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School and Workshop on Highly Frustrated Magnets and Strongly Correlated Systems: From Non-Perturbative Approaches to Experiments

30 July - 17 August, 2007

2D quantum magnetism and spin liquids

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Outline

- What are the possible ground-states of 2D Heisenberg models, when magnetic long-range order has been destroyed by the zero-point fluctuations ?
- Propose/discuss 3 three definitions of "spin liquids":
  - A spin liquid is a state without mag. long range order
  - A spin liquid is a state without any spontaneously broken symmetry
  - A spin liquid is a state which sustains spin-½ excitations (spinon)
- Some basic ideas about spinon fractionalization
- Introduce a (fermionic) formalism to discuss some mean-field theories for spin liquids, and investigate fluctuations effects (gauge fields, confinement/deconfinement, etc).

What is specific to D=2 ?

- D=1: Mermin-Wagner ⇒ no magnetic LRO.
Powerful results and methods. LSM theorem: a spin chain is either i) gapped and ordered, or ii) critical. Bethe Anstaz. Bosonization. Tomonaga-Luttinger liquids. Conformal field theory. DMRG.
- D ≥ 3: Spin liquid are theoretically possible (and interesting!), but a priori more difficult to find in real systems, mostly because Néel ordered state are more stable in higher dimensions.
- D=2: Many phases are possible: different kinds of ordered states, different kinds of **spin liquids** (gapped & gapless). There is no *unique* method which is efficient to attack *all* problems/models.

Which spin models are we talking about ?

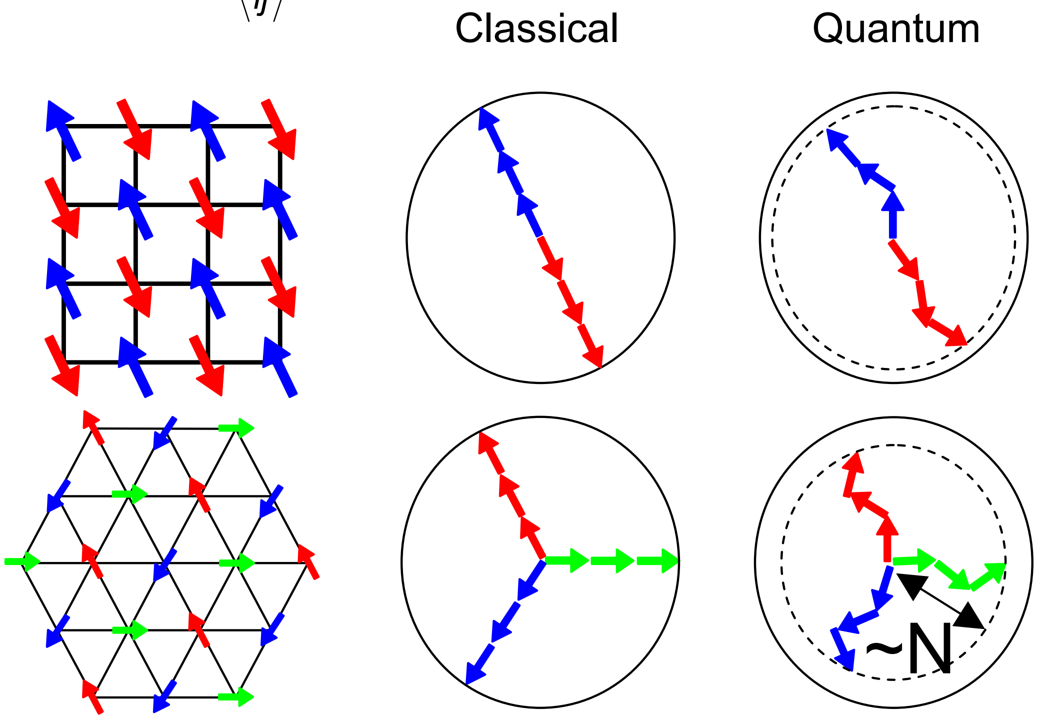
- T=0
- Spin-½
- SU(2) symmetric models: Heisenberg, competing interactions (J_1-J_2), ring exchange and other multiple-spin interactions.

$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle ijkl \rangle} \left(P_{ijkl} + H.c \right) + \ldots$$

"Moderate" quantum fluctuations ⇒ Néel states

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Anderson, PR 1953
Bernu *et al.*, PRL 1992, PRB 1994
Lhuillier, cond-mat/0502464

- The lattice breaks up in sub-lattices
- Spontaneously broken SU(2) symmetry
  - Goldstone theorem
  - ⇒ Gapless spin waves (ΔS^z=1)
- The classical ground-state is "dressed" by **zero-point fluctuations**. But each sub-lattice keeps an **extensive magnetization**
- Possible description using a "1/S" expansion

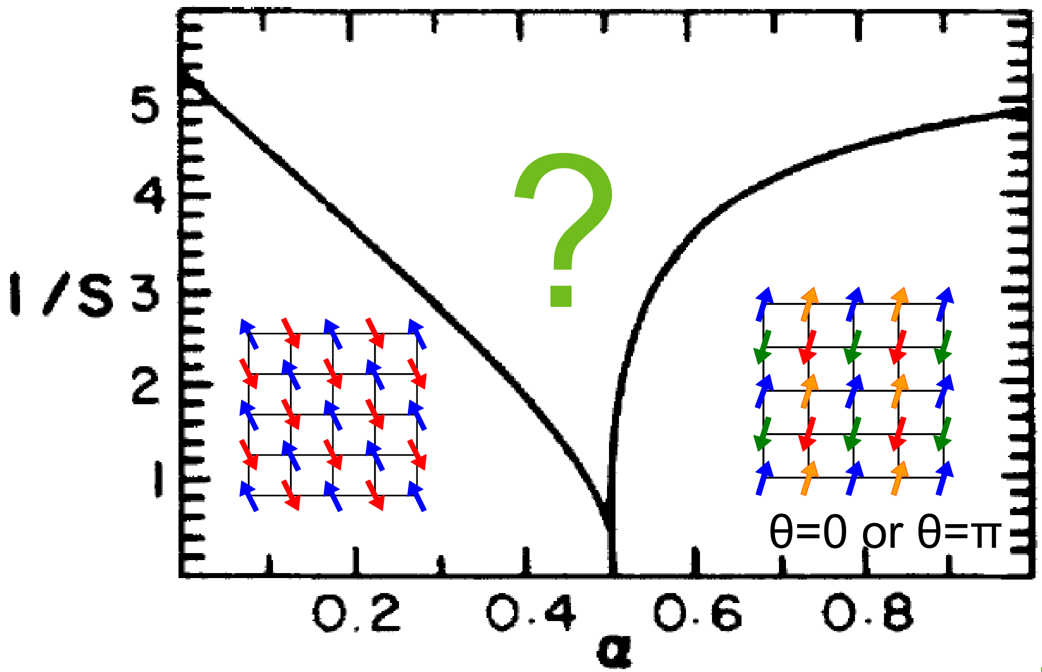
What happens if quantum fluctuations are strong enough to destroy the magnetic order ?

Mechanisms to destroy the mag. long range order

- Small spin S
- High density of low-energy classical modes:
  - Low space dimension
  - Low coordination
  - Frustration
  - Big (continuous) rotation symmetry group (SU(2), U(1), U(N), Sp(2N)).

Spin wave theory for the J_1-J_2 model
Chandra & Douçot, Phys. Rev. B **38**, 9335 (1988)

$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \alpha \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$



Spin liquid – definition 1

A spin liquid is a state without magnetic long-range order

- More precisely, the structure factor S(**q**) never diverges, whatever **q**.

