MHD

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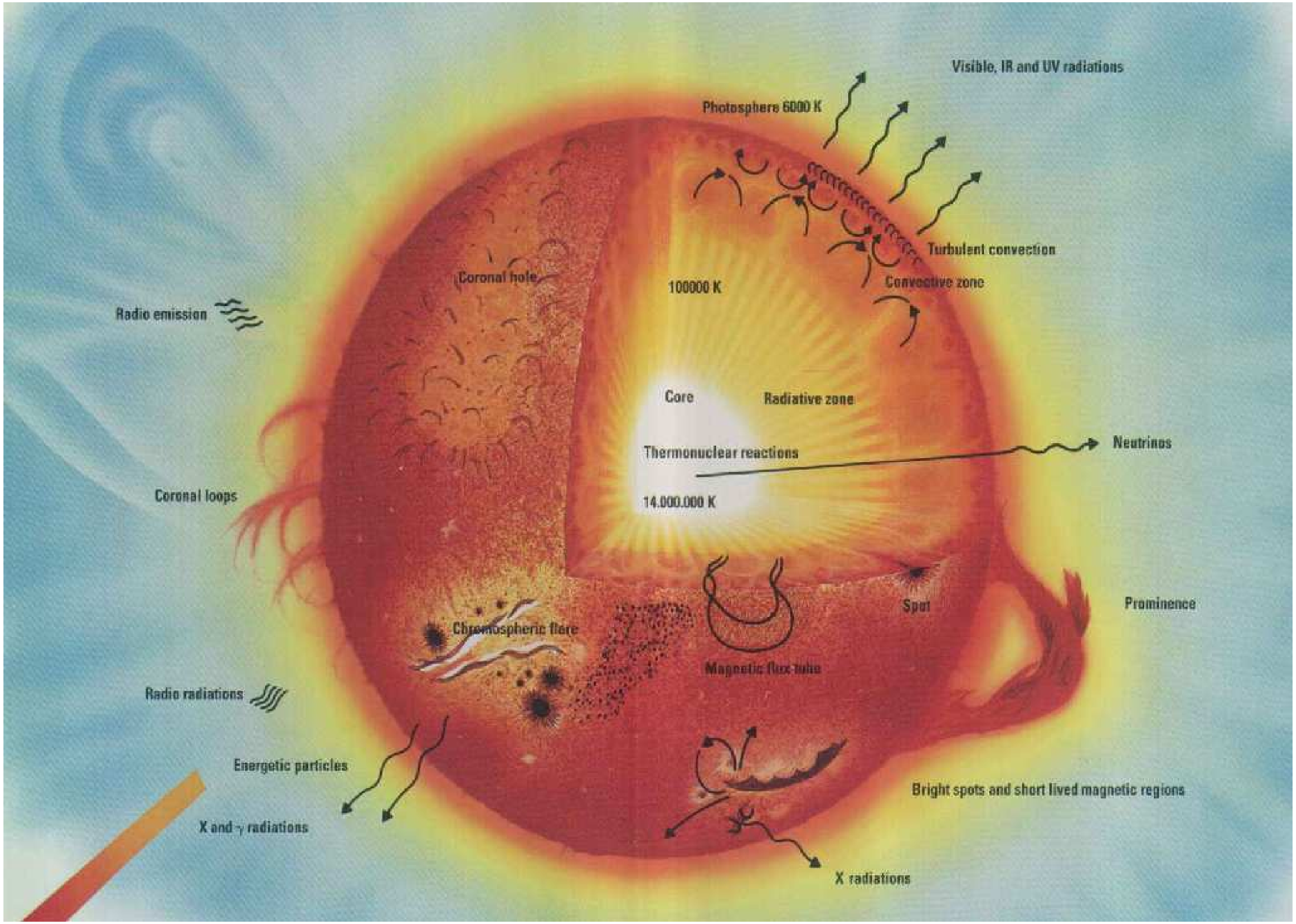
Plan

