The Abdus Salam
International Centre for Theoretical Physics

United Nations
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International Workshop on QCD at Cosmic Energies III

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Lecture Notes

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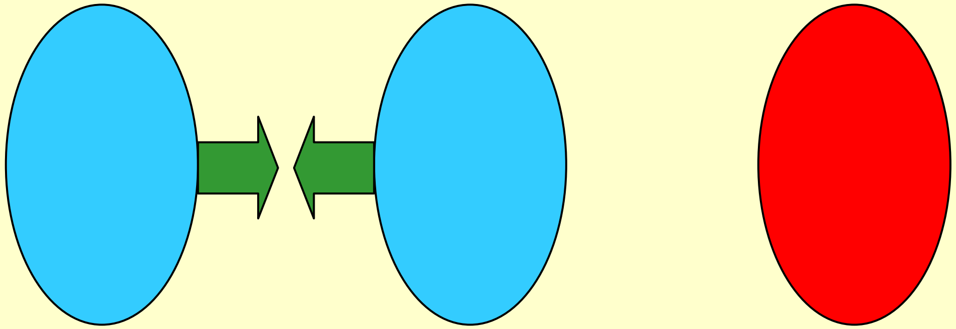
Open questions of the statistical approach to relativistic HIC's

E. Zabrodin, Univ. of Oslo, Norway

QCD at Cosmic Energies – III, ICTP, Trieste, 28.05.2007

Fermi statistical model

E. Fermi, Phys. Rev. 81 (1951) 683



(1) (2)

1. Momentary stopping within the overlapped volume
2. The whole CMS energy is liberated in this volume
3. The vacuum boils up and we get a hot system of particles in thermal and chemical equilibrium

$$V_F \propto M_h/\sqrt{s}$$ ← Volume

$$\epsilon_F \propto \sqrt{s}/V_F \propto s$$ ← Energy density

$$\epsilon = \sigma T^4 \;, \; => \; T \propto s^{1/4}$$ ← Temperature

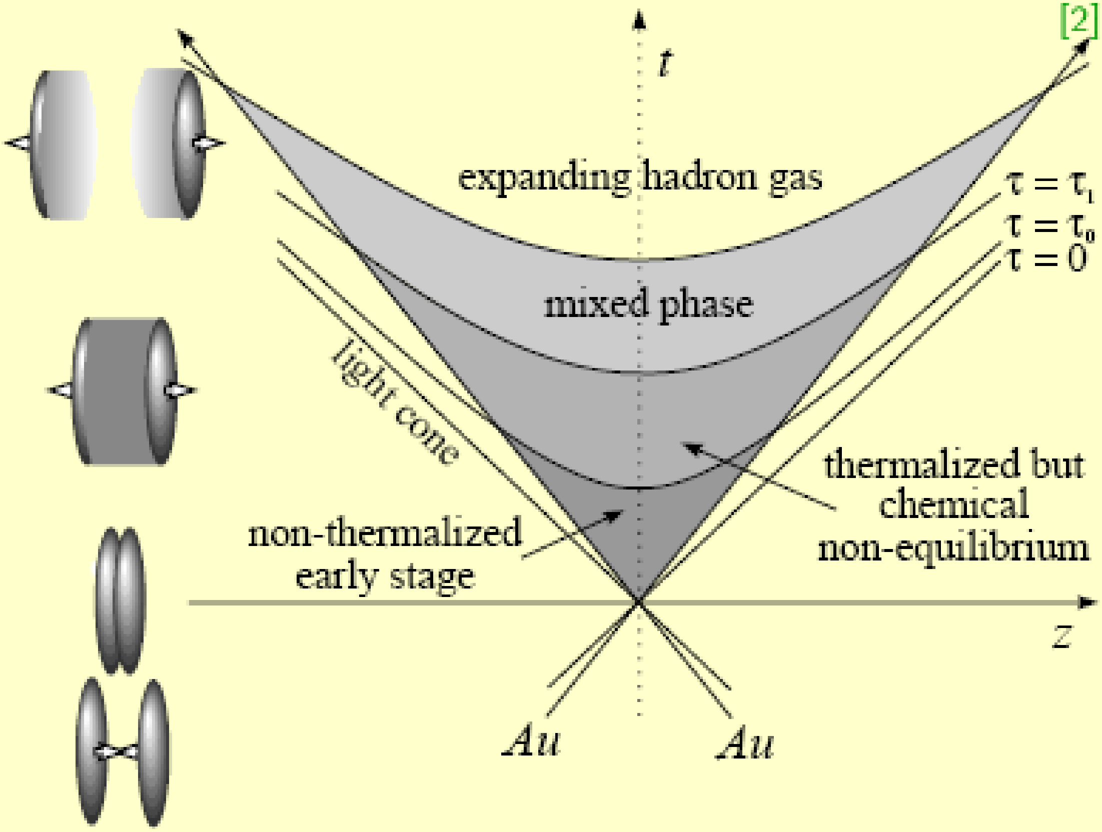
$$\langle n \rangle \propto s^{1/4}$$ ← Average multiplicity (correct)

$$n_p : n_\pi = 8 : 3$$ → Wrong!

$$n_p : n_\pi = 0.1 \div 0.01$$ → from cosmic ray experiments

Relativistic Heavy Ion Collisions - Tool to Probe Hot and Dense Nuclear Matter

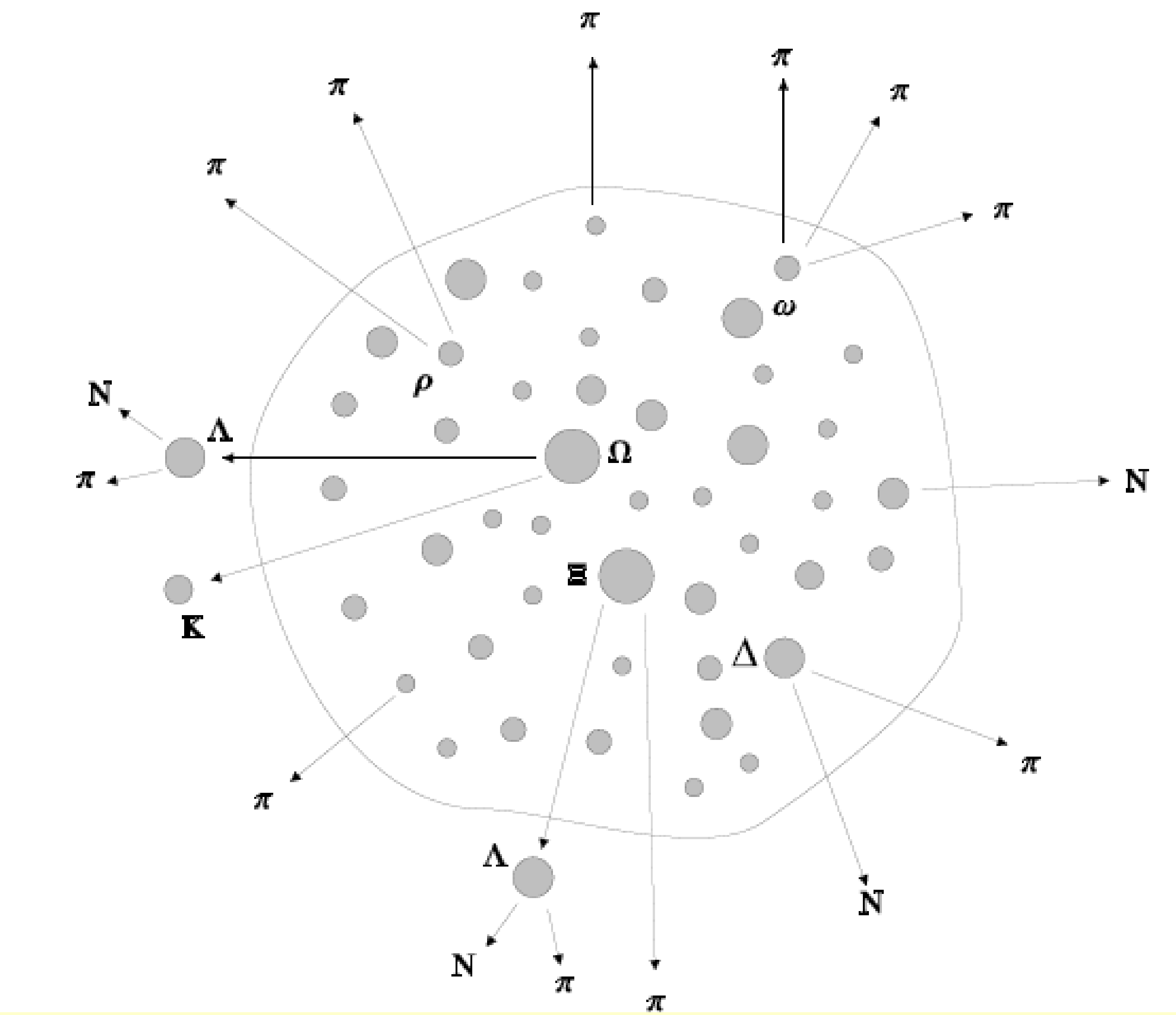
[1] E. Fermi, Phys. Rev. **81** (1951) 683
[2] L. Landau, Izv. Acad. Nauk USSR **17** (1953) 51



Ideas behind macroscopic description:

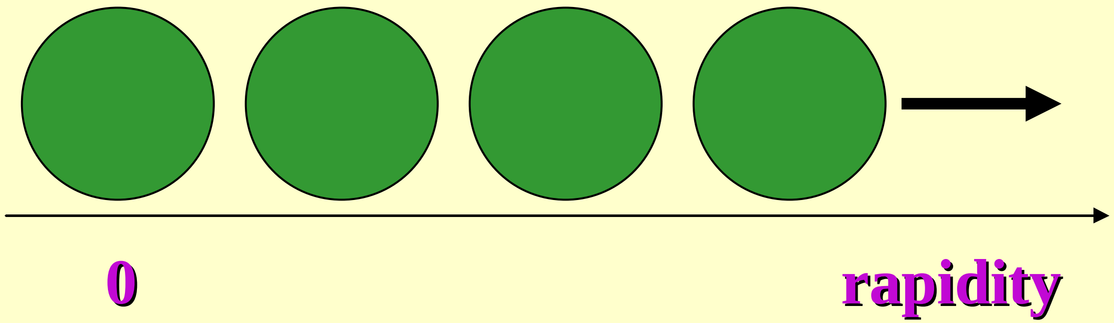
- **two head-on colliding nuclei stop within their overlapped Lorentz-contracted volume and form a fluid of unceasingly appearing and annihilating hadrons ("boiling liquid of operators")**
- **further expansion of the pattern proceeds adiabatically and is governed by the relativistic hydrodynamics**
- **when the fluid cools down to the temperature $T \approx m_\pi$, it decays into the gas of final particles.**

Basic assumptions:



1. Formation of a single fireball
2. All hadrons are in thermal and chemical equilibrium
3. After the chemical freeze-out only thermal equilibrium is maintained, but hadronic yields are frozen
4. Thermal freeze-out: see transverse mass spectra

Expanding source or several fireballs

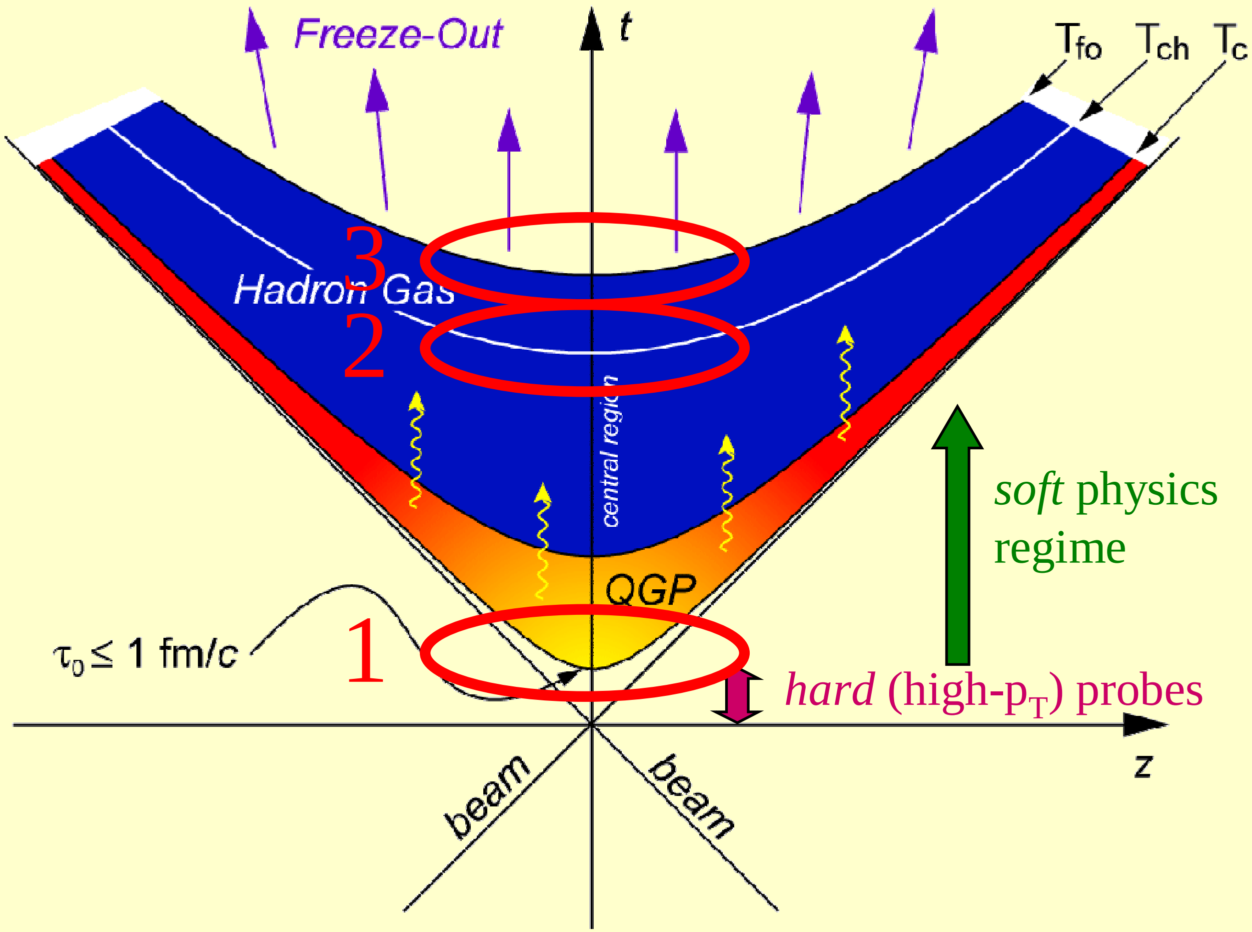


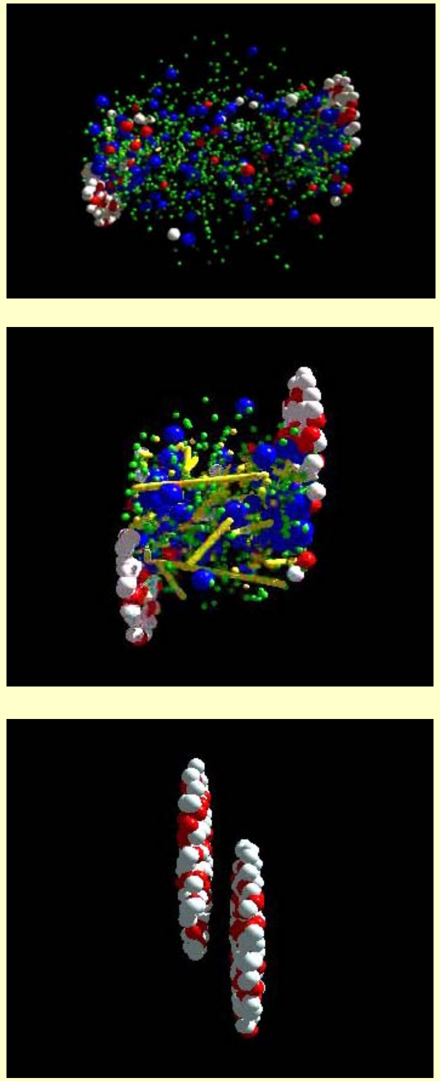
$$\frac{dN_i/dy}{dN_j/dy} = \frac{\int dy\, dN_i/dy}{\int dy\, dN_j/dy} = \frac{N_i}{N_j}$$

$$\frac{dN_i}{dy} = \int d^2p_T \frac{dN_i}{d^2p_T\, dy}$$

If the temperature and the chemical potentials of the fireballs are the same at the FO, the particle yields and ratios coincide with those obtained for the single static fireball

Collision picture





Chemical freezeout ($T_{ch} \leq T_c$): inelastic scattering ceases

Kinetic freeze-out ($T_{fo} \leq T_{ch}$): elastic scattering ceases

Statistical (thermal) model of an ideal hadron gas

If the system is in chemical and thermal equilibrium, its macroscopic characteristics are derived via the set of distribution functions

$$f(p, m_i) = \left\{ \exp\left(\frac{E_i - \mu_i}{T}\right) \pm 1 \right\}^{-1}$$

where "+" is for fermions and "−" for bosons,

$$E_i = (p_i^2 + m_i^2)^{1/2} \, , \qquad \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$$

Particle density : $$n_i = \frac{g_i}{2\pi^2} \int_0^\infty p^2 f(p, m_i) dp$$

Energy density : $$\varepsilon_i = \frac{g_i}{2\pi^2} \int_0^\infty p^2 \sqrt{p^2 + m_i^2}\, f(p, m_i) dp$$

Pressure : $$P = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 \frac{p^2}{3E_i} \, f(p, m_i) dp$$

Gibbs thermodynamical identity

$$\varepsilon = Ts + \mu_B \rho_B + \mu_S \rho_S - P$$

Statistical (thermal) model of an ideal hadron gas

Using the power series expansion

$$[\exp(\alpha) \pm 1]^{-1} = \exp(-\alpha) \sum_{n=0}^{\infty} (\mp 1)^{-n} \exp(-n\alpha)$$

we obtain

$$n_i = \frac{g_i m_i^2 T}{2\pi^2} \sum_{n=1}^{\infty} (\mp 1)^{n+1} \exp\left(\frac{n\mu_i}{T}\right) \frac{1}{n} K_2\left(\frac{n m_i}{T}\right)$$

$$\varepsilon_i = \frac{g_i m_i^2 T^2}{2\pi^2} \sum_{n=1}^{\infty} (\mp 1)^{n+1} \exp\left(\frac{n\mu_i}{T}\right) \frac{1}{n^2} \left[3K_2\left(\frac{n m_i}{T}\right) + \frac{n m_i}{T} K_1\left(\frac{n m_i}{T}\right)\right]$$

$$P = \sum_i \frac{g_i m_i^2 T^2}{2\pi^2} \sum_{n=1}^{\infty} (\mp 1)^{n+1} \exp\left(\frac{n\mu_i}{T}\right) \frac{1}{n^2} K_2\left(\frac{n m_i}{T}\right)$$

where K_1 and K_2 are modified Bessel functions of first and second order, respectively. $n = 1$ corresponds to the Maxwell-Boltzmann statistics

$$f^{MB}(p, m_i) = \exp\left(\frac{\mu_i - E_i}{T}\right)$$

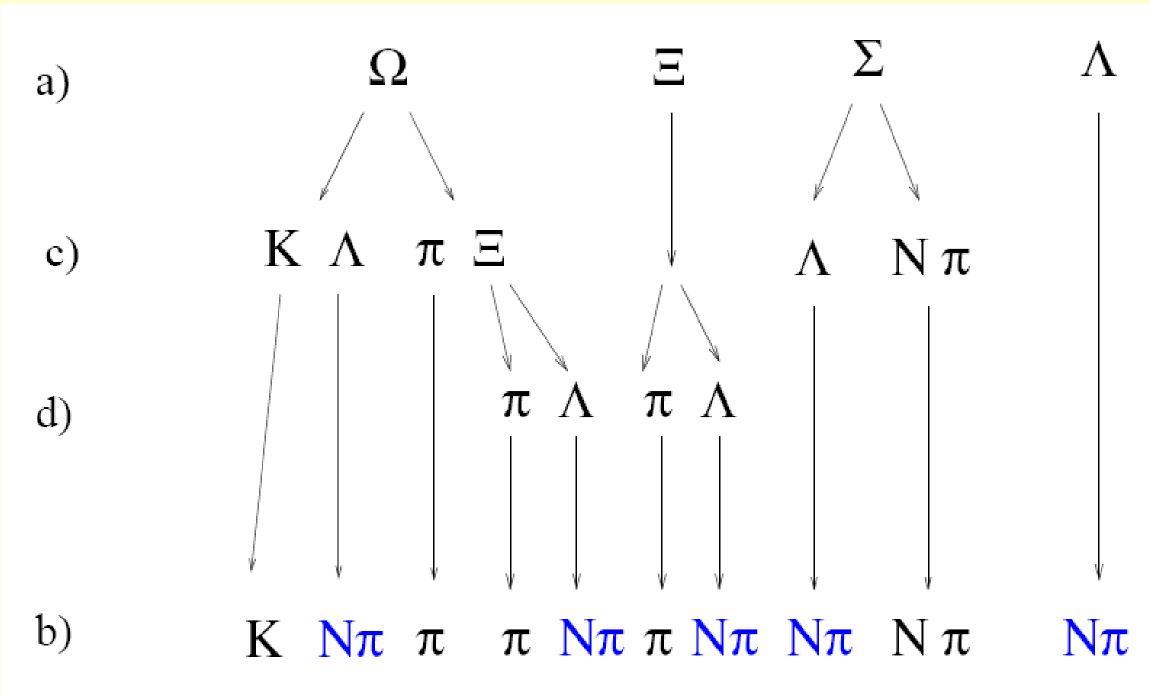
Attractive feature:

$$T,\ V,\ \rho_B$$
$$\rho_S = 0$$

3 free parameters only !

Problems:

- **Yields of pions are underestimated**
- **Abundances of strange hadrons are overpredicted**
- **Number and energy densities are too large**



Modifications:

1. **Excluded volume effects**
2. **Strangeness suppression**
3. **Canonical strangeness suppression**
4. **Light quarks are not in chemical equilibrium**
5. **Feed-down from weak decays**

Modifications:

Excluded volume effects (van der Waals-type of the EOS)

$$P^{excl}(T,\mu) = P^{id.gas}(T,\mu')$$

$$\mu' = \mu - V_h P^{excl}(T,\mu)$$

These corrections have no impact on the fitted values of the temperature and baryon chemical potential

Strangeness suppression

$$n_i = \frac{g_i}{2\pi^2}\int_0^\infty dp\,p^2\left[\gamma^{-s_i}\exp\left(\frac{E_i-\mu_i}{T}\right)\pm 1\right]^{-1}$$

$$(0 < \gamma \le 1)$$

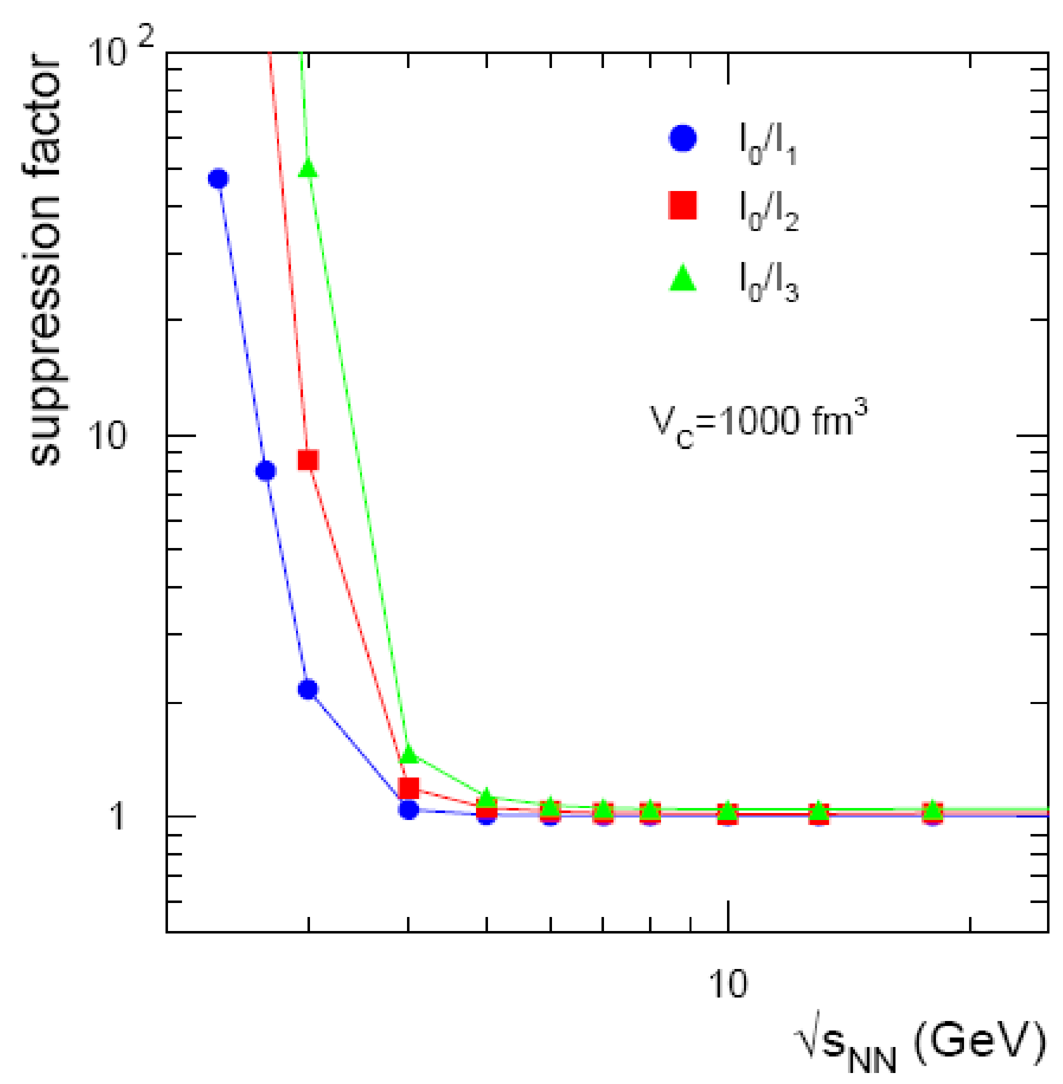
Here

$$\gamma_s \approx 1 \text{ at } y = 0$$

$$\gamma_s \approx 0.7 \text{ for } 4\pi - \text{data}$$

Modifications:

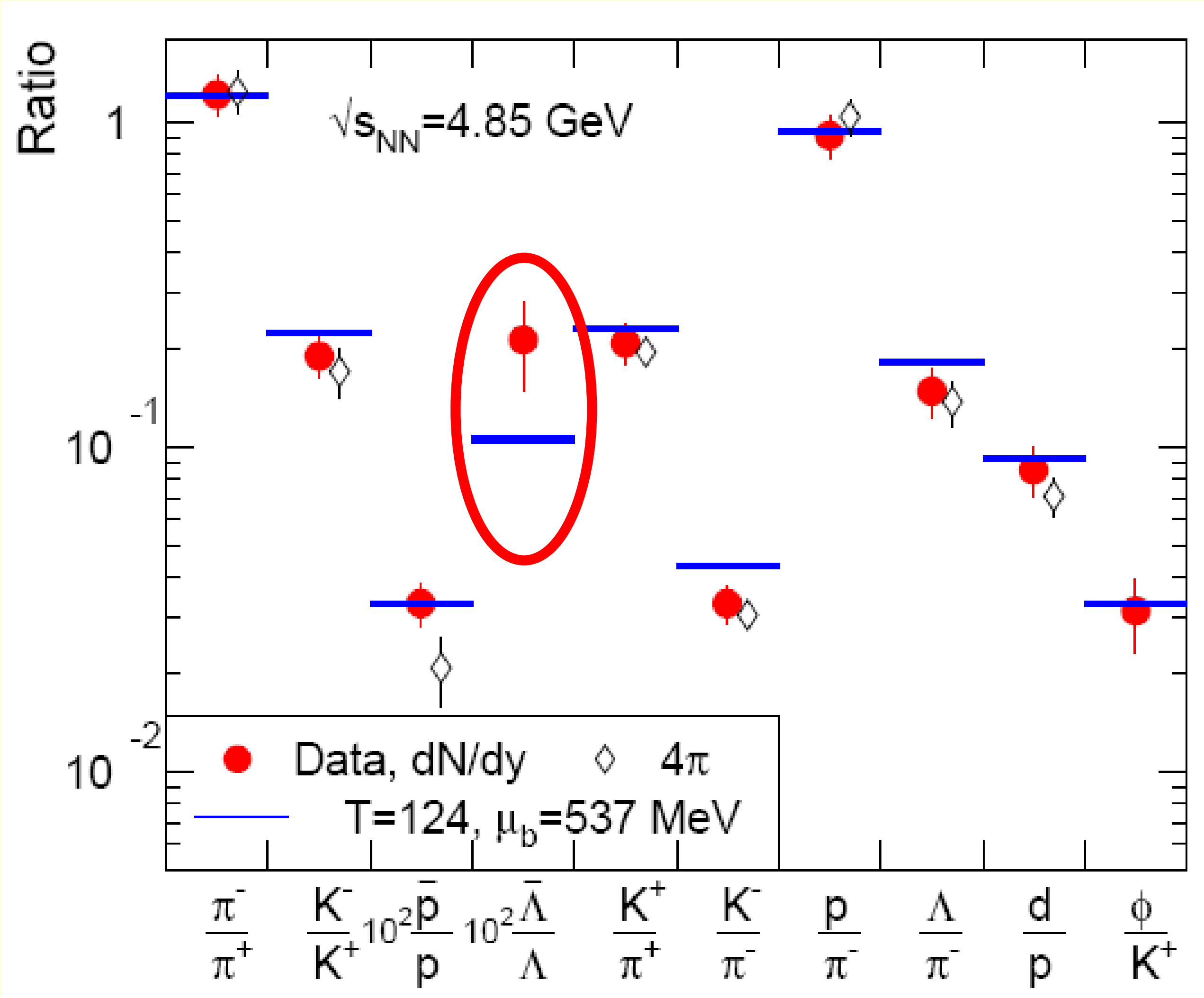
Canonical strangeness suppression



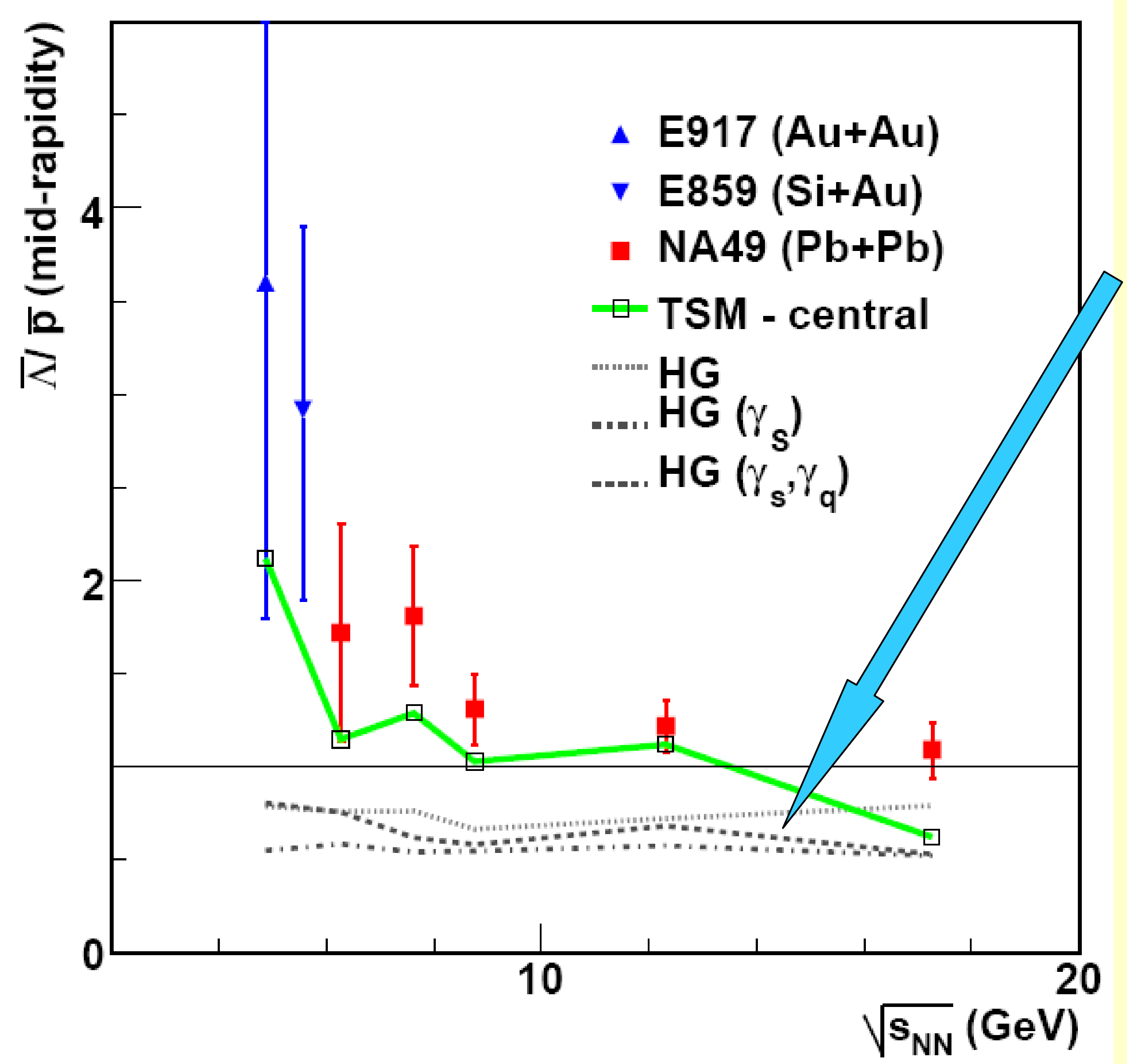
For small systems (or peripheral A+A collisions) and for low energies in case of strangeness production, a <u>canonical ensemble</u> treatment is mandatory. For a canonical volume about 1000 fm3 this effect is negligibly small**

Particle ratios at AGS (Au+Au @ 10.7 AGeV)

Andronic, Braun-Munzinger, Stachel, NPA 772 (2006) 167



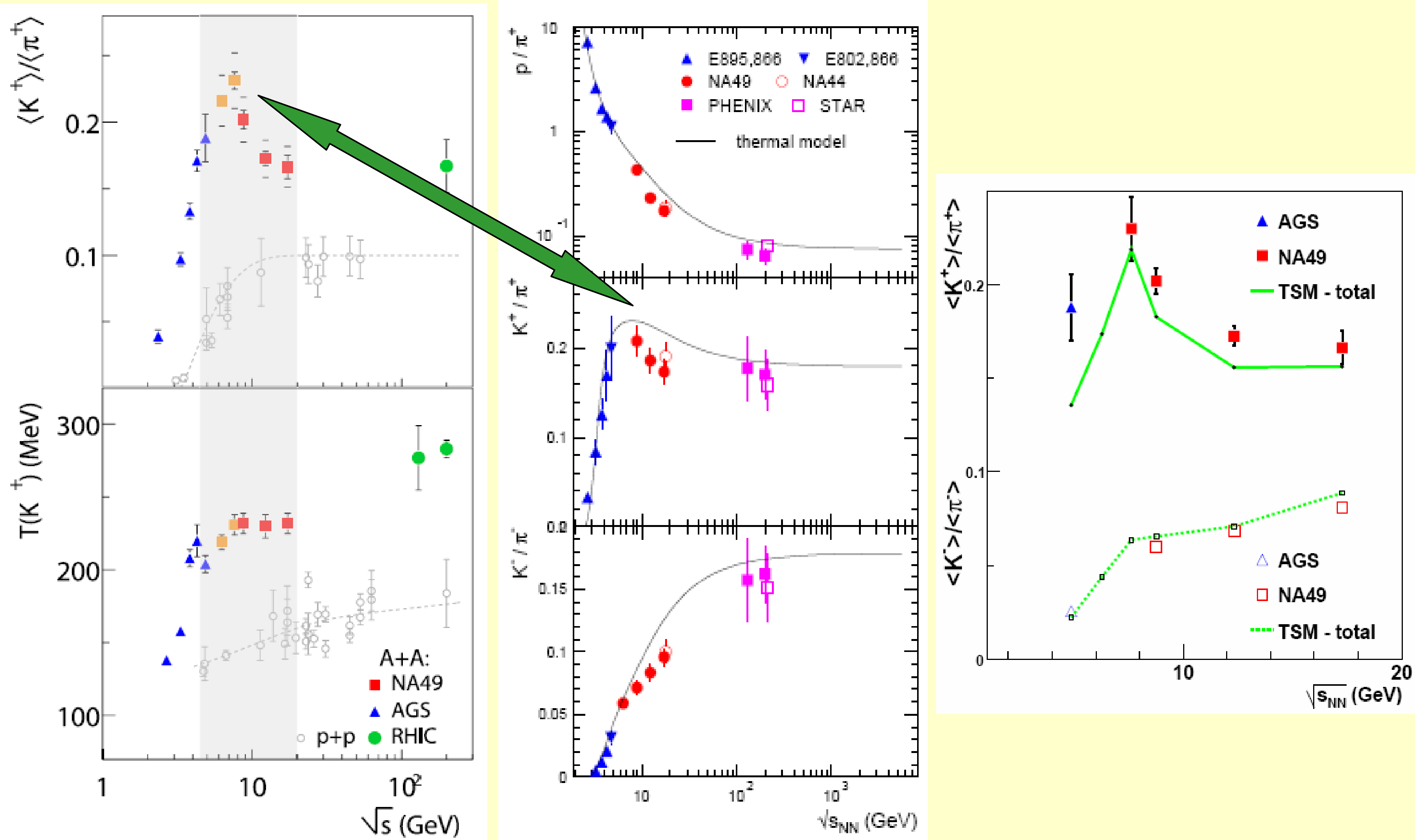
Open problem: $\overline{\Lambda}/\overline{p}$ puzzle



None of the SM versions can explain the high values of the antilambda-to-antiproton ratio

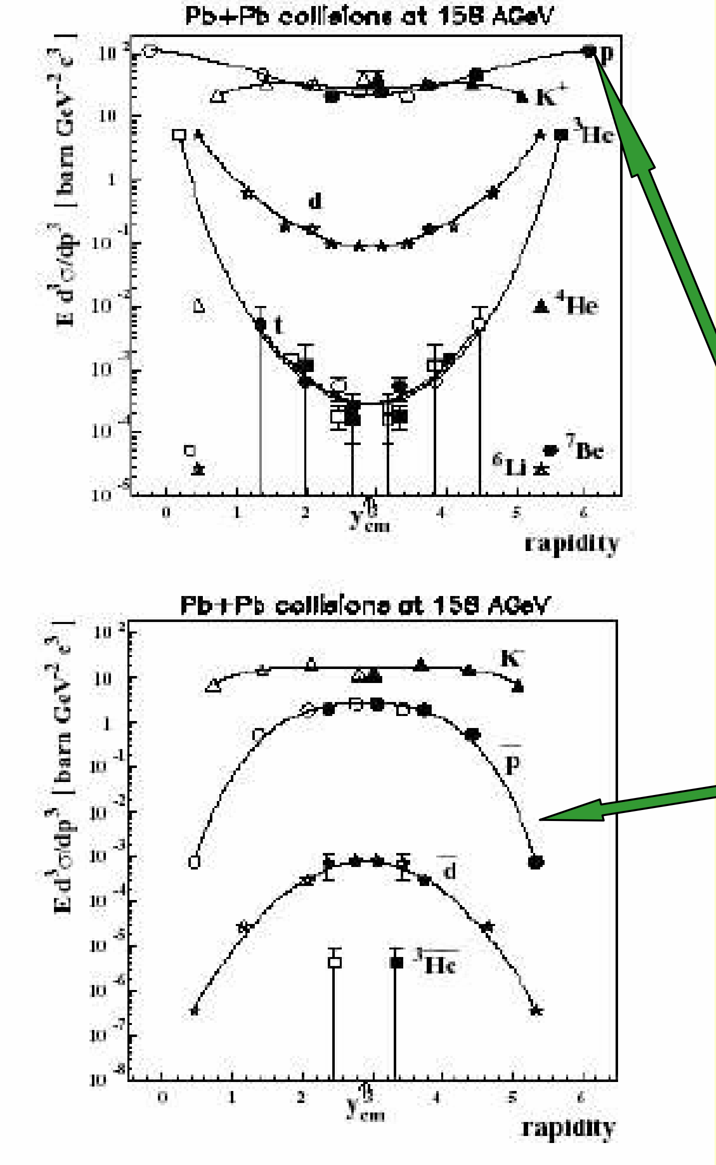
The problem can be solved if we assume the presence of two sources, core and halo, each being in equilibrium

Open problem: K$^+$/ p$^+$- horn



M. Gazdzicki, nucl-th/0512034 A. Andronic et al., NPA 772(06) 167 K.Tywoniuk et al., PRC(submitted)

Rapidity distributions at SPS



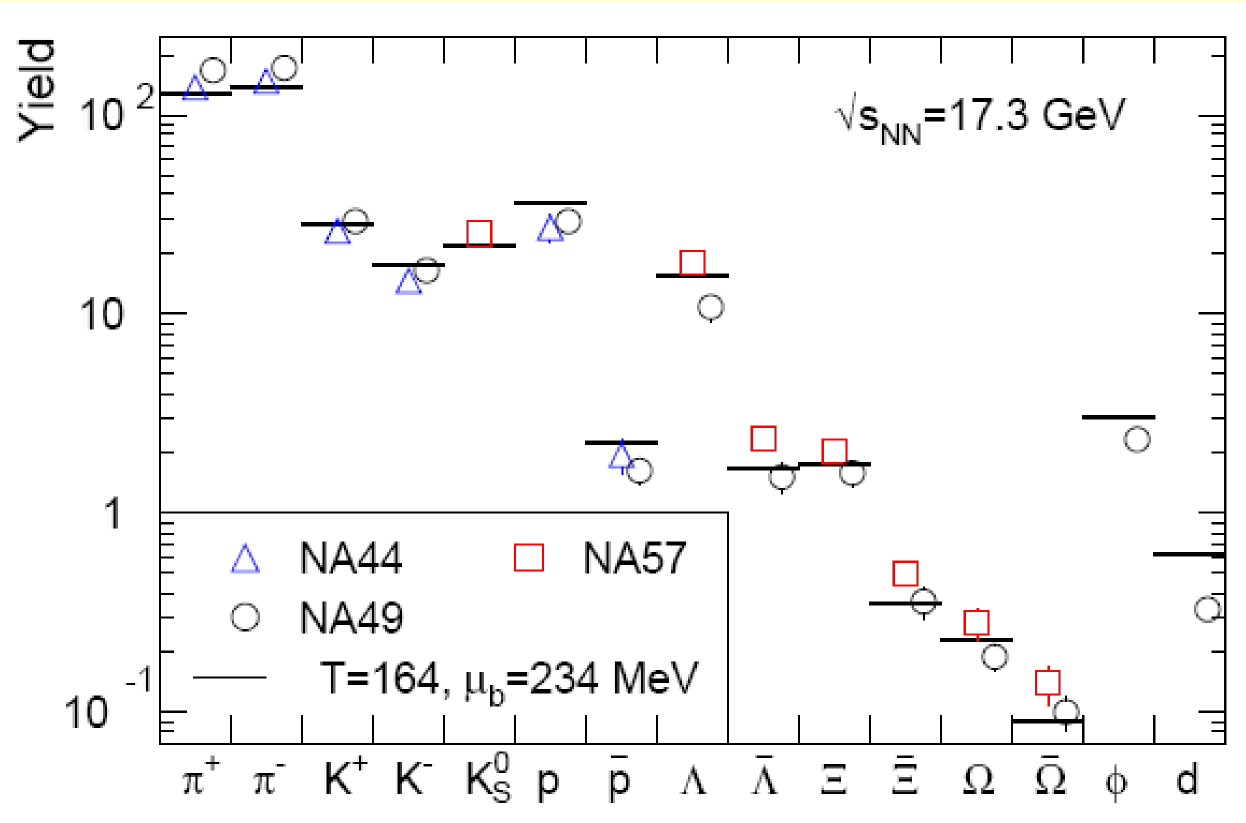
[10] J. B. Kinson, J. Phys. G 25 (1999) 143

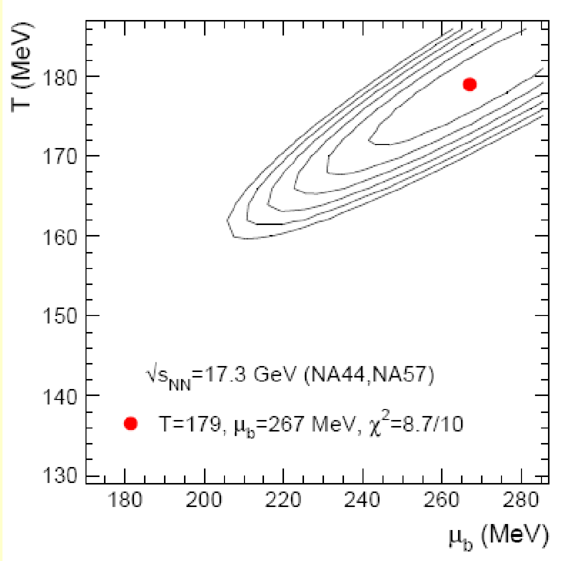
Rapidity distributions (NA52) for minimum bias Pb+Pb collisions at zero p_T (SPS 158 AGeV).

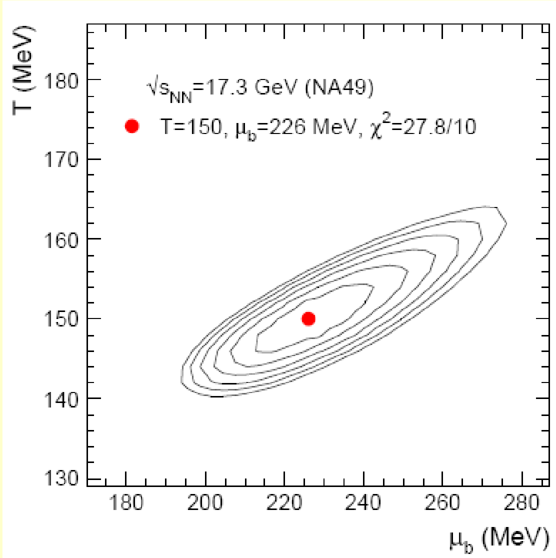
Baryons and antibaryons have very different distributions in rapidity. For the protons, the distribution resembles a "seagull"-shape with a minimum for center-of-mass rapidity, while the distribution of antiprotons is at its peak there. The shapes of the different distributions signal that the net-baryon charge is not distributed homogeneously in the volume of the fireball.

Particle ratios at SPS (Pb+Pb @ 158 AGeV)

Andronic, Braun-Munzinger, Stachel, NPA 772 (2006) 167

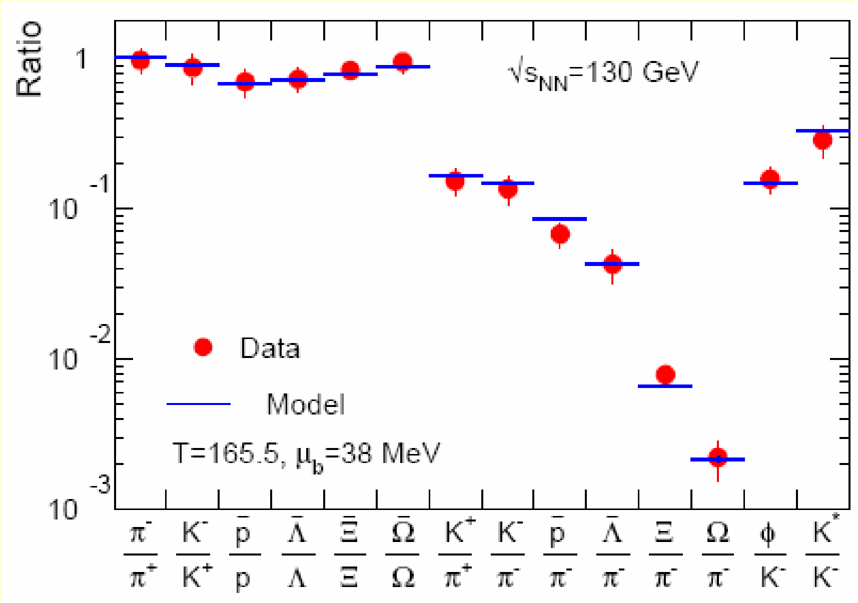


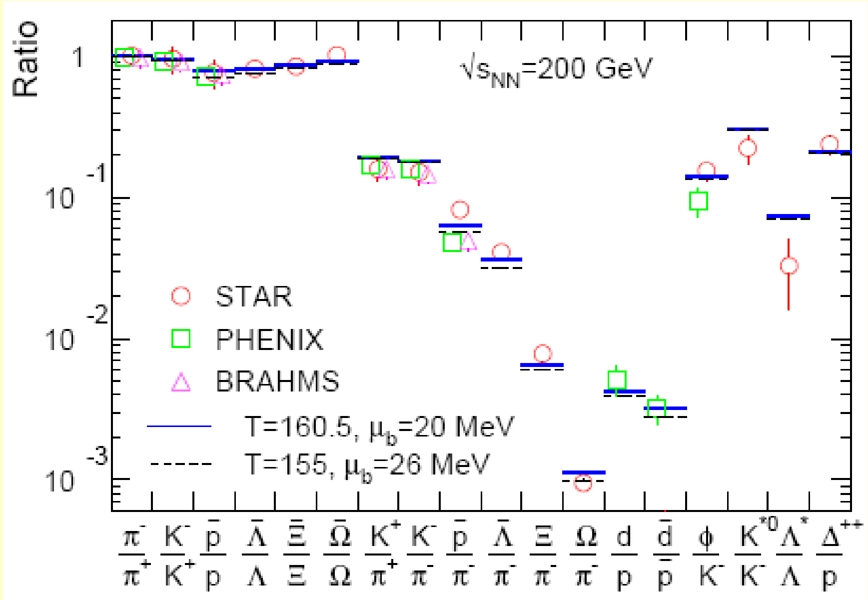




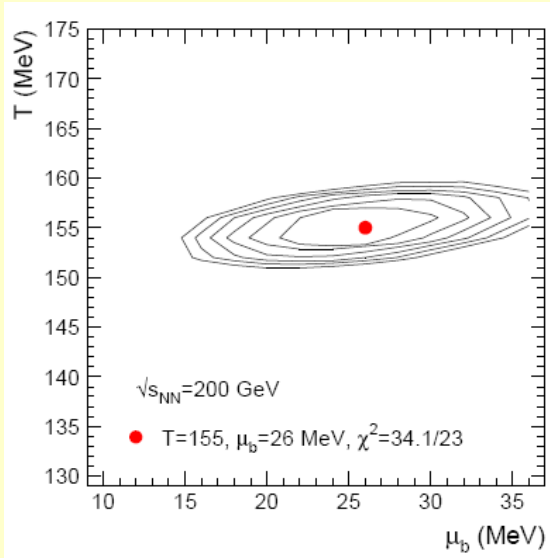
Particle ratios at RHIC (Au+Au @ 130, 200 AGeV)

Andronic, Braun-Munzinger, Stachel, NPA 772 (2006) 167



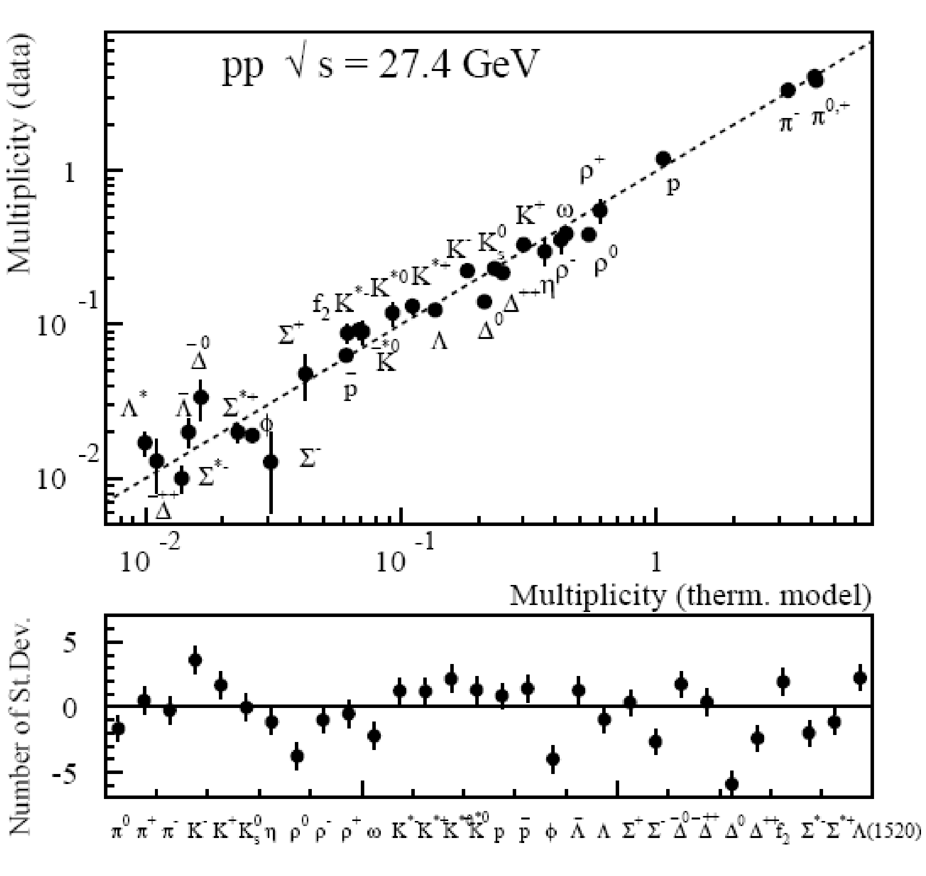


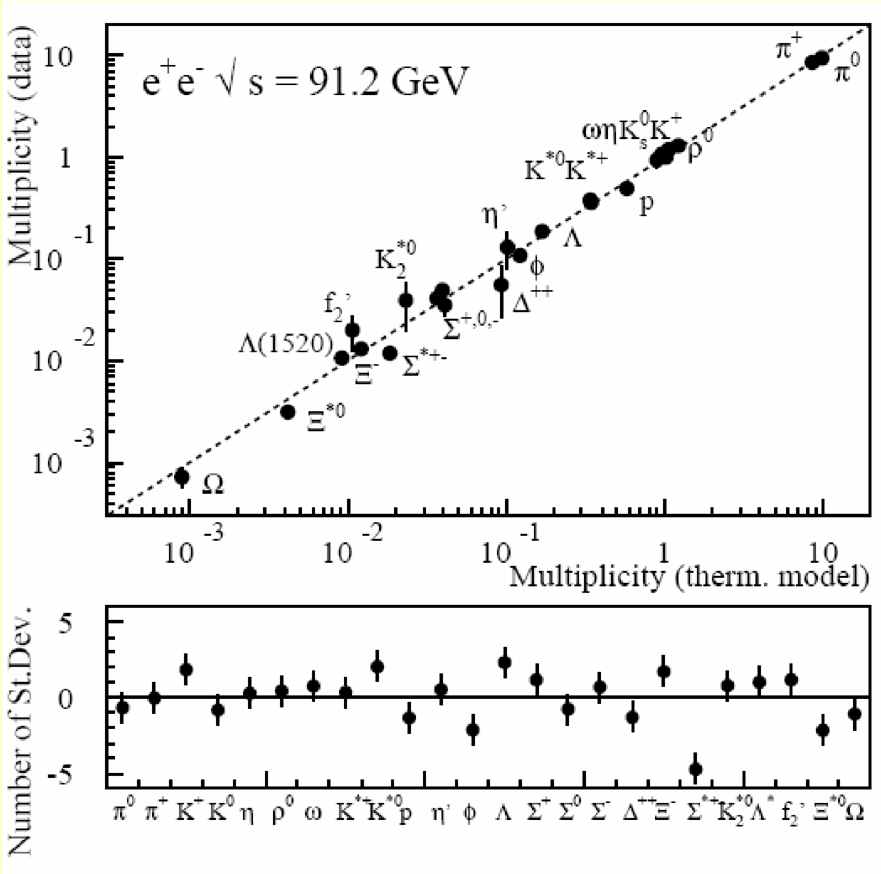
Statistical model seems to work well



Particle yields in elementary collisions

F. Becattini, U. Heinz, Z.Phys.C 76 (1996) 269

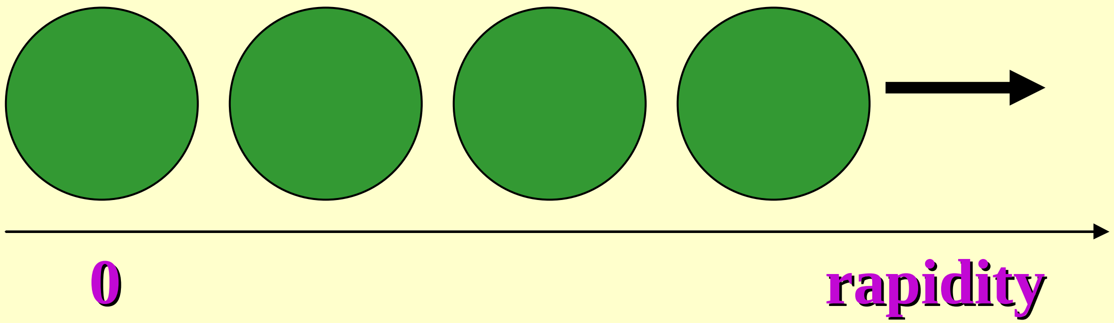




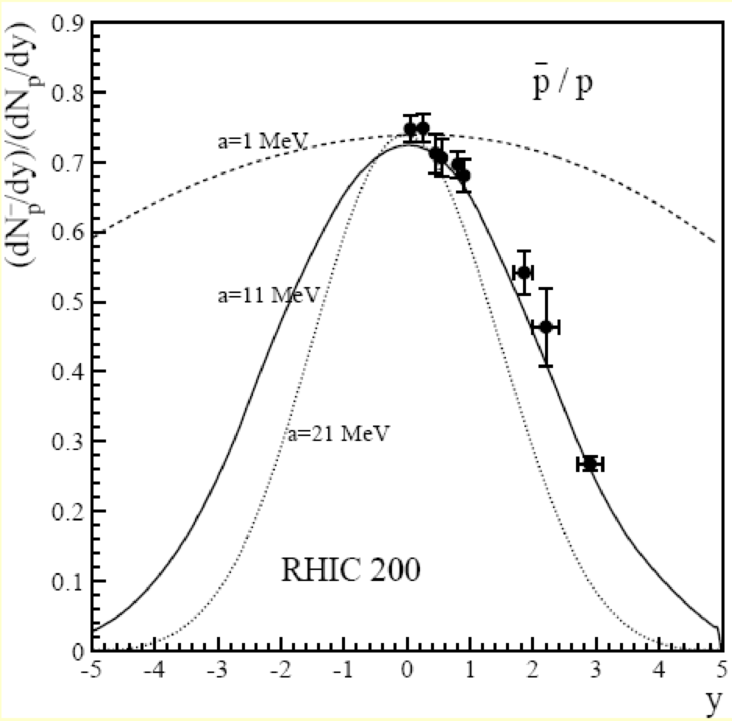
Statistical model seems to work even for pp or e+e⁻ reactions

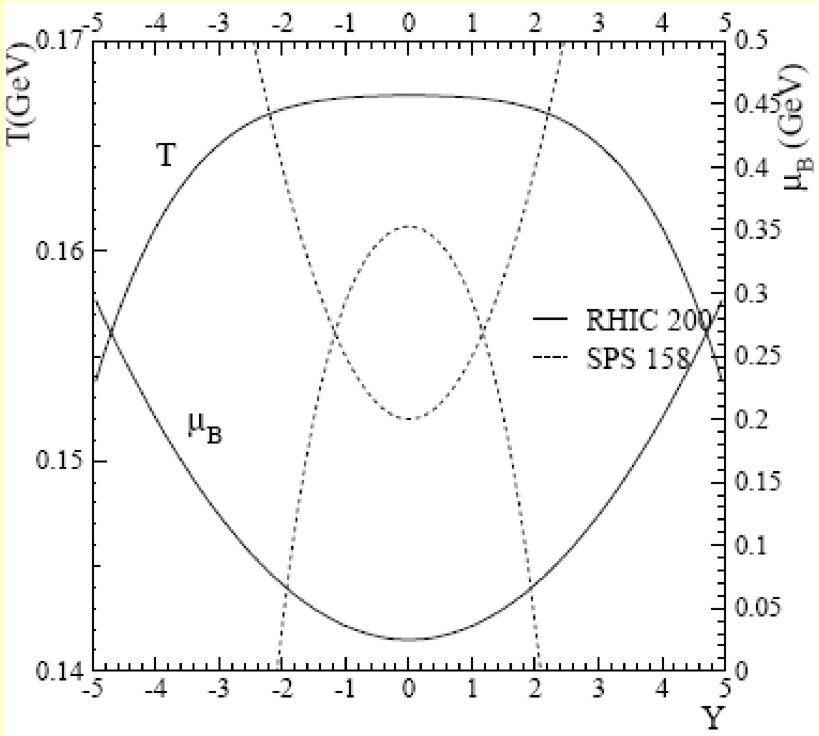
Are there any thermalized sources?

Rapidity distributions at RHIC



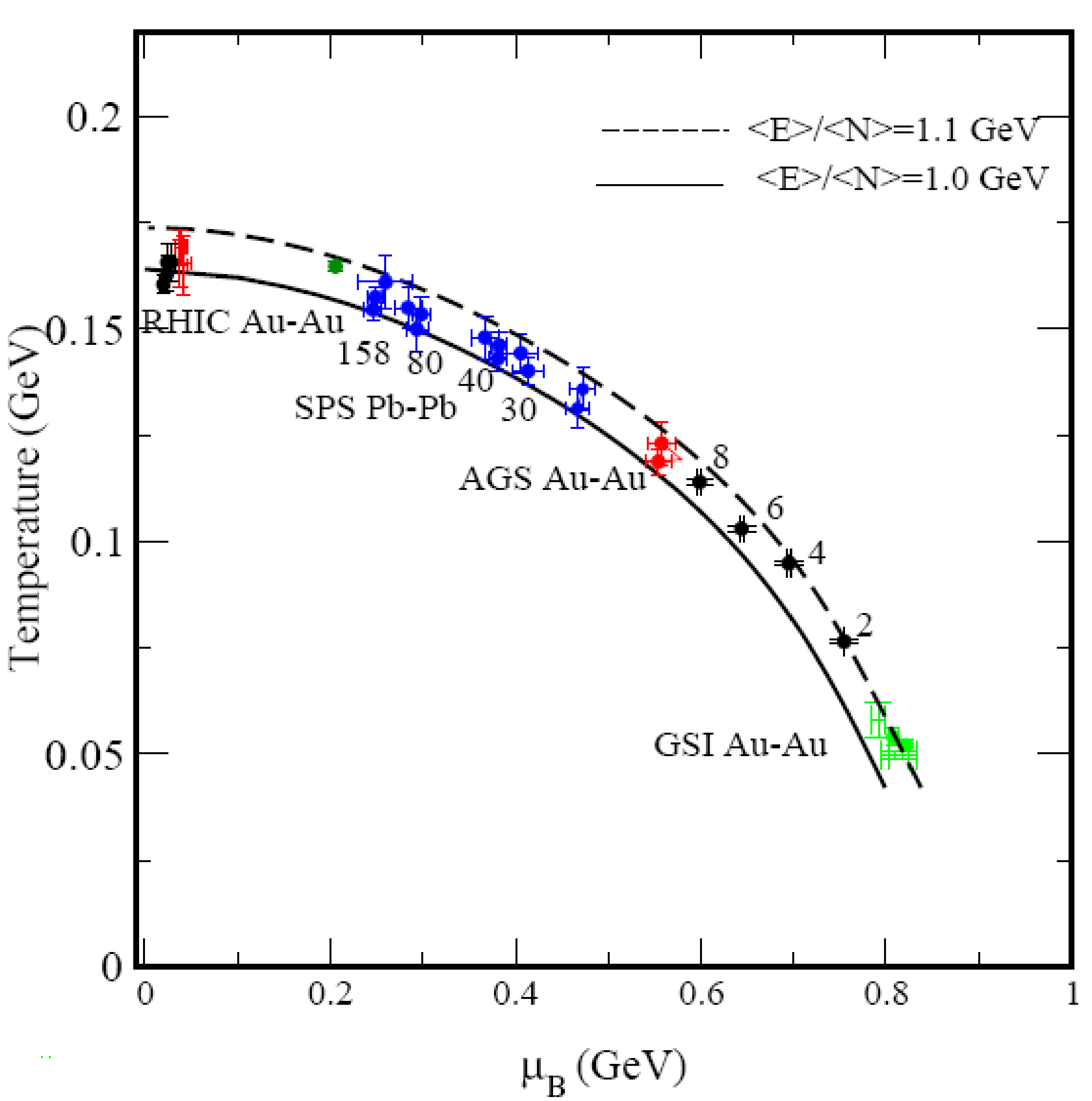
F. Becattini, J. Cleymans, hep-ph/0701029





The net baryon charge is not uniformly distributed

Energy-per-particle scaling



J. Cleymans, K. Redlich, PRL 81 (1998)5284

Again, even pp or e^+e^- reactions are in line with this universality

One can make predictions for particle ratios at LHC

... and predictions for Pb+Pb at LHC

I. Kraus et al, J. Phys. G 32 (2006) S495

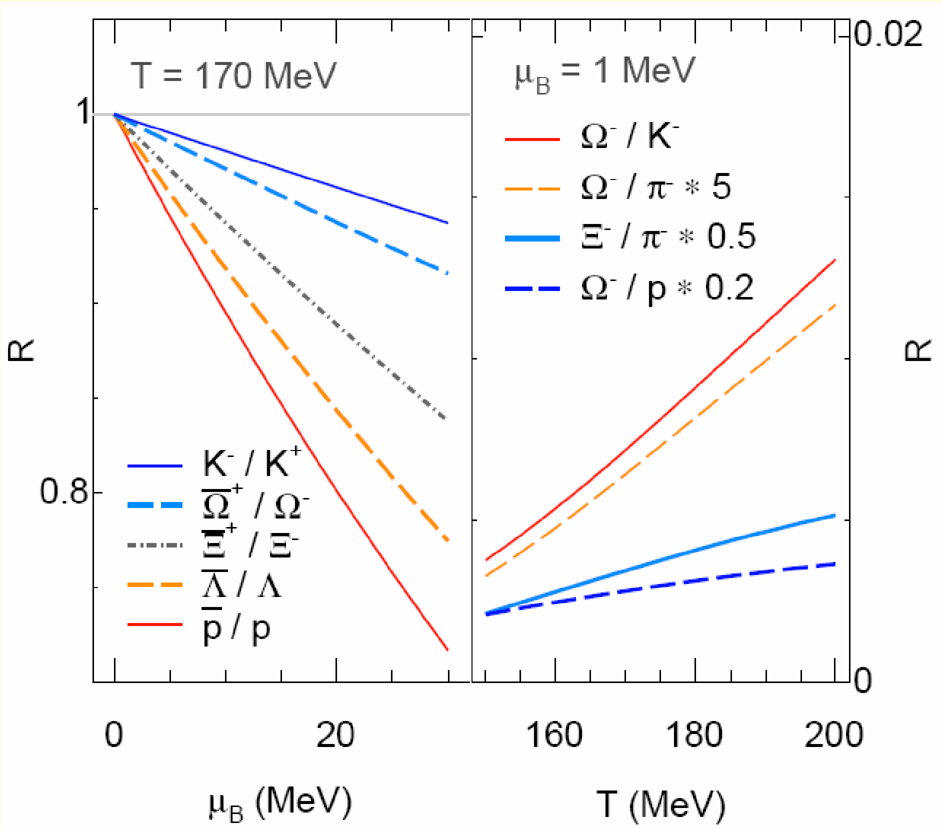
h/h Ratio		mixed Ratio	
π^+/π^-	$0.9998^{+0.0002}_{-0.0010}$	K^+/π^+	$0.180^{+0.001}_{-0.001}$
K^+/K^-	$1.002^{+0.008}_{-0.002}$	K^-/π^-	$0.179^{+0.001}_{-0.001}$
$\bar{p}/p$	$0.989^{+0.011}_{-0.045}$	p/π^-	$0.091^{+0.009}_{-0.007}$
$\bar{\Lambda}/\Lambda$	$0.992^{+0.009}_{-0.036}$	Λ/p	$0.473^{+0.004}_{-0.006}$
$\bar{\Xi}^+/\Xi^-$	$0.994^{+0.006}_{-0.026}$	Ξ^-/Λ	$0.160^{+0.002}_{-0.003}$
$\bar{\Omega}^+/\Omega^-$	$0.997^{+0.003}_{-0.015}$	Ω^-/Ξ^-	$0.186^{+0.008}_{-0.009}$

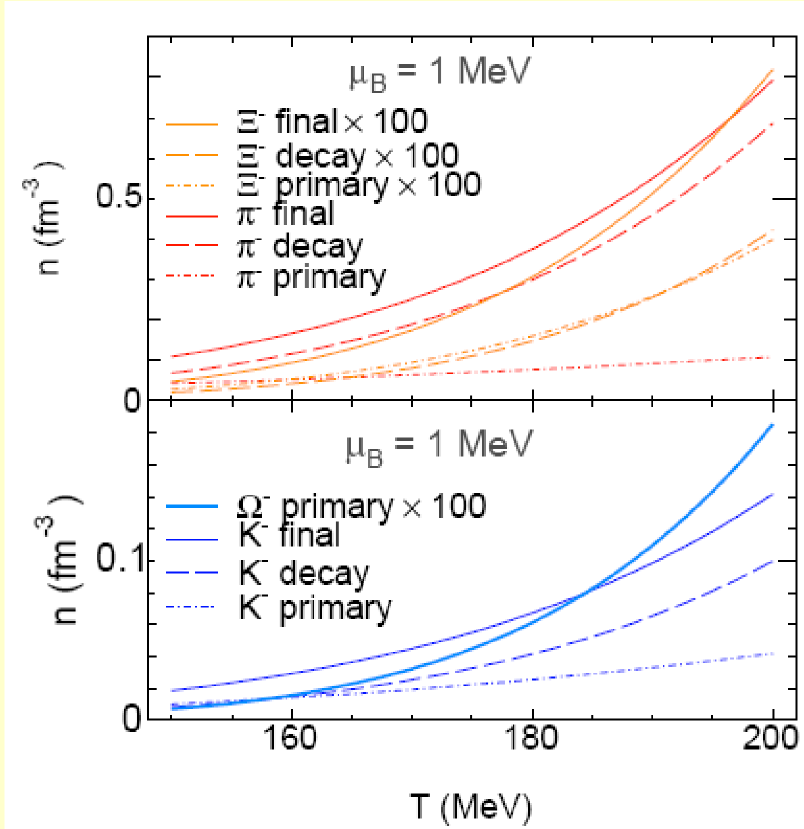
Table 1: Particle ratios in central Pb-Pb collisions at freeze-out conditions expected at the LHC: $T = (170\pm5)$ MeV and $\mu_B = 1^{+4}_{-1}$ MeV. The given errors correspond to the variation in the thermal parameters. Additional, systematic uncertainies in the ratios of the right column arise from unknown decay modes. They are smaller than 1% in general, but reach 3% in the Ξ^-/Λ ratio and 7% in the p/π^- and the Λ/p ratios.

Main feature: particle-antiparticle symmetry

Sensitivity to the main parameters

I. Kraus et al, J. Phys. G 32 (2006) S495

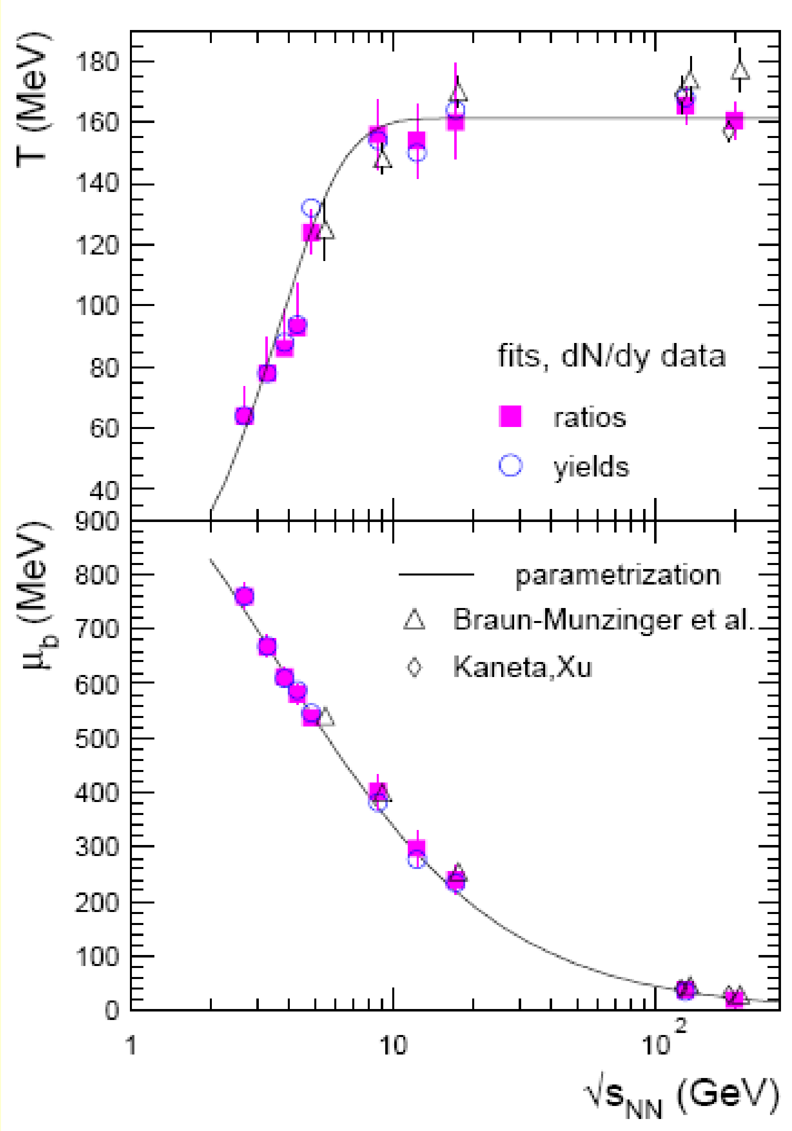




Results are very sensitive to both T and m

Another extrapolation

A. Andronic, P. Braun-Munzinger, J. Stachel, NPA 772 (2006) 167



$$T[\mathrm{MeV}] = T_{\mathrm{lim}} \left(1 - \frac{1}{0.7 + (\exp(\sqrt{s_{NN}}(\mathrm{GeV})) - 2.9)/1.5} \right)$$

$$\mu_b[\mathrm{MeV}] = \frac{a}{1 + b\sqrt{s_{NN}}(\mathrm{GeV})}$$

$$a = 1303 \pm 120\,\mathrm{MeV}$$

$$b = 0.286 \pm 0.049\,\mathrm{GeV}^{-1}$$

and predictions for LHC:

Comparison of predictions for Pb+Pb at LHC

(T=161 MeV, μ_b=0.84 MeV)

A. Andronic et al., NPA 772 (2006) 167

π^-/π^+	K^-/K^+	$\bar{p}/p$	$\bar{\Lambda}/\Lambda$	$\bar{\Xi}/\Xi$	$\bar{\Omega}/\Omega$
1.00	0.99	0.95	1.00	1.00	1.00
p/π^+	K^+/π^+	K^-/π^-	Λ/π^-	Ξ^-/π^-	Ω^-/π^-
0.074	0.180	0.179	0.039	0.0058	0.00106
ϕ/K^-	K^{*0}/K^0_S	Δ^{++}/p	$\Sigma(1385)^+/\Lambda$	Λ^*/Λ	$\Xi(1530)^0/\Xi^-$
0.136	0.312	0.216	0.140	0.075	0.396

I. Kraus et al, J. Phys. G 32 (2006) S495

h/h Ratio		mixed Ratio	
π^+/π^-	$0.9998^{+0.0002}_{-0.0010}$	K^+/π^+	$0.180^{+0.001}_{-0.001}$
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$\bar{\Omega}^+/\Omega^-$	$0.997^{+0.003}_{-0.015}$	Ω^-/Ξ^-	$0.186^{+0.008}_{-0.009}$

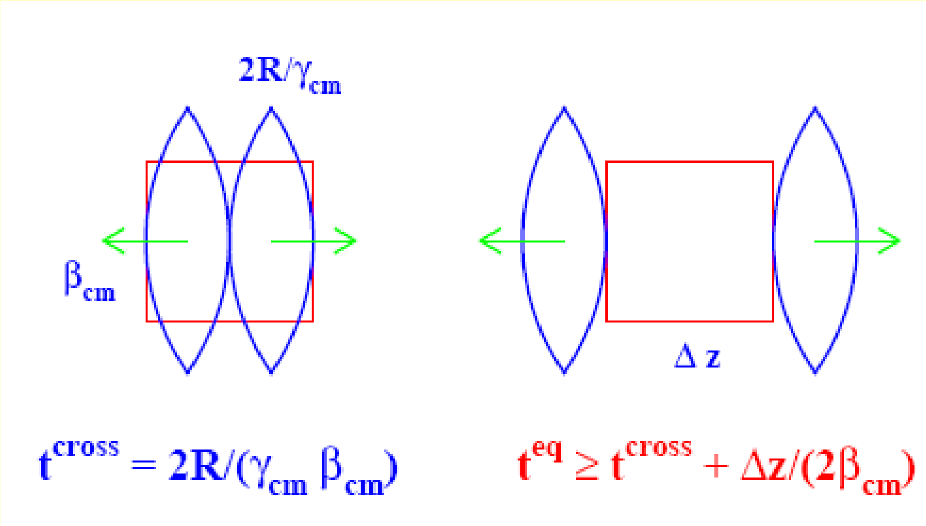
10%

T = 170 MeV, m = 1 MeV

Microscopic model:

Relaxation to equilibrium

Equilibration in the Central Cell



Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equlibrium:

Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

$$N_i = \frac{V g_i}{2\pi^2\hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values **output values**

$$\varepsilon^{\rm mic} = \frac{1}{V}\sum_i E_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm B}^{\rm mic} = \frac{1}{V}\sum_i B_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm S}^{\rm mic} = \frac{1}{V}\sum_i S_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}).$$

Multiplicity

Energy

Pressure

Entropy density

$$N_i^{\rm SM} = \frac{Vg_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p,m_i)dp,$$

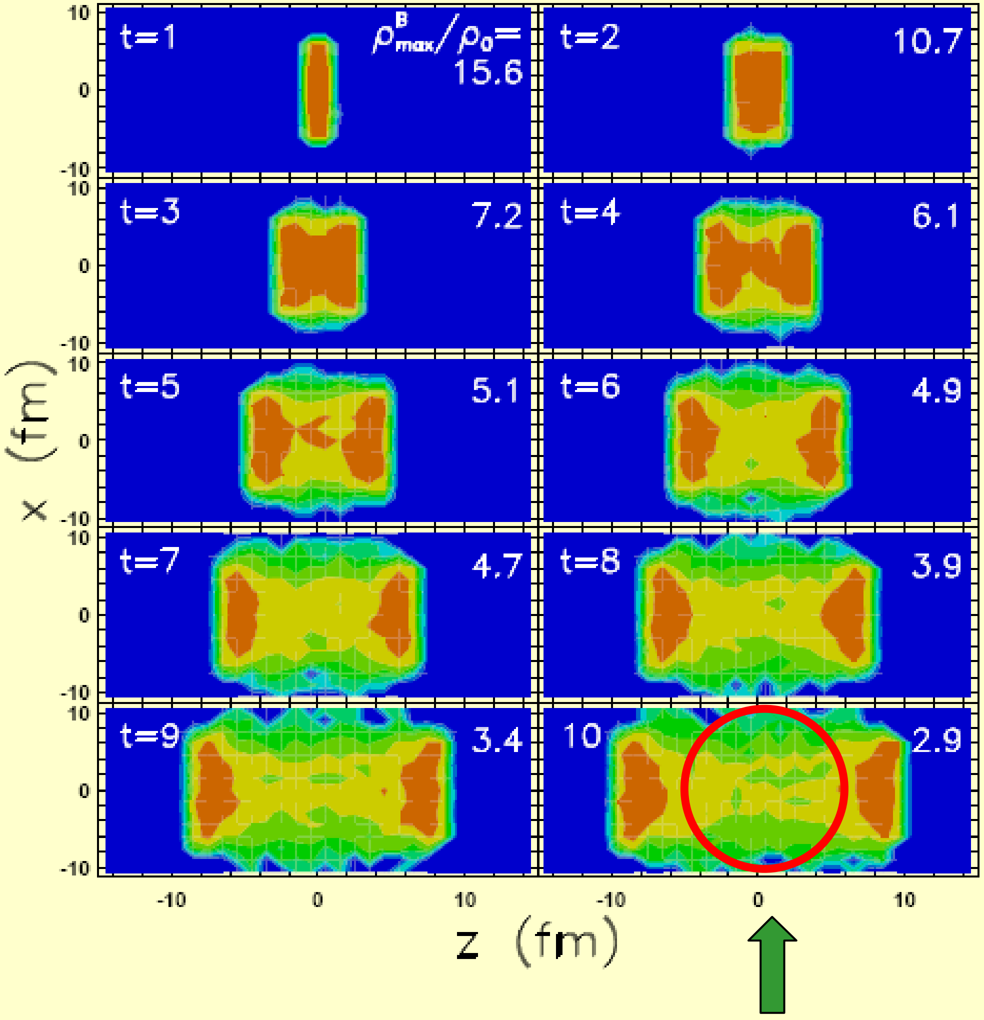
$$E_i^{\rm SM} = \frac{Vg_i}{2\pi^2\hbar^3}\int_0^\infty p^2\sqrt{p^2+m_i^2}\, f(p,m_i)dp$$

$$P^{\rm SM} = \sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \frac{p^2}{3(p^2+m_i^2)^{1/2}}\, f(p,m_i)dp$$

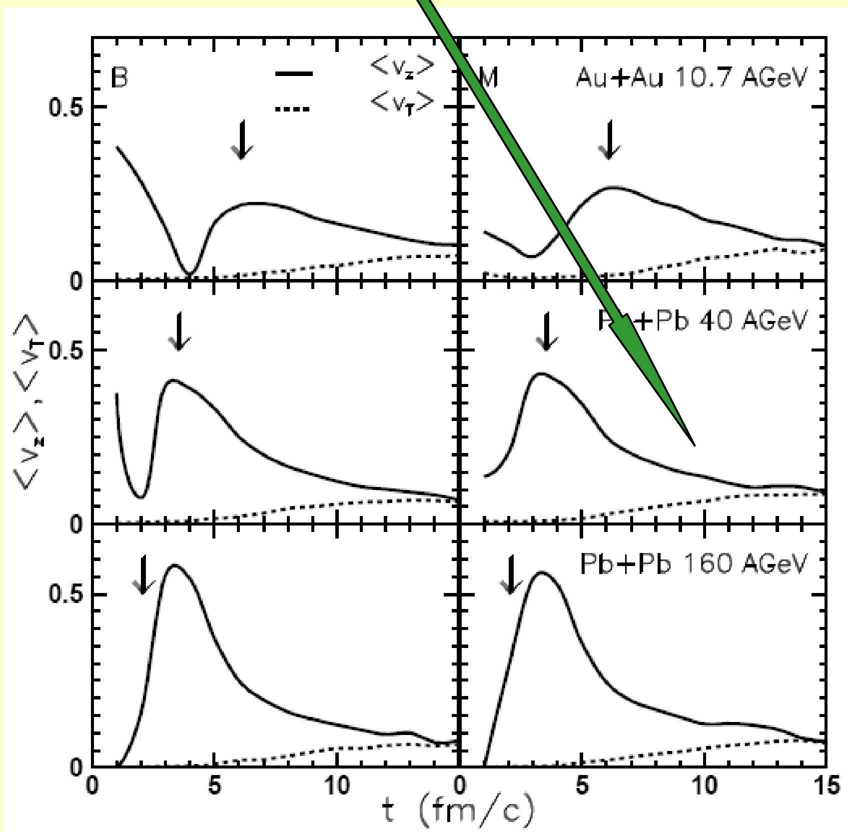
$$s^{\rm SM} = -\sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p,m_i)\,\left[\ln f(p,m_i)-1\right]\, p^2 dp$$

Pre-equilibrium Stage

Homogeneity of baryon matter



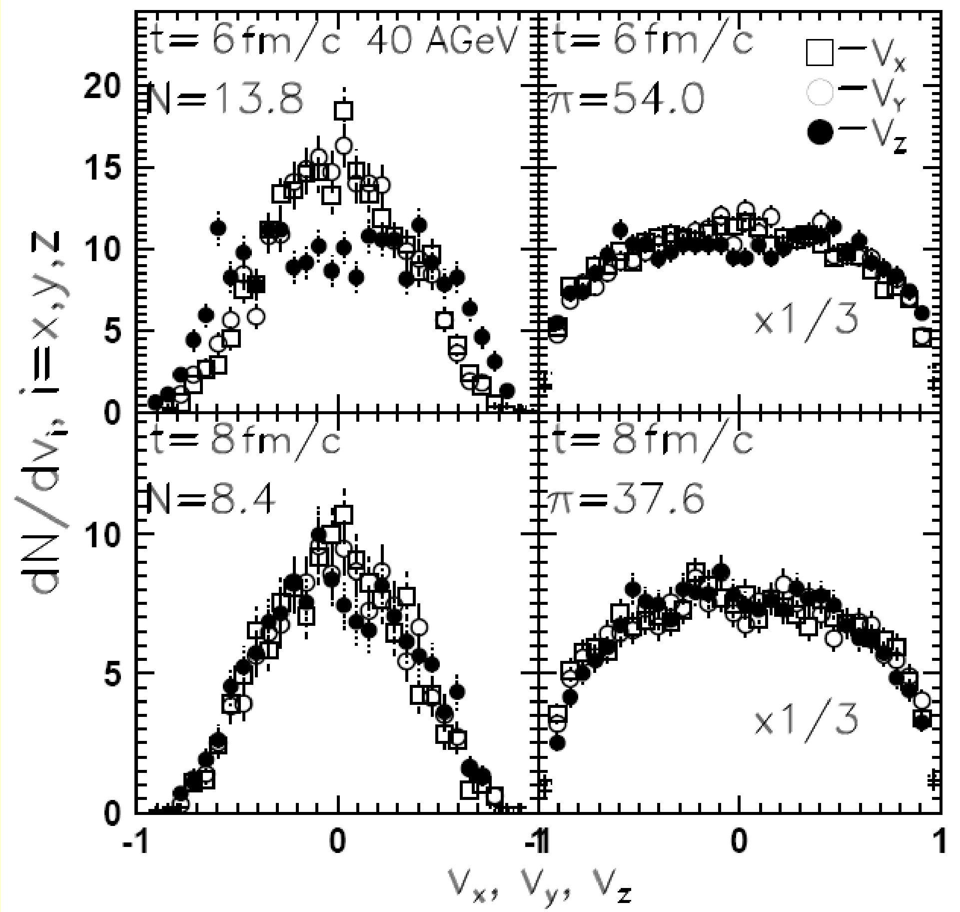
Absence of flow



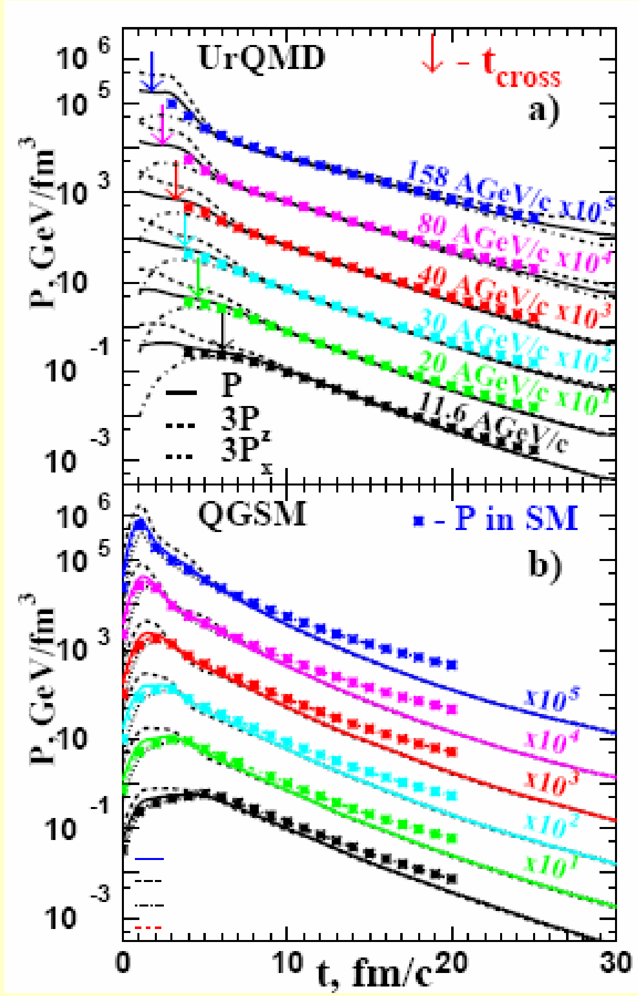
The local equilibrium in the central zone is quite possible

Kinetic Equilibrium

Isotropy of velocity distributions



Isotropy of pressure

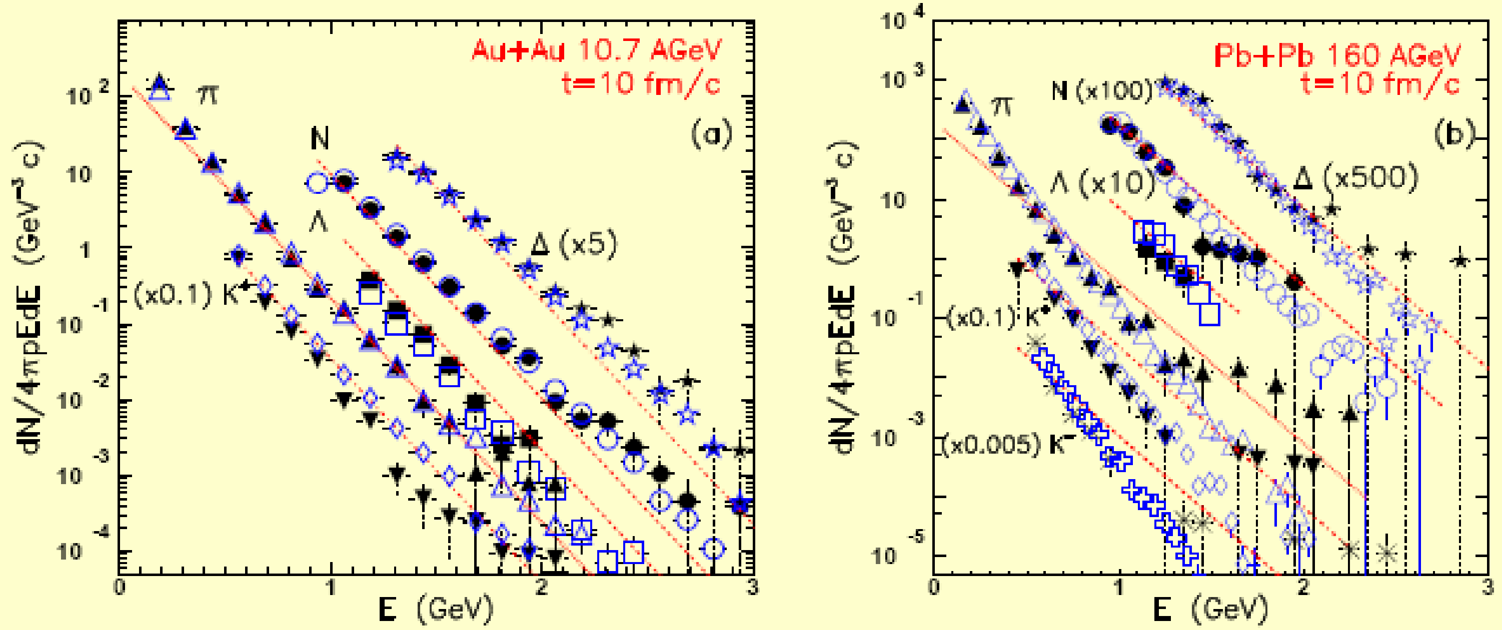


Velocity distributions and pressure become isotropic at t=9 fm/c (for 40 AGeV)

Thermal and Chemical Equilibrium

energy spectra of particles are described by Boltzmann distribution with temperature $\mathbf{T}$ and chemical potential $\mu_i = \mathbf{B_i}\mu_\mathbf{B} + \mathbf{S_i}\mu_\mathbf{S}$:

$$\frac{\mathbf{dN_i}}{\mathbf{4\pi pEdE}} = \frac{\mathbf{Vg_i}}{(\mathbf{2\pi\hbar})^\mathbf{3}} \exp\left(\frac{\mu_\mathbf{i}}{\mathbf{T}}\right) \exp\left(-\frac{\mathbf{E_i}}{\mathbf{T}}\right)$$



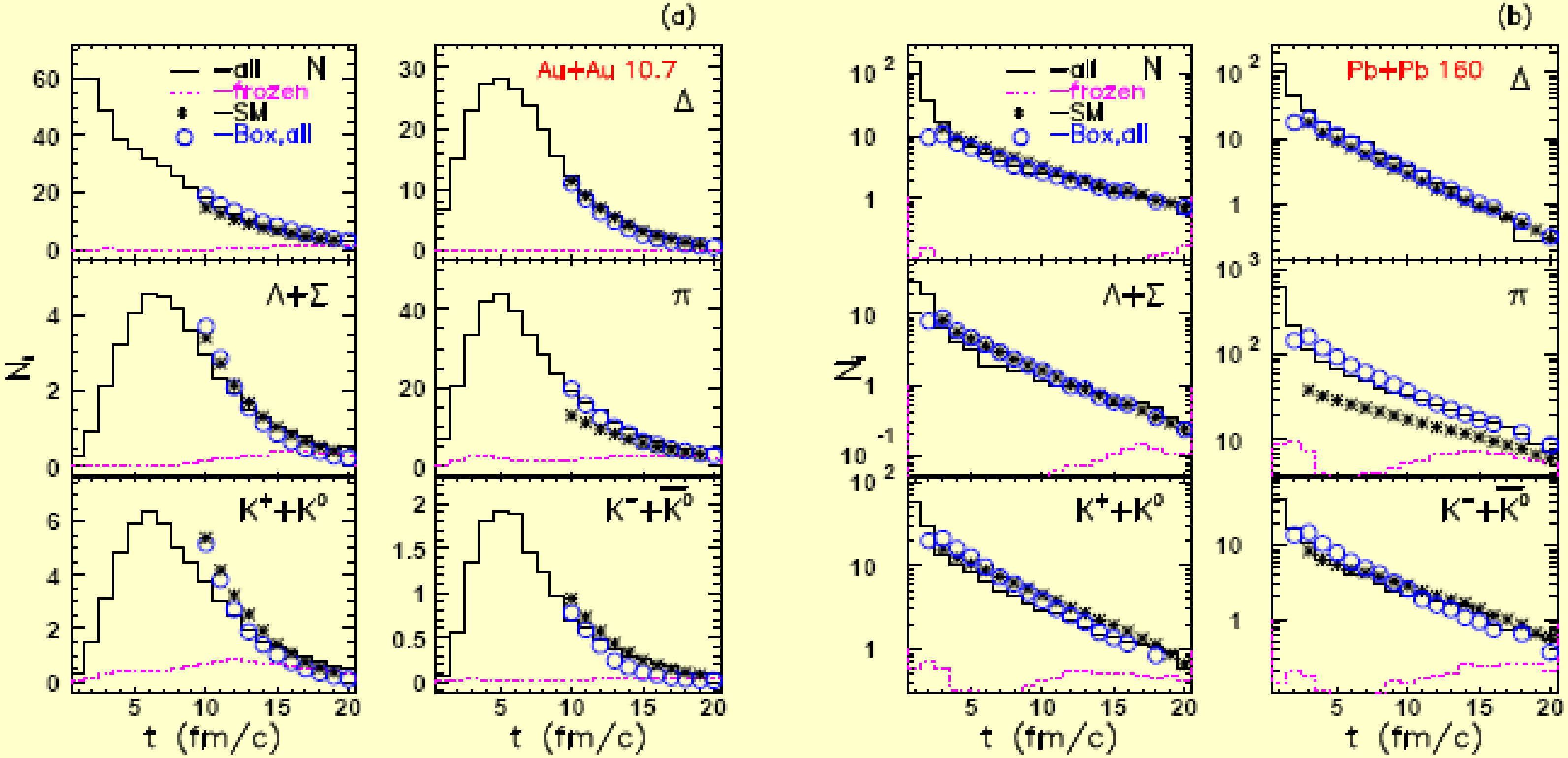
Energy spectra of hadrons in the cell and box compared with the SM results

Thermal equilibrium seems to be reached

Thermal and Chemical Equilibrium

the yields of particles are reproduced by the SM with the same parameters $(\mathbf{T},\ \mu_\mathbf{B},\ \mu_\mathbf{S})$:

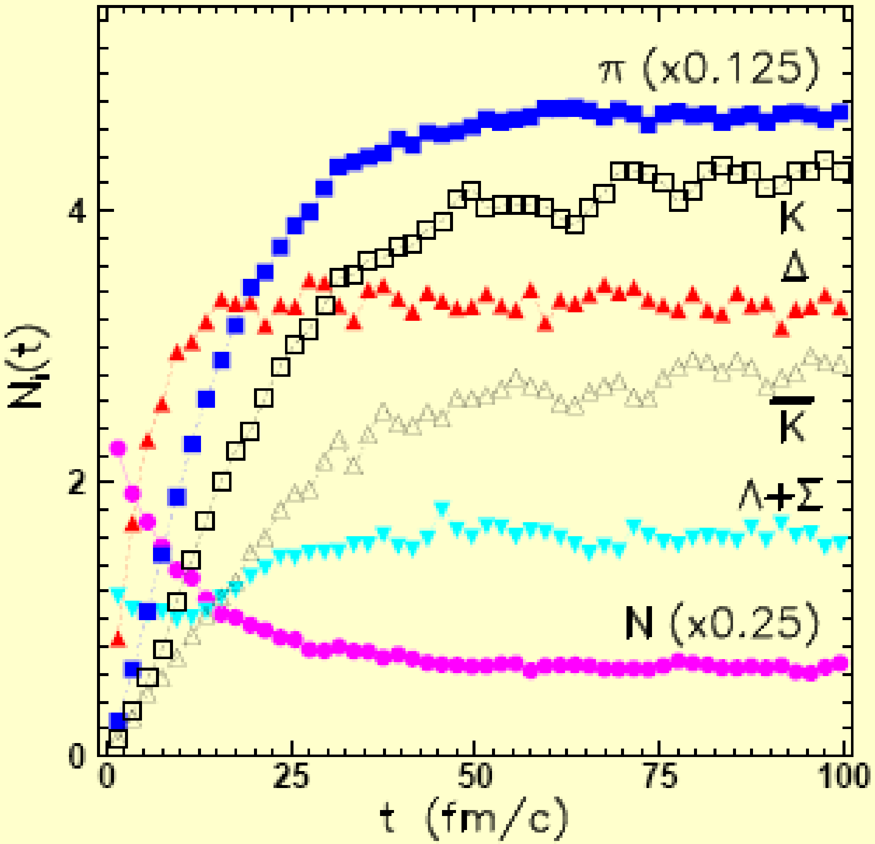
$$\mathbf{N_i} = \frac{\mathbf{V g_i}}{\mathbf{2\pi^2 \hbar^3}} \int_0^\infty \mathbf{p^2 dp} \exp\left(\frac{\mu_\mathbf{i}}{\mathbf{T}}\right) \exp\left(-\frac{\mathbf{E_i}}{\mathbf{T}}\right)$$

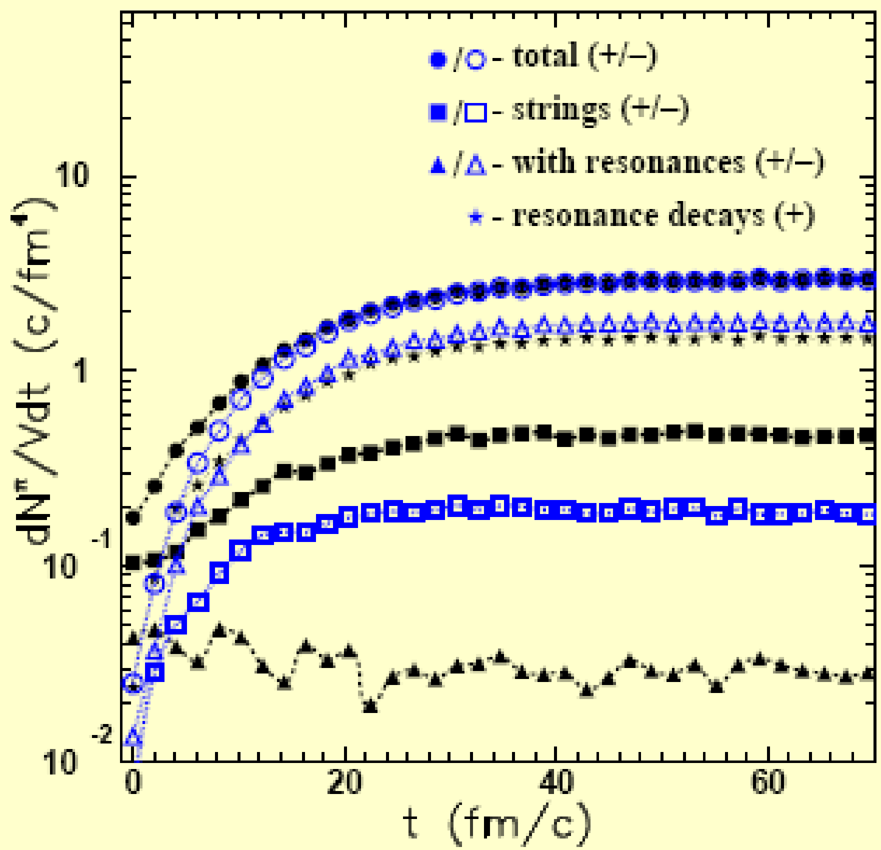


Yields of hadrons in the cell and box compared with the SM results

Chemical equilibrium seems to be reached

M. Belkacem et al., Phys. Rev. C 58 (1998) 1727
L. Bravina, E.Z., et al., Phys. Rev. C 62 (2000) 064906
E. Bratkovskaya et al., Nucl. Phys. A675 (2000) 661

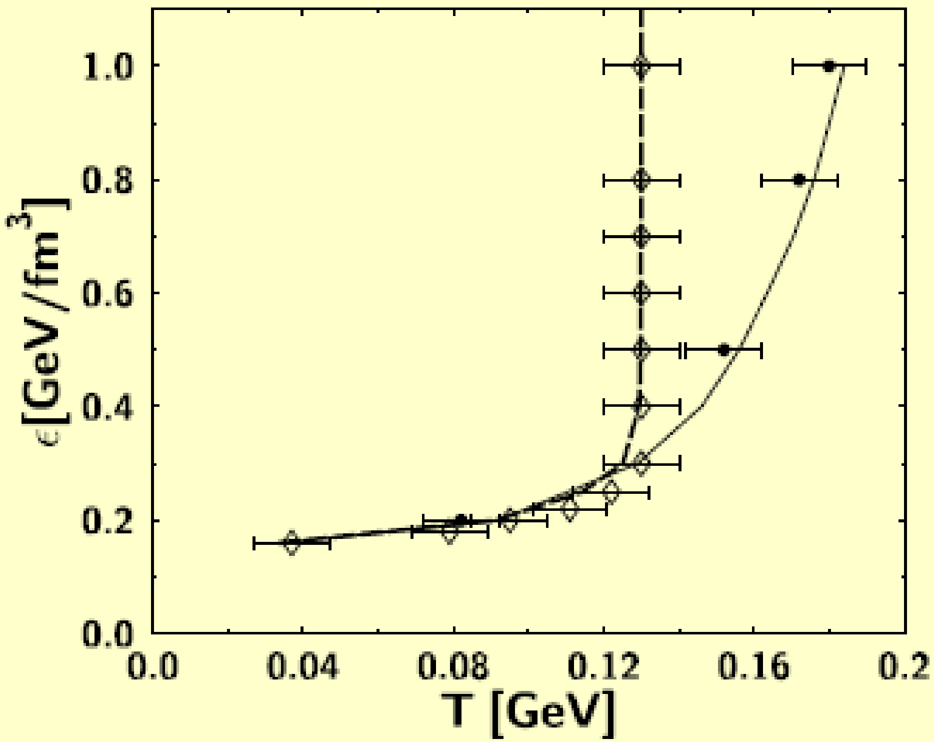




Hadron yields in the box with $V = 125\,\mathrm{fm}^3$, $\varepsilon = 468\,\mathrm{MeV/fm}^3$, $\rho_B = 0.0924\,\mathrm{fm}^{-3}$, and $\rho_S = -0.00987\,\mathrm{fm}^{-3}$.

Production (full symbols) and absorption (open symbols) rates for pions in the box

Hadronic matter in the box reaches steady-state instead of thermal and chemical equilibrium



UrQMD box calculations with (open diamonds) and without (solid circles) strings and many-body $(n > 2)$ decays. Solid line - SM, dashed line - SM with continuous mass spectrum. The calculations are done at $\rho_B = 0.16\,\mathrm{fm}^{-3}$ and $\rho_S = 0\,\mathrm{fm}^{-3}$.

The EOS appears to be Hagedorn-like, with the limiting temperature $T_H \approx 130 \pm 15\,\mathrm{MeV}$. At high densities the UrQMD agrees well with the SM if strings and other many-body decays are switched off.

The time-reversal symmetry is broken for the processes

$$2 \longrightarrow N\,, \quad N \geq 3$$

and this circumstance drives system out of equilibrium.

Summary

- *Statistical description of HIC's works very good in a broad energy range. However, several issues should be clarified.*
- *How many sources do we have? What are their conditions at freeze-out?*
- *Why even elementary reactions obey the statistical description?*
- *Full list of parameters of a particular model should be presented*
- *Reversibility of multi-particle reactions*