



The *Abdus Salam*
International Centre for Theoretical Physics



SMR/1855-3

**School and Workshop on Highly Frustrated Magnets and Strongly
Correlated Systems: From Non-Perturbative Approaches to
Experiments**

30 July - 17 August, 2007

Highly Frustrated Magnets: Materials and Materials Preparation

John E. Greedan
*McMaster University
Hamilton, Canada*

Highly Frustrated Magnets: Materials and Materials Preparation

**John E. Greedan
McMaster University
Hamilton, Canada**

Abdus Salam ICTP August 30, 2007

Outline:

I. Families of geometrically frustrated materials.

A. Triangles, tetrahedra and magnetic frustration.

(Subversion of the 3rd Law by suppression of long range magnetic order)

- 1. The frustration index $f \sim |\theta|/T_{\text{ord}}$**
- 2. Role of the spin quantum number.**

B. Common frustrated lattices in 2 and 3 dimensions.

- 1. Triangular planar**
- 2. Kagomé**
- 3. Face-centred cubic**
- 4. Pyrochlore**

C. Real materials: transition metal compounds

1. Triangular planar

a. “ordered rock salt”



b. anhydrous alums



2. Kagomé

a. Jarosites



b. Herbertsmithites



3. F.c.c.

A. B-site ordered “double perovskites”



4. Pyrochlore

- a. pyrochlore oxides: $A_2M_2O_7$, ex $Y_2Mo_2O_7$**
- b. Spinel oxides: AB_2O_4 . ex $ZnCr_2O_4$**

5. Other frustrated 3D lattices.

- a. “SCGO” $SrCr_{12-x}Ga_xO_{19}$**
- b. Garnet: $A_3M_5O_{12}$. ex $Gd_3Ga_5O_{12}$**
- c. “ordered rock salt” ex $Li_3Mg_2RuO_6$**
- d. $BaM_{10}O_{15}$: M = V^{3+} , Cr^{3+}**

II. Materials preparation and crystal growth.

A. Control of transition metal oxidation state and oxygen stoichiometry.

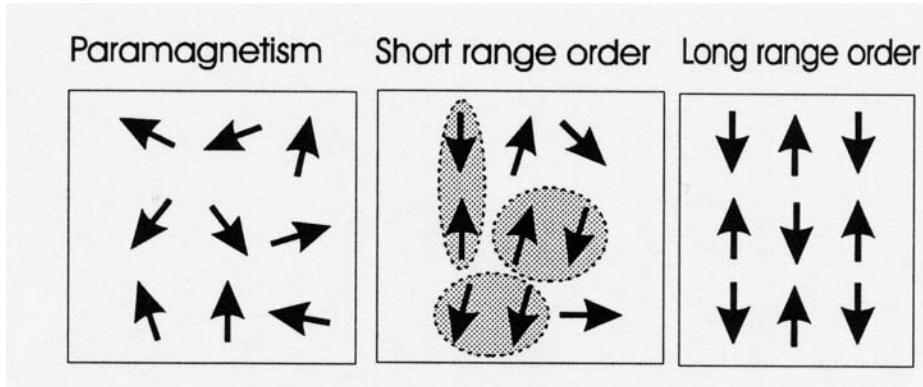
1. oxygen affinity, pO, buffer gases

B. Crystal growth methods.

- 1. Bridgeman**
- 2. Czochralski**
- 3. Floating zone**
- 4. Flux**
- 5. Hydrothermal**

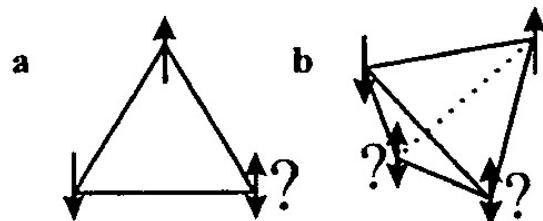
C. “Soft chemical” routes to metastable phases

A. Triangles, tetrahedra and magnetic frustration.



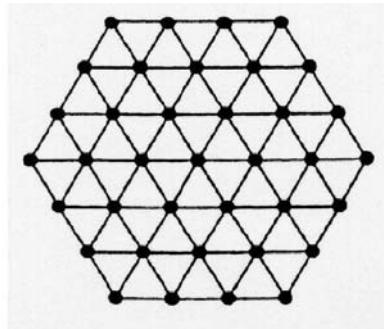
3rd Law: $S \rightarrow 0$ as $T \rightarrow 0$

triangular topology frustrates LRO

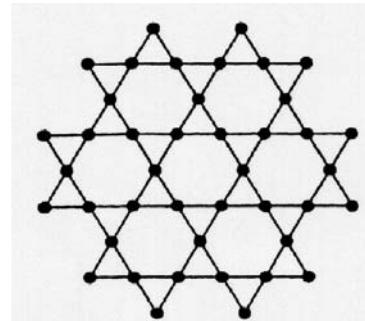


Subversion of the 3rd Law ?

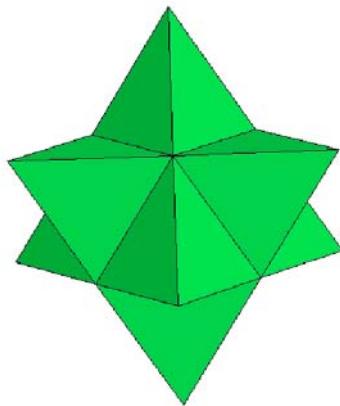
Realization of frustrated topology in real(common)lattices, 2D and 3D



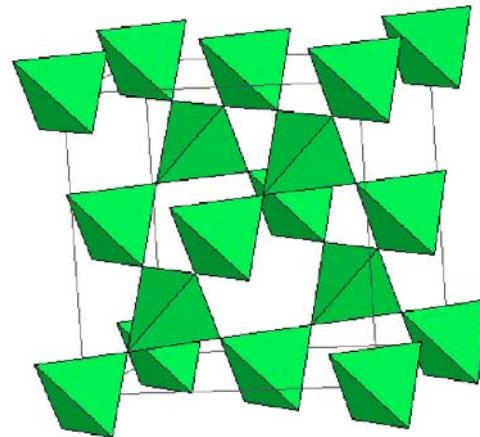
a. triangular



b. Kagome



c. face centered
cubic



d. pyrochlore

The frustration index: $f \sim |\theta| / T_{\text{ord}}$

P. Schiffer, A.P. Ramirez, Comm. Cond. Matter Phys. 10 (1996) 21.

from mean field theory:

$$\theta = \frac{2S(S+1)}{3k} \sum_{m=1}^{N'} z_m J_m$$

algebraic, weighted, sum of all pairwise exchange interactions

$$H_{\text{ex}} = -2JS_n \cdot S_m$$

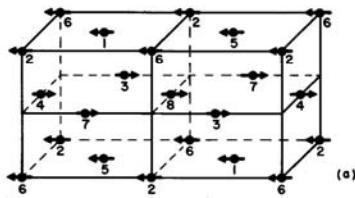
sets energy (temperature) scale for the magnetic exchange.

$$T_{\text{ord}} = T_c, T_N, T_g$$

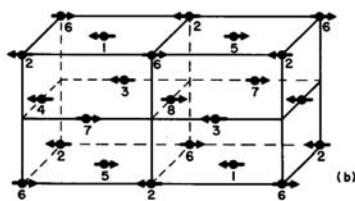
if $f \gg 5$, evidence for frustration

MFT phase diagram for f.c.c. lattice

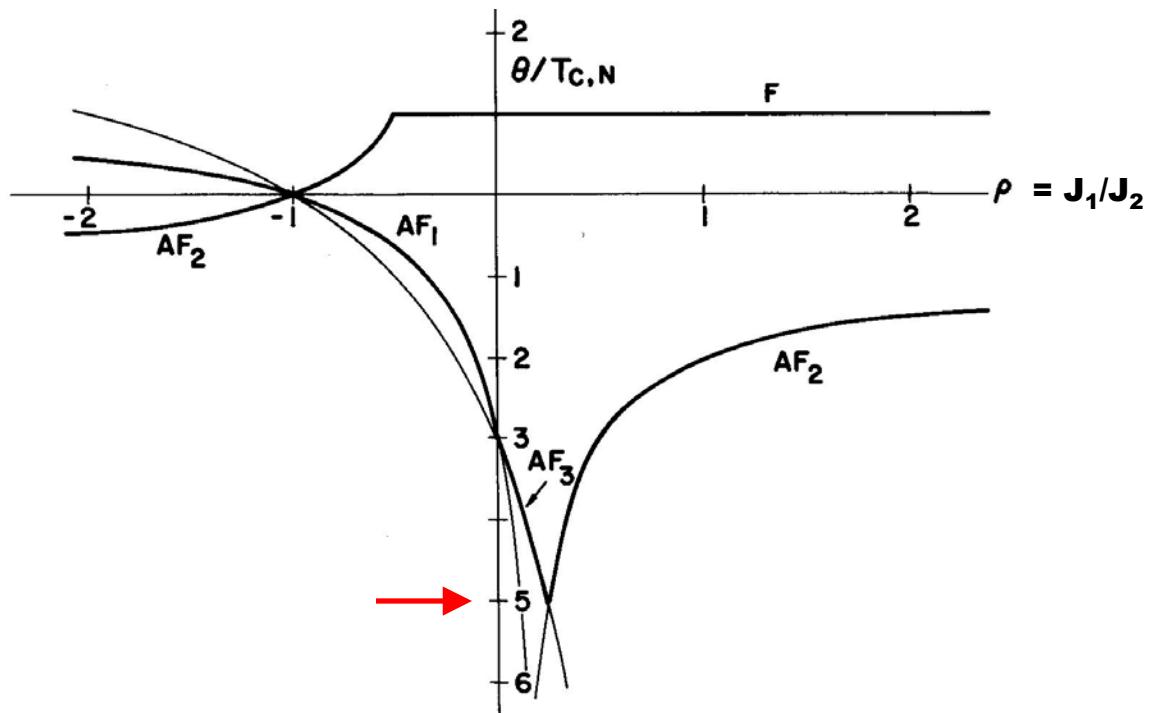
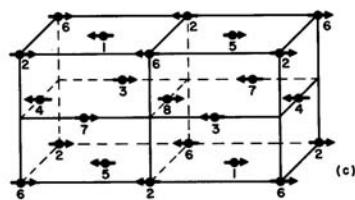
AF₁