- Observing the B field
- Batteries and Seed fields
- Dynamos

A. Brandenburg & K. Subramanian, Physics Reports, 417, 1-205 (2005)

Sun: A testbed for HD and MHD

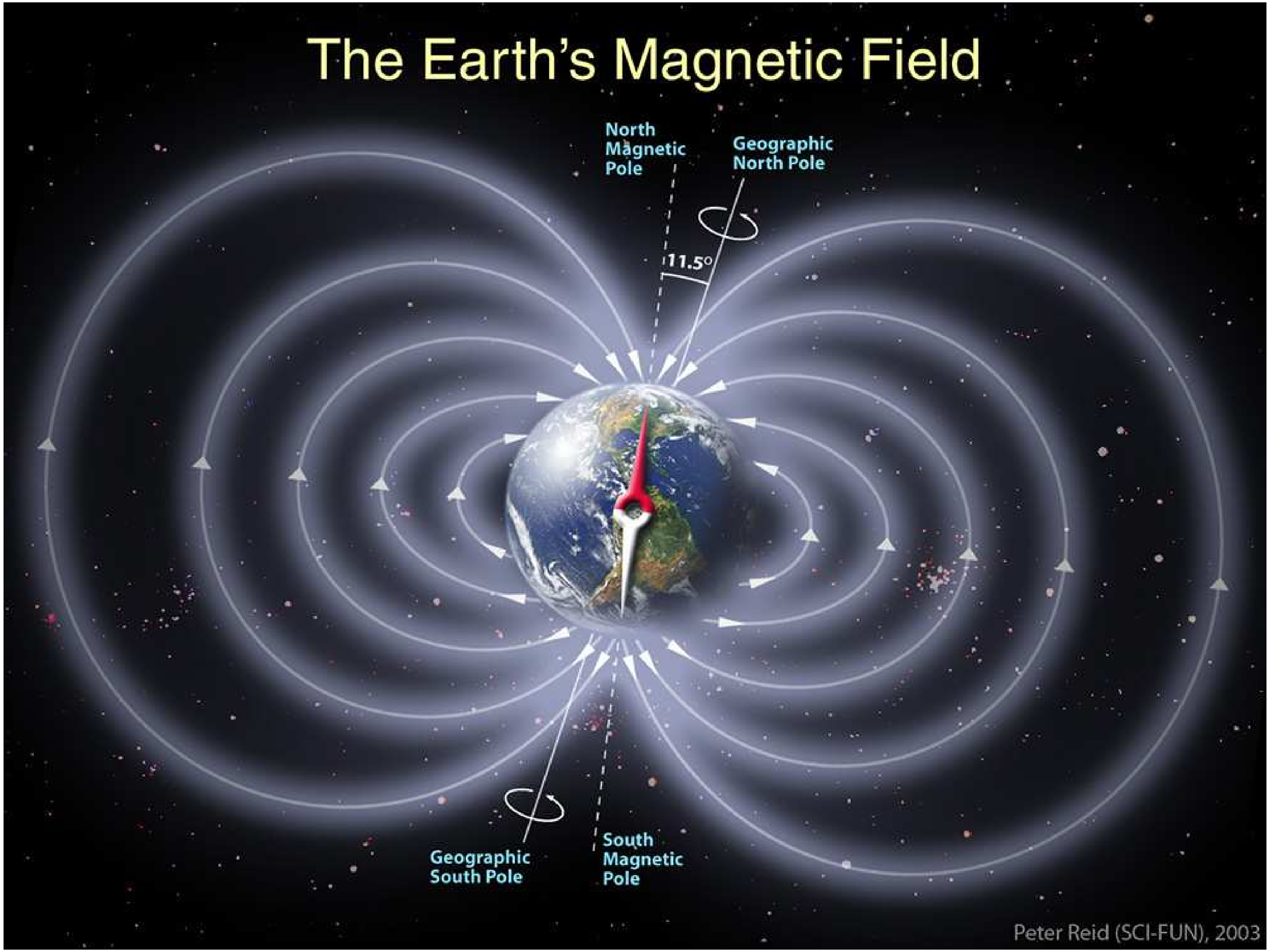


The Magnetic Universe

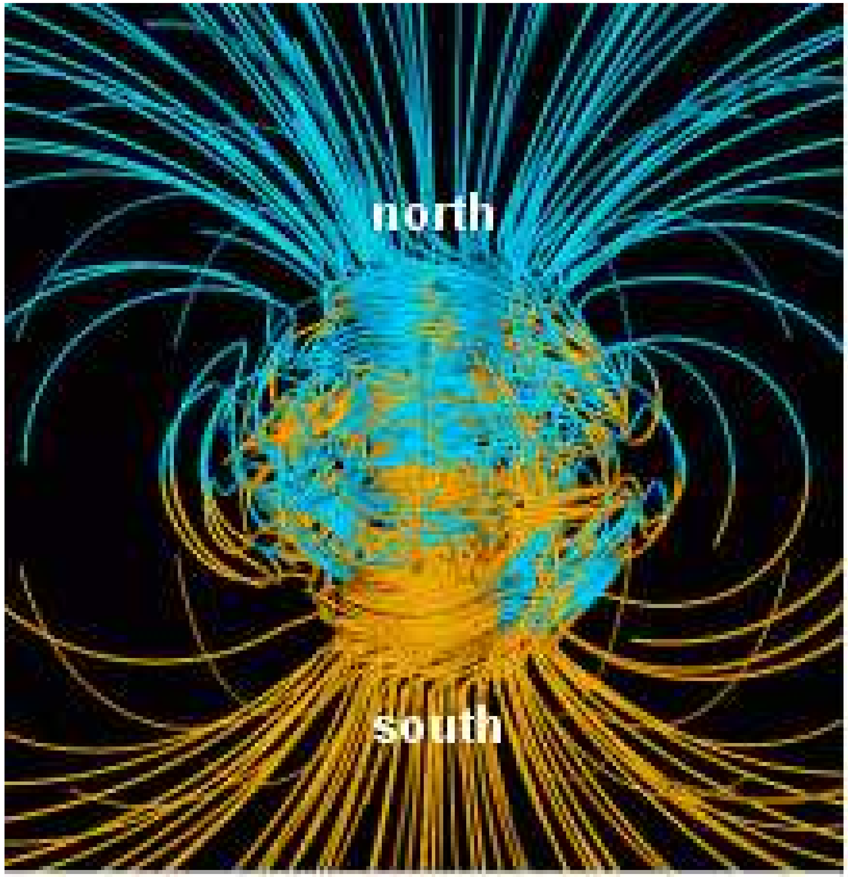
Why Magnetic Universe?

- Electrically Neutral Universe
  - Both $+$ve and $-$ve Charges Present
- Free Electic charges + No magnetic charges $\Rightarrow$
  - Can short out strong E in plasma rest frame
- Non Relativistic Velocities $\Rightarrow$
  - Simpler to think interms of B than E