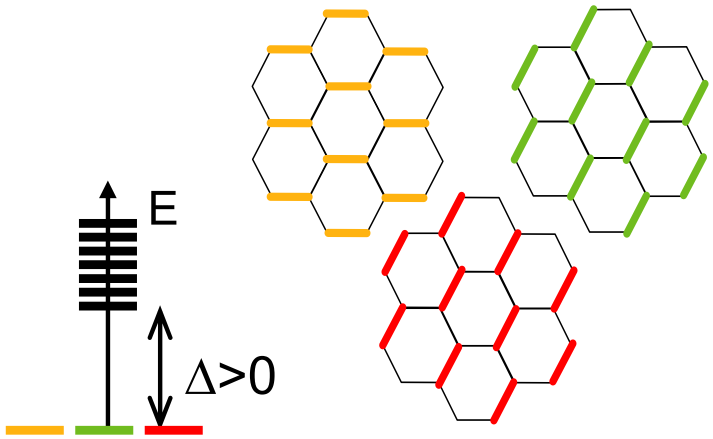
$$S(\mathbf{q}) = \frac{1}{N}\langle 0 \left| \sum_i \vec{S}_i \exp(i\mathbf{q}\cdot\mathbf{r}_i) \right|^2 | 0 \rangle$$

$$= \frac{1}{N}\sum_{ij}\langle 0 | \vec{S}_i \cdot \vec{S}_j | 0 \rangle \exp(i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j))$$

$$= \begin{cases} \approx O(1)\ \forall \mathbf{q} \Leftrightarrow \text{short-range mag. order} \\ \exists \mathbf{q}_0 / S(\mathbf{q}_0) \approx O(N) \Leftrightarrow \text{long-range mag. order} \end{cases}$$

- Can be checked using neutron scattering. But also, µ-SR, NMR, …
- Mermin-Wagner theorem ⇒ *any* 2D Heisenberg model at T>0 is a S.L. according to this def. ☹

Valence-bond crystals



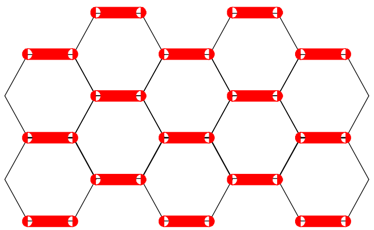
J_1-J_2 Heisenberg model
(hexagonal lattice)
Fouet *et al*. EPJB 2001

 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ Singlet, total spin S=0

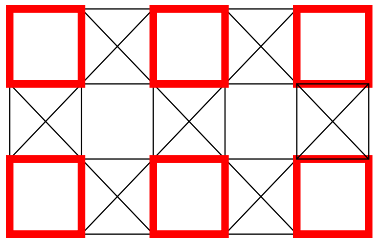
Properties:

- Short-ranged spin-spin correlations
- Spontaneous breakdown of some lattice symmetries
  - ⇒ Ground-state degeneracy
- Gapped $\Delta S=1$ excitations ("magnons" or "triplons")

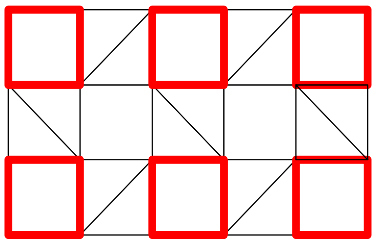
Valence-bond crystals (examples in2D, from numerical studies)



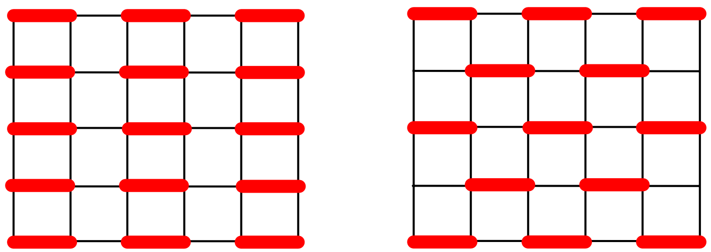
J_1-J_2-J_3 model
Fouet *et al*. EPJB 2001



Fouet *et al*. PRB 2003

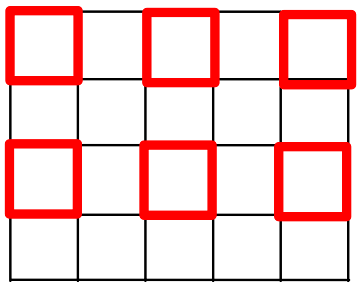


Shastry-Sutherland lattice
Koga & Kawakami, PRL 2000
Läuchli, Wessel & Sigrist PRB 2002

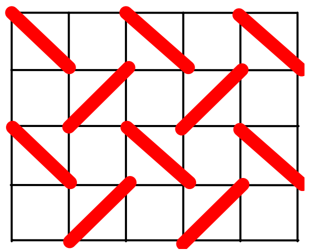


Heisenberg model & 4-spin "ring" exchange
Läuchli *et al*. PRL 2005

+ others…

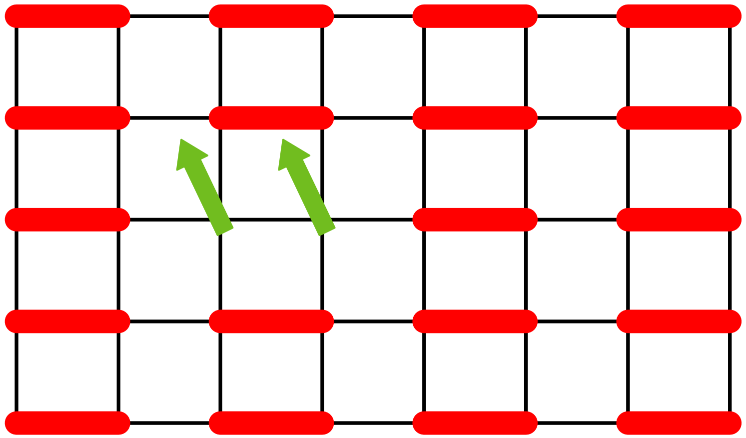


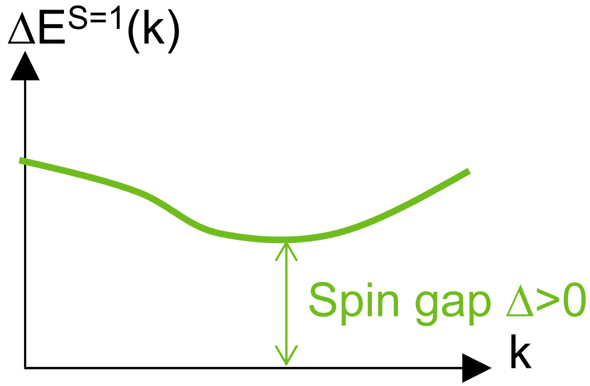
J_1-J_2-J_3 model
Mambrini *et al*., cond-mat/0606776



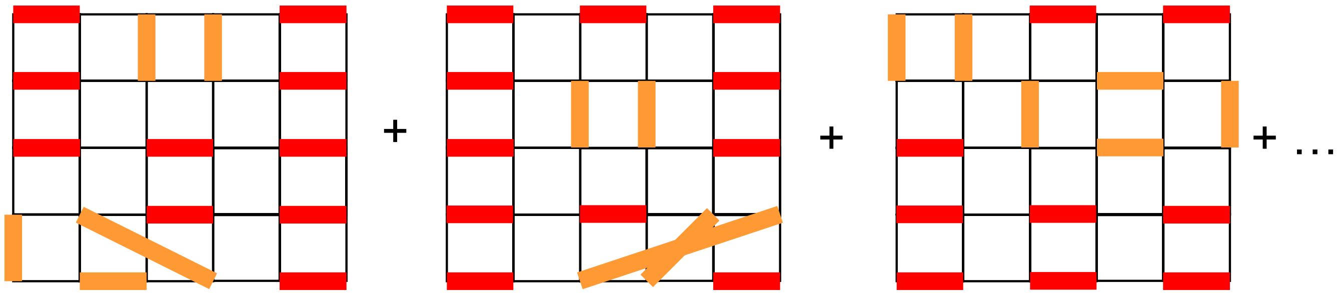
Gellé *et al*. arxiv:0704.2352
(⇒tutorial)

Magnetic excitations in a valence-bond crystal





A simple (tensor) product of singlet is usually not an exact eigenstate for realistic Hamiltonians. The true VBC ground-state is a regular singlet arrangement « dressed » by **fluctuations** :



Remark: Comparing a typical v.-bond configuration with the appropriate "parent" columnar state, one gets a collection of *small loops* (length of order one).

