The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA
International Atomic Energy Agency

SMR/1842-23

International Workshop on QCD at Cosmic Energies III

28 May - 1 June, 2007

Lecture Notes

T. Rogers

The Pennsylvania State University
Eberly College of Science
University Park, USA

Fully Unintegrated Parton Correlation Functions

Ted Conant Rogers
The Pennsylvania State University

(In collaboration with J.C. Collins and A.M. Staśto.)

QCD at Cosmic Energies 3,
Trieste, Italy 2007

May 29, 2007

Overview

- Looking at details of final state interactions requires precise kinematics.

  (Already noted by, e.g., Watt, Martin, and Ryskin Eur.Phys.J. C31,73 (2003))

- Exact kinematics forces us to consider PCFs (non-perturbative objects) in both the initial and final states.

  (fully unintegrated PDFs, soft factor, jet factors)

- Without usual approx., standard methods for disentangling soft/collinear gluons do not work.

- Problems even at lowest order.

Relevance to Cosmic Rays

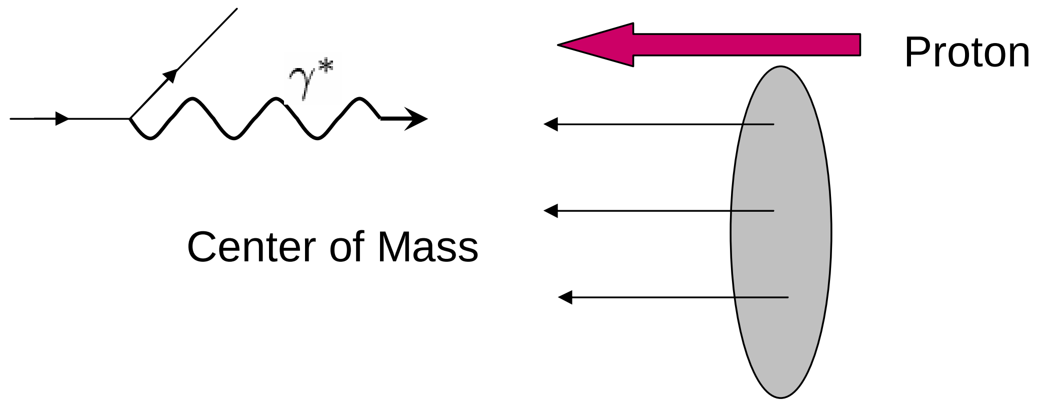
- Standard approximations used in event generators.

  (Pythia, Herwig, DPMJET, QGSJET, etc…)

- How does extrapolation of cross sections to higher energies work?

Leading Order DIS:

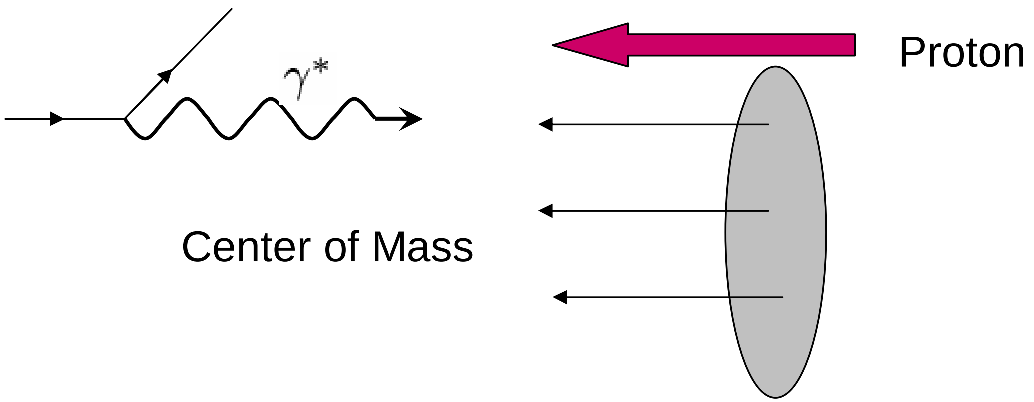
Conventional Intuition

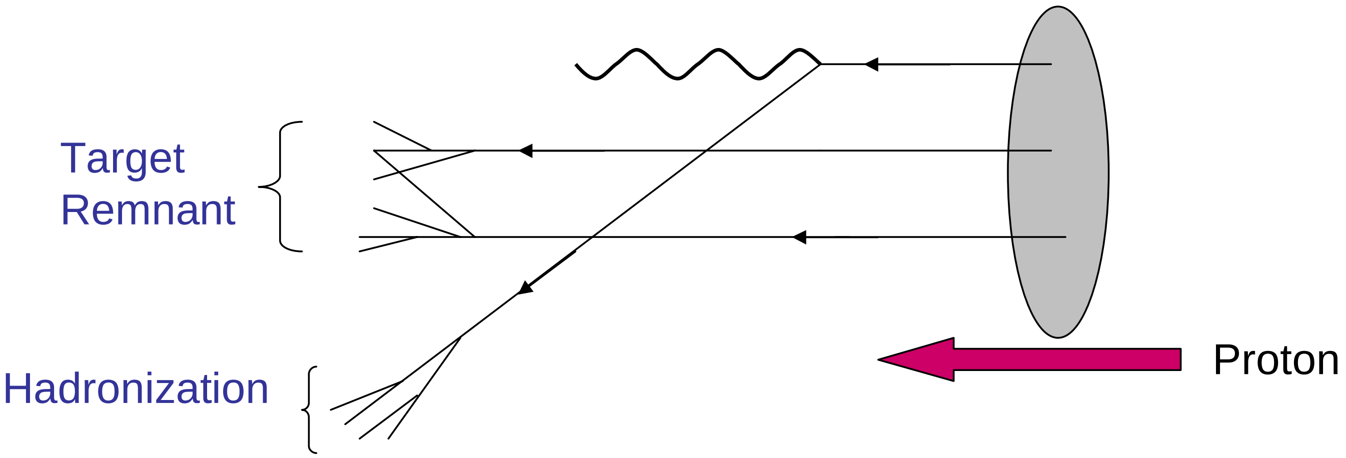


The Parton Model

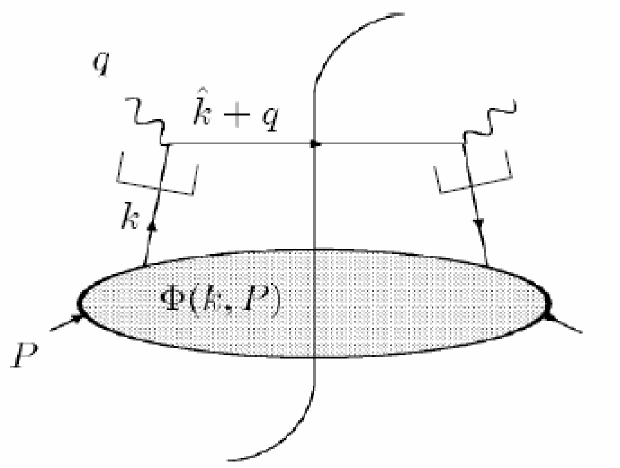
Leading Order DIS:

Conventional Intuitive





Conventional Diagrammatic Formalism



Parton Model / Handbag Diagram

$$F_{T/L} = \sum_j H_j(\hat{k},q) f_{j/h}(x_{\rm Bj})$$

$$\hat{k} \equiv (x_{\rm Bj} P^+, 0, \mathbf{0}_T)$$

$$H_j(\hat{k},q) = \frac{e_j^2 P_{\mu\nu}^{T/L}}{2} {\rm Tr}\left((\hat{\not k} + \not q)\gamma^\mu \hat{\not k} \gamma^\nu \right)$$

On-Shell Partonic Structure Function

Cannot be the complete picture even at LO...

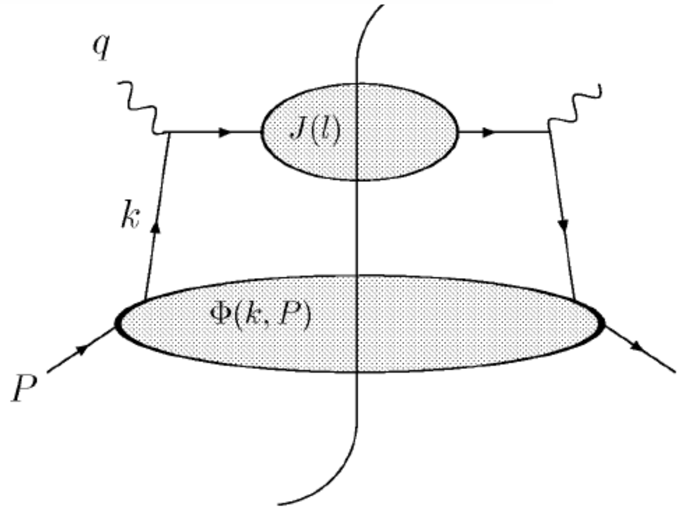
Struck Quark Must Hadronize

(at least…)

$$W^{\mu\nu}(q,P) \;=\; \sum_j \frac{e_j^2}{4\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\gamma^\mu J_j(k+q)\gamma^\nu \Phi_j(k,P)\right)$$

- Massless, Collinear approx:

$$k^+ = xP^+ + \frac{M_J^2 + k_T^2}{2(k^- + q^-)} \;\rightarrow\; xP^+$$



Steps to Reproduce Parton Model:

- Substitute hatted variables in hard scattering (e.m. vertex).

- For performing integrals, make a substitution in the bubbles:

$$k \longrightarrow (x_{\rm Bj}P^+, k^-, \mathbf{k}_T)$$

$$l \longrightarrow \left(l^+, \frac{Q^2}{2x_{\rm Bj}P^+}, \mathbf{0}_T\right)$$

- Integrate over small components:

$$W^{\mu\nu}(q,P) \simeq \frac{e_j^2}{4\pi\hat{k}^+}\underbrace{\left\{\int \frac{dk^- d^2\mathbf{k}_T}{(2\pi)^4}\Phi_j^+(x_{\rm Bj}P^+, k^-, \mathbf{k}_T)\right\}}_{\text{Parton Distribution???}}\underbrace{\mathrm{Tr}\left(\gamma^\mu\gamma^+\gamma^\nu\hat{\not{k}}\right)}_{\text{Reproduces Partonic LO structure functions.}}\underbrace{\left\{\int dl^+ J_j^-(l^+, q^-, \mathbf{0}_T)\right\}}_{\text{Set to one by unitarity argument}}$$

Parton Distribution???

UV divergent – requires renormalization.
Not gauge invariant.

Reproduces Partonic LO structure functions.

Compare final state P.S. before and after approx!

Set to one by unitarity argument

The Standard PDFs

Operator definition:

(Reproduces integral form up to c.t.)

$$f_j(x_{\rm Bj},\mu) = \int \frac{dy^-}{4\pi} e^{-ix_{\rm Bj}p^+y^-} \langle p|\bar{\psi}(0,y^-,\mathbf{0}_T)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|p\rangle_R$$

Light-like Wilson lines for gauge invariance:

$$V_y(n) = P\exp\left(ig_s\int_0^\infty d\lambda\, n\cdot A(y+\lambda n)\right)$$

$$n \equiv (0,1,\mathbf{0})$$ → ***Light-like!***

$$V_y^\dagger(n)V_0(n) = P\exp\left(ig_s\int_0^{y^-} d\lambda\, n\cdot A(\lambda n)\right)$$

Summary of LO Deeply Inelastic Scattering in the Conventional Treatment:

- There is a re-assignment of final state kinematics.

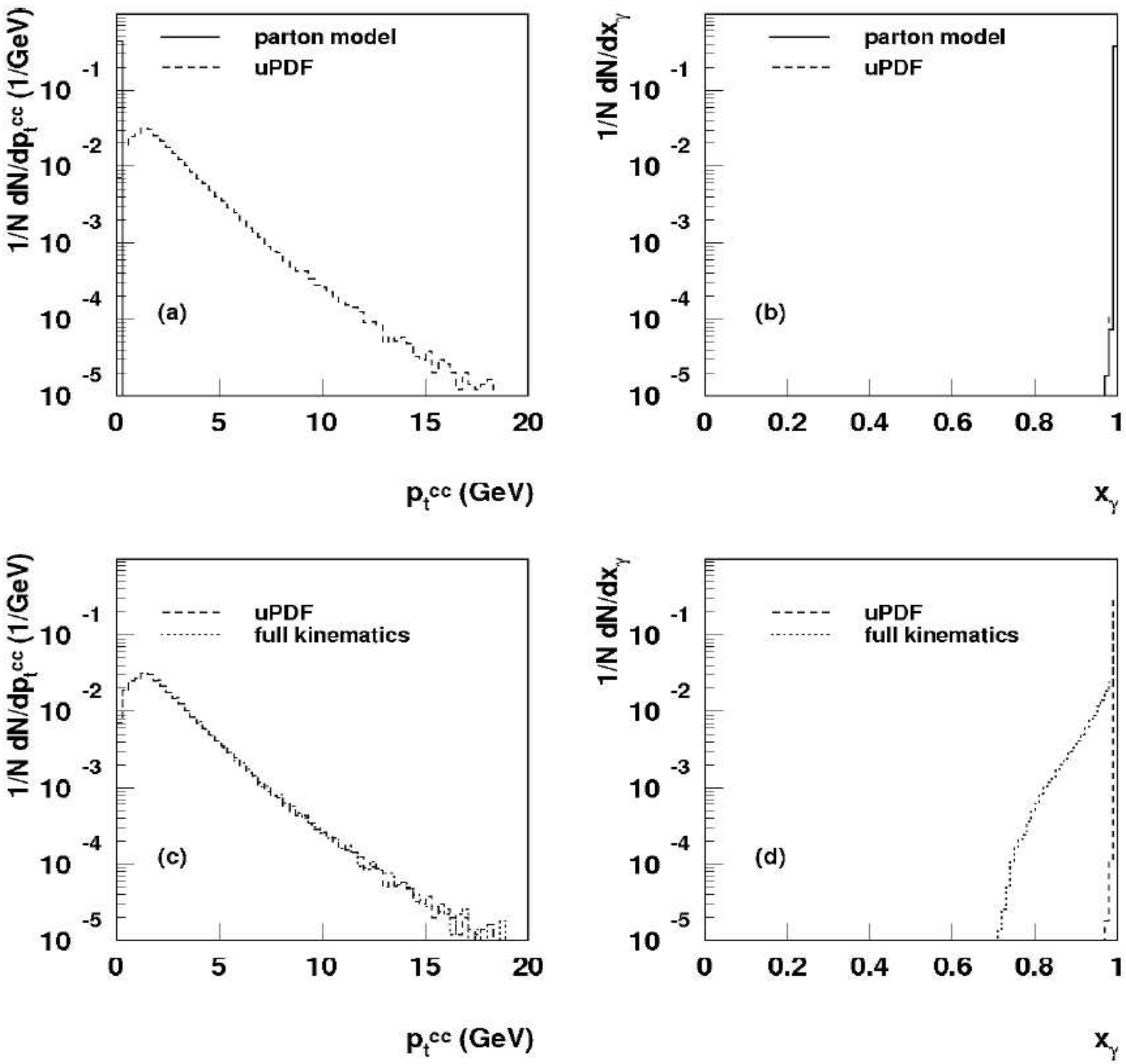
- Can be large.

- These kinematical approximations are necessary for reproduction of standard LO DIS expression (parton model).

Important Distinctions!

- Integrated PDFs:
  - Standard PDFs of classic LT factorization theorems.
- Unintegrated PDFs:
  - Depend on k_T, *but still integrated over invariant energy*.
- Parton Correlation Functions (Including *Fully Unintegrated* PDFs):
  - Differential in all components of four-momentum.

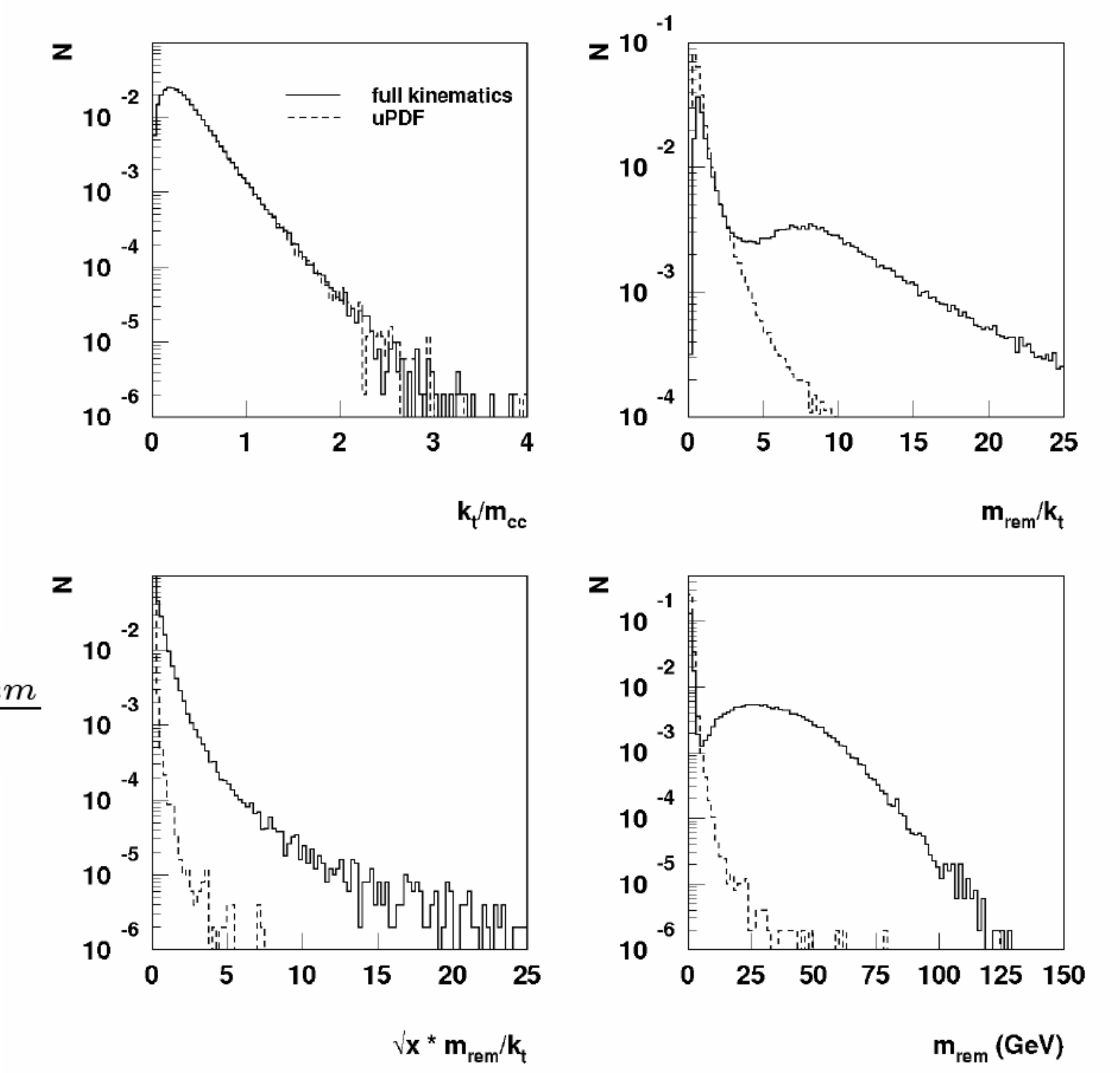
$c\bar{c}$
Pair-production
In DIS.



(Collins and Jung: hep-ph/0508280)

$c\bar{c}$
Pair-production

$$k^2 = -\frac{k_T^2 + x m_{rem}^2}{1-x}$$



What is Needed?

- Exact overall kinematics of initial and final states.
- Explicit factors representing final states.
- NP factors differential in all components of four-momentum.
- Hard scattering calculated with on-shell Feynman graphs.
- Factorization formula.
- Approximations should be consistent with gauge invariance! (Ward identities.)

Strategy Overview

- Define gauge invariant PCFs.
- Consider extra soft/collinear gluon attachments.
  - Characterize regions, R, of gluon momentum.
  - Apply *Consistent* Approximations.
  - Sum over graphs, Γ, apply Ward identities.
  - Obtain factorized form:

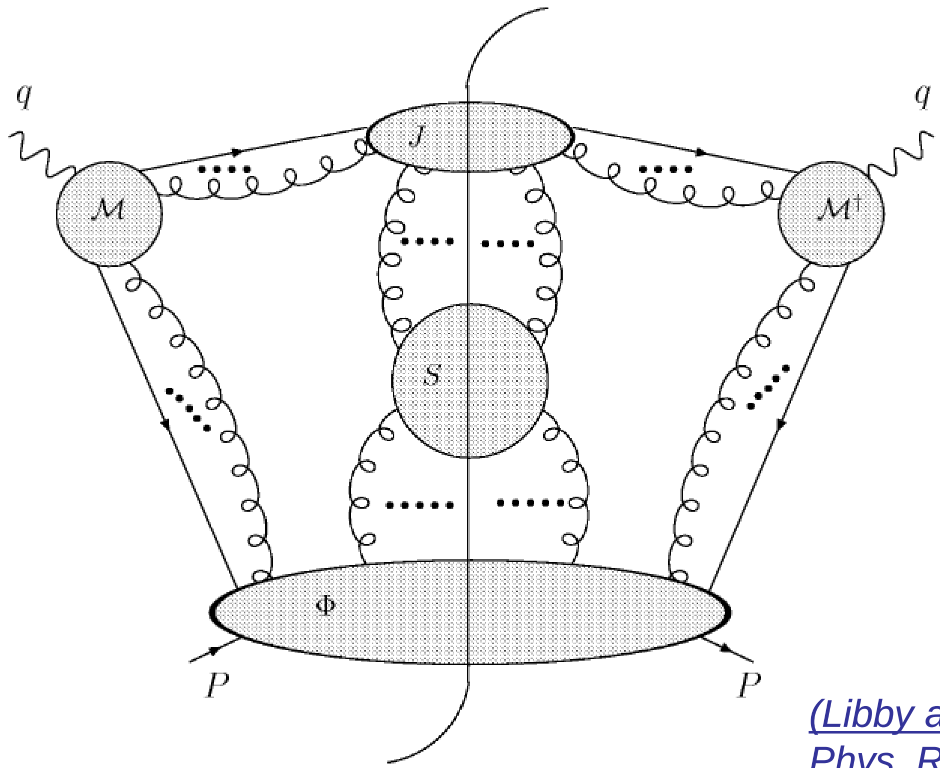
$$\sum_{R,\Gamma} C_R \Gamma$$ = factorized form + p.s. corrections

  - Identify contributions to PCFs.

- Reproduce, BFKL, DLGAP, etc… in appropriate kinematical limits.

Generalized Factorization:

Should start with:

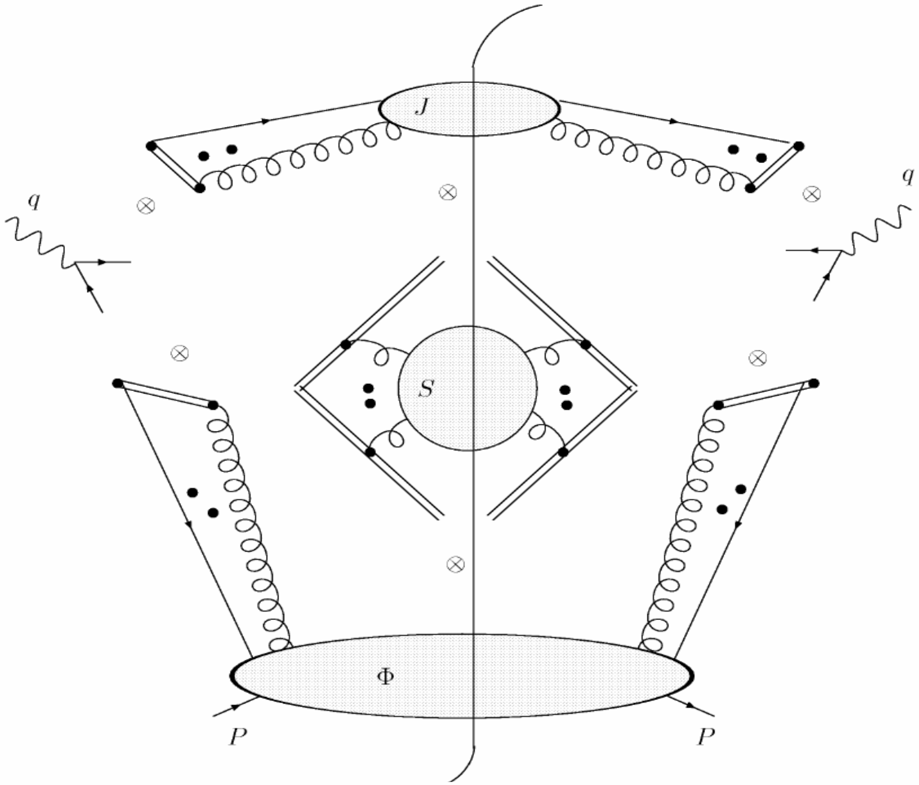


(Libby and Sterman, Phys. Rev. D18, 4337 (1978))

Must disentangle soft and collinear gluons to get…

Generalized Factorization:

At lowest order in hard scattering…



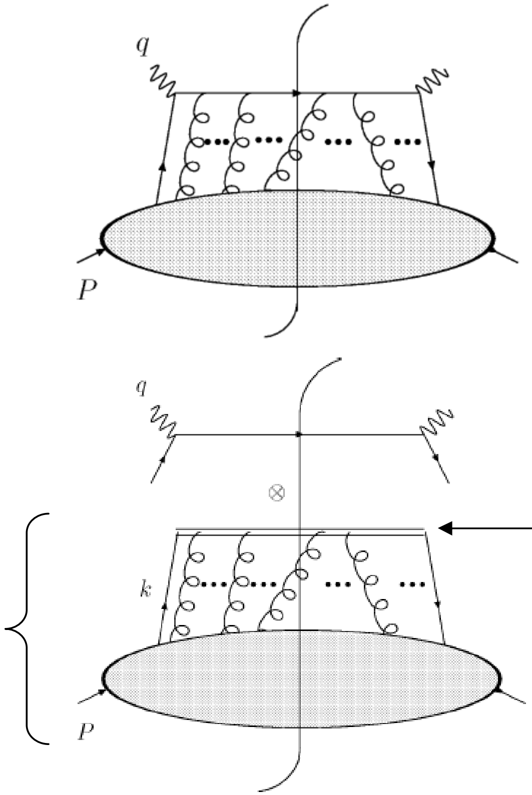
$$\sigma = \hat{H} \otimes F \otimes J \otimes S + \mathcal{O}((\Lambda/Q)^a|\sigma|), \qquad a > 0$$

Compare Classic DIS Factorization

Graphical structure in arb. gauge:

Factorization

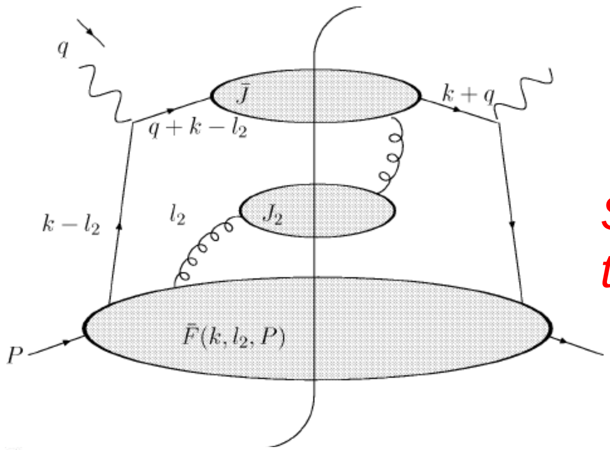
LO Feynman graph in DIS.



Eikonal line from gauge link.

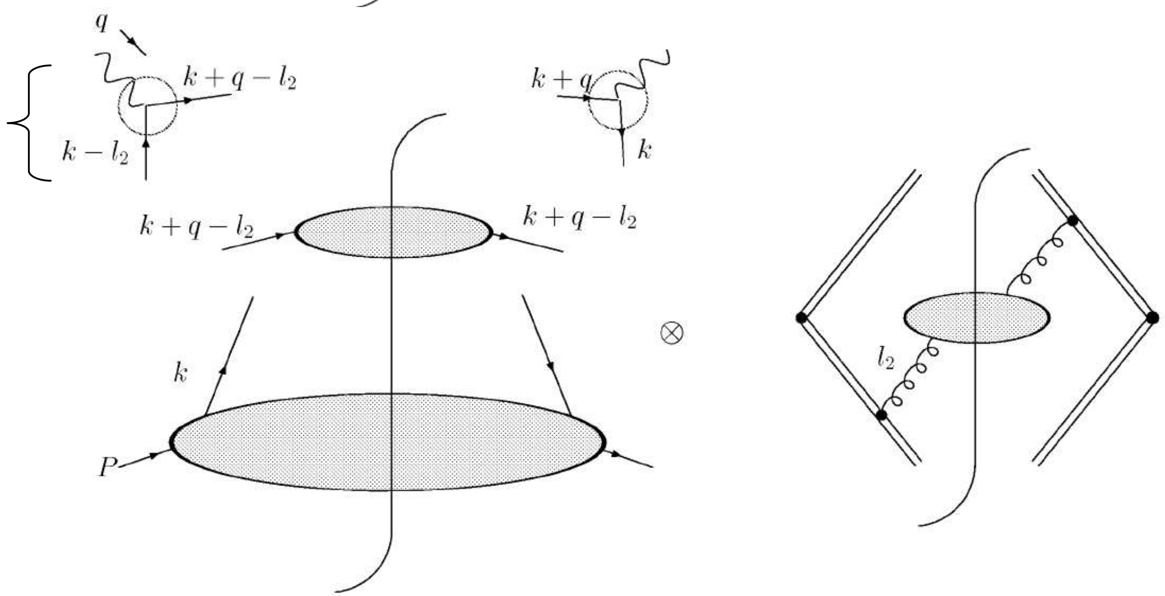
$$f_j(x_{\rm Bj},\mu) = \int \frac{d\,y^-}{4\pi} e^{-ix_{\rm Bj}p^+y^-} \langle p|\bar{\psi}(0,y^-,\mathbf{0}_T)V_y^\dagger(n)\gamma^+V_0(n)\psi(0)|p\rangle_R$$

Example: a single soft gluon:



Sum up diagrams with this topology.

Apply Parton model kinematics inside circles.

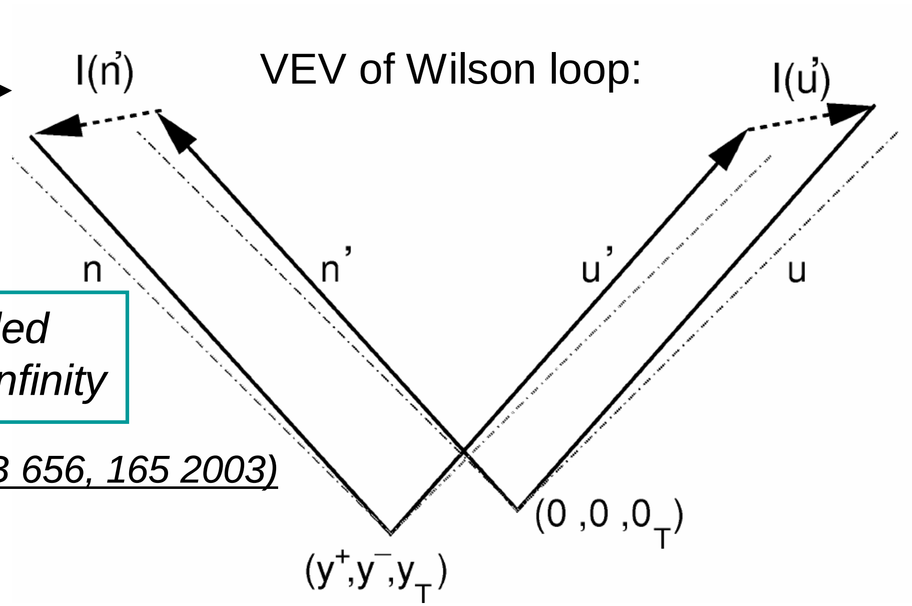


Definitions

Start with smallest region: *All components of parton momentum are small.*

Soft factor (coordinate space)

$$\tilde{S}(y,\eta_1,\eta_2,\mu) = \langle 0| I^{\dagger}_{u';y,0} V_y(u') V_y^{\dagger}(n') I_{n';y,0} V_0(n') V_0^{\dagger}(u') |0\rangle_R$$



Wilson line needed to link points at infinity

(Belitsky et. al Nucl. Phys.B 656, 165 2003)

Definitions

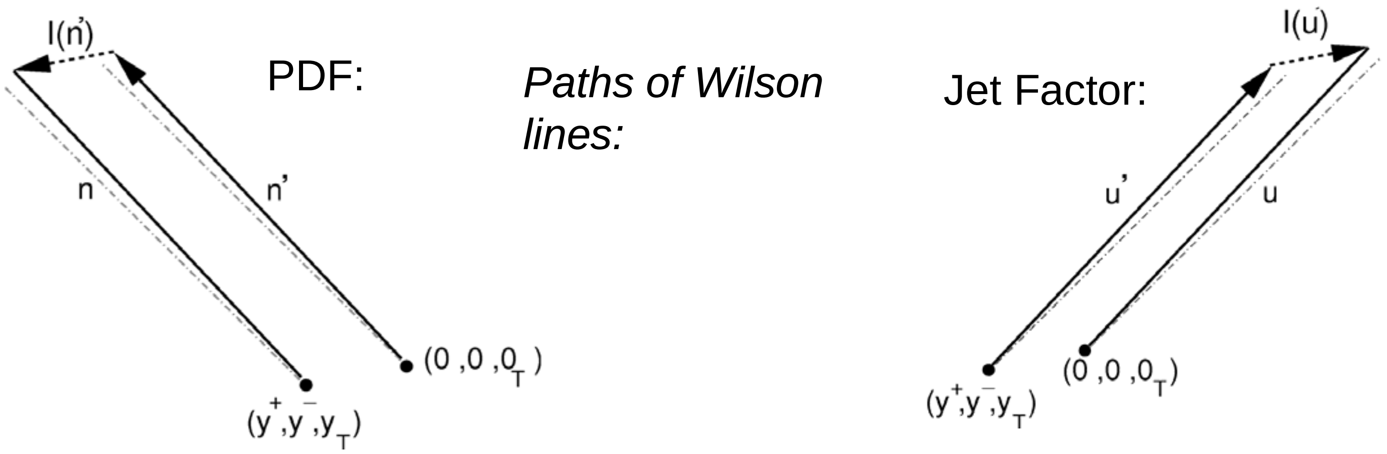
PDF:

$$\tilde{F}^{+}(y,\eta_1,\eta_2,\mu)=\frac{\langle p|\bar{\psi}(y)V_y^{\dagger}(n')I_{n';y,0}\gamma^{+}V_0(n')\psi(0)|p\rangle_R}{\langle 0|I^{\dagger}_{u';y,0}V_y(u')V_y^{\dagger}(n')I_{n';y,0}V_0(n')V_0^{\dagger}(u')|0\rangle_R}$$

Jet Factor:

$$\tilde{J}^{-}(y,\eta_1,\eta_2,\mu)=\frac{\langle 0|\psi(0)V_0(u')\gamma^{-}I_{u';y,0}V_y^{\dagger}(u)\bar{\psi}(y)|0\rangle_R}{\langle 0|I^{\dagger}_{u';y,0}V_y(u')V_y^{\dagger}(n')I_{n';y,0}V_0(n')V_0^{\dagger}(u')|0\rangle_R}$$

Paths of Wilson lines:



Notes on Ward Identities and Factorization

- The above arguments work for Abelian gauge theory.
- Ward identity relations need to be more explicit for non-Abelian case:
  - Classic factorization theorems rely on unitarity cancellations.
  - Ward identities not always exact – extra terms possibly violate factorization.

(See recent work of Collins and Qiu.)

Conclusions:

- Exact kinematics needed. (Unintegrated PDFs not enough.)
  (Basic program outlined for scalar theory by Collins and Zu (2005))

- Requires exact definitions for parton correlation functions.

- We have defined parton correlation functions and derived a factorization formula for the case of an Abelian gauge theory.
  (Strongly suggestive of a structure for the non-Abelian case.)

Outlook:

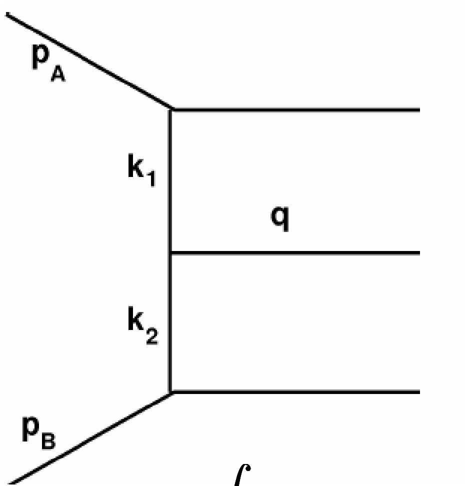
- Much work needed:
  - Full factorization theorem and implementation at NLO in non-Abelian case?
  - Evolution?
  - Unitarity?
  - Large-x, small-x, implementation in MCEGs?

Backup slides

Kinematics and Final States

Small-x: Multi-Regge kinematics

(Need for doubly integrated PDFs:
Martin, Watt, Ryskin Eur.Phys.J.C31,73(2003))



$$k_1 = (\alpha_1 p_A, \beta_1 p_B, k_{1T}) \; , \; k_2 = (\alpha_2 p_A, \beta_2 p_B, k_{2T})$$

$$0 \ll \alpha_2 \ll \alpha_1 \ll 1, \quad 0 \ll |\beta_1| \ll |\beta_2| \ll 1$$

$$\int d(P.S.) = \frac{1}{4(2\pi)^8 s} \int d^2k_{1T}\, d^2k_{2T} \ln \frac{s}{k_{2T}^2}$$

$$\int d(P.S.) = \frac{1}{4(2\pi)^8 s} \ln \frac{s}{m^2} \int d^2k_{1T}\, d^2k_{2T}$$

Hadronic Mass Scale

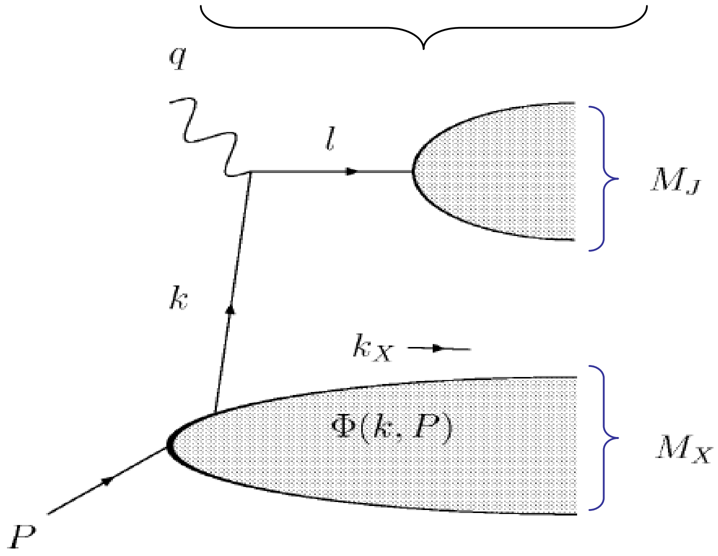
Standard Approx.

Poor for large k_{2T}.

Kinematics and Final States

Large-x

Two jets with fixed invariant energy in final state:



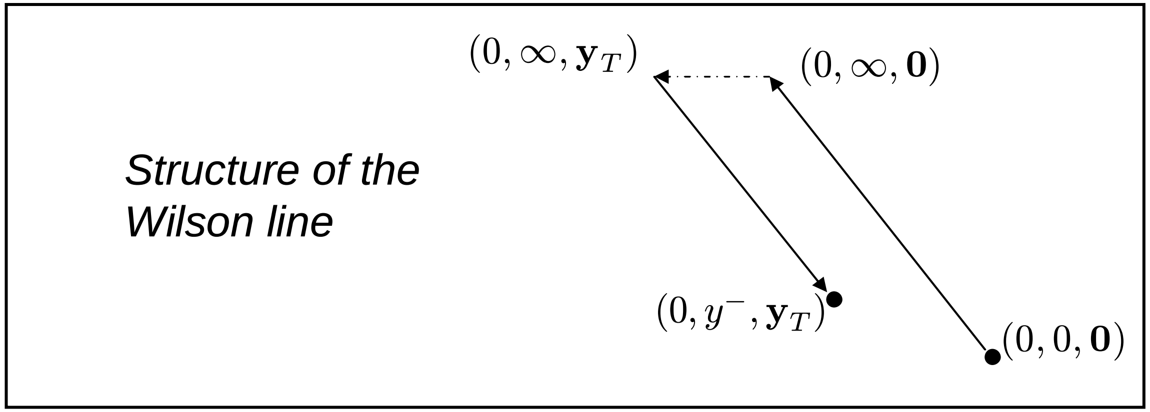
$$s = (1-x)M_p^2 + \frac{Q^2}{x}(1-x)$$

$$\Rightarrow \quad k_T^2 < \frac{(1-x)}{4}M_p^2 + \frac{Q^2}{4x}(1-x)$$

But k_T runs to order Q^2 in the def. of the PDF!

Structure of the complete gauge link:

$$\mathcal{F}(x,\mathbf{k}_T,\mu) \stackrel{??}{=} \int \frac{dy^- d\mathbf{y}_T}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_T\cdot\mathbf{y}_T} \times \langle p|\bar{\psi}(0,y^-,\mathbf{y}_T) V_y^\dagger(n) I_{n;y,0} \gamma^+ V_0(n) \psi(0)|p\rangle$$



- Definition has divergences at infinite '-' rapidity even with massive gluon!!

- Not an appropriate definition.

Dealing with light cone divergences:

- Divergence is result of using light-like Wilson line in definition
- Use non-light-like Wilson lines?
- Use a type of generalized renormalization?
- We want to use fully unintegrated objects (PCFs) rather than the unintegrated PDF anyway.

Complications with typical unintegrated PDFs

$$\mathcal{F}(x, k_T^2) \stackrel{??}{=} Q^2 \frac{\partial}{\partial Q^2} xg(x, Q^2)$$

$$xg(x, Q^2) \stackrel{??}{=} \int_0^{Q^2} \frac{dk^2}{k^2} \mathcal{F}(x, k_T^2)$$

- Positivity: $\mathcal{F}(x, k_T^2) > 0\,?$
- Scale dependence in $\mathcal{F}(x, k_T^2)$?
- Consistent operator definitions of UI PDFs? Will discuss later…