AF₂

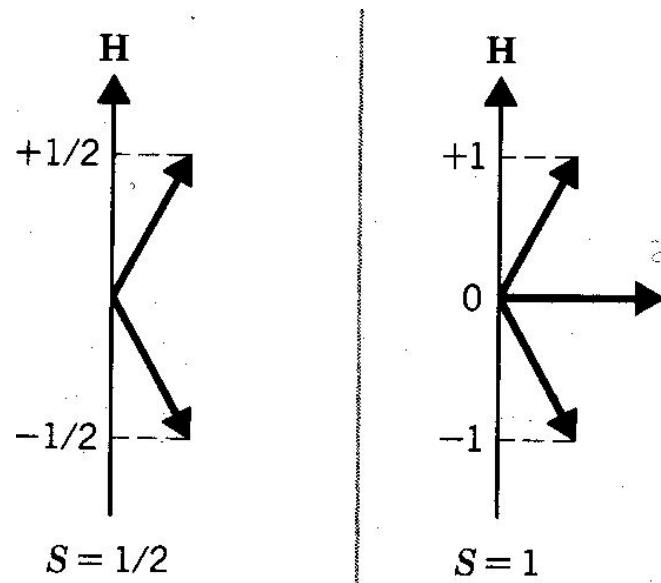


AF₃



J.S. Smart, "Effective field theories of magnetism"
W.B. Saunders 1966

**Spin quantum number and quantum fluctuations:
As $S \downarrow$, system becomes more quantum mechanical.**

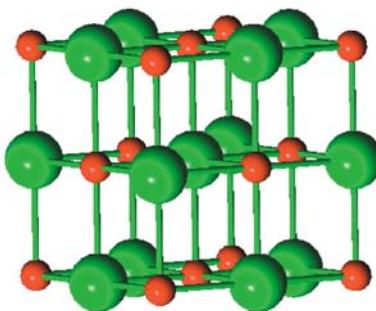


**Spin state fluctuations have major effect on attempt
to establish long range order.**

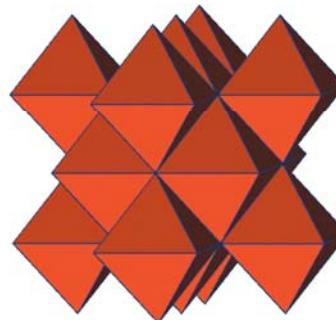
C. Real materials: transition metal compounds

1. Triangular planar

a. “ordered rock salt”



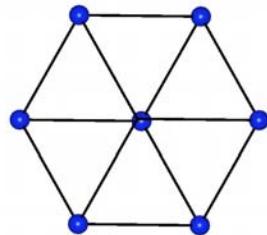
NaCl



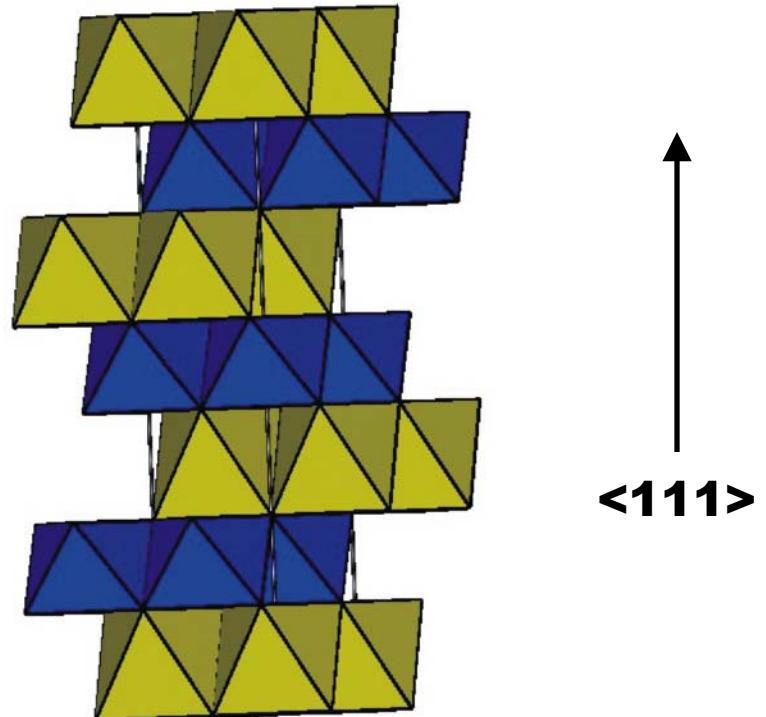
octahedra sharing all edges



**M⁺ and M³⁺ order
into layers
normal to the body
diagonal <111> of
the NaCl cubic cell
(Fm3m → R-3m)**



M³⁺ layer

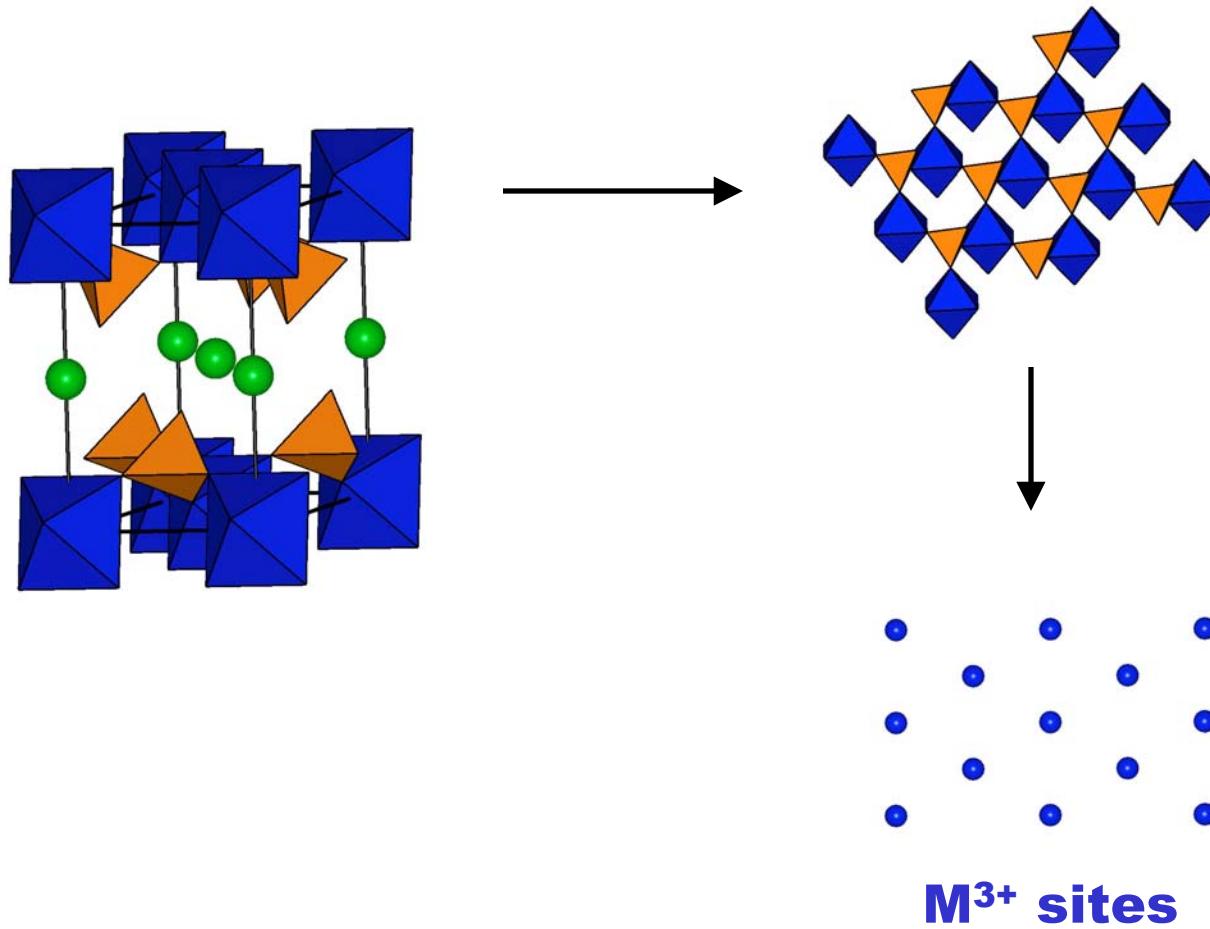


| <u>Compound</u> | <u>θ(K)</u> | <u>T_c(K)</u> | <u>θ/T_c</u> | <u>J_{intra}</u> | <u>J_{inter}</u> | <u>Grd.State</u> |
|--------------------------|--------------------------------------|-----------------------------------|---------------------------------------|--------------------------------------|--------------------------------------|-------------------------|
| NaNiO₂ | +100 | 23 | 4.4 | F | AF | AF LRO |
| LiNiO₂ | + 20 | 9* | 2.2 | F? | AF? | spin glass |
| LiCrO₂ | - 600 | 62 | 9.7 | AF | AF | AF LRO |

