School on Physics, Technology and Applications of Accelerator Driven Systems (ADS)

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ADS Dynamics
"Accelerator-Driven System Dynamics. Part II"

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Accelerator-driven and advanced system dynamics

Part II
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Outline

• New challenges in the simulation of the neutron dynamics of ADS
• Models and methods
• Time-dependent transport models
• An advanced application: dynamics of fluid fuel systems
• Simulation of source experiments
The role of delayed neutrons

- Time-dependent analysis of nuclear systems can be done only taking account of delayed emissions from fission
- On the basis of elementary physics considerations, a multiplying system evolution is regulated by the exponential law \( \exp((\delta k/\Lambda)t) \), where ....
The role of delayed neutrons

$\Lambda$ is a “characteristic” time
- No delayed neutrons: $10^{-4}$ - $10^{-6}$ s
- With delayed neutrons: $\Lambda + \beta/\lambda \sim 10^{-1}$ s
  ($\lambda \sim 10^{-1}$ s$^{-1}$)

Evolution is dominated by delayed neutrons
(for sub-prompt-critical systems)

**Note:** $\beta$ is an important dynamic parameter
(the *physical* fraction $\beta$ is: for U235: 0.0065, for Pu239: 0.0022)
Time-scales in the dynamics of nuclear reactors

- Prompt neutron (very fast) scale, connected to the lifetime of prompt neutrons ($10^{-4} - 10^{-6}$ s)
- Delayed emission scale, connected to evolution of delayed neutron precursors ($10^{-1} - 10^{1}$ s)
- Thermal-hydraulic scale (feedback), connected to the evolution of temperatures and hydraulic parameters ($10^{-1} - 10^{2}$ s)
- Control scale, connected to the movement of masses in the system (control rods, poisons)
- Nuclide transmutation scale, connected to neutron transmutation phenomena ($>10^{2}$ s)
Time-scales in the dynamics of nuclear reactors

Very different time-scales

the physico-mathematical problem is stiff
Time-scales in the dynamics of nuclear reactors

- We now focus our interest on the dynamics of nuclear systems during operational and accidental transients
  - Nuclide transmutation can be neglected, but still
    - Delayed emissions
    - Thermal feedback
  are to be considered.
Basic equations for neutron dynamics (1)

Boltzmann transport equation in presence of delayed emissions:

\[
\frac{\partial n(r, E, \Omega, t)}{\partial t} = \hat{B}(t)n(r, E, \Omega, t) + \sum_{i=1}^{6} \lambda_i \frac{\chi_i(E)}{4\pi} C_i(r, t) + S(r, E, \Omega, t)
\]

\[
\frac{\partial (\chi_i(E)C_i(r, t)/4\pi)}{\partial t} = \hat{M}_i(t)n(r, E, \Omega, t) - \lambda_i \frac{\chi_i(E)}{4\pi} C_i(r, t)
\]
Basic equations for neutron dynamics (2)

\[
\frac{\partial n(r, E, \Omega, t)}{\partial t} = \hat{B}(t)n(r, E, \Omega, t) + \sum_{i=1}^{6} \lambda_i \frac{\chi_i(E)}{4\pi} C_i(r, t) + S(r, E, \Omega, t)
\]

\[
\hat{L}(t) = -\Omega \cdot \nabla v(E) - \Sigma_s(r, E, t) v(E) + \int dE' \int d\Omega' v(E') \Sigma_s(r, E' \rightarrow E, \Omega' \cdot \Omega, t)
\]

\[
\hat{M}_p(t) = \sum_j \frac{\chi_p(E)}{4\pi} \int dE' \int d\Omega' v(E')(1 - \beta^j) \times \nu^j(E') \Sigma_f^j(r, E', t)
\]

**Leakage**  **Balance operator**  **Prompt multiplication**

\[
\hat{B}(t) = \hat{L}(t) + \hat{M}_p(t)
\]

November 2007  New challenges in the simulation of the neutron dynamics of ADS
Basic equations for neutron dynamics (3)

\[
\frac{\partial (\chi_i(E)\tilde{C}_i(r, t)/4\pi)}{\partial t} = \hat{M}_i(t)n(r, E, \Omega, t) - \\
\lambda_i \frac{\chi_i(E)}{4\pi} C_i(r, t)
\]

Delayed multiplication

\[
\hat{M}_i(t) = \sum_j \frac{\chi_i(E)}{4\pi} \int dE' \int d\Omega' \nu_i(E') \beta_i^j \times \\
\nu_j(E') \sum_f^j(r, E', t)
\]

Operators can be time-dependent because of:

- effects independent of neutron flux (perturbations)
- non-linear effects (feedback)
Challenges in the simulation of neutron dynamics

- The Boltzmann equation is a very challenging problem

  Example: 3D calculation of a nuclear reactor
  - Space: $\sim (10^2)^3 = 10^6$ meshes
  - Angle: $\sim 10^2$ directions ($S_8$ in 3D)
  - Energy: $\sim 10^1 - 10^2$ groups
  $\Rightarrow \sim 10^9 - 10^{10}$ unknowns for a steady-state calculation
  - Time: $\Delta t \sim 10^{-6}$ s
  $\Rightarrow \sim 10^6$ pseudo-stationary calculation per second in time-dependent evaluation

  It yields too much physical detail

  In real systems only integral quantities can be observed
Challenges in the simulation of neutron dynamics

- Need to construct simplified models (multigroup, diffusion...) based on physical assumptions
- Need of numerical algorithms (discretizations, expansions)

Development of approximate models and algorithms

Important: establish adequateness of approximations for the problem considered (benchmarks)
Models and methods for neutron dynamics

• Point kinetics
  - Derivation of the model and physical interpretation

• Quasi-static method
  - Improved quasi-statics
  - Predictor-Corrector quasi-statics

• Multipoint kinetics
  - Features of MPK approach
Point kinetics

the neutron distribution is factorized in an amplitude (time-dependent) and a shape (time independent)

\[ n(r, E, \Omega, t) = P(t)\varphi(r, E, \Omega; \lambda) \]

**Critical systems**
Shape: fundamental eigenfunction of the model

\[ \left( \hat{L}_0 + \frac{1}{k} \hat{M}_0 \right) \varphi = 0 \]

**Subcritical systems**
Shape: steady-state solution, dominated by the source

\[ \left( \hat{L}_0 + \hat{M}_0 \right) \varphi + S_0 = 0 \]
Point kinetics

The factorized form is introduced into the balance equations

\[ \begin{align*}
    P \frac{\partial \varphi}{\partial t} + \varphi \frac{dP}{dt} &= P \hat{B} \varphi + \sum_{i=1}^{6} \lambda_i \left( \frac{X_i}{4\pi} C_i \right) + S \\
    \frac{\partial (X_i C_i / 4\pi)}{\partial t} &= P \hat{M}_i \varphi - \lambda_i \left( \frac{X_i}{4\pi} C_i \right)
\end{align*} \]

and is projected on a weighting function \( w \):

\[ \begin{align*}
    \langle w | \varphi \rangle \frac{dP}{dt} &= \langle w | \hat{B} \varphi \rangle P + \sum_{i=1}^{6} \lambda_i \langle w | \left( \frac{X_i}{4\pi} C_i \right) \rangle + \langle w | S \rangle \\
    \langle w | \frac{\partial (X_i C_i / 4\pi)}{\partial t} \rangle &= \langle w | \hat{M}_i \varphi \rangle P - \lambda_i \langle w | \left( \frac{X_i}{4\pi} C_i \right) \rangle
\end{align*} \]
Point kinetics

Weight \( w \rightarrow \) solution of the adjoint steady-state problem

Critical systems

\[
\left( \hat{L}_0^\dagger + \frac{1}{k} \hat{M}_0^\dagger \right) N_0^\dagger = 0
\]

Subcritical systems

\[
\left( \hat{L}_0^\dagger + \hat{M}_0^\dagger \right) N_0^\dagger + S_0^\dagger = 0
\]

The procedure is standard for critical reactors, while for subcritical source-driven systems the question on the adjoint source arises

definition can be given on the basis of

physical consideration and variational principles
Integral quantities are evaluated and the differential equations for the amplitudes are derived:

\[
\begin{align*}
\left\langle N_0^+ \right| \left\{ \frac{dP(t)}{dt} = \frac{\rho(t) - \tilde{\beta}}{\Lambda} P(t) + \sum_{i=1}^{6} \lambda_i \tilde{C}_i(t) + \tilde{S} \right\rangle + \left\langle N_0^+ \right| S \right\rangle \\
\left\langle N_0^+ \right| \left\{ \frac{dC_i(t)}{dt} = \frac{\tilde{\beta}}{\Lambda} P(t) - \lambda_i \tilde{C}_i(t) - \tilde{C}_i \right\rangle
\end{align*}
\]

having introduced the definition of the kinetic parameters

\[
\rho(t) = \frac{\left\langle N_0^+ | \delta \hat{K} \varphi \right\rangle}{\left\langle N_0^+ | \hat{M} \varphi \right\rangle}, \quad \tilde{\beta}_i = \frac{\left\langle N_0^+ | \hat{M}_i \varphi \right\rangle}{\left\langle N_0^+ | \hat{M} \varphi \right\rangle}, \quad \Lambda = \frac{\left\langle N_0^+ | \varphi \right\rangle}{\left\langle N_0^+ | \hat{M} \varphi \right\rangle}
\]
Point kinetics

• Characteristics of the point kinetic approximation:
  - no space distortion during the transient
  - the evolution is space-time separable
  - any point is representative of the whole system

The approximation is poor when localized phenomena (e.g. control rod insertion) are concerned
Point kinetics - results

• Transient following extraction of a control device in a critical system

- Simplified 1D system
- Exact solution vs point kinetic results
Point kinetics - results

- Transient following extraction of a control device in a critical system
  - Simplified 1D system
  - Exact solution vs point kinetic results
Point kinetics - results
Point kinetics - results

- Transient following extraction of a control device in a critical system
  - Simplified 1D system
  - Exact solution vs point kinetic results
Point kinetics

- Results produced with point kinetics underestimate real power evolution
  
  not reliable for safety assessment

- Spatial/spectral effects are neglected

- Need for a more sophisticated method, able to take into account these effects...

