



*The Abdus Salam  
International Centre for Theoretical Physics*



**SMR/1845-16**

**Conference on Structure and Dynamics in Soft Matter and  
Biomolecules: From Single Molecules to Ensembles**

*4 - 8 June 2007*

**Structure and dynamics in the physics of foams**

Denis WEAIRE  
*School of Physics  
Trinity College  
Dublin 2  
IRELAND*

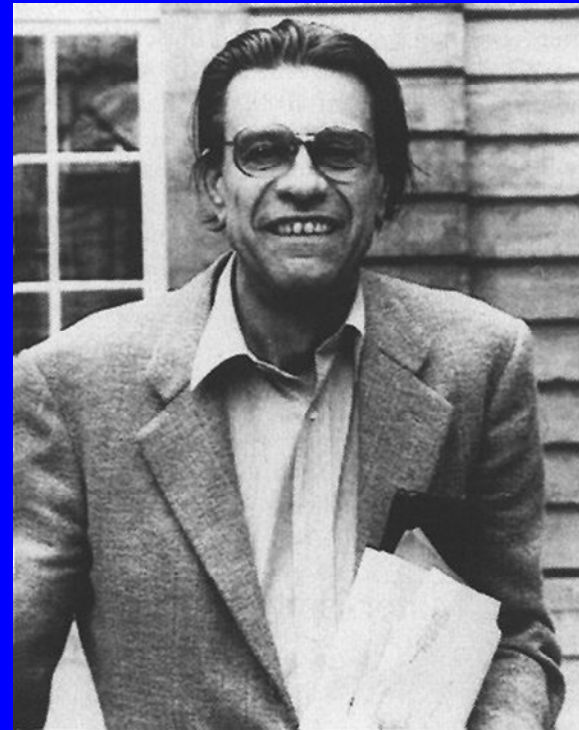
In memoriam

# Pierre Gilles de Gennes

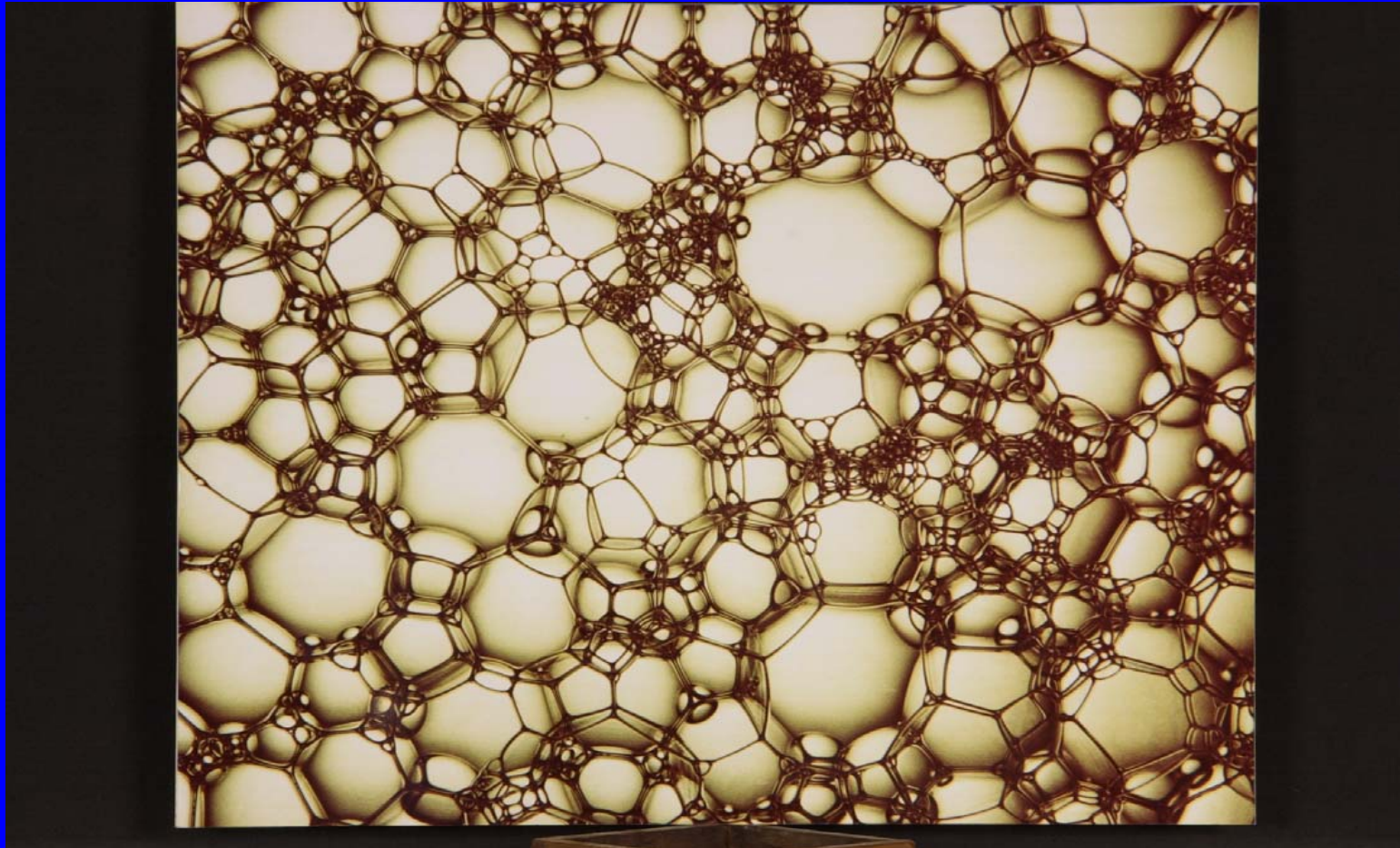
Nobel Prize Lecture

*Amusons-nous. Sur la terre et sur  
l'onde  
Malheureux qui fait son nom!  
Richesse, Honneurs, faux éclat de ce  
monde,  
Tout n'est que boules de savon.*

[Let's have fun. On land and sea,  
Fame brings nought but troubles,  
Riches, honours, vain celebrity,  
Are only soapy bubbles.]



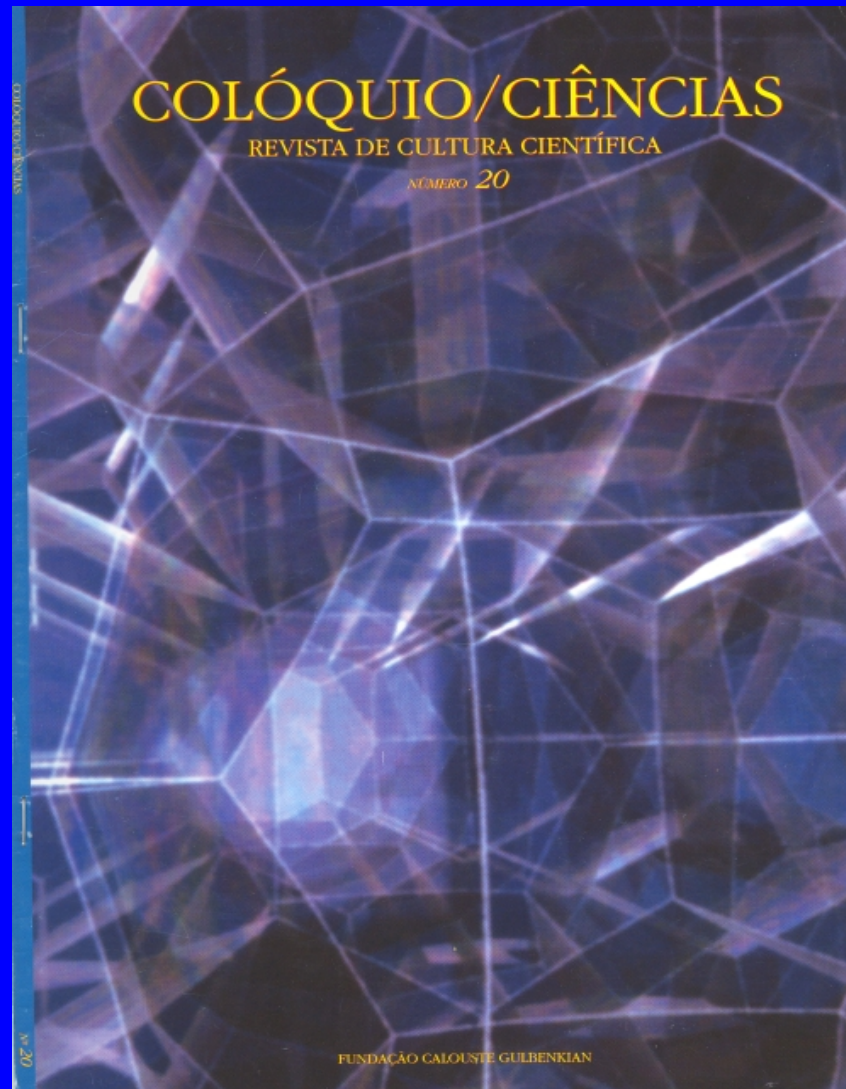
# Foams (in particular, rheology)



# Success story: statics/quasistatics

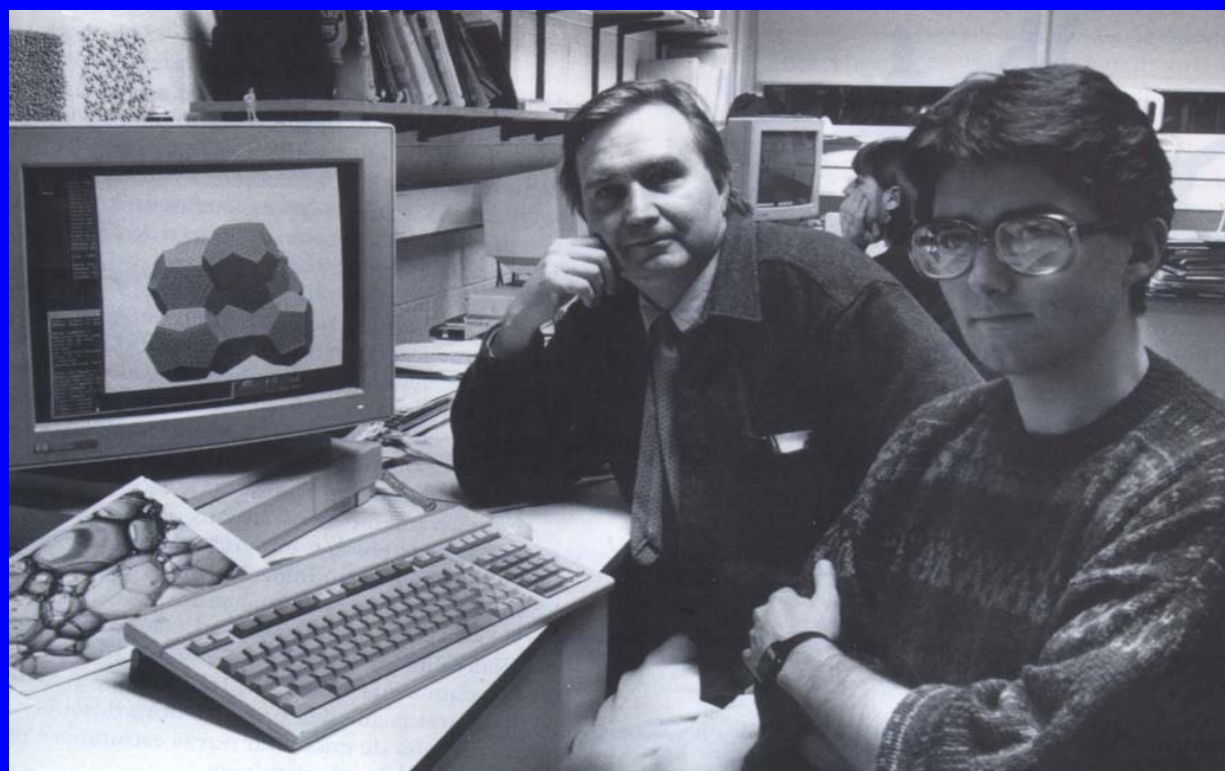
We understand many aspects of :