— ***spin freezing**

b. anhydrous alums : $\text{AM}(\text{SO}_4)_2$

$\text{A}^{+1} = \text{K}, \text{Rb}, \text{Cs}$ $\text{M}^{3+} = \text{Fe}, \text{Ti}$



Selected data for some anhydrous alums

| Compound | S | θ_c/K | T_c/K | $ \theta_c /T_c$ |
|----------------|-----|--------------|---------|------------------|
| $CsFe(SO_4)_2$ | 5/2 | -34.17 | 4.4 | 7.8 |
| $RbFe(SO_4)_2$ | 5/2 | -29.12 | 4.2 | 6.9 |
| $KFe(SO_4)_2$ | 5/2 | -55.5 | 8.3 | 6.7 |
| $KTi(SO_4)_2$ | 1/2 | -10.2 | <1.2 | >10 |

- **relatively large f values**
- **Ti phase does not order to 1.2K**

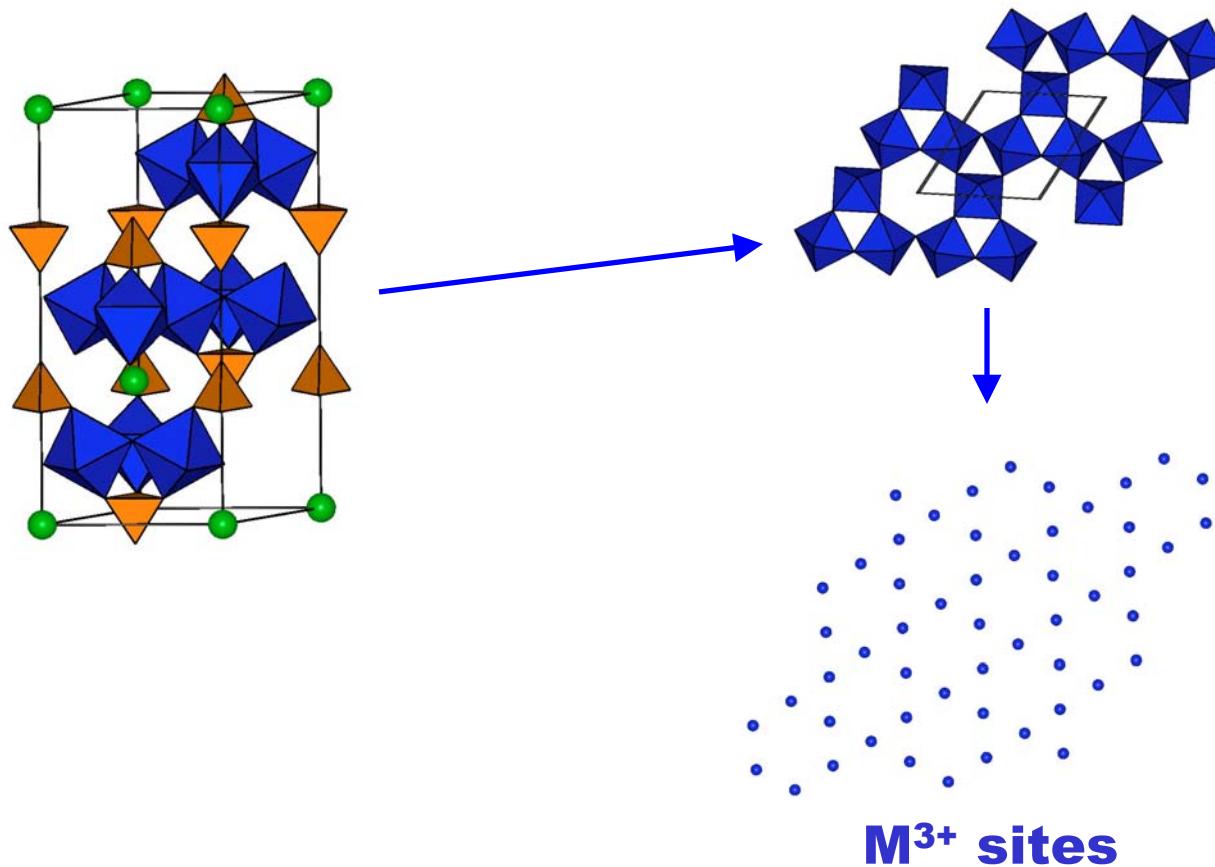
2. Kagomé

a. Jarosites

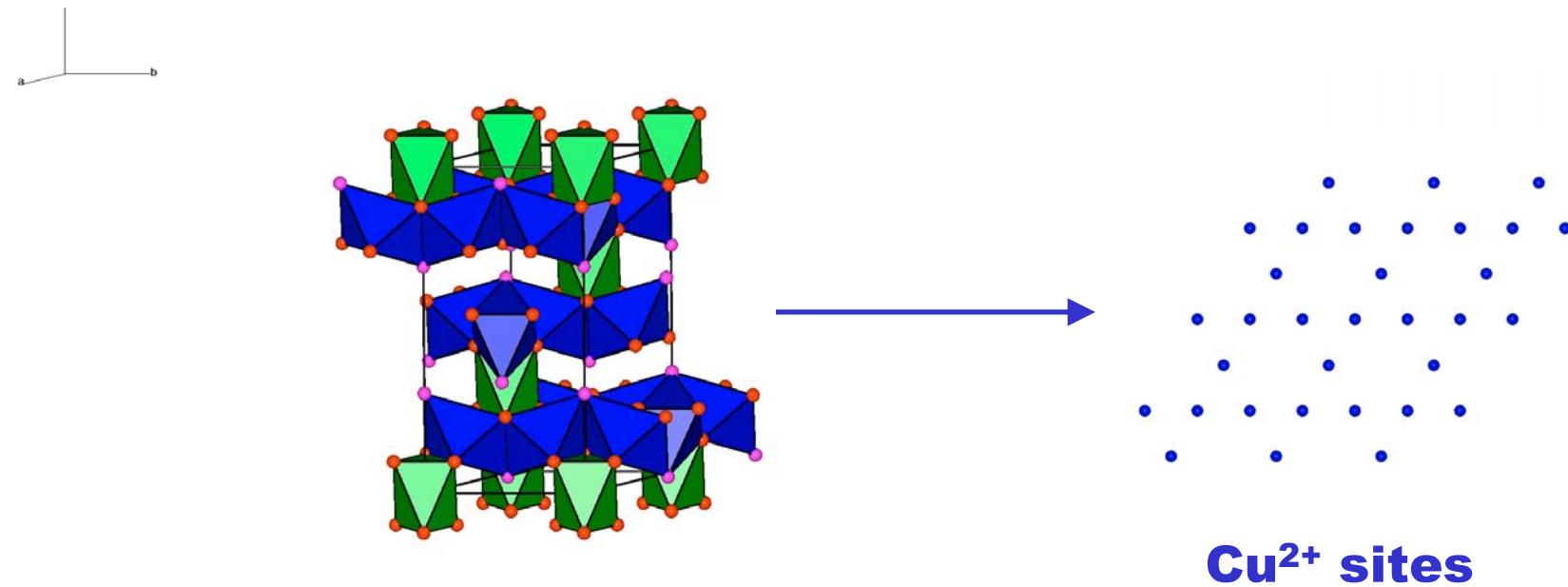
AM₃(SO₄)₂(OH)₆: ex

A⁺ = Na, K, Rb, NH₄, H₃O, Ag, Tl • • •

M³⁺ = Fe, Cr, V



b. Herberthsmithite $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$



AFe₃(SO₄)₂(OH)₆
[PRB 67 (2003) 064401]
[NatureMater. 4 (2004) 323]

| A^+ | C^a | Θ_{CW}/K^a | T_N/K | T_D/K^b | f^c | $\frac{c}{3}(=d)/\text{\AA}$ |
|------------------------------|-------|-------------------|---------|-----------|-------|------------------------------|
| Na ⁺ | 5.91 | -825 | 61.7 | ~58 | 13.5 | 5.535 |
| K ⁺ | 5.77 | -828 | 65.4 | ~53 | 12.7 | 5.728 |
| Rb ⁺ | 5.82 | -829 | 64.4 | ~53 | 12.9 | 5.856 |
| NH ₄ ⁺ | 5.84 | -812 | 61.8 | ~53 | 13.1 | 5.767 ^d |



No LRO to 1.4K
spin glassy ground state.
disorder due to H₃O⁺ - OH⁻
proton exchange.

