Nonlinear dynamics for signal identification

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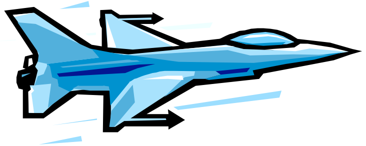
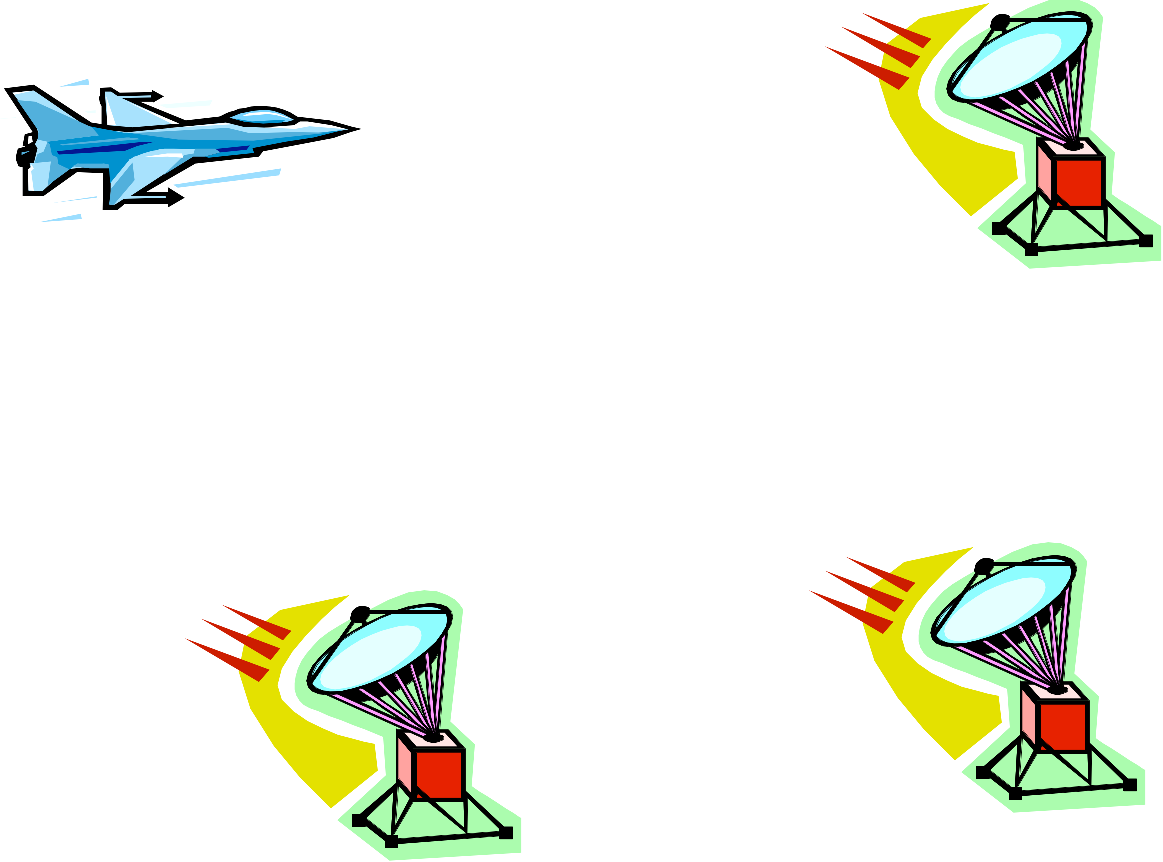


Multiple radars: how many transmitters are there?

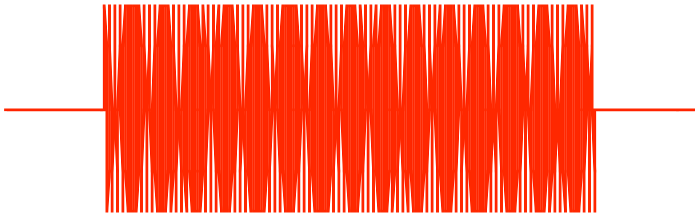


Specific Emitter Identification

Older transmitters

Transients on leading and trailing edges: use linear signal processing to get signature

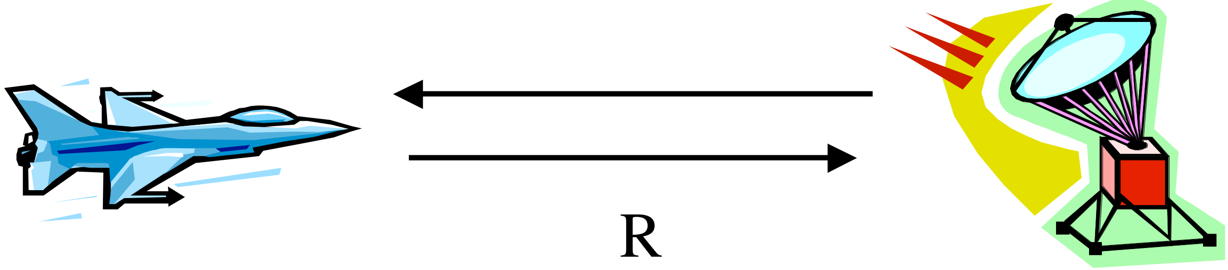
Modern transmitters



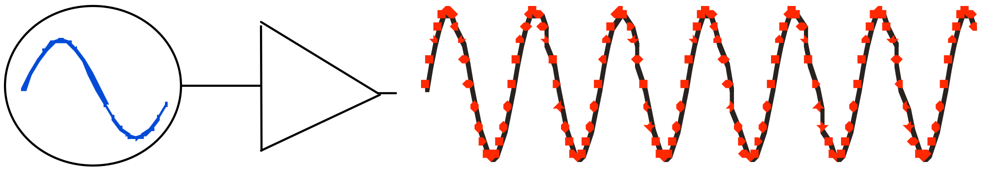
No transients: linear signal processing can’t find signature