  Quasi-static method
Quasi-statics

- The factorization procedure is generalized as:

\[ n(r, E, \Omega, t) = P(t) \varphi(r, E, \Omega; t) \]

No approximation introduced

Amplitude: fast evolving phenomena
Shape: slowing evolving phenomena

inserted into the t-d model and projected on a weight

\[
\begin{align*}
\langle w | \frac{\partial \varphi}{\partial t} \rangle P + \langle w | \varphi \rangle \frac{dP}{dt} &= \langle w | \hat{B} \varphi \rangle P + \sum_{i=1}^{6} \lambda_i \langle w | \left( \frac{\chi_i}{4\pi} C_i \right) \rangle + \langle w | S \rangle \\
\langle w | \frac{\partial (\chi_i C_i / 4\pi)}{\partial t} \rangle &= \langle w | \hat{M}_i \varphi \rangle P - \lambda_i \langle w | \left( \frac{\chi_i}{4\pi} C_i \right) \rangle
\end{align*}
\]

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Models and methods for neutron dynamics
Quasi-statics

- Again, the weight is the solution of the adjoint model:

\[
\frac{d}{dt} \langle N^\dagger_0 | \varphi \rangle P + \langle N^\dagger_0 | \varphi \rangle \frac{dP}{dt} = \langle N^\dagger_0 | \hat{B} \varphi \rangle P + \sum_{i=1}^{6} \lambda_i \langle N^\dagger_0 | \left( \frac{\chi_i}{4\pi} C_i \right) \rangle + \langle N^\dagger_0 | S \rangle \\
\langle N^\dagger_0 | \frac{\partial (\chi_i C_i / 4\pi)}{\partial t} \rangle = \langle N^\dagger_0 | \hat{M}_i \varphi \rangle P - \lambda_i \langle N^\dagger_0 | \left( \frac{\chi_i}{4\pi} C_i \right) \rangle
\]

and a normalization condition is introduced to make the factorization unique

\[
\frac{d}{dt} \langle N^\dagger_0 | \varphi \rangle = 0
\]
Quasi-statics

- The final form of the equation for the amplitude is the well-known point model:

\[
\begin{aligned}
\frac{dP(t)}{dt} &= \frac{\rho(t) - \tilde{\beta}}{\Lambda} P(t) + \sum_{i=1}^{6} \lambda_i \tilde{C}_i(t) + \tilde{S} \\
\frac{dC_i(t)}{dt} &= \frac{\tilde{\beta}}{\Lambda} P(t) - \lambda_i \tilde{C}_i(t)
\end{aligned}
\]

but the kinetic parameters depend on the shape function, which is the other unknown of the problem.
Improved quasi-statics

- The solution is obtained on a two-scale frame:
  - Evaluation of the kinetic parameters with the shape at time $t_0$ (if $t_0=0$, the initial shape is used)
  - Solution of the point model on time interval $[t_0, T]$ with a fine time mesh $\Delta T_A$
  - Solution of the shape model (computationally expensive) on $\Delta T_\varphi = T-t_0$ to update shape function

\[ \rho, \beta, \Lambda \]

\[ t_0 \quad \Delta T_A \quad \Delta T_\varphi \quad T=t_0+\Delta T_\varphi \]
Improved quasi-statics

• Characteristics of the algorithm:
  - The model is non linear

\[
\begin{align*}
  \langle N_0^\dagger & \mid \varphi \rangle \frac{dP}{dt} = \langle N_0^\dagger \mid \hat{B} \varphi \rangle P + \sum_{i=1}^{6} \lambda_i \langle N_0^\dagger \mid \left( \frac{\chi_i}{4\pi} C_i \right) \rangle + \langle N_0^\dagger \mid S \rangle \\
  \langle N_0^\dagger & \mid \frac{\partial (\chi_i C_i/4\pi)}{\partial t} \rangle = \langle N_0^\dagger \mid \hat{M}_i \varphi \rangle P - \lambda_i \langle N_0^\dagger \mid \left( \frac{\chi_i}{4\pi} C_i \right) \rangle
\end{align*}
\]

- The normalization condition needs to be fulfilled

Iterations on the solution of the shape model are performed
Improved quasi-statics

• Iterative procedure for the shape update (1)
  - Solution of the shape model with known $P$ and $\frac{dP}{dt}$:

$$
\frac{\varphi^{(n+1)}(T) - \varphi^{(n)}(T)}{\Delta T_\varphi} P(T) + \varphi^{(n+1)} \frac{dP}{dt} \bigg|_T = \hat{B}_\varphi^{(n+1)} P(T) + \sum_{i=1}^{6} \lambda_i \left(\frac{\chi_i C_i^{(n+1)}}{4\pi}\right) + S^{(n+1)} \\
\left(\chi_i C_i^{(n+1)}/4\pi\right) - \left(\chi_i C_i^{(n)}/4\pi\right) = \hat{M}_i \varphi^{(n+1)} P(T) - \lambda_i \left(\frac{\chi_i C_i^{(n+1)}}{4\pi}\right)
$$

  - Renormalization of the shape

$$
\tilde{\varphi}^{(n+1)} = \frac{\left\langle N_0^\dagger \big| \varphi_0 \right\rangle}{\left\langle N_0^\dagger \big| \varphi^{(n+1)} \right\rangle} \varphi^{(n+1)}
$$

Check on error on shape
Improved quasi-statics

- Iterative procedure for the shape update (2)
  - Computation of kinetic parameters with $\tilde{\varphi}^{(n+1)}$
  - Modification of $P$ (continuity of total power)
    $$\left\langle \hat{M}^{(n+1)} \tilde{\varphi}^{(n+1)} \right\rangle P^{(n+1)} = \left\langle \hat{M}^{(n)} \varphi^{(n)} \right\rangle P^{(n)}$$
  - Modification of $dP/dt$ (fulfillment of point model with updated kinetic parameters)
    $$\begin{align*}
    \frac{dP^{(n+1)}}{dt} &= \frac{\rho^{(n+1)} - \tilde{\beta}^{(n+1)}}{\Lambda} P^{(n+1)} + \sum_{i=1}^{6} \lambda_i \tilde{C}_i + \tilde{S} \\
    \frac{dC_i^{(n+1)}}{dt} &= \frac{\tilde{\beta}^{(n+1)}}{\Lambda} P^{(n+1)} - \lambda_i \tilde{C}_i
    \end{align*}$$
  - Substitution into the shape model and...
Improved quasi-statics - Results

- Improvement of the dynamic simulation of the transient discontinuities of time derivative

\[ \Delta T_\phi \]
Improved quasi-statics

• Characteristics of the method:
  - Spatial and spectral effects can be taken into account
  - Solution converges to reference when $\Delta T_\varphi$ is reduced
  - The method can allow to obtain high quality results with reduced computational time

**BUT**

- The definition of the interval $\Delta T_\varphi$ largely influences the quality of the results (need of adaptive procedure)
- The convergence of the shape is not always ensured
- The iterative procedure of the shape update can be time consuming when large modifications of the shape are involved
- The procedure can become too expensive computationally

Needs for alternative numerical schemes to avoid the non-linearity of the problem

**Predictor-corrector quasi-statics**
Predictor-Corrector quasi-statics

- Scheme for the solution of the quasi-static equation, avoiding the non linearity of the model

\[ \frac{\partial n}{\partial t} = \hat{B}n + \sum_{i=1}^{6} \lambda_i \frac{X_i}{4\pi} C_i + S \]

\[ \partial (\chi_i C_i / 4\pi) = \hat{M}_i n - \lambda_i \frac{X_i}{4\pi} C_i \]

\[ \Delta T_\varphi \]

\[ t_0 \quad \text{T} = t_0 + \Delta T_\varphi \]
Predictor-Corrector quasi-statics

- Scheme for the solution of the quasi-static equation, avoiding the non linearity of the model (2)
  
  - Renormalization if the flux in order to obtain a proper shape function
    \[ \varphi^{(n+1)} = \frac{\langle N_0^+ | \varphi_0 \rangle}{\langle N_0^+ | n^{(n+1)} \rangle} n^{(n+1)} \]
  
  - Evaluation of kinetic parameters and point kinetic solution with time mesh \( \Delta T_A \)

\[ T = t_0 + \Delta T_\phi \]

Models and methods for neutron dynamics

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Predictor-Corrector quasi-statics

- Characteristics of PC quasi-statics
  - No iterations to fulfil normalization are required
  - Kinetic parameters used for point-kinetic calculations are more suitable to describe the transient during $\Delta T\phi$ and can provide more accurate results
  - The computational effort can be effectively reduced with respect to IQM

- When transients with large power effects and small shape modifications are involved, PCQM can fail in reducing computational time (point-like transients)
Convergence of IQM and PCQM to the reference solution

Different performance of kinetic parameters generated with IQM and PCQM
Further improvements

• The factorization procedure can be improved, subdividing the domain in several regions of the phase space.