Herbertsmithite $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

$\Theta \sim -300 \text{ K}$ but

- no LRO from neutrons to 1.4K !**
- no LRO specific heat to 50 mK !**
- spin liquid ground state?**

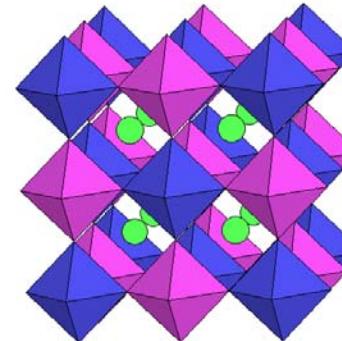
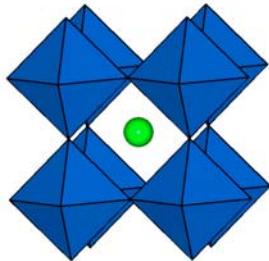
PRL 98 (2007) 107204

3. F.c.c. B-site ordered “double perovskites”

A_2BBO_6 : A = Ca, Sr, Ba, La

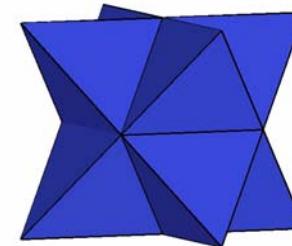
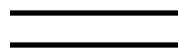
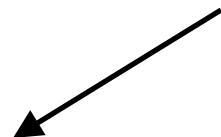
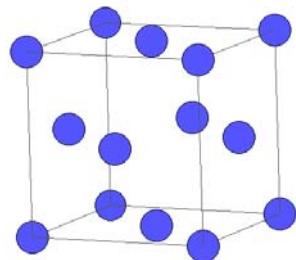
B = Y, Mg, Ca, Li

B = Mo, Re, Ru



ABO_3 - simple perovskite

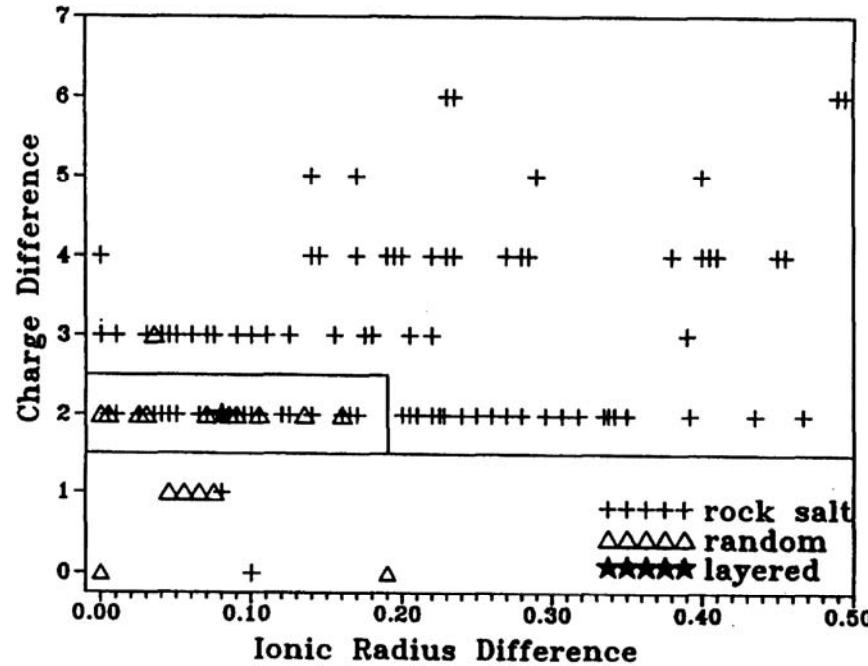
A_2BBO_6 - ordered double perovskite



B (magnetic) sublattice
f.c.c.

edge-sharing tetrahedra:
unfashionable but still
frustrated lattice

Factors controlling B-site (**BB**) ordering

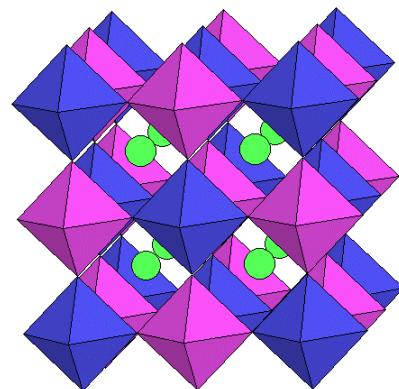


- **BB radius difference**
- **BB formal charge difference**

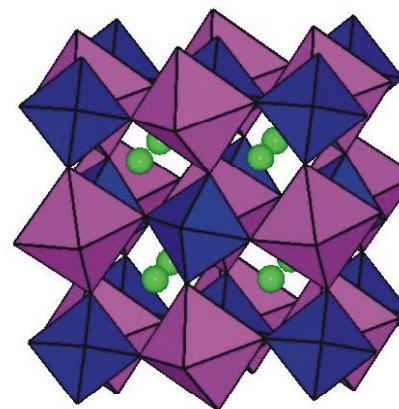
- **Ordered double perovskites - flexibility in crystal engineering.**
- **Control of crystal structure symmetry - cubic vs monoclinic**

Goldschmidt tolerance factor, $t = (A - O) / 2^{1/2} (\langle B - O \rangle)$

$t \sim 1$ (Fm3m)



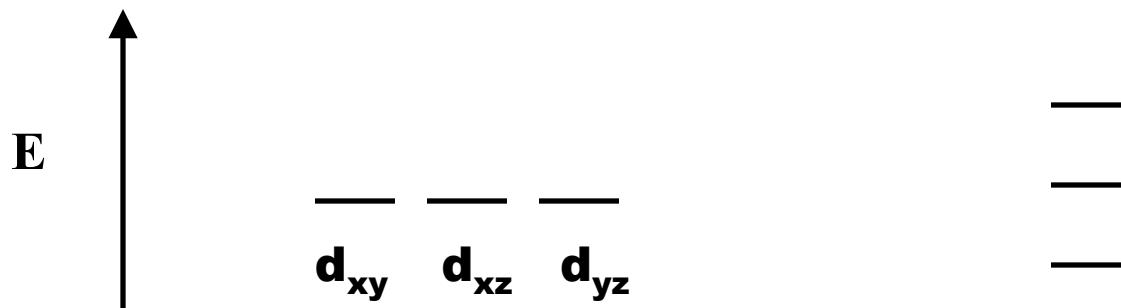
$t \sim 0.9$ (P2₁/n)



- Control of site symmetry of magnetic ion/orbital degrees of freedom

**Fm3m (m3m or O_h)
(t_{2g}ⁿ) retain orbital
degeneracy**

**P2₁/n (-1)
(t_{2g}ⁿ) degeneracy lifted
“orbital ordering”**



- Control of spin state of magnetic ion within same crystal symmetry



B = Ca²⁺
B⁶⁺ = Re⁶⁺ (S = 1/2)
= Os⁶⁺ (S = 1)
B = Y³⁺, Lu³⁺
B⁵⁺ = Mo⁵⁺ (S = 1/2)
= Re⁵⁺ (S = 1)
= Ru⁵⁺ (S = 3/2)



B = Li⁺
B⁵⁺ = Mo⁵⁺ (S = 1/2)
= Re⁵⁺ (S = 1)
= Ru⁵⁺ (S = 3/2)



B = Ca²⁺
B⁶⁺ = Re⁶⁺ (S = 1/2)

B = Y³⁺Lu³⁺
B⁵⁺ = Ru⁵⁺ (S = 3/2)

| <u>Compound</u> | <u>S</u> | <u>θ(K)</u> | <u>T_c(K)</u> | <u>f</u> | <u>Ground State</u> |
|--|------------|-------------------------------|----------------------------|----------------|---------------------|
| Ba₂YRuO₆ | 3/2 | -571 | 36 | 16 | AF LRO |
| La₂LiRuO₆ | 3/2 | -207 | 23.8 | 9 | AF LRO |
| Ba₂YReO₆ | 1 | -480 | 40* | 12 | ? |
| La₂LiReO₆ | 1 | -204 | 33* | 6 | ? |
| Ba₂YMoO₆ | 1/2 | -91 | < 2 | > 45 | ? |
| Sr₂CaReO₆ | 1/2 | -443 | 14 | 32 | spin glass |

* broad χ (max), ZFC/FC divergence

4. Pyrochlore

a. pyrochlore oxides: $\text{A}_2\text{B}_2\text{O}_7$ or $\text{A}_2\text{B}_2\text{O}_6\text{O}$

