+1}^{(0)}$
- compute one-loop corrections to *n*-point Born amplitude: $M_n^{(1)}$
- sum and check that all divergences are canceled at the (proper) inclusive level

NLO calculation strategies

- semi-numerical methods
- analytical methods
 - unitarity
 - reduction to elements (box + triangle + bubble + constant)
 - recurrence relations
 - supersimmetry: $QCD = \{SYM, N = 4\} 4\{SYM, N = 1\} + \{SYM, N = 0\}$

Feynman diagrams have unexpected simple structures

+ another 15 pages

 $F_{11111}^{[1]} = 0$. (3.10) +46-1 (A.11) $H^{1}_{1,k+1} = \frac{13}{15}S + \frac{1}{2} + \left\{1 + 0\right\},$ (A.12) $H^{[4]}_{1+1+1} = \frac{1}{12}(13\tilde{\chi} - 33\tilde{Y}) + \frac{1}{2},$ (A.D) $T_{\tau + \tau + \tau}^{IT} = -\frac{1}{6}\tilde{X} + \{\tau \rightarrow \pi\},$ 13.10 $T_{m+1}^{[0]} = -\frac{1}{2}(\hat{X} - 2\hat{Y})$ (3.15) where z, y, X, Y, X and F are defined in sys. [317] and (3.18). For ----- the functions are $-\left\{\frac{21}{2}(x-y)+\frac{13}{24}\frac{2}{2}\right\}S + \frac{11}{10}\frac{1-rs}{rs}+\left\{t-s\right\}$ 13.20 $D^{(4)}_{r \to r \pi} = -\frac{1 + \pi^2}{2 \pi \pi^2} (\bar{\lambda}^4 + \tau^2) + \frac{5 - 3 \pi}{3 \pi^2} \bar{\lambda} + \frac{2}{5} \bar{\tau} + \frac{1 - \pi \eta}{\pi^2}$ 14.201 $D^{(4)}_{-\infty+1} = -\frac{1}{16} g^2 \Big((X-Y)^2 + \pi^2 \Big) + \frac{1}{16} d^2 g - b k l +$ $+\frac{1-xy}{yty}+\{1-y\}$ 14.30 $E^{(4)}_{max} = \frac{p^2}{12\pi}X$ (5.20) $E_{-1+++}^{(0)} = \frac{1}{1\log_2} X + \left[1 \cdots + \right],$ (A.91) ptt 10.197 Phase a. (A.28) $-\frac{1}{100} \left(-\frac{10}{4\pi} + \frac{3}{2} (15\pi^2 - 2\delta_T + 8g^2)\right) X - \frac{11}{4} + \frac{354\pi}{22845} +$ $H^{(l)}_{a b + a} = \frac{1}{10} \Big(2 \frac{g^2}{a} + \frac{2 \pi (1 - 2 z)}{a^3} \Big) (\hat{X}^{\dagger} + a^2) + \Big(\frac{(1 - 2^2)}{2} - \frac{1}{2} \Big) \hat{X} +$ + (+-- +) $+\frac{1}{2m}\Big(-4\frac{\beta^2}{2}+2\pi p\Big)\Big((X-Y)^2+t^2\Big)+\Big\{t\to t\Big)\,,$ (A.25) $B^{(0)}_{111+4} = - A^{(101)0}_{11+4} + A^{(0)}_{11+4} + C^{(0)}_{11+4} + E^{(0)}_{11+4} + \frac{1}{2} \Big\{ \frac{g^2}{2r} + g^2 \Big\} X^2 +$ $H^{22}_{-k+1} = \frac{3}{10} \Big(2 \frac{y^2}{x} - 3 \frac{2y^2 - x}{x^2} \Big) (X^2 + x^2) + \frac{1}{11} \Big(2 \frac{x^2}{x} - 3 \frac{y - y}{x^2} \Big) (Y^2 + x^2) -$ $+\frac{2}{3}\left(\frac{\mu^{2}}{2}-\frac{1}{2}+\frac{\mu^{2}}{2}\right)\dot{T}^{2}\dot{T}^{2}-\frac{1}{3}\left(\frac{\mu}{2}-1+3\mu_{D}+\frac{\mu^{2}}{2}\right)\dot{T}\dot{T}^{2}+$ $-\frac{1}{12}\Big(2\frac{x^2}{\pi}+15\pi y-2y^2-2\frac{x^2}{\pi}\Big)\Big((X-Y)^2+x^2\Big)+$ $+\frac{g^2}{h}(2-t_0)(X - W) = \frac{2}{h}(\frac{g^2}{h} + g^2)U^2 =$ $+\frac{1}{15}\left(22\frac{\mu^2}{2}+1\right)\overline{X}+\frac{1}{5}\left(11\frac{\mu^2}{2}-10\right)\overline{Y}$ (8.29) $-\frac{1}{16} \Big(\frac{11}{\pi} + 3(\alpha-p) \Big) (\lambda^{2} + i\pi^{2}) + \\ + \frac{1}{16} \Big(\frac{11}{16} + \frac{1}{16} (\alpha-p) \Big) (\lambda^{2} + i\pi^{2}) + \\ + \frac{1}{16} \Big) (\lambda^{2} + i\pi^{2}) +$ 13.391 112- 12 .141 -10