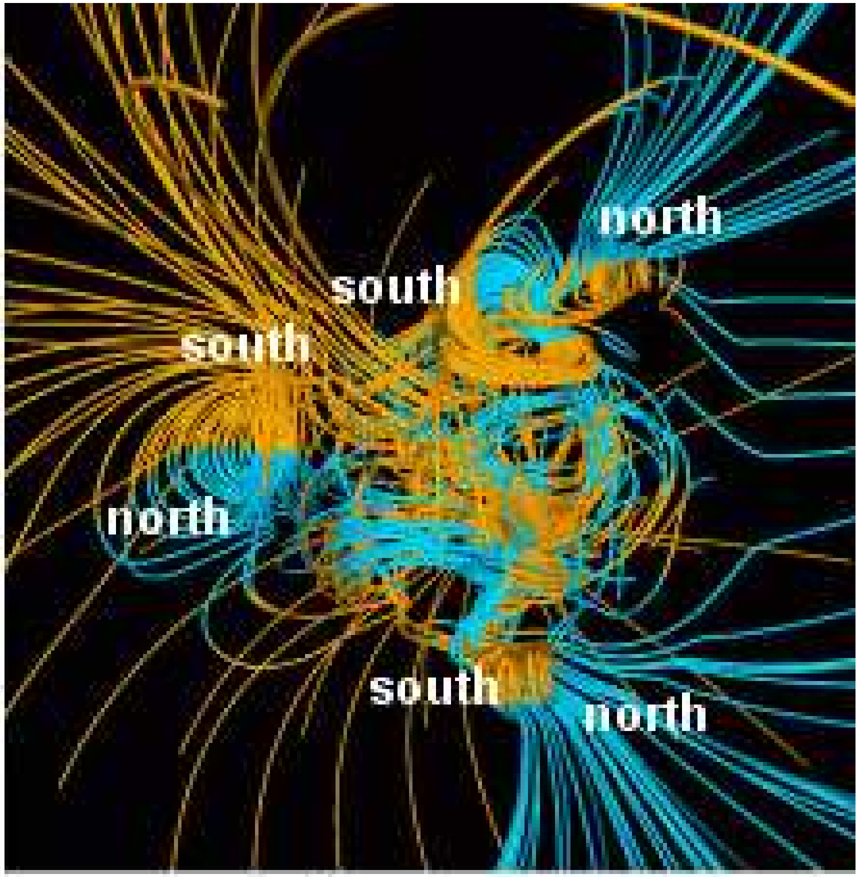
The Earth's Magnetic Field



The Earth´s Magnetic Field



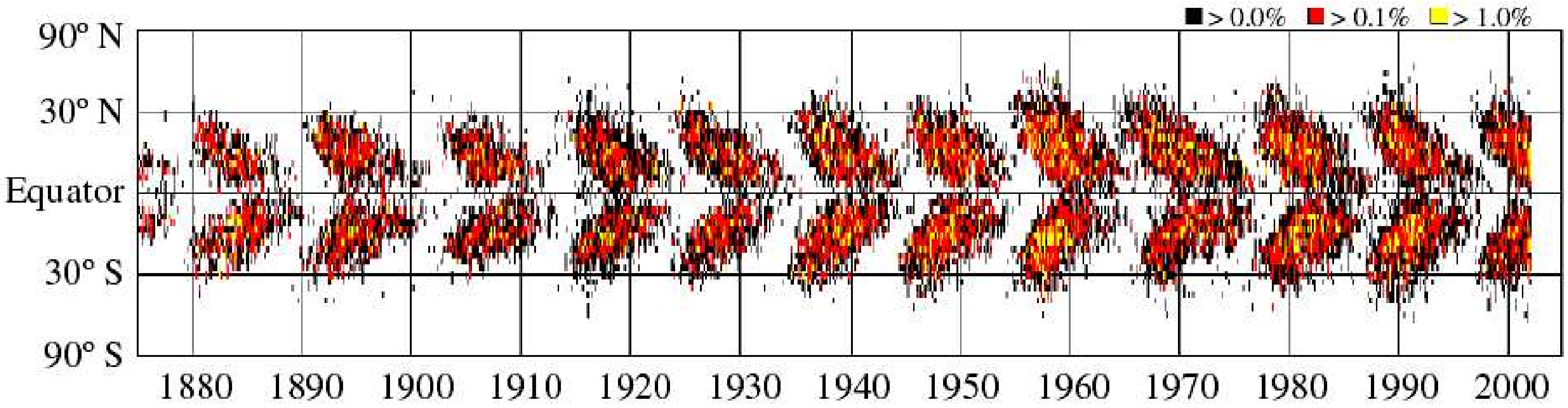
between reversals

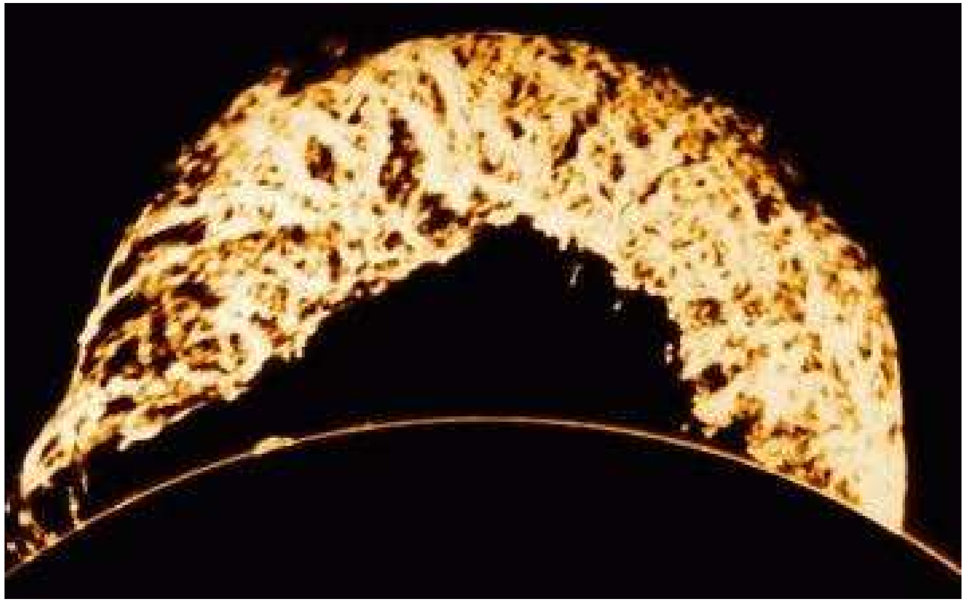