Spin liquid – definition 2

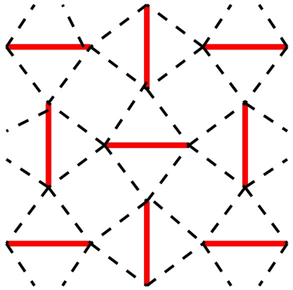
A spin liquid is a state without any spontaneously broken symmetry

- This def. excludes Néel ordered states, which break the SU(2) sym. (also spin nematics)
- This def. excludes valence-bond crystals, which break some lattice sym.

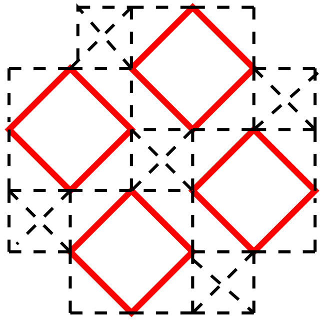
Quantum paramagnets

- Some magnetic insulators without any broken sym.

$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ S=0 spin singlet, or dimer

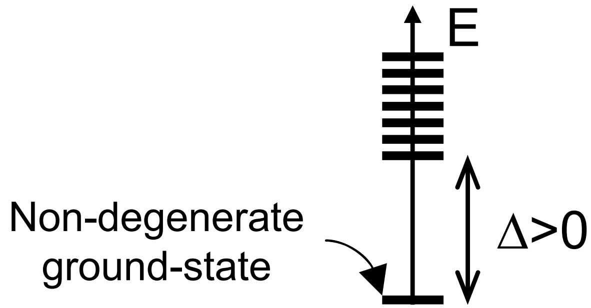


$SrCu_2(BO_3)_2$ Kageyama *et al.* (1999)



CaV_4O_9 Taniguchi *et al.* J. Phys. Soc. Jpn (1995)

$\Delta \approx 100$ K - 1st 2D spin-gap system



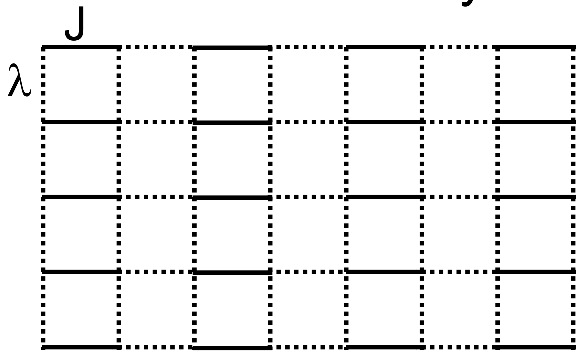
Other examples: coupled dimer systems: $TlCuCl_3$, **etc.**

- Properties:
  - **Even number of spin-½ in the crystal unit cell**
  - No broken symmetry
  - Adiabatically connected to the (trivial) limit of *decoupled* blocks
  - No phase transition between T=0 and T=∞

    $\Rightarrow$ "simple" quantum paramagnet at T=0

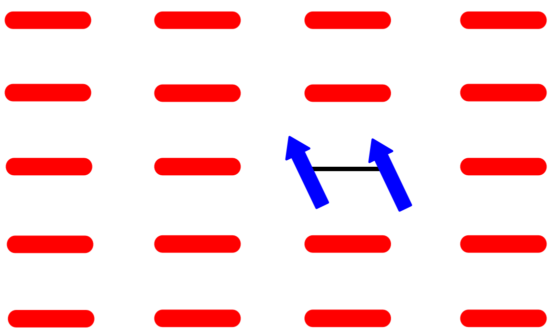
Quantum paramagnets – perturbation theory

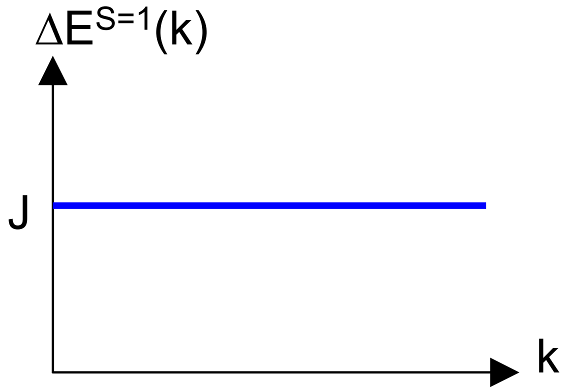
Perturbation theory about a decoupled limit



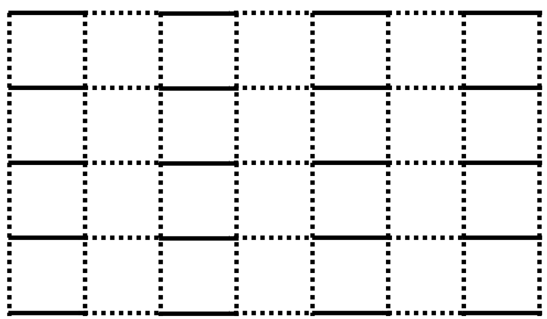
$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

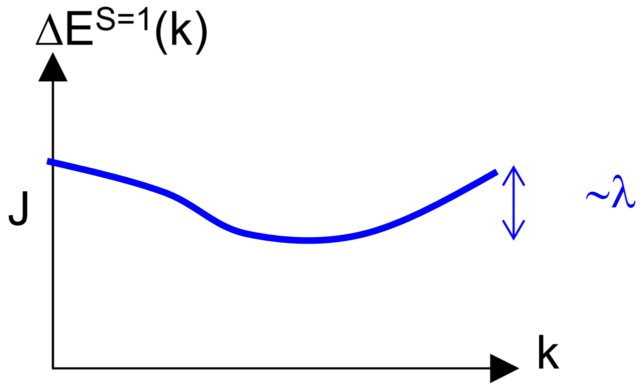
$\lambda=0$



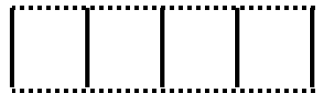


$\lambda \ll J$





Exercise:
compute $\Delta E(k)$ to first order in λ for a two-leg ladder. What about a three-leg ladder ?