• This approach can be very effective when loosely coupled systems are concerned.

*Multipoint method*
The multipoint method

- The method can be viewed as an extension of the point kinetic model
- The domain considered in the phase space is subdivided in $K$ (reasonably small) regions (points)
- The neutron density in each region is factorized in a product of amplitude and shape
- $K$ point-like systems for the amplitudes $P_k$ are obtained, all coupled by integral coefficients obtained by the projection technique
The multipoint method

- Example of the multipoint philosophy

![Initial neutron density](image)
The multipoint method

- Example of the multipoint philosophy

Localized perturbation

\[ \phi \]

\[ x/H \]
The multipoint method

• Example of the multipoint philosophy

Different regions are affected differently by the perturbation...
The multipoint method

- Example of the multipoint philosophy

...and are simulated with different amplitude functions
The multipoint method

- Example of the multipoint philosophy

... and are simulated with different amplitude functions
The multipoint method

Balance equations in discretized form and phase-space subdivision:

\[
\frac{1}{v_m} \frac{d\phi_{nm}}{dt} = \sum_{n'} \sum_{m'} k_{nm,n'm'} \phi_{n'm'} + \\
\sum_{i=1}^{6} \lambda_i \chi_{i,m} C_{i,n} + S_{nm} \\
\frac{dC_{i,n}}{dt} = \beta_i \sum_{m'} f_{nm'} \phi_{nm'} - \lambda_i C_{i,n} \quad i = 1, 2, \ldots, 6
\]

\[
\phi_{nm}(t) = \phi(r_n, V_m, t) \quad C_{i,n}(t) = C_i(r_n, t)
\]

\[
\phi_{nm}(t) = A_{NM}(t)\varphi_{nm}(t) \quad r_n, V_m \in \Gamma_{NM}
\]
The multipoint method

Regionwise inner products

\[ \langle w \mid g \rangle = \left[ \sum_n \sum_m \right]_{NM} w_{nm} g_{nm} \]

Introduce factorization (shape equations - known amplitudes):

\[
\begin{align*}
\frac{1}{v_m} \varphi_{nm} \frac{dA_{NM}}{dt} + \frac{1}{v_m} A_{NM} \frac{d\varphi_{nm}}{dt} = \\
\sum_{N'} \sum_{M'} \left[ \sum_{n'} \sum_{m'} \right]_{N'M'} k_{nm,n'm'} \varphi_{n'm'} A_{N'M'} + \\
\sum_{i=1}^{6} \lambda_i x_{i,m} c_{i,n} + s_{nm} \\
\frac{dC_{i,n}}{dt} = \beta_i \sum_{M'} \left[ \sum_{m'} \right]_{M'} f_{nm'} \varphi_{nm'} A_{N'M'} - \lambda_i C_{i,n}
\end{align*}
\]
The multipoint method

Project on weight (amplitude equation – known shape):

\[
\begin{align*}
\frac{dA_{NM}}{dt} &= \sum_{N'} \sum_{M'} K_{NM,N'M'} A_{N'M'} + \\
&\quad \sum_{i=1}^{6} \lambda_i C_{i,NM} + S_{NM} \\
\frac{dC_{i,NM}}{dt} &= \beta_i \sum_{M'} F_{i,NM,M'} A_{NM'} - \lambda_i C_{i,NM} \\
i &= 1, 2, \ldots, 6,
\end{align*}
\]
The multipoint method

Normalization condition (its application may require iteration):

\[
\frac{d}{dt} \left[ \sum_n \sum_m w_{nm} \frac{1}{v_m} \varphi_{nm}(t) \right]_{NM} = \frac{d}{dt} \gamma_{NM} = 0
\]

Kinetic effective parameters and source are introduced.
Multipoint effective terms

\[ K_{NM, N'M'} = \]
\[ \frac{1}{\gamma_{NM}} \left[ \sum_n \sum_m \right]_{NM} \left( w_{nm} \left[ \sum_{n'} \sum_{m'} \right]_{N'M'} k_{nm,n'm'} \phi_{n'm'} \right) \]

coupling terms

effective source

\[ S_{NM} = \frac{1}{\gamma_{NM}} \left[ \sum_n \sum_m \right]_{NM} w_{nm} S_{nm} \]

effective delayed concentration

\[ C_{i, NM} = \frac{1}{\gamma_{NM}} \left[ \sum_n \sum_m \right]_{NM} w_{nm} \chi_{i,m} C_{i,n} \]

delayed fission term

\[ F_{i, NM, M'} = \]
\[ \frac{1}{\gamma_{NM}} \left[ \sum_n \sum_m \right]_{NM} w_{nm} \chi_{i,m} \beta_i \left[ \sum_{m'} \right]_{M'} f_{nm'} \phi_{nm'} \]
Multipoint features

Multipoint can be used in quasi-statics
Graph to show features of multipoint:

Circle (●) : PK
Square (■): exact
Diamond (▲): 2-point
Effect of choice of points

Different subdivision of the phase space has influence on the accuracy of the results

Bold: exact
Circle (•): PK
Square (□): 2-point
Triangle (△): 2-point

• and △ are characterized by different subdivisions of the spatial domain
Effect of choice of points

- The different subdivision of the spatial domain are evidenced

  Bold: exact
  Circle (●): PK
  Square (■): 2-point
  Triangle (△): 2-point

- Update of shape functions through quasi-static procedure

  Continuity of fluxes

Models and methods for neutron dynamics
Time-dependent transport models

• Transport effects in source-driven experiments
  - study of basic phenomena
  - evaluation of limits of diffusion theory referring to
    reference configurations
  - study of propagation phenomena in pulsed experiments:
    • Analysis of sharp space-energy wave-front appearance
    • Identification of limits of low-order transport models
    • Separate analysis of model- and numerically-induced effects
    • Analytical validation tools for numerical schemes in the solution
      of time-dependent transport problems
    • Application to subcritical source-driven systems -> neutron
      pulsed-source
Analytical approach to kinetic models

Propagation phenomena can be properly accounted for by solving transport models

Signal transmission at finite velocity
(while diffusion models propagate signals at infinite velocity)

The analytical approach allows to produce benchmark solutions, extremely useful when dealing with innovative systems (e.g. Accelerator-Driven Systems)
Objective of the work

• Study the propagation of a neutron pulse adopting different transport models
  - analytical approach for the time integration of the problem
  - analytical treatment of space and angle dependences (when possible!)
• Define a “reference” solution (exact), useful to evaluate the approximations introduced by different transport models
• Perform the analysis of the effect of space and angle discretization without any time-discretization effect
Objective of the work

• Separate physical and numerically-induced effects:
  - Physical effects
    • Propagation at finite speed
    • High frequency effects
  - Model effects
    • Wave-like behavior of $P_N$ models (telegrapher’s equation)
      Time-dependent ray-effects
  - Numerically-induced effects
    • Ray-effects in space (multi-D $S_N$)
    • Oscillations due to spatial discretization schemes
A reference solution: the space asymptotic method

- Approach to transport equation in the frequency domain by superposition of spatial waves
  - Series involving Helmholtz eigenfunctions vanishing at the physical boundary of the system (complete set!)

- Unable to correctly describe the exact boundary behavior of the neutron flux, but...
A reference solution: the space asymptotic method

... can describe exactly the propagation of a localized pulse for all times shorter than the time taken by neutrons to reach the boundary

\[ t < \frac{x_1}{v} \]
A reference solution: the space asymptotic method

Consider the transport equation in 1D:

\[
\frac{1}{v} \frac{\partial \varphi(x, \mu, t)}{\partial t} + \mu \frac{\partial \varphi(x, \mu, t)}{\partial x} + \sigma \varphi(x, \mu, t) = \frac{c \sigma}{2} \int_{-1}^{1} d\mu' \varphi(x, \mu', t) + \frac{1}{2} S(x, t)
\]

perform the Laplace transform with respect to the time variable

\[
\frac{s}{v} \varphi(x, \mu, s) + \mu \frac{\partial \varphi(x, \mu, s)}{\partial x} + \sigma \varphi(x, \mu, s) = \frac{c \sigma}{2} \int_{-1}^{1} d\mu' \varphi(x, \mu', s) + \frac{1}{2} S(x, s)
\]

and the Fourier transform for the space variable (infinite medium)

\[
\varphi(B, \mu, s) \left[ \left( \sigma + \frac{s}{v} \right) - iB \mu \right] = \frac{c \sigma}{2} \int_{-1}^{1} d\mu' \varphi(B, \mu', s) + \frac{1}{2} S(B, s)
\]
A reference solution: the space asymptotic method

Starting from this formulation

\[ \varphi(B, \mu, s) \left[ \left( \sigma + \frac{s}{\nu} \right) - iB \mu \right] = \frac{c\sigma}{2} \int_{-1}^{1} d\mu' \varphi(B, \mu', s) + \frac{1}{2} S(B, s) \]

different approaches are possible:

- Expansion of angular variable \( \rightarrow \) spherical harmonics (approximated to order \( L \))
- Integration of the equation over \( \mu \), after recasting (no further approximation introduced)

\[ \int_{-1}^{1} d\mu \varphi(B, \mu, s) \Phi(B, s) = \frac{A_{00}(B, s)}{1 - c\sigma A_{00}(B, s)} S(B, s) \varphi(B, \mu', s) + \int_{-1}^{1} d\mu' S(B, s) + \frac{1}{2} S(B, s) \]