(4.4)

(A.5)

14.6)

14.71

(4.8)

14.01

 $C_{a+++}^{[1]} = \frac{1}{34} \left(-13\bar{X} + 2\frac{y^2}{z} + 10 \right)$

 $D_{n+1}^{[1]} = -\frac{1}{2}(\hat{K} - 2\hat{Y}) + \frac{1}{2}.$

 $D_{a+++}^{[1]} = -\frac{1}{2}S + \frac{1}{2} + \{t \rightarrow u\},$

 $E_{1+++}^{H} = \frac{1}{10} \left(2 \overline{X} - \frac{y^2}{2} + 1 \right),$

 $C_{n+n+1}^{H} = \frac{1}{2N} \left(-12\hat{X} + 2\frac{N}{2} + 10^2 \right) + \left\{ t - x \right\}.$

 $\mathcal{L}^{\mathrm{H}}_{\mathrm{hom}} = \frac{1}{22} \Big(4 \mathcal{K} - \frac{1}{2N} + 1 \Big) + \big\{ l - n \big\},$

 $-\frac{4}{2}\left(\frac{4^{2}}{12^{2}}-\frac{4}{2}+1\right)(S^{2}+t^{2})-\frac{4^{2}}{12^{2}}\left(64+18etitr-m\right)$ $- \left| \frac{5}{2} \left(\frac{\pi}{2} - p_{2}^{2} \right) + \frac{67}{126} \right| S - \frac{\pi}{26} - \frac{25}{22} + \frac{1000}{640}$ 14.55 $\mathcal{X}^{(2)}_{1+1+1} = -\pi^{2}\frac{3-2\pi}{2}\chi(X-Y')^{2} - \frac{\pi^{2}}{12}\frac{(1-\pi)^{2}}{12}(X-Y)^{2} + \frac{\zeta_{X}}{2\pi}\tilde{X} - \frac{\pi^{2}}{12\pi}$ $-\frac{11}{12m}\left(t_{110} - r \right) - \chi t_{10} (-r) - \frac{1}{2} X^2 - \frac{r}{4} r^2 \Sigma + \frac{2 \chi_0}{12} - r \frac{r^2}{24} \right) +$ $+\left(\frac{11}{2\pi}\frac{d^2}{2}-\frac{2}{4}(\tau-g)\right)\left((X-Y)^2X-\frac{T^2}{4}Y\right) -\frac{11}{\pi}a_{1}\frac{(1-xy)^{2}}{\pi}\left[(X-Y)^{2}+\pi^{2}\right]+\frac{\pi^{2}}{2}(2\pi^{2}-2xy)+38\lambda I+$ $+ \frac{11}{10} \left(\frac{11}{\pi} - \otimes s - g_0 \right) (\bar{X}^2 + e^0) + \frac{1}{10} (m - \otimes a_0) \left((X - V)^2 + e^0 \right)$ $+\pi^2 \frac{1+42(1-\eta)}{96\eta} + \left(\frac{10\eta}{27\eta} + 5\eta + \frac{10}{11} + \frac{240}{6\eta r}\right)S + \frac{5}{\eta} - \frac{10000}{1400} +$ 11.125 $\boldsymbol{x}^{[1]}, \quad = \boldsymbol{D}^{(m_1, q)} + \boldsymbol{D}^{[1]}, \quad + i\boldsymbol{n}^{[1]}, \quad - \boldsymbol{n}^{[1]}, \quad + i^{[1]}, \quad - \boldsymbol{D}^{[1]}, \quad + i^{[2]}, \quad -\frac{1}{2}(x-y)\left(2\frac{y^2}{2}-3\right)\hat{X}^2-\frac{1}{2}\left(2\frac{y^2}{2}-3yx^2\right)\hat{X}^2+\frac{1}{2}\left(2\frac{y^2}{2}-3yx-\frac{y^2}{2}\right)\hat{X}^2\hat{Y}+$ $+\frac{1}{2}\Big(9\frac{\mu^2}{2}+x-30\eta y+0y+10\frac{\mu^2}{2}\Big)\tilde{X}\tilde{T}^{*1}-\frac{\mu}{2}(x-y)\Big(2\frac{\mu^2}{2}-y\Big)\tilde{T}^{*4}+$ $+\frac{2^{2}}{3}(t-k_{10})(\hat{X}-3^{0})-\frac{3}{3}(11\frac{s^{2}}{2}+s-10)\hat{X}^{0}+$ $+\frac{4}{10}\Big(11\frac{|t|^2}{2}+s-14\Big)\hat{T}^2+\frac{2}{10}\Big(\frac{2t}{4t}-18(t-|t|+\frac{10}{4t}\Big)\hat{T}\hat{T}+$