- structure
- elastic behaviour, onset of plasticity
- coarsening
- drainage
- electrical/thermal conductivity
- *etc*



WEAIRE-PHELAN STRUCTURE

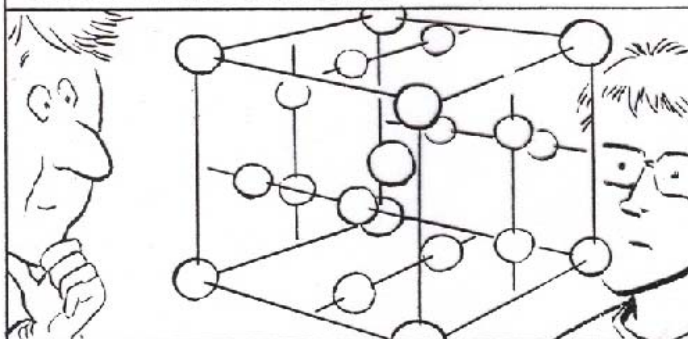




BUT SINCE KELVIN, HUGE STRIDES HAVE BEEN MADE IN CRYSTALLOGRAPHY. AFTER ALL, HE WAS WORKING BEFORE X-RAYS, BEFORE COMPUTERS...



IN LATE 1993 IRISH PHYSICISTS **DENIS WEAIRE** AND **ROBERT PHELAN** BUILT A NEW FOAM WHOSE SKELETON WAS A CRYSTAL STRUCTURE CALLED **BETA-TUNGSTEN**.

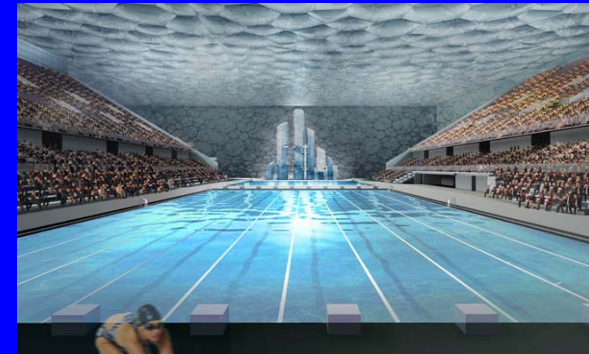
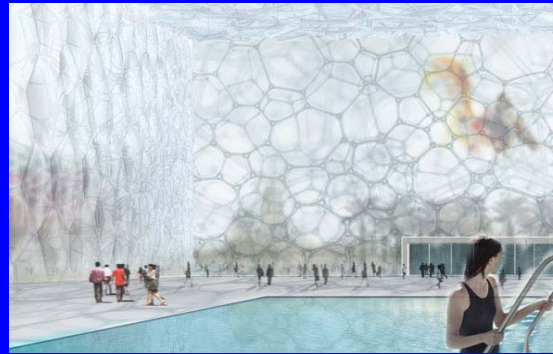


DRAWING A CELL AROUND EACH ATOM, WEAIRE AND PHELAN THEN USED "SURFACE EVOLVER" SOFTWARE TO EQUALIZE CELL VOLUMES AND FIT THEM TOGETHER PERFECTLY.

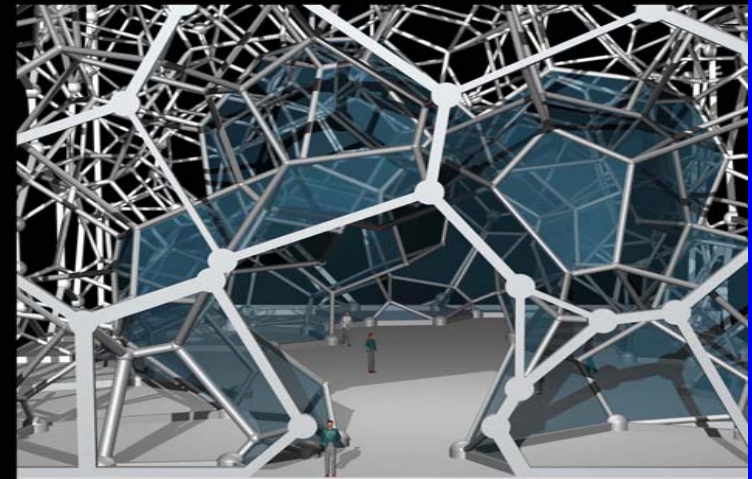
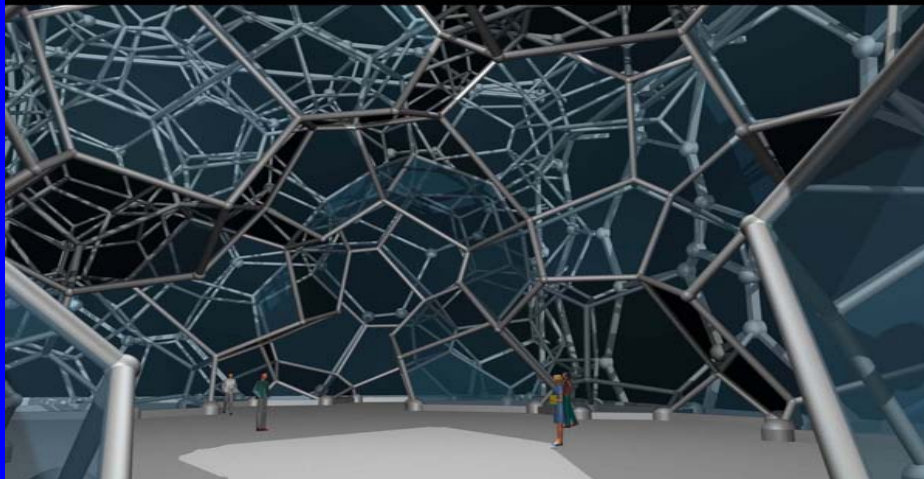
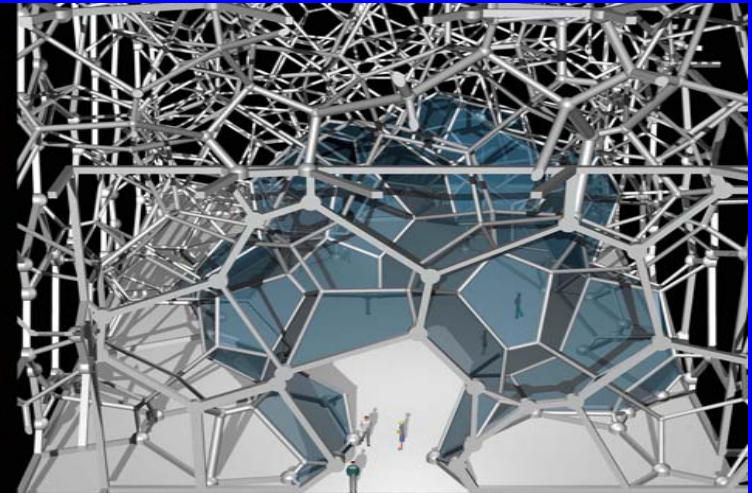
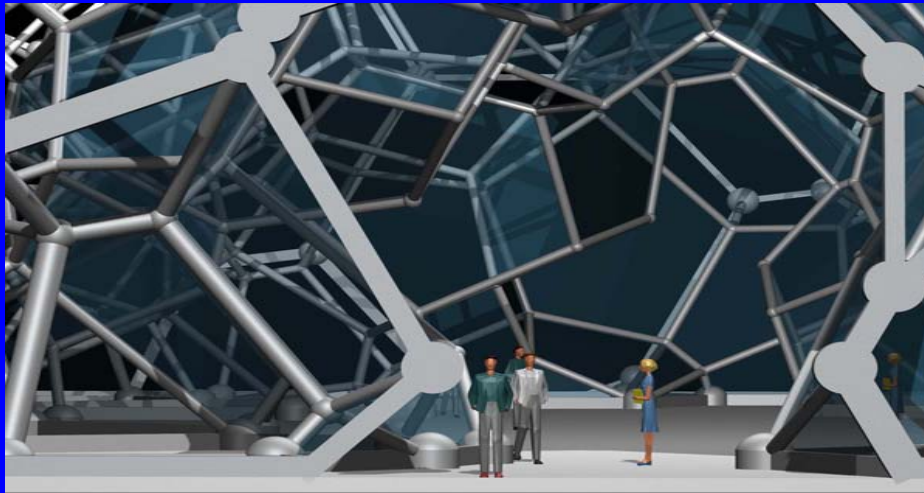


# The Water Cube

Arup's winning design for the 2008 Beijing Olympics,  
National Swimming Centre







Beijing National Swimming Centre

Tristram Carfrae

6500 tonnes of steel

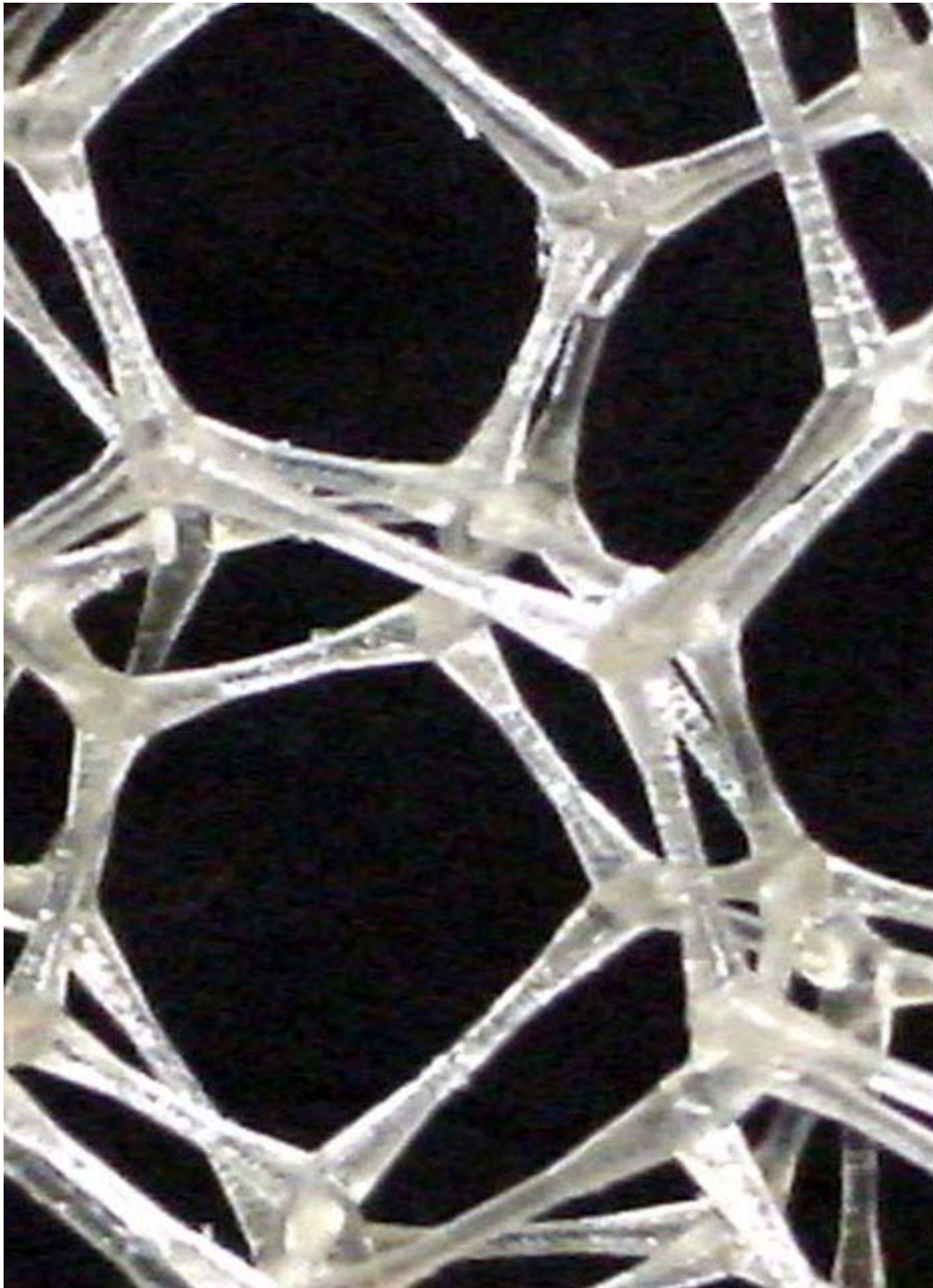
Steel beams would stretch for 90kms

4,000 bubbles !