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Time-dependent transport models
A reference solution: 
the space asymptotic method

• Considering a 1D symmetric system, the source term can be expanded in terms of Fourier-transformed Helmholtz eigenfunctions (vanishing on the physical boundary of the system)

\[ S(B, s) = \sum_{n=1}^{\infty} s_n(s) \sqrt{\frac{2}{a}} \left( \delta(B - B_n) + \delta(B + B_n) \right) \]

and perform the inverse transforms:

**Fourier**

\[ \Phi(x, s) = \sum_{n=1}^{\infty} \frac{A_{00}(B_n, s)}{1 - c \sigma A_{00}(B_n, s)} s_n(s) \sqrt{\frac{2}{a}} \cos(B_n x) \]

**Laplace**

\[ \Phi(x, t) = \sum_{n=1}^{\infty} \left\{ \int dt' \left[ \frac{1}{2\pi i} \int_{c_n - i\infty}^{c_n + i\infty} ds \frac{A_{00}(B_n, s)}{1 - c \sigma A_{00}(B_n, s)} e^{s(t-t')} \right] s_n(t') \right\} \sqrt{\frac{2}{a}} \cos(B_n x) \]
A reference solution: 
the space asymptotic method

- Focusing on the Laplace transform, the function:

\[ \Gamma(B_n, s) = \frac{A_{00}(B_n, s)}{1 - c\sigma A_{00}(B_n, s)} \]

where

\[ A_{00}(B, s) = \frac{1}{2} \int_{-1}^{1} d\mu \frac{1}{\left(\sigma + \frac{s}{v}\right) - iB\mu} = \frac{1}{B} \arctan \frac{B}{\sigma + \frac{s}{v}} = \frac{1}{2i} \log \left[ \frac{\sigma + s/v + iB_n}{\sigma + s/v - iB_n} \right] \]

needs to be studied, to determine its singularities:

- Location of poles (discrete spectrum) \( 1 - c\sigma \frac{\arctan \frac{B_n}{\sigma + \frac{s}{v}}}{B_n} = 0 \)
- Presence of continuum spectrum

\[ \text{Im} \left[ \frac{\sigma + s/v + iB_n}{\sigma + s/v - iB_n} \right] = 0 \quad \text{Re} \left[ \frac{\sigma + s/v + iB_n}{\sigma + s/v - iB_n} \right] < 0 \]
A reference solution: the space asymptotic method

Integral in the complex plane corresponding to each $B_n$:

- Real pole $s_n$;
- Cut from $(-v\sigma-vB_n)$ to $(-v\sigma+vB_n)$

**NOTE 1:** for increasing $n$
the singularity moves towards the cut and disappears

**NOTE 2:** critical condition is obtained when the pole corresponding to the first harmonics is equal to zero:

\[
1 - c\sigma A_{00}(B_1, 0) = 1 - c\sigma \frac{1}{B_1} \arctan \frac{B_1}{\sigma} = 0
\]
Space asymptotic method: propagation of a source pulse

- Slab geometry
- Isotropic space-localized pulsed source (initial condition for the flux)
- Isotropic scattering
- The solution obtained represents the “reference” case, useful to compare to different transport approximations and diffusion model
Space asymptotic method: propagation of a source pulse

\[ \sigma = 1 \]
\[ v = 1 \]
\[ c = 0.9 \]

\[ x = v^* \]

\[ t = 1.25 \text{ mft} \]
\[ x = 1.33 \text{ mfp} \]
Solution of $P_N$ and $S_N$ models

• $S_N$ and $P_{N-1}$ are proved to be equivalent in 1D configurations (with special care to the choice of the boundary conditions for the $P_N$ model)

Differences are introduced by the discretization/solution schemes adopted

⇒ The solution of the $P_N$ model can be approached in the asymptotic framework starting from the telegrapher’s equation

⇒ The $S_N$ model is solved by spatial discretization of the unknown and analytical time integration
Solution of $S_N$ model in 1-D configurations

- $S_N$ model

\[
\begin{align*}
\frac{1}{v} \frac{\partial \varphi_{j+}}{\partial t} + \mu_j \frac{\partial \varphi_{j+}}{\partial x} + \sigma \varphi_{j+} &= \frac{c\sigma}{2} \sum_{j=1}^{N/2} w_j (\varphi_{j+} + \varphi_{j-}) + \frac{S}{2} \\
\frac{1}{v} \frac{\partial \varphi_{j-}}{\partial t} - \mu_j \frac{\partial \varphi_{j-}}{\partial x} + \sigma \varphi_{j-} &= \frac{c\sigma}{2} \sum_{j=1}^{N/2} w_j (\varphi_{j+} + \varphi_{j-}) + \frac{S}{2}
\end{align*}
\]

\[j = 1, 2, ..., N/2,\]

With initial and boundary conditions (vacuum):

\[
\varphi_{j+}(-H/2, t) = 0, \quad \varphi_{j-}(H/2, t) = 0, \quad j = 1, 2, ..., N/2
\]
Solution of $P_N$ model in 1-D configurations

- **$P_1$ model**

\[
\begin{align*}
\frac{1}{v} \frac{\partial \phi_0(x, t)}{\partial t} + \sigma_a \phi_0(x, t) + \frac{\partial \phi_1(x, t)}{\partial x} &= S(x, t) \\
\frac{3D}{v} \frac{\partial \phi_1(x, t)}{\partial t} + D \frac{\partial \phi_0(x, t)}{\partial x} + \phi_1(x, t) &= 0
\end{align*}
\]

- **Boundary conditions:**
  - **Marshak (zero incoming current)**
    \[
    \phi_0(\pm \frac{a}{2}, t) = 2\phi_1(\pm \frac{a}{2}, t) = 0
    \]
    \[
    \int_{0}^{1} \left( \frac{1}{2} \phi_0(-\frac{a}{2}, t) + \frac{3}{2} \mu \phi_1(-\frac{a}{2}, t) \right) \mu d\mu = \int_{-1}^{0} \left( \frac{1}{2} \phi_0(\frac{a}{2}, t) + \frac{3}{2} \mu \phi_1(\frac{a}{2}, t) \right) \mu d\mu = 0
    \]
  - **Mark (zero incoming flux)**
    \[
    \phi_0(\pm \frac{a}{2}, t) = \sqrt{3} \phi_1(\pm \frac{a}{2}, t) = 0
    \]
    \[
    \frac{1}{2} \phi_0(\frac{a}{2}, t) + \frac{\sqrt{3}}{2} \phi_1(-\frac{a}{2}, t) = \frac{1}{2} \phi_0(\frac{a}{2}, t) - \frac{\sqrt{3}}{2} \phi_1(\frac{a}{2}, t) = 0
    \]

Consistent with BC in $S_2$ model

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Time-dependent transport models
Solution of $S_N$ model in 1-D configurations

- Spatial discretization of the model:

1. Diamond difference

\[
\begin{align*}
\frac{1}{v} \frac{d\varphi_{j+}^{(i)}}{dt} + \frac{2\mu_j}{\Delta_i} \left( \varphi_{j+}^{(i)} - \varphi_{j+}^{(i-1/2)} \right) + \sigma\varphi_{j+}^{(i)} &= \frac{c\sigma}{2} \sum_{j=1}^{N/2} w_j \left( \varphi_{j+}^{(i)} + \varphi_{j-}^{(i)} \right) + \frac{S^{(i)}}{2} \\
\frac{1}{v} \frac{d\varphi_{j-}^{(i)}}{dt} - \frac{2\mu_j}{\Delta_i} \left( \varphi_{j-}^{(i+1/2)} - \varphi_{j-}^{(i)} \right) + \sigma\varphi_{j-}^{(i)} &= \frac{c\sigma}{2} \sum_{j=1}^{N/2} w_j \left( \varphi_{j+}^{(i)} + \varphi_{j-}^{(i)} \right) + \frac{S^{(i)}}{2}
\end{align*}
\]

\[j = 1, 2, ..., N/2, \quad i = 1, 2, ..., I,\]

with transmission relation:

\[
\varphi_{j\pm}^{(i)} = \frac{1}{2} \left( \varphi_{j\pm}^{(i-1/2)} + \varphi_{j\pm}^{(i+1/2)} \right)
\]
Solution of $S_N$ model in 1-D configurations

- Spatial discretization of the model:

2. Linear discontinuous

\[
\frac{1}{v} \frac{d}{dt} \left( \varphi_{j+}^{i+1/2,R} + \varphi_{j+}^{i-1/2,L} \right) + \mu_j \frac{\varphi_{j+}^{i+1/2,L} - \varphi_{j+}^{i-1/2,L}}{\Delta_i} + \sigma_i \frac{\varphi_{j+}^{i+1/2,L} + \varphi_{j+}^{i-1/2,R}}{2} = \frac{c \sigma}{2} \sum_{j=1}^{N/2} \omega_j \left( \left( \varphi_{j+}^{i+1/2,L} + \varphi_{j+}^{i-1/2,R} \right) + \left( \varphi_{j-}^{i+1/2,L} + \varphi_{j-}^{i-1/2,R} \right) \right) + \frac{S_{i+1/2,L} + S_{i-1/2,R}}{4},
\]

NOTE: time derivative is not discretized

→ system of first-order differential equations

→ solution of the problem in matrix form
Section of $S_N$ model in 1-D configurations

