Nonlinearly Induced Diffraction - a New Model Equation

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Abstract

A new effect called nonlinerly induced diffraction is proposed and discussed. This effect is due to the nonlinearity in the div **D** term in Maxwell equations and prevents in a natural way the "catastrophical collapse" known from the cubic nonlinear Schroedinger equation. A new model equation is derived and its solitary solutions are discussed.

Nonlinear wave equation

Let us consider propagation of a light-beam $\boldsymbol{E} = (E_X, 0, E_Z)$ of frequency ω along the z-axis in a bulk isotropic Kerr-medium with nonlinear polarization $\boldsymbol{P}^{NL} = \varepsilon_0 \chi (E_X^2 + E_Z^2)\boldsymbol{E}$. The wave equation [1,2, 3] can be written in the dimensionless form

$$
\begin{aligned}
i\frac{\partial E}{\partial z} + \Delta_\perp E + |E|^2 E + 4\frac{\partial^2 |E|^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} + \frac{8}{3}\left|\frac{\partial E}{\partial x}\right|^2 E - \frac{4}{3}E^*\left(\frac{\partial E}{\partial x}\right)^2 \\
+ \frac{4}{3}\frac{\partial}{\partial x}\left(E^2 \frac{\partial E^*}{\partial x}\right) - \frac{8}{3}\frac{\partial}{\partial x}\left(|E|^2 \frac{\partial E}{\partial x}\right) = 0
\end{aligned}
\tag{1}
$$

where $E \equiv \gamma E_X$, $\gamma^2 = (3\omega^2\chi)/(16\beta^2 c^2)$, $\Delta_\perp \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and the co-ordinates are normalized by $2\beta x \to x$, $2\beta y \to y$, $2\beta z \to z$; with $\beta = \frac{\omega}{c}\sqrt{\varepsilon_L}$ for the linear wavenumber. Equation (1) accounts for a vector, nonparaxial model (see, e.g., [10]) of self-focussing in (2+1)D.

First three terms in (1) correspond to the NLSE obtained in a paraxial approximation:

$$
i\frac{\partial E}{\partial z} + \Delta_\perp E + |E|^2 E = 0
$$

In (1+1) dimension this reads

$$
i\frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} + |E|^2 E = 0.
$$

This equation is fully integrated, and has an infinite number of soliton solutions. One of the predictions of NLSE is the self-focusing. The beam focusing increases the field intensity.

The higher field intensity – the higher value of $\mathbf{P^{NL}}$ which results in further focusing and so on. This leads, however, to an unphysical collapse of the beam, known as the *catastrophycal collap*se. A natural limitation of the focusing are the next nonlinear terms introduced first by Pushkarov, Pushkarov and Tomov (1979) [1]. They derived qubic-quintic NLSE which possesses soliton-like solutions too, even if the equation itself is not fully integrated. The behavior of the solutions depends on the sign of the fifth-order term, and, for the appropriate sign, prevents beam collapse.

There is however a fundamental problem whether a restriction exists which follows from the internal properties of Maxwell eqs. and does not depend on model properties. We show here that such a mechanism exists due to the nonlinearity in the $\operatorname{div}\mathbf{D}$ term (neglected in the paraxial approximation). To find the effect of this nonlinearity one has to consider the vector model (taking into account all components of **E**). This leads to Eq. (1). It can be shown [2,3] that the leading terms in Eq. (1) are included in the following "simplified" version:

$$i\frac{\partial E}{\partial z} + \Delta_\perp E + |E|^2 E + 4\frac{\partial^2\left(|E|^2 E\right)}{\partial x^2} = 0 \tag{2}$$

This equation does not contain free parameters. It is written in a dimensionless form and can serve as a **model equation** which describes the nonlinear induced diffraction (NLID) with the same generality as NLSE. Note, that even if the NLID equation was derived by means of the vector model, it represents a scalar effect.

Solitary wave solutions

In order to see the difference between the vector model Eq. (1) and the NLID Eq. (2) we consider them in parallel. We look for solitary-wave solutions of the form

$$E(x,y,z) = F(x,y)\exp(i\mu z),$$

where μ is the nonlinear wavenumber shift. Then Eqs. (1) and (2) reduce to one and the same equation, but with different values of the coefficients:

$$(1 + aF^2)\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} - kF + F^3 + bF\left(\frac{\partial F}{\partial x}\right)^2 = 0\,. \tag{3}$$

