[1] Show that the direct product of the dimer singlet

$$\left| \Psi_{s} \right\rangle = \bigotimes_{i} \left| s \right\rangle_{i}$$

is an eigenstate of the Shastry-Sutherland Hamiltonian irrespectrive of the value of J'/J.



Tutorial: Shastry-Sutherland model II

[2] Consider two orthogonal dimers. Show that

$$J'(\mathbf{s}_{1} + \mathbf{s}_{2}) \cdot \mathbf{s}_{3} |s_{a}\rangle |t_{mb}\rangle = 0 \quad (m = 1, 0, -1)$$
$$J'(\mathbf{s}_{1} + \mathbf{s}_{2}) \cdot \mathbf{s}_{3} |t_{1}\rangle_{a} |s\rangle_{b} = \frac{J'}{2} (|t_{1}\rangle_{a}|t_{0}\rangle_{b} - |t_{0}\rangle_{a}|t_{1}\rangle_{b})$$

Explain the difference between this and the result for a frustrated ladder.





 $|s\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$ $|t_1\rangle = |\uparrow\uparrow\rangle$ $|t_0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$ $|t_{-1}\rangle = |\downarrow\downarrow\rangle$

[3] consider a dimer with exchange and DM interaction in a magnetic field.

$$H = J \mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{D} \cdot (\mathbf{S}_1 \times \mathbf{S}_2) - g\mu_B H (S_1^z + S_2^z) \qquad D \ll J$$

Let us define the uniform and the staggered moment as

$$\mathbf{m}_{u} = \frac{\langle \mathbf{S}_{1} + \mathbf{S}_{2} \rangle}{2}, \quad \mathbf{m}_{s} = \frac{\langle \mathbf{S}_{1} - \mathbf{S}_{2} \rangle}{2}$$



Derive the following to the lowest order in D.

$$\mathbf{m}_{\rm s} = \frac{g\mu_B}{2J^2} \big(\mathbf{D} \times \mathbf{H} \big)$$

[4] In the previous question, suppose magnetic ions have NMR active nuclei with isotropic on-site hyperfine coupling *A*. Describe the angle dependence of the NMR shift. What is the distinctive feature compared with the ordinary shift due to uniform susceptibility.