 $+\frac{1}{n}\Big(\frac{11}{n^2}+4\frac{2}{n}+20\frac{1}{2}(X^2+\pi^2)-\frac{T^2}{2n^2}\Big(20\frac{T^2}{n}-\frac{30}{n}+30\delta_0-101\Big)+$ $+ \frac{1}{100} \left(\pi \tau \frac{r + 2y}{100} + \frac{111}{100} \right) \hat{\tau} - \frac{\beta}{10} + \frac{110}{100} - \frac{3849}{10007}$ 14.56 $e^{\frac{14}{2}}_{m++} = -\frac{1}{46} e^{2} ((X-Y)^{2} + e^{2}) (4(X-Y)^{2} - e^{2}) +$ $+\frac{1}{2m}\left[1h(-\pi)-X(t_{0}t-\pi)-\frac{3}{2}X^{4}-i\frac{\pi}{2}X(X-Y)-\frac{3}{2}\pi^{2}X-\right]$ $-\frac{2}{12}(z_1 + e^{\frac{2\pi}{3}}) - \frac{1}{12}\left(2\frac{e^2}{2} + \frac{e^2}{2}(1)e + 4e\right)\hat{X}^2\hat{Y},$ $-\frac{1}{2k!}\left(6\frac{k^2}{2}+11(2t^2+q^2)\right)(61)(t^2+t^2)+$ $+\frac{r^{2}}{12}\left(r\frac{\theta^{2}}{2}-1+4r\right)\dot{x}-\frac{1}{22}\left(\frac{43}{2}-26(t-2)\right)(\theta^{2}+r^{2}) -\frac{1}{24}(\overline{\pi}p^2 + 4\pi p)((X - Y) + r^2) - r^2 \frac{12 + 366(1 - r_0)}{428\pi r_0}$

 $qq \rightarrow qq$

11.36

High order QCD amplitudes as curves in twistor space

1986 Park+Taylor: the many page long 4-gluon amplitude (MHV and tree level) can be reduced to a small single line formula

General multi-gluon amplitude:

$$\mathcal{M}_n = g_s^{n-2} \sum_{\text{perm}} \text{Tr}(t^{a_1} t^{a_2} \cdots t^{a_n}) \cdot M(p_1 \lambda_1; p_2 \lambda_2; \cdots p_n \lambda_n)$$

 $M(\lambda_1\lambda_2\cdots\lambda_n)$ colour ordered amplitude

MHV configuration $\lambda_i = \lambda_j = -1, \ \lambda_k = +1$

$$M_{\rm tree}^{\rm MHV}(+-++-++) = \frac{\langle p_i p_j \rangle^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \cdots \langle p_n p_1 \rangle},$$
$$\langle p_i p_j \rangle = \sqrt{2p_i p_j} e^{i\phi_{ij}}$$

Space-like singlet anomalous dimension at three loops Vogt Moch Vermaseren, Nucl. Phys. B691(2004) 129; B688(2004)101

$$\begin{split} & \sum_{i=1}^{n} \sum_{j=1}^{n} \left[-\frac{1}{2} \left[-\frac{1}{2}$$

$$\begin{split} & = - m_{1} \left(\frac{1}{2} \left(1 + m_{1} + m_{2}^{2} \right) \right) & = - m_{1} \left(1 + m_{1} + m_{2}^{2} \right) \\ & = - m_{1} \left(\frac{1}{2} \left(1 + m_{1} + m_{2}^{2} \right) + m_{1} \left(1 + m_{1} + m_{2}^{2} \right) + m_{2} \left(1 + m_{1} + m_{2}^{2} \right) + m_{1} \left(1 + m_{1} + m_{2}^{2} \right) \\ & = - m_{1} \left(1 + m_{1} + m_{2} + m_{2}$$