- Spatial discretization of the model:

\[
\frac{1}{v} \frac{d}{dt} \left( \frac{\varphi_{j-1/2,L}^{i+1/2} + \varphi_{j-1/2,R}^{i-1/2}}{2} - \mu_j \frac{\varphi_{j-1/2,R}^{i+1/2} - \varphi_{j-1/2,L}^{i-1/2}}{\Delta_i} + \sigma_j \frac{\varphi_{j-1/2,L}^{i+1/2} + \varphi_{j-1/2,R}^{i-1/2}}{2} \right) = \frac{c\sigma}{2} \sum_{j=1}^{N/2} \omega_j \left( \frac{\varphi_{j-1/2,L}^{i+1/2} + \varphi_{j-1/2,R}^{i-1/2}}{2} + \frac{\varphi_{j-1/2,R}^{i+1/2} + \varphi_{j-1/2,L}^{i-1/2}}{2} \right) + \frac{S_{i+1/2,L}^{i+1/2} + S_{i-1/2,R}^{i-1/2}}{4} \\
\frac{1}{v} \frac{d}{dt} \left( \frac{\varphi_{j-1/2,L}^{i+1/2} + 2\varphi_{j-1/2,R}^{i-1/2}}{3} - \mu_j \frac{\varphi_{j-1/2,R}^{i+1/2} - \varphi_{j-1/2,L}^{i-1/2}}{\Delta_i} + \sigma_j \frac{\varphi_{j-1/2,L}^{i+1/2} + 2\varphi_{j-1/2,R}^{i-1/2}}{3} \right) = \frac{c\sigma}{2} \sum_{j=1}^{N/2} \omega_j \left( \frac{\varphi_{j-1/2,L}^{i+1/2} + 2\varphi_{j-1/2,R}^{i-1/2}}{3} + \frac{\varphi_{j-1/2,R}^{i+1/2} + 2\varphi_{j-1/2,L}^{i-1/2}}{3} \right) + \frac{S_{i+1/2,L}^{i+1/2} + 2S_{i-1/2,R}^{i-1/2}}{6}
\]

NOTE: time derivative is not discretized

→ system of first-order differential equations
→ solution of the problem in matrix form
Solution of $S_N$ model in 1-D configurations

Matrix form of the problem:

$$\frac{dX}{dt} + \hat{A}X = S$$

$X$ is the unknown flux vector

- DD: $X=\{\phi^{(i)}_{j \pm}\}$, dimension $I \times N$
- LD: $X=\{\phi^{(i \pm 1/2)}_{j \pm}\}$, dimension $2I \times N$

- In DD the interface fluxes are eliminated through the transmission relation
The solution for an initial condition is obtained analytically in terms of the eigenvectors of matrix $\hat{A}$:

$$X(t) = \sum_{k=1}^{I \times N} c_k(t) u_k = \sum_{k=1}^{I \times N} (c_k(0) \exp(-\omega_k t)) u_k$$

$$= \sum_{k=1}^{I \times N} (u_k^+ \cdot X(0) \exp(-\omega_k t)) u_k.$$
Solution of $P_1$ model in 1-D configurations

- $P_1$ model

\[
\begin{align*}
\frac{1}{v} \frac{\partial \phi_0(x, t)}{\partial t} + \sigma_a \phi_0(x, t) + \frac{\partial \phi_1(x, t)}{\partial x} &= S(x, t) \\
\frac{3D}{v} \frac{\partial \phi_1(x, t)}{\partial t} + D \frac{\partial \phi_0(x, t)}{\partial x} + \phi_1(x, t) &= 0
\end{align*}
\]

with Mark boundary conditions:

\[
\phi_0(\pm \frac{a}{2}, t) = \sqrt{3} \phi_1(\pm \frac{a}{2}, t) = 0.
\]

This problem can be given a different formulation: telegrapher’s equation
The telegrapher’s equation

Consider the second equation of the P₁ model:

\[
\frac{3D}{v} \frac{\partial \phi_1(x, t)}{\partial t} + D \frac{\partial \phi_0(x, t)}{\partial x} + \phi_1(x, t) = 0
\]

- Steady state \(\rightarrow\) Fick’s law \(\rightarrow\) diffusion model
- Time-dependent situation \(\rightarrow\) integrate with respect to time:

\[
\phi_1(x, t) = \phi_1(x, 0) e^{-\frac{v}{3D} t} - D \int_0^t \frac{\partial \phi_0(x, t')}{\partial x} \frac{v}{3D} e^{-\frac{v}{3D} (t-t')} dt'
\]

- If \(v \rightarrow \infty\) we obtain the original Fick’s law (infinite speed signal transmission)
- The current at time \(t\) depends on the gradient of the flux at preceding times, through the kernel \(v/3D \exp(-(v/3D)t)\)
The telegrapher’s equation

Substituting the integral in the first equation of the $P_1$ model we obtain the integro-differential form of the telegrapher’s equation:

$$
\frac{1}{v} \frac{\partial \phi_0(x, t)}{\partial t} + \sigma_\alpha \phi_0(x, t) - \frac{v}{3D} \int_0^t D \frac{\partial^2 \phi_0(x, t')}{\partial x^2} e^{-\frac{v}{3D} (t-t')} \, dt' = S(x, t)
$$

Alternatively, the telegrapher’s equation can be derived in second-order formulation:

$$
\frac{1}{v} \frac{\partial}{\partial t} \left( 1 + \frac{3D}{v} \frac{\partial}{\partial t} \right) \Phi(x, t) = D \frac{\partial^2 \Phi(x, t)}{\partial x^2} + \left( 1 + \frac{3D}{v} \frac{\partial}{\partial t} \right) [S(x, t) - \sigma_\alpha \Phi(x, t)]
$$
The telegrapher's equation

- Second-order time derivative \( \rightarrow \) wave propagation at speed \( \nu/\sqrt{3} \) (projection of the velocity on the x-axis)

  - The solution of the problem in asymptotic theory is equivalent to directly project the Laplace-transformed flux in the \( P_1 \) model on the Helmholtz eigenfunctions;
  - The solution for the projection coefficients for the pulse propagation comes out as:

\[
A_n(s) = \frac{3D}{\nu^2} p + \frac{1}{\nu} \frac{3D}{\nu} p + \left( \sigma + DB_n^2 \right) A_n(0)
\]

\( \Phi(x, s) = \sum_{n=0}^{\infty} A_n(s) \varphi_n(x) \)

this expression can be derived alternatively....

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The telegrapher’s equation

... from the exact kernel of the transport equation and introducing $P_1$
hypothesis!!!!

\[ \Gamma(B_n, s) = \frac{A_{00}(B_n, s)}{1 - c\sigma A_{00}(B_n, s)} \]

\[ A_{00}(B_n, s) = \frac{1}{2} \int_{-1}^{1} d\mu \frac{1}{(\sigma + \frac{s}{v}) - iB_n\mu} \approx \sum_{m=1}^{M/2} \frac{(\sigma + \frac{s}{v})w_m}{(\sigma + \frac{s}{v})^2 + (B_n\mu_m)^2} \]

Then, for $M=2$, introducing the definitions of $D$ and $\sigma_a$ the kernel becomes:

\[ \Gamma(B_n, s) = \frac{\left(\frac{3D}{v}s + 1\right)}{\frac{3D}{v^2}s^2 + (1 + 3D\sigma_a)\frac{s}{v} + (\sigma_a + DB_n^2)} \]

Number of directions of $P_{M-1-S_M}$

Number of directions of $P_{M-1-S_M}$

No continuum spectrum

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Time-dependent transport models
The diffusion model

- Starting from the P1 model, the diffusion case can be derived, by adding some further hypotheses:
  - Infinite-velocity $v \to \infty$
  - Collision-dominated medium $c \to 1$

\[
\Gamma(B_n, s) = \frac{1}{s \frac{1}{v} + (\sigma + DB_n^2)}
\]

1 discrete pole in $\mathbb{R}$ for each $n$

We now compare the "reference" result with $P_1, S_2$ (spatially discretized) and diffusion results
Propagation of a source pulse: asymptotic method vs. diffusion

Diffusion is totally inadequate to describe this phenomenon
Superposition of real exponential $\Leftrightarrow$ Propagation at infinite speed

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Propagation of a source pulse: asymptotic method vs. diffusion

Diffusion is totally inadequate to describe this phenomenon

Superposition of real exponential ⇔ Propagation at infinite speed
Propagation of a source pulse: asymptotic method vs. diffusion

Diffusion is totally inadequate to describe this phenomenon
Superposition of real exponential $\Leftrightarrow$ Propagation at infinite speed

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Propagation of a source pulse: asymptotic method vs. $P_1$

$P_1$ presents a wave-like propagation of the uncollided part + forward collided part of the pulse

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Propagation of a source pulse: asymptotic method vs. $P_1$

$P_1$ presents a wave-like propagation of the uncollided part + forward collided part of the pulse
Propagation of a source pulse: asymptotic method vs. $P_1$

$P_1$ presents a wave-like propagation of the uncollided part + forward collided part of the pulse.
Propagation of a source pulse: $P_1$ vs. $S_2$

Since $P_1$ and $S_2$ in 1D are equivalent, comparisons are made to evidence the effect of spatial discretization of $S_2$
Propagation of a source pulse: $P_1$ vs. $S_2$

Since $P_1$ and $S_2$ in 1D are equivalent, comparisons are made to evidence the effect of spatial discretization of $S_2$.
Propagation of a source pulse: $P_1$ vs. $S_2$

Since $P_1$ and $S_2$ in 1D are equivalent, comparisons are made to evidence the effect of spatial discretization of $S_2$.
Propagation of a source pulse: \( P_1 \) vs. \( S_2 \)

