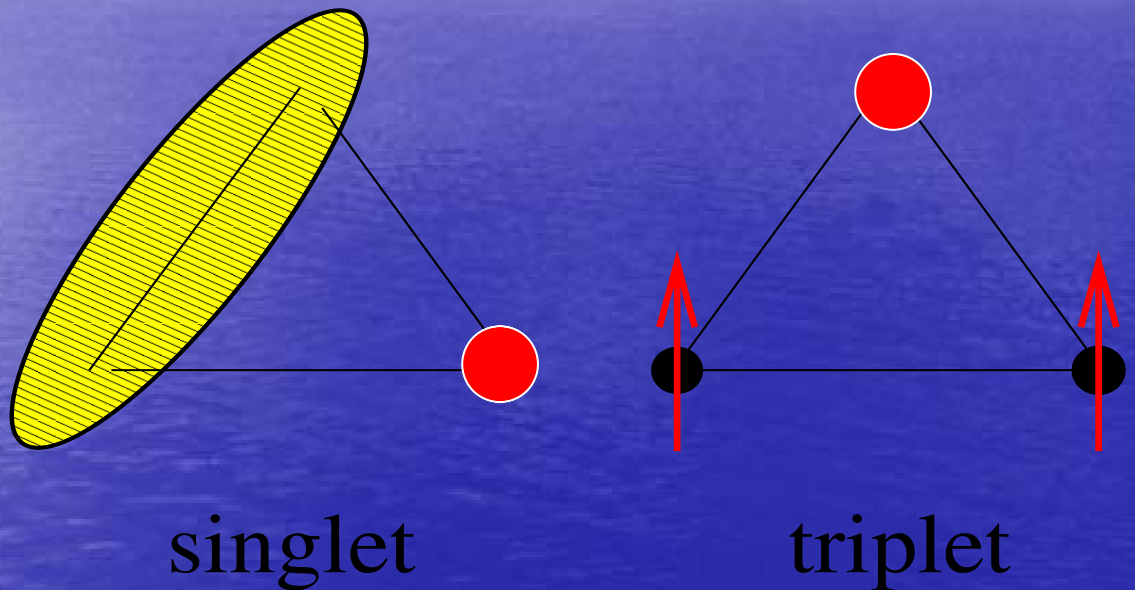
Mobile holes in frustrated quantum magnets

Tutorial

1. Exact diagonalisation of the 3-site U=∞ Hubbard model

J=0



$$\mathcal{H} = -t \sum_{i,j} \mathcal{P}_G \left(c^\dagger_{i\sigma} c_{j\sigma} + h.c. \right) \mathcal{P}_G$$

2. Single hole Green's function computed by Lanczos algorithm

- Show that the GF of a single hole in a quantum frustrated magnet has the following form:

$$I(\omega) = -\frac{1}{\pi} \lim_{\epsilon \to 0} \mathrm{Im}\, \tilde{C}(\omega + i\epsilon)$$

with:

$$\tilde{C}(z) = \langle \Psi_0 | A \frac{1}{z - H + E_0} A^\dagger | \Psi_0 \rangle$$

- Re-write $\tilde{C}(z)$ as:

$$\tilde{C}(z) = \langle \Psi_0 | A A^\dagger | \Psi_0 \rangle \langle \tilde{\Phi}_1 | (z' - H)^{-1} | \tilde{\Phi}_1 \rangle$$

- Assuming $|\tilde{\Phi}_1\rangle$ is computed, what is the procedure to get the matrix below ?

$$z' - H = \begin{matrix} z' - \tilde{e}_1 & -\tilde{b}_2 & \dots & 0 \\ -\tilde{b}_2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\tilde{b}_M \\ 0 & \dots & -\tilde{b}_M & z' - \tilde{e}_M \end{matrix}$$

Δ_n is the $(M-n+1)\times(M-n+1)$ matrix:

$$\Delta_n = \begin{matrix} z' - \tilde{e}_n & -\tilde{b}_{n+1} & \ldots & 0 \\ -\tilde{b}_{n+1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\tilde{b}_M \\ 0 & \ldots & -\tilde{b}_M & z' - \tilde{e}_M \end{matrix}$$

- Then, show that:

$$\tilde{C}(z) = \langle \Psi_0 | A A^\dagger | \Psi_0 \rangle \frac{D_2}{D_1}$$

- Prove that:

$$D_n = (z' - \tilde{e}_n)D_{n+1} - \tilde{b}^2_{n+1}D_{n+2}$$

for $1 \leq n \leq M - 2$.

- Deduce the following continued-fraction form:

$$\tilde{C}(z) = \cfrac{\langle \Psi_0 | A A^\dagger | \Psi_0 \rangle}{z + E_0 - \tilde{e}_1 - \cfrac{\tilde{b}_2^2}{z + E_0 - \tilde{e}_2 - \cfrac{\tilde{b}_3^2}{z + E_0 - \tilde{e}_3 - \dots}}}$$