 $\begin{array}{l} \begin{array}{l} p_{1} = m_{1} + m_{2} + m_{1} + m_{2} + m$

and a second sec $1 + 1 + 1 + \frac{1+1}{2} + 1 + 1 + \frac{1}{2} + 1 + 1 + \frac{1}{2} + 1 + 1 + 1 + 1 + \frac{1+1+1}{2} + 1 + 1 + 1 + 1 + \frac{1}{2} + 1$ The state of the state of the state of the state of to be fitted to be a first a first a state and a state of the and a state of the reconstruction because the second state of the $\left\| x_{i} - \frac{V_{i}}{2} x_{i}^{2} \right\|_{1}^{2} + \left\| m_{i} \frac{h_{i}}{h_{i}} \right\|_{1}^{2} \left\| M_{i} + m_{i} - M_{i} - M_{i} + 0 \right\|_{1}^{2} \frac{h_{i} \left(m_{i} - M_{i} \right)}{h_{i} \left(m_{i} - M_{i} \right)} + \left\| m_{i} - M_{i} + 0 \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} - M_{i} - M_{i} - M_{i} \right\|_{1}^{2} + \left\| m_{i} - M_{i} -$ Constantial a second a second se moltemate francisco e francisco - franc March - March Attoors (2) To a Moor + (2) The Attoor + (2) The Attoors (2) The Attoor + (2) n an - Shan 🖓 na mar - Shan an ana mara an an mean been among an an an and a part of feedbal means (1999) - Serie - Marca tente de la composición de la composicinde la composición de la composición de la composición de la co a franciska – fran $a_1 + \frac{12}{3}a_2 + (1-2)a_1^2 + \frac{12}{3}a_2(1-\frac{12}{3}a_2)a_1 + \frac{12}{3}a_2(1-\frac{12}{3}a_2)a_2 + \frac{12}{3}a_2(1-\frac{12}{3}a_2)a_2$ Freedown States Processing a Freedom and a States Street $\frac{10}{2}(0, s) = \frac{10}{2}(0, s) + 2(0, s, s) = \frac{1}{2}(0, s) + 10(0, s) + 2(0, s) + \frac{1}{2}(0, s) + 2(0, s) + 10(0, s)$ (1.1.1) (1.1.1) (1.1.1) (1.1.1) (1.1.1) (1.1.1) (1.1.1) han basal 👾 a Basal Basabada Basaligan Ba

are provident to the state of the state of

$$\begin{split} & \frac{1}{2} \left[\left[a_{1} - a_{1} \cos \left(a_{1} - a_{2} + a_{2} - a_{2} + a_{1} + a_{2} +$$