$\text{A}^{3+} = \text{Ln}^{3+}(\text{La-Lu,Y}), \text{Tl}, \text{Bi}$

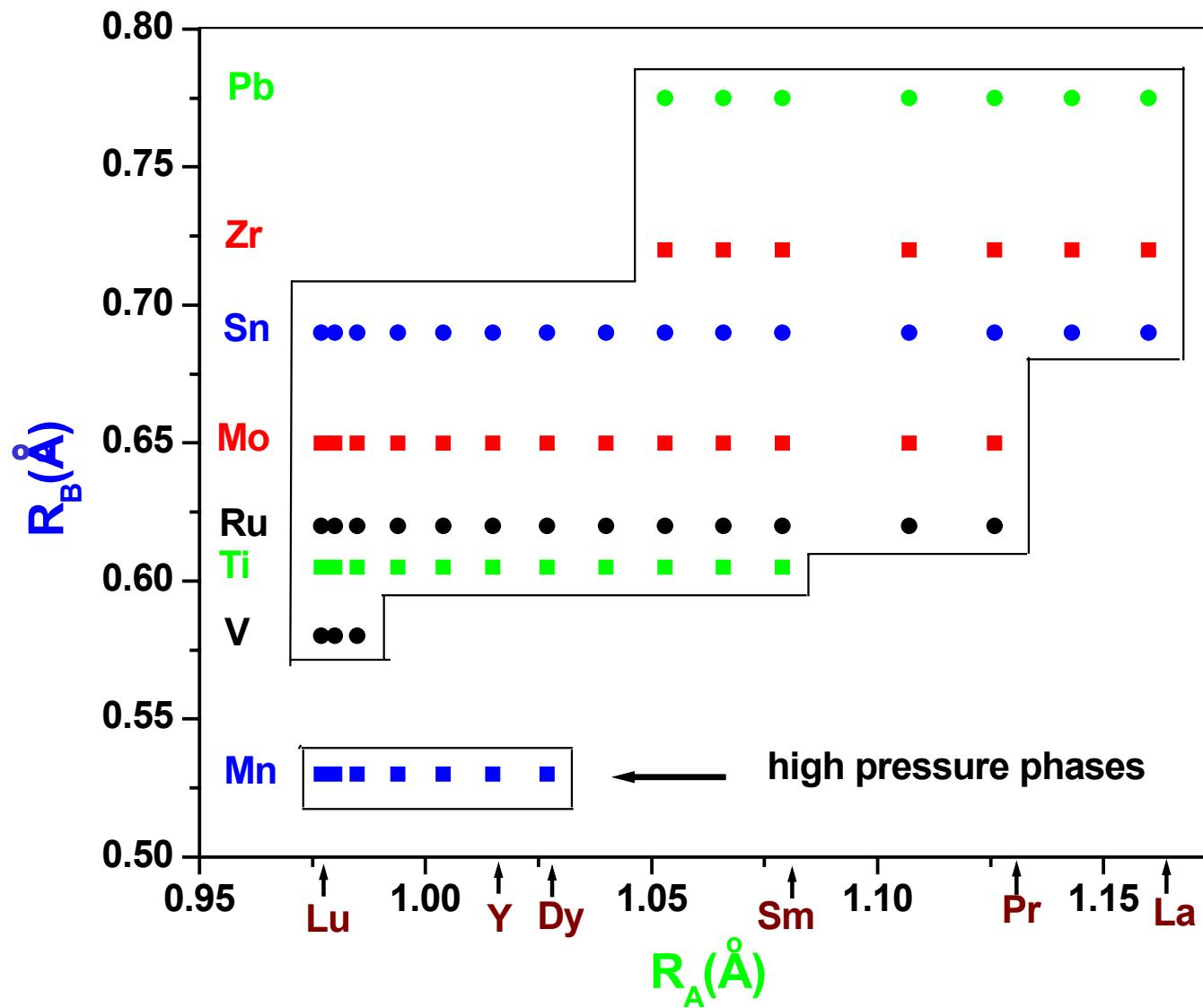
$\text{B}^{4+} = 3\text{d}, 4\text{d}, 5\text{d}$ transition metals, Sn, Pb

The Periodic Table

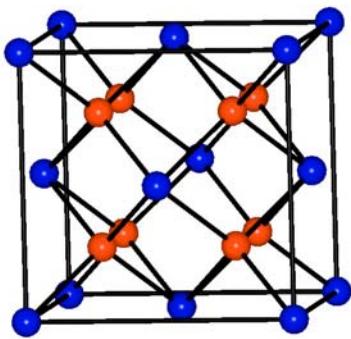
Periodic Table diagram showing the distribution of elements into s, p, d, and f blocks. The s block is at the top left, p block is at the top right, d block is in the middle, and f block is at the bottom. Lanthanides are shown as a green horizontal band between the d and f blocks, and Actinides are shown as a green vertical band below the f block.

| Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 13/III | 14/IV | 15/V | 16/VI | 17/VII | 18/VIII | | | | |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--|--|
| 2 | Li 6.941 | Be 9.012 | | | | | | B 10.81 | C 12.01 | N 14.01 | O 16.00 | F 19.00 | He 4.003 | | | | |
| 3 | Na 22.99 | Mg 24.30 | Sc 44.96 | Cr 52.00 | | | | Al 26.98 | P 28.09 | Si 30.97 | S 32.07 | Cl 35.45 | Ne 20.18 | | | | |
| 4 | K 39.10 | Ca 40.08 | Sc 44.96 | Cr 52.00 | Fe 55.85 | Co 58.93 | Ni 58.69 | Cu 63.55 | Zn 65.39 | Ga 69.72 | Ge 72.61 | As 74.92 | Se 78.96 | Br 79.90 | | | |
| 5 | Rb 85.47 | Sr 87.62 | | Nb 92.91 | Tc 98.91 | Rh 102.9 | Pd 106.4 | Ag 107.9 | Cd 112.4 | In 114.8 | | Sb 121.8 | Te 127.6 | I 126.9 | Xe 131.3 | | |
| 6 | Cs 132.9 | Ba 137.3 | La-Lu | Ta 180.9 | W 183.8 | Re 186.2 | | Au 197.0 | Hg 200.6 | | | Po 210.0 | At 210.0 | Rn 222.0 | | | |
| 7 | Fr 223.0 | Ra 226.0 | Ac-Lr | Unq | Unp | Unh | Uns | Uno | Une | | | | | | | | |
| | s block | | d block | | | | | | | | | | p block | | | | |
| | Lanthanides | | | | | | | | | | | | | | | | |
| | Actinides | | | | | | | | | | | | | | | | |
| | f block | | | | | | | | | | | | | | | | |

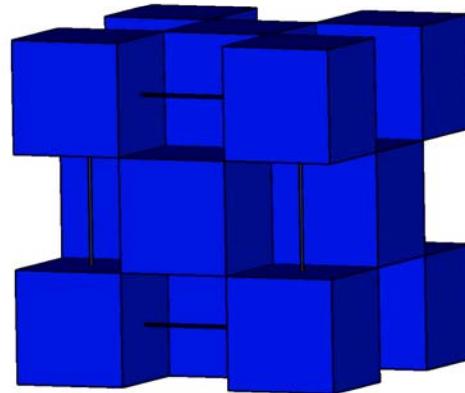
Pyrochlore “structure - field map”

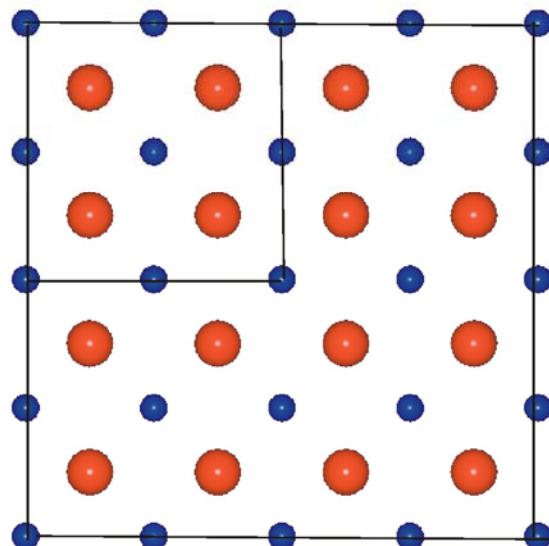
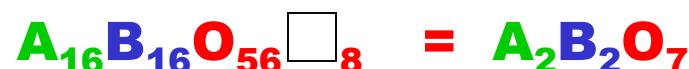
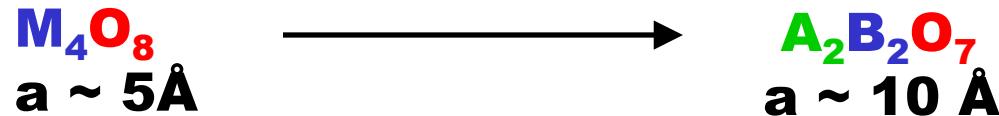


Pyrochlore as an ordered defect CaF_2 (fluorite)