Since \( P_1 \) and \( S_2 \) in 1D are equivalent, comparisons are made to evidence the effect of spatial discretization of \( S_2 \)

\( x = \mu^* v^* \uparrow \)
Propagation of a source pulse: asymptotic method vs. $P_1$ (influence of $c$)

$c=0.9$

$c=0.1$

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Propagation of a source pulse: asymptotic method vs. $P_1$ (influence of $c$)

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Propagation of a source pulse: asymptotic method vs. $P_1$ (influence of $c$)

c=0.9

c=0.1
Propagation of a source pulse: spectrum of the asymptotic method (influence of $c$)

$\text{Re}(p)$

$\text{Im}(p)$

$c=0.9$

$c=0.1$

No discrete spectrum

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Time-dependent transport models
Solution of the $P_3$-$S_4$ problem

- Higher order $P_N$-$S_N$ methods are analyzed with a similar procedure:
  - $P_3$: space asymptotic method
  - $S_4$: spatial discretization with DD and LD and analytical time integration

- The propagation of pulses at different velocities (corresponding to the directions $\mu_1$ and $\mu_2$) is observed
Solution of the $S_4$ problem - DD

Influence of space discretization $\rightarrow$ numerically-induced oscillation

$x = \mu_1 \ast v^* \uparrow$

$x = \mu_2 \ast v^* \uparrow$

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Solution of the $S_4$ problem - LD

Influence of space discretization $\rightarrow$ smoothening of oscillation with linear discontinuous

$\star \quad x = \mu_1 \times v \times \dagger$

$\star \quad x = \mu_2 \times v \times \dagger$

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Solution of $P_1$-$S_2$ models in 2-D configurations

• $P_1$ and $S_2$ model are no longer equivalent:
  - $P_1$: time ray effects (model effects)
  - $S_2$: space and time ray effects (effects due to the model itself and the angular discretization)

• Solution methods:
  - $P_1$: Laplace transform
  - $S_2$: space discretization and analytical time integration
Solution of $P_1-S_2$ models in 2-D configurations
Solution of $P_1$-$S_2$ models in 2-D configurations

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Solution of $P_1$-$S_2$ models in 2-D configurations

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Solution of $P_1$-$S_2$ models in 2-D configurations
An application: dynamics of fluid fuel systems

- The multiplying medium is in a fluid phase, flowing through the reactor core.

  - The fissile salt acts both as nuclear fuel and system coolant.

- The presence of a velocity field in the multiplying medium affects its neutronic behavior → need to develop suitable models and tools for reactor physics analysis.
An application: dynamics of fluid fuel systems
Neutronic model for MSR (1)

- Balance equations for neutrons in presence of delayed emissions:

\[
\begin{aligned}
\frac{\partial n(r, E, \Omega, t)}{\partial t} &= \left[ \hat{L}(t) + \hat{M}_p(t) \right] n(r, E, \Omega, t) + \sum_{i=1}^{R} \mathcal{E}_i(r, E, t) + S(r, E, \Omega, t) \\
\frac{1}{\lambda_i} \frac{\partial \mathcal{E}_i(r, E, t)}{\partial t} + \frac{1}{\lambda_i} \nabla \cdot (u(r, t) \mathcal{E}_i(r, E, t)) &= \hat{M}_i(t) n(r, E, \Omega, t) - \mathcal{E}_i(r, E, t) \\
&\quad i = 1, 2, \ldots, R
\end{aligned}
\]

where the delayed emissions are defined as

\[
\mathcal{E}_i(r, E, t) = \lambda_i C_i(r, t) \frac{\chi_i(E)}{4\pi}
\]
Neutronic model for MSR (2)

• Equation for neutron density is unchanged

\[ \frac{\partial n(r, E, \Omega, t)}{\partial t} = \left[ \hat{L}(t) + \hat{M}_p(t) \right] n(r, E, \Omega, t) + \sum_{i=1}^{R} \mathcal{E}_i(r, E, t) + S(r, E, \Omega, t) \]

the time scale of prompt neutron production is much faster than fluid-dynamic phenomena

• A streaming term appears in the precursor equations

\[ \frac{1}{\lambda_i} \frac{\partial \mathcal{E}_i(r, E, t)}{\partial t} + \frac{1}{\lambda_i} \nabla \cdot (u(r, t) \mathcal{E}_i(r, E, t)) = \hat{M}_i(t) n(r, E, \Omega, t) - \mathcal{E}_i(r, E, t) \]

the mathematical nature of the equation is changed
Neutronic model for MSR (3)

Appropriate boundary conditions must be introduced for the precursors:

\[ \mathcal{E}_i(r, E, t)u(r, t) (-n(r)) = \int_{A_{in}} \mathcal{E}_i(r', E, t - \tau(r' \to r)) e^{-\lambda_i \tau(r' \to r)} \times \]

\[ u(r', t - \tau(r' \to r)) \cdot n(r') \mathcal{F}(r' \to r) dA' \]

The flow function is a \textit{probability density function}, normalized as:

\[ \int_{A_{in}} \mathcal{F}(r' \to r) dA = 1, \quad \forall r' \in A_{out}. \]
Effects of fuel motion

- STEADY STATE
  - Dependence of the multiplication eigenvalue on the velocity field and delayed neutron characteristics
  - Spatial redistribution of the delayed neutron precursors importance and reduction of their importance
  - Reactivity loss

- TIME-DEPENDENT
  - Reduced effective delayed neutron fraction
  - Prompter dynamic response
  - Peculiar effects connected to the undecayed fraction of precursors re-entering in the system
  - New definition of $\beta_{\text{eff}}$
Extension of quasi-statics to fluid fuel systems

In order to properly study the physical phenomena typical of fluid fuel systems, the standard tools for dynamic analysis need to be consistently extended and adapted.

The point kinetic model is reformulated consistently for fluid-fuel systems, applying Henry procedure of factorization and projection of the neutron density and delayed emissions.

The quasi-static scheme is applied, taking particular care of the normalization conditions to be fulfilled.
Point Kinetics

Point models are based on the factorization of the unknowns and the projection of the factorized problem on a weighting function. A reference configuration is considered:

\[
\begin{cases}
\left[ \hat{L}_0 + \hat{M}_{P,0} \right] N_0(\mathbf{r}, E, \Omega) + \sum_{i=1}^{R} \mathcal{E}_{i,0}(\mathbf{r}, E) + S_0(\mathbf{r}, E, \Omega) = 0 \\
\frac{1}{\lambda_i} \nabla \cdot (u_0 \mathcal{E}_{i,0}(\mathbf{r}, E)) = \hat{M}_{i,0} N_0(\mathbf{r}, E, \Omega) - \mathcal{E}_{i,0}(\mathbf{r}, E), \quad i = 1, 2, \ldots, R
\end{cases}
\]

Subcritical systems

Still differential!
A proper inner product definition is introduced:

$$\mathbf{w} = (n, \varepsilon_1, \ldots, \varepsilon_R)^t$$

$$\mathbf{w}^\dagger = (n^\dagger, \varepsilon^\dagger_1, \ldots, \varepsilon^\dagger_R)$$

$$\langle \mathbf{w}^\dagger, \mathbf{w} \rangle = \sum_{n=1}^{R+1} \langle w_n^\dagger | w_n \rangle = \sum_{n=1}^{R+1} \int dV \int dE \int d\Omega w_n^\dagger w_n$$

And the \textit{solution of the consistently derived steady-state adjoint problem} is used as weighting function.
Point Kinetics

Direct matrix operator

\[
\begin{bmatrix}
\hat{L}_0 + \hat{M}_{p,0} & \frac{1}{4\pi} \int d\Omega & \cdots & \frac{1}{4\pi} \int d\Omega \\
\hat{M}_{1,0} & -1 - \frac{1}{\lambda_1} \nabla \cdot (u_0) & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\hat{M}_{R,0} & & \cdots & -1 - \frac{1}{\lambda_R} \nabla \cdot (u_0)
\end{bmatrix}
\]

Adjoint matrix operator (transpose with adjoints)

\[
\begin{bmatrix}
\hat{L}^\dagger_0 + \hat{M}^\dagger_{p,0} & \hat{M}^\dagger_{1,0} & \cdots & \hat{M}^\dagger_{R,0} \\
\frac{1}{4\pi} \int d\Omega & -1 + \frac{1}{\lambda_1} \nabla \cdot (u_0) & \cdots & \cdots \\
\frac{1}{4\pi} \int d\Omega & \vdots & \ddots & \vdots \\
\frac{1}{4\pi} \int d\Omega & \vdots & \cdots & -1 + \frac{1}{\lambda_R} \nabla \cdot (u_0)
\end{bmatrix}
\]
Point Kinetics

The adjoint equations take the form:

\[
\begin{align*}
\left[ \hat{L}^\dagger_0 + \hat{M}^\dagger_{p,0} \right] N^\dagger_0(r, E, \Omega) + \sum_{i=1}^{R} \hat{M}^\dagger_{i,0} \mathcal{E}^\dagger_{i,0}(r, E) + S^\dagger_0(r, E, \Omega) &= 0 \\
\frac{1}{4\pi} \oint d\Omega N^\dagger_0(r, E, \Omega) + \frac{1}{\chi^\circ} u_0 \cdot \nabla \left( \mathcal{E}^\dagger_{i,0}(r, E) \right) - \mathcal{E}^\dagger_{i,0}(r, E) &= 0, \quad i = 1, 2, \ldots, R
\end{align*}
\]

with associate boundary conditions for adjoint delayed emissions:

\[
\mathcal{E}^\dagger_i(r, E) = \int_{A_{\text{in}}} \mathcal{E}^\dagger_i(r', E) e^{-\lambda_i \tau(r\rightarrow r')} \tilde{f}(r \rightarrow r') \, dA', \quad r \in A_{\text{out}}
\]

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Dynamics of fluid fuel systems
Point Kinetics

Physical interpretation of the adjoint functions $E_{i}^{\dagger}(r,E)$ in terms of importance:

Importance of a delayed neutron emitted isotropically by a precursor of the $i$-th family

$u(z)e_{z}$

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Dynamics of fluid fuel systems
Point Kinetics

Neutron density and delayed emissions are factorized as follows:

\[ n(r, E, \Omega, t) = A(t)\phi(r, E, \Omega; t) \]

\[ E_i(r, E, t) = G_i(t)e_i(r, E; t) \quad i = 1, 2, ..., R \]

These expressions are then introduced into the balance equations:

\[
\begin{align*}
A(t) \frac{\partial \phi}{\partial t} + \phi \dot{A} &= \left[ \hat{L} + \hat{M}_p \right] \phi A + \sum_{i=1}^{R} G_i e_i + S \\
\frac{\partial e_i}{\partial t} G_i + e_i \dot{G}_i + \nabla \cdot (ue_i) G_i &= \lambda_i \hat{M}_i \phi A - \lambda_i e_i G_i \\
i &= 1, 2, ..., R,
\end{align*}
\]
Point Kinetics

The system is projected on the adjoint functions

\[
\frac{d}{dt} \left< N_0^\dagger | \phi \right> A + \left< N_0^\dagger | \phi \right> \dot{A} = \left< N_0^\dagger \left[ \hat{L} + \hat{M}_p \right] | \phi \right> A + \sum_{i=1}^{R} \left< N_0^\dagger | e_i \right> G_i + \left< N_0^\dagger | S \right>
\]

\[
\frac{d}{dt} \left< \varepsilon_{i,0}^\dagger | e_i \right> G_i + \left< \varepsilon_{i,0}^\dagger | e_i \right> \dot{G}_i + \left< \varepsilon_{i,0}^\dagger | \nabla \cdot (u e_i) \right> G_i = \left< \varepsilon_{i,0}^\dagger | \lambda_i \hat{M}_i \phi \right> A - \left< \varepsilon_{i,0}^\dagger | \lambda_i e_i \right> G_i
\]

\[
i = 1, 2, ..., R
\]

and normalization conditions are imposed, to make the factorization unique:

\[
\frac{d}{dt} \left< N_0^\dagger | \phi \right> = \frac{d \gamma_0}{dt} = 0,
\]

\[
\frac{d}{dt} \left< \varepsilon_{i,0}^\dagger | e_i \right> = \frac{d \eta_{i,0}}{dt} = 0, \quad i = 1, 2, ..., R
\]
Point Kinetics

A point model is obtained:

\[
\begin{align*}
\frac{dA}{dt} &= \alpha A + \sum_{i=1}^{R} \mu_i G_i + \tilde{S}, \\
\frac{dG_i}{dt} &= \phi_i A - \lambda_i G_i, \quad i = 1, \ldots, R
\end{align*}
\]

with a different definition of the kinetic parameters with respect to a solid fuel reactor.

**Note:** the kinetic parameters are functions of the neutron and precursor shapes!!!
Reformulated kinetic parameters

New definition of the effective delayed neutron fraction:

\[ \tilde{\beta}_i = \frac{\langle \mathcal{E}_{i,0}^\dagger | \hat{M}_i \phi \rangle}{\mathcal{F}} \]

- weighted on the adjoint delayed emissions
- takes into account the modification introduced by the fuel motion
- in absence of motion reduces to the standard definition

<table>
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<tr>
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<th>1</th>
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<td>117</td>
<td>263</td>
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<td>45</td>
<td>680</td>
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<td>(\tilde{\beta}_i \ [pcm])</td>
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<td>4</td>
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</table>
Quasi Statics

• To perform quasi-static calculations, the point model solution is calculated over a time interval $\Delta T_{\phi}$, using a fine time discretization $\Delta T_A$;

\[ T = t_0 + \Delta T_{\phi} \]
Quasi Statics

- To perform quasi-static calculations, the point model solution is calculated over a time interval $\Delta T_\varphi$, using a fine time discretization $\Delta T_A$;

- At time $T$ the shape functions are updated, solving the full space-time problem on the time interval $t_0-T$:

\[
\begin{align*}
A(t) \frac{\partial \phi}{\partial t} + \phi \dot{A} &= \left[ \hat{L} + \hat{M}_p \right] \phi A + \sum_{i=1}^{R} G_i e_i + S \\
\frac{\partial e_i}{\partial t} G_i + e_i \dot{G}_i + \nabla \cdot (ue_i) G_i &= \lambda_i \hat{M}_i \phi A - \lambda_i e_i G_i \\
i &= 1, 2, \ldots, R,
\end{align*}
\]

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Quasi Statics

- To perform quasi-static calculations, the point model solution is calculated over a time interval $\Delta T_\phi$, using a fine time discretization $\Delta T_A$;

- At time $T$ the shape functions are updated, solving the full space-time problem on the time interval $t_0-T$:

  Discretized with an implicit Euler scheme

\[
\begin{align*}
\frac{\phi_j^{n+1} - \phi_j^n}{\Delta T_\phi} + \phi_j^{n+1} \frac{\dot{A}}{A} & = \left[ \hat{L} + \hat{M}_p \right] \phi_j^{n+1} + \sum_{i=1}^{R} e_{i,j}^{n+1} \frac{G_i}{A} \bigg|_{T,j} + \frac{S}{A} \bigg|_{T,j}, \\
\frac{e_{i,j}^{n+1} - e_i^n}{\Delta T_\phi} + e_{i,j}^{n+1} \frac{\dot{G}_i}{G_i} & = \nabla \cdot \left[ (ue_{i,j})^{n+1} \right] = \lambda_i \hat{M}_i \phi_j^{n+1} \frac{A}{G_i} \bigg|_{T,j} - \lambda_i e_{i,j}^{n+1} \bigg|_{T,j},
\end{align*}
\]

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Normalization of the solution

- The solutions obtained for the shape functions must be rescaled to fulfill normalization conditions:

\[
\phi_{j+1/2}^{n+1} = \frac{\gamma_0}{\langle N_0^\dagger \phi_j^{n+1} \rangle} \phi_j^{n+1}
\]

\[
e_{i,j+1/2}^{n+1} = \frac{\eta_{i,0}}{\langle \mathcal{E}_{i,0}^\dagger e_{i,j}^{n+1} \rangle} e_{i,j}^{n+1}
\]

\[i = 1, ..., R\]

and are used for the recalculation of the kinetic parameters.

This non linear procedure implies the need to perform iterations on the solution obtained
Iterations on quasi static solution

- Neutron amplitude $A$ (imposed continuity of system power during the transient):

$$\left( \sum_f \phi_{j+1/2}^{n+1} \right) A|_{T,j+1} = \left( \sum_f \phi_j^{n+1} \right) A|_{T,j}$$

- Delayed amplitude $G_i$ (imposed fulfillment of boundary conditions):

$$G_i|_{T,j+1} e_i^{n+1/2} u \cdot (-n) = \int_{A_{out}} G_i(T - \tau(r' \rightarrow r)) e_i(r', E; T - \tau(r' \rightarrow r)) e^{-\lambda_i \tau(r' \rightarrow r)} \times$$

$$u(r', T - \tau(r' \rightarrow r)) \cdot n(r') \tilde{g}(r' \rightarrow r) \, dA'.$$

- Time derivatives of $A$ and $G_i$ (update of kinetic parameters):

$$\begin{cases}
\dot{A}|_{T,j+1} = \alpha^j A|_T + \sum_{i=1}^R \mu_i^j G_i|_{T,j+1} + \tilde{S}_j, \\
\dot{G}_i|_{T,j+1} = \phi_i^j A|_T - \Lambda_i^j G_i|_{T,j+1}, & i = 1, \ldots, R
\end{cases}$$

Dynamics of fluid fuel systems
Numerical results

Non compensated fuel velocity transient in a critical system

$\Delta \rho = -131 \text{pcm}$

Dynamics of fluid fuel systems
Numerical results

Non compensated fuel velocity transient in a critical system

Shape functions used for kinetic parameter calculation at time $t=5$ s

Modification of shape functions

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Dynamics of fluid fuel systems
Numerical results

Non compensated fuel velocity transient in a critical system

Computational effort

<table>
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<tr>
<th>recalculation</th>
<th>1</th>
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<th>50</th>
<th>100</th>
<th>1000</th>
<th>reference</th>
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<td>Rel. time</td>
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<td>0.043</td>
<td>0.158</td>
<td>0.337</td>
<td>0.481</td>
<td>0.515</td>
<td>1.000</td>
</tr>
</tbody>
</table>
• Experiments are under way to study the physics of source-driven systems
• Reactivity reconstruction from local flux measurements is an important aspect
• Pulsed experiments are considered
• Flux interpretation needs to account for spectral and spatial effects
Simulation of source experiments

- Inverse methods are easily implemented from point kinetics models.
- It has been observed that satisfactory results are obtained if the global system power is used in inverse methods associated to *global* kinetic parameters. However,
- Only signals from local flux detectors are available.
Simulation of source experiments