$$\begin{split} & = \frac{1}{2} (x_1 - \frac{y_1}{2} x_2 + 2 \pi m) (x_1 - 1) (x_2 + \frac{y_1}{2} x_1 + 2 \pi m) (x_2 - \frac{y_1}{2} x_2 + 2 \pi m) (x_1 - \frac{y_1}{2} \pi) (x_2 - \frac{y_1}{2} x_2 + 2 \pi m) (x_1 - \frac{y_1}{2} x_2$$
the farmer of the second strategies - Second a fron - non a non- nan ma 🔤 🖬 a fina- a (mar - froa mol والإستانية والمراجع أوراده وأسترك والتنازي والمعالية والمتعارية والمترار ويحتاج والتقالية ومعتمد والمتكر والمتكر والمتكر - The garden contract of the second model and the - See another mode a Second Second Second research and Second Se Second Sec and a Property of the second state of the seco and and a first and a sign of the state of the second second second second second second second second second s atter med aber alate the standard from the state a Bear Second Hear a Second Second Second and Second (Sum (Sec.), Sum (Sec.), Charles United Street, Control (Sec.), Control (Se $Eq_{1,r-1} + E Eq_{1,r-1} - \frac{1}{2} q_{1,r,0} - E Eq_{1,r} - \frac{1}{2} E E_{1,r} + \frac{1}{2} E E_{1,r} - \frac{1$ den men Bernen men men Kan este Van der $\frac{1}{12}\frac{m}{2}q + \frac{2}{3}q_{-1} + \frac{2}{3}R_{0} + R_{0,1} + \frac{R}{3}q_{-1} + R_{0} \Big] + \left[-R_{0} \right] \left[\frac{R}{3}R_{0} + \frac{2}{3}\frac{m}{3}r_{-1} + R_{0} \Big] \frac{1}{3}$ $+\frac{1}{2} t_{1,2} + \frac{10}{16} t_{1,-2} + \frac{10}{16} t_{1,-2} + \frac{10}{16} t_{1,2} + \frac{10}{16} t_{1,-2} + \frac{10}{16} t_{1,2} + \frac{$ $+\frac{10^{10}}{10^{10}}\eta_{10}+\frac{110^{10}}{10^{10}}\eta_{1}+\frac{110^{10}}{10^{10}}\eta_{1}+\frac{110^{10}}{10^{10}}\eta_{10}+\frac{110^{10}}{10^{10}}\eta_{10}+\frac{100^{10}}{10^{10}}\eta$ $+ 2 \eta_{1,0,1} + \frac{100}{2} \eta_{1,0,2} - 2 \eta_{1,0,1} + 2 \eta_{1,0,1} + \frac{100}{2} \eta_{1,1} + \frac{100}{20} \eta_{1,1} + \frac{100}{2} \eta_{1,1}$ $- m_{1}^{(1)} + m_{1} + \frac{1}{2} ([\mathbf{n}_{1} + \mathbf{n}_{2} - \mathbf{n}_{1} - \mathbf{n}_{1}] + m_{1} + m_{1}$ $-[\mathbf{x}_{1}-\mathbf{x}_{2}]\frac{2\pi i m}{2\pi i n}\mathbf{q} - \frac{2\pi i}{10}\mathbf{x}_{1} - \frac{2\pi i}{10}\mathbf{q} + \frac{2\pi i}{10}\mathbf{q} - \frac{2\pi i}{10}\mathbf{x}_{2} + 2\mathbf{x}_{1}\Big] + [1-\mathbf{x}_{2}]\Big[\frac{2\pi i}{10}\mathbf{q} + 2\mathbf{x}_{2}$ $+\frac{2\pi T}{2\pi T}\mathbf{n}_{1}\cdot\frac{T}{2}\mathbf{n}_{2}\cdot\frac{T}{2}\mathbf{n}_{3}\cdot\frac{T}{2}\mathbf{n}_{4}-\frac{T}{2}\mathbf{n}_{4}+\frac{T}{2}(\mathbf{n}_{1}-1)\mathbf{n}_{4}^{2}+2\mathbf{n}_{4}^{2}\mathbf{n}_{2}^{2}(\mathbf{n}_{1}-1)\left[\mathbf{n}_{1,n-1}\right]$