Quantum paramagnets – magnetic excitations in $TlCuCl_3$

PHYSICAL REVIEW B, VOLUME 63, 172414

Magnetic excitations in the quantum spin system $TlCuCl_3$

N. Cavadini,[1] G. Heigold,[1] W. Henggeler,[1] A. Furrer,[1] H.-U. Güdel,[2] K. Krämer,[2] and H. Mutka[3]

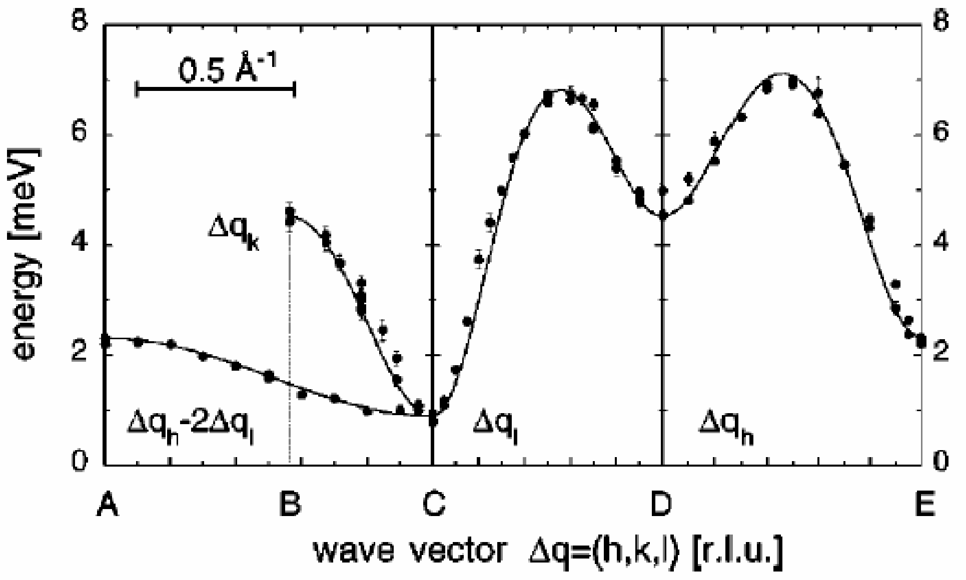
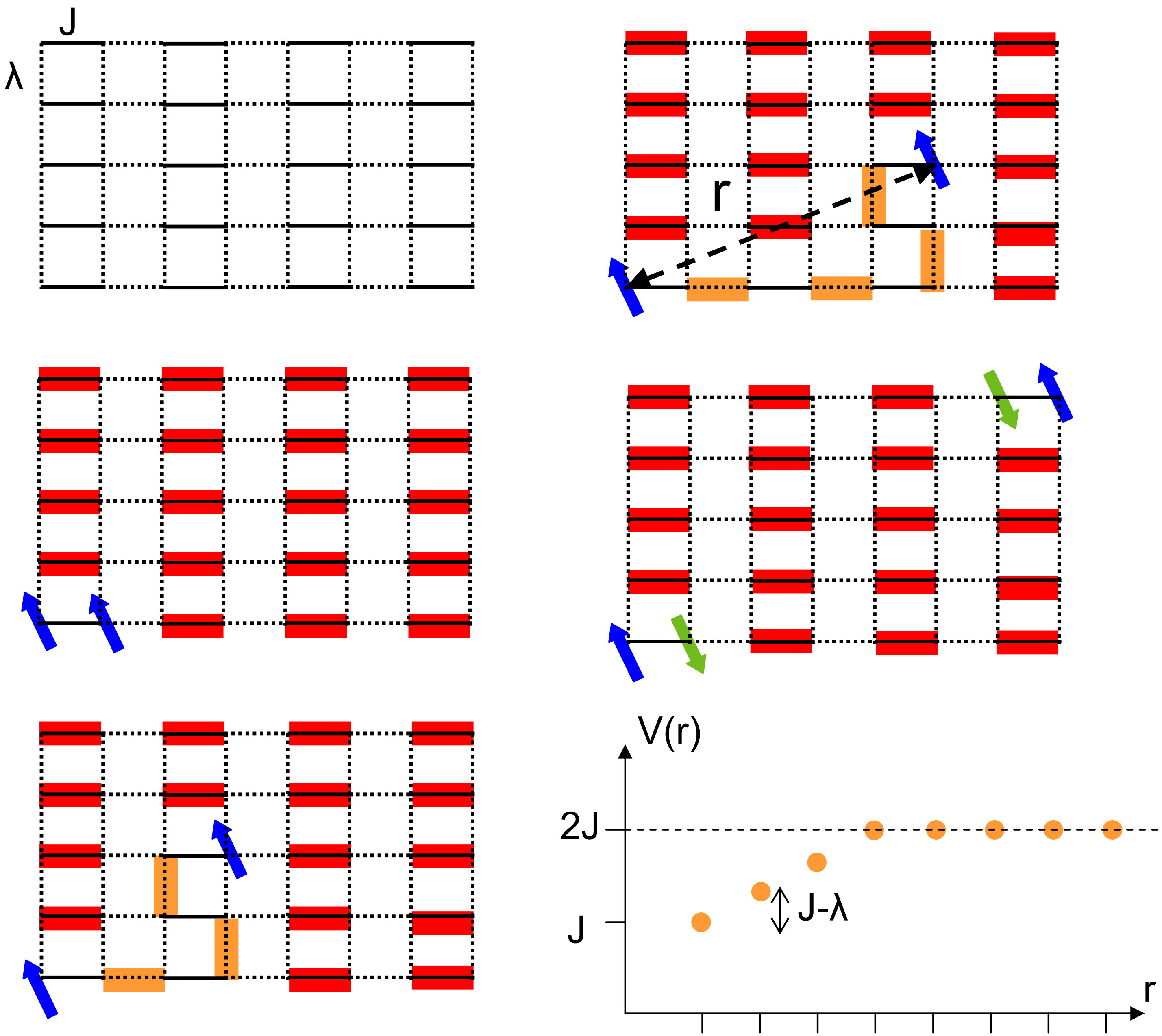


FIG. 2. Observed energy dispersion of the magnetic excitation modes in $TlCuCl_3$ at $T=1.5$ K. Full circles from the relevant directions of reciprocal space are arranged in a reduced scheme representation, with $A=E=(1/2,0,0)$, $B=(0,1,0)$, $D(0,0,0)$ [r.l.u.]. Zone centering corresponds to $C=(0,0,1)$ for $\Delta q=(h,0,l)$, $C=(0,0,0)$ for $\Delta q=(0,k,0)$. Lines are fits to the model expectations explained in the text with the parameters reported in Table I.

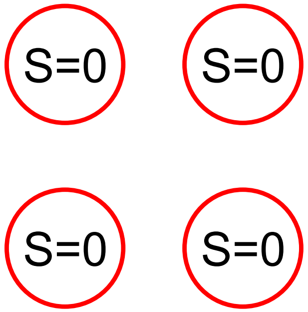
Quantum paramagnets (and VBC) are *not* fractionalized

Sachdev & Vojta, cond-mat/0009202



Remark: the same picture applies to valence-bond crystals.

VBC, paramagnets, and what else ?