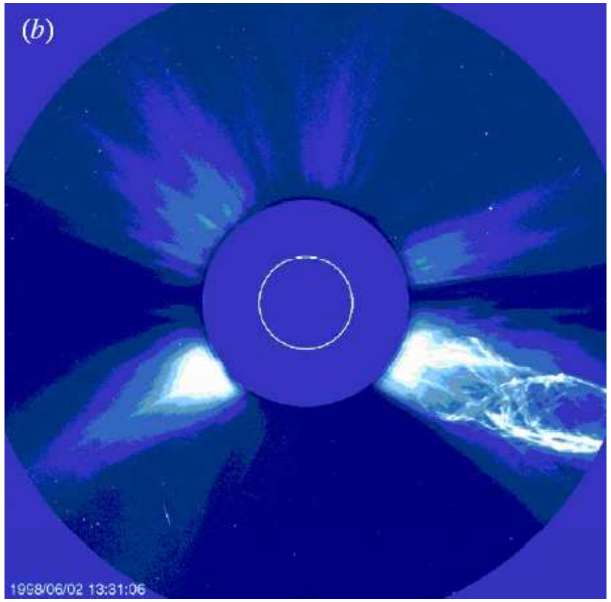
during a reversal

Simulations by Glatzmaier and Roberts, 1995

The Solar Cycle

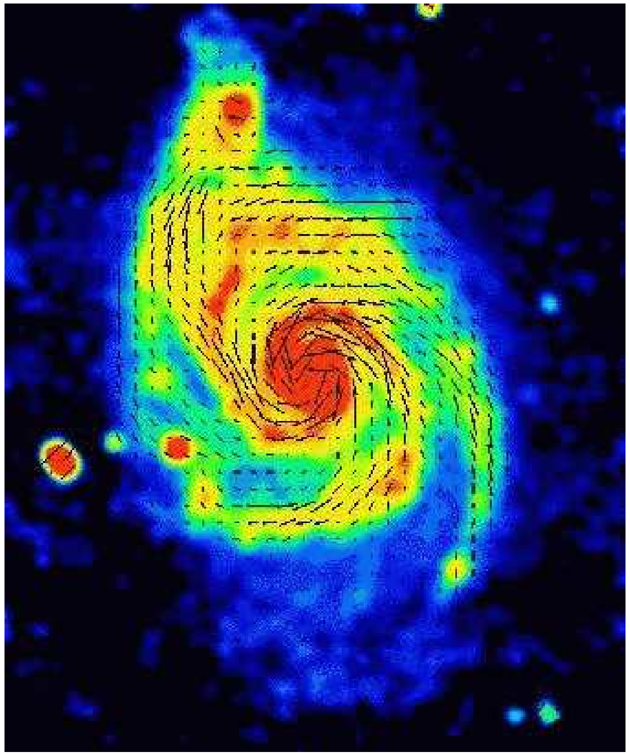