a Sina - Sina basa a Sina basa a basha mar 🖉 Sina a a - Sina $+\frac{H}{N} H u - \frac{H}{N} + \left(\mathbf{N} \cdot \mathbf{1} - \mathbf{N} \mathbf{N} - \mathbf{N} \right) + \mathbf{N} u + \mathbf{N} \left(\frac{H}{1 + N} \mathbf{n} - \frac{H}{N} \mathbf{n} \right) - \frac{1}{N} \left(\mathbf{N} - \mathbf{N} \right) \left[\mathbf{n} \right]$ $-m_1 + n + (n_1 + n_2 - n_3) \left[\frac{1}{n_1} \frac{1}{n_2} - \frac{1}{n_2} \frac{1}{n_3} \frac{1}{n_1} - \frac{1}{n_1} \frac{1}{n_2} \frac{1}{n_1} - \frac{1}{n_2} \frac{1}{n_1} \frac{1}{n_2} \frac{1}{n_2} \frac{1}{n_2} \frac{1}{n_1} \frac{1}{n_2} \frac{1}{n_2} \frac{1}{n_1} \frac{1}{n_2} \frac{1}{n$ e (Nata 1964). No select an $\left\{\frac{222}{22}$ is a major $\frac{1}{2}$ is the $\frac{12}{2}$ is the rest of $a \frac{1}{2}$ in b $-\frac{1}{2}(t_{1}-t_{2})+\frac{1}{2$ -100 $= \frac{10}{2}$ ($n_1 + \frac{1}{2}$) $(n_2 + \frac{10}{2})^{2}$ ($n_2 + 100$ $+ \frac{10}{2}$) $(n_1 + \frac{10}{2})^{2}$ $= \frac{10}{2}$ $(n_1 - 10)\frac{100}{2}$ and and - Sector Sector (a) Sector Sector ($-2\pi a \left[\frac{1}{2} m_{0}^{2} + \left[\frac{1}{2} m_{0} - 2m_{0}^{2} + \left[\frac{1}{2} m_{0} + 2m_{0}^{2} + \frac{1}{2} m_{0} + \frac{1}{2} m_{0} + \frac{1}{2} m_{0} + 2m_{0}^{2} + \frac{1}{2} m_{0}^{2} + 2m_{0}^{2} + 2m_{0}^{2}$ 4 1 mar - 2 mar 12 mar - 12 mar 12 mar - 2 mar $= \frac{10}{2} m_1 - \frac{10}{2} m_2 + \frac{1}{2} m_1 \frac{1}{2} (m_1^2) + 2 m_1^2 \frac{1}{2} (m_1 - m_2 - m_3 + m_3 + 1) \left[\frac{1000}{200} m_1 - m_3 + 1 \right]$ a finan a mana a finan ana ann an ann an fifinn a marair annos - more annos ¹²no annos e mar a¹²na annos - ma $-\frac{10}{2}$ is $-10.04 - \frac{10}{2}$ in a single $-\frac{10}{2}$ in all $-10.04 + \frac{10}{2}$ in a limit $-\frac{10}{2}$ in a lin a limit $-\frac{10}{2}$ in a limit $-\frac{10}{2}$ $= 2223 + 2233 + 2233 + 3(1 + 23 + 23)(1 + 23 + 23) + \frac{123}{1 + 23} + \frac{12}{1 + 23} + \frac{12}{$ Sector and the sector sector and $-100 + 10 + \frac{10}{2} + 10 + 100 + \frac{100}{2} + 100 + 10 + \frac{10}{2} + 100 + 10$ $+ \left[{{{\mathbf{N}}_{\rm{s}}}_{\rm{s}}} \right]\left[{{{\mathbf{N}}_{\rm{s}}}_{\rm{s}}}_{\rm{s}} + \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}}_{\rm{s}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}_{\rm{s}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}_{\rm{s}}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}_{\rm{s}}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}_{\rm{s}}}}_{\rm{s}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}_{\rm{s}}}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}_{\rm{s}}}}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}}}}_{\rm{s}} - \frac{{{{\mathbf{N}}_{\rm{s}}}}}}{{{{\mathbf{N}}}}$ $+W_{k_1,1}-\frac{m_1^2}{\pi}v_{k_1,1}+iv_{k_2}+iw_{k_1}+iw_{k_2}\Big|+|t-S_k|\Big|\frac{4\pi m_1}{(1+k_1)}v_1+\frac{m_2}{\pi}v_{k_1,1}+\frac{4\pi m_2}{m}v_1.$ - No. 1 and some ^{1,4} in a second ^{1,9} for the allocate ^{1,9} for the second $\left|\frac{H_{1}}{2}h_{1}^{2}-\frac{H_{1}}{2}+2A_{1}+2A_{1}+\frac{M_{1}}{2}h_{1}+2A_{1}+2A_{1}+2A_{1}+2A_{1}+\frac{M_{1}}{2}h$ $= \frac{1}{2} \nabla_{A_{1}+1} + (\nabla_{A_{1}+1} + (\nabla_{A_{1}+1} - \frac{1}{2}))_{i=1} + \frac{1}{2} \nabla_{A_{1}} - \frac{1}{2} \nabla_{A_{1}} + \frac{1}{2} \nabla_{A_{1}} + \frac{1}{2} \nabla_{A_{1}} - \frac{1}{2} \nabla_{A_{1}}$ $- = i_{1,1,2} - \frac{m}{2} a_{11} + \frac{m}{2} a_{12} - m_{1,12} + \frac{m}{2} a_{1,12} + a_{1,12} - \frac{m}{2} a_{1,2} + m_{1,12,12} + \frac{m}{2} a_{1,2}$

$$\begin{split} & + \frac{1}{2} | a_{1} | b_{1} | b_{1} | a_{2} | b_{2} - b | b_{1} - b_{2} | b_{1} | b_{2} | b_{2} - b | b_{1} - b_{2} | b_{2} - b | b_{2} - b_{2} | b_{2} -$$

$$\begin{split} &+ \sum_{i=1}^{N} (-1,0) + (-$$

may prove our spectrum many means the second state of a sign of the state τ_{i} (σ_{i}) and the prove state of the s

The state of the

The transmission of the strain product of the transmission of the strain $\frac{1}{2}e^{-\frac{1}{2}}$ is a set of the strain of the st

where δ is the matrix matrix matrix the set indication in general $\alpha \phi^{2}$ is a set

For *expected* simple structure here see Yuri's talk

Second stage: Generate initial state bremsstrahlung radiation $k_1 k_2 \cdots k_n$

Use DGLAP evolution equation



$$P_1 \rightarrow p_1 + k_1 k_2 \cdots, \qquad p_1 = x_1 P_1$$
$$\partial_{Q^2} F(x, Q^2) = \int_0^1 \frac{dz}{z} P(z, \alpha_s) F\left(\frac{x}{z}, Q^2\right)$$

Collinear cutoff Q_0 required

Subtleties:

- evolution variable: k_t (coherence)
- splitting function: $P(z, \alpha_s)$ (known at three-loop VVM 2004)

• running coupling argument: k_t , and physical scheme

Space-like evolution variable (coherence)



$$\frac{-k_i^2}{k_{i,+}} = \frac{-k_{i-1}^2}{k_{i-1,+}} + \frac{{k'}_i^2}{k'_{i,+}} + \frac{k_{i,+}k'_{i,+}}{k_{i-1,+}} \left(\frac{\vec{k}_{it}}{k_{i,+}} - \frac{\vec{k'}_{it}}{k'_{i,+}} \right)$$
mass singularities factorize

$$-k_{i-1}^2 \ll -k_i^2 z_i^{-1}, \qquad \frac{k_{i,+}}{k_{i-1,+}} \equiv z_i$$

Actually, cancellation (coherence) leading to k_t -ordering pre-QCD observation by Gribov 1960's

$$-k_i^2 < -k_{i-1}^2 < -k_i^2 z_i^{-1} \quad \Rightarrow \quad -k_{i-1}^2 < -k_i^2 \quad \simeq \quad k_{t,i-1}^2 < k_{t,i}^2$$

Yuri's talk for connection with crossing symmetric case (fragmentation function)

Third stage: Final state emission



Radiation off the *primary partons* given by:

 $p_3, p_4 + \text{bremsstrahlung:} k_1 + \cdots \text{ and } k'_1 + \cdots$

$$P_1 \rightarrow p_1 + k_1 \cdots$$
 and $P_2 \rightarrow p_2 + k'_1 \cdots$

Consider first the fragmentation function of a primary parton



Mass-singularity phase space + coherence \Rightarrow angular-ordering: $Q = \theta E$

$$Q^2 \partial_{Q^2} D(x, Q) = \int_0^1 \frac{dz}{z} P(z, \alpha_s) D\left(\frac{x}{z}, zQ\right)$$

Generalization to parton branching $a \rightarrow b_1 b_2$ Bassetto, Ciafaloni+GM 1983

$$Q^{2}\partial_{Q^{2}}G_{a}(Q) = \frac{1}{2}\int_{\epsilon}^{1-\epsilon} \frac{dz}{z} P_{a}^{b_{1}b_{2}}(z,\alpha_{s}) \{G_{b_{1}}(zQ)G_{b_{2}}((1-z)Q) - G_{a}(Q)\}$$

HERWIG 1983

$p\bar{p} \rightarrow W^+ + X \,, \quad W^+ \rightarrow t\bar{b} :$ a MC event



 G^{ab}_{μ} : gluon with two indexes

$$SU(N_c) \to U(N_c)$$

Figure 1: Colour structure of a $\bar{p}p \rightarrow W^+ + X, \ W^+ \rightarrow t\bar{b}$ event.

As a consequence of factorization, one can construct a single Monte Carlo program, such as



Forth stage: hadronization

QCD radiation of partons

how to go to hadrons?

beyond perturbative QCD

Preconfinement (D.Amati, G.Veneziano, PL83(79)87) :

• colour connection \rightarrow Sudakov suppression in mass distribution

 \rightarrow colour connected partons at small mass

• colour connected \rightarrow small mass colour singlet \rightarrow model \rightarrow hadrons

results not too sensitive to hadronization model

Cluster Hadronization Model



- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. $t \rightarrow bW$
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster \rightarrow hadrons
- hadronic decays

QCD Monte Carlo simulation:

 e^+e^- annihilation lepton-hadron DIS hadron-hadron (large E_T)

• Herwig (B.Webber&GM, 1984): e^+e^- , ℓ^-h and h^-h

• Pythia (T.Sjostrand 1987): e^+e^- , ℓ -h and h-h

• Ariadna (L. Lonnblad 1992): e^+e^- mostly

others for specific processes



Improvements

- library of hard processes: Heavy quarks, Electroweak, Higgs, Sypersymmetry...
- matching with exact fix order results: LO, NLO, NNLO....
- include QCD results in high energy scattering (BFKL, CCFM, ...)
 H.Jung 1992
- include low p_t radiation: underlying event
- include large angle soft emission

Include large angle soft emission

Generating functional (couting emitted partons)

$$G_{ab}[E, u] = \sum_{n} \int \prod_{i} \left(\bar{\alpha}_{s} dq_{i} u_{i} \right) M_{ab}^{2}(q_{1} \cdots q_{n}), \qquad \bar{\alpha}_{s} = \frac{N_{c} \alpha_{s}}{\pi}$$

Multi-soft gluon distribution (planar limit): Bassetto, Ciafaloni&GM 1981

$$M_{ab}^{2}(q_{1}\cdots q_{n}) = w_{ab}(q_{\ell}) \cdot M_{a\ell}^{2}(q_{1}\cdots q_{\ell-1}) \cdot M_{\ell b}^{2}(q_{\ell+1}\cdots q_{n}), \qquad w_{ab}(q) = \frac{(ab)}{(aq)(qb)}$$

Give evolution equation (virtual corrections included) Banfi, Smye&GM 2002

$$E\partial_E G_{ab}[E,u] = \int \frac{d\Omega_q}{4\pi} \bar{\alpha}_s w_{ab}(q) \Big\{ G_{aq}[E,u] \cdot G_{qb}[E,u] - G_{ab}[E,u] \Big\}$$

To solve by Monte Carlo introduce Sudakov form factor (with a cutoff Q_0)

$$\ln S_{ab}(E) = -\int_{Q_0}^{E} \frac{d\omega_q}{\omega_q} \int \frac{d\Omega_q}{4\pi} \,\bar{\alpha}_s w_{ab}(q) \cdot \theta(q_{tab} - Q_0)$$

"Solution" of the evolution equation as a dipole branching process (ab)
ightarrow (aq)(qb)

$$G_{ab}[E,u] = S_{ab}(E,Q_0) + \int d\mathcal{P}_{ab}(q) \ G_{aq}[\omega_q,u] \cdot G_{qb}[\omega_q,u]$$

$$d\mathcal{P}_{ab}(q) = \left\{ \frac{d\omega_q}{\omega_q} \frac{S_{ab}(E)}{S_{ab}(\omega_q)} \right\} \left\{ \frac{d\Omega_q}{4\pi} \bar{\alpha}_s w_{ab}(q) \right\} \cdot \theta(q_{tab} - Q_0)$$

$$r_{ab}(E, \omega_q) = \frac{S_{ab}(E)}{S_{ab}(\omega_q)} \qquad \int dr_{ab}(E, \omega_q) = 1 - S_{ab}(E)$$

$$dR_{ab}(\Omega_q) = \mathcal{N}_{ab} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s w_{ab}(q) \qquad \int dR_{ab}(\Omega_q) = 1$$

- 1. *ab*-dipole emits? Take a random number $0 < \rho < 1$:
 - $\rho < S_{ab}(E)$ no emission
 - $\rho > S_{ab}(E)$ emission with energy ω_q with $\rho \cdot S_{ab}(\omega_q) = S_{ab}(E)$
- 2. obtain Ω_q by sampling $dR_{ab}(\Omega_q)$
- 3. repeat procedure for each new generated dipole till no dipole emits within the resolution Q_0 .

Strong energy ordering: $\omega_n \ll \omega_{n-1} \cdots \ll \omega_1 \ll E_a, E_b$ How to implement energy conservation and the full splitting function?



Final consideration

- MC (for QCD jets) needs hard scale $Q \gg \Lambda_{QCD}$
- MC as "summa" of most hard QCD studies
- MC not a solution of \mathcal{L}_{QCD} , but universal features in e^+e^- , DIS and hh-collisions
- MC enormous impact on experimental analysis and planing of future beyond SM investigations
- MC continuously improved to include new theoretical results
- MC future (not planed) developments
 - ♦ $1/N_c^2 = 1/9$ corrections: beyond parton model?
 - include single collinear and singe infrared logarithms
 - include unbiased high energy scattering (BFKL dynamics)
- large-angle-soft-emission: amazing connection to BFKL dynamics A.Mueller&GM,PhysLettB(2003)