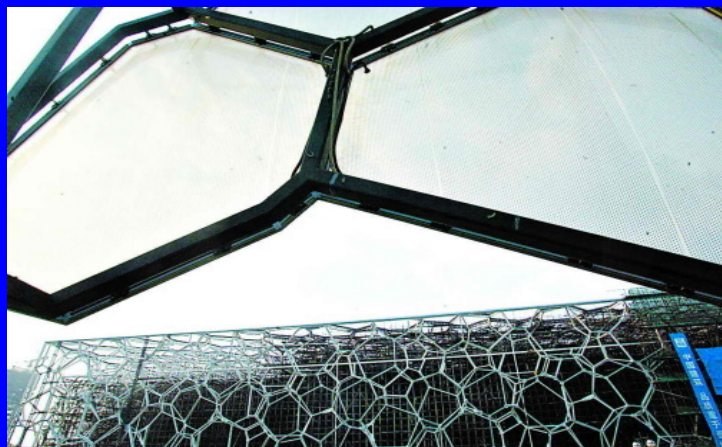
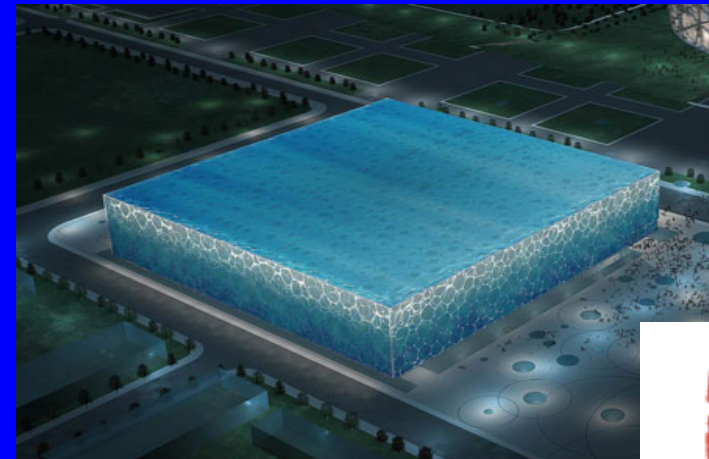








It even looks like it is  
supposed to do...