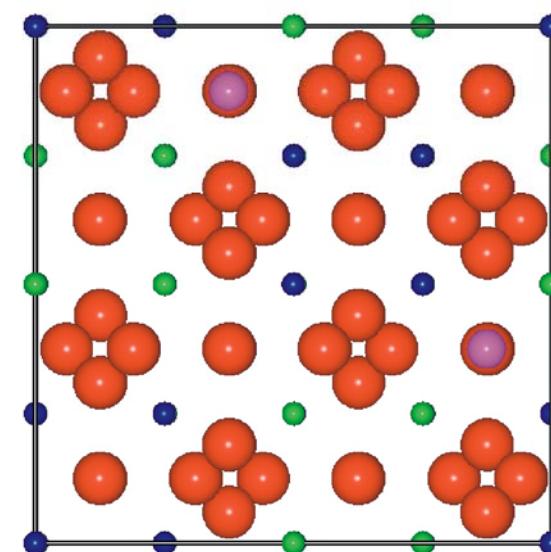


**M^{4+} coordinated
by 8 O^{2-} , perfect cube**

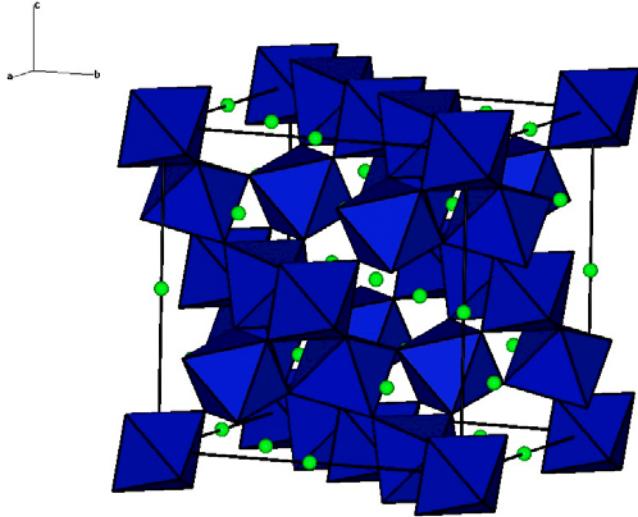




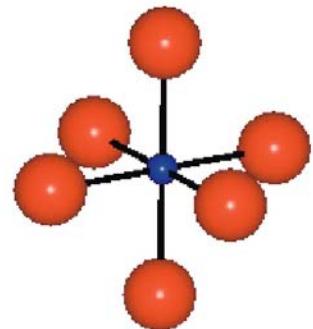
M₄O₈ face projection



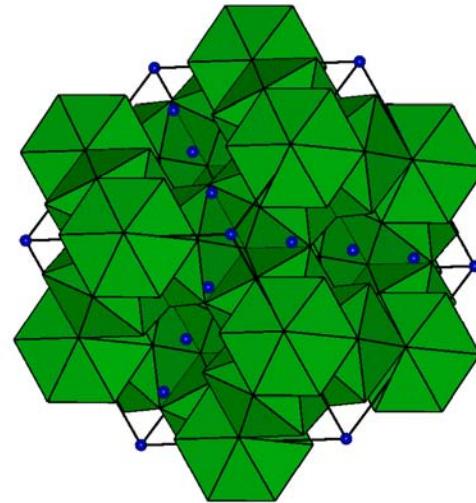
pyrochlore face projection



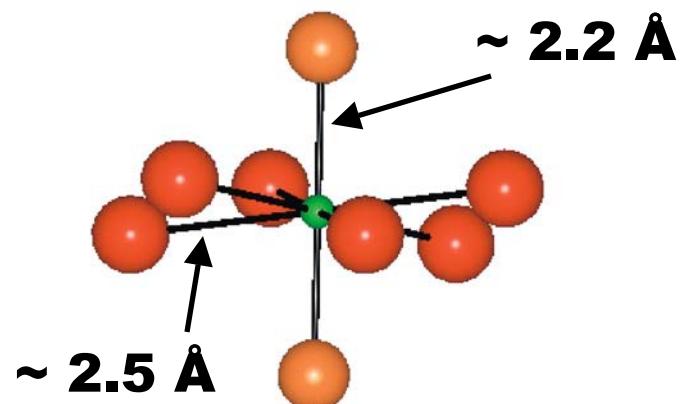
B - site octahedra

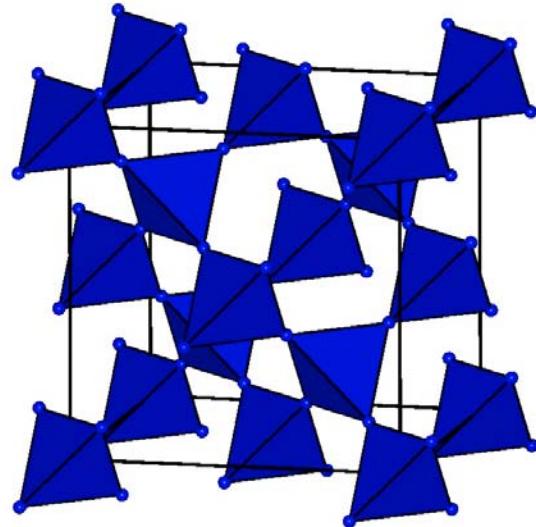


6 equiv B - O \sim 2.0 Å



A - site hexagonal bipyramids

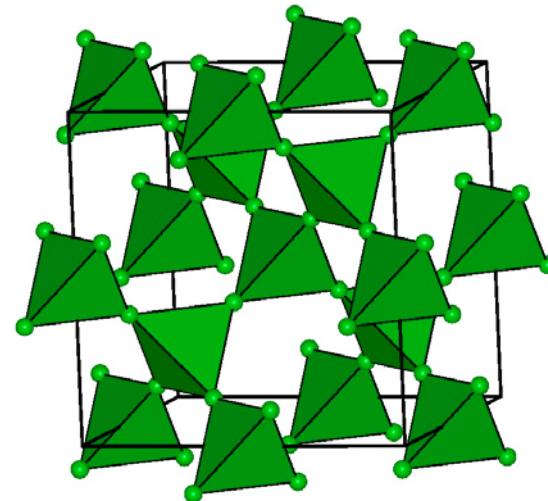




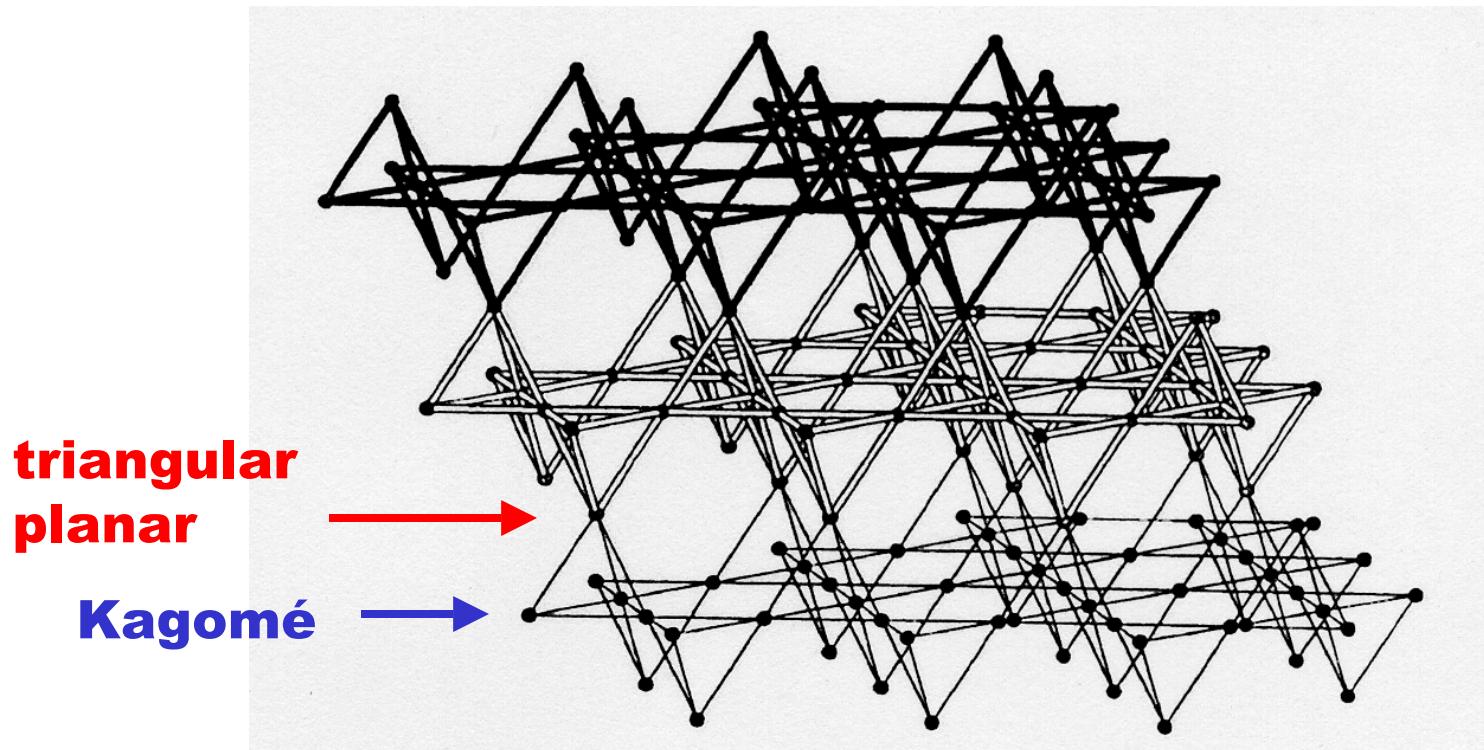
B-sites

A-sites

**Both A and B sites
pyrochlore lattices !!**



Another view: pyro lattice as alternating stacking of Kagomé and triangular planar layers



Possibilities:

- **A-site only magnetic**

A = Ln³⁺ (\neq La, Y, Lu) , B = Ti⁴⁺, Sn⁴⁺, Zr⁴⁺ ...

- **B - site only magnetic**

A = Y, Lu, B = V⁴⁺, Mn⁴⁺, Mo⁴⁺, Ir⁴⁺ ...

- **both A and B sites magnetic**

A = Ln³⁺ (\neq La, Y, Lu) , B = V⁴⁺, Mn⁴⁺, Mo⁴⁺, Ir⁴⁺ ...

A-site only

- $\text{Tb}_2\text{Ti}_2\text{O}_7$ - spin liquid
- $\text{Dy}_2\text{Ti}_2\text{O}_7$ - spin ice
- $\text{Gd}_2\text{Ti}_2\text{O}_7$ - two AF orderings

B-site only

- $\text{Y}_2\text{Mo}_2\text{O}_7$ - spin glass, subtle disorder
- $\text{Y}_2\text{Mn}_2\text{O}_7$ - F - SRO, complex ground state
- $\text{Lu}_2\text{V}_2\text{O}_7$ - F insulator

A and B site

- $\text{Ln}_2\text{Mo}_2\text{O}_7$ and $\text{Ln}_2\text{Ir}_2\text{O}_7$ - metal insulator transitions
(ferromagnetic metal to insulating spin glass)
- $\text{Nd}_2\text{Mo}_2\text{O}_7$ - anomalous Hall effect

Disorder Levels in Pyrochlores?

“antisite” **A** ⇔ **B** and **O** vacancies

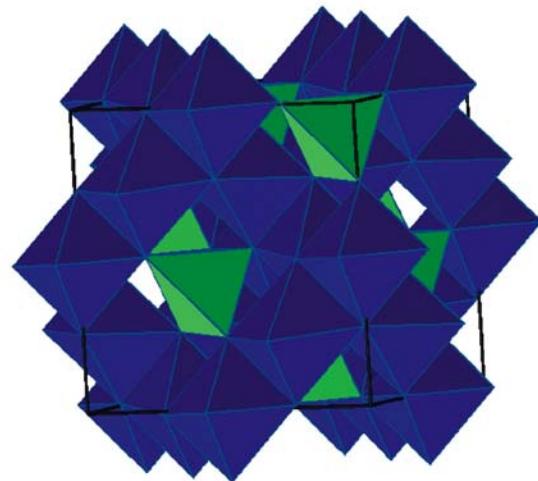
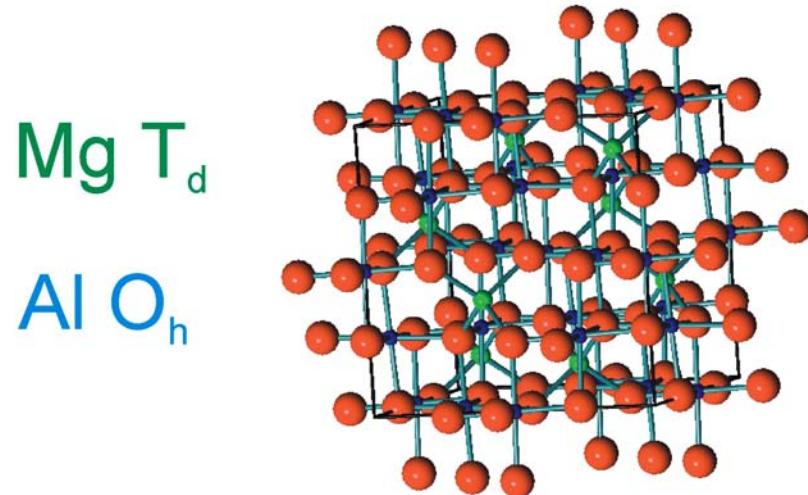
Predictions: Phil. Mag. 82 (2002) 123

| | % antisite | | | | | | % O vacancies | | | | | |
|-----------|------------|-----------|-----------|-----------|-----------|-----------|---------------|-----------|-----------|-----------|-----------|-----------|
| A | Ti | Ru | Mo | Sn | Zr | Pb | Ti | Ru | Mo | Sn | Zr | Pb |
| Lu | .71 | .94 | 1.7 | 4.4 | | | .24 | .31 | .57 | 1.5 | | |
| Yb | .66 | .87 | 1.5 | 3.9 | | | .22 | .29 | .51 | 1.3 | | |
| Er | .58 | .74 | 1.2 | 3.0 | | | .19 | .25 | .41 | 1.0 | | |
| Y | .54 | .68 | 1.1 | 2.6 | | | .18 | .23 | .36 | .87 | | |
| Gd | .50 | .58 | .83 | 1.8 | 3.1 | 8.3 | .17 | .19 | .28 | .61 | 1.0 | 2.8 |
| Sm | .52 | .57 | .76 | 1.6 | 2.7 | 6.4 | .17 | .19 | .25 | .53 | .88 | 2.1 |
| Nd | | .59 | | 1.4 | 2.2 | 4.9 | | .20 | | .47 | .75 | 1.6 |
| La | | | | 1.3 | 1.8 | 2.8 | | | | .43 | .60 | .94 |

b. Spinel oxides: AB_2O_4 .

- The O^{2-} lattice is cubic close packed (f.c.c.)
- Typically, A^{2+} and B^{3+} but others are possible
e.g. A^{1+} , B^{3+} , B^{4+} or A^{3+} , B^{2+} , B^{3+}
- A occupies a tetrahedral site (T_d) and B an octahedral site (O_h)
- In a close-packed lattice, there are
per close-packed ion
2 T_d sites and 1 O_h site
- Thus, the occupation rates are:
 $1/8 T_d$ by A^{2+} $1/2 O_h$ by B^{3+}

Spinel MgAl_2O_4



Edge-shared
octahedra

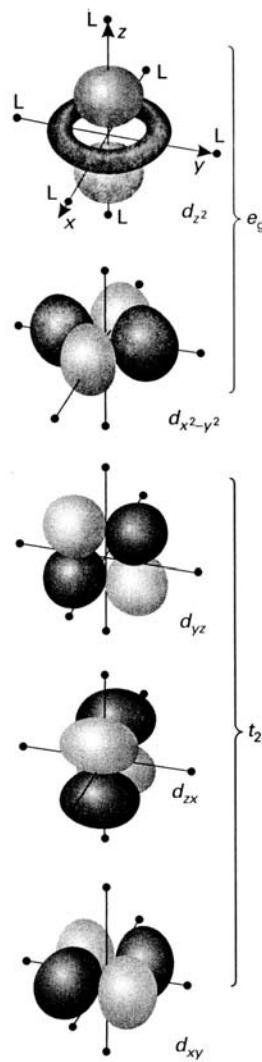
Factors determining site occupation in spinels?

- Unlike perovskites and pyrochlores, **A²⁺** and **B³⁺** have similar ionic radii
- size differences not usually critical
- major factor is “O_h site preference energy”

O.S.P.E.

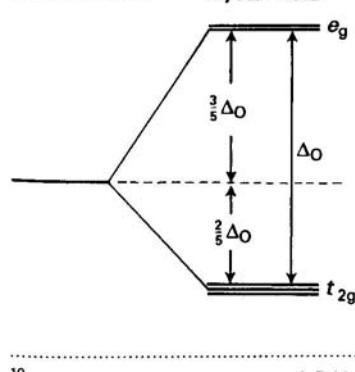
derived from simple crystal field theory

O_h xtal field



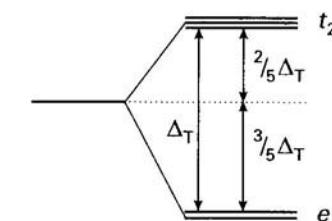
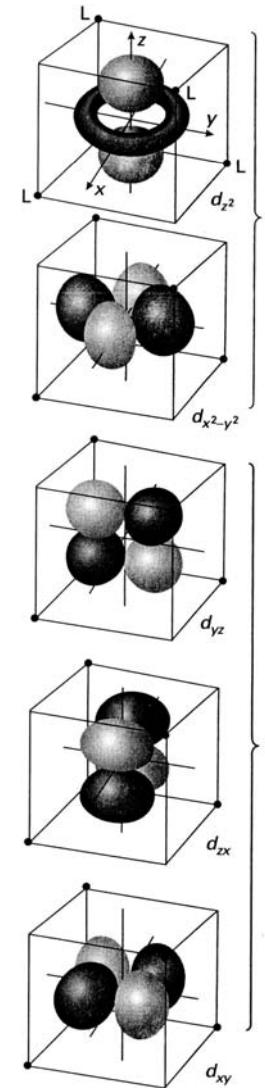
Spherical environment

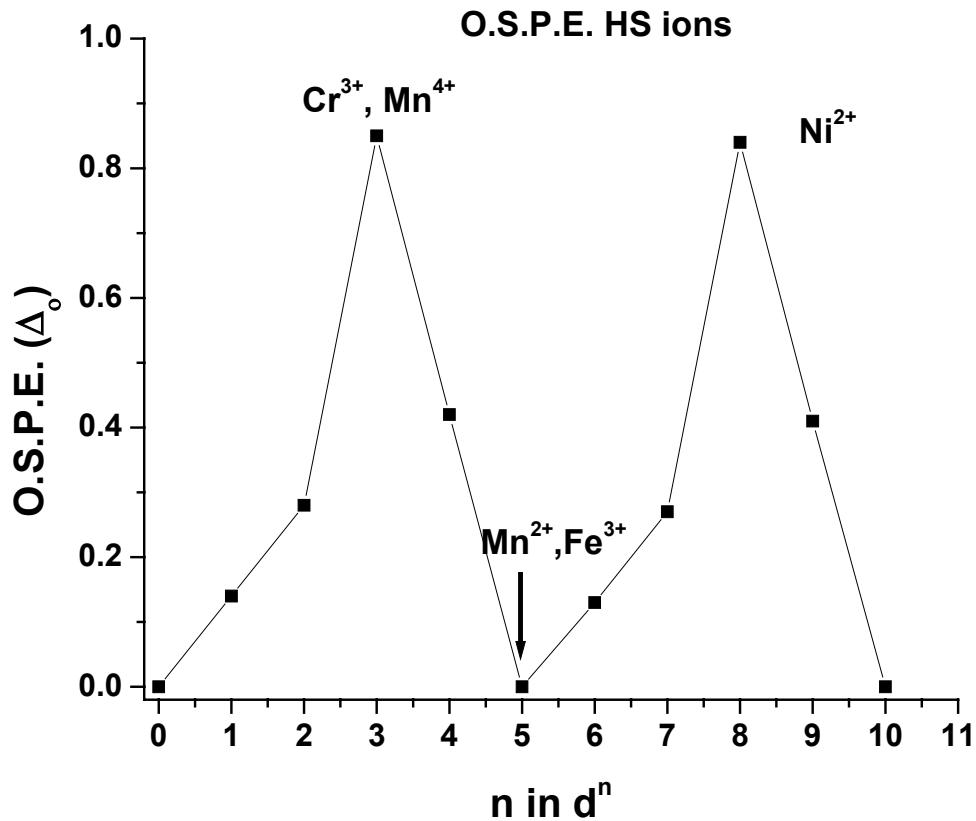
In octahedral crystal field



$$\Delta_O \sim 2 \Delta_T$$

T_d xtal field





Note:

large OSPE
 $\text{Cr}^{3+}(t_{2g}^3)$ $\text{Ni}^{2+}(t_{2g}^6 e_g^2)$

OSPE = 0

$\text{Mn}^{2+}, \text{Fe}^{3+}(t_{2g}^3 e_g^2)$

**OSPE even
larger for
4d and 5d metal
ions which are
LS**

Classification of spinels:

- **normal** $\text{A}^{2+}(\text{T}_d)$, $\text{B}^{3+}(\text{O}_h)$
ex ZnCr_2O_4 , LiMn_2O_4
- **inverted** $\text{B}^{3+}(\text{T}_d)$ A^{2+} , $\text{B}^{3+}(\text{O}_h)$
ex Fe_3O_4 - $\text{Fe}^{3+}\text{Fe}^{2+}\text{Fe}^{3+}\text{O}_4$
- **mixed**

Spinels of interest:

ZnCr_2O_4 - frustration driven cubic to tetragonal phase transition

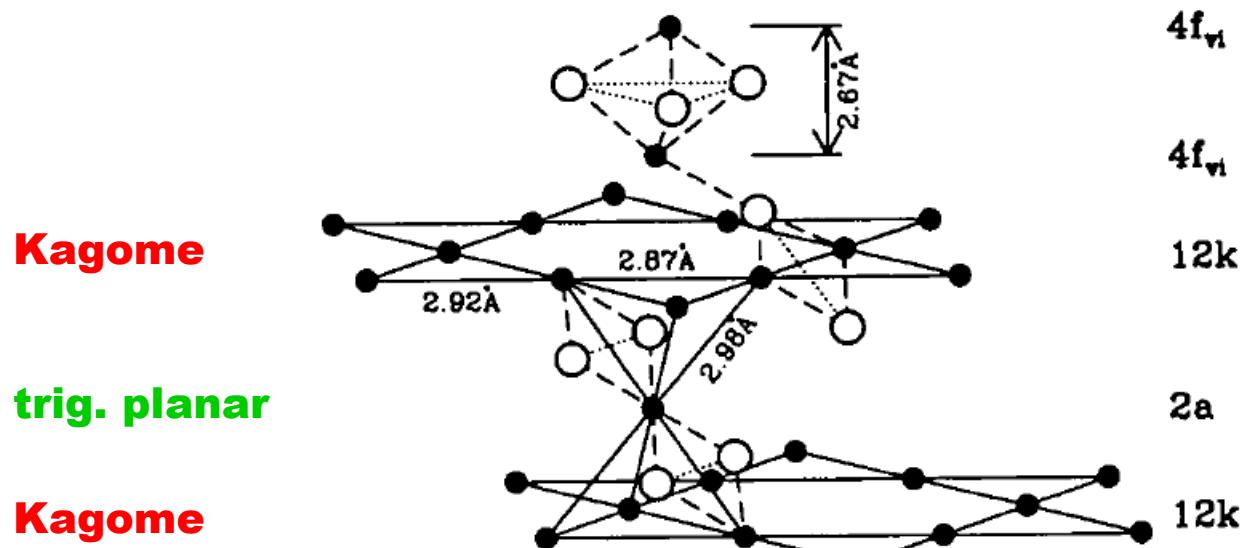
LiMn_2O_4 - co-existence AF SRO/LRO

$\text{Li}_2\text{Mn}_2\text{O}_4$, MgMn_2O_4 - only 2D magnetic correlations

5. Other frustrated 3D lattices.

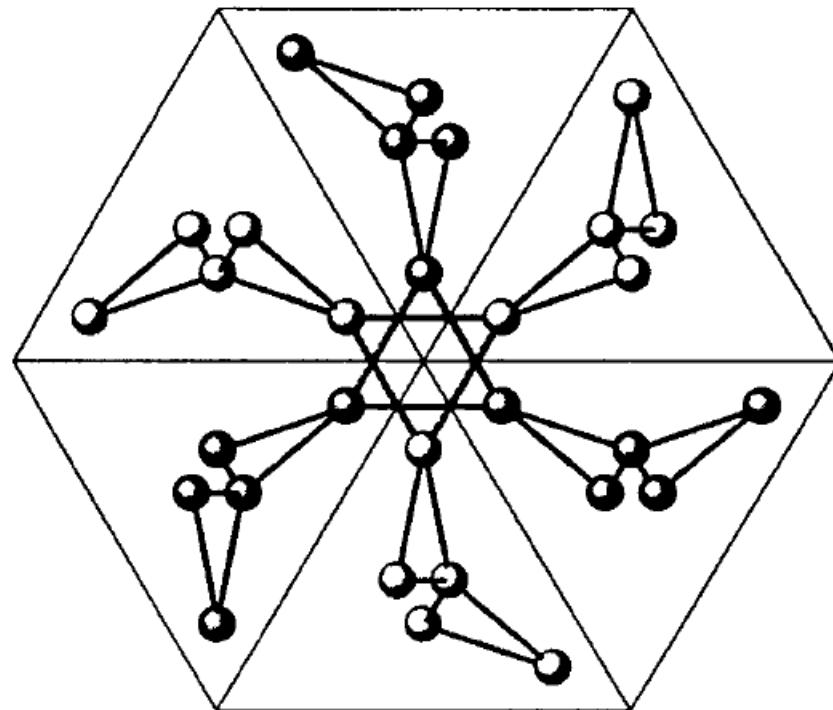
- a. “**SCGO**” $\text{SrCr}_{12-x}\text{Ga}_x\text{O}_{19}$
- b. **Garnet**: $\text{A}_3\text{M}_5\text{O}_{12}$. ex $\text{Gd}_3\text{Ga}_5\text{O}_{12}$
- c. $\text{BaM}_{10}\text{O}_{15}$: $\text{M} = \text{V}^{3+}, \text{Cr}^{3+}$
- d. “**ordered rock salt**” ex $\text{Li}_3\text{Mg}_2\text{RuO}_6$

a. SCGO decoupled “pyrochlore slabs”



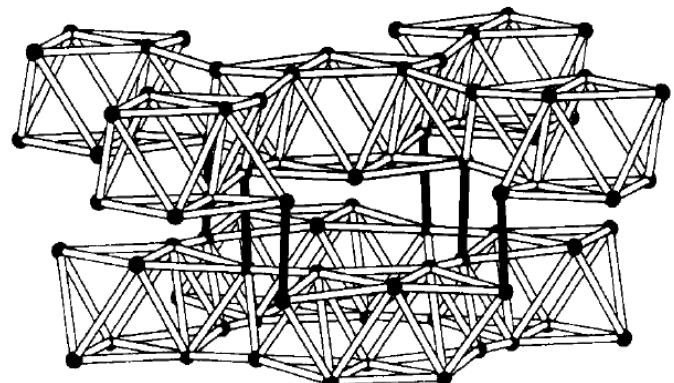
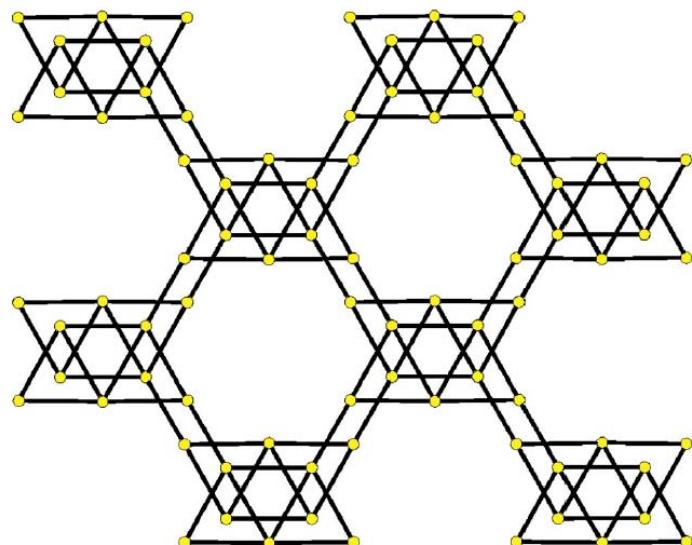
b. “GGG” Gd^{3+} lattice in $\text{Gd}_3\text{Ga}_5\text{O}_{12}$

corner sharing triangles

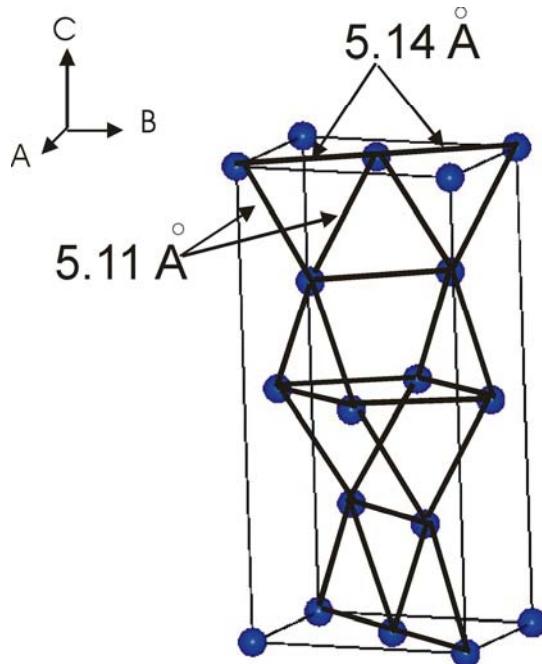


c. $\text{BaM}_{10}\text{O}_{15}$ - $\text{M} = \text{V}, \text{Cr}$

M_{10} clusters of edge-sharing tetrahedra



d. “ordered NaCl” $\text{Li}_3\text{Mg}_2\text{RuO}_6$



- **ribbons of edge-sharing triangles linked by corners**

$$\text{Ru-Ru-Ru} = 59.81^\circ \times 2, 60.39^\circ$$

Ru-Ru distances and angles
in $\text{Li}_3\text{Mg}_2\text{RuO}_6$