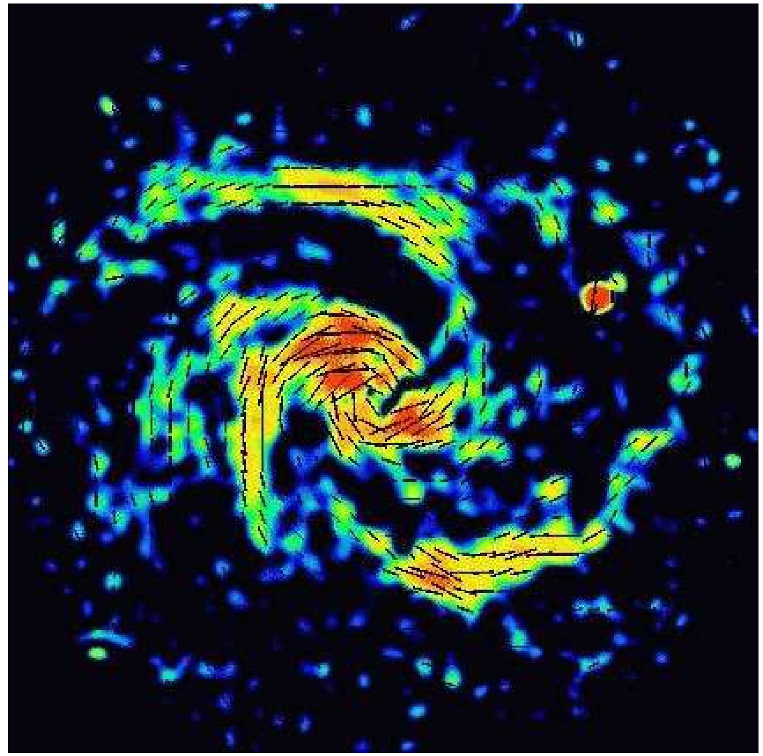


Galactic Magnetic Fields: Observations

M51 in 6 cm with **B** fields



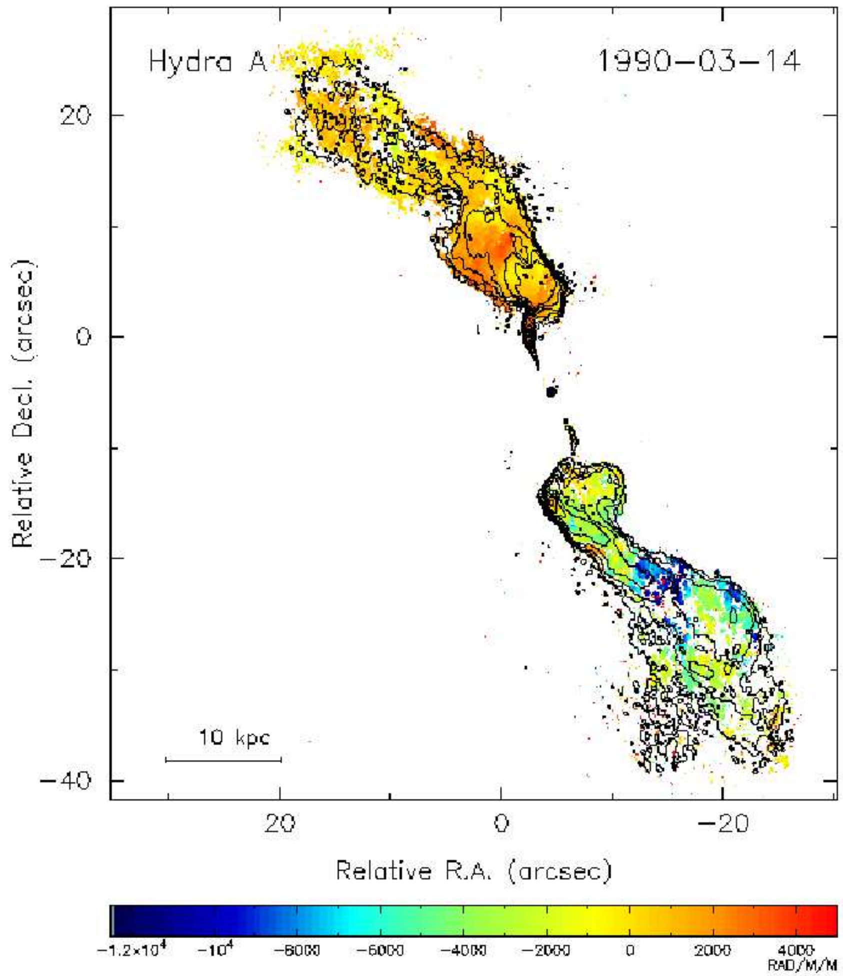
NGC 6946 in polarization, 6 cm (R. Beck)

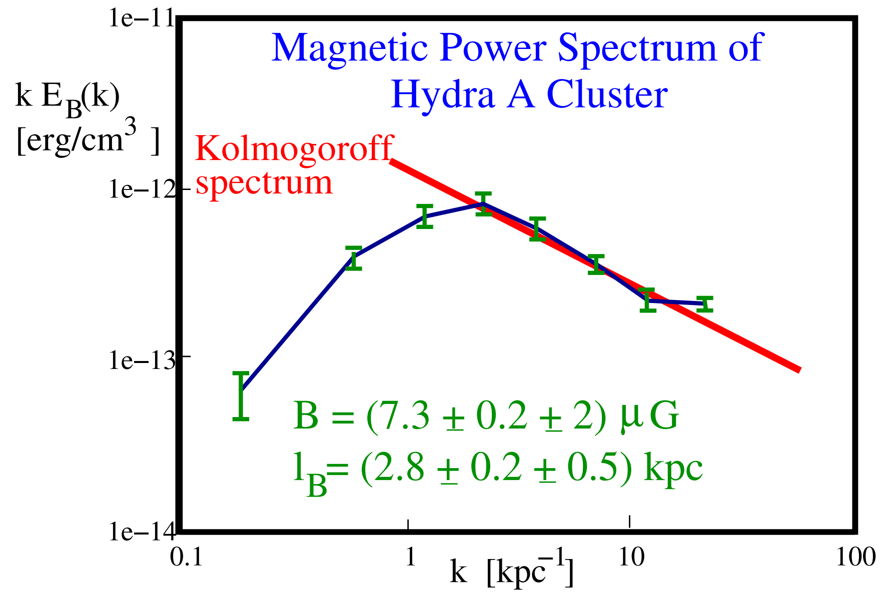


- $\langle B_{total} \rangle = 9\mu G$ **in sample of 74 spirals**
- **Mean fields about** $0.5 - 1$ **smaller than random field**
- **Coherent fields correlated on** 10 **kpc scales**

How do such large scale galactic fields arise?