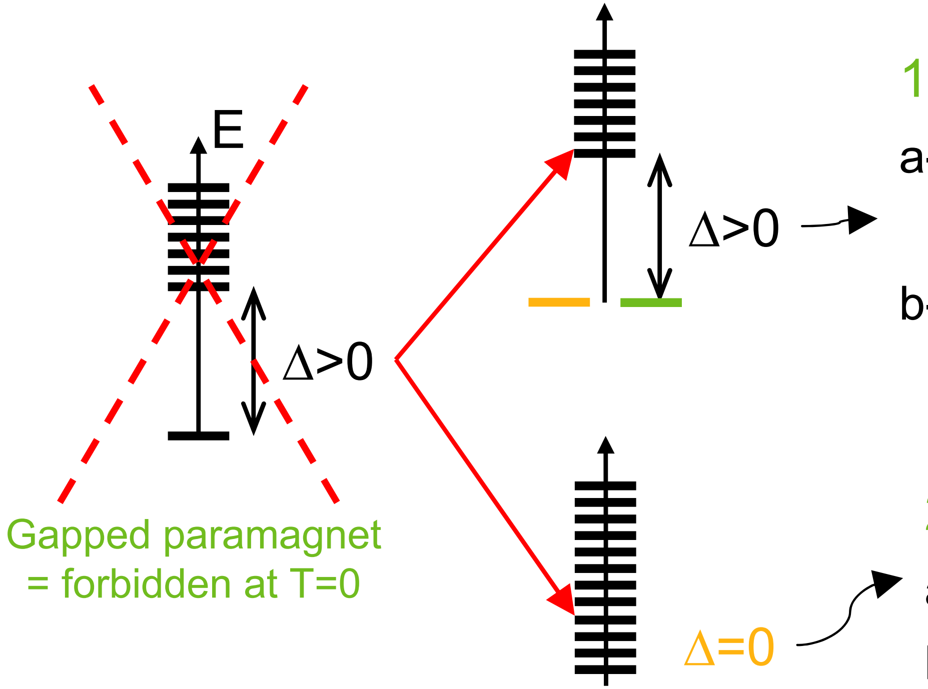


- In a quantum paramagnet, the unit cell contains an ***even*** number of spin-½
- In a VBC, the unit cell is spontaneously enlarged to enclose a ***even*** number of spin-½
- Are there other types of wave-functions with short-range spin-spin correlations ?
(with just *one* spin ½ per unit cell in particular?)

Lieb-Schultz-Mattis-Hastings theorem

- Lieb-Schultz-Mattis (D=1: 1961) Hastings (D>1: Phys. Rev. B 2004; Europhys. Lett. 2005)
  [See also: Affleck 1988; Bonesteel 1989; Oshikawa 2000; GM *et al. 2002*]

> *"A system with a half-odd-integer spin in the unit cell*
> *(+ periodic boundary conditions, + dimensions $L_1 \times L_2 \times \ldots \times L_D$ with $L_2 \times \ldots \times L_d$=odd)*
> *cannot have a gap and a unique ground-state*
> *(in the thermodynamic limit). "*



1) Ground-state degeneracy

a- "Conventional" broken symmetry
(valence bond crystal for instance)

b- Resonating valence bond liquid (Z_2)
or more complex topological structure

2) Gapless spectrum

a- Continuous broken sym. (Néel order)

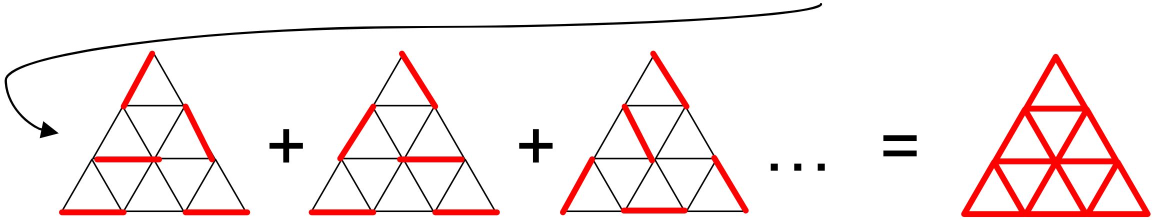
b- Critical state

spin liquids with fractional excitations

Short-range RVB picture

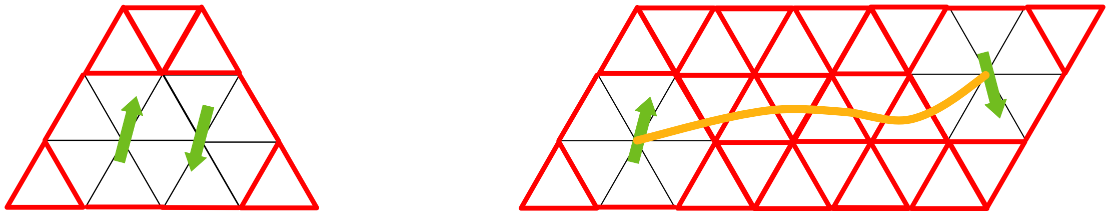
- P. W. Anderson's idea (1973) : (short-ranged) resonating valence-bond (RVB)

Linear superposition of many (exponential) low-energy short-range valence-bond configurations

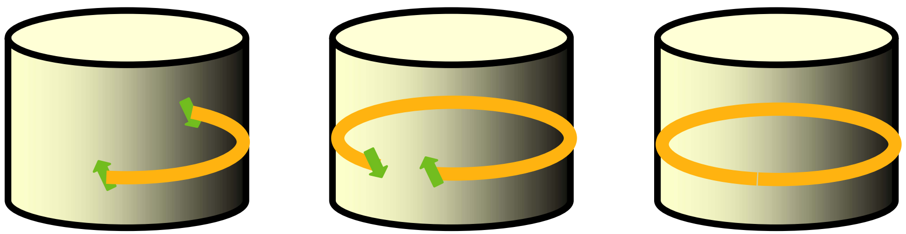


Spatially *uniform* state

- Spin-½ excitations ?
  VBC ⇒ linear potential between spinons
  no dimer order ⇒ we *may expect* deconfined spinons

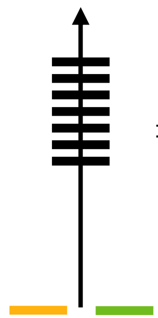


- Topological degeneracy & spinon fractionalization



adiabatic process → New ground-state

Topological degeneracy
(X. G. Wen 1991) ↔
fractionalization
See also Oshikawa & Senthil
PRL 96, 060601 (2006)



2-fold degeneracy
⇒ Satisfies LSMH

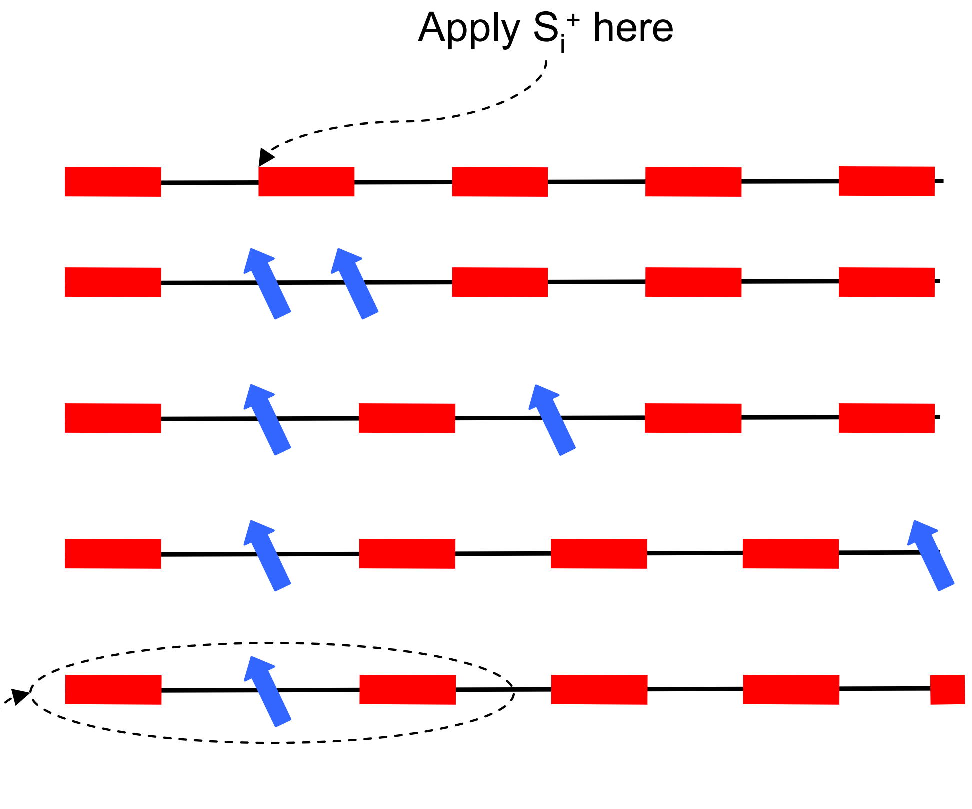
Definition 3:

A spin liquid is a state which sustains fractional (spin-½) excitations

- What is fractionalization ?
  - Existence of (finite energy) excitations with quantum number(s) which are fraction of the elementary degrees of freedom.
    Most famous example: charges q=e/3 in the fractional quantum hall effect.
  - In magnetic systems:
    A spinon is a neutral spin-½ excitation, “one half” of a $\Delta S^z=1$ spin flip.
    (it has the same spin as an electron, but is has no *charge*)
  - Spinons can only be created by pairs in finite systems (combining S^+ and S^- operators can only change S^z by some integer) The question is to understand if they then can propagate at large distances from each other, as two elementary particles.

What is a fractional excitation ? (very) simple example in 1D

- Majumdar-Gosh chain $H=\sum_i \vec{S}_i\cdot\vec{S}_{i+1}+\frac{1}{2}\sum_j \vec{S}_j\cdot\vec{S}_{j+2}$



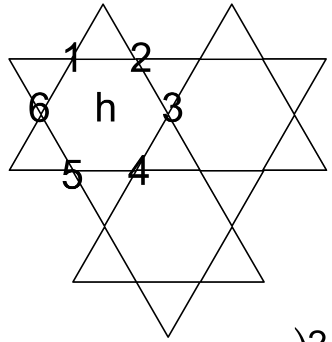
The initial $S^z=+1$ excitation can *decay* into *two* spatially separated spin-½ excitations (spinons)

Finite-energy state with an *isolated* spinon (the other is far apart)

A few examples of fractionalized systems in D>1

- Easy-axis kagome model

  Balents, Fisher & Girvin, Phys. Rev. B **65**, 224412 (2002)
  Sheng & Balents, Phys. Rev. Lett. **94**, 146805 (2005)



$$H = J_z \sum_{h \text{ hexagon}} \left(S^z_{h1} + \cdots + S^z_{h6}\right)^2$$

$$+ J_\perp \sum_{h \text{ hexagon}} \left(\left(S^x_{h1} + \cdots + S^x_{h6}\right)^2 + \left(S^y_{h1} + \cdots + S^y_{h6}\right)^2\right)$$

$$J_\perp << J_z$$

- Kitaev's "toric code" model⇒ tutorial
- SU(2) symmetric spin models

  GM *et al.*, 1999

  Raman-Moessner-Sondhi, Phys. Rev. B **72**, 064413 (2005)

- Experiments ? Some candidates:
  - Cs_2CuCl_4 [Anisotropic S=1/2 triangular lattice, Coldea *et al.* 2003]
  - κ-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ [Shimizu *et al.* 2003]
  - $NiGa_2S_4$ [Spin-1 on a triangular lattice, Nakatsuji *et al.*, 2005]
  - $ZnCu_3(OH)_6Cl_2$ [Helton *et al.* 2007, Mendels *et al.* 2007, Ofer *et al.* 2007, Imai *et al.* 2007]
  - $Na_4Ir_3O_8$ [3D lattice of corner sharing triangles, "hyper kagome", Okamoto *et al.* 2007]
  - He^3 films [Nuclear magnetism on a triangular lattice, Masutomi *et al.* 2004]

How to detect deconfined spinons ?

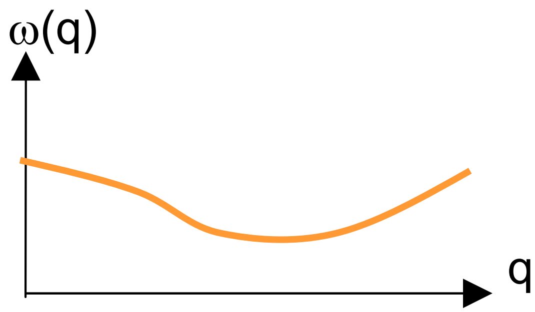
- Neutron scattering
- Non magnetic impurities $\Rightarrow$ tutorial

Inelastic neutron scattering – spinon continuum

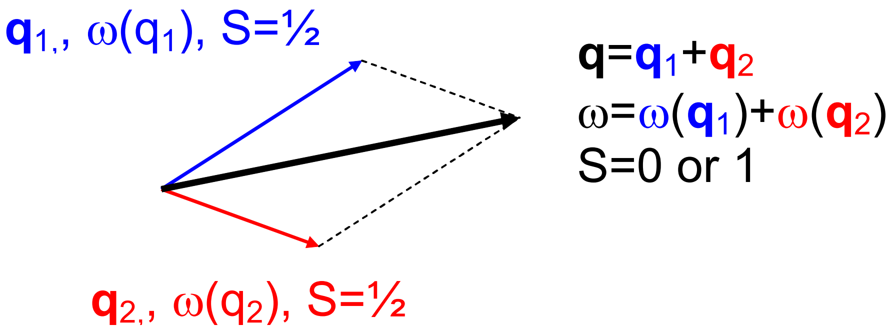
Inelastic neutron scattering : probe for the dynamical structure factor S(**q**,ω).

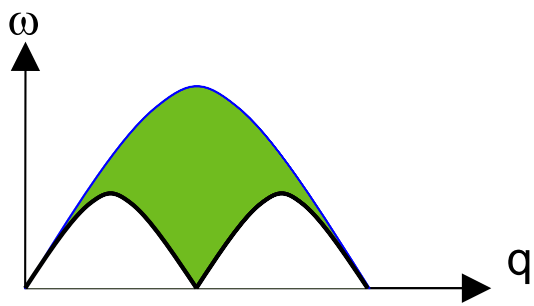
$$S(\mathbf{q},\omega)=\int dt \langle 0 | S^{-}_{-\mathbf{q}}(t) S^{+}_{\mathbf{q}}(0) | 0 \rangle e^{-i\omega t}$$

- If the elementary excitations are spin-1 magnons :
  S(**q**,ω) has single-particle **pole** at ω= ω(**q**)