The quantities a=12, b=24 and $k=\mu$ refer to the solution of Eq. (2) whereas a=32/3, b=68/3 and $k=\mu+\mu^2$ refer to Eq. (1). A variational method [7, 11] leads to the following form of Eq. (3) for NLID:

$$\frac{\partial}{\partial x}\left(\frac{\partial L_0}{\partial F_X}\right)+\frac{\partial}{\partial y}\left(\frac{\partial L_0}{\partial F_Y}\right)-\frac{\partial L_0}{\partial F}=-4\frac{\partial^2 F^3}{\partial x^2}, \tag{4}$$

where $L_0=\frac{|\nabla F|^2}{2}-\frac{F^4}{4}+\frac{kF^2}{2}$ is the Lagrangian associated with the stationary NLS equation. For a trial function $F(x,y)=Aexp\left[-\left(x^2/a^2\right)-\left(y^2/b^2\right)\right]$ (see, e.g., [7]) the parameters of the solution are related to each other according to:

$$\frac{1}{a^2}=\frac{A^2}{8+42A^2}, \quad \frac{1}{b^2}=\frac{1}{a^2}+\frac{9}{2}\frac{A^2}{a^2}, \quad k=\frac{A^2}{4}+\frac{3A^2}{a^2}. \tag{5}$$

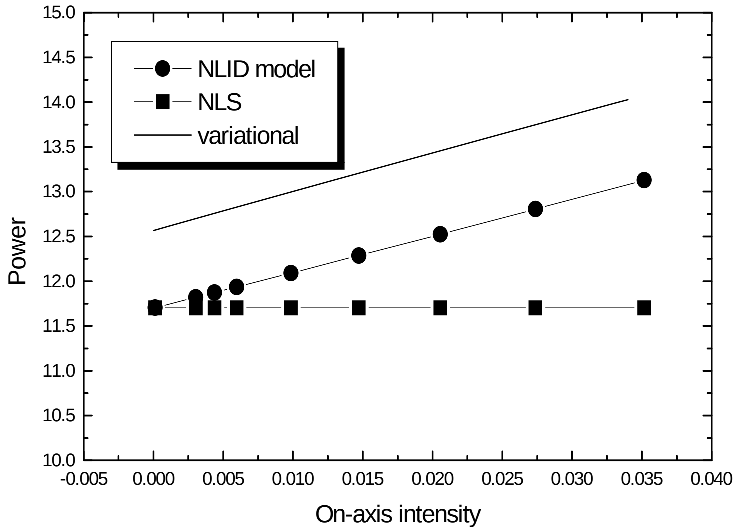
A perturbation scheme is another approach to solving Eq. (3). With transformation $x=r\cos\varphi$, $y=r\sin\varphi$ the solution can be written in the form $F(r,\varphi)=F_0(r)+f_0(r)+f_1(r)\cos 2\varphi$, $k=k_0+\Delta k$, where f_0, f_1 and Δk are the first order corrections, and Eq. (3) reduces to:

$$\frac{\partial^2 F_0}{\partial r^2}+\frac{1}{r}\frac{\partial F_0}{\partial r}-k_0F_0+F_0^{\,3}=0 \tag{6}$$

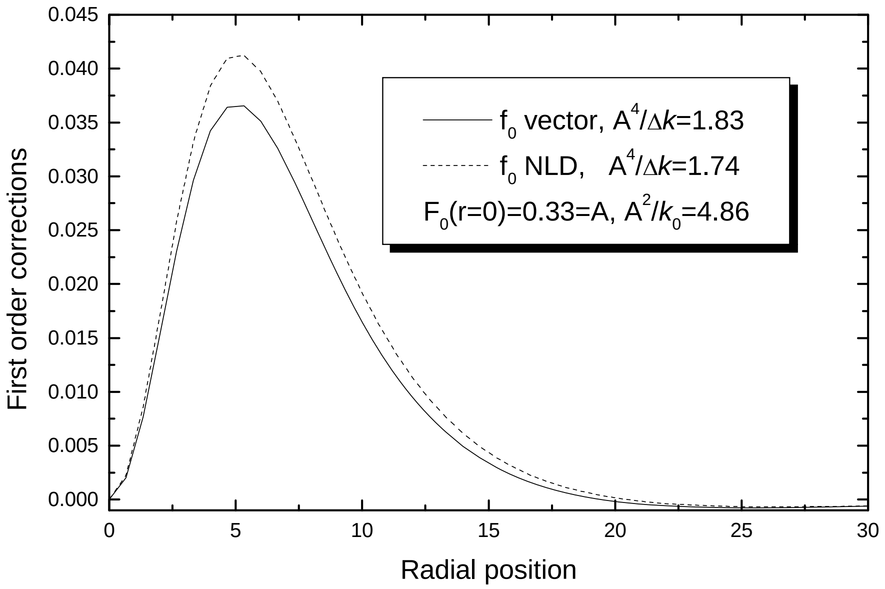
$$\frac{\partial^2 f_0}{\partial r^2}+\frac{1}{r}\frac{\partial f_0}{\partial r}+\left(3F_0^{\,2}-k_0\right)f_0=\Delta kF_0-\frac{aF_0^{\,2}}{2}\left(\frac{\partial^2 F_0}{\partial r^2}+\frac{1}{r}\frac{\partial F_0}{\partial r}\right)-\frac{bF_0}{2}\left(\frac{\partial F_0}{\partial r}\right)^2 \tag{7}$$