Cluster Magnetism: Observations

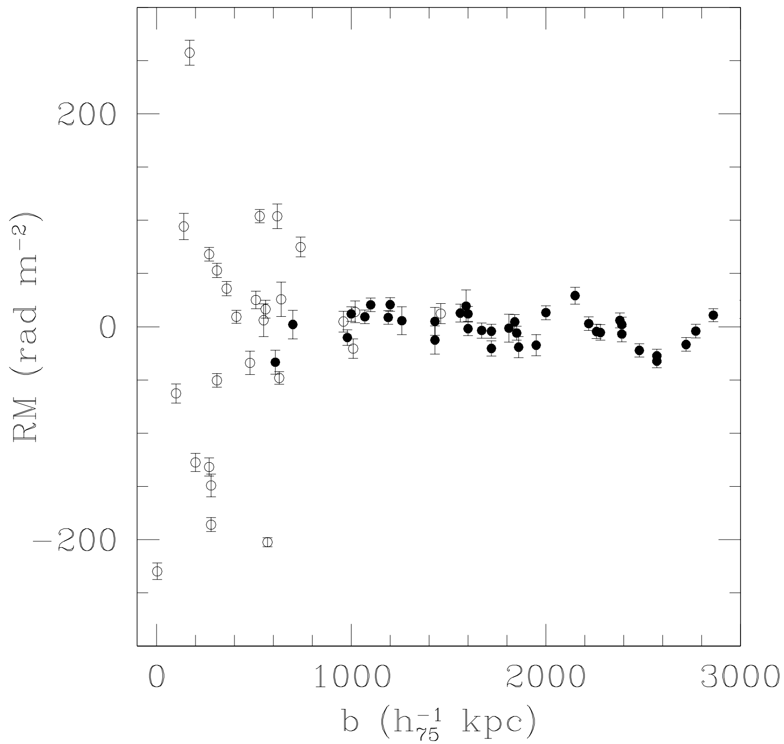


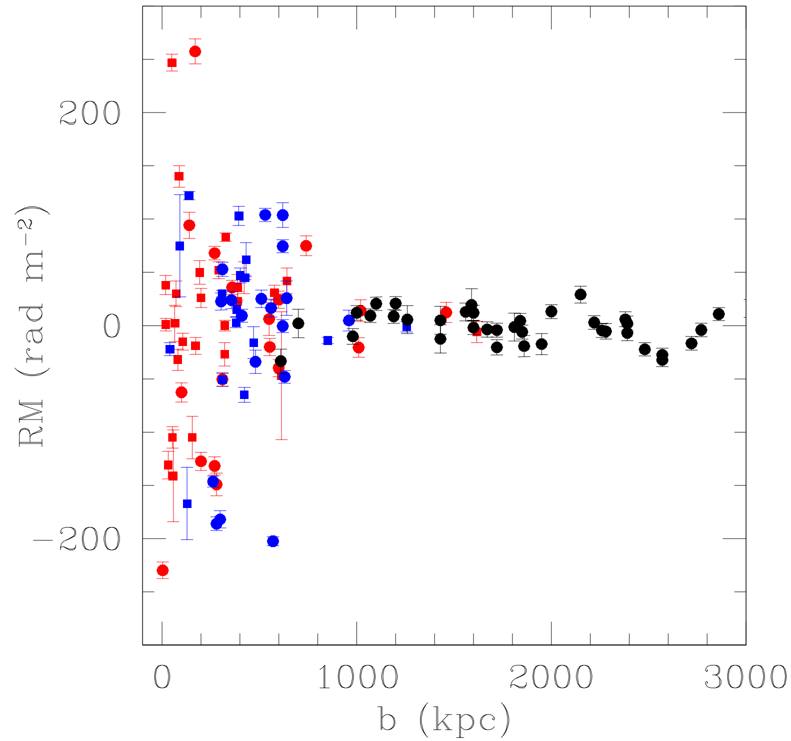


Vogt & Ensslin, A&A, 434, 67, 2005

Cluster Magnetism: Observations

Clarke et al., ApJ, 547, L111, 2001





- **Statistical RM study**
- $B \sim 5(l/10kpc)^{-1/2}\mu G$

- **embedded sources**
- **background sources**

How are cluster fields generated/maintained against turbulent decay?

Faraday's 6 principles

Faraday's six principles

1. Have a little pad and take notes at all times
2. Exchange letters with other scientists
3. Have collaborations
4. Check everything
5. Avoid controversy
6. Never make general assumptions too quickly, speak and write as precisely as possible

MHD Basics

- Maxwell equations + Ohms law $\Rightarrow$ Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \, .$$

  - $\mathbf{U} = 0 \Rightarrow$ pure diffusion and decay
  - If $\eta \to 0$, the flux $\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$ is 'frozen' $\to d\Phi/dt \to 0$.
  - Magnetic Reynolds number $R_{\rm M} = (UB)/(\eta B/L) = UL/\eta$
  - For Astro systems $R_{\rm M} \gg 1$
- $\mathbf{B} = 0$ is solution! $\Rightarrow$ Need Batteries to generate B_{seed}
- $B_{seed} \ll B_{observed} \Rightarrow$ Need $\mathbf{U}$ to act as Dynamo
- Lorentz force $\mathbf{J} \times \mathