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Transparencies

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The casuality and/or energy-momentum conservation constraints on QCD amplitudes in small x regime.

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based on B. Blok and L. Frankfurt Phys. Rev. D75 074001 (2007) hep-ph/0611062

① The problem:

can traditional eikonal (Glauber) approximation be justified in QCD?

We show that the answe is „no".
The eikonal (planar) contribution to scattering amplitudes is not increasing with energy
As aresult, the energetic parton can undergo at most one inelastic collision.
In other words, even in the formalism where the entire energy ($Y = \ln s$) dependence is in the ladders (Gribov formalism) the number of constituents in the wave function increases with energy.

In particular: Mandelstam arguments about AFS cancellation can be easily generalised to QCD, and one cannot iterate ladders.

These results follow in two ways:

a) from casuality (dispersion relation)

b) energy-momentum conservation

closely related questions were considered by Bartels, Ryskin (1995); Bartels, Lipatov (2005), but using different approach.

Contents of the talk

1. What is an eikonal, when is it used and the problem with it in QFT
2. Review of Mandelstam idea (ϕ^3)
3. Generalisation to QCD
4. higher cuts, AGK
5. Energy-momentum conservation constraints

6. conclusion & comments

) Introduction

what is eikonal-?

comes from Quantum mechanics
take Klein-Gordon/Dirac
equations.

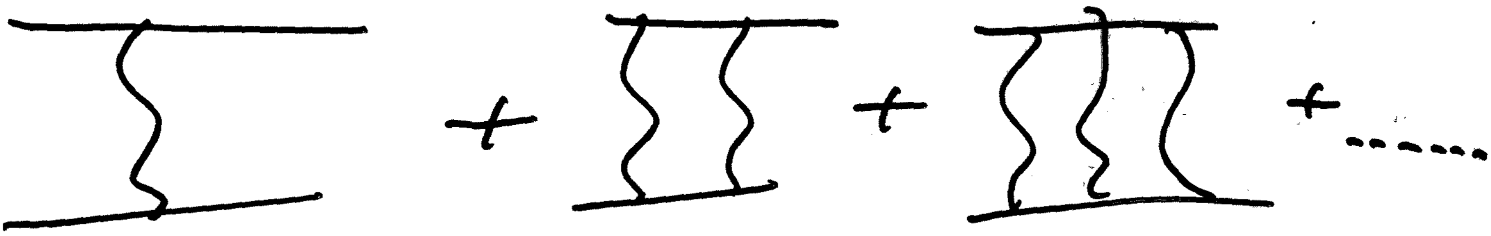
$$(\Box_A - m^2)\phi = 0 \qquad (i\not\partial - eA - m)\Psi = 0$$

Then it can be shown say

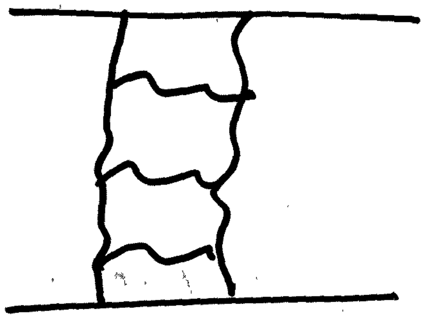
$$\Psi \sim u_0 e^{ipx + i\chi_0 + \frac{1}{E}\chi_1 + \dots}$$ for high energies

$$\chi_0 \sim e \int_L A_\mu dx^\mu$$ - eikonal phase

The solution corresponds to ite-
rating one-photon exchange:



In field theory (QCD) however one needs to iterate ladders. indeed, the ladder contribution say in DIS grows much faster than one-gluon exchange



up to $x \sim 10^{-4}$ (for $Q^2 \sim 10$ GeV^2) - ladder approximation is quite good.

BFKL (LO + NLO) $\approx$ DGLAP (LO + NLO)

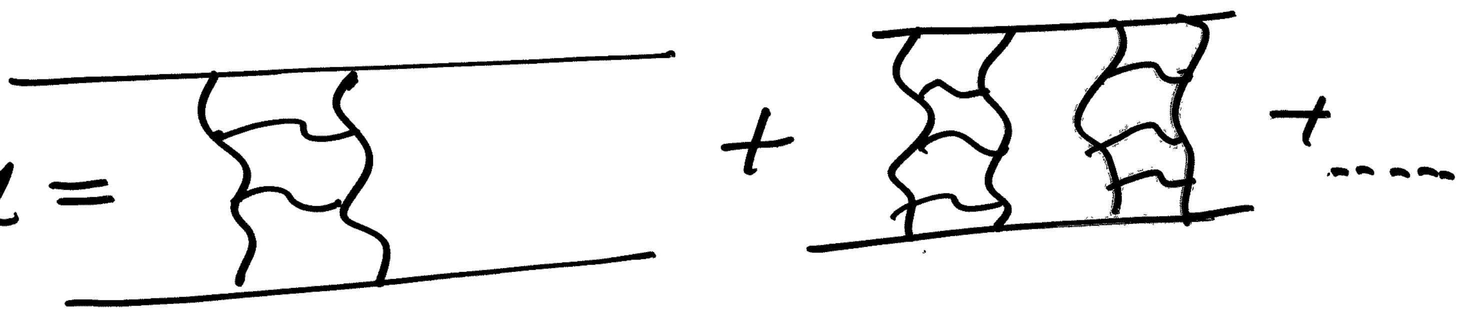
Lipatov - Fadin
Ciafaloni - Cursi

This equivalence - "resummation models"

(Salam et al
Altarelli et al

however at $x \sim 10^{-4}$ LT breaks down ($f_0 \sim 1$)
- we approach the black disk (Froissart) limit

Natural way - to use Glauber approach, similar to Cheng-Wu (1969, ϕ^3) - but iterate ladders

$\chi =$ 

Then:

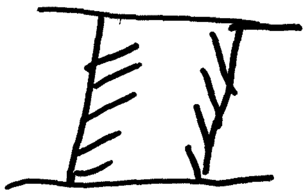
$$\sigma_T = 2\int d^2b\,(1 - e^{-a(s,\vec{b})}) \quad \text{total}$$

$$\sigma_{El} = \int d^2b\,(1 - e^{-a(s,\vec{b})})^2 \quad \text{elastic}$$

However: in QFT in difference ≠ to QM there is particle creatio

⇓

intuitive problem with eikonal—

— 

The eikonal diagrams mean that particles are created in big numbers, and then reabsorbed in a similar configuration, that looks very improbable.

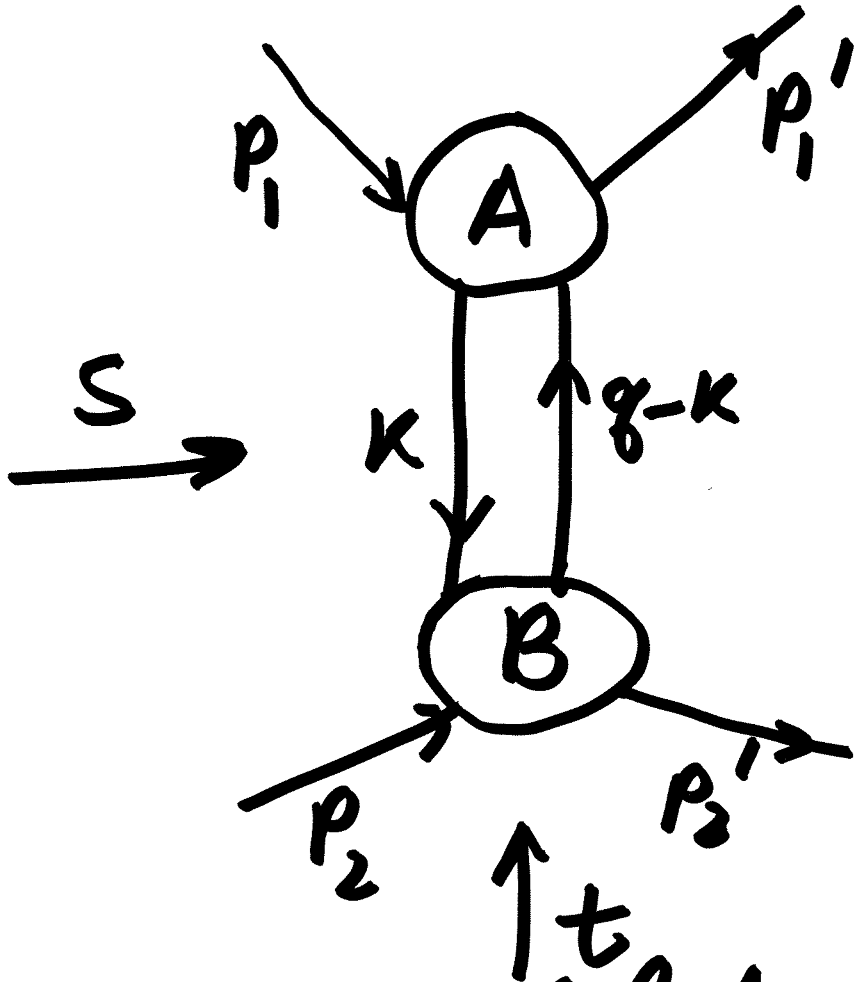
more rigorously—

a) casuality (Mandelstam)

b) Energy-momentum conservation arguments

show that indeed these diagrams are zero.

④ Casuality arguments (Mandelstam-Gribov)



Consider the blob A as a function of invariant mass $s=(p_1+k)^2$

$k = \alpha p_2 + \beta p_1 + k_t$ (Sudakov)

$t = q^2 \sim q_t^2$, $s_1 = (p_1+k)^2 \sim \alpha s$

$s_2 = (p_2-k)^2 \sim -\beta s$

Then consider 2 particles exchanged in t-channel

In multiRegge kinematics

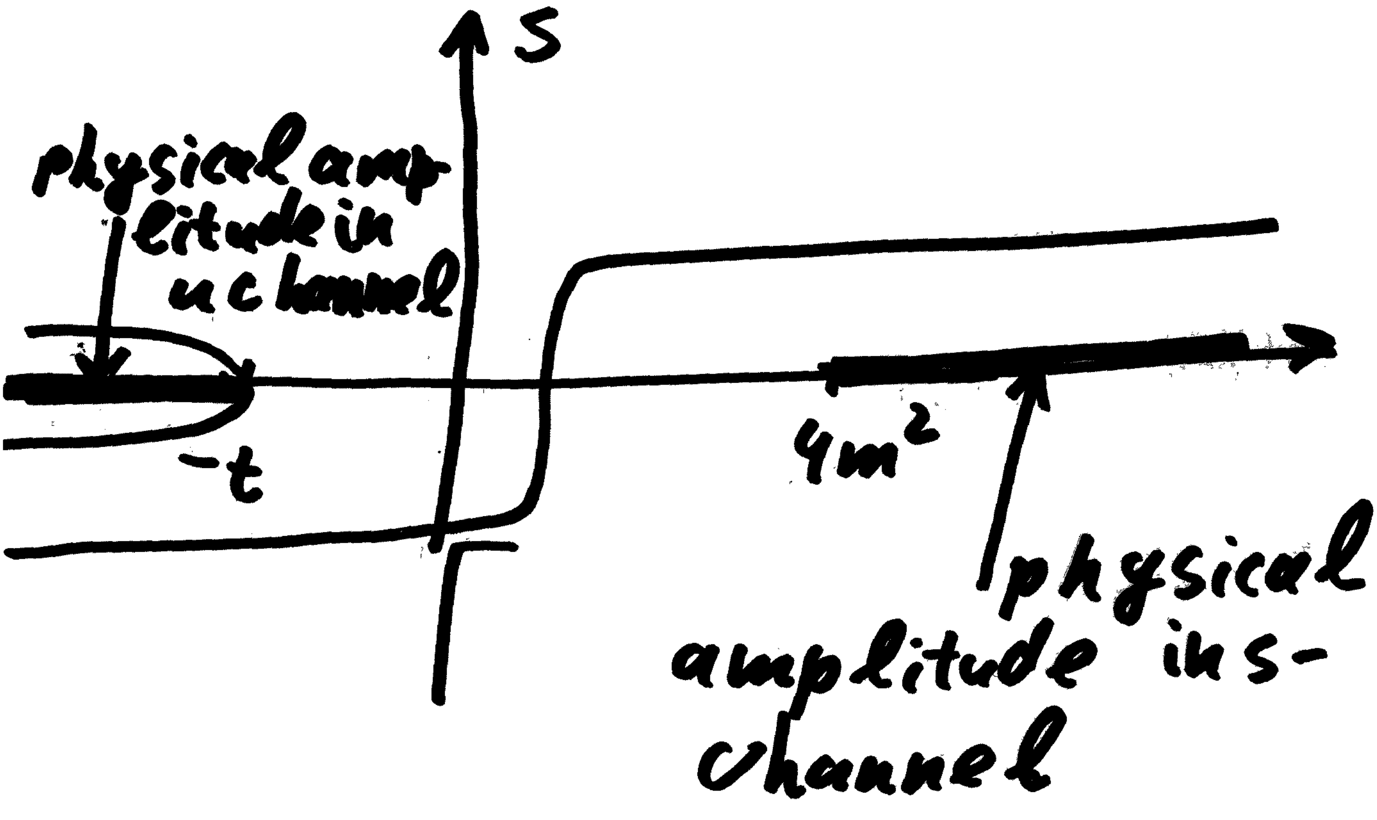
$$\frac{1}{(k^2-m^2)\left((k-q)^2-m^2\right)} \sim \frac{1}{\alpha\beta s+k_t^2-m^2}\cdot$$

$$\frac{1}{\alpha\beta s+(q-k)_t^2+m^2} \sim \frac{1}{k_t^2+m^2}\,\frac{1}{(q_t-k_t)^2+m^2}$$

So the full amplitude is:

$$M=\frac{i}{4s}\int\frac{d^2k_t}{(2\pi)^2}\,\frac{1}{(m^2+k_t^2)\left(m^2+(k-q)_t^2\right)}$$