$$\frac{\partial^2 f_1}{\partial r^2}+\frac{1}{r}\frac{\partial f_1}{\partial r}+\left(3F_0^{\,2}-\frac{4}{r^2}\right)f_1=-\frac{aF_0^{\,2}}{2}\left(\frac{\partial^2 F_0}{\partial r^2}-\frac{1}{r}\frac{\partial F_0}{\partial r}\right)-\frac{bF_0}{2}\left(\frac{\partial F_0}{\partial r}\right)^2. \tag{8}$$

Equations (6), (7) and (8) are solved by using the shooting technique. The solution of Eq. (6) is checked by comparison with the result in Ref. [13].



NLID model
NLS
variational
Power
On-axis intensity



f_0 vector, $A^4/\Delta k$=1.83
f_0 NLD, $A^4/\Delta k$=1.74
F_0(r=0)=0.33=A, A^2/k_0=4.86
First order corrections
Radial position

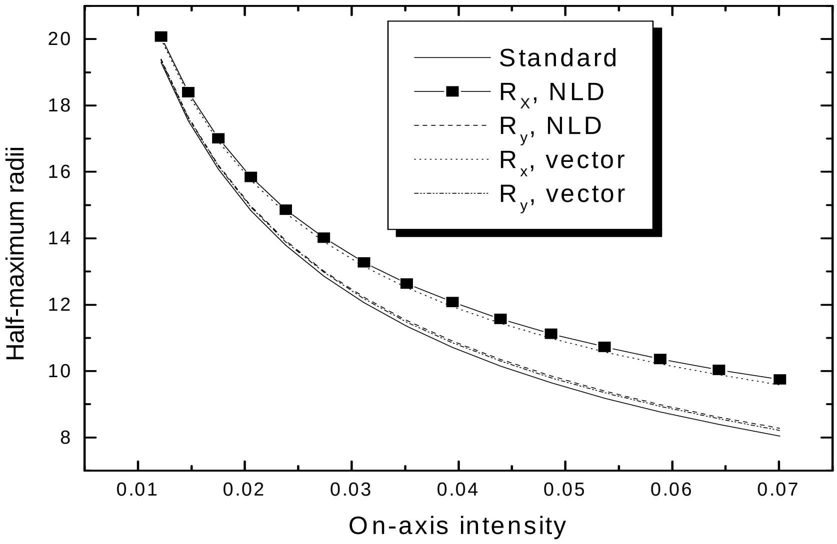


Fig. 1. The solutions of Eq. (3) show that the NLID prevails over the longitudinal-field-component effect. In the NLID model (Eq. (2)) the obtained nonlinear wavenumber shift is $\mu_{NLID} \approx \left(A^2/4.86\right)+\left(A^4/1.74\right)$

(up to the A4–order), whereas the full – vector and nonparaxial model (Eq. (1)) – gives

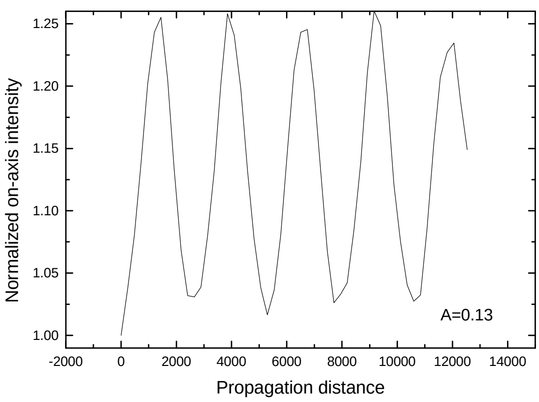
$\mu_{VN} \approx \left(A^2/4.86\right)+\left(A^4/1.74\right)\left[1-(1/20.52)-(1/13.52)\right]$.

The first correction in the second term is due to the longitudinal field component and the second one is related to the nonparaxiality, thus validating the use of Eq. (2) instead of Eq.(1).

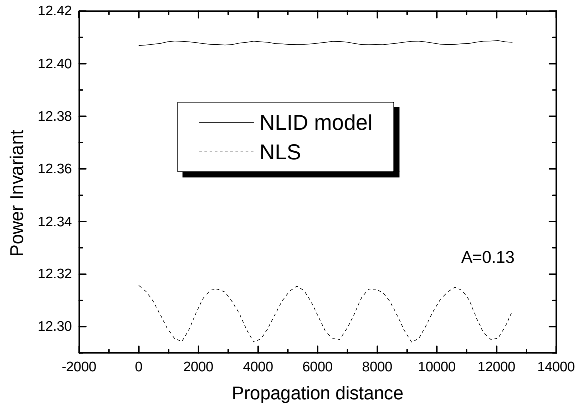
Beam propagation analysis

With an assumption for a small deviation from a circular symmetry of the beam (which is equivalent to not too strong NLID) the power invariant of Eq. (4) is given by $P=\int_0^{2\pi}\int_0^{\infty} r(|E|^2+|E|^4)drd\varphi$, up to $|E|^4$–order.

The solitary wave beams display oscillations (periodical focusing and defocusing) as Fig 2(a), (b) shows. The period of the oscillations rapidly decreases with the increase of the input power. The oscillating behaviour can not be associated *only* with the deviations of the initial condition (obtained from the perturbation scheme) from the exact solution of Eq. (3) because if this is the case, the decrease of the input power increases the accuracy of the perturbative approach and, thus, the amplitude of the oscillations should decrease.



(a)



(b)

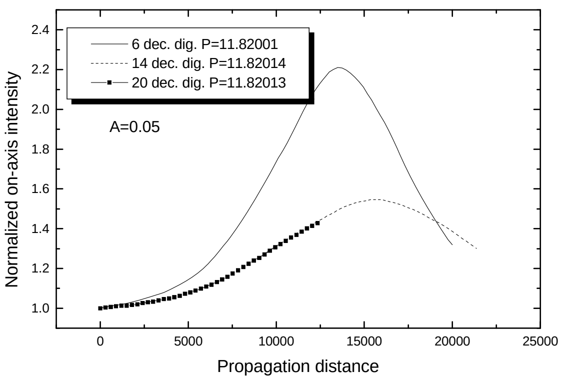


Fig. 2 (c)

However, the situation is the opposite one: the stabilizing action of NLID increases with the power increase. The sensitivity of the beam behaviour to small changes of the input shape, at lower power values, is demonstrated in Fig. 2(c). This sensitivity is a leftover from the collapse instability, removed by NLID. The increase of the power (more narrow beams) makes the solitary waves less sensitive to deviations of their shape from that of the exact solutions of Eq. (3), i.e. they are more stable.

Conclusions

Bright spatial solitary waves in bulk self-focusing Kerr medium influenced by NLID are studied both with respect to their stationary (amplitude, width, power etc.) and dynamical (stability) properties. The stabilizing role of NLID is demonstrated.

Acknowledgements. The work is supported by the National Fund for Scientific Research and the Foundation, grant F-1517.

References

[1] Kh.I. Pushkarov, D.I.Pushkarov and I.V.Tomov (1979) *Opt.Quant.Electr.* **11** 471
[2] A.D. Boardman, K. Marinov, D.I. Pushkarov and A. Shivarova (2000) *Opt. Quant. Electr.* **32**, 49.
[3] K. Marinov, D.I. Pushkarov and A. Shivarova (2000) *Phys. Scripta* **T84**, 197.
[4] P. L. Kelley (1965) *Phys. Rev. Lett.* **15**, 1005.
[5] E. L. Dawes and J. H. Marburger (1969) *Phys. Rev.* **179**, 862.
[6] S. Vlasov, V. Petrishtev and V. Talanov (1971) *Izv. VUZ Radiofizika* **14**, 1353.
[7] M. Karlsson (1992) *Phys. Rev. A* **46**, 2726.
[8] J. T. Mannasah and B. Gross (1992) *Opt. Lett.* **17**, 976.
[9] M. D. Feit and J. A. Fleck, Jr. (1988) *J. Opt. Soc. Am. A* **5**, 633.
[10 S. Chi and Qi Guo (1995) *Opt. Lett.* **20**, 1598
[11] Y.S. Kivshar and W. Krolikowski (1995) *Opt. Commun.* **114**, 353.
[12] D. Anderson, F. Cattani and M. Lisak (1999) *Phys. Scripta* **T82**, 32.
[13] R.Y. Chiao, E. Garmire and C.H. Townes (1964) *Phys. Rev. Lett* **13**, 479.