<http://www.flickr.com/search/?q=watercube&m=text>

# VIDEO CLIP

April 2007 Beijing Olympic buildings

<http://educatedearth.net/video.php?id=3152>

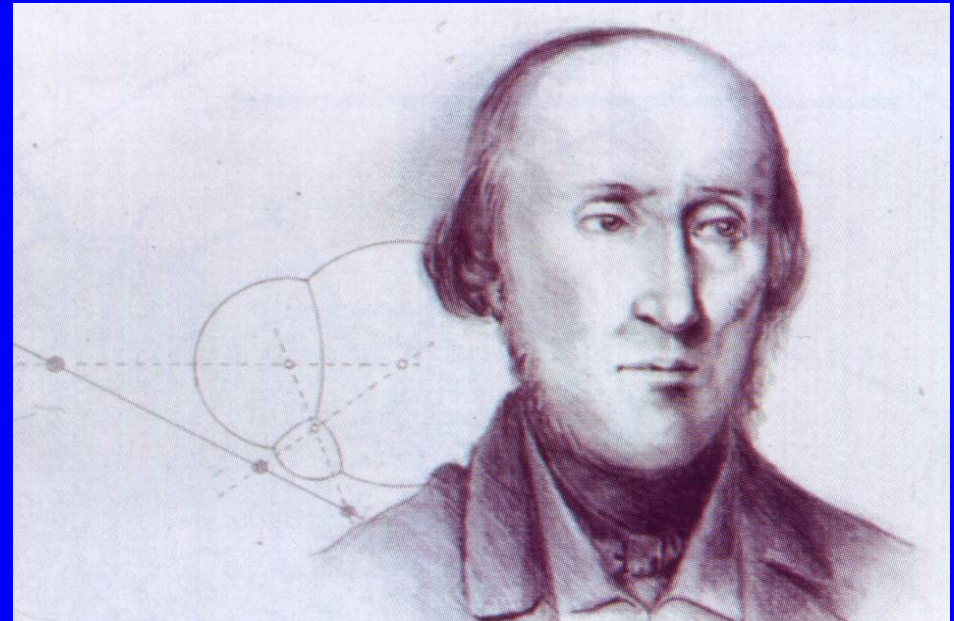
# Joseph Plateau

Blind experimentalist

Classic text 1873

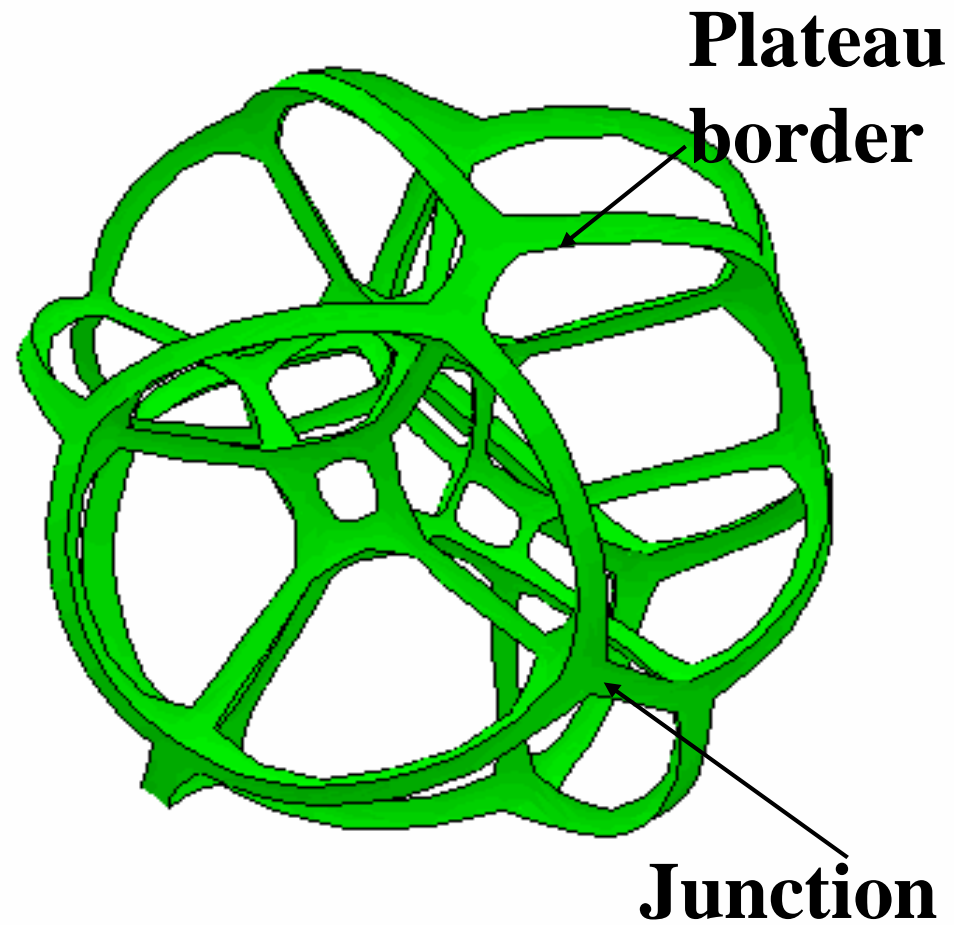
Rules of equilibrium

Wire frame demos

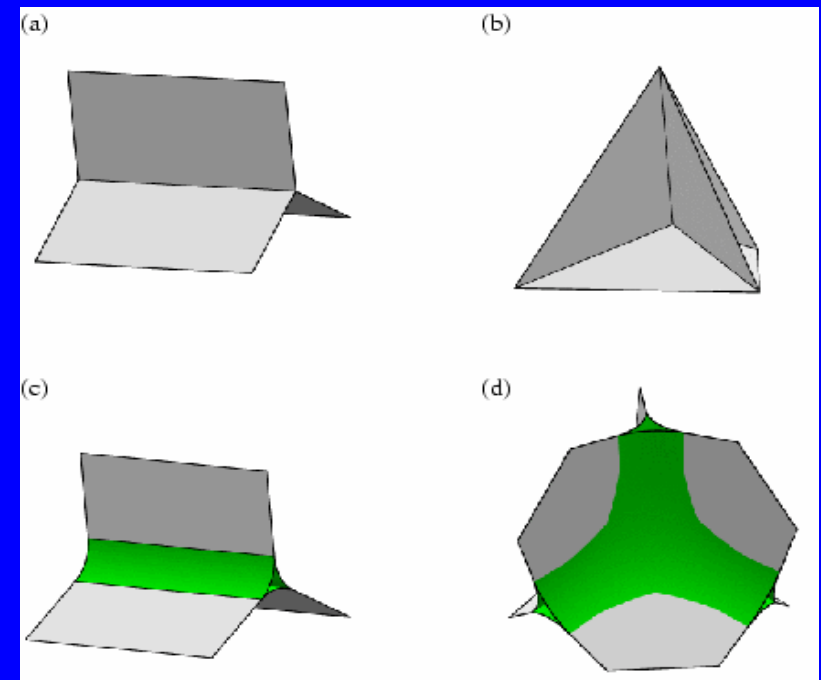




# Elements of the foam structure



## Dry Foam

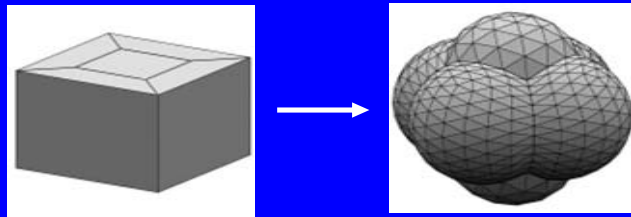


## Wet Foam

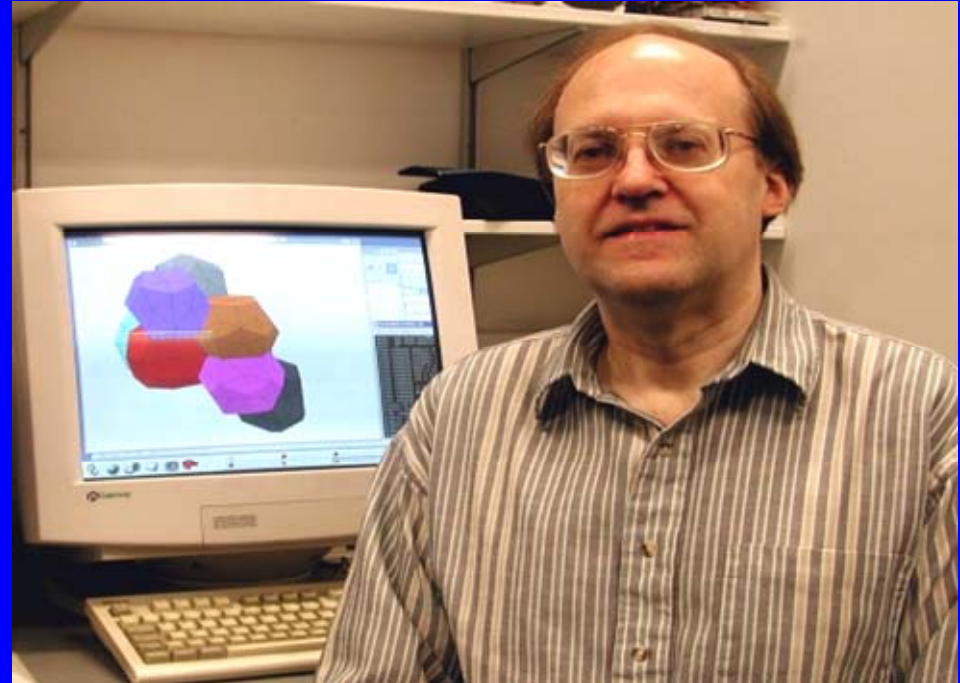
# Ken Brakke

Mathematician

Surface Evolver

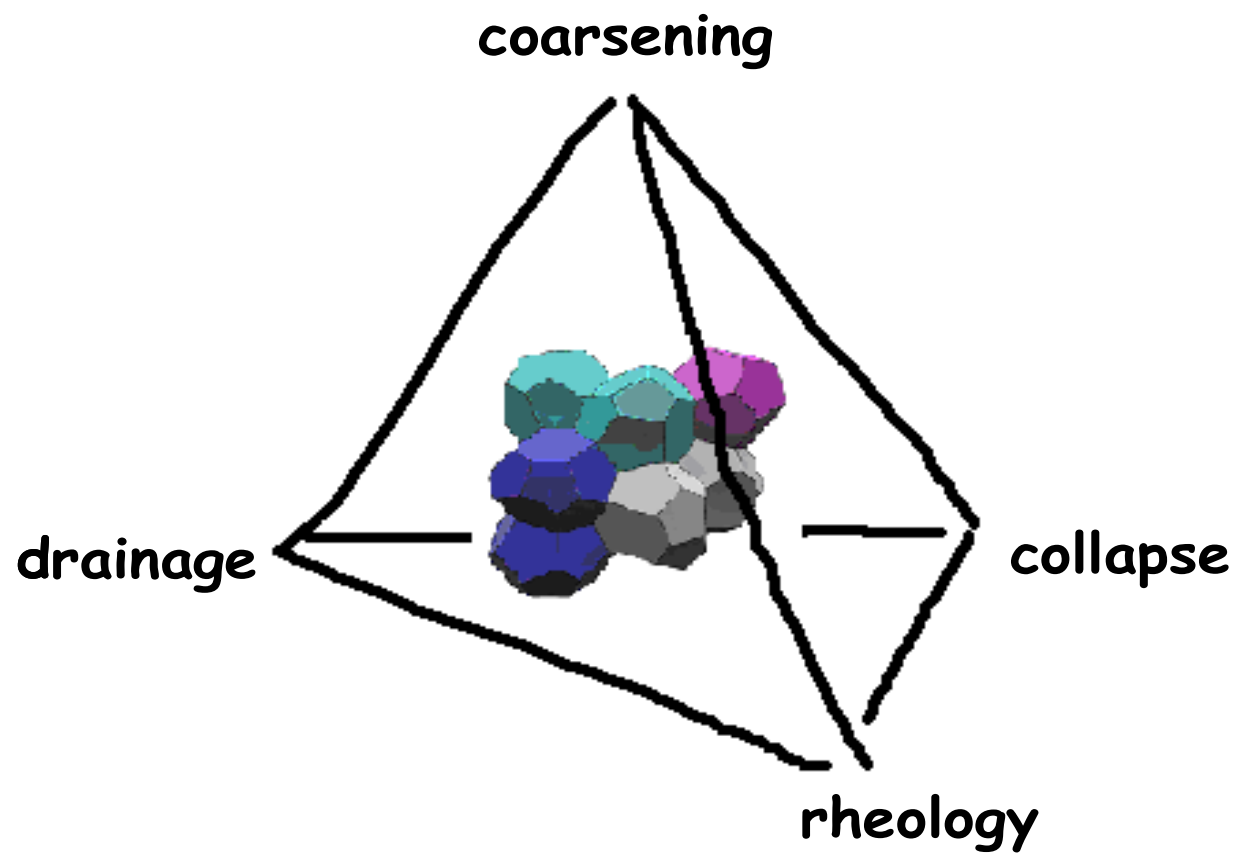


<http://www.susqu.edu/facstaff/b/brakke/>



John Sullivan, Andy Kraynik

*The evolver is a spectacular example of the effects of a gift to science which advances a whole field.* Alan L. Mackay





# Cyril Stanley Smith

Metallurgist

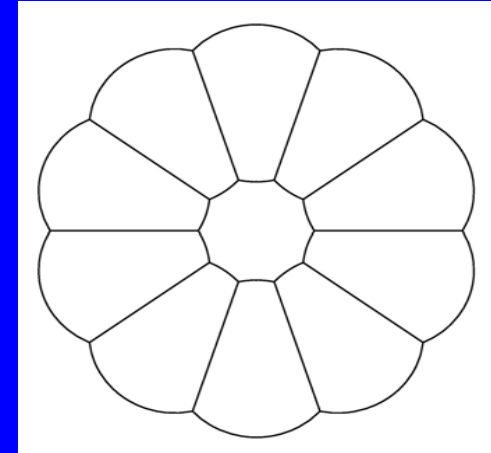
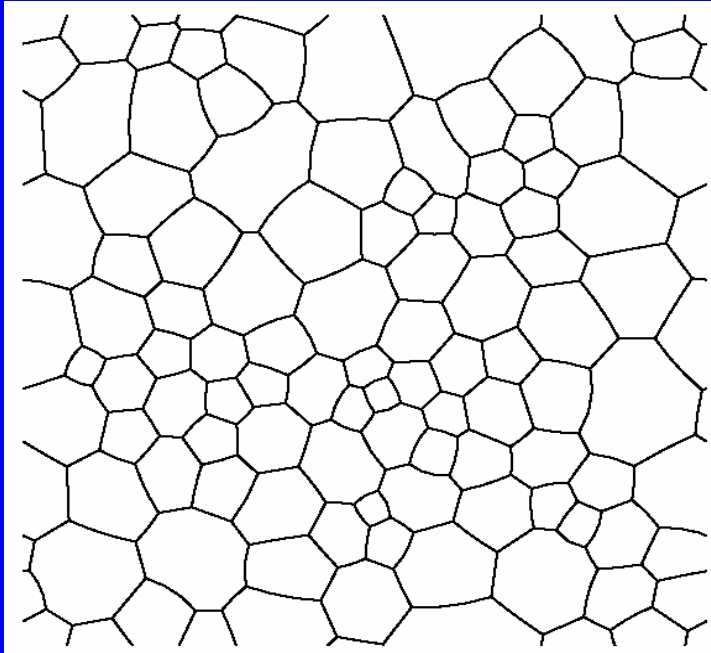
2D soap froth

Model for grain growth

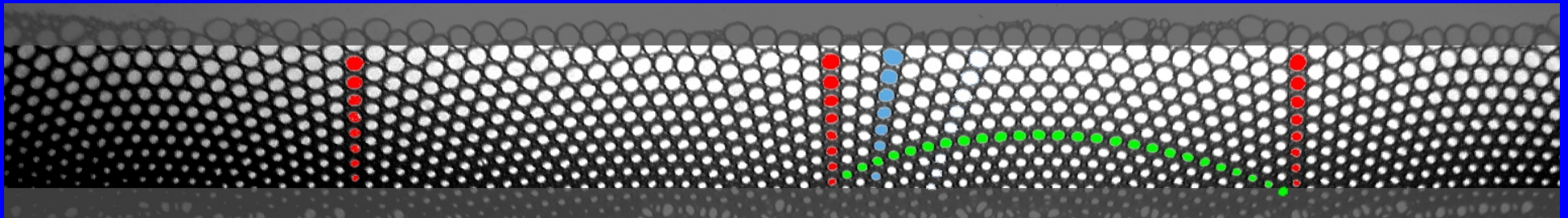
Inspiring influence!



# Two – dimensional foams



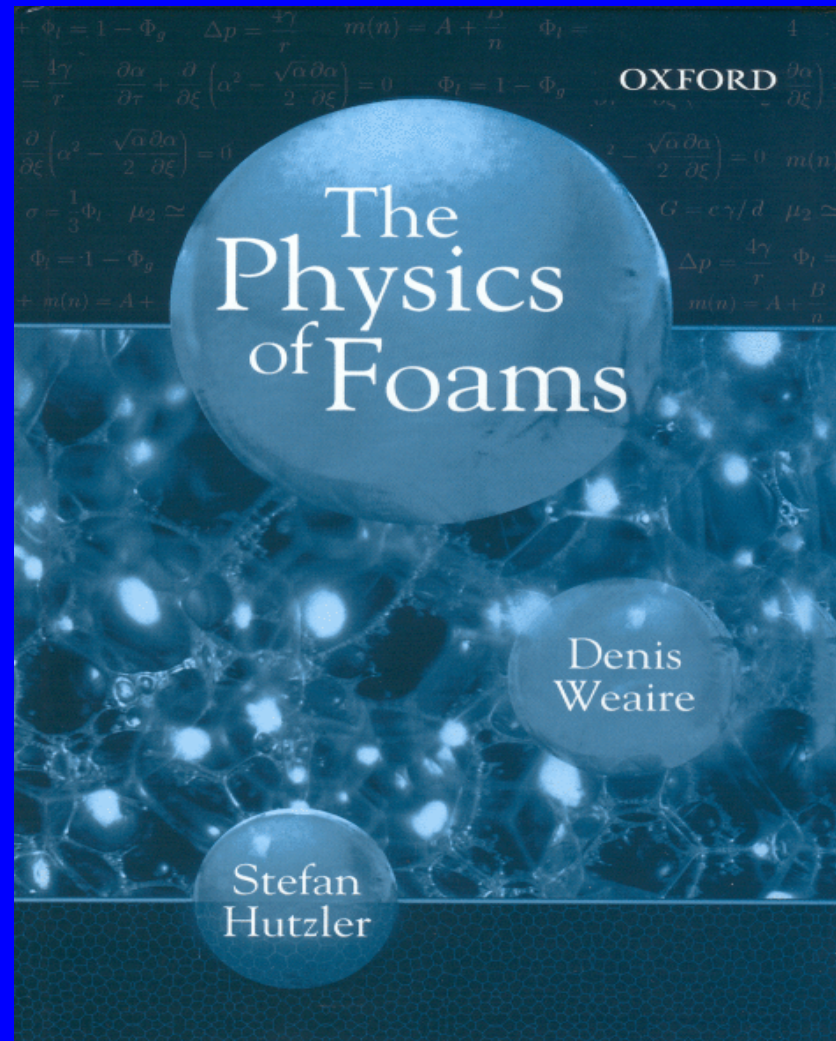
Gravity's rainbow



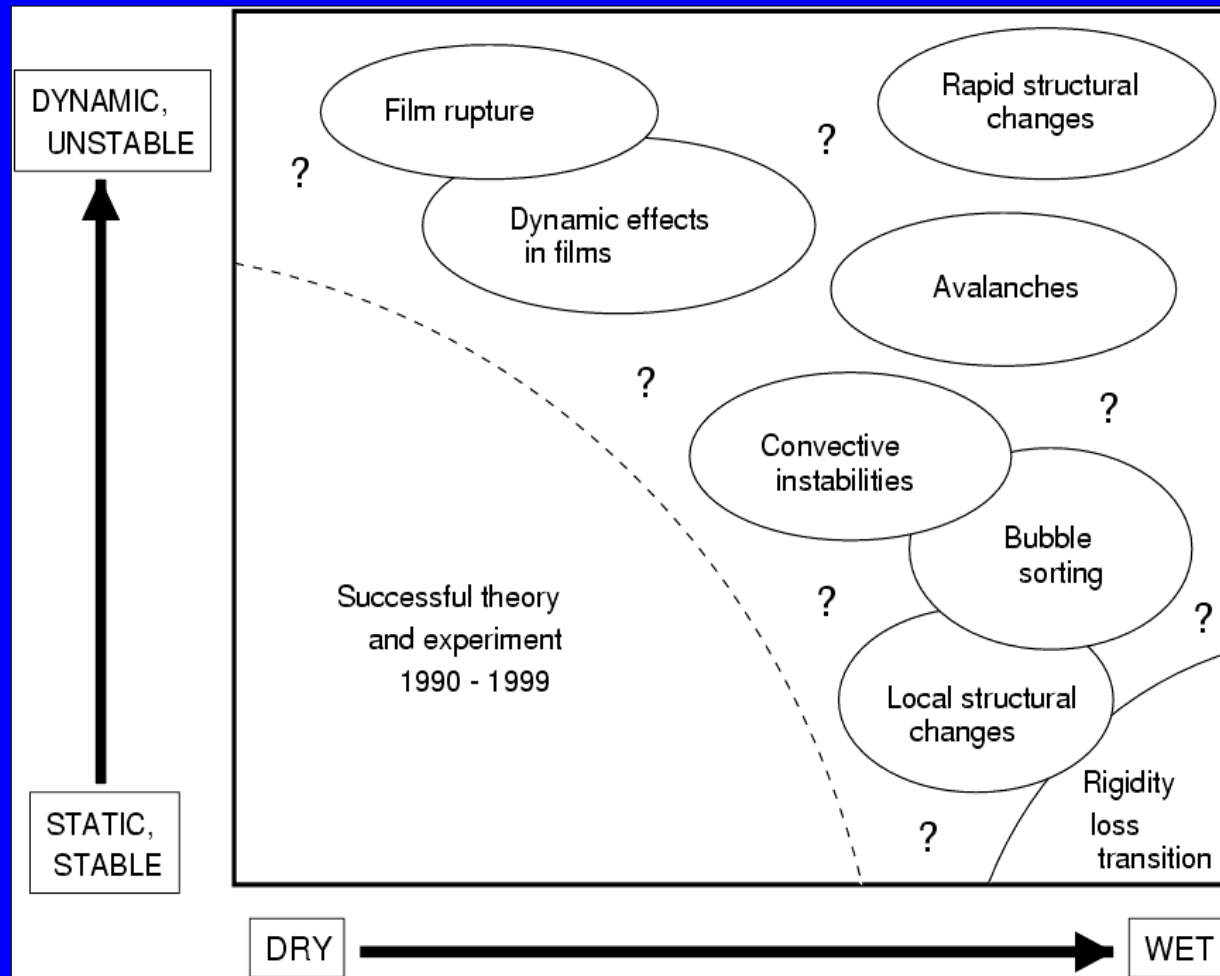
# From statics to dynamics...

Statics and  
quasistatics:  
structure, elasticity,  
coarsening

Dynamics: drainage  
and rheology



# New frontiers in foams





# The Challenge to Theory and Simulation

?

Local dynamics of bulk liquid, surfaces and gases

Chemistry, surface chemistry

?

Dynamics of structured elements: films, Plateau borders and junctions

Physics, local rheology

?

Continuum description of bulk properties: drainage, **rheology** (coarsening, collapse)

*RHEOLOGY*





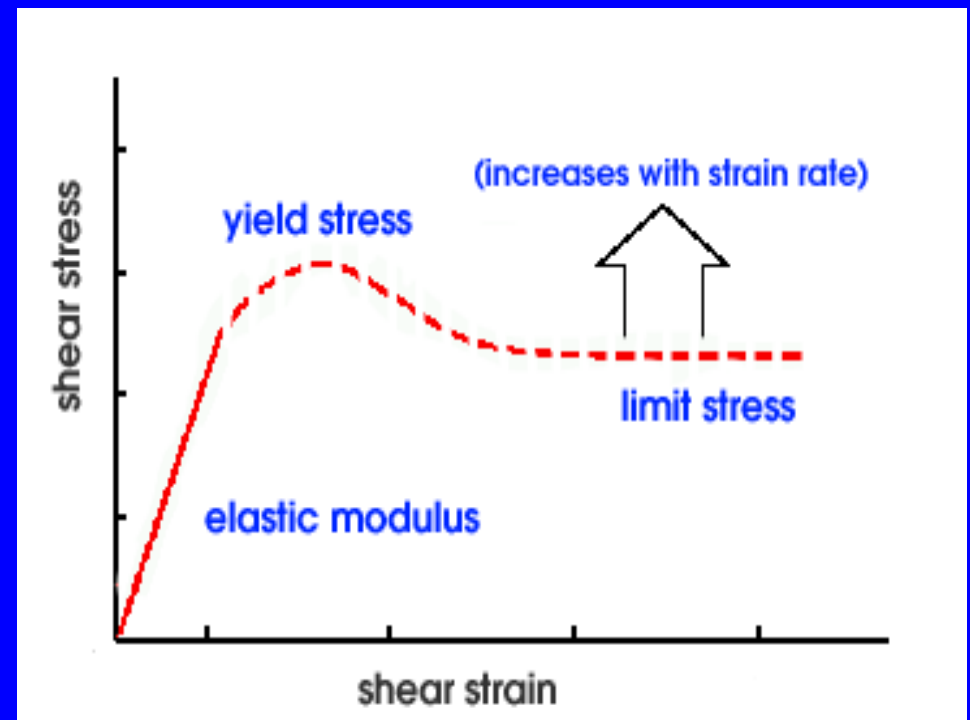
# Rheology with yield stress

Steady shear

Yield stress=limit stress?

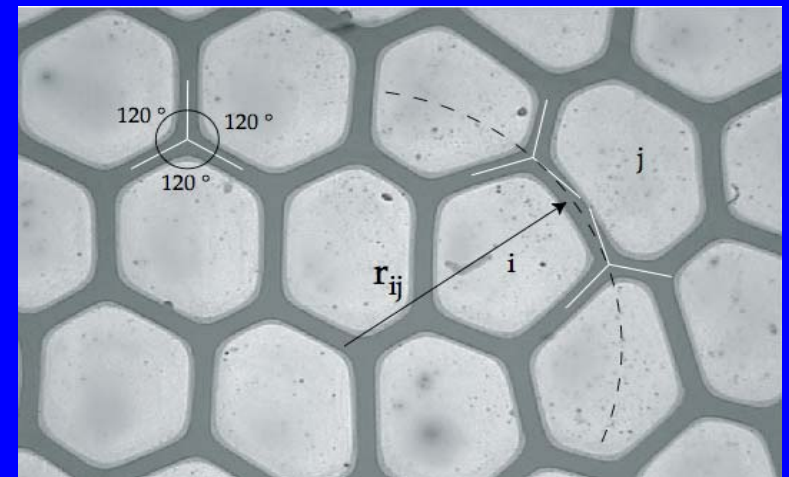
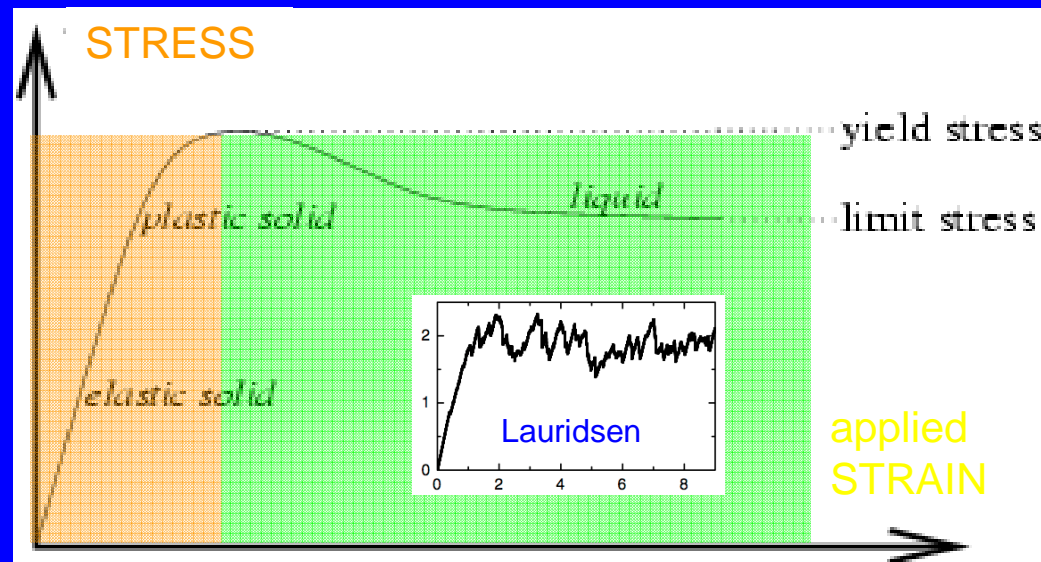
Stress = limit stress + X

$X \sim (\text{strain rate})^n$ ,  $n = ?$

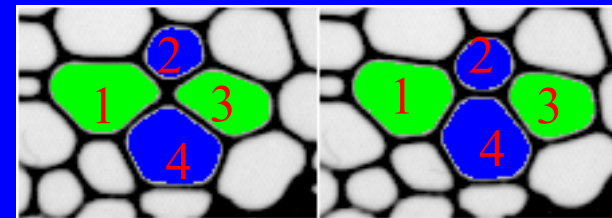


# (quasi) 2D foams as a model system for foam rheology

- simpler and easier than 3D foams!
- same generic mechanical properties
  - elastic at small strain
  - flow at large strain (hysteretic!)
  - funneous?



T1 events: plasticity



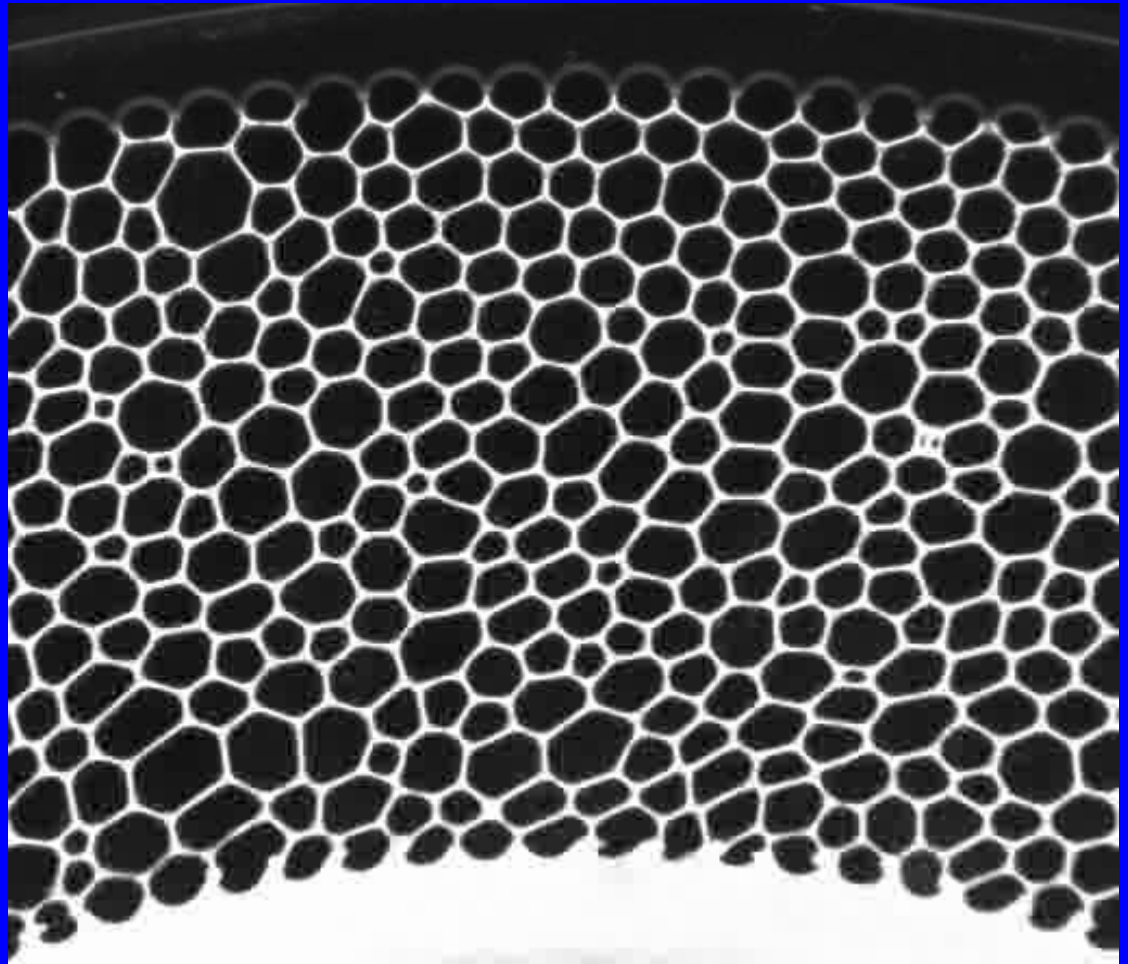


# The Experiment of Debregeas

Between glass plates

Localisation

G.Debregeas *et al*,  
Physical Review Letters  
8717,8305 (2001)



# A 2d effect: wall drag

$$F \sim v^n$$

Mobile Surface

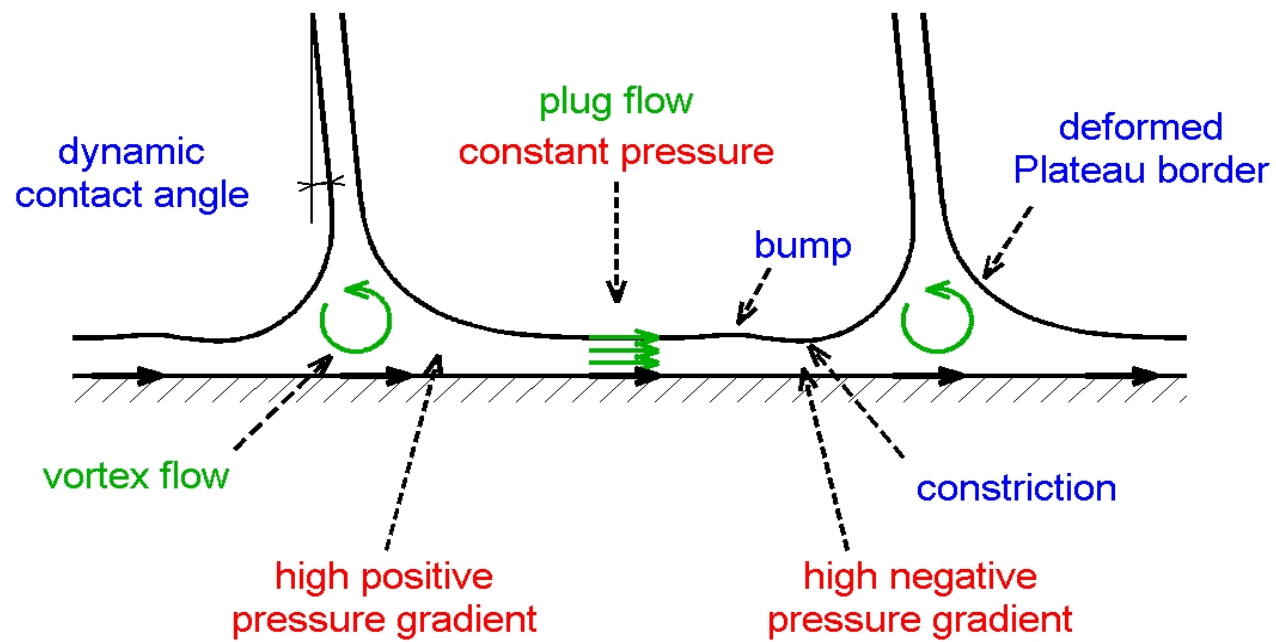
$$n = 2/3$$

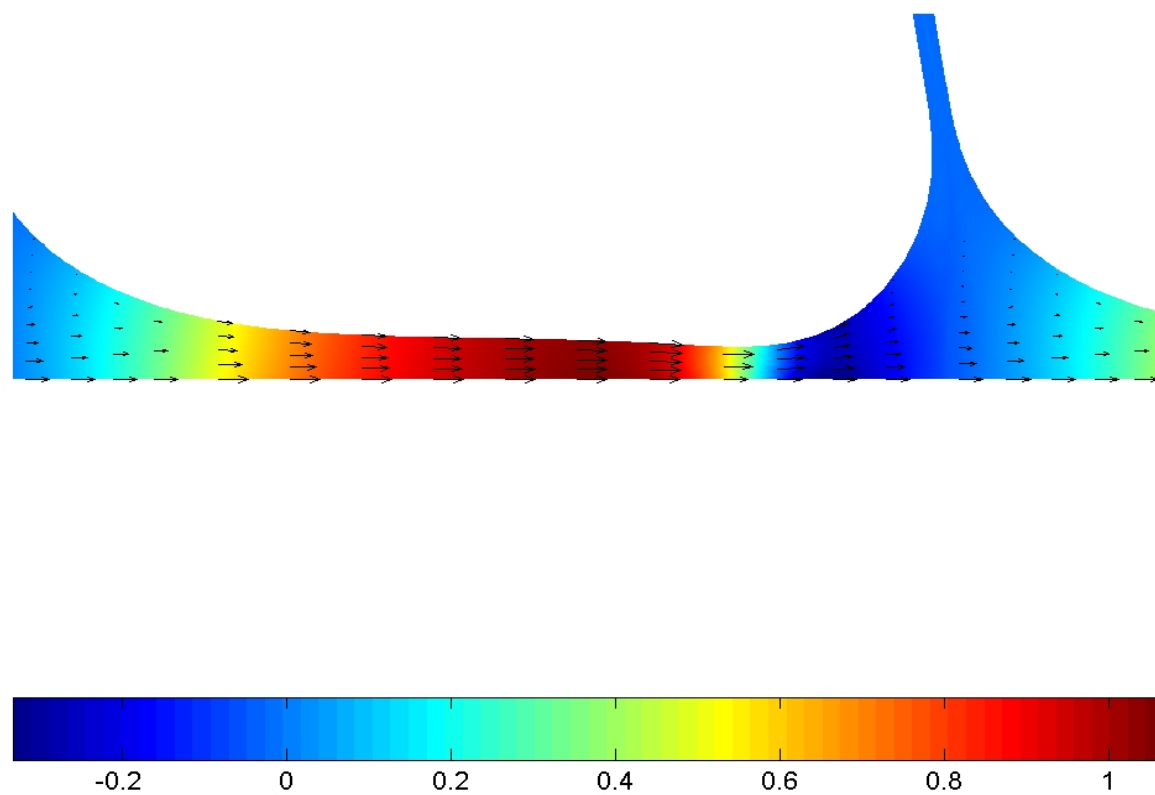
Bretherton

Immobile Surface

$$n = 1/2$$

Denkov *et al.*







# A continuum model?

Detailed quasistatic simulations have shown localisation (shear banding) but with some anomalous features.

Why not attempt a continuum description?

# Four key ingredients:

Bingham-like constitutive equation:

$$\sigma = \sigma_0 f(\gamma/\gamma_0) + \eta \gamma$$

Elastic

Plastic

viscous

$$f(x) = \tanh(x)$$

(linear) Viscous drag:

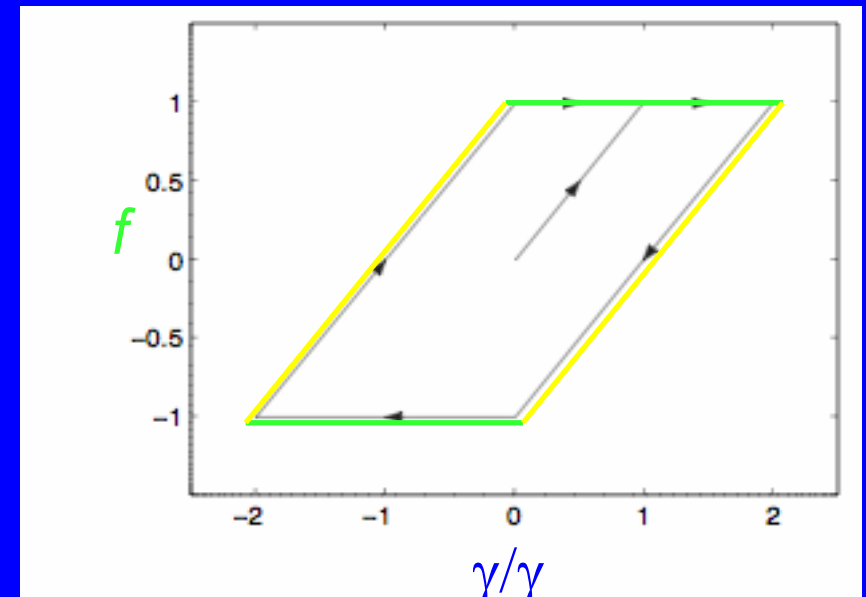
$$F = \beta v$$

$\sigma_0$  yield stress

$\eta$  Bingham viscosity

$\beta$  viscous drag coefficient

hysteretic cycle



Scaling parameters of the fluid:

characteristic length:

$$L_0 = \sqrt{\frac{\eta}{\beta}}$$

characteristic velocity:

$$V_0 = \frac{\sigma_Y}{\sqrt{\eta\beta}}$$

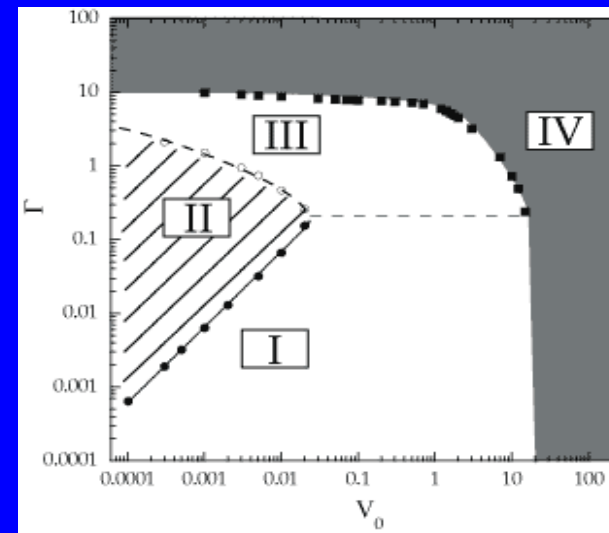
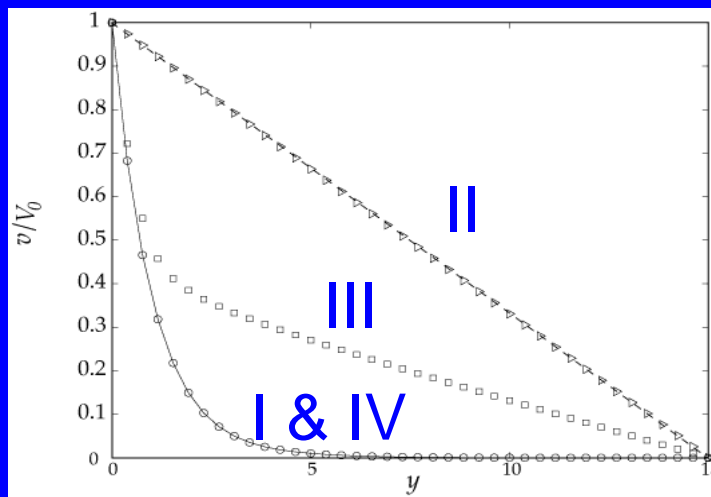
# Hence a simple continuum model for quasi 2D foams

- simple constitutive equation

$$\sigma = \sigma_y \tanh \gamma + \eta \dot{\gamma}$$

- adding viscous drag for quasi 2D foams:

$$\operatorname{div} \sigma = \beta v$$

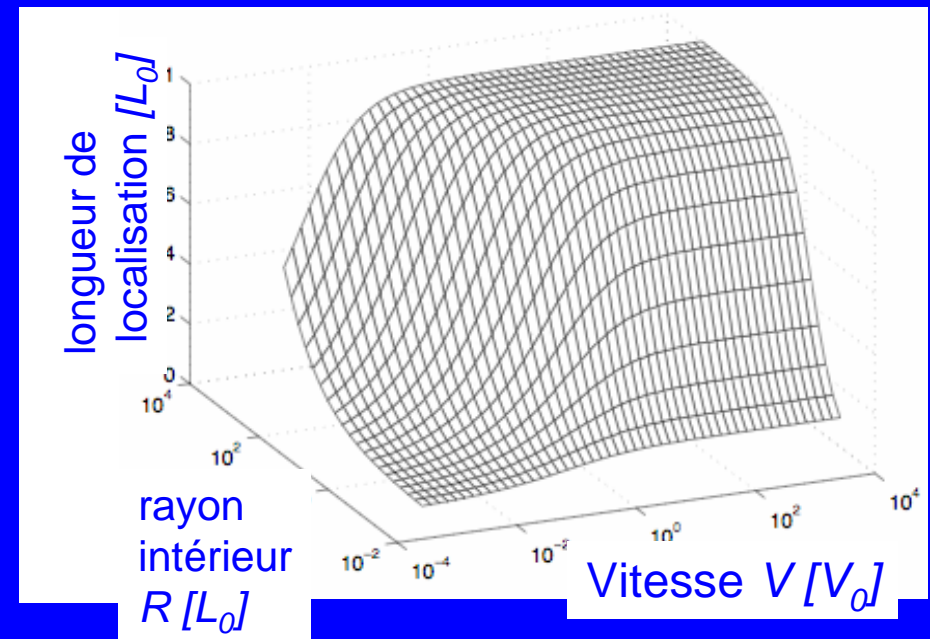
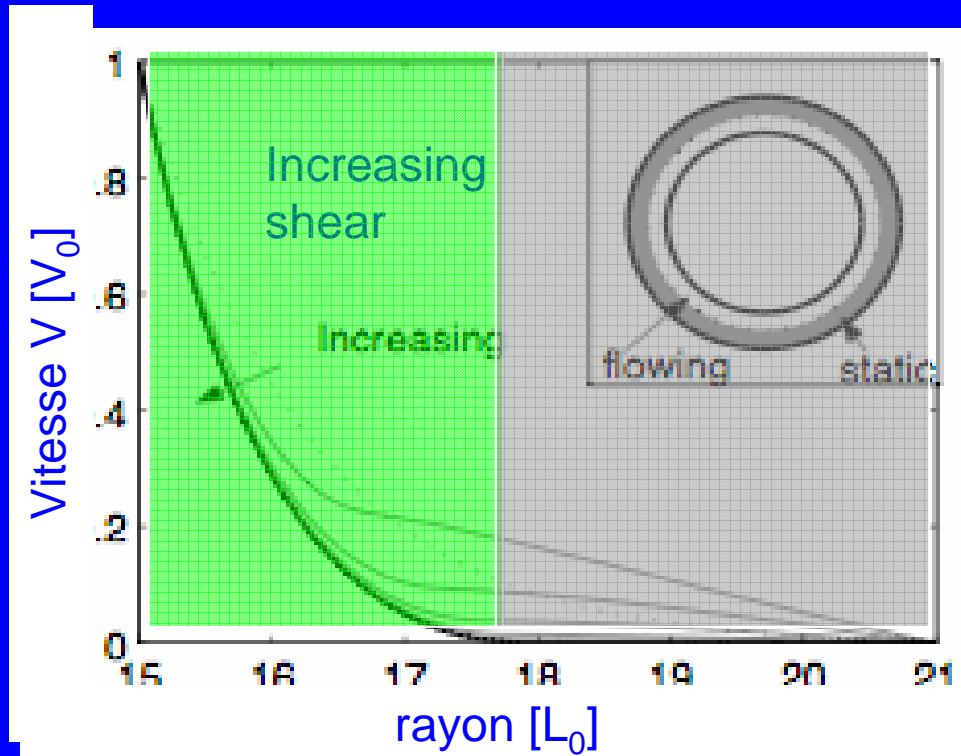


D. Weaire, E. Janiaud, S. Hutzler, PRL



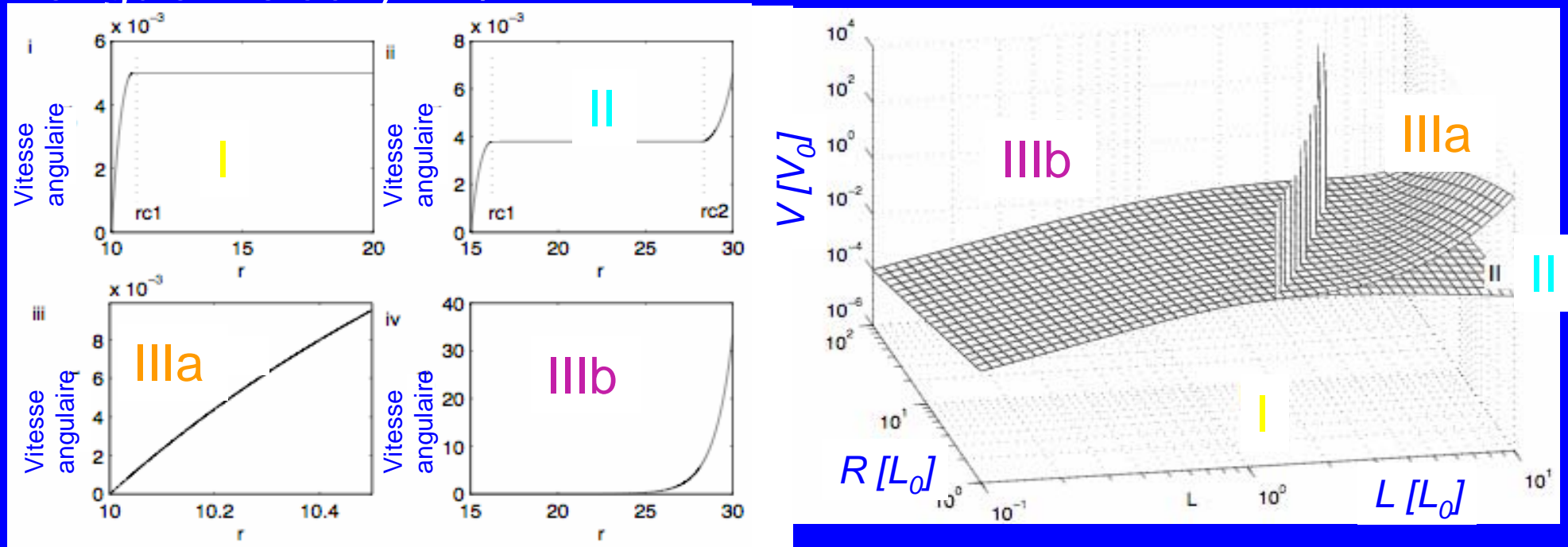
# Couette geometry: inner rotating boundary

Localisation length varies



# Outer moving boundary

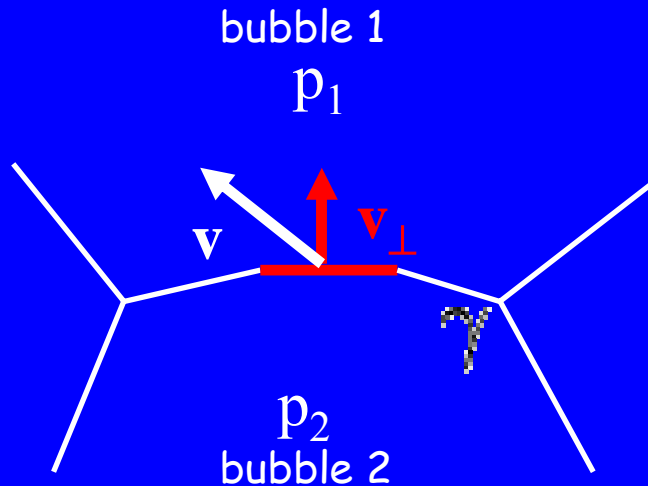
angular velocity =  $v/r$



I et IIIa experimentally observed (Lauridsen prl 2004 & 2002)

IIIa et IIIb correspond to the straight edge case

# Dynamics with drag?



$P_i$  - pressure in bubble I

$\gamma$  - surface tension

$c$  - curvature

$\lambda$  - drag coefficient

$\mathbf{v}_\perp$  - normal velocity

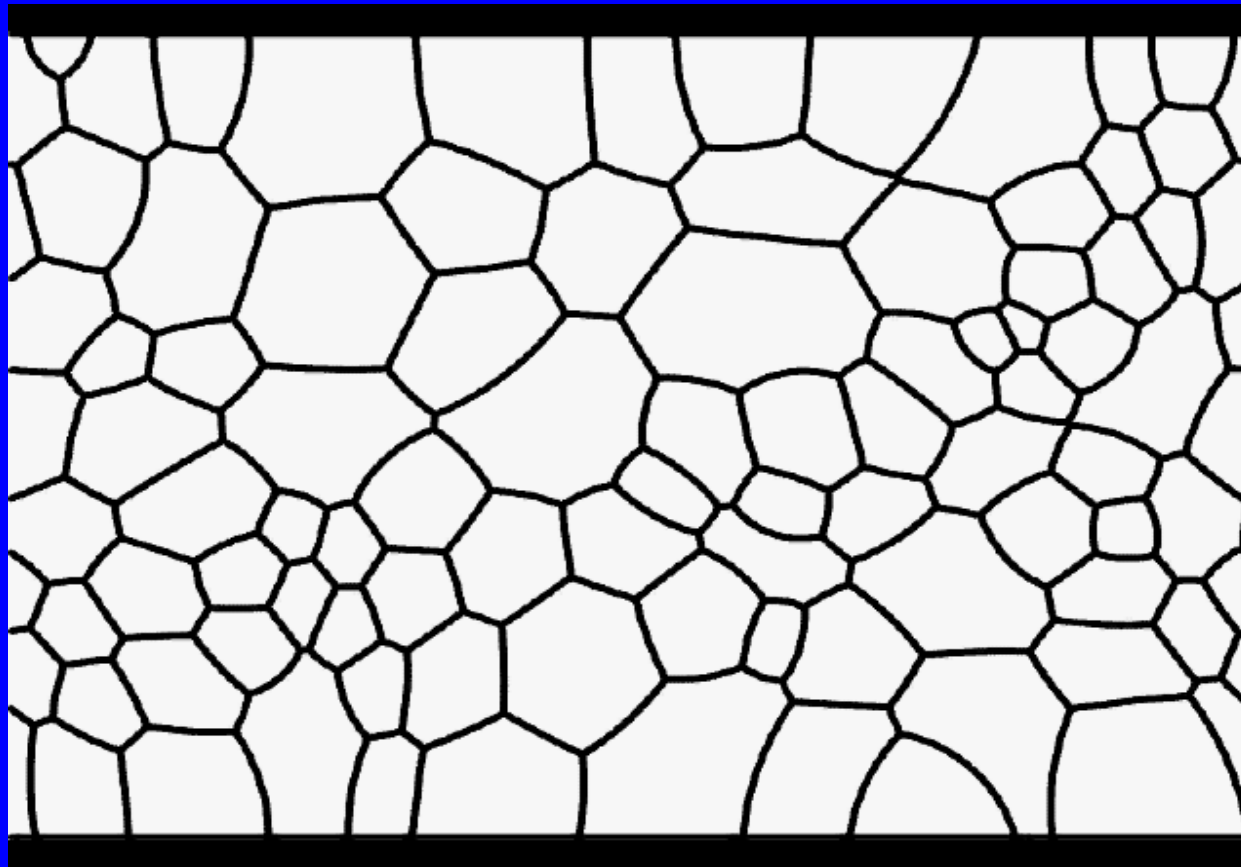
Laplace law:

$$P_1 - P_2 = \gamma c$$

Viscous froth:

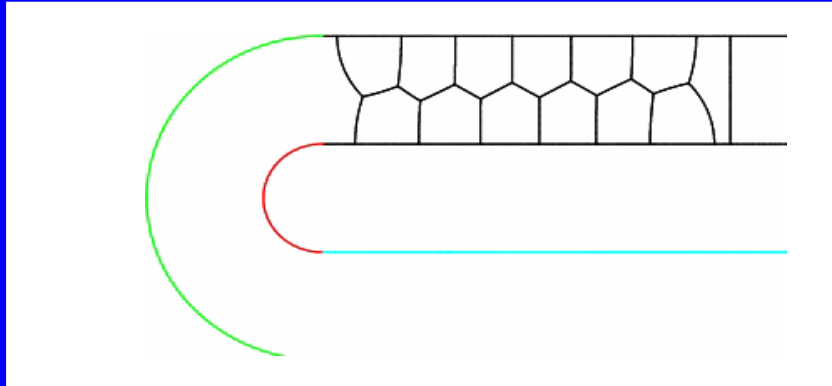
$$P_1 - P_2 = \gamma c - \lambda \mathbf{v}_\perp$$

## Viscous Froth 2

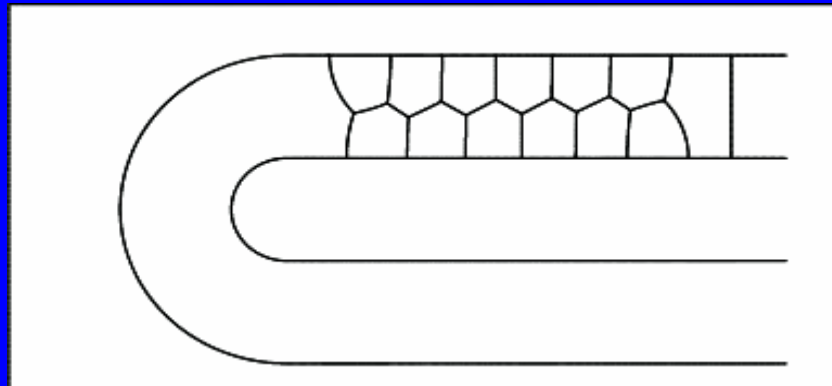




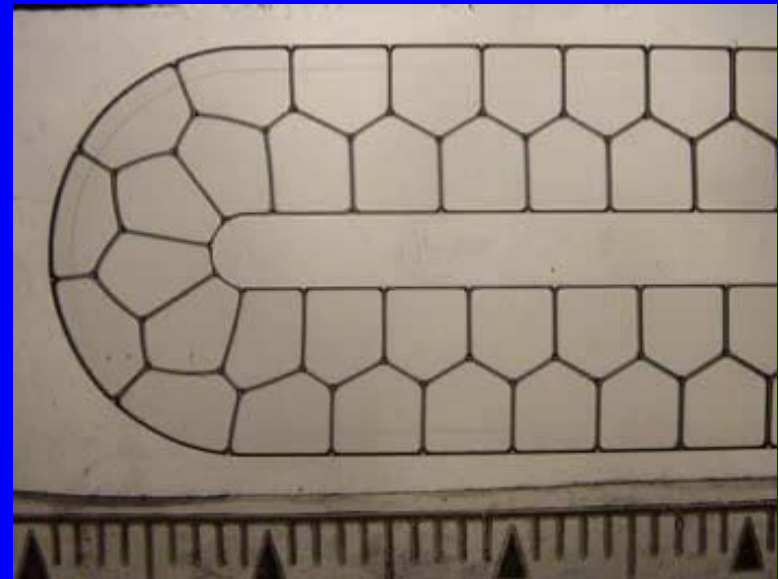
# Geometric effects in channels



quasi – static --> no T1

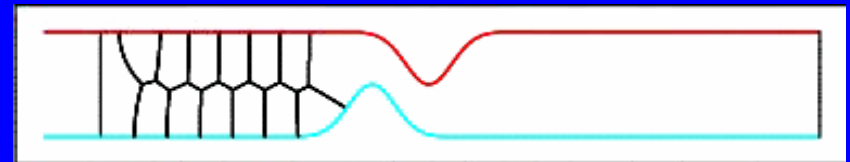
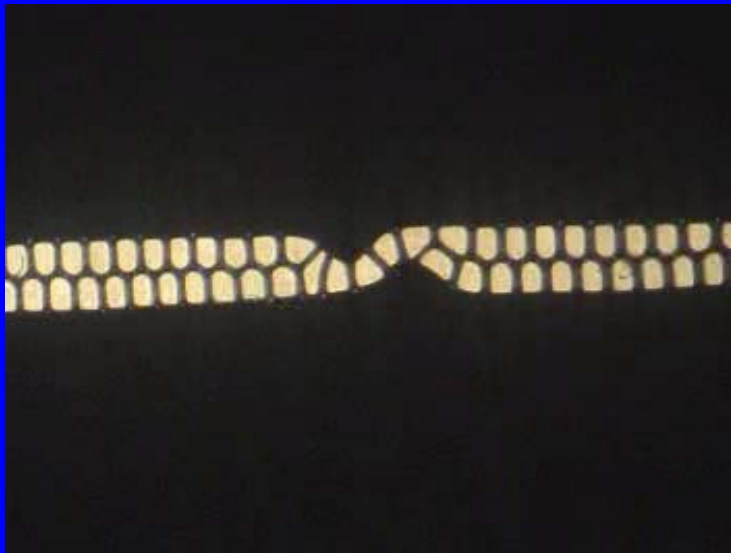
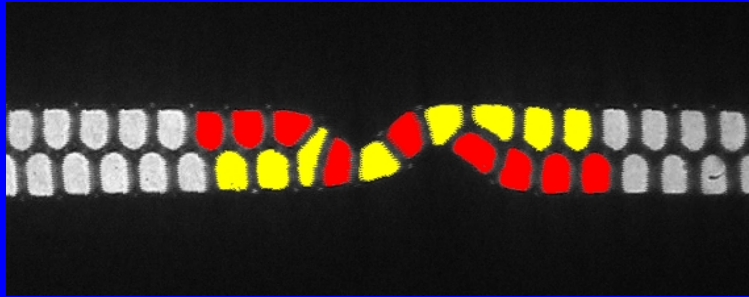


viscous froth --> T1

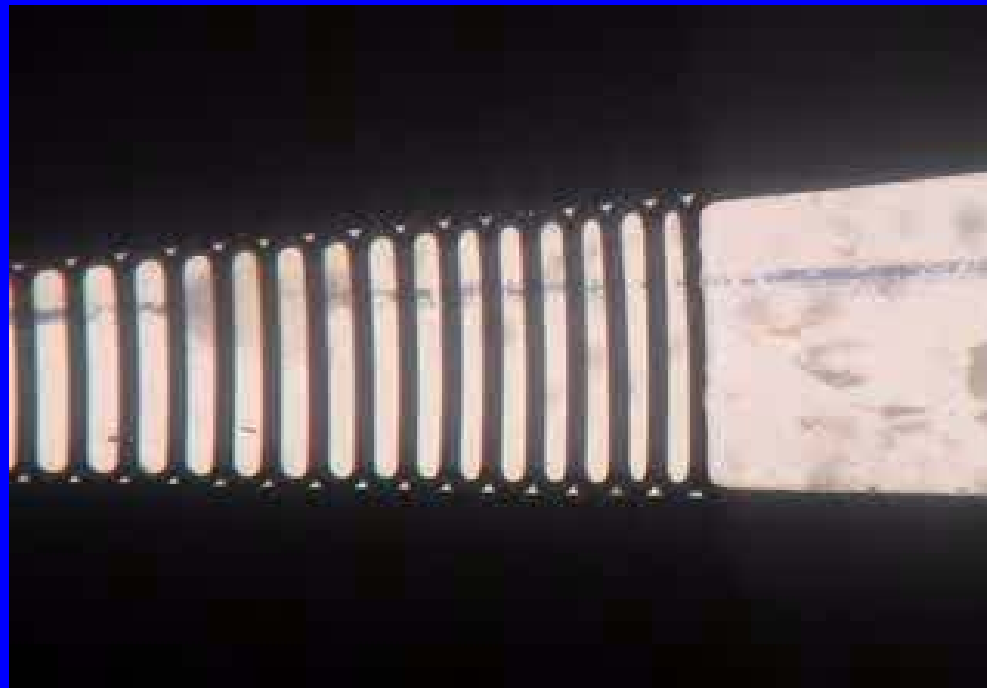


Experiment  
increase velocity

# The Flipper



# The Zipper



# The Y-junction