Good test problem for NLD approach:

- Radar reflection goes as $1/R^4$, so S/N large at target



Radar amplifier runs at high power- very nonlinear

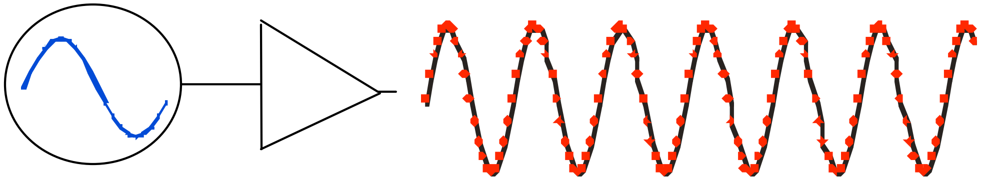


All amplifiers based on nonlinear devices

- consider problem as driven nonlinear system

Specific Emitter ID: Data Driven

Amplifier is driven nonlinear system

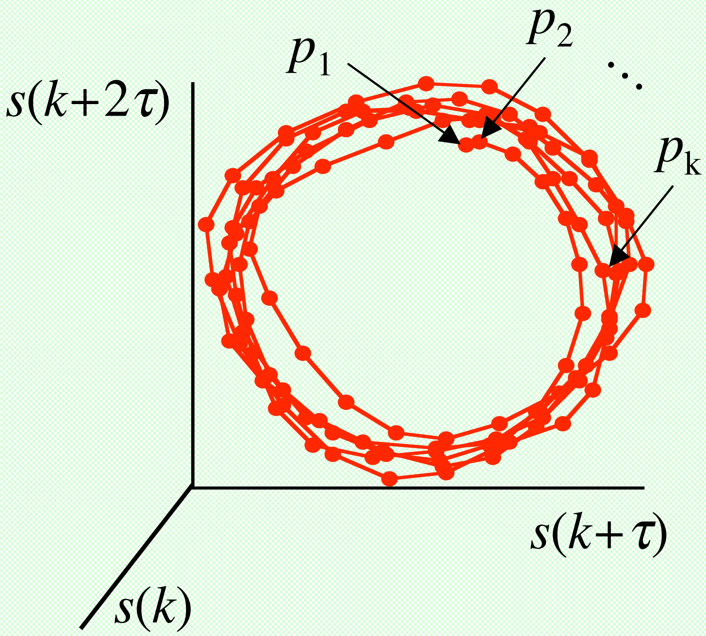


Digitize output signal

Create vectors:

$$p_1 = \begin{bmatrix} s(1) \\ s(1+\tau) \\ s(1+2\tau) \end{bmatrix}, \quad p_2 = \begin{bmatrix} s(2) \\ s(2+\tau) \\ s(2+2\tau) \end{bmatrix}, \quad \ldots$$

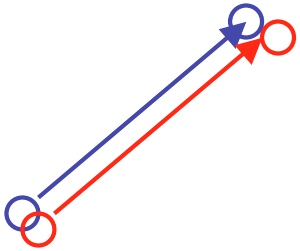
"**Embedding**" of original signal



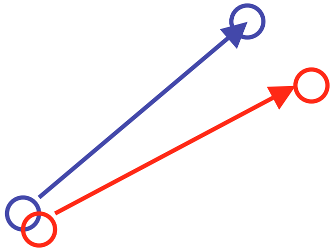
Has same geometrical properties as original attractor

Specific Emitter ID: Data Driven

- If 2 dynamical systems identical, same point in phase space should have same derivatives



Identical systems



Non-identical systems

Quantify this difference: average over signal

Average = <Δ>

Simple numerical example

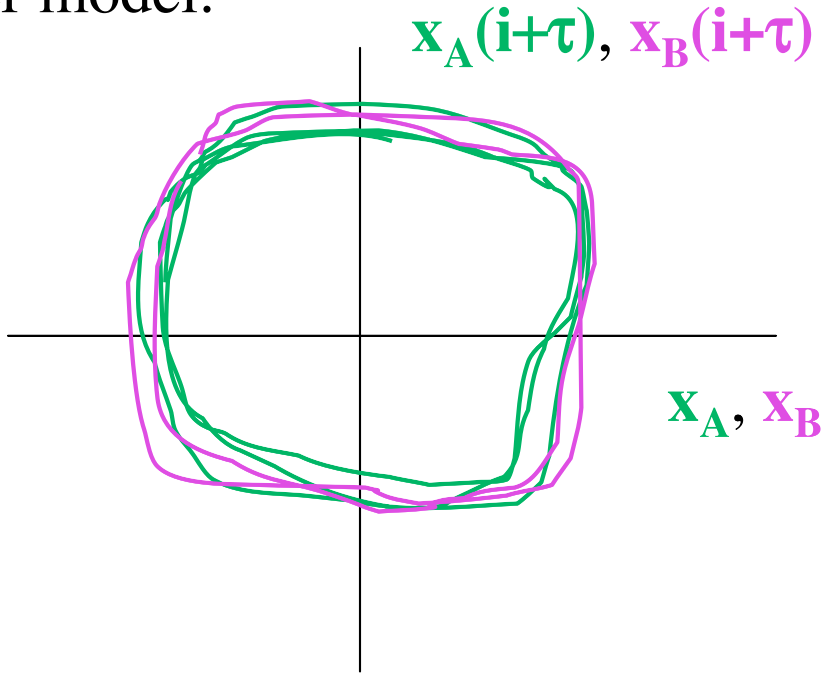
Amplifier model:

$$\xi(t)=\sin(\omega t)$$

$$\frac{dx_A}{dt}=\gamma_A\left(g(\xi)-x_A\right)$$

$$\frac{dy_A}{dt}=\gamma_A\left(\frac{dx_A}{dt}-y_A\right)$$

$$g(x)=x+\rho_A x^3$$



$$\xi(t)=\sin(\omega t)$$

$$\frac{dx_B}{dt}=\gamma_B\left(g(\xi)-x_B\right)$$

$$\frac{dy_B}{dt}=\gamma_B\left(\frac{dx_B}{dt}-y_B\right)$$

$$g(x)=x+\rho_B x^3$$

Set different parameters for A, B

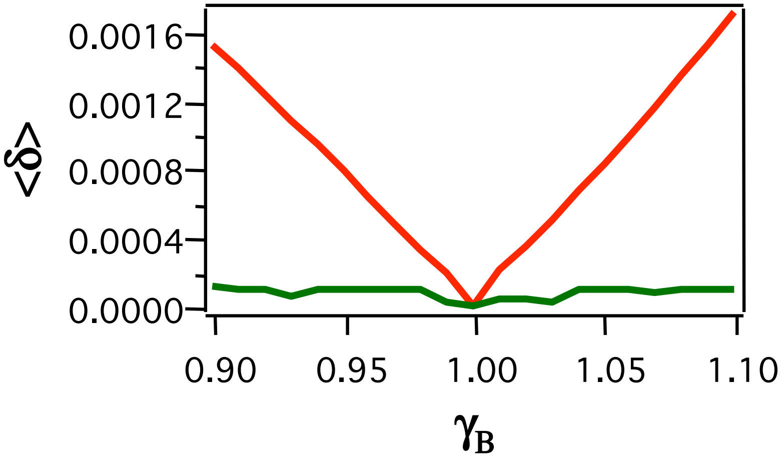
Embed x_A, x_B in 2-d phase space

Derivative difference $\delta = | [x_A(i+1)-x_A(i)] - [x_B(i+1)-x_B(i)] |$

Take average for many points

Simulation results:

Different time constant, linear or nonlinear

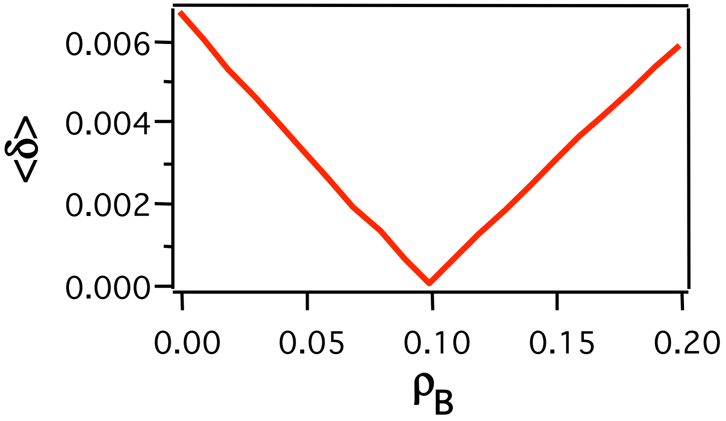


$\gamma_A = 1$

$\rho_A = \rho_B = 0$ (linear)

$\rho_A = \rho_B = 0.1$ (nonlinear)

Same time constant, different nonlinearity



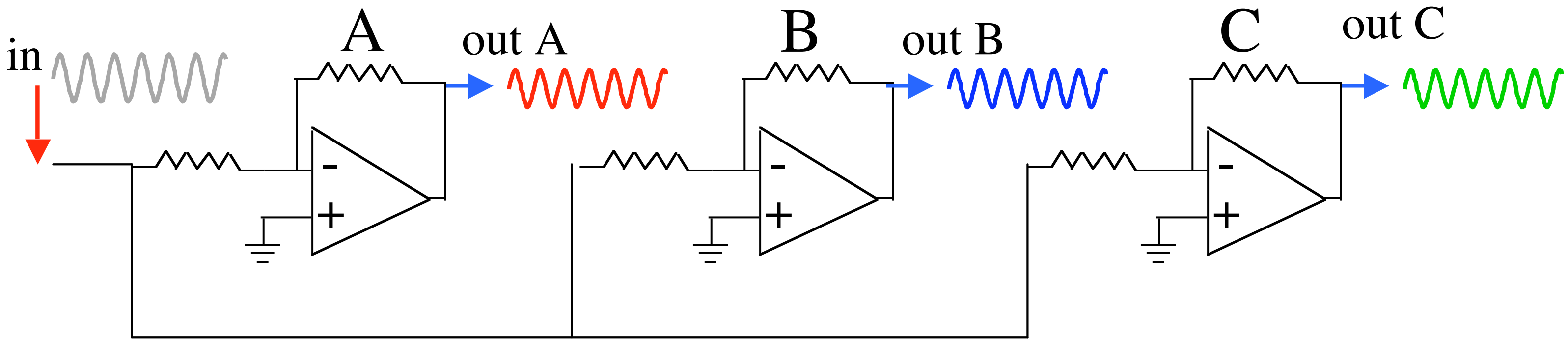
$\gamma_A = \gamma_B = 1$

$\rho_A = 0.1$

Specific Emitter ID: Data Driven

Experiment

3 nominally identical op amps driven with same signal



- Save a signal from each amp as a reference
- Take an unknown signal - compare to each reference
- Which reference gives smallest phase space difference?

Experiment

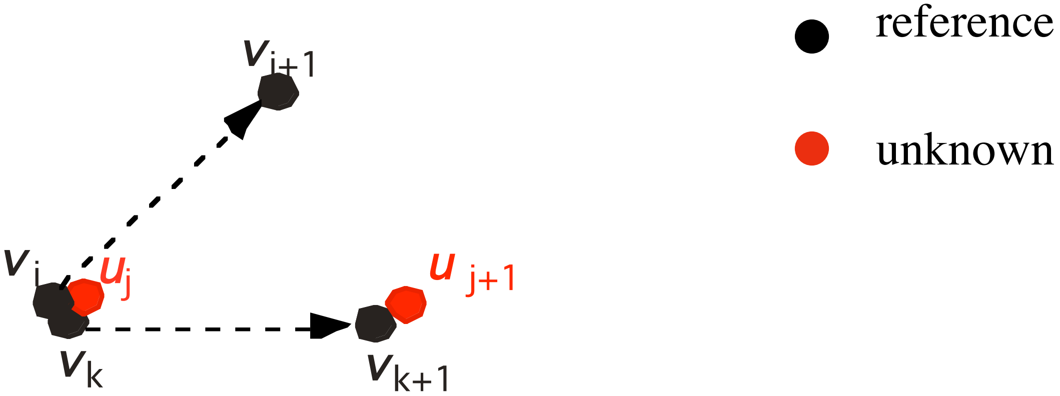
Embed reference
Embed unknown
Pick index point u_j on unknown
Nearest reference point is v_k

● reference

● unknown

Derivatives $u_{j+1}-u_j$, $v_{k+1}-v_k$

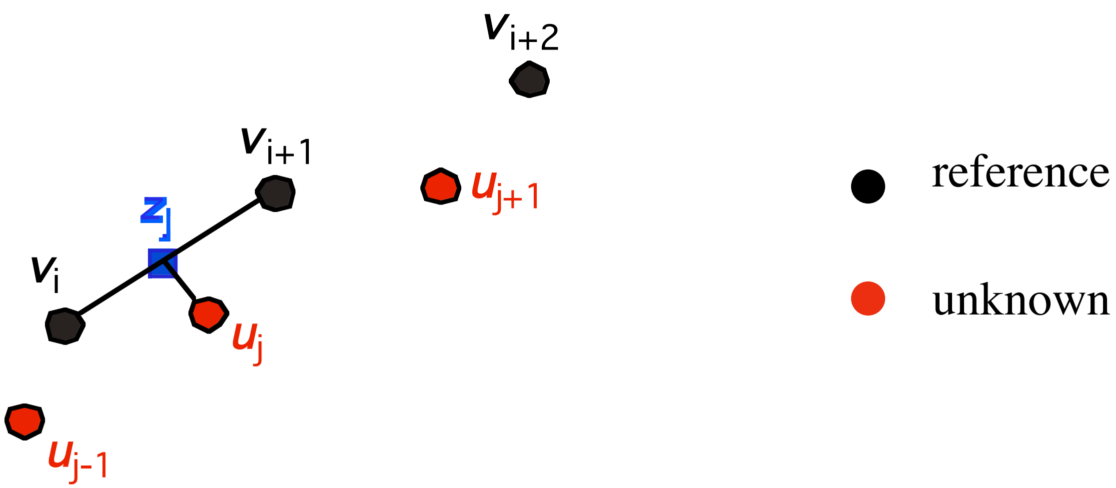
Real Data: complications



Frequency modulation: reference may have 2 very different derivatives

- search for pairs of points (v_i, v_{i+1})

Real Data: complications



Unknown signal sampled at different phase than reference

- Draw line between v’s
- z_j is closest approach of line to u_j
- z_j is estimate for reference point v_i
- difference in derivatives proportional to distance between u and z: normalize by this distance

Phase space statistic:
k is vector component

$$\Delta = \frac{\sum_{k=1}^{d}\left|\overbrace{\left(z_{j+1}^{k} - z_{j}^{k}\right)}^{\text{Reference derivative}} - \overbrace{\left(u_{j+1}^{k} - u_{j}^{k}\right)}^{\text{Unknown derivative}}\right|}{\sqrt{\sum_{k=1}^{d}\left[\left(z_{j+1}^{k} - u_{j+1}^{k}\right)^{2} + \left(z_{j}^{k} - u_{j}^{k}\right)^{2}\right]}} \quad \longleftarrow \text{distance}$$

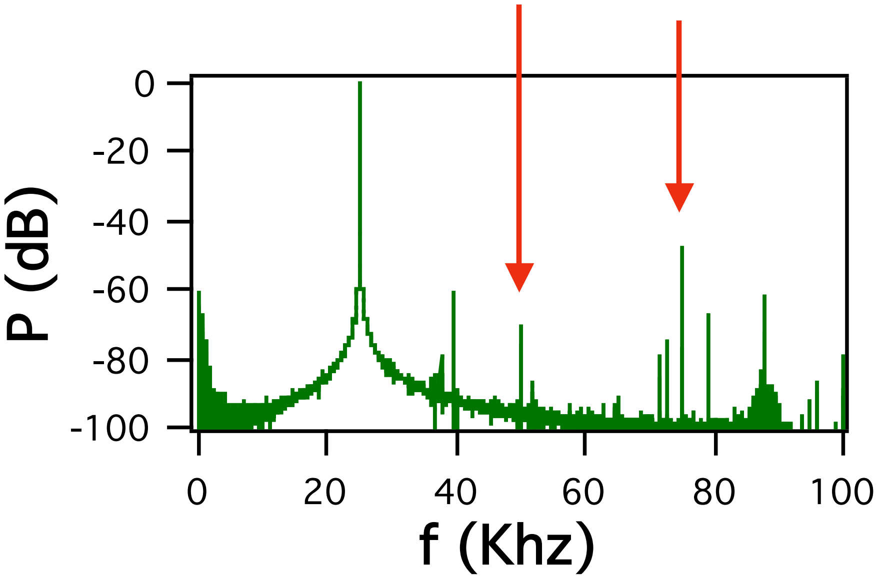
Average Δ over signal: <Δ>

Summary of algorithm

1. Record reference signals from several amplifiers
2. Record unknown signal
3. Embed unknown and reference signals
4. For each reference:
    A. Pick index pair on unknown: search for nearest reference pair.
    B. Interpolate reference point to correct for phase error
    C. Compute unknown and reference derivatives: take difference
    D. Normalize difference by distance between unknown and reference pairs
    E. Average over trajectory.

Experiment:

Drive 3 op amps with 25 kHz sine, amplitude 1V


harmonics

Divide signal into many segments

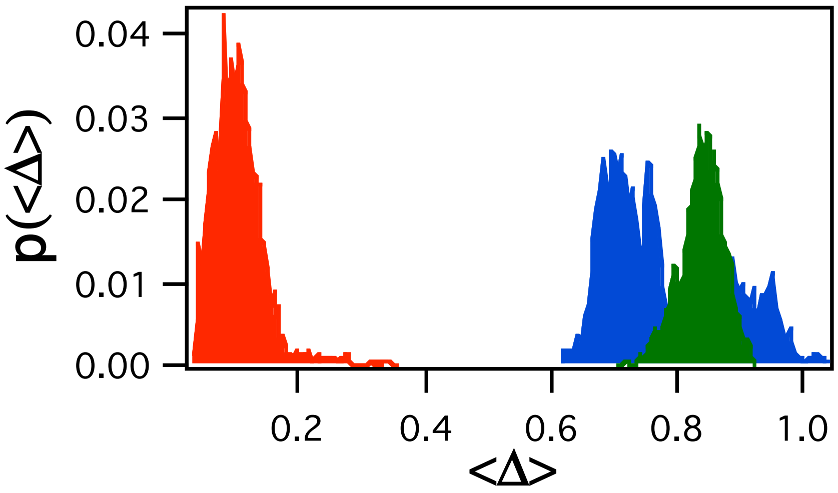
Measure <Δ> for each

Compute histogram

Experiment
Histogram of phase space difference

Compare unknown to:

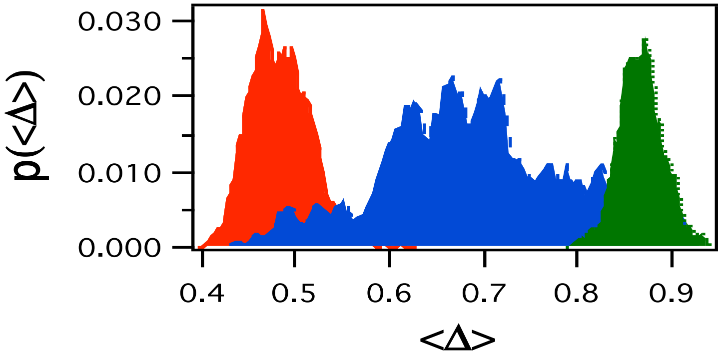
- 🟥 Reference A
- 🟦 Reference B
- 🟩 Reference C



(unknown came from amplifier A)

Experiment:
frequency modulated (FM) signals
Randomly shift signal between 22.5 kHz and 27.5 kHz

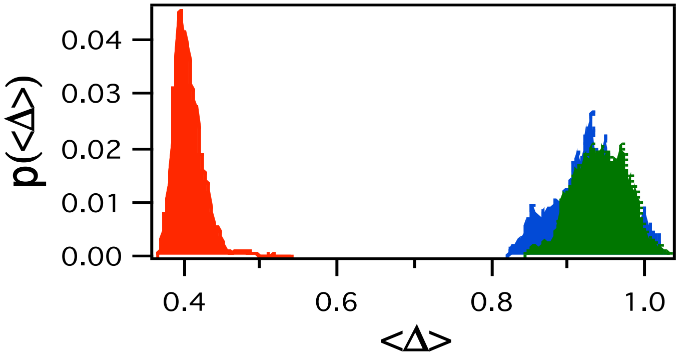
Compare unknown to: 🟥 Reference A 🟦 Reference B 🟩 Reference C



Drive amplitude:

1 V

(reference amplitude = 1V)



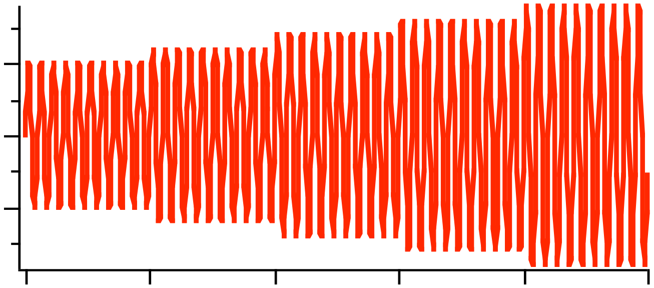
2 V (nonlinear effects larger)

(reference amplitude = 2V)

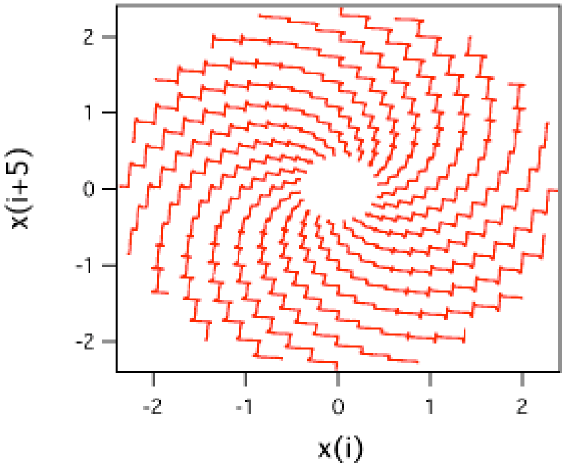
(unknown came from A)

What about signals with different amplitudes?

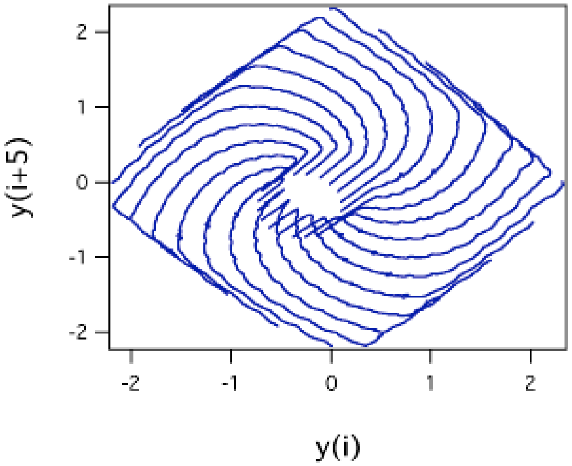
Need information from all parts of phase space
Step up amplitude of driving signal: sine wave



input signal

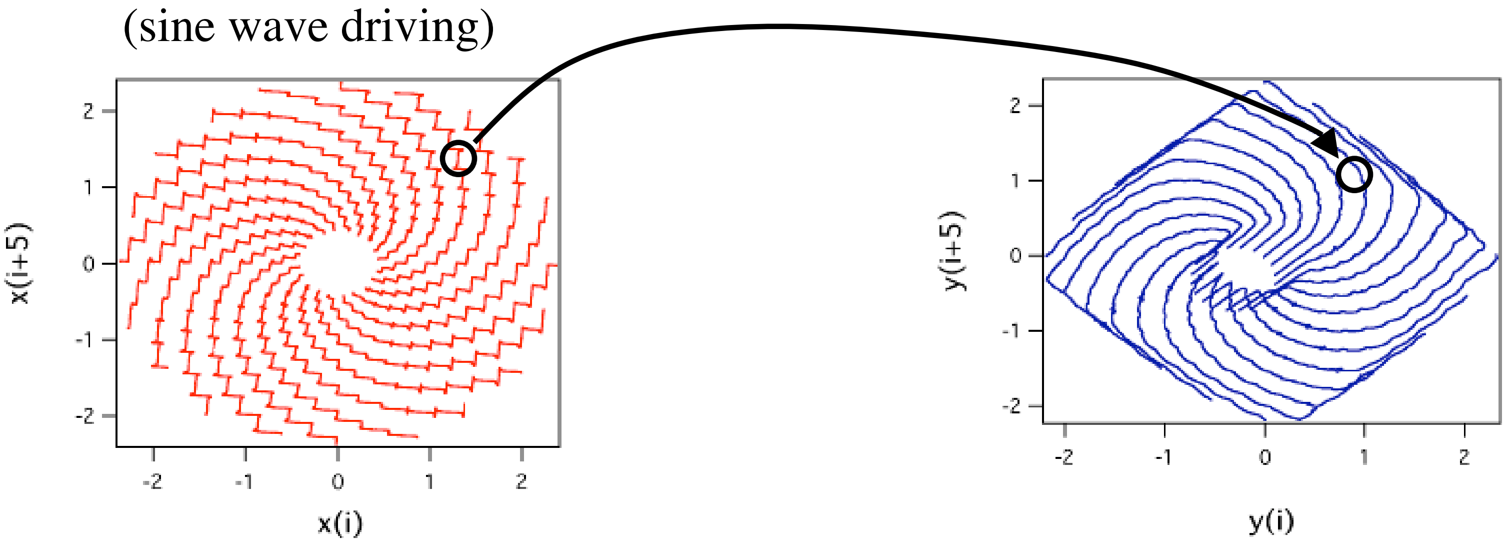


output signal



At each point in phase space, find output derivative as function of input derivative

$$\frac{d\vec{y}\left(\vec{x}\right)}{dt}=a_1\left(\vec{x}\right)+a_2\left(\vec{x}\right)\frac{d\vec{x}}{dt}\bigg|_1+a_3\left(\vec{x}\right)\frac{d\vec{x}}{dt}\bigg|_2+\ldots$$



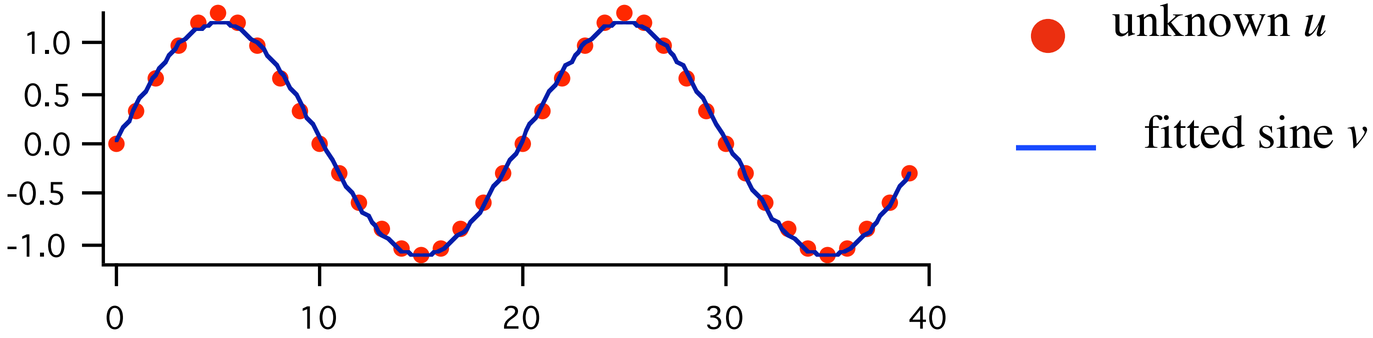
Store coefficients as function of x locations
This is the phase space model

Using phase space model

Record unknown signal output signal u

Assume unknown input signal was sine

Fit sine to u: fitted sine is assumed input signal v

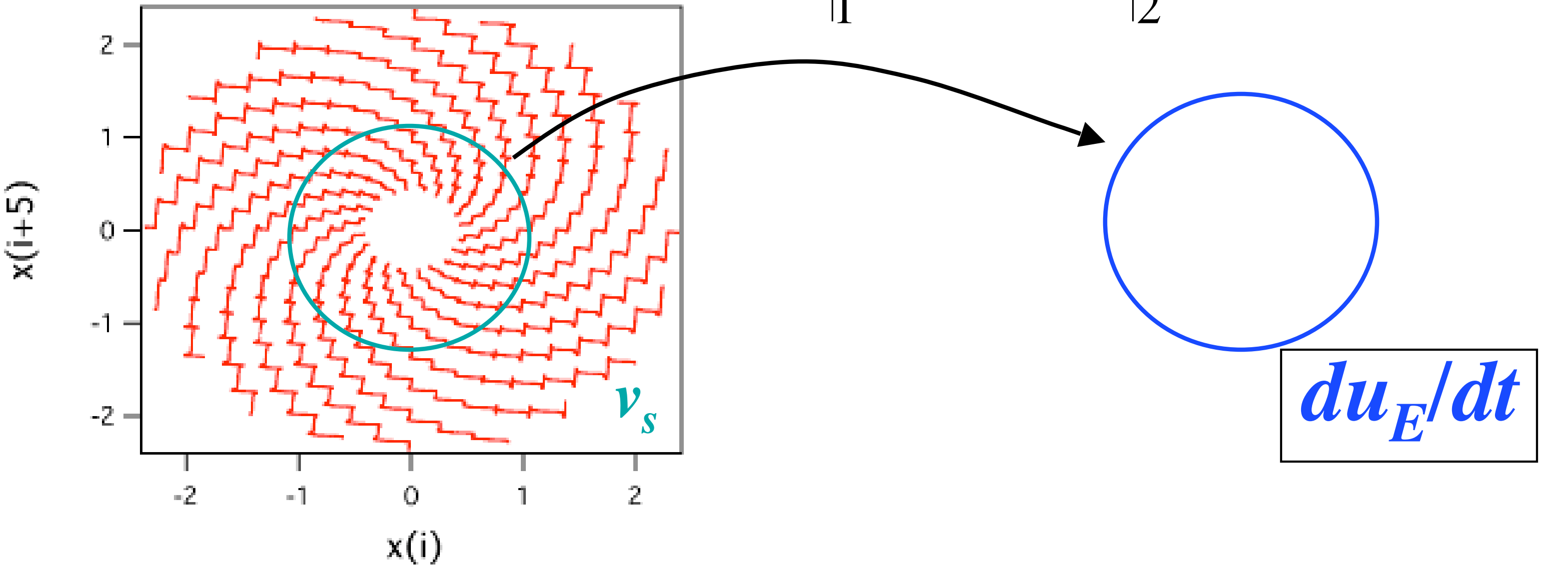


Amplitude of real driving signal not known

Rescale assumed drive signal $v_s = \alpha v$

Use phase space model: estimate unknown du/dt based on v_s

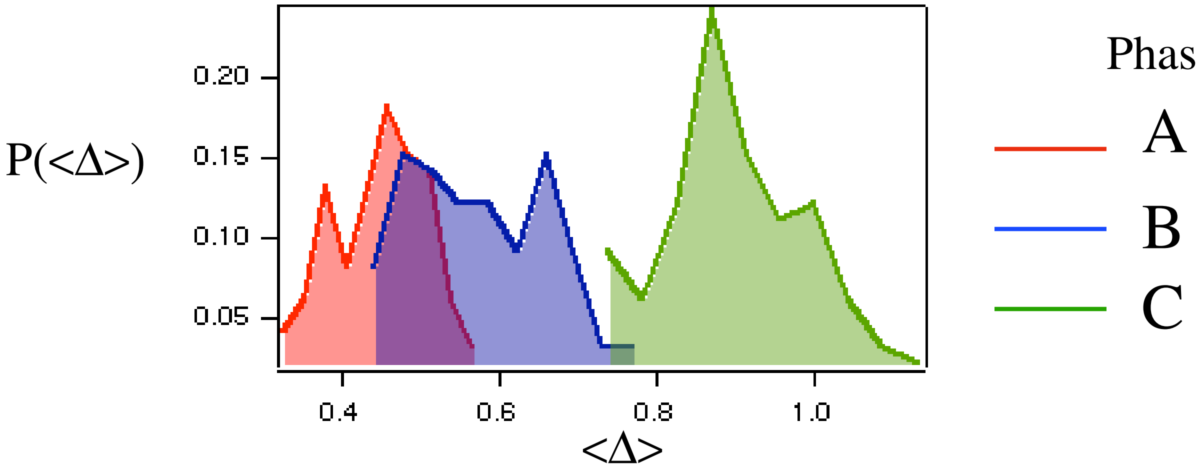
$$\frac{d\vec{u}_E\left(\vec{v}_s\right)}{dt} = a_1\left(\vec{v}_s\right) + a_2\left(\vec{v}_s\right)\left.\frac{d\vec{v}_s}{dt}\right|_1 + a_3\left(\vec{v}_s\right)\left.\frac{d\vec{v}_s}{dt}\right|_2 + \ldots$$



Find value of α to minimize $\alpha du/dt - du_{E/}dt = \Delta$

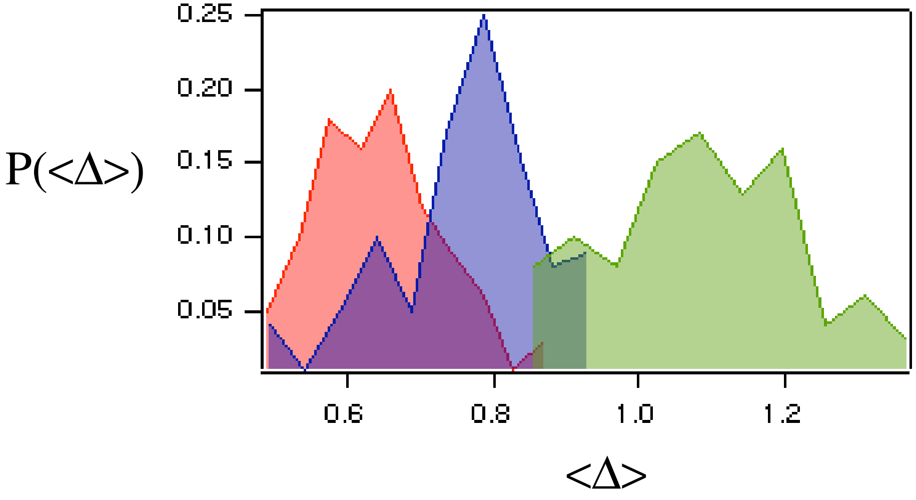
Unknown amplitudes: results- sinusoidal driving
Use phase space model from **A**, **B**, or **C**

histogram



Phase space model:

Drive amp = 1V



Same model
Drive amp = 2V

Unknown was from A

Amplifier ID based on phase space methods is possible

- larger nonlinearity- easier to ID

Phase space modeling:

- need better models- should reflect physics of problem
- need adjustable parameters for different amplifiers of same class.