- Signals from local detectors have to be (importance) averaged
- Strong importance effects can be observed in source-driven problems
- More suitable and problem-oriented weighting procedures must be used in inverse frameworks to construct ad-hoc kinetic parameters
Scope of the analysis

• *Study importance weighting procedures to generate point models that are adequate to accurately simulate local flux signal evolutions in a pulsed experiment*

• *Re-define time-dependence of source to better represent the physical response*
Methodology

Integral parameters are derived by taking the factorized balance equations

\[ n(x,t) = A(t)\varphi(x;t) \]

\[ \begin{align*}
\frac{\partial (A \varphi)}{\partial t} &= \hat{L} \varphi A + \hat{F}_p \varphi A + \lambda C + S \\
\frac{\partial C}{\partial t} &= -\lambda C + \hat{F}_d \varphi A 
\end{align*} \]

project them on a weight

\[ \begin{align*}
\langle \psi | \varphi \rangle \frac{dA}{dt} &= \langle \psi | \hat{L} \varphi \rangle A + \langle \psi | \hat{F}_p \varphi \rangle A + \lambda \langle \psi | C \rangle + \langle \psi | S \rangle \\
\frac{d\langle \psi | C \rangle}{dt} &= -\lambda \langle \psi | C \rangle + \langle \psi | \hat{F}_d \varphi \rangle A 
\end{align*} \]
Methodology

obtaining a system of equations in time for amplitude functions with a point-like structure

\[
\begin{align*}
\frac{dA}{dt} &= \frac{\langle \psi | \hat{L} \varphi \rangle A + \langle \psi | \hat{F}_p \varphi \rangle}{\langle \psi | \varphi \rangle} A + \lambda \tilde{C} + \frac{\langle \psi | S \rangle}{\langle \psi | \varphi \rangle} \\
\frac{d\tilde{C}}{dt} &= -\lambda \tilde{C} + \frac{\langle \psi | \hat{F}_d \varphi \rangle}{\langle \psi | \varphi \rangle} A
\end{align*}
\]

Questions:

• Choice of the shape function \( \varphi \) ? Initial reference configuration

• Choice of the weighting function ? \( \rightarrow \) definition of various point kinetic models with different objectives

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Simulation of source experiments
Global and Local Point Kinetics \((gpk)\) and \((lpk)\)

weighting function \(\rightarrow\) adjoint problem

Source-driven adjoint \(\leftrightarrow\) critical adjoint (eigenvalue)

- the adjoint source can be assumed as the local detector where the flux measurement is taken \(\rightarrow lpk\)
- global weighting, i.e. assuming as adjoint source the fission productivity \(v\Sigma_f\), suitable to predict power evolution \(\rightarrow gpk\)
Pulsed-source experiments

- Localized source in space and time
- Analysis of the evolution of the flux after the source pulse
- Comparison of different options for the construction of the point model adopted in the interpretation of the results
Pulsed-source experiments

• Analytical and semi-analytical study of a source pulse in a 1D system:
  - Comparison of exact results with point kinetic models;
  - Influence of the distance of the detector from the source:
    • Time-delay of the response of the system
    • Modification of the time behavior of the source
Pulsed-source experiments

• Point kinetic models:
  - Adjoint functions:
    • Local: \( S^\dagger = \delta(x - x_0) \rightarrow \text{lpk} \)
    • Global: \( S^\dagger = \nu \Sigma_f \rightarrow \text{gpk} \)
    • Critical problem \( \rightarrow \text{cpk} \)

• Shape functions (initial ref. configuration is \( \varphi=0 \))
  • Equilibrium configuration with external source \( \rightarrow \varphi_S \)
  • Critical configuration \( \rightarrow \varphi_{cr} \)
Pulsed-source experiments

• Results for a simplified configuration, adopting analytical approach:
  - Similar performances of all options in reproducing the power evolution
  - Results with poor accuracy in flux prediction when the detector is placed far from the source
  - Strong higher harmonic effects for the flux for detectors near the source
  - $l_{pk}$ shows better performance in preserving areas (important when area methods are used in the interpretation of measurements)
Pulsed-source experiments
Flux at detector position

• Flux response at spatial points far from the source is mainly influenced by the time delay of the neutron signal detected

It is necessary to modify the time behavior of the source in order to simulate the real signal
• Evaluation of time of flight to detector position
• Evaluation of mean travel time (which distribution?)
• Convolution of external source with system response (use of Green function)
Treatment of source delay

• Time of flight $\Rightarrow t_{x_0} = \frac{x_0}{v}$

• Mean travelling time $\Rightarrow t_{x_0} = \int t' \cdot f(x_0, t') dt'$

$S(x, t) = \delta(x)\vartheta(t) \rightarrow \tilde{S}(x, t) = \delta(x)\vartheta(t - t_{x_0})$

• Source convolution:

$\tilde{\vartheta}(t) = \int \vartheta(t')G(x_0, t' \rightarrow t) dt'$

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Simulation of source experiments
Treatment of source delay
Treatment of source delay

\[ x_0 = 4.4L \]

Exact solution

Point kinetics

\[ t_{x_0} = \frac{x_0}{v} = 6.15 \mu s = 0.46 \text{ m.a.t.} \]

\[ t_{x_0} = \int t' \cdot f(x_0, t') dt' = 36 \mu s = 2.7 \text{ m.a.t} \]

Convolution with Green function
Treatment of source delay

\[ x_0 = 0.9L \]

**Exact solution**

**Point kinetics**

\[ t_{x_0} = \frac{x_0}{v} = 1.16 \mu s = 0.087 \text{ m.a.t.} \]

\[ t_{x_0} = \int t' \cdot f(x_0, t') dt' = 12.3 \mu s = 0.92 \text{ m.a.t.} \]

**Convolution with Green function**

Simulation of source experiments
Use of lpk in transient evaluation

- 3-group solution in a MUSE-like configuration (1D)
- Localized perturbations of cross sections
- Evaluation of power and local flux using gpk and lpk
Use of lpk in transient evaluation

Transient 1: $\Delta \Sigma_{a,1} < 0$

- $\Delta \rho = 729\,\text{pcm}$
- $\Delta P_{gpk} = (P_{gpk} - P) / P \% = -1.78$
- $\Delta \Phi_{gpk} \% = -8.33$
- $\Delta \Phi_{lpk} \% = -0.46$

$\alpha$, $\beta$, $\gamma$

Monochromatic detectors $d_{\alpha,g}$

Improved performances with lpk

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Simulation of source experiments
• $\Phi(x_{\alpha,2}) / \Phi_0(x_{\alpha,2}) = 1.316$
  $\Delta \Phi_{gpk}[\%] = -4.05$
  $\Delta \Phi_{lpk}[\%] = -1.45$

• $\Phi(x_{\alpha,3}) / \Phi_0(x_{\alpha,3}) = 1.309$
  $\Delta \Phi_{gpk}[\%] = -3.56$
  $\Delta \Phi_{lpk}[\%] = -1.56$

• $\Phi(x_{\beta,1}) / \Phi_0(x_{\beta,1}) = 1.316$
  $\Delta \Phi_{gpk}[\%] = -4.05$
  $\Delta \Phi_{lpk}[\%] = -1.45$

• $\Phi(x_{\beta,2}) / \Phi_0(x_{\beta,2}) = 1.300$
  $\Delta \Phi_{gpk}[\%] = -2.92$
  $\Delta \Phi_{lpk}[\%] = -1.63$

• $\Phi(x_{\beta,3}) / \Phi_0(x_{\beta,3}) = 1.297$
  $\Delta \Phi_{gpk}[\%] = -2.68$
  $\Delta \Phi_{lpk}[\%] = -1.69$

• $\Phi(x_{\gamma,1}) / \Phi_0(x_{\gamma,1}) = 1.368$
  $\Delta \Phi_{gpk}[\%] = -7.72$
  $\Delta \Phi_{lpk}[\%] = -0.65$

• $\Phi(x_{\gamma,2}) / \Phi_0(x_{\gamma,2}) = 1.326$
  $\Delta \Phi_{gpk}[\%] = -4.78$
  $\Delta \Phi_{lpk}[\%] = -1.34$

• $\Phi(x_{\gamma,3}) / \Phi_0(x_{\gamma,3}) = 1.320$
  $\Delta \Phi_{gpk}[\%] = -4.36$
  $\Delta \Phi_{lpk}[\%] = -1.45$
Use of lpk in transient evaluation

Transient 1: \( \Delta \Sigma_{a,1} < 0 \) \( \Delta \rho = 729 \text{pcm} \)

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
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<td>-1.45</td>
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</table>

Simulation of source experiments
Use of $\text{lpk}$ in transient evaluation

Transient 2: $\Delta \Sigma_{a,2} < 0 \quad \Delta \rho = 1157 \text{pcm}$

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Use of lpk in transient evaluation

Transient 3: $\Delta \Sigma_{a,3} < 0 \quad \Delta \rho = 499\text{pcm}$

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<td><strong>-0.79</strong></td>
<td><strong>-0.75</strong></td>
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</table>
Reactivity evaluation in pulsed-source experiments

- 3-group evaluation of a subcritical system
- Evaluation of reactivity from $l_{pk}$ for different detectors (at 3 spatial positions a-b-c and within each of 3 energy groups)
Reactivity evaluation in pulsed-source experiments

Simulation of source experiments

$k_{\text{eff}} = 0.97$

- 1° group
- 2° group
- 3° group
Reactivity evaluation in pulsed-source experiments

Simulation of source experiments

$k_{\text{eff}} = 0.99$

- **1° group**
- **2° group**
- **3° group**

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