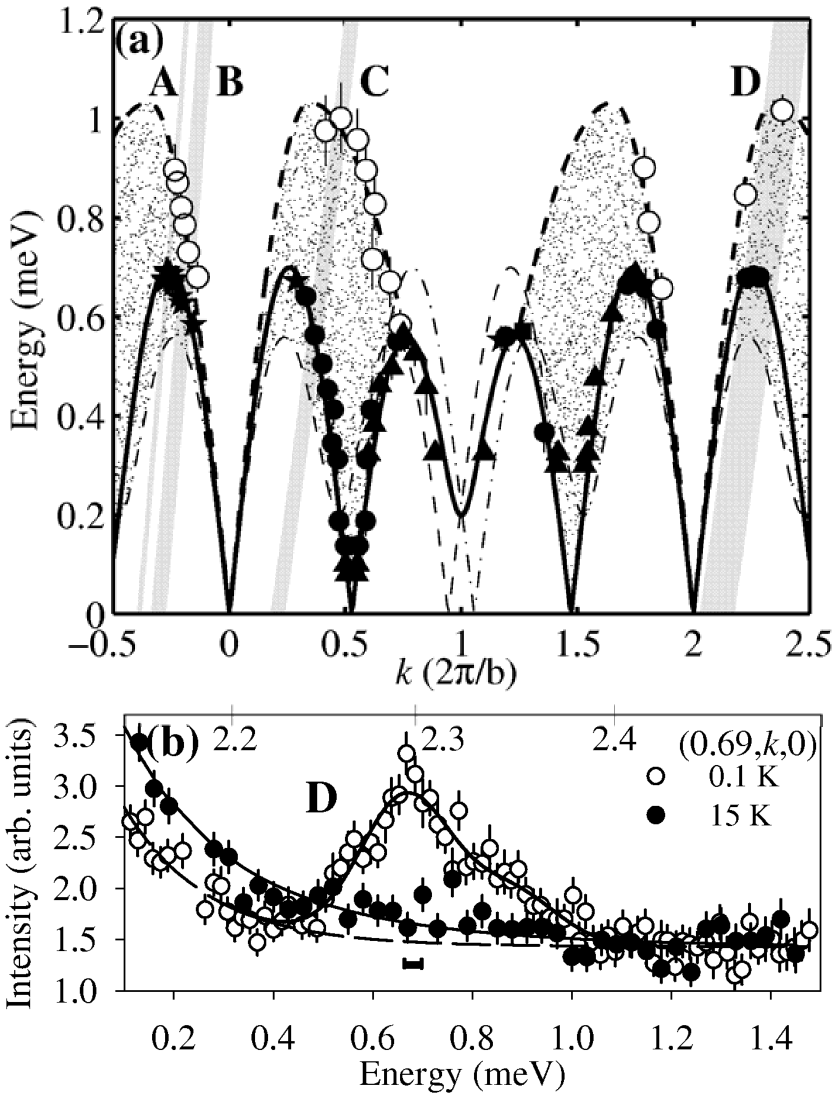


- If the spin flip decays into two spin-½ excitations
  S(**q**,ω) exhibits a two-particle **continuum**

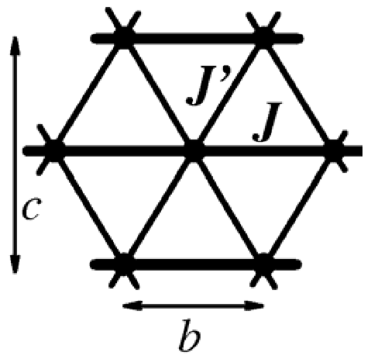




Inelastic neutron scattering – spinon continuum



- Neutron scattering on Cs_2CuCl_4
  R. Coldea *et al.* (2000)



Formalisms and methods to investigate fractionalized spin liquids ?

- Numerics (exact diag., quantum Monte-Carlo, …)
- Effective models (dimer models)
- Large-N/slave particle approaches:
  - Bosonic mean-field (Schwinger bosons) + fluctuation effects
  - Fermionic mean-field (Abrikosov fermions) + fluctuation effects

  See Lee, Nagaosa & Wen, Rev. Mod. Phys. 78, 17 (2006)

Fermionic representation of a spin-1/2

$$S^z = \tfrac{1}{2}\left(c_\uparrow^+ c_\uparrow - c_\downarrow^+ c_\downarrow\right)$$

$$S^+ = c_\uparrow^+ c_\downarrow \qquad S^- = c_\downarrow^+ c_\uparrow$$

$$c_\uparrow^+ c_\uparrow + c_\downarrow^+ c_\downarrow = 1$$

$$S^a = c_\mu^+ \sigma_{\mu\nu}^a c_\nu \quad a = x,y,z \quad \mu,\nu = \uparrow,\downarrow$$

$c_\uparrow^+$ (or $c_\downarrow^+$) changes S^z by $+\tfrac{1}{2}$ ($-\tfrac{1}{2}$) creates a "spinon"

- Compact notations using a 2x2 matrix $\psi_i = \begin{bmatrix} c_{i\uparrow} & c_{i\downarrow} \\ c_{i\uparrow}^+ & -c_{i\downarrow}^+ \end{bmatrix}$

$$S^a = \frac{1}{2}\mathrm{Tr}\left[\psi_i^+ \psi_i \left(\sigma^a\right)^T\right]$$

$$\vec{S}_i \cdot \vec{S}_j = \frac{1}{4}\sum_A \mathrm{Tr}\left[\psi_i^+ \psi_i \left(\sigma^a\right)^T\right]\mathrm{Tr}\left[\psi_j^+ \psi_j \left(\sigma^a\right)^T\right]$$

$$= \frac{1}{8}\mathrm{Tr}\left[\psi_i \psi_j^+ \psi_j \psi_i^+\right]$$

Mean-field decoupling

$$\vec{S}_i \cdot \vec{S}_j = \frac{1}{8}\mathrm{Tr}\left[\psi_i \psi_j^+ \psi_j \psi_i^+\right]$$

$$\psi_i \psi_j^+ \psi_j \psi_i^+ \to \left\langle \psi_i \psi_j^+ \right\rangle \psi_j \psi_i^+ + \psi_i \psi_j^+ \left\langle \psi_j \psi_i^+ \right\rangle - \left\langle \psi_i \psi_j^+ \right\rangle \left\langle \psi_j \psi_i^+ \right\rangle$$

Mean-field approx.

$$J_{ij} \vec{S}_i \cdot \vec{S}_j \to \mathrm{Tr}\left[U_{ij}^0 \psi_j \psi_i^+ + \psi_i \psi_j^+ \left(U_{ij}^0\right)^+ - U_{ij}^0 U_{ij}^{0+}\right]$$

$$U_{ij}^0 = \frac{J_{ij}}{8}\left\langle \psi_i \psi_j^+ \right\rangle = \frac{J_{ij}}{8}\begin{bmatrix} \left\langle c_{i\uparrow} c_{j\uparrow}^+ + c_{i\downarrow} c_{j\downarrow}^+ \right\rangle & \left\langle c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow} \right\rangle \\ \left\langle c_{i\downarrow}^+ c_{j\uparrow}^+ - c_{i\uparrow}^+ c_{j\downarrow}^+ \right\rangle & \left\langle c_{i\downarrow}^+ c_{j\downarrow} + c_{i\uparrow}^+ c_{j\uparrow} \right\rangle \end{bmatrix} = \begin{bmatrix} -\chi_{ij}^+ & \eta_{ij} \\ \eta_{ij}^+ & \chi_{ij} \end{bmatrix}$$

Mean-field parameters

$$H_{MF} = \sum_{\langle ij \rangle} \chi_{ij}\left(c_{i\downarrow}^+ c_{j\downarrow} + c_{i\uparrow}^+ c_{j\uparrow}\right) + \eta_{ij}\left(c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow}\right) + H.c$$

Spinon “hopping” Spinon “pairing”

Spin rotation symmetry

$V \in$ SU(2), global spin rotation

$$\psi_i = \begin{bmatrix} c_{i\uparrow} & c_{i\downarrow} \\ c_{i\uparrow}^+ & -c_{i\downarrow}^+ \end{bmatrix} \rightarrow \psi_i V$$

$$\left.\begin{array}{l} \vec{S}_i \cdot \vec{S}_j = \frac{1}{8}\mathrm{Tr}\left[\psi_i \psi_j^+ \psi_j \psi_i^+\right] \rightarrow \vec{S}_i \cdot \vec{S}_j \\ U_{ij}^0 = \frac{J_{ij}}{8}\left\langle \psi_i \psi_j^+ \right\rangle \rightarrow U_{ij}^0 \\ H_{MF} \rightarrow H_{MF} \end{array}\right\} \text{rotation invariant}$$

⇒ Mean-field Hamiltonian and its ground-state are rotation invariant (can describe a “spin liquid”)

Beyond mean-field

- **Is the mean field state *stable* under the inclusions of fluctuations ?**
- Spinon are deconfined (free fermions!) at the mean-field level, but is this a robust property ? Will the inclusion of fluctuations confine the spinons ?
- This is usually a difficult question…
  - If yes: fluctuations are strong, they induce long-range interactions between spinons and this mean-field is not a very useful starting point.
  - If no (fluctuations do not confine the spinon), the mean-field approximation is a good starting point to describe the spin liquid.