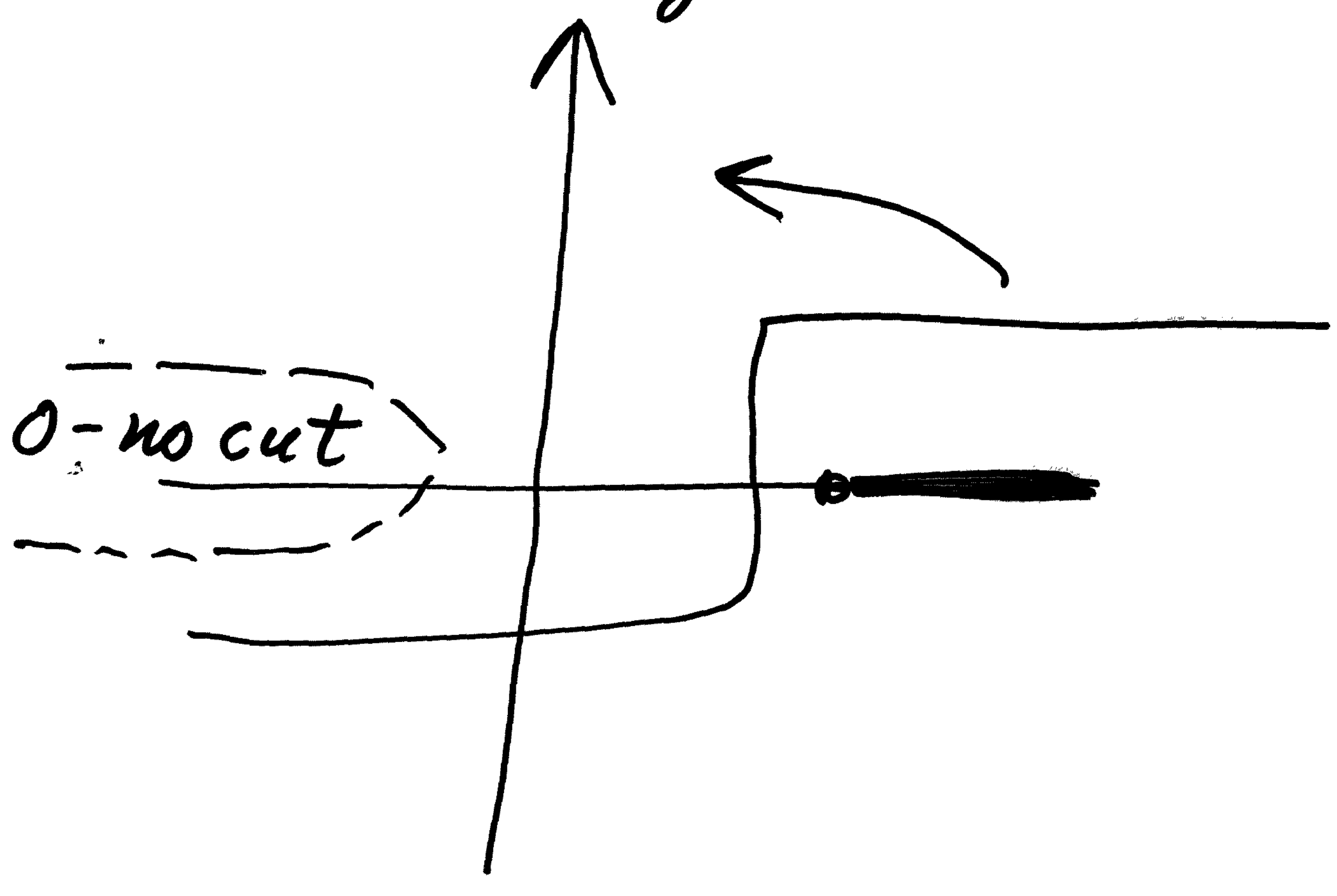
$$\int_\Gamma \frac{ds_1}{2\pi i}A(s_1)\int_\Gamma\frac{ds_2}{(2\pi i)}A(s_2)$$



$$A(s,t) = \int_{\Gamma_1} \frac{\rho_{st}(s',t)}{s-s'} + \int_{\Gamma_2} \frac{\rho_{su}(s',t)}{s'-s}$$

But suppose that: 1) $\rho_{su} = 0$ (no left cut, i.e. planar diagram for Impact-factor A)

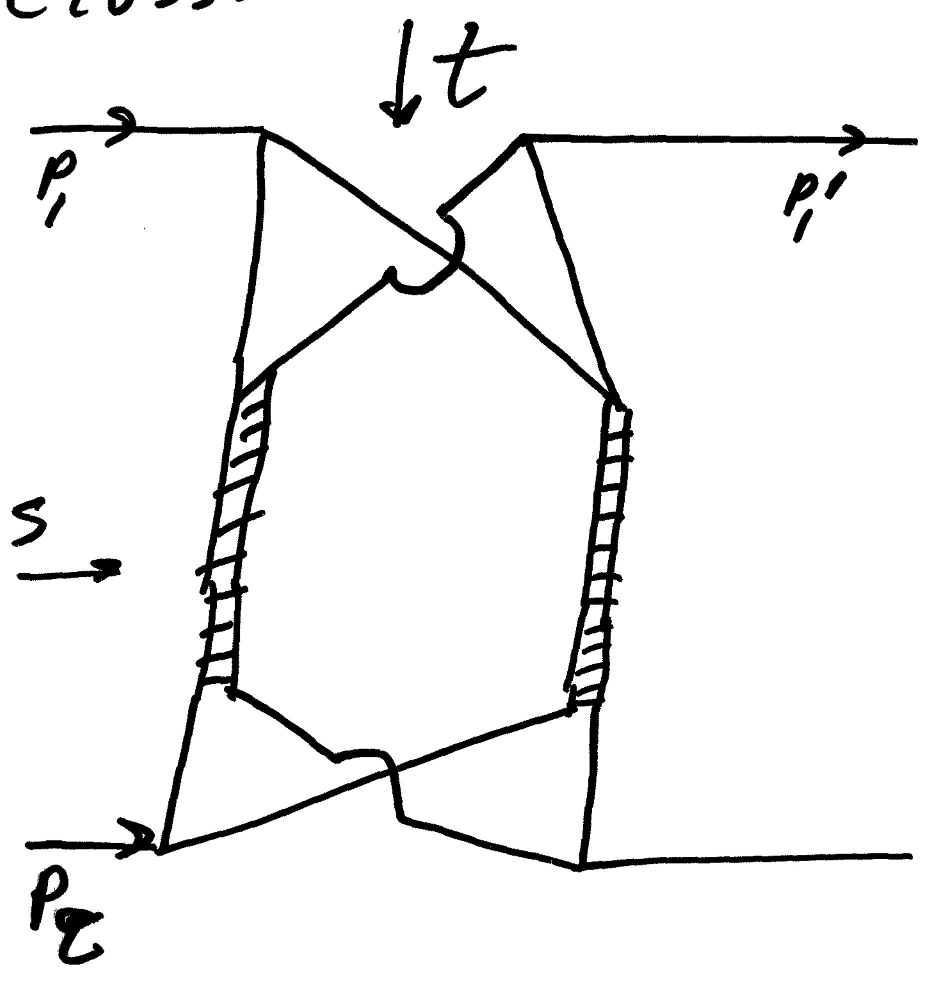
2) A decreases with s, more rapid than s_1. Then we deform the contour and get zero:



In reality - not true - point b) for 2-particle exchange.

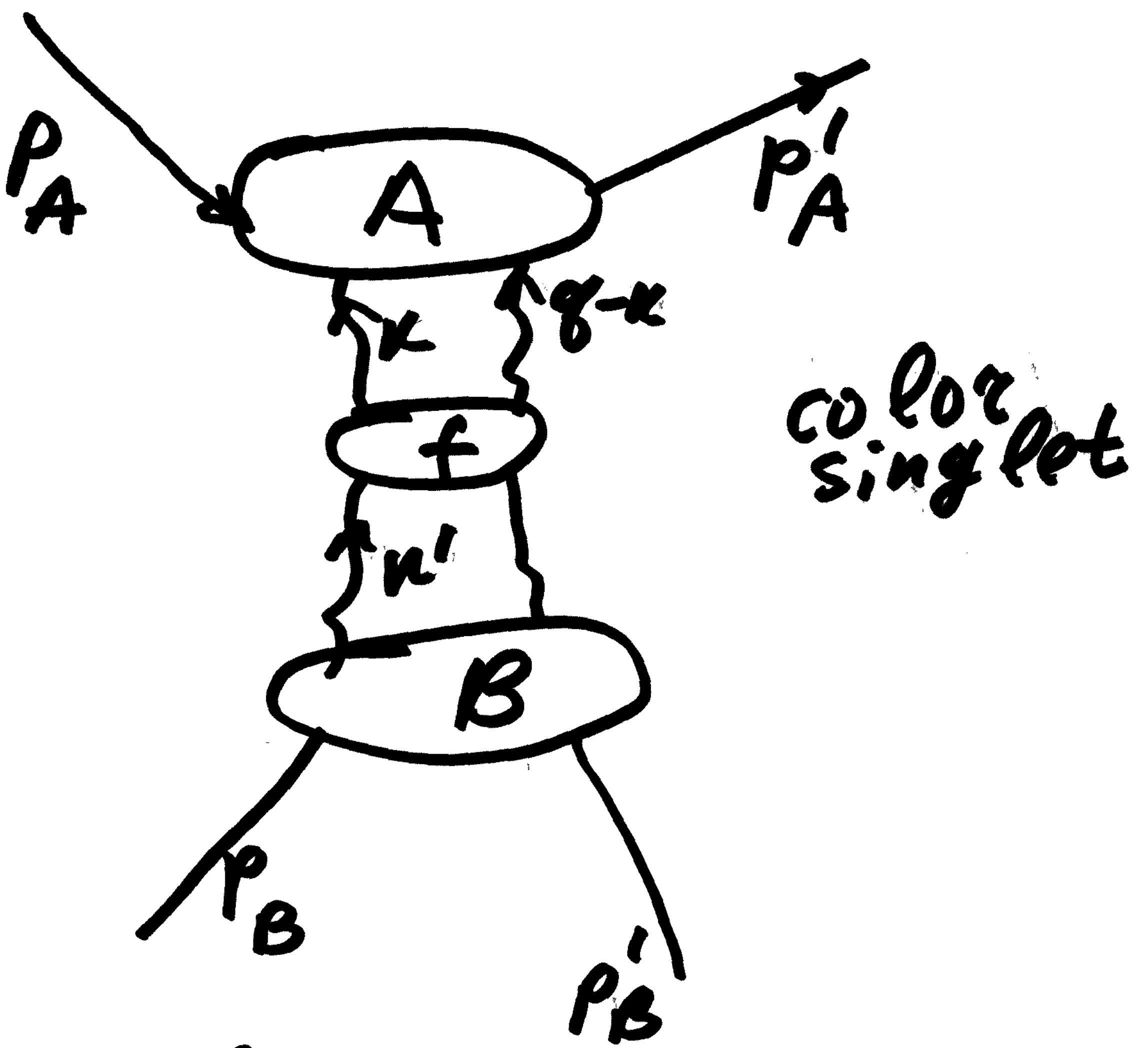
But for the 2-reggeon (2-ladder exchange) there appeares additional formfactor that leads to additional power of $\frac{1}{s}$, - and the two-ladder diagram becomes zero (Mandelstam, 1961)

The nonzero diagram - Mandelstam cross: - nonplanar



5) Casuality and QCD

in QCD for high energies one has a similar pPR factorisation as in ϕ^3 (although more complicated)



$$A(s,t) = \int d^2k \int d^2k' \, \phi_1(p_A, k, q-k) f(s, k, q-k; k', q-k') \phi_2(p_B, k', q-k')$$

i.e. f depends on transverse momenta

$\Phi_{1,2}$ - impact factors

$\Phi_1 = \int \frac{dM^2}{s^2} p_B^\mu p_B^\nu f^{\mu\nu} = \int \frac{dM^2}{M^4} f_{\mu\nu} k_t^\mu (q-k)_t^\nu$

M^2 - mass squared of a diffractionally produsced state

and we use here Ward identities (<=> color singlets)

The momentum factorisation in the vertex - similar to ϕ^3 - is in the BFKL regime (Lipatov), for DGLAP - no such factorisation

Consider 2 ladders exchanged in t - channel. The amplitude is

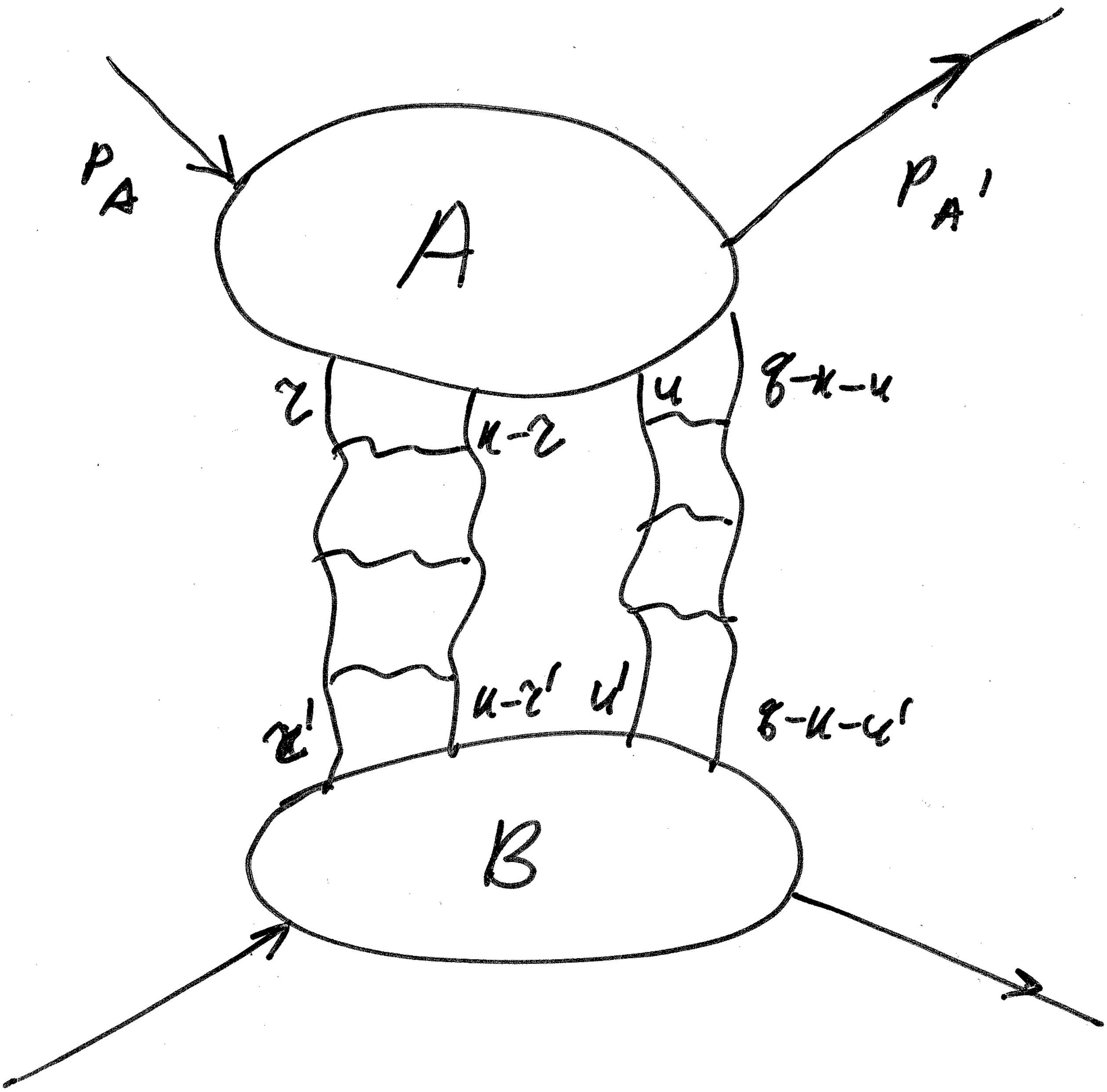
$A = \int d^2 r_t \int d^2 k_t \int d^2 u_t \int d^2 u_t' \int d^2 r_t' \int d\alpha \int d\beta$

$\phi_1(p_A, r, k, q, u) \phi_2(p_B, r', k', q, u') f_1(r, r', k_1, s_1) f_2(q-k, u, u', s_2)$.

f_1, f_2 - exchanged reggeons; s_1, s_2 - their squared energies, $s_{12} \sim (p_A+r)^2 \sim \alpha_r s_1$, $s_{34} \sim (p_A+k)^2 \sim \alpha_k s_2$, $s_\alpha \sim (p_B+k)^2 \sim \alpha s$

$s_\beta \sim (p_B-k)^2 \sim -\beta s$

- the relevant invariant masses
($\alpha, \beta, \ldots$ - Sudakov variables)



$$A = \int d^2 r_t\, d^2 u_t\, d^2 k_t \int ds_a\, \Phi_1(p_A, u, r, k, q, s_a)$$
$$\int d^2 r_t'\, d^2 k_t'\, d^2 u_t' \int ds_B\, \Phi_2(p_B, u', r', k, q, s_B)$$
$$f_1(r, r', k, s_1)\, f_2(q-k, u, u', s_2)$$

The latter integral - has the same

factorised structure as the one in ψ

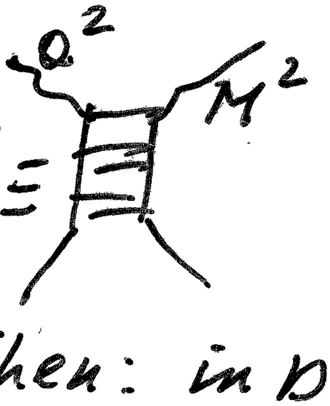
so it is zero, if

a) one of impact factors is eikonal ($planar \equiv \rho_{sa}=0$)

b) dependence on $S_a \equiv M_a$ is the decrease mor rapid than $\frac{1}{M^2}$

1) in RFT this dependence is $\frac{1}{(M^2)^{2\Delta+1}}$ (Frankfurt, Strikman 1989)

2) in QCD one needs to find

$A \equiv$ 

decrease of $A(M^2, Q^2)$ when $M^2 \gg Q^2$.

Then: in DGLAP for $M^2 \gg Q^2$

$$A \sim \frac{1}{(Q^2+M^2)^{3/4}}$$ (Frankfurt, Strikman)

BFKL must be continuosly connected to DGLAP $\Rightarrow$ also rapid decrease

! $\frac{1}{M^2}$ already have from eikonal intermidiate state

) arguments inside BFKL (Ciafaloni)

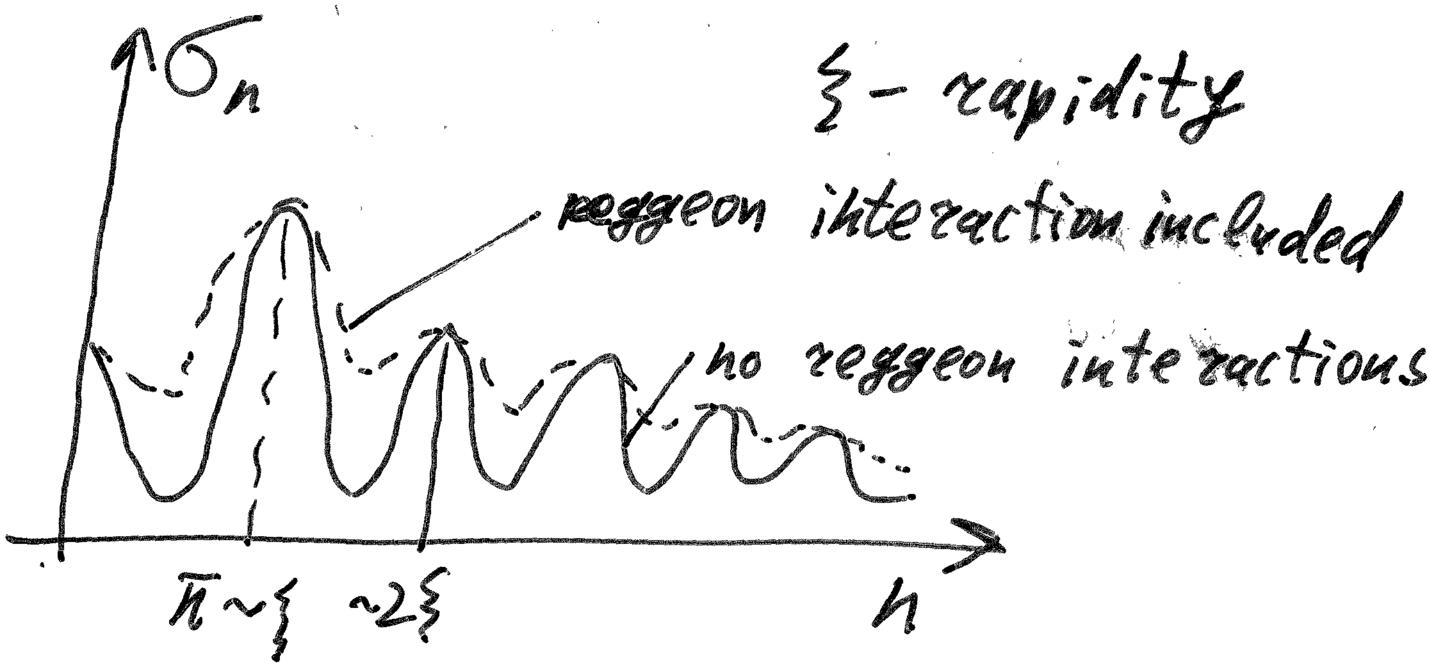
$$A \sim \frac{1}{M^2} S\left(\frac{M^2}{\theta_1^2}\right)$$

S- Sudakov formfactor

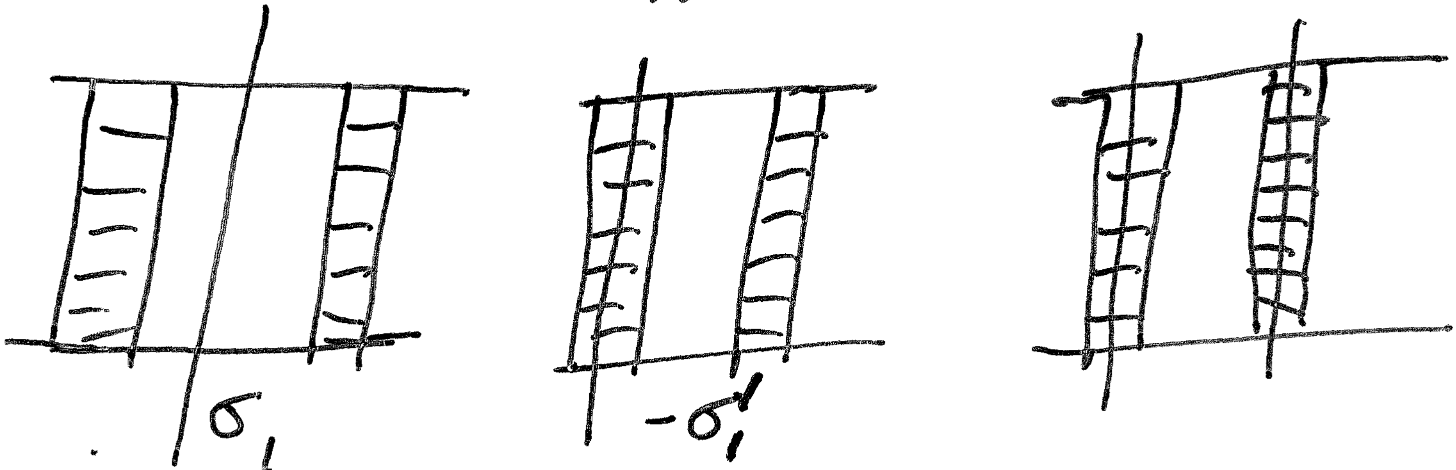
note: our approach can not be applied to octet exchanges, since the correspondent formfactors do not decrease.

Indeed, Bartels & Lipatov (2005) argued that such ladders (Reggeized gluons) can be iterated

6) cut ladders, fluctuations & AGK



Consider cross-sections of productions of n particles σ_n.



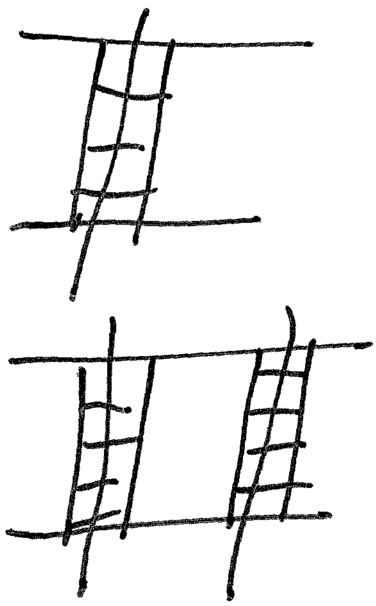
Then 2-ladder exchange corresponds to ~~process~~ 1, ~~screening~~ correction $-\sigma_{\bar{n}}'$ & $2\bar{n}$ particle production

$4\sigma_1 = -\sigma_{\bar{n}}' = 2\sigma_{2\bar{n}}$ - AGK rules)

(AGK - Abramowski, Gribov, Kancheli 197

These rules are valid for QCD
(Bartels, Ryskin, 1995
Treleani
The formalism – at least for color singlet ladders)
in this way multiparticle distributions are calculated

7) Energy-momentum constraints



in eikonal approach we have production of kn particles with the same energy distributions, but total energy the same ($\sqrt{s}$, instead of $k\sqrt{s}$)

– no way to fulfill energy conservation law

thus cut color singlet diagrams must be zero (another way – energy sum rule is violated for $\alpha(0) > 1$, where we assume Mueller vertex for inclusive distribution (Mueller, 1970)

note: in ϕ^3 theory the energy-momentum nonconservation was noticed by Cheng & Wu; Capella & Kaidalov suggested to include it by hand, but there is no justification for this in perturbation theory (Iengo, 1976)

note: also cut eikonal diagrams are zero.

9) conclusions

1. the cancellation of planar (in particular eikonal) diagrams found within the RFT by Mandelstam is valid in QCD.
2. The use of energy-momentum considerations supports this results, & shows that DGLAP ladders also cannot be iterated
3. To obtain nonzero result by n ladders the incident parton must develop a configuration with at least n partons before the collision.
In other words any participant can participate in the ladder exchange at most once.
4. eikonal can be used at moderate x and large transverse momenta where scattering amplitudes are predominantly real.