



**The Abdus Salam  
International Centre for Theoretical Physics**



**SMR/1842-15**

**International Workshop on QCD at Cosmic Energies III**

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**Lecture Notes**

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# Hadron Nucleus collisions in Air Showers

- Introduction
- air shower simulations
- hh scattering
- hA scattering
- High density effects at high densities/energies
- Model comparison/air-shower properties

# Why are ultra high energy cosmic rays interesting?

- **Origin unclear**

- Bottom-up: acceleration
  - Study astrophysical objects,
- Top-down: decay of some heavy object
  - Study exotic physics

- **Propagation:**

- interaction with CMB
- Interaction with magnetic fields (IGMF, GMF)

- **Interaction in the atmosphere**

- Determine primary
- Learn about high energy physics

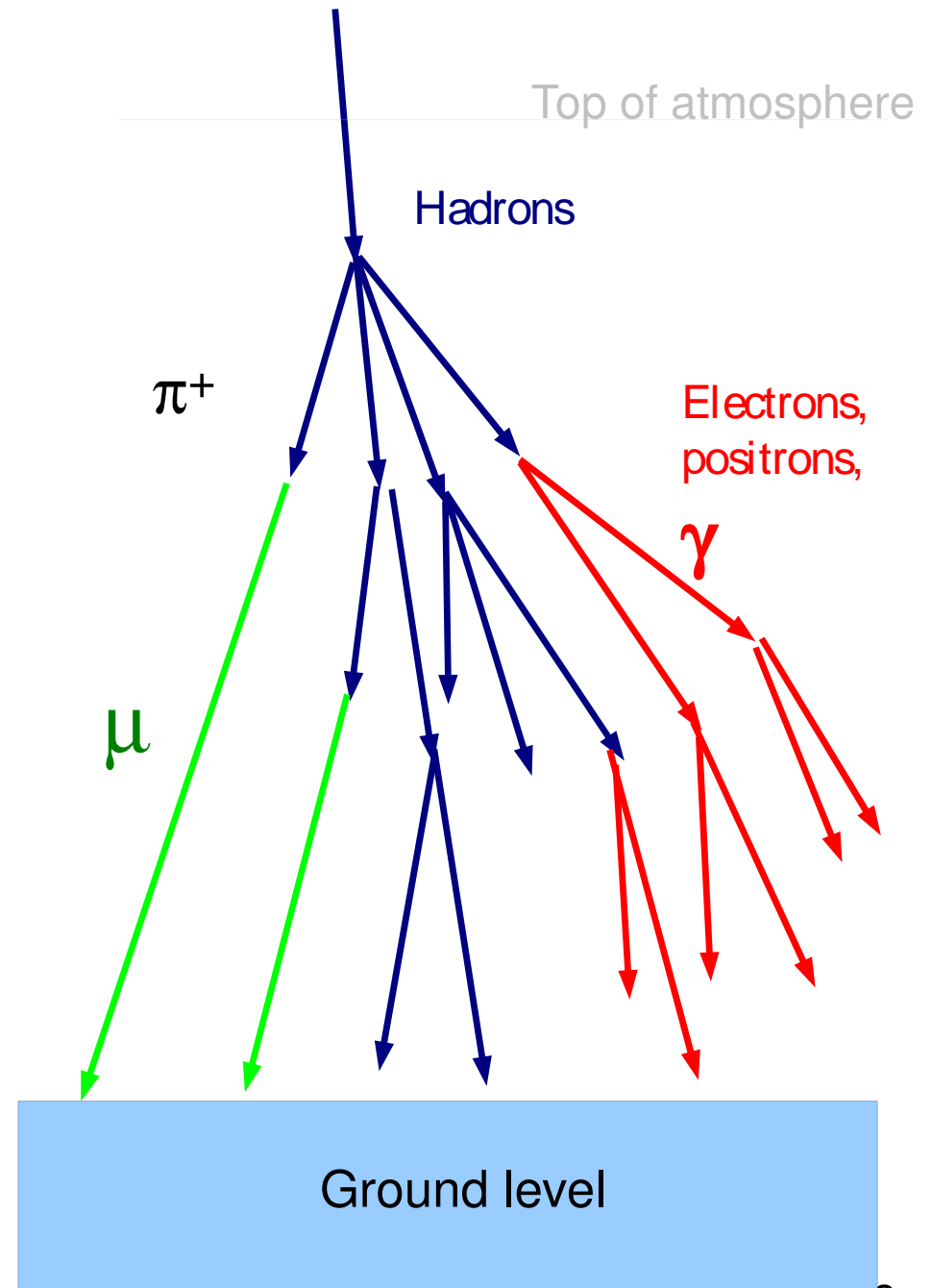
# Detection of UHECR

Direct detection not possible:  
Flux very low:  
 $E > 10^{20}$  eV  
-> 1 particle/km<sup>2</sup>/century

indirect detection via  
AIR SHOWERS  
induced by UHECRs

Reconstruct primary  
from shower properties:

- Energy,
- Arrival direction,
- Particle type



# Air shower interactions

Hadrons: hadronic interactions  
p-Air  $\pi$ -Air, K-Air  
decays

Electrons/Gammas:  
Bremsstrahlung  
Pair Creation  
inverse Compton  
energy-loss  
photo-nuclear effect  
LPM

.....  
Muons: energy loss  
bremsstrahlung ...

- Main theoretical uncertainty:
- higher energies
  - $\pi$  induced reactions
  - phase space different from collider experiments
  - inelastic cross section

## Gribov Regge Theory for pp interaction

Watson Sommerfeld transform of Amplitude  $A(s,t)$ ,  
(similar to partial wave expansion),  
has poles at  $l = \alpha(t)$

$$A(s, t) = \beta(t) \eta(t) s^{\alpha(t)}$$

Assume one dominating pole for  $s \rightarrow \infty$

$$\alpha(t) = \alpha(0) + \alpha' t$$

Regge trajectory

$$\beta(t) = \beta(0) e^{-B_0 t/2}$$

residue

$$\eta(t) = \frac{-1 + \xi e^{-i\alpha(t)}}{\sin(\pi \alpha(t))}$$

signature factor

Optical theorem  $\sigma_{tot} = \frac{1}{s} \text{Im} A(s, t=0) \sim s^{\alpha(0)-1}$

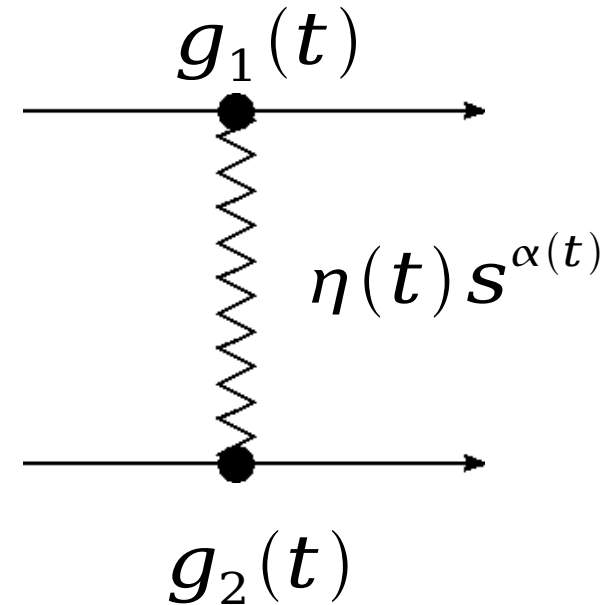
Reggeon  $\alpha(0) \sim .5$

Pomeron for  $\alpha(0) > 1$   
slowly rising cross section

$$A(s, t) = \eta(t) g_1(t) g_2(t) s^{\alpha(t)}$$

$$\frac{d\sigma_{el}}{dt} \sim s^{2\alpha(0)-2} e^{-B|t|}$$

$$B = B_0 + 2\alpha' \ln s$$



Shrinkage of diffractive peak  
increase of interaction range

$$\sigma_{tot} \sim s^{\alpha(0)-1}$$

$$\alpha(0) \approx 1.07$$

Violation of Froissart bound  
-> Multiple Pomeron exchange

## Eikonal formalism recovers unitarity

$$\begin{aligned} \sigma_{tot} &= 2 \int d^2 b [1 - e^{-\chi(s,b)}] && \chi(s,b) \\ \sigma_{el} &= \int d^2 b [1 - e^{-\chi(s,b)}]^2 \\ \sigma_{inel} &= \int d^2 b [1 - e^{-2\chi(s,b)}] \end{aligned} \quad \text{Eikonal}$$

$$\begin{aligned} \chi(s,b) &\sim \int d^2 q_t e^{-iq_t b} \text{Im}(A(s, q_t)) \\ &\sim \frac{g_1 g_2 s^{\alpha(0)-1}}{4\pi B} e^{-b^2/2B} \end{aligned}$$

Poisson distribution: 
$$P_n = \frac{(2\chi(s,b))^n}{n!} e^{-2\chi(s,b)}$$



## String end distributions

Gribov-Regge only gives number of (cut) Pomerons, nothing about their mass.

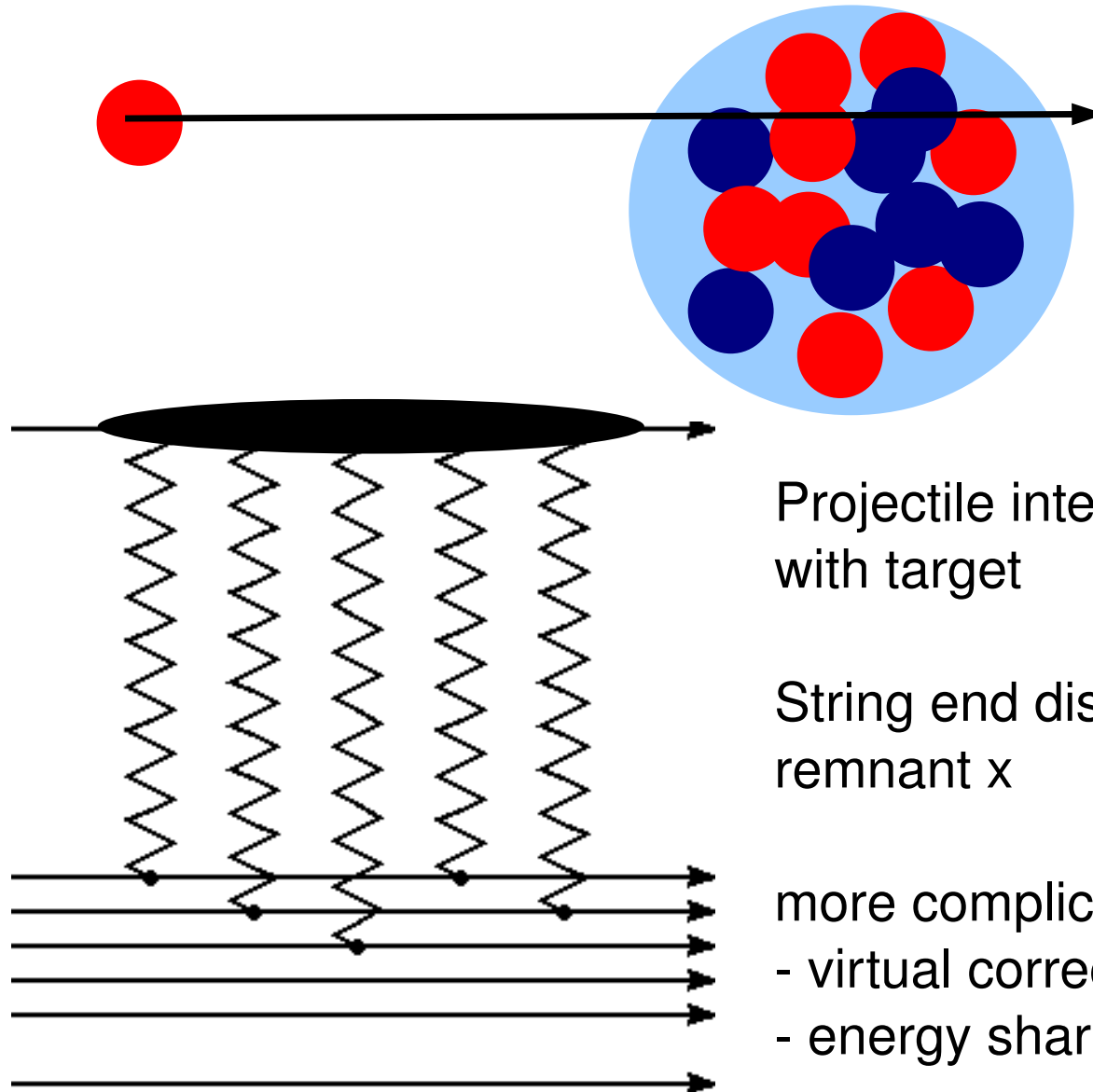
Assume following distribution of string ends (or Pomeron ends )

$$f(\mathbf{x}) \sim x_1^{\alpha_q} x_2^{\alpha_q} \dots \left(1 - \sum_i x_i\right)^{\alpha_{remn}}$$

$$\alpha_q \approx -\frac{1}{2} \quad \text{Momentum fraction distribution of (anti-) quarks}$$

$$\alpha_{remn} \approx \frac{3}{2} \quad \text{Momentum fraction distribution of remnant}$$

# Generalization to hadron Nucleus collisions with Glauber Gribov



Projectile interacts coherently with target

String end distributions influence remnant  $x$

more complicated:

- virtual correction (elastic rescattering)
- energy sharing

# Perturbative processes

$\alpha_s(Q^2)$  running coupling constant small at large  $p_t$

Inclusive jet cross section:

$$\sigma_{jet} = K \sum_{i,j} \int_{p_t^2 > p_{t,0}^2} dp_t^2 \int dx_1 dx_2 \frac{d\sigma_{i,j}(x_1 x_2 s, p_t^2)}{dp_t^2} \times f_{A,i}(x_1, Q^2) f_{B,j}(x_2, Q^2)$$

$K$   $K \sim 2$  factor accounts for NLO corrections

$d\sigma(\hat{s}, p_t^2)/dp_t^2$  Differential parton-parton cross section

$f_{A,i}(x_1, Q^2)$  Parton distribution function

# Multiple jet production if a single event

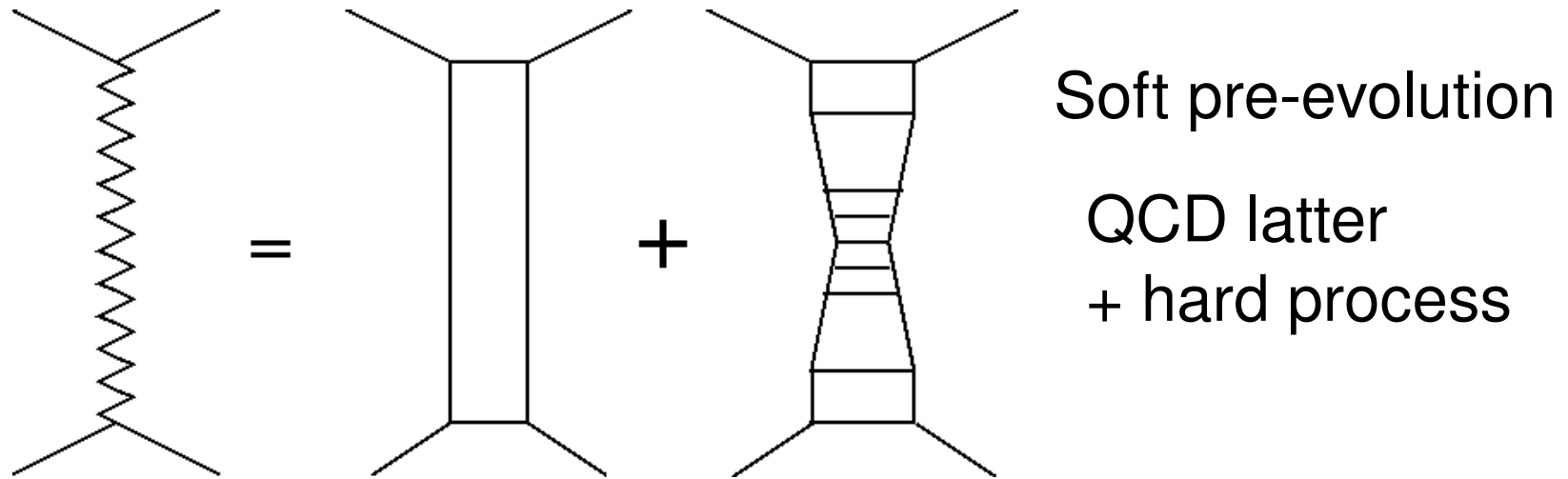
- Inclusive jet cross section  $\sigma_{jet} > \sigma_{tot}$
- Multiple jets

Define hard eikonal:  $\chi_{hard}(s, b) = \frac{1}{2} \sigma_{jet} A(s, b)$

$A(s, b)$  Overlap function of hadrons

Total inelastic cross-section:  $\sigma_{inel} = \int d^2 b \left[ 1 - e^{-2\chi_{soft}(s, b) - 2\chi_{hard}(s, b)} \right]$

# Matching soft and hard scattering with the semihard Pomeron



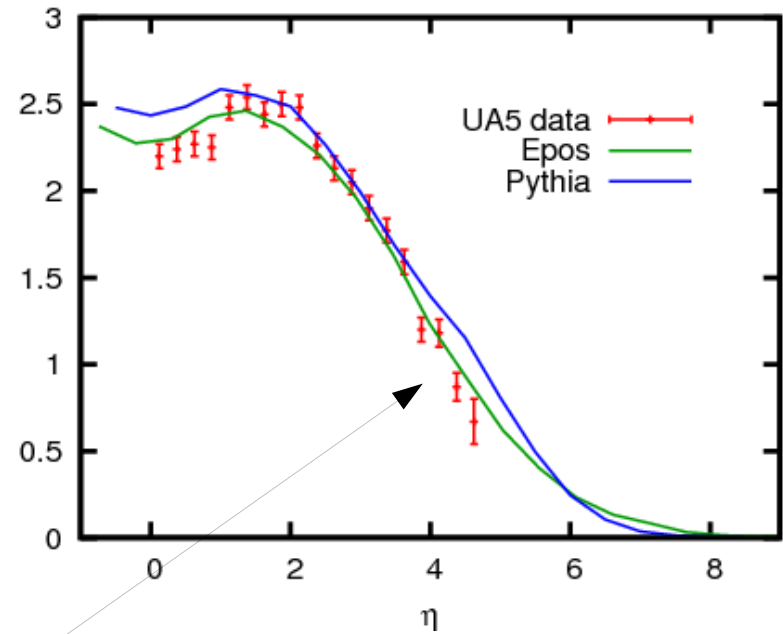
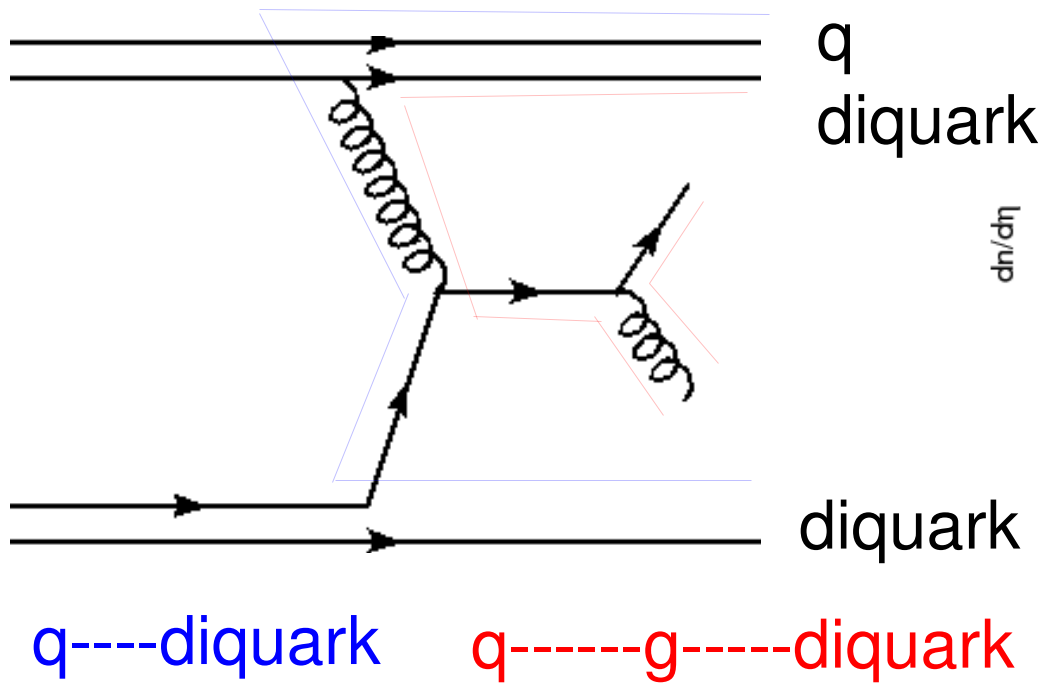
Soft pre-evolution: string ends for hard partons

Used in QGSJet, Nexus, Epos

# String ends for hard scatterings

No information from pQCD how to match hard partons to soft strings.

Usually employed: color flow

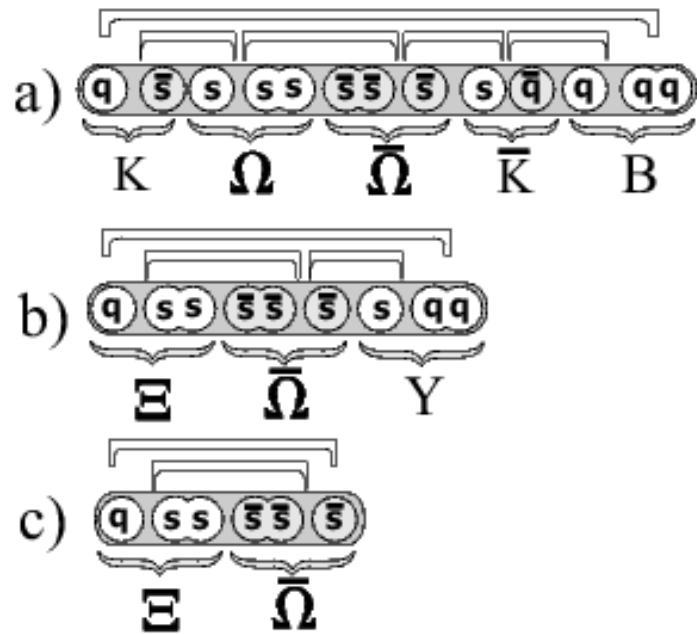


Better soft pre-evolution with  $1/\sqrt{x}$  stringends

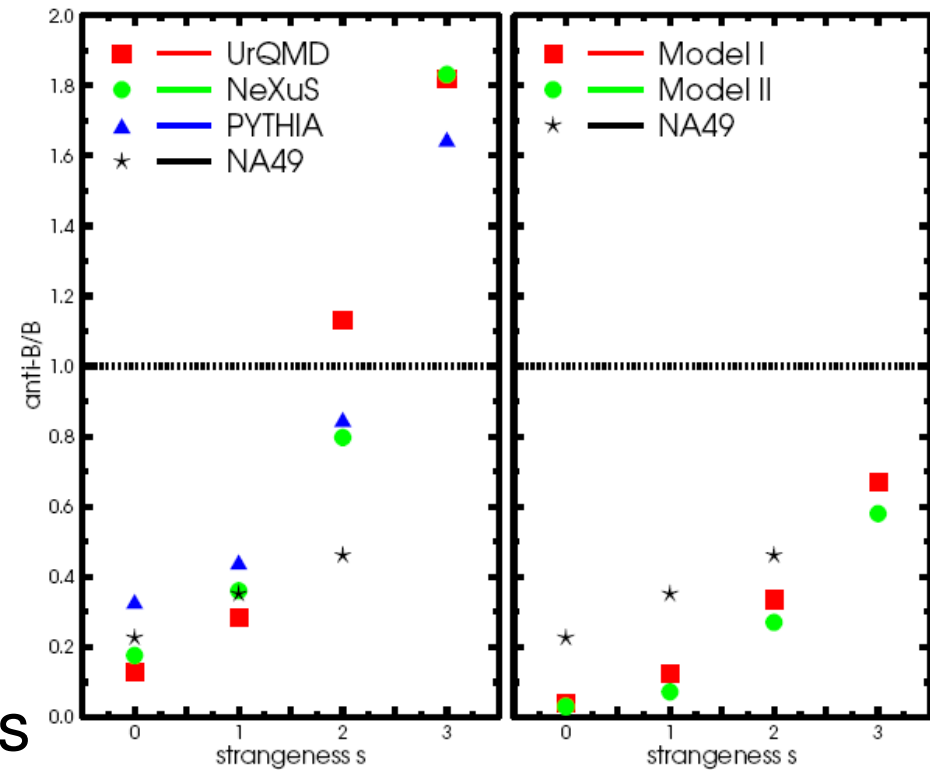
Another argument for soft pre-evolution:  $\frac{\bar{\Omega}}{\Omega}$

Bleicher et al.,

Phys.Rev.Lett.88:202501,2002.



Omega production  
 suppressed in di-quark strings



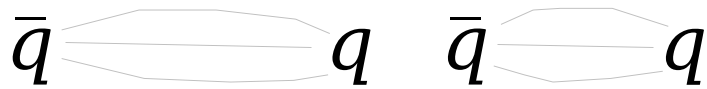
# Fragmentation of partons into hadrons

Pomeron = Cylinder  $\dashrightarrow$  cut  $\dashrightarrow$  2 Strings

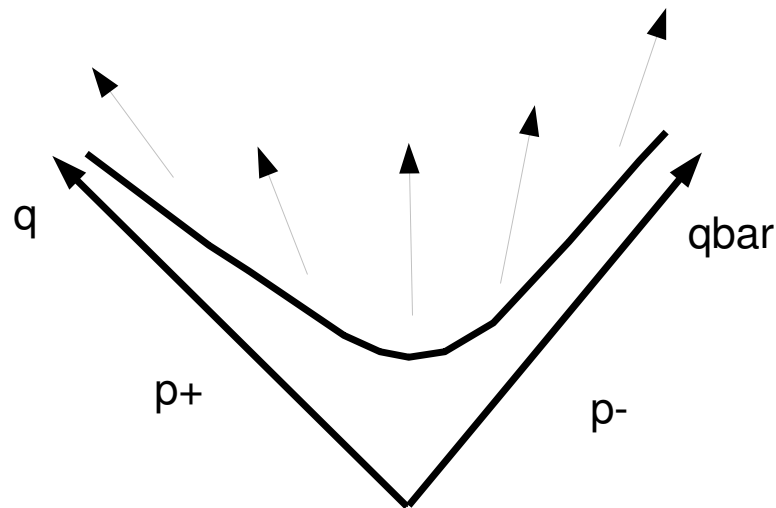
Inspired from QCD-flux tube between color charges



string dynamics governed by Nambu-Goto Lagrangian



Fragmentation via fragmentation function or area law (Atrtu-Menessier)



Break probability determines multiplicity

hard partons (gluons) are mapped onto string as kinks”.



# Scattering on a dense target

At high energies, dense systems the independent interaction picture is not valid any more.

Partons overlap - re-interact

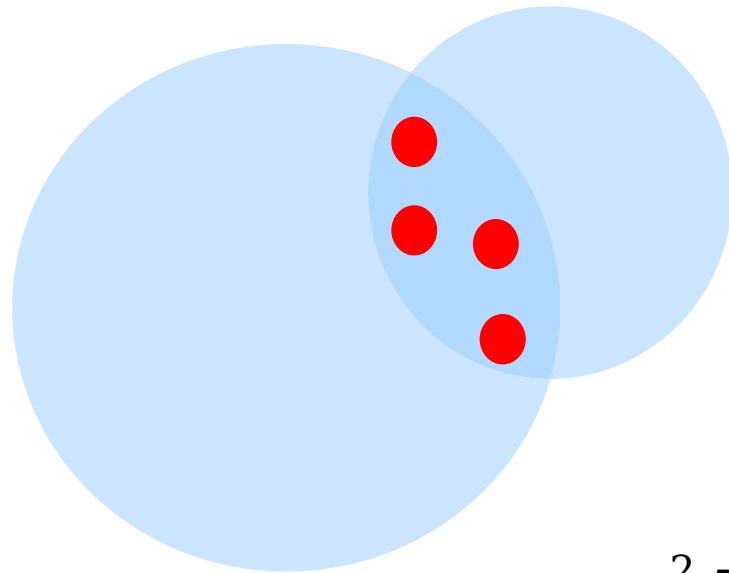
How to deal with rescattering?

Many approaches :

string fusion	(DPMJET, Armesto, Ranft )
enhanced diagrams	(QGSJet-II S.Ostapchenko)
s-dependent pt-cut-off	(Sibyll, R.Engel)
Black Disk/CGC	(BBL, HJD, Dumitru Strikman)
effective treatment	(Epos, K.Werner)
shadowing of PDF	(Hijing)

Pajares, EPJC43 (05) 9 (hep-ph/0501125);  
Armesto et al., PRL77 (96) 3736 (hep-ph/9607239)

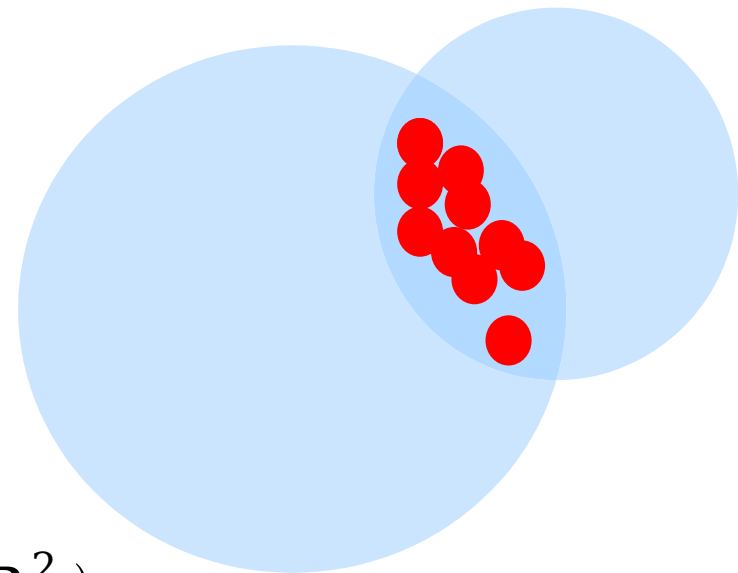
## String fusion



Low energy

$$\eta = \pi r^2 N / (\pi R_A^2)$$

$$\eta_c = 1.12 - 1.5$$



High energy/density

- String fusion takes place when the parent partons overlap
- Only fusion of pairs is allowed, only soft strings
- Implements non-linear effects on the string level (a posteriori)

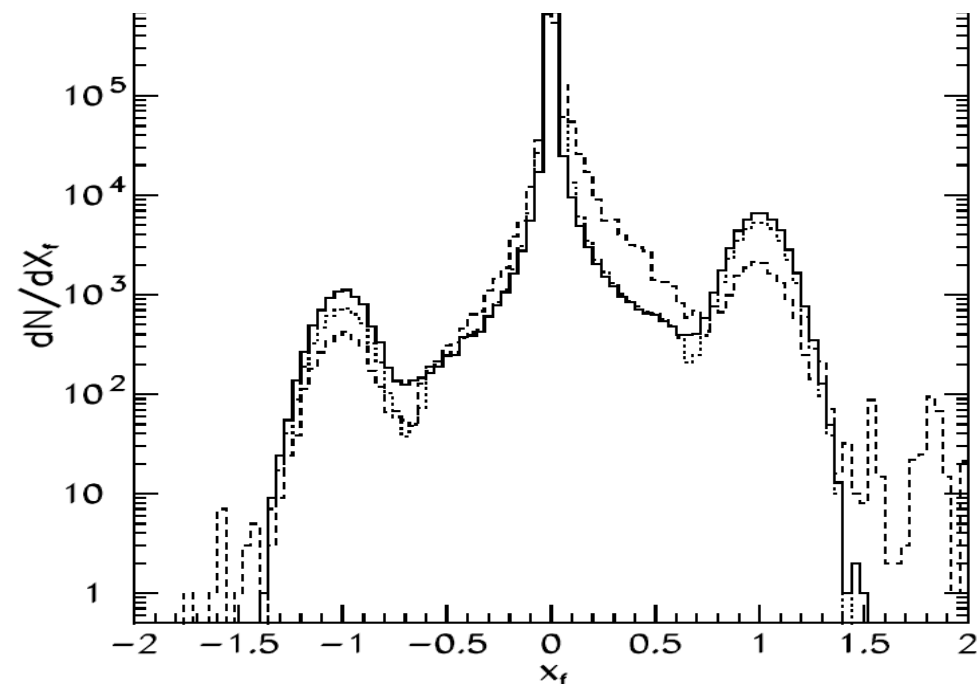
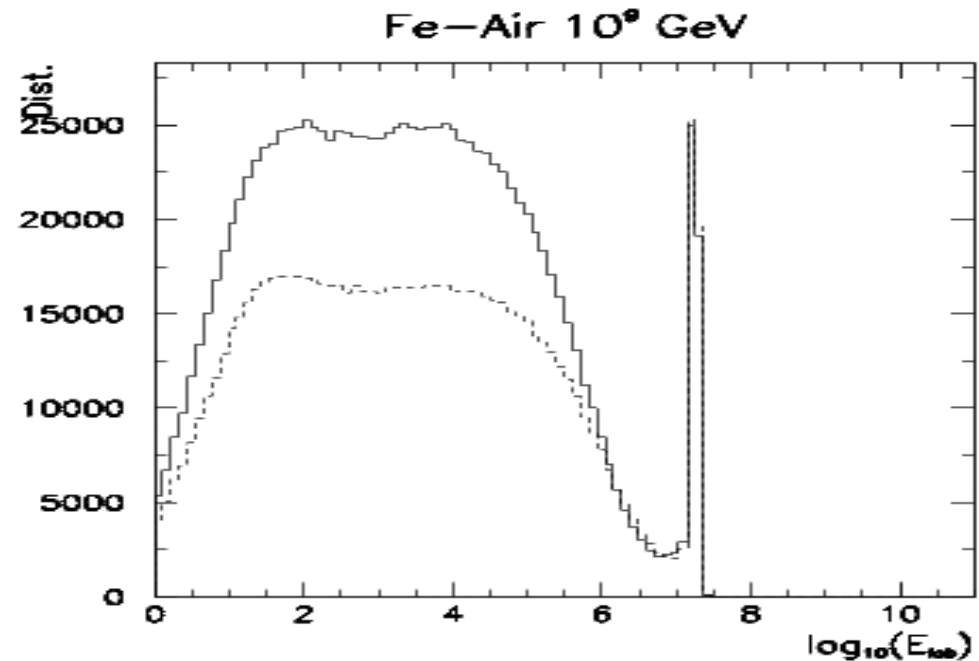
## Consequences of string fusion

- reduction of multiplicity
- enhanced strangeness
- enhanced baryon production
- enhanced correlations backward/forward

But:

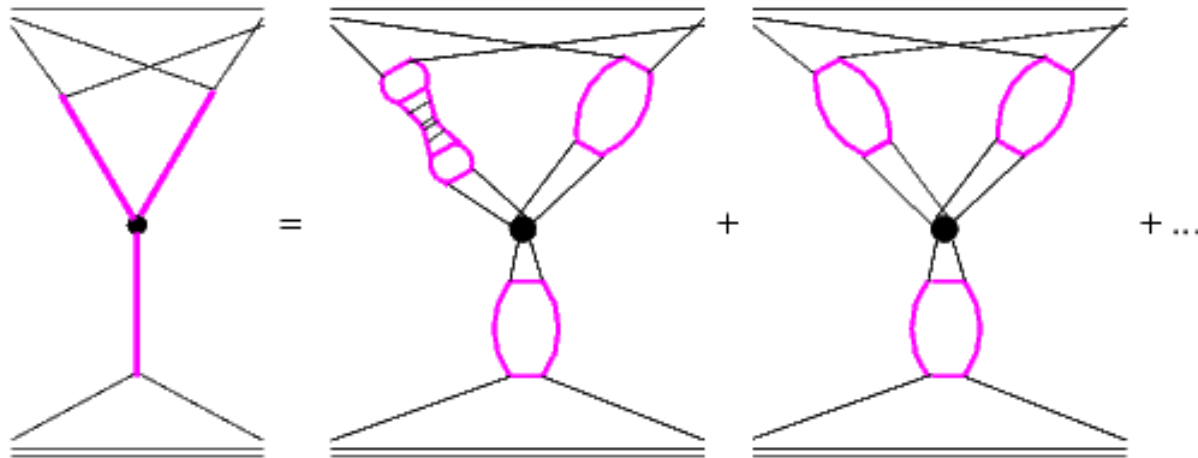
Effects are small  
for Air showers

e.g. no  $X_{\text{max}}$  change



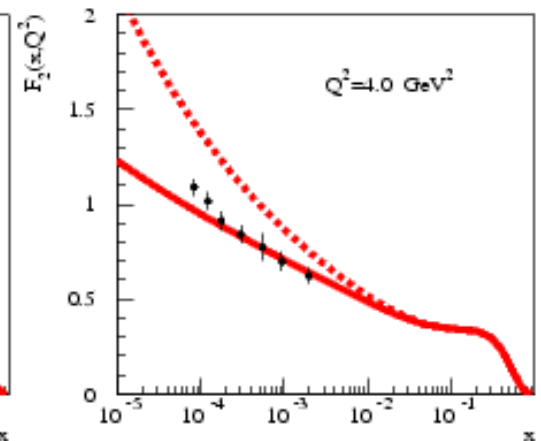
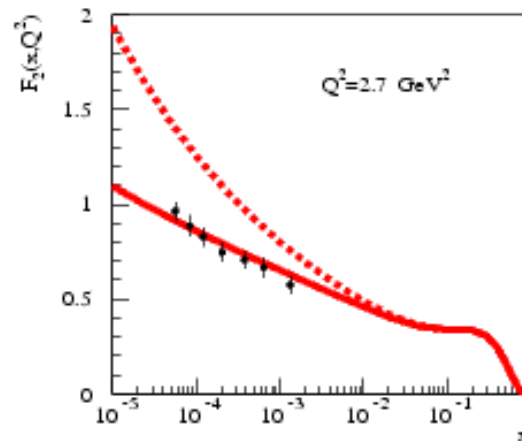
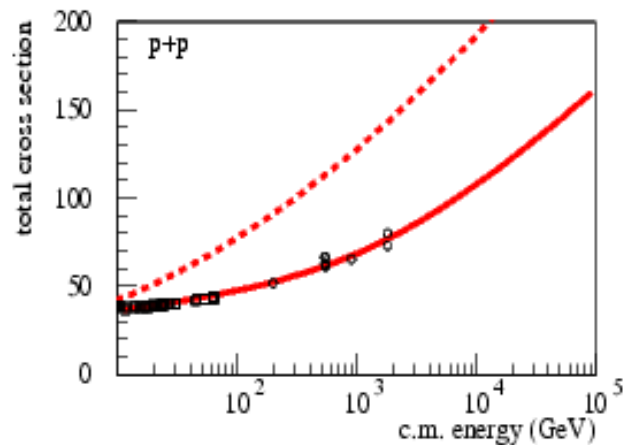
# Enhanced Diagrams

Non-linear effects by interaction between soft Pomerons  
 resummation of all orders



Reduces cross section, multiplicity, enhances inelasticity

Total cross section and SF  $F_2(x, Q^2)$  with (without) enhanced graphs:



# Shadowing structure functions

Wang, Li,  
Phys.Lett.B527:85-91,2002.

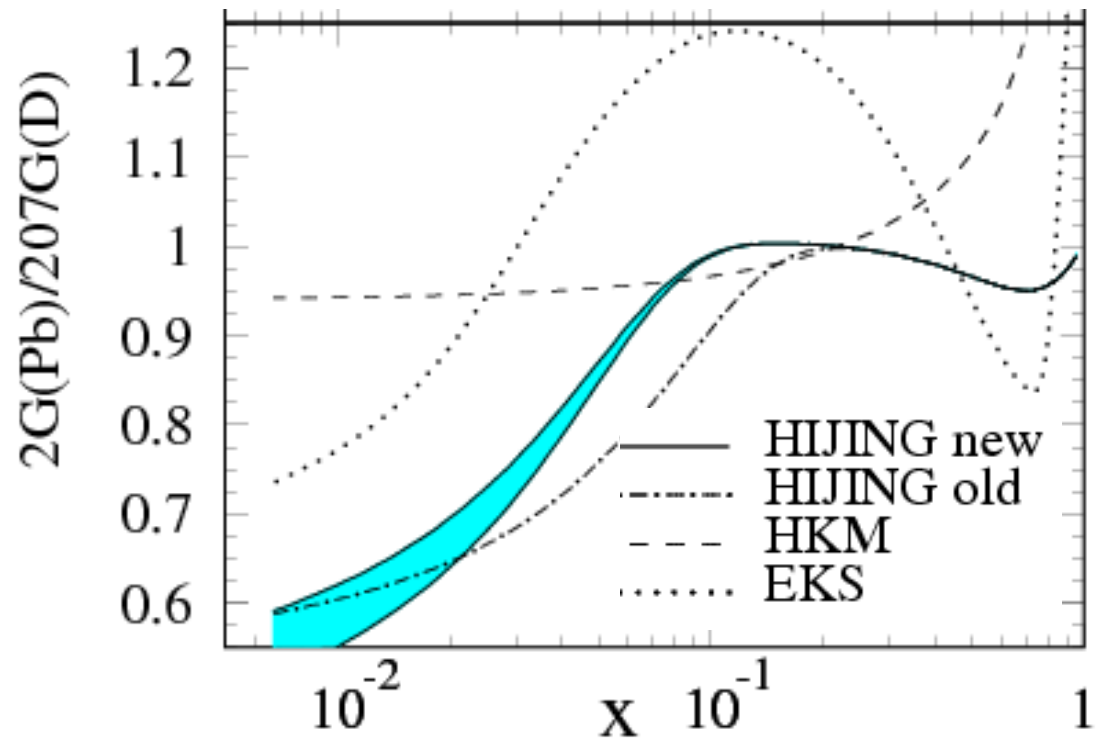
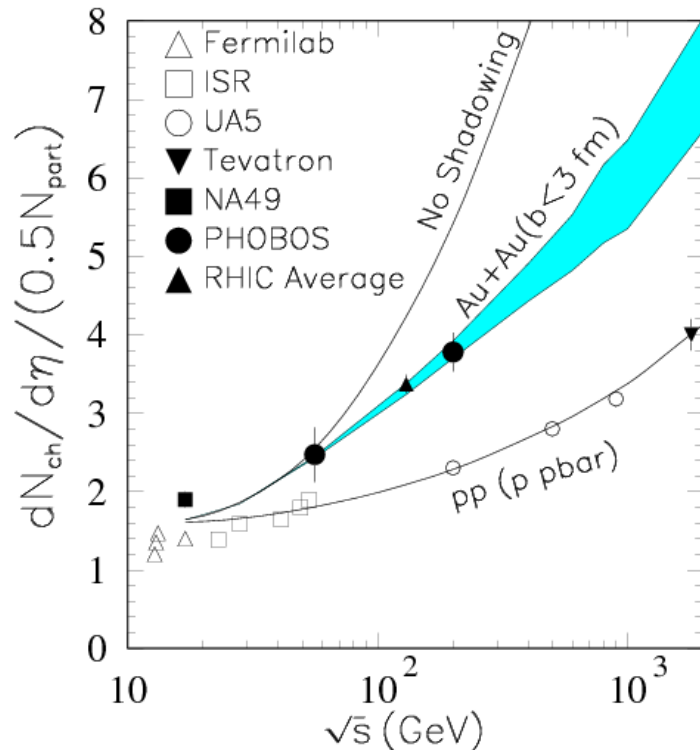
$$f_a^A(x, Q^2) = AR_a^A(x, Q^2) f_a^N(x, Q^2)$$

$$R_q^A(x) = 1.0 + 1.19 \log^{1/6} A (x^3 - 1.2x^2 + 0.21x)$$

$$- s_q (A^{1/3} - 1)^{0.6} (1 - 3.5\sqrt{x}) \exp(-x^2/0.01)$$

$$R_g^A(x) = 1.0 + 1.19 \log^{1/6} A (x^3 - 1.2x^2 + 0.21x)$$

$$- s_g (A^{1/3} - 1)^{0.6} (1 - 1.5x^{0.35}) \exp(-x^2/0.004),$$



# Energy dependent pt cutoff

PQCD cross section diverges for low  $p_t$  cutoff needed, but energy dependent.

e.g. Sibyll uses:

$$p_{t,min} = 1 + 0.065 e^{0.9\sqrt{\ln s}}$$

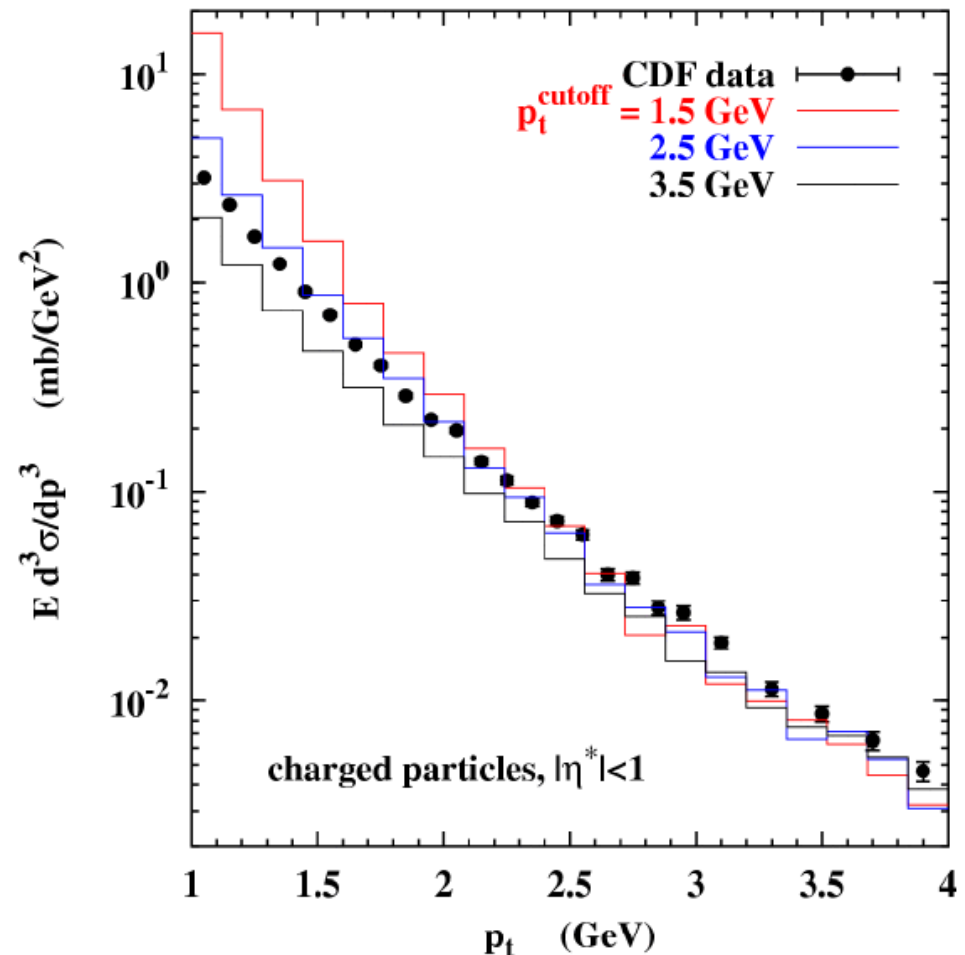
Saturation momentum  $Q_s(s)$  can be associated with  $p_t$ -cutoff

But:

- Independent of centrality
- independent of  $A$

Many others have  $p_t$ -cutoff like Pythia, Herwig, a.s.o.

From R.Engel:



# Empirical treatment of high parton density in EPOS

Ansatz for parton density in target/projectile:

$$Z_T(j) = \sum_i Z_T(i, j)$$

$$Z_T(i, j) = z_0 \exp(-b_{ij}^2/2b_0^2) \quad \text{Partons interact in nucleon } i \text{ and } j$$
$$+ \sum_{\text{target nucleons } j' \neq j} z'_0 \exp(-b_{ij'}^2/2b_0^2), \quad \text{Additional split ladders}$$

$$b_0 = w_B \sqrt{\sigma_{\text{inel } pp}/\pi}$$

$$z_0 = w_Z \ln s/s_M,$$

$$z'_0 = w_Z \sqrt{(\ln s/s_M)^2 + w_M^2},$$

Coefficients for density depend on  $\log(s)$

Treat high density effects (enhanced diagrams) effectively instead of explicitly

K.Werner, T.Pierog, F.Liu

Phys.Rev.C74:044902,2006.

## Quantities that depend on Z:

Amplitude changes due to the density

elastic screening:

parameterized Amplitude:

$$\alpha(x_1)^\beta (x_2)^\beta \rightarrow \alpha(x_1)^\beta (x_2)^{\beta+\epsilon}$$

$$\epsilon = \alpha_s \beta_s Z$$

Reduces small-x contributions as a function of Z

collective fragmentation:

absorb into remnant

change hadronization parameters

as a function of Z

$$p_B \rightarrow p_B - \alpha_B Z$$

$$p_D \rightarrow p_D (1 + \alpha_D Z)$$

.....





## Approach of the black disk limit

Gluon structure function rises fast at small  $x$

A hadron hitting a dense target  $x_2 = 4 p_t^2 / (x_1 s)$   
interaction probability of quarks interacting inelastically  
reaches unity

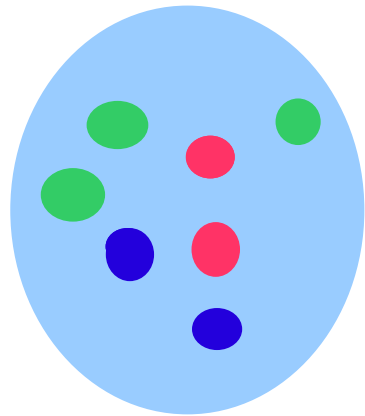
scattering incoming quarks off dense target,  
projectile breaks up -> no leading particle effect

importance for air showers:

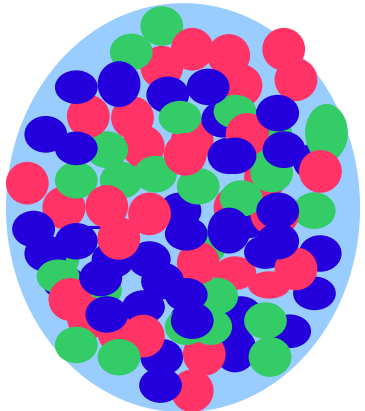
- reduced forward scattering

- faster absorption in atmosphere (lower  $X_{max}$ )

# Color Glass Condensate



Low energy  
dilute parton gas



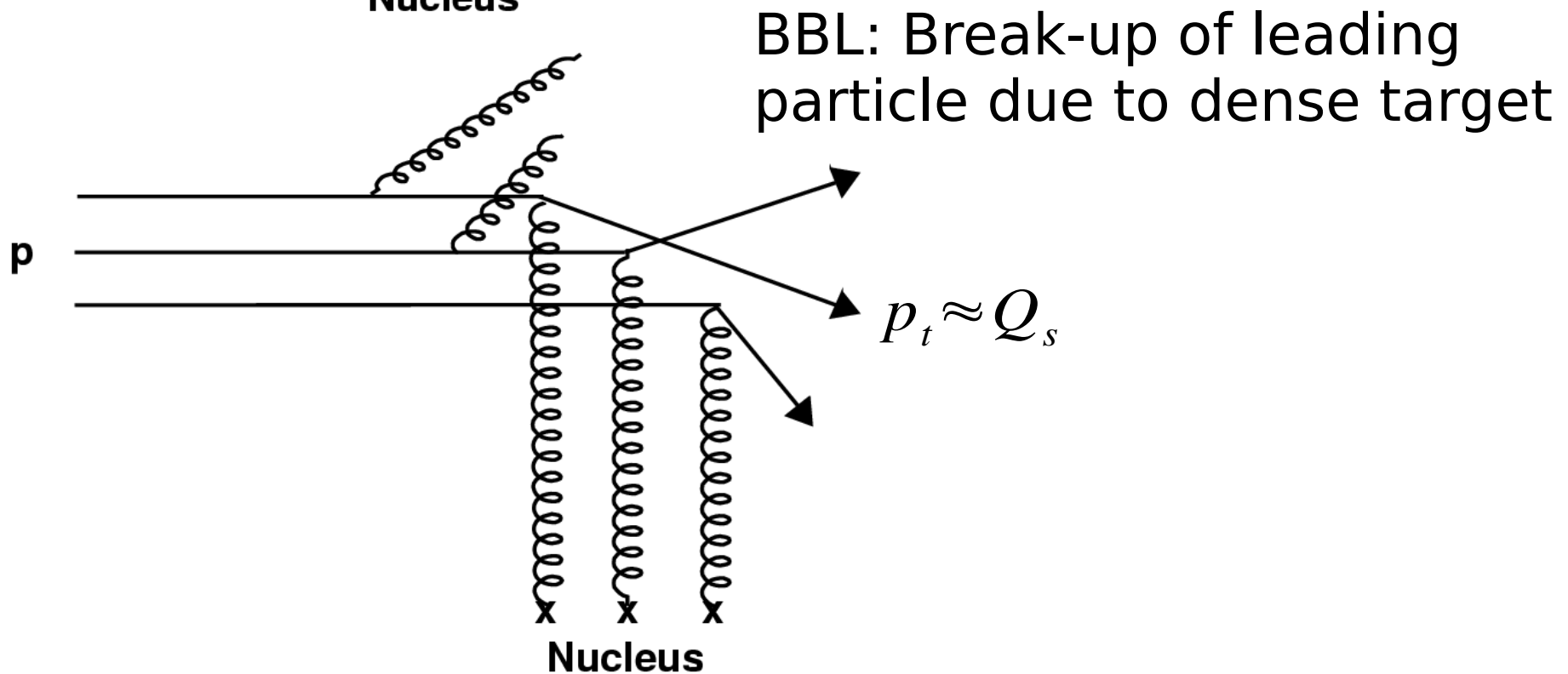
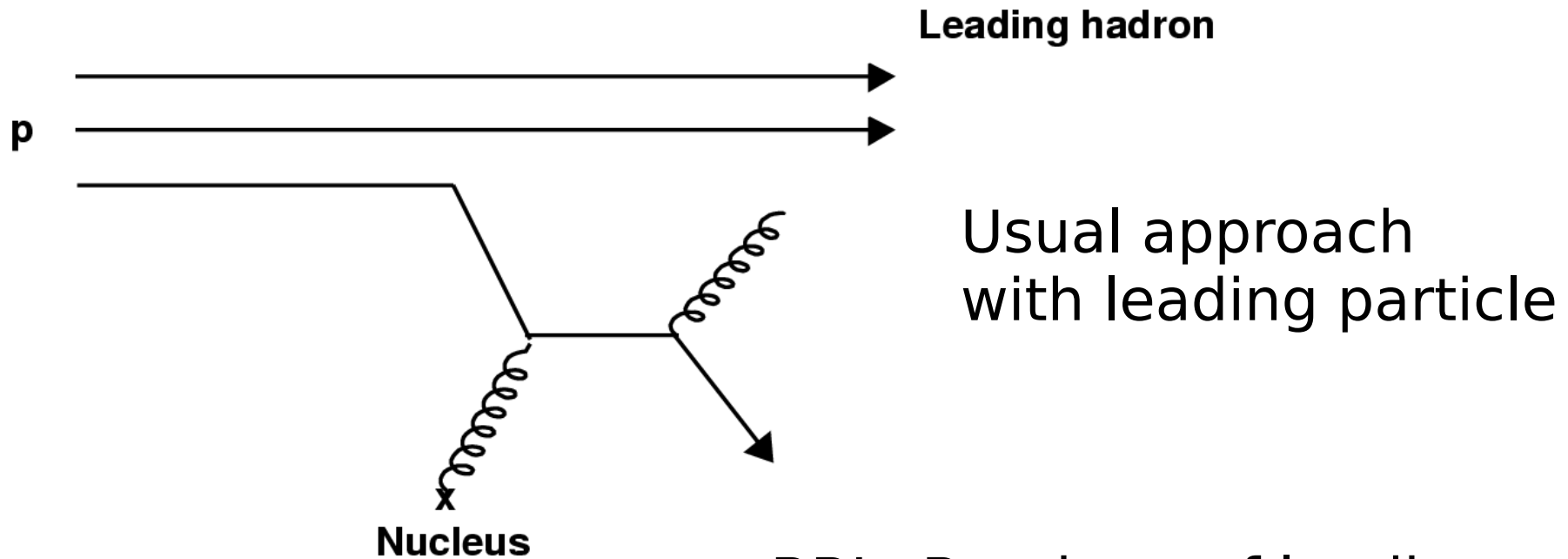
High energy  
saturated color field

Gluon density grows  
rapidly with smaller  $x$

partons saturate at  
 $\rho \sim 1/\alpha_s$

typical Momentum  
is  $Q_s(x)$

$\alpha(Q_s) \ll 1$   
McLerran-Venugopalan  
(MV Model)



Forward quark-Nucleus scattering:

**Dumitru, Jalilian-Marian  
PRL 89 (2002)**

$$\frac{d\sigma^{qA}}{dq^- d^2q_t d^2b} = \delta(q^- - p^-) C(q_t)$$

$$C(q_t) = \int \frac{d^2r_t}{(2\pi)^2} e^{iq_t r_t} \left\{ \exp \left[ -2Q_s^2 \int_{\Lambda} \frac{d^2p_t}{(2\pi)^2} \frac{1}{p_t^4} (1 - e^{ip_t r_t}) \right] - 2 \exp \left[ -Q_s^2 \int_{\Lambda} \frac{d^2p_t}{(2\pi)^2} \frac{1}{p_t^4} \right] + 1 \right\}$$

--->

$$d\sigma^{\text{el}}/d^2b = \left[ 1 - e^{-Q_s^2/4\pi\Lambda^2} \right]^2, \quad d\sigma^{\text{tot}}/d^2b = 2 \left[ 1 - e^{-Q_s^2/4\pi\Lambda^2} \right]$$

Limits for qA cross section :

$$C(q_t) =$$

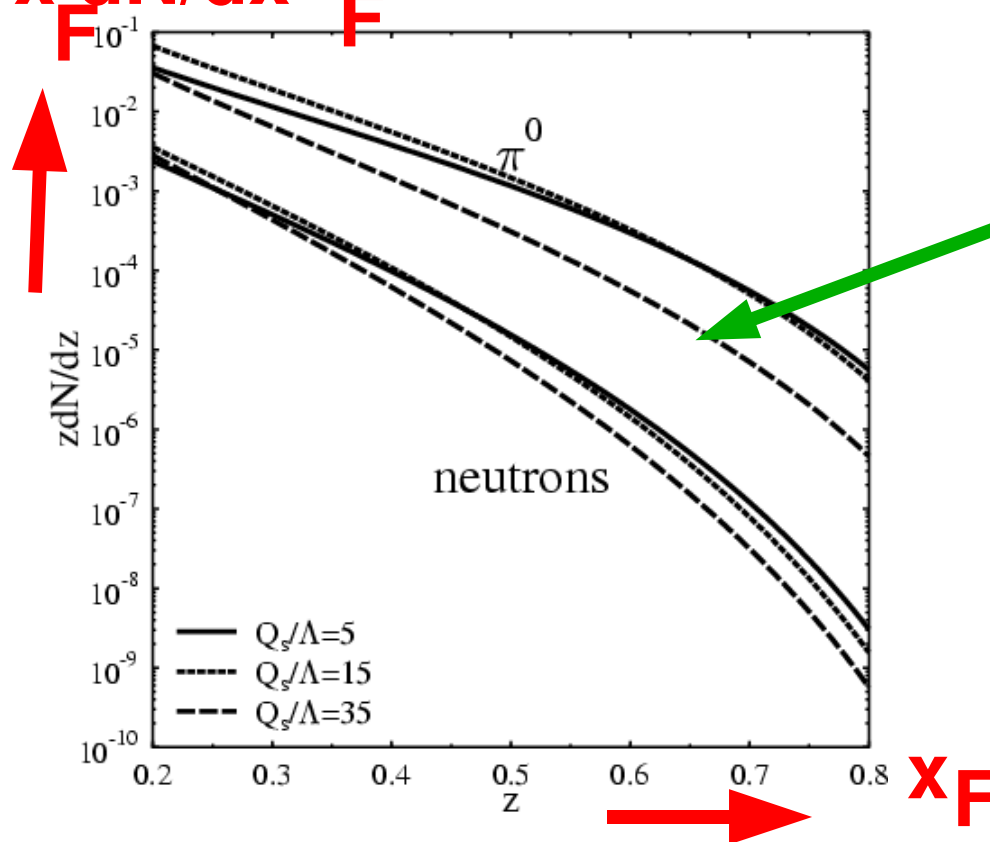
$$q_t \gg Q_s : \quad \frac{1}{2\pi^2} \frac{Q_s^2}{q_t^4} \left[ 1 + \frac{4}{\pi} \frac{Q_s^2}{q_t^2} \log \frac{q_t}{\Lambda} + \mathcal{O} \left( \frac{Q_s^2}{q_t^2} \right) \right]$$

$$q_t \lesssim Q_s : \quad \frac{1}{Q_s^2 \log Q_s/\Lambda} \exp \left( -\frac{\pi q_t^2}{Q_s^2 \log Q_s/\Lambda} \right)$$

If one assumes indep. fragm. of scattered partons :

$$x_F \frac{d\sigma^{pA \rightarrow hX}}{dx_F d^2k_t d^2b} = \int_{x_F}^1 dx \frac{x}{x_F} f_{q/p}(x, Q_s^2) D_{h/q}\left(\frac{x_F}{x}, Q_s^2\right) \frac{d\sigma^{qA}}{d^2q_t d^2b}$$

**x<sub>F</sub> dN/dx<sub>F</sub>** **“Limiting Fragmentation” curve in BBL**



Long. distribution steepens

Gerland, A.D., Strikman  
PRL 90 (2003)

see also

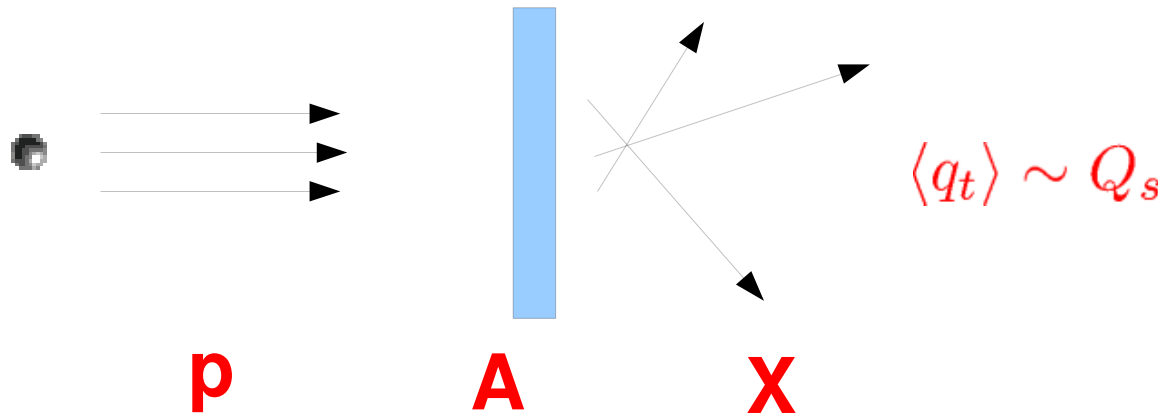
Frankfurt, Guzey, McDermott,  
Strikman: PRL 87 (2001)

# Shattering the proton

Probability for quark to be scattered to  $q_t \sim 0$  (with color exchange !):

$$\int_0^\Lambda d^2 q_t \frac{d\sigma^{\text{in}}}{d^2 b d^2 q_t} \simeq 1 - \exp\left(-\frac{\pi\Lambda^2}{Q_s^2 \log Q_s/\Lambda}\right) \simeq \frac{\pi\Lambda^2}{Q_s^2 \log Q_s/\Lambda}$$

--> **suppression of “beam-jet remnants”**  
**(soft physics) in the BBL**



**All partons resolved at scale  $Q_s$ , coherence of proton destroyed completely.**

# Monte Carlo implementation

- Choose model as function of density, energy

$$Q_s(b, x_F = 0.001) > 1 \text{ GeV}$$

→ BBL

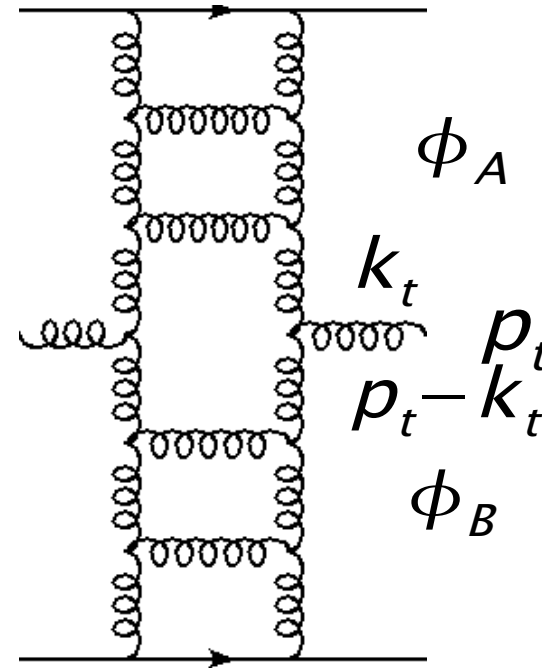
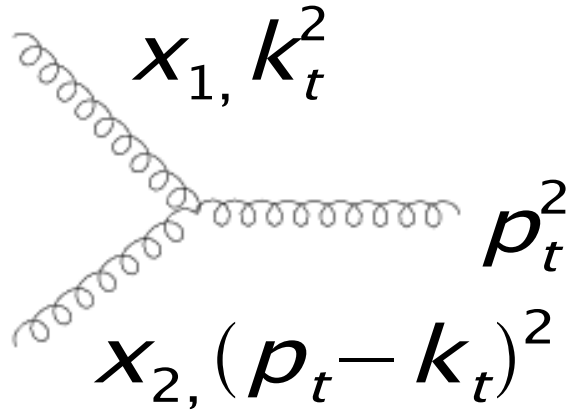
else

→ Sibyll (standard pQCD EG,  $p_t(s)$  cut-off)

- Generate leading partons according to PDF
- Generate gluons from kt-factorization formula
- Valence quarks and gluons form strings with kinks:
  - Collinear g absorbed (low  $q_t$ )
  - Low invariant mass of quarks forms diquark  
recovers leading particle effect for low  $Q_s$

# Kt factorization

GLR 1983



$$\frac{dN}{dy d^2 p_t d^2 x} = \frac{4 \pi N_c}{N_c^2 - 1}$$

$$\times \frac{1}{p_t^2} \int^{p_t^2} dk_t^2 \alpha_s \phi_A(x_1, k_t^2) \phi_B(x_2, (p_t - k_t)^2)$$

$\phi_A(x_1, k_t^2)$

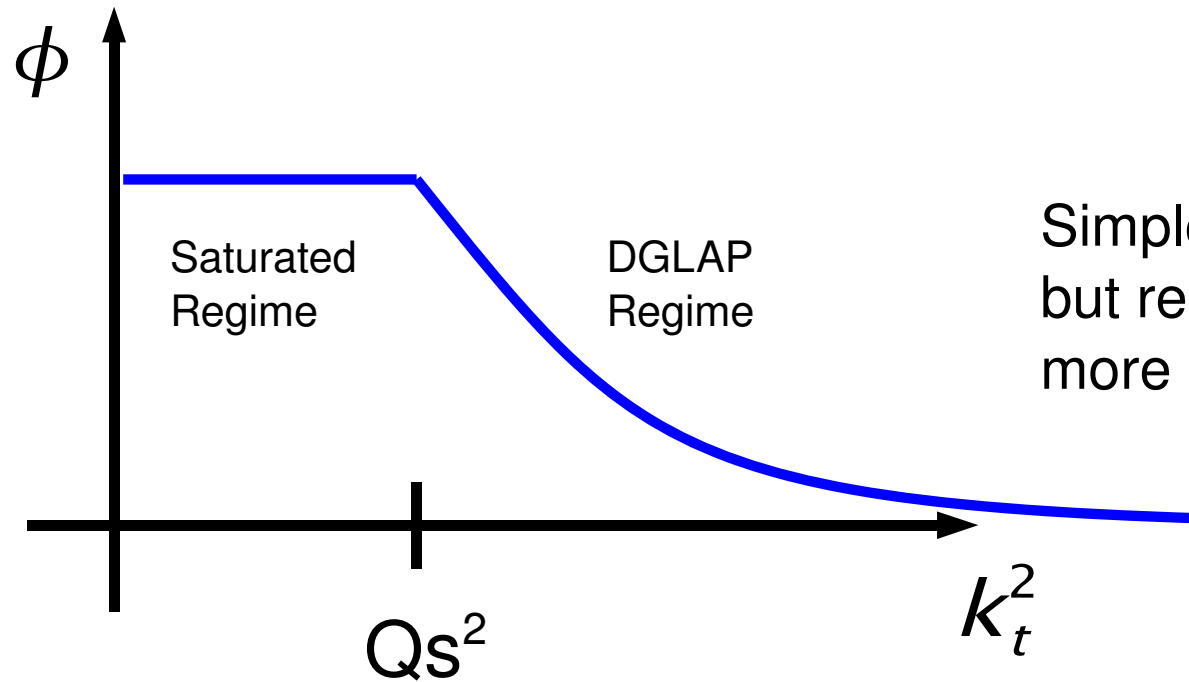
Unintegrated gluon distribution function  
use numerical integration instead of  
approximations



# Unintegrated gluon distribution functions

e.g.  
Kharzeev et al.  
Nuc.Phys.A 747

$$\phi_A(x, k_t^2) \sim \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2}{\max(Q_s^2, k_t^2)} (1-x)^4$$



Simple Ansatz  
but results are similar to  
more sophisticated choices

# Saturation scale

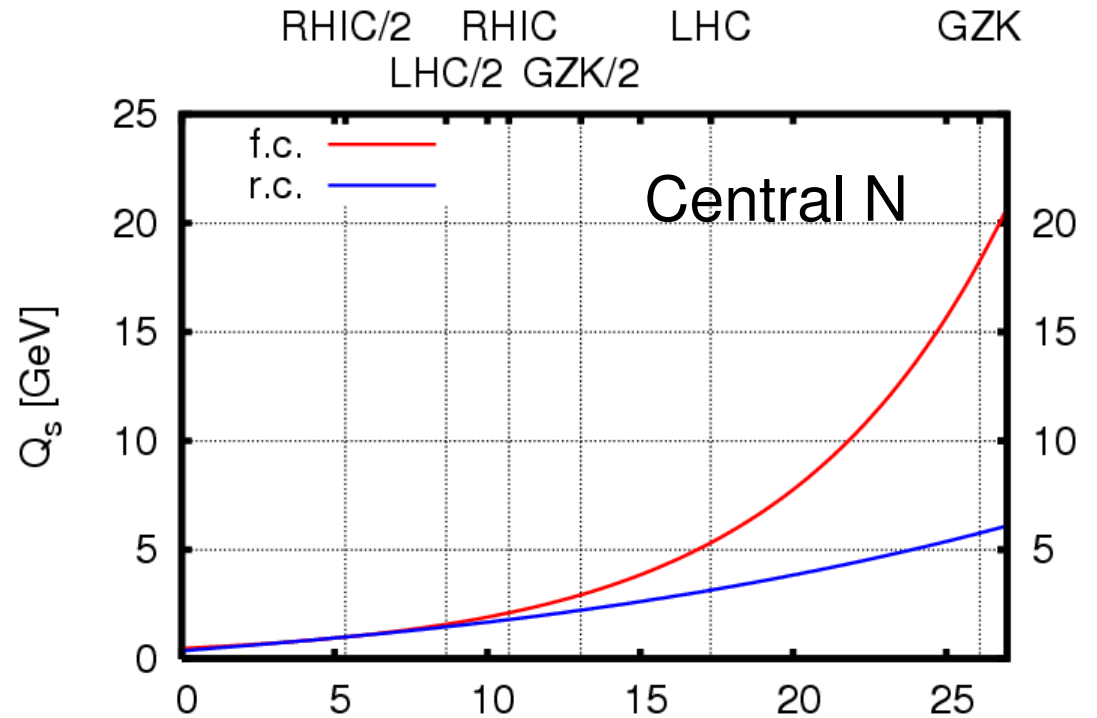
$$Q_s^2(\mathbf{x}) = 2\text{GeV}^2 \left( \frac{n_{part}}{1.53} \right) \left( \frac{0.01}{x} \right)^\lambda \quad \text{Fixed coupling evolution}$$

$$n_{npart,A} = A T_A (1 - (1 - \sigma T_B)^B) \quad \text{From Glauber model}$$

$\lambda = 0.2-0.3$ : determines growth with energy

$n_{part}$  depends on properties of both nuclei  $\rightarrow$  saturation scale not universal ???

Running coupling evolution:

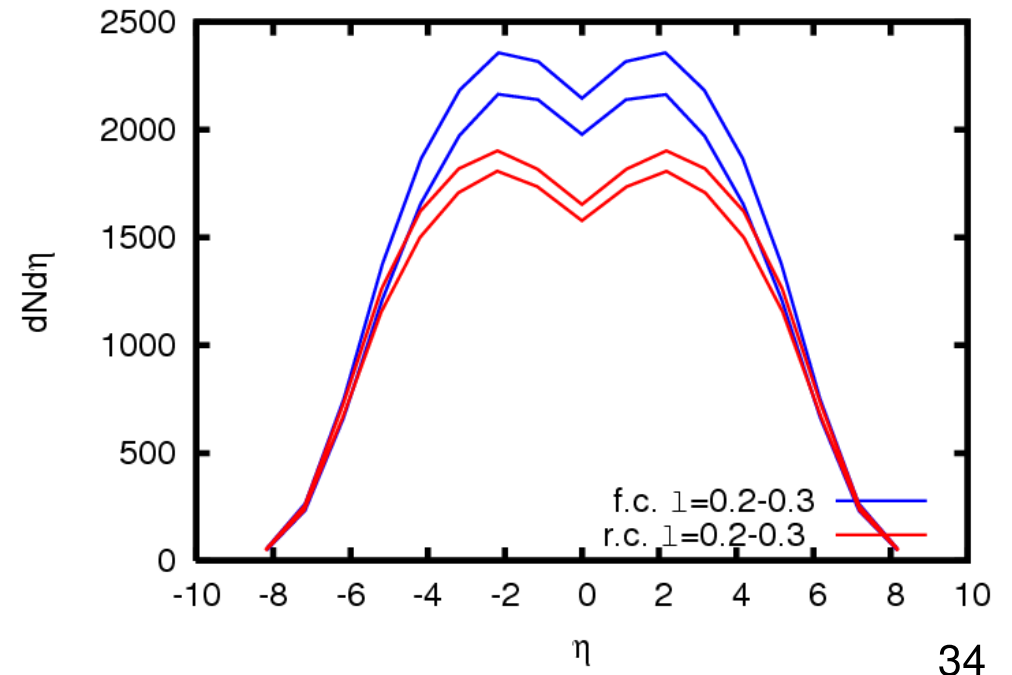
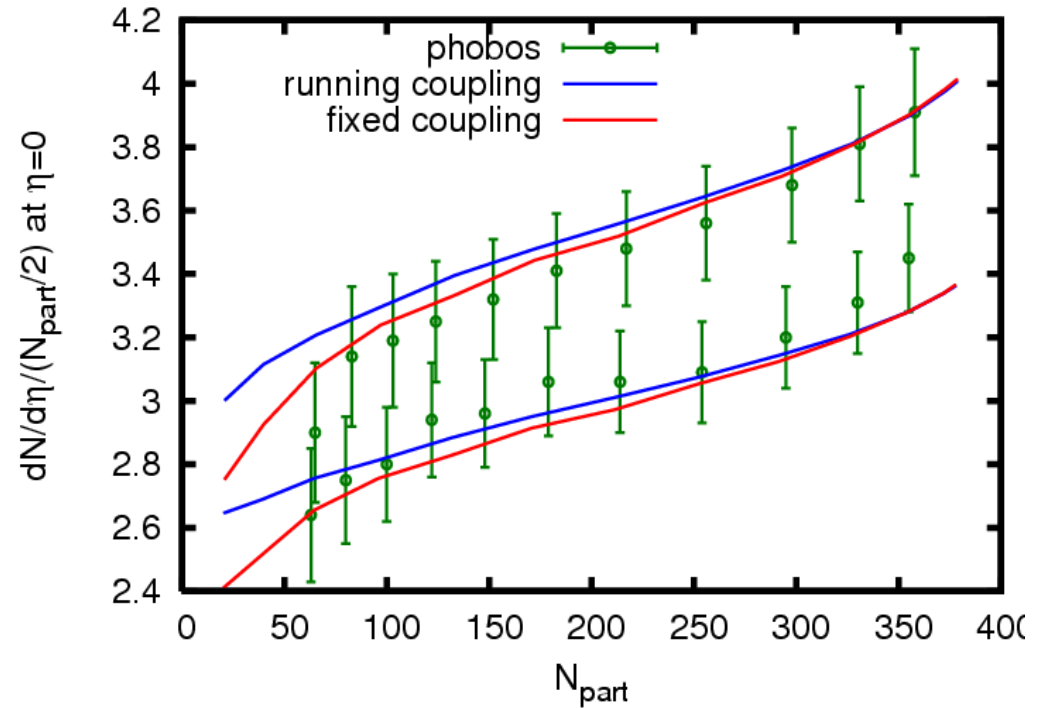


$$Q_s^2(\mathbf{x}) = \Lambda^2 \exp(\log(Q_0^2/\Lambda^2) \sqrt{1 + 2c\alpha y}) \quad y = \log(1/x)$$

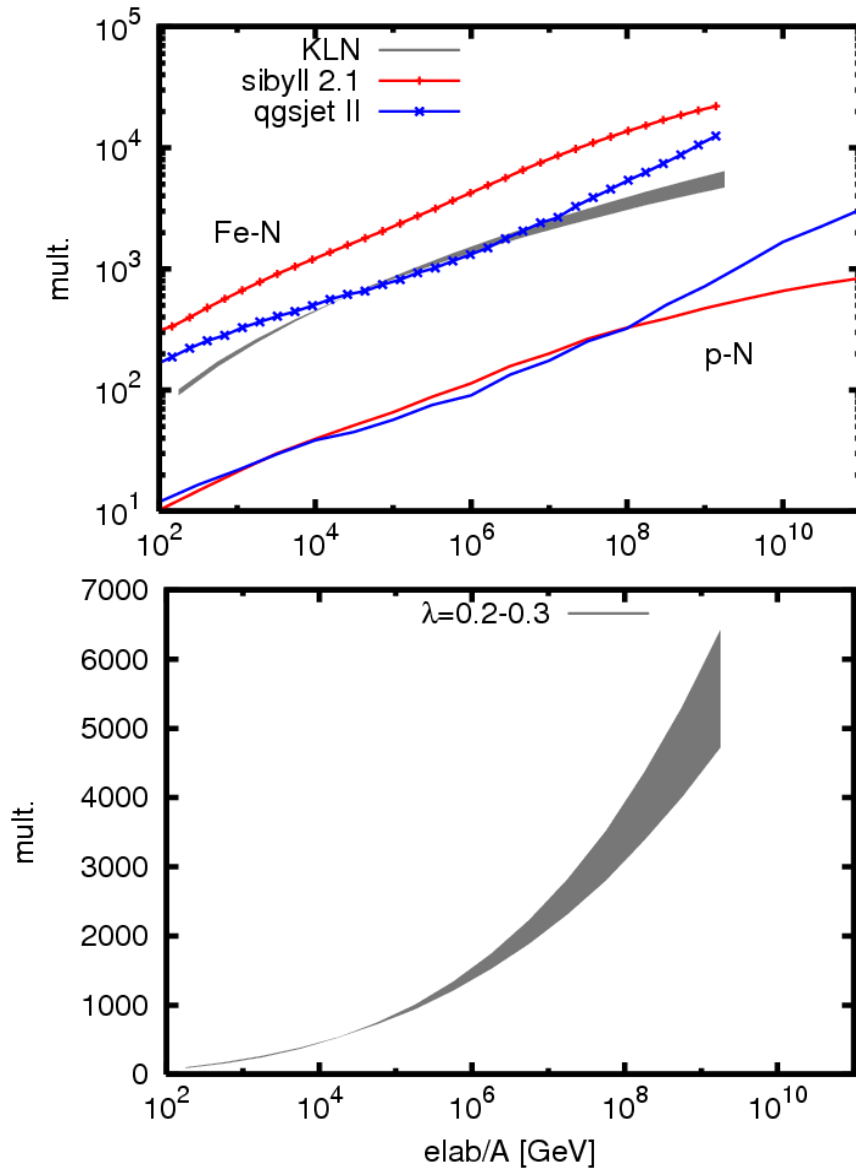
# kt Factorization results

Good description  
of Multiplicity at  
mid rapidity for all  
centralities  
200 and 130 GeV

Pb-Pb at 5500 GeV,  
 $b=2.4$  fm



# Iron Air: total multiplicities



Central Fe-N collisions  
(running coupling)

total multiplicity as  
function of Elab/A

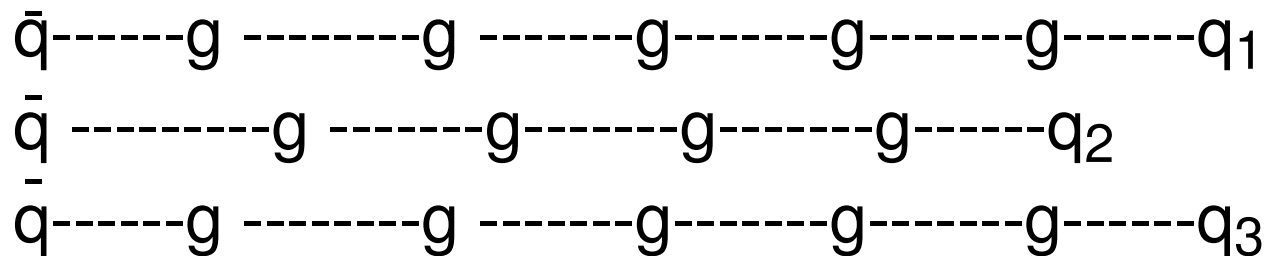
Uncertainty due to  $\lambda = 0.2-0.3$

# Generation of leading quarks and strings in BBL

x-distribution:  $P_i(x) = f_i(Q_s^2(x), x)$

$f_i(Q^2, x)$ : GRV98 parton distribution functions

$p_t$  distribution given by  $C(p_t)$   $p_t \approx Q_s^2(x)$

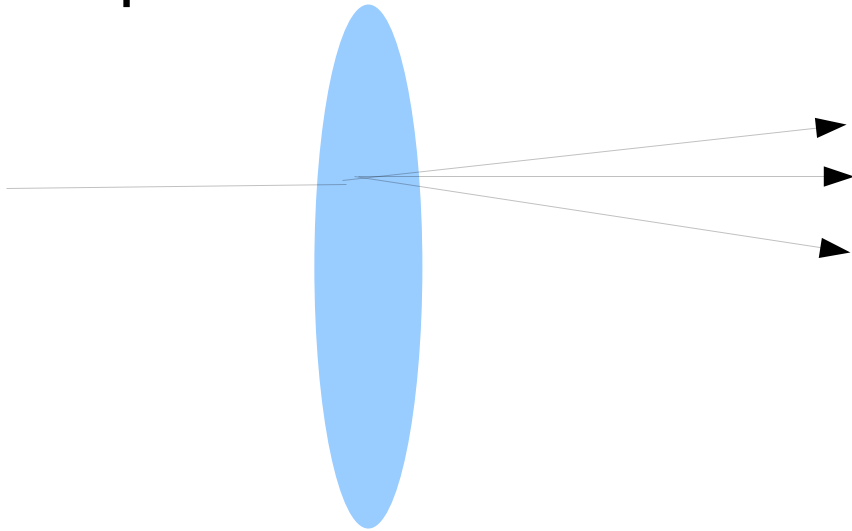


Gluons ordered in rapidity (x)

No energy loss of partons (L.Frankfurt,yesterday)

- would make effect even stronger

# Diquark recombination at low transverse momentum

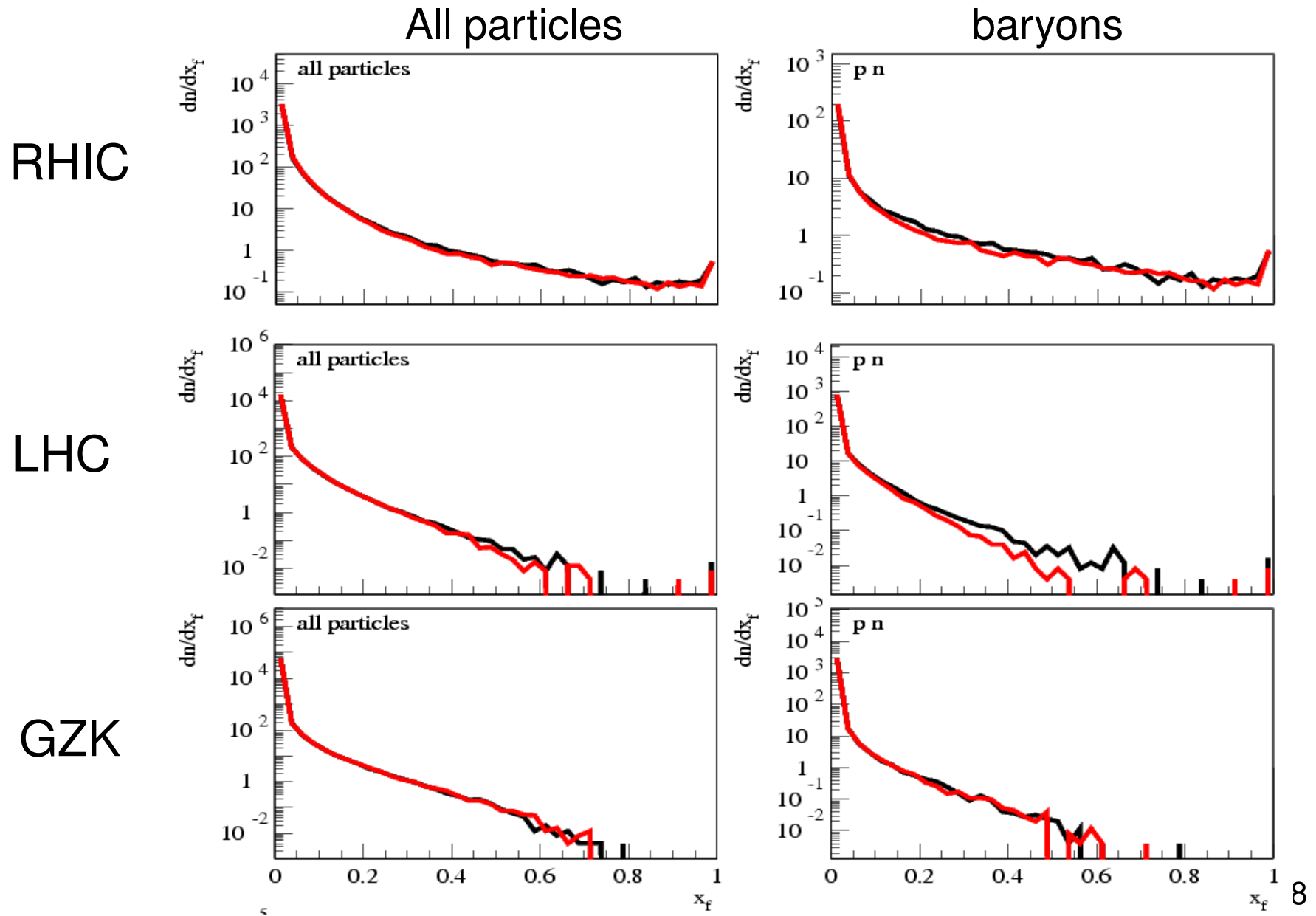


Check invariant mass between  
two quarks

if  $M_{\text{diquark}} < m_{\rho} = 0.77 \text{ GeV}$

Recombine quarks to diquark  
recovers leading particle effect

# Diquark recombination: central pN



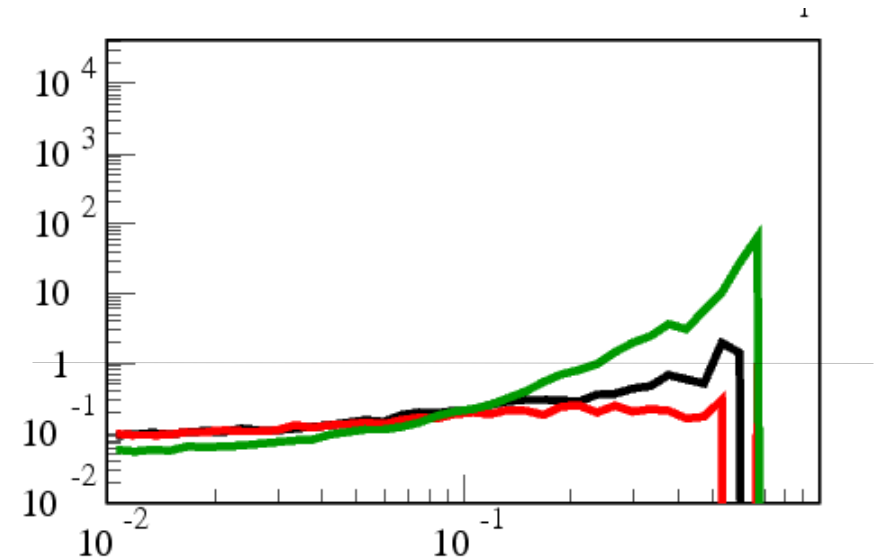
# Baryon to meson ratio for forward direction central pN at LHC energies

Ratio of baryon to meson  
production in forward region

Green: QGSJet-II

Red: BBL w.o. recombination

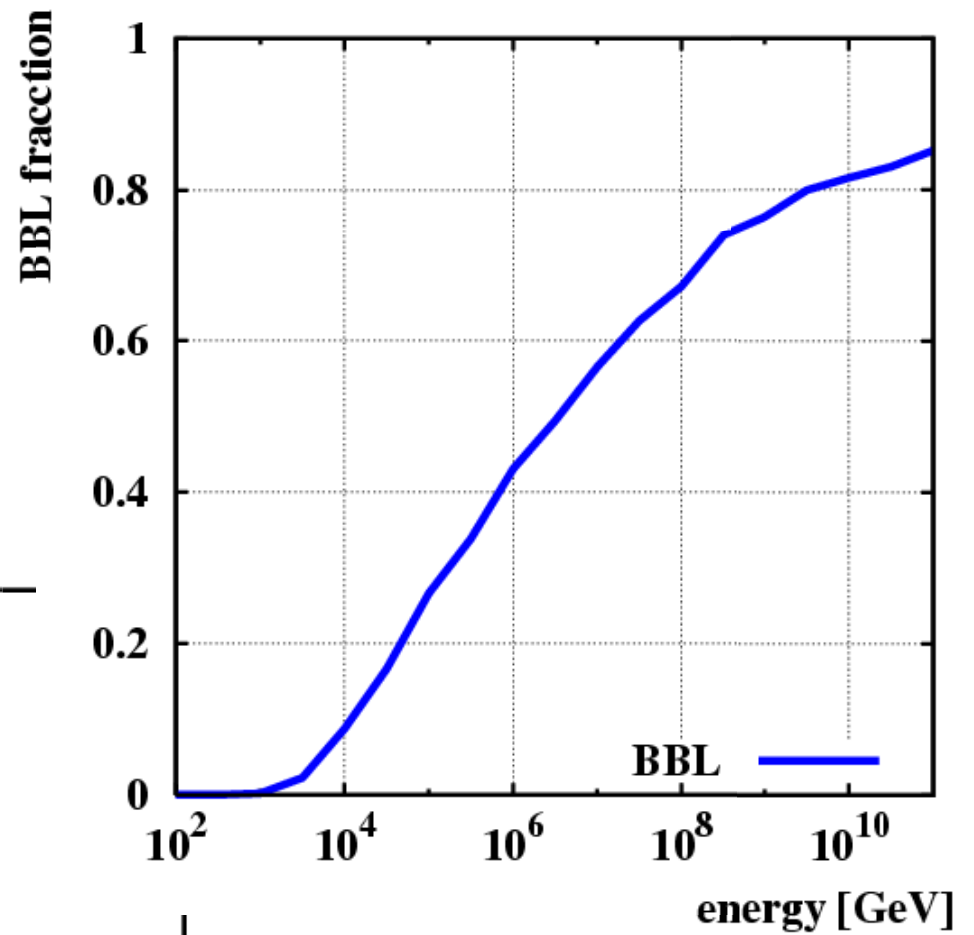
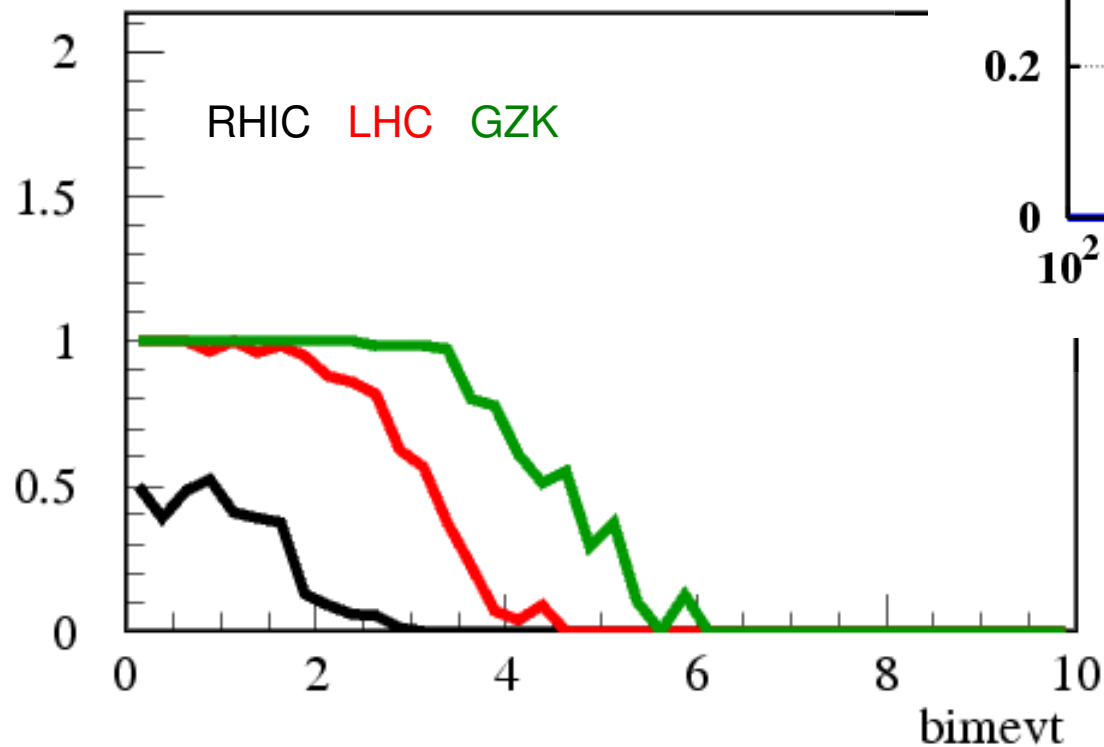
Black: BBL w recombination



Without recombination: leading baryon  
completely suppressed

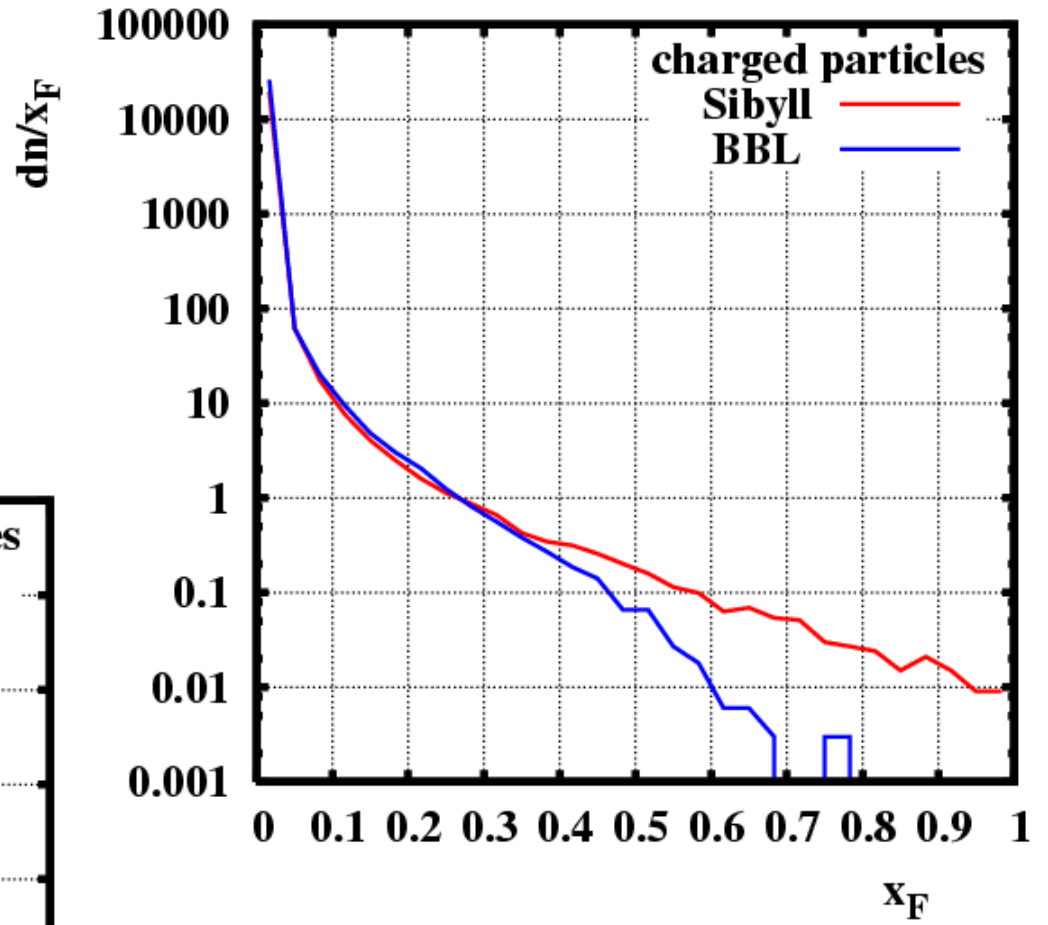
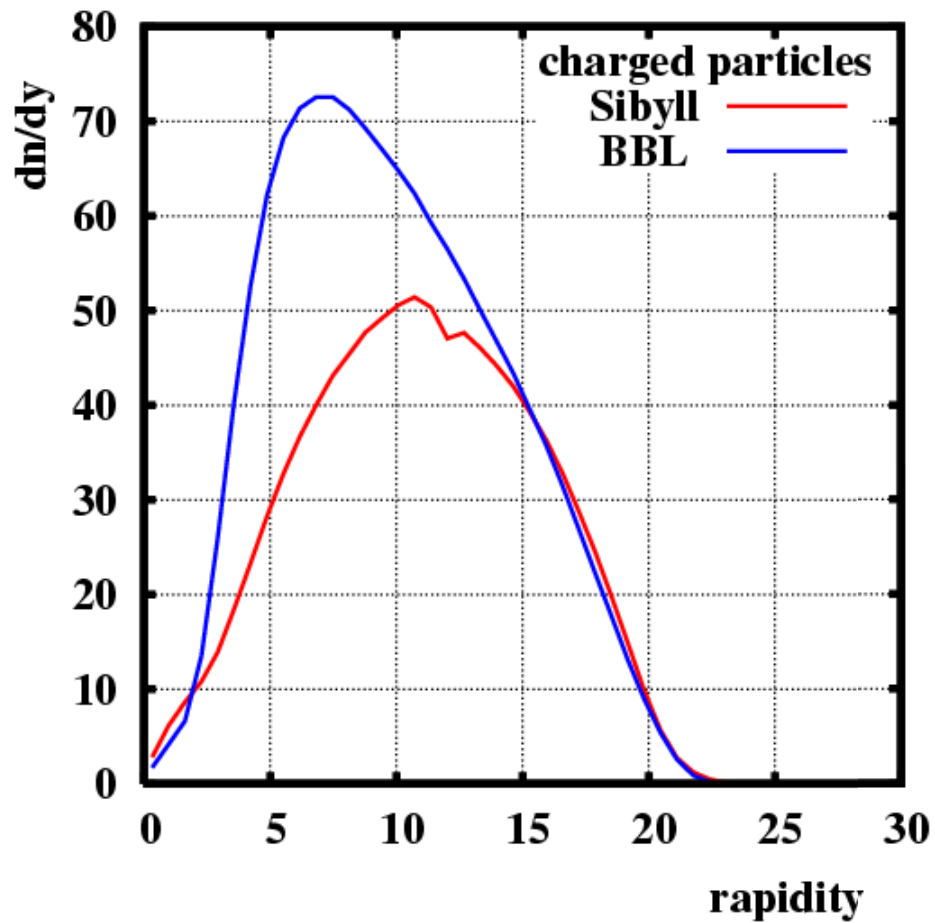


# Fraction of BBL events versus Sibyll events for min bias p-Air

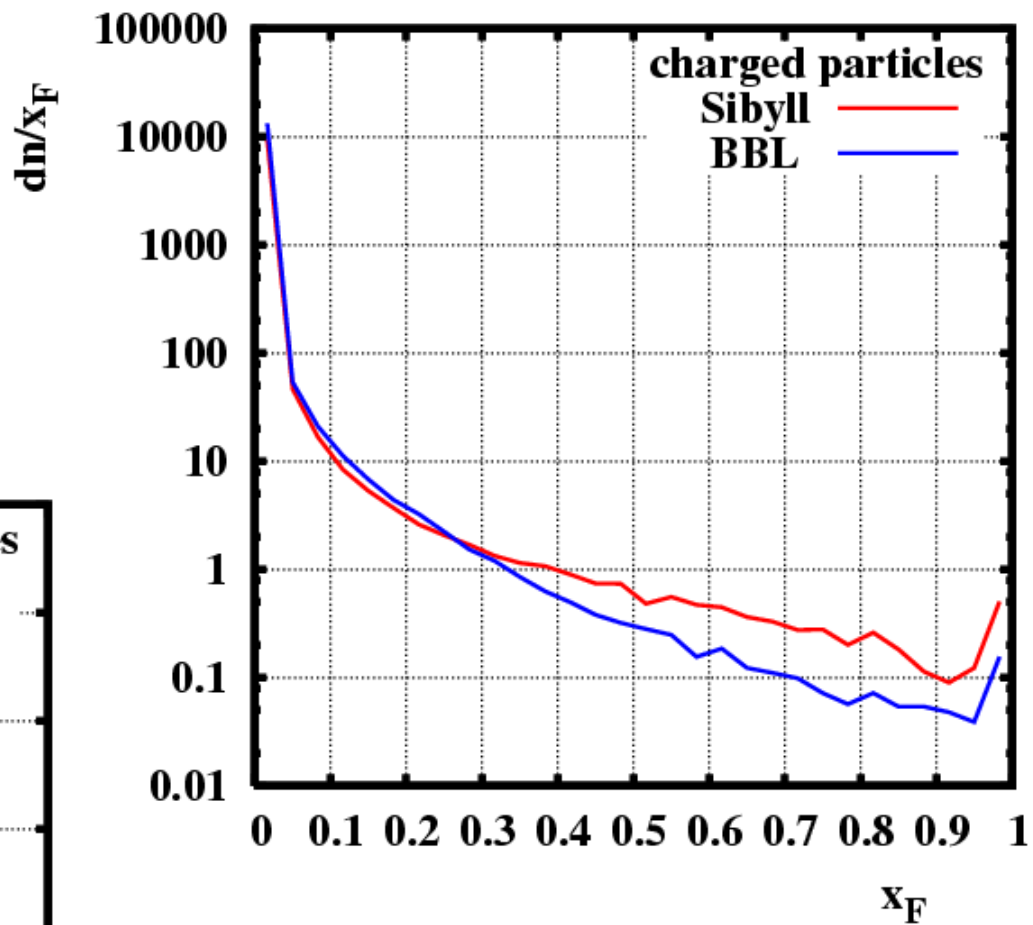
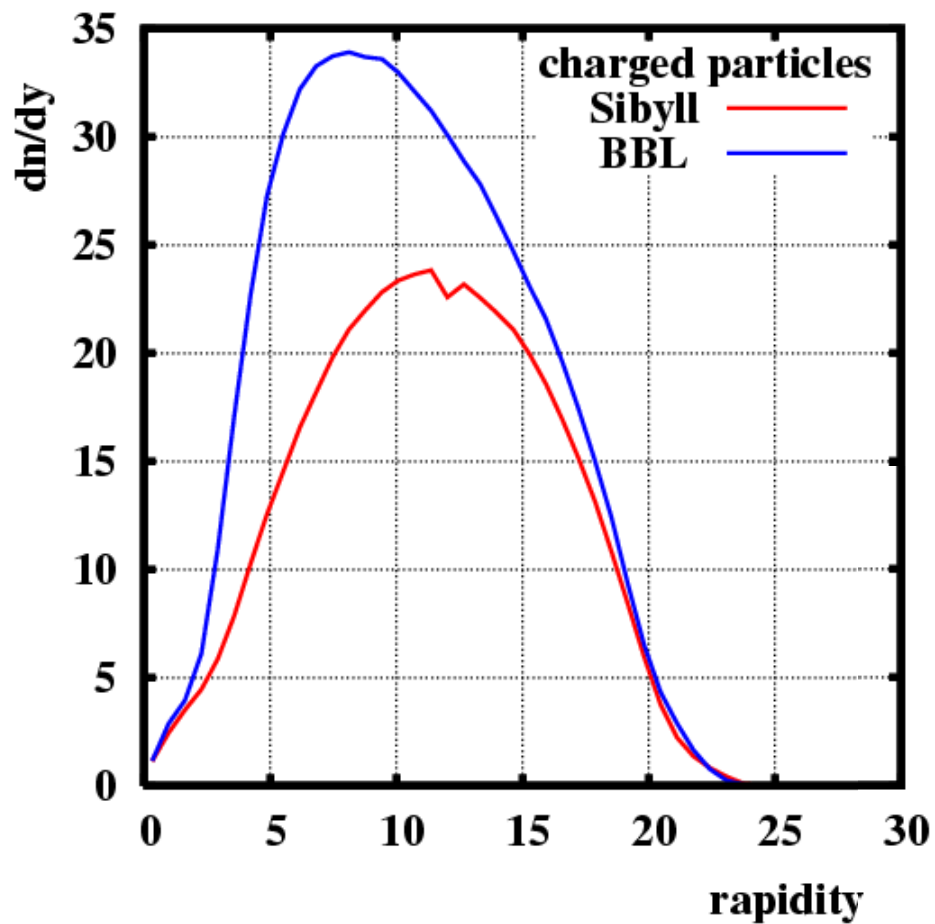


Central p-Air events

$E=10^{10}$  GeV

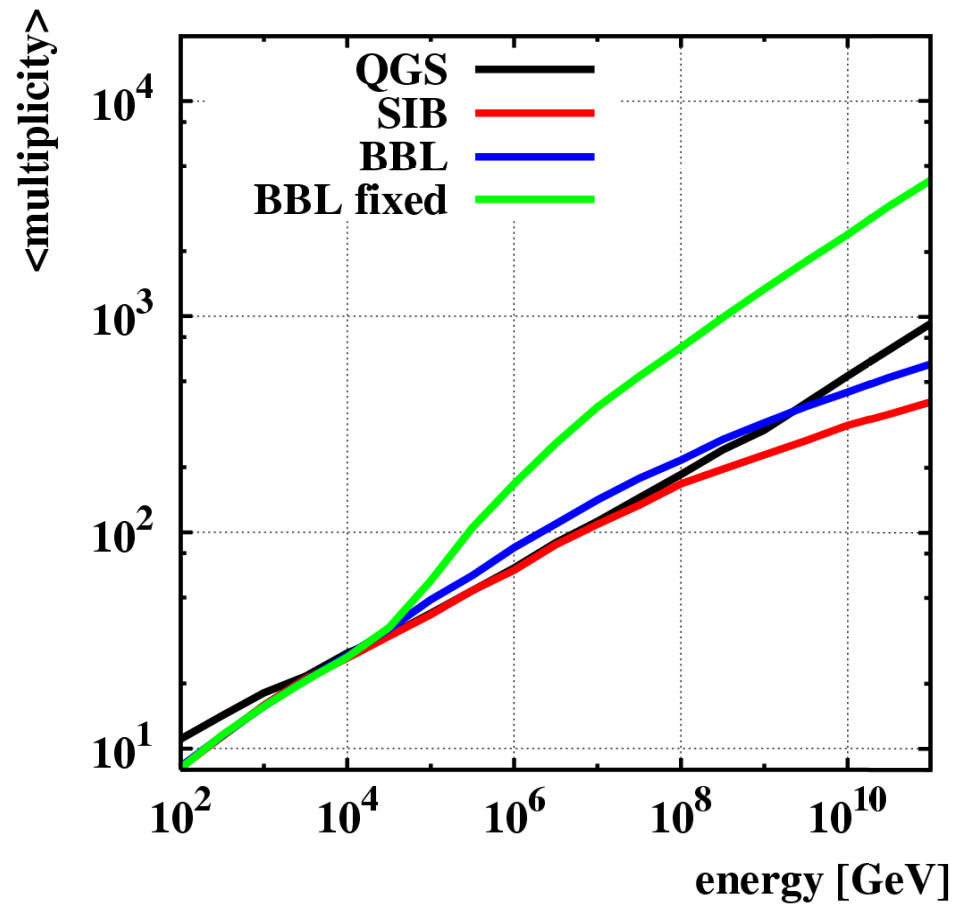


Minimum bias  
p-Air events



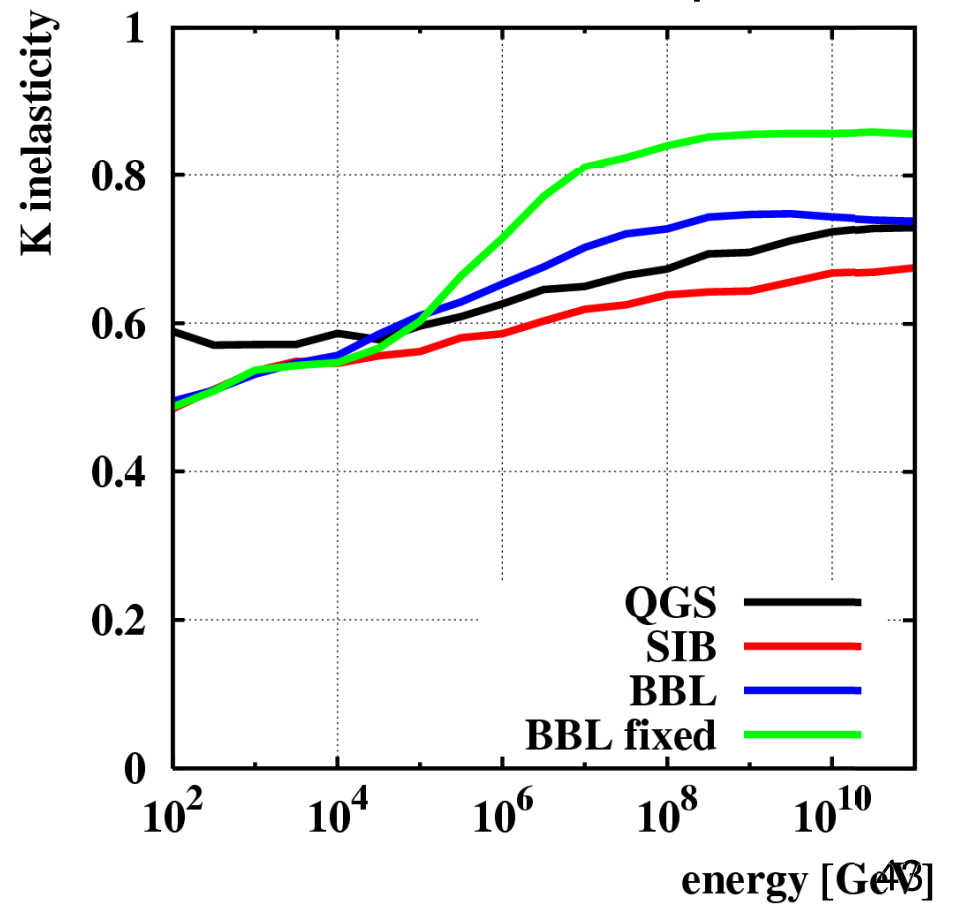
# Event shape

## Multiplicity

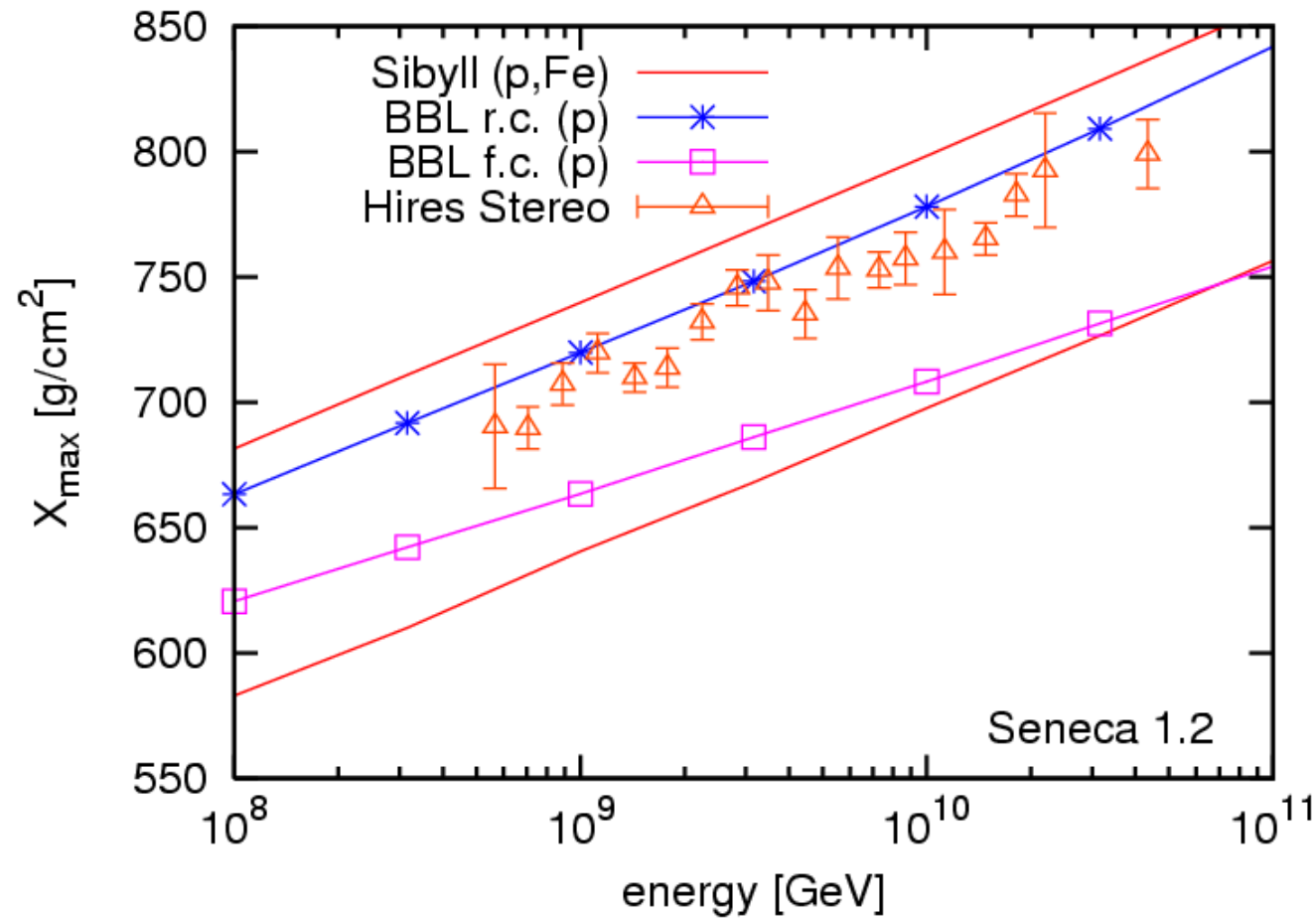


## Inelasticity

$K = 1 - \langle x_F \text{ of fastest particle} \rangle$



# $X_{\max}$ plot for fixed and running coupling



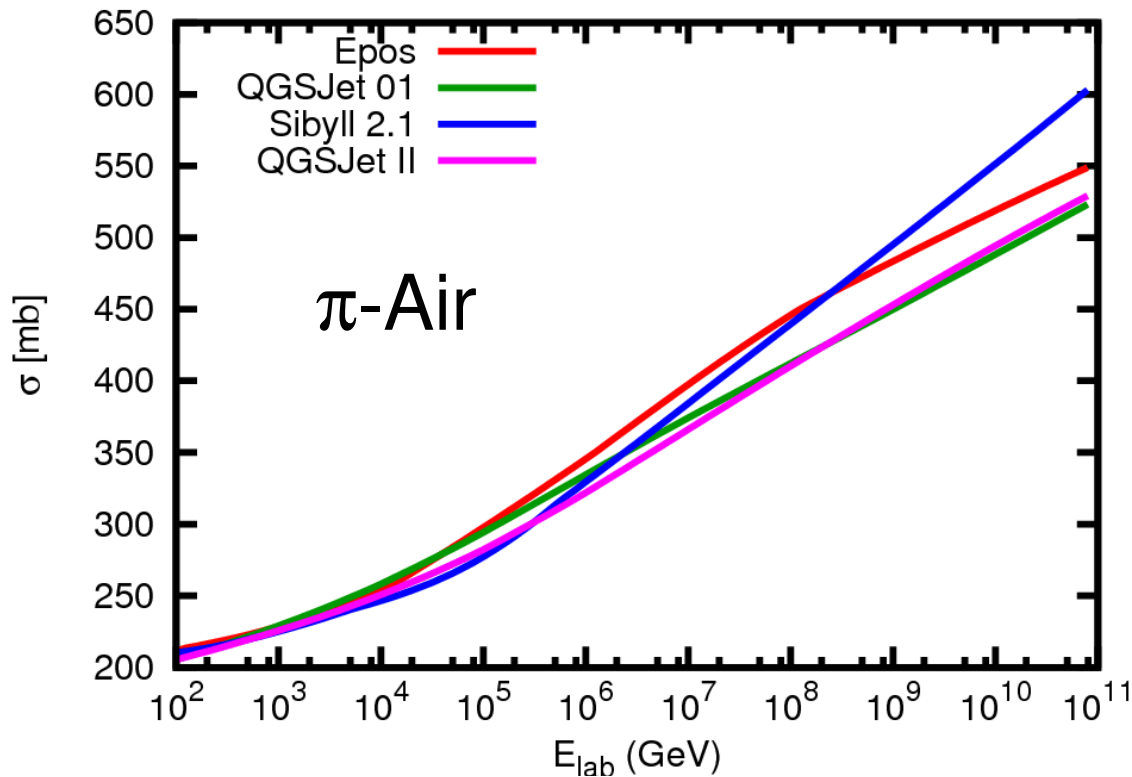
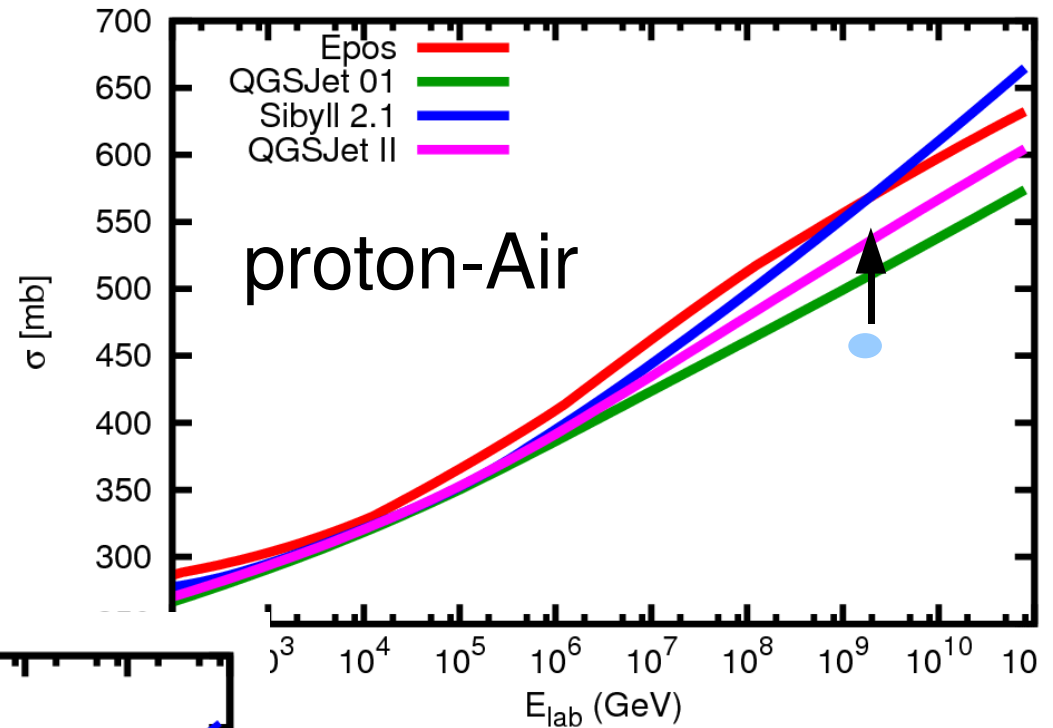
- $X_{\max}$  sensitive to evolution scenario

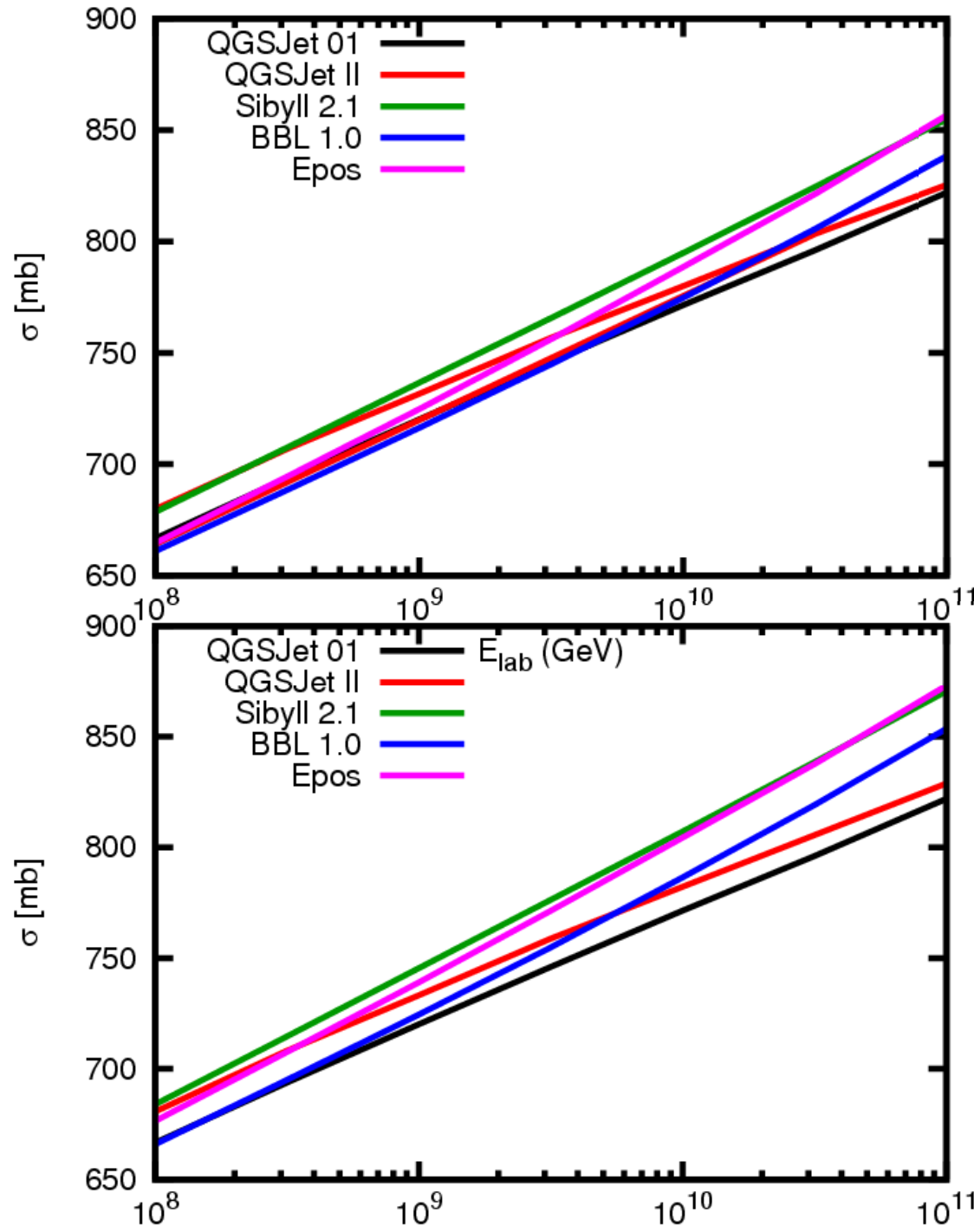
Model comparisons show density effects  
in observables relevant for Air Showers:

- QGSJet 01 versus QGSJet-II
- Sibyll versus Sibyll/BBL
- Epos

# Model Comparison

inelastic cross section  
proton/pion-Air



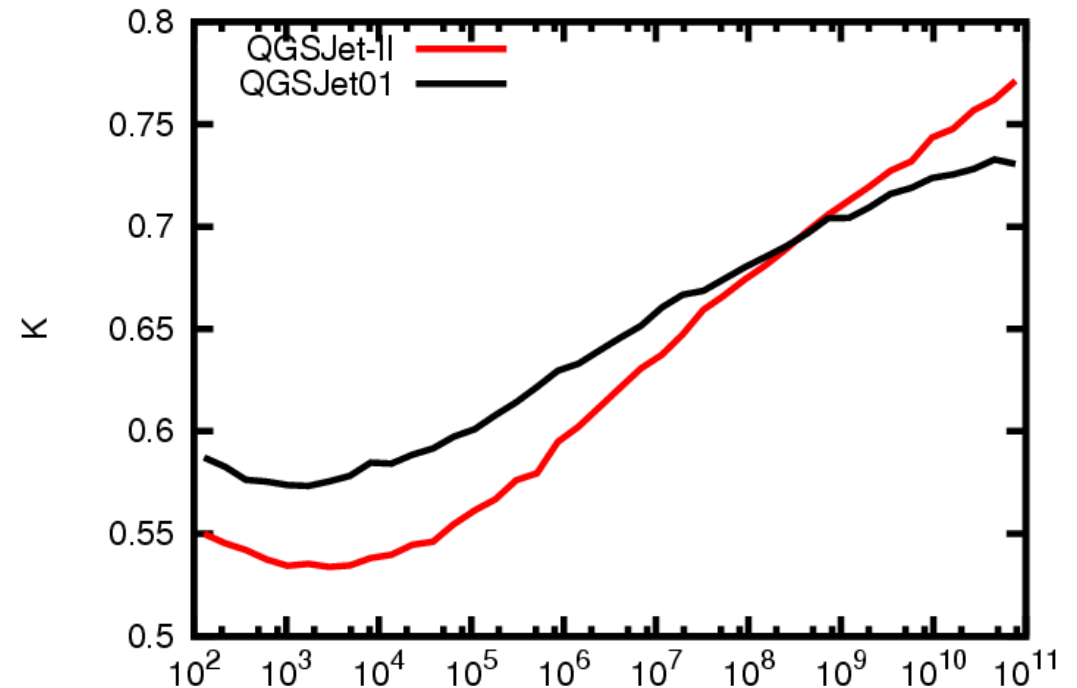


$X_{\text{max}}$  of models

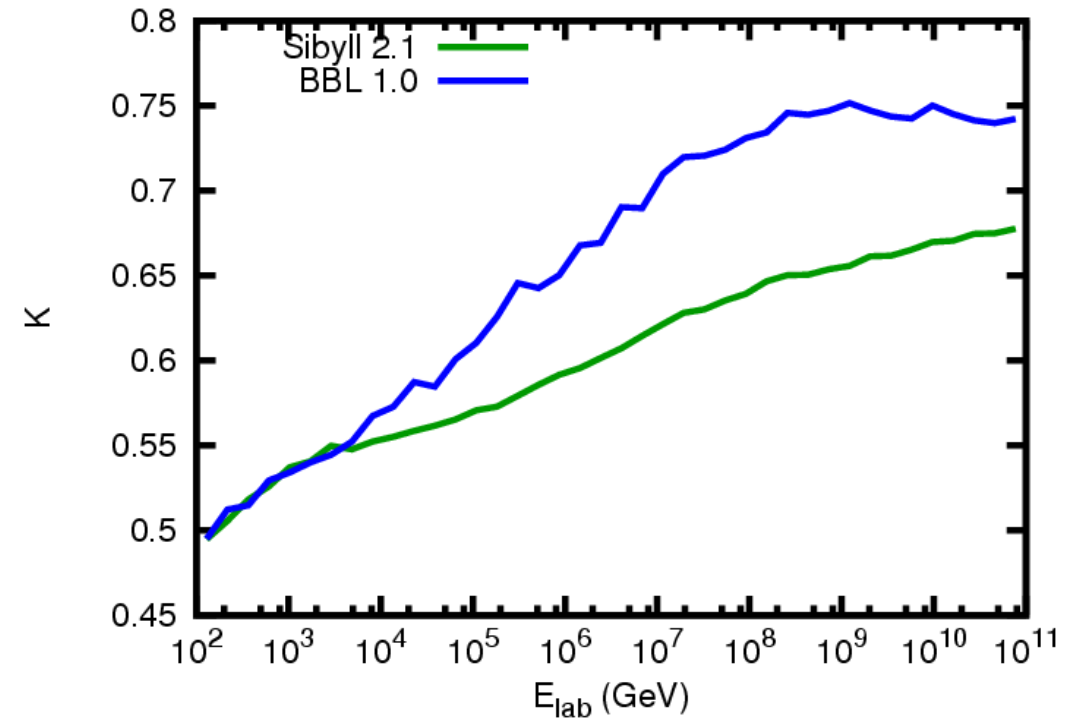
$X_{\text{max}}$  of models  
for same  
inelastic cross  
section  
from QGSJet01  
particle production  
even more different



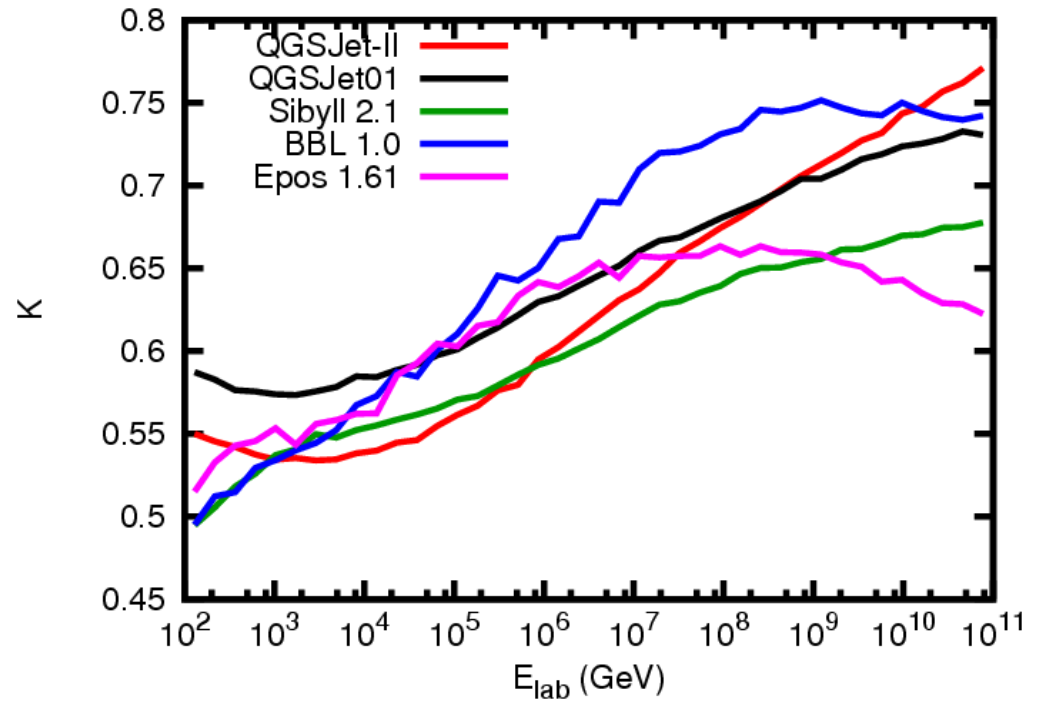
Inelasticity  
QGSJET01 –  
QGSJet- II



Inelasticity BBL  
– Sibyll-2.1

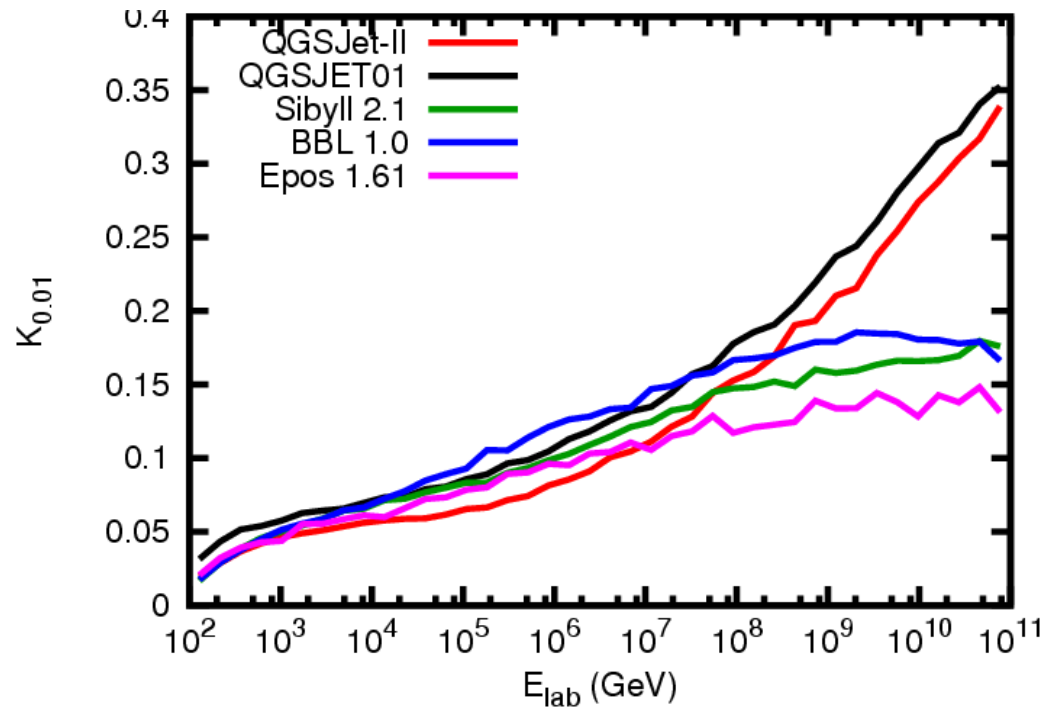


Inelasticity of all models

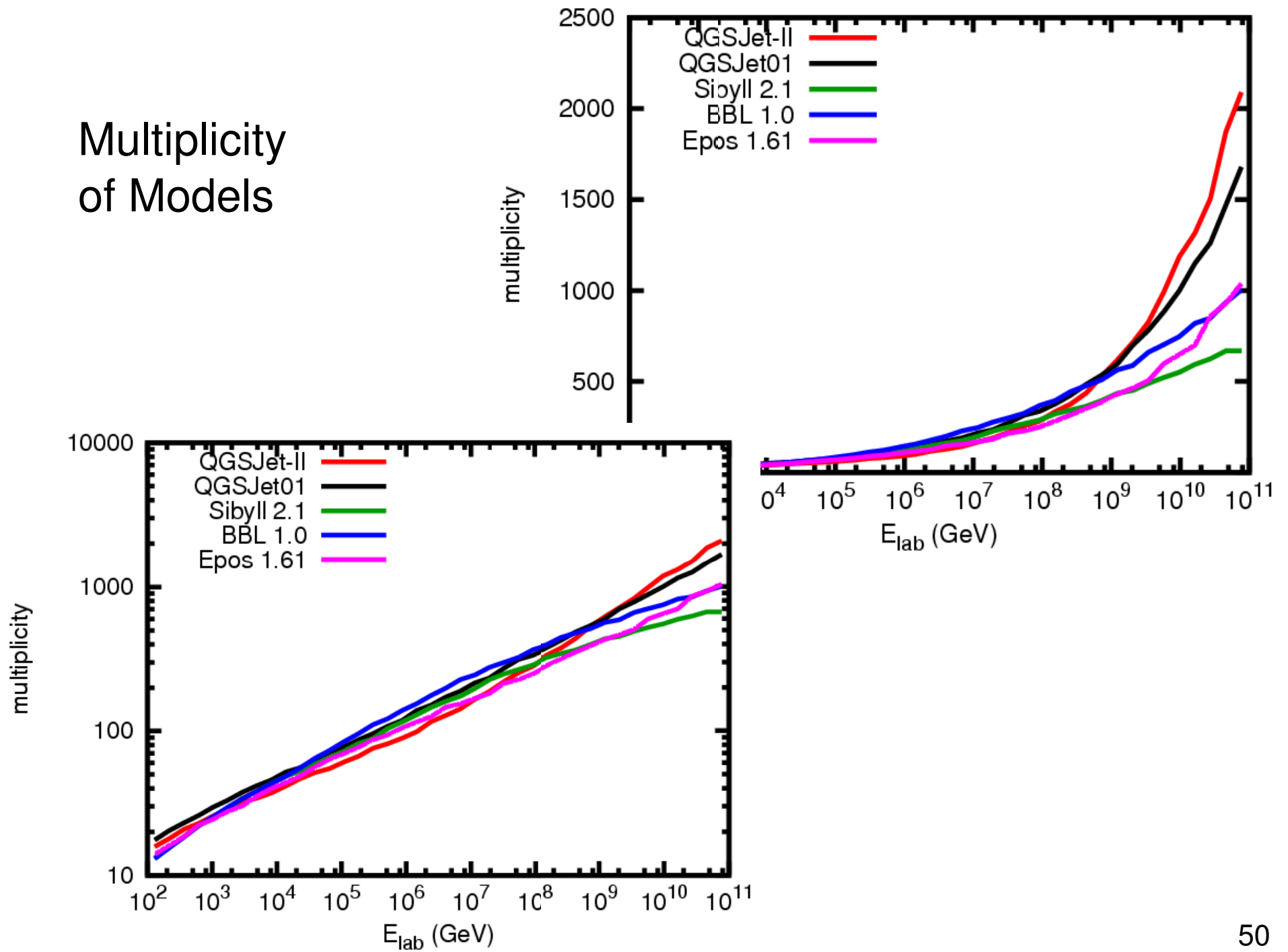


Inelasticity  $K_{0.01}$   
of all models

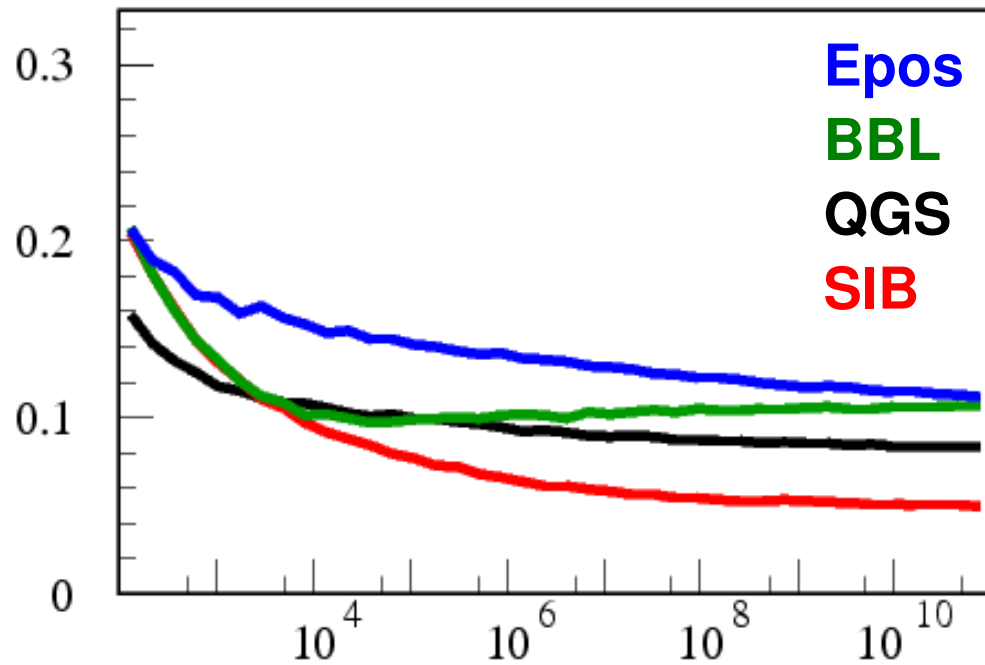
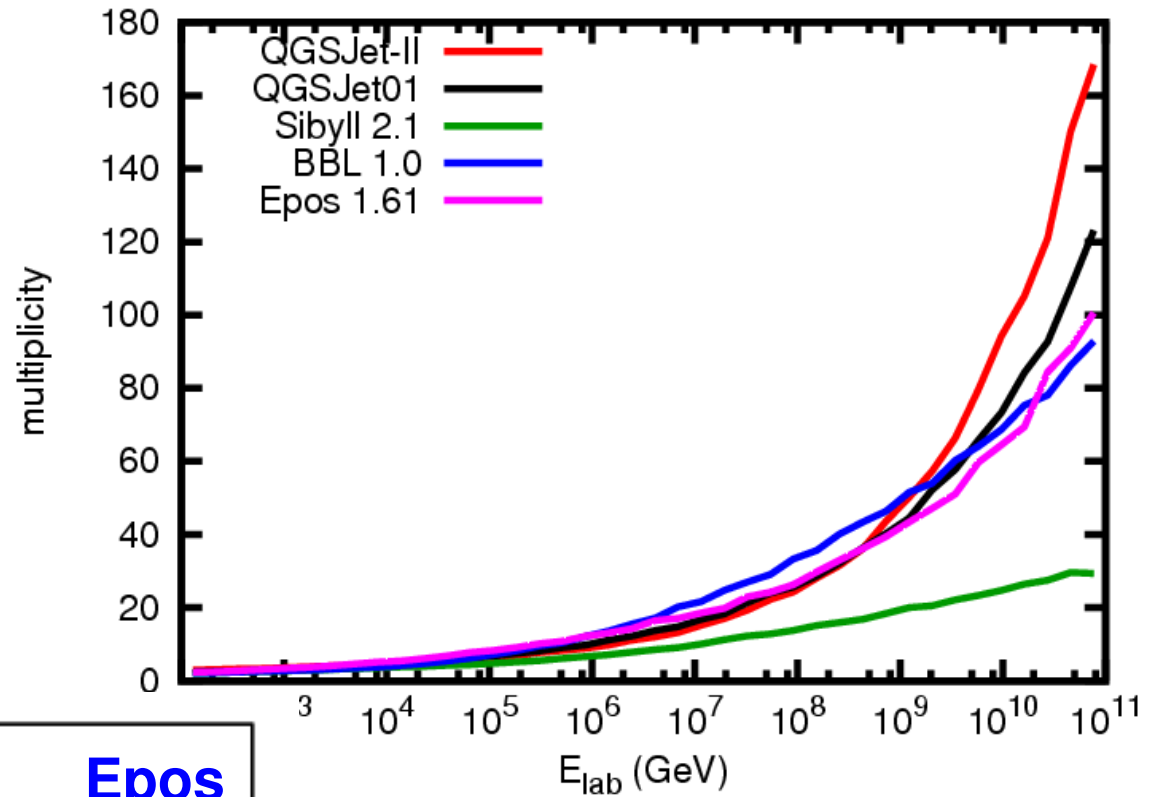
$$K_{0.01} = 1 - \int_{0.01}^1 x_F \frac{dn}{dx_F} dx_F$$

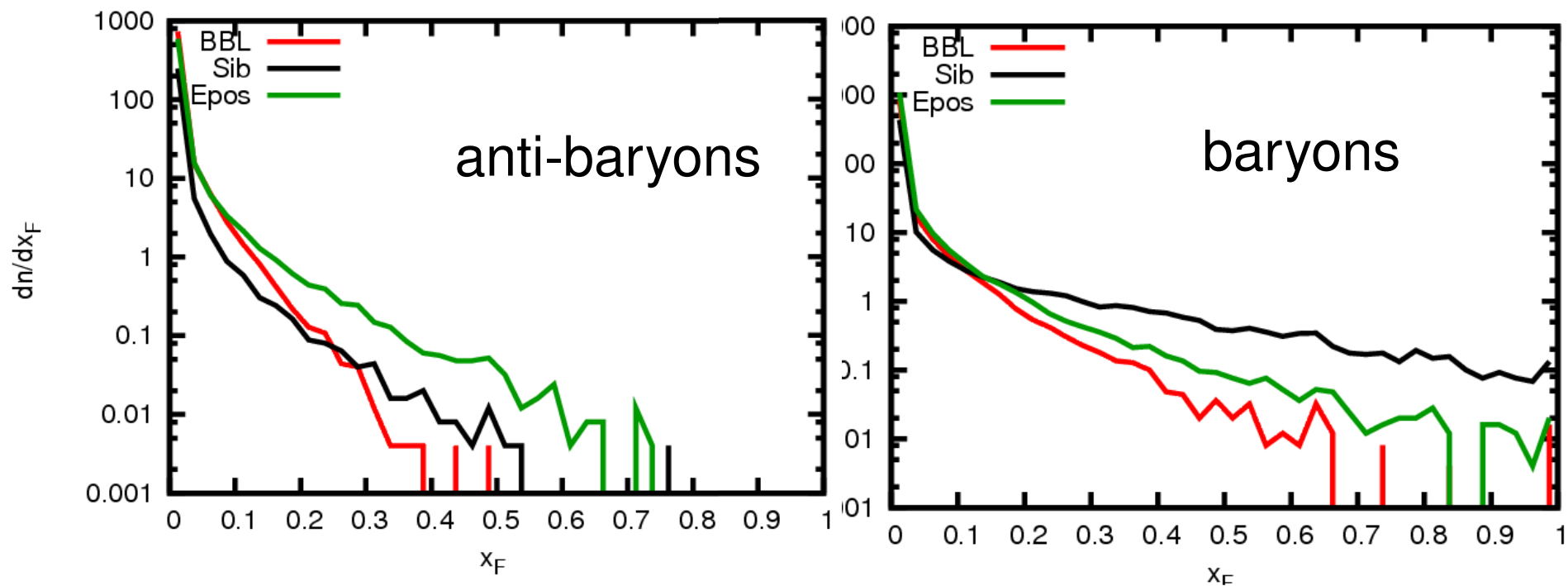
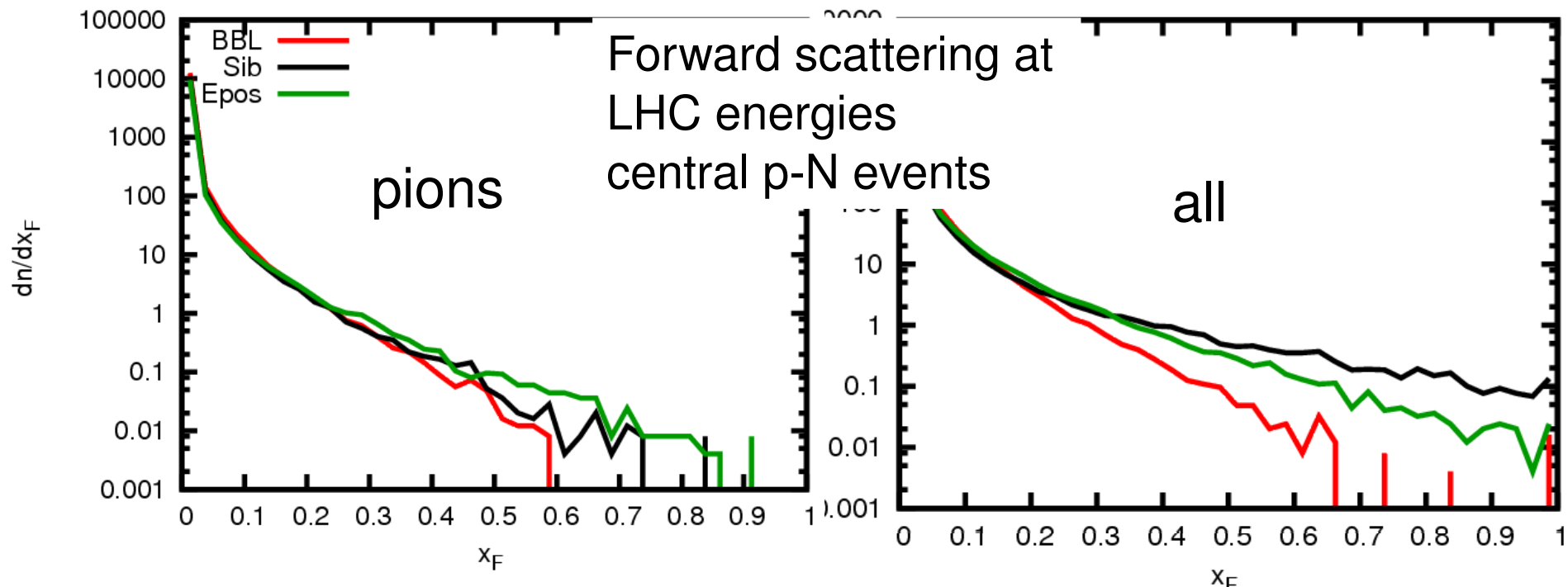


# Multiplicity of Models



# Baryons and baryon/meson ratio

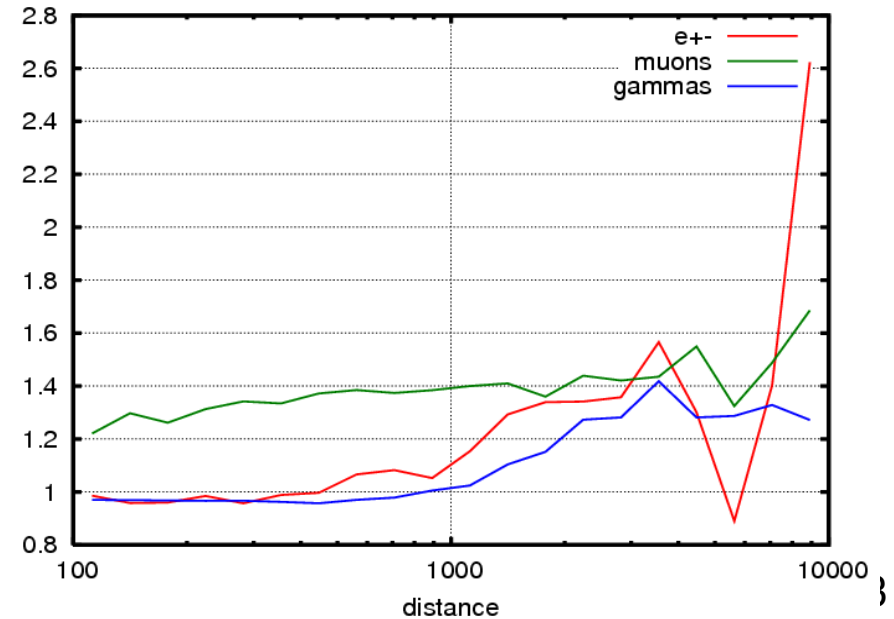
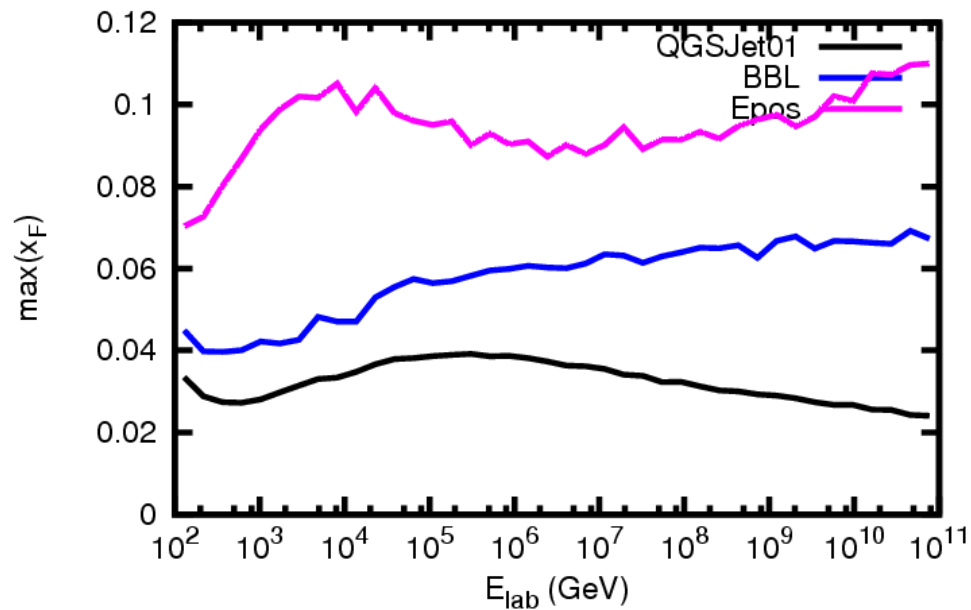




More Muons from Epos due to enhanced production of baryons

Produce more particles which stay in the hadronic channel of an air shower ( p/n instead of pi0 )

max(x<sub>F</sub> of baryon for  $\pi^\pm$  - Air)  
forward production of baryons



## Conclusion

- hadron Nucleus collisions main uncertainty for Air-shower simulations
- many approaches for high density/small  $x$
- leading particle suppression in black disk limit
- checking models thoroughly with data is good (Epos)
- extrapolation to high energies still unclear