- One way to address these questions: numerical Gutzwiller projection

  Example of recent study on the kagome lattice: Ran, *et al.*, PRL (2007).

- Other point of view: analyze the qualitative structure of the (potentially important) low-energy mode/fluctuations about the mean-field state. Some important modes for the long-distance physics are *gauge* modes.

Redundancy – gauge transformations

W_i arbitrary $SU(2)$ matrix at each site i :

$$\left.\begin{array}{l} \psi_i \to W_i \psi_i \\ \vec{S}_r \to \vec{S}_r \end{array}\right\} \text{local redundancy}$$

$$U^0_{ij} = \frac{J_{ij}}{8}\left\langle \psi_i \psi_j^+ \right\rangle \to W_i U^0_{ij} W_j^+$$

$$\psi_i = \begin{bmatrix} c_{i\uparrow} & c_{i\downarrow} \\ c^+_{i\uparrow} & -c^+_{i\downarrow} \end{bmatrix}$$

$$S^a = \frac{1}{2}\mathrm{Tr}\left[\psi_i^+ \psi_i \left(\sigma^a\right)^T\right]$$

$$\vec{S}_i \cdot \vec{S}_j = \frac{1}{8}\mathrm{Tr}\left[\psi_i \psi_j^+ \psi_j \psi_i^+\right]$$

Remark: can be extended to *time-dependent* gauge transformations – see the Appendix and Affleck, Zou, Hsu & Anderson, Phys. Rev. B 38, 745 (1988)

- The same physical mean-field state can be represented by many different sets of parameters U^0 (differing from each-other by some gauge-transformation).
- The bond parameters U are a *redundant* way of labeling physical states. This redundancy is unavoidable if we want to use a formalism which include spinon operators.
- **This redundancy has important consequences on the structure of the fluctuation modes around a given mean-field state ⇒ gauge modes.**

Projective symmetry group (PSG)

X.-G Wen, Phys. Rev. B 65, 165113 2002

- Two sets of U_{ij} which differ by a gauge transformation describe the *same* physical spin wave-function.
- Definition of PSG

T : lattice symmetry

W : gauge transformation

$$U_{ij}^{\ 0} \xrightarrow[\text{Lattice sym.}]{} U^0_{T(i)T(j)} \xrightarrow[\text{gauge transf.}]{} W_i U^0_{T(i)T(j)} W_j^+ = U_{ij}^{\ 0}$$

$$(\mathrm{T},\mathrm{W}) \in \mathrm{PSG}$$

$$\Leftrightarrow W_i U^0_{T(i)T(j)} W_j^+ = U_{ij}^{\ 0}$$

- We have defined the PSG of the *mean-field* state, but the PSG is in fact a universal property of the whole *phase.* Including fluctuations should not affect the PSG (unless an instability or phase transition occurs).
- Even in the absence of any spontaneously broken symmetry, the PSG is generally non-trivial. It characterizes 'how' the lattice symmetries are realized in the wave-functions. Distinct spin-liquid phase can have the same lattice symmetries (they can be completely symmetric for instance), but different PSG. The PSG of a fractionalized phase plays a role analog to that of the symmetry group for usual ordered phases.

Invariant gauge group (IGG) - definition

X.-G Wen, Phys. Rev. B 65, 165113 (2002)

- The invariant gauge group of a mean-field state is defined as the set of all gauge transformations which leaves the mean-field parameter U^0 invariant.

$$W : i \mapsto W_i \in SU(2)$$

$$W \in IGG$$

$$\Leftrightarrow W_i U_{ij}^0 W_j^+ = U_{ij}^{\ 0} \quad \forall ij$$

- Why is this useful ?

The IGG of a mean-field state is the gauge group associated to the fluctuations around this mean-field state.

Invariant gauge group & and gauge fluctuations

- Consider some mean-field state defined by the bond parameters U_{ij}^0
- Assume, for simplicity, that the associated IGG is ~U(1) and that its elements can be parameterized in the following way (but the final result is in fact general):

$$W^\theta \in \mathrm{IGG}:\ W_j^\theta = \underbrace{\exp\left(i\theta\vec{\sigma}\cdot\vec{n}\right)}_{\text{global, indep of j}}\ ,\ \theta \in \left]-\pi,\pi\right], \vec{n}^2 = 1$$

$$W_i^\theta U_{ij}^0 W_j^{\theta+} = U_{ij}^0 \quad \forall ij\ , \forall\theta \qquad \text{(by definition of the IGG)}$$

- We will now show that some fluctuations about this mean field state are described by a **U(1) gauge field**.
- Consider the following fluctuations:

$$\psi_i\psi_j^+ = \underbrace{U_{ij}^0}_{\langle\psi_i\psi_j^+\rangle} \exp(i \underbrace{A_{ij}}_{\text{fluctuation field}} \vec{\sigma}\cdot\vec{n})$$

Remark: Aij is rotation invariant ⇒ describes S=0 modes

- Now we perform the following U(1) gauge transformation : $\psi_i \to \exp(i \underbrace{\theta_i}_{\text{local angle}} \vec{\sigma}\cdot\vec{n})\psi_i$ and see how the field A_{ij} transforms:

$$\psi_i\psi_j^+ \to \exp(i\theta_i\vec{\sigma}\cdot\vec{n})\psi_i\psi_j^+\exp(-i\theta_j\vec{\sigma}\cdot\vec{n})$$

$$= \exp(i\theta_i\vec{\sigma}\cdot\vec{n})U_{ij}^0\exp(iA_{ij}\vec{\sigma}\cdot\vec{n})\exp(-i\theta_j\vec{\sigma}\cdot\vec{n})$$

$$= U_{ij}^0\exp\left[i\left(A_{ij}+\theta_i-\theta_j\right)\vec{\sigma}\cdot\vec{n}\right]$$

$$A_{ij} \to A_{ij} + \theta_i - \theta_j$$

⇒ ***A* is a gauge field**

Fluctuations (about mean-field) and gauge fields

- The IGG gives the gauge structure of the fluctuations about a given mean-field state.
- The associated gauge field(s) may or may not provide gapless excitations, may or may not confine the spinon.
- Non-trivial (non-perturbative) results about gauge theories coupled to matter may be 'imported' to discuss the stability/instability of a given mean-field state.
- For instance (D=2):
  - Z_2 gauge field + gapped spinons may be in a stable deconfined phase.
    =short-range RVB physics, Read & Sachdev PRL 1991
  - U(1) gauge field + gapped spinon: instability
    usually toward confinement and VBC, Read & Sachdev PRL 1989
  - U(1) gauge field + Dirac gapless spinons: may be stable
    (so-called "algebraic spin liquids") Hermele, Senthil, Fisher, Lee, Nagaosa, Wen, PRB 2004
- Remark: gauge modes are not the only source of instability: interactions between fermions should also be investigated. The projective symmetry group (PSG) constrains the possible interaction terms.

Summary

- There are several possible definitions for “spin liquids”
  - A spin liquid is a state without mag. long range order
  - A spin liquid is a state without any spontaneously broken symmetry
  - A spin liquid is a state which sustains spin-½ excitations (spinon)
- From the theoretical point of view, the richest structures are found in “fractionalized” spin liquids.
- Gauge theories are the natural language to describe these fractionalized phases.
- There are many kinds of fractionalized spin liquids, they are characterized by different gauge groups, and different PSG.
- In the recent years, many spin models are be shown to be fractionalized.
- Several compounds have emerged as ‘candidates’ but there is not yet any definitive experimental evidence for a fractionalized spin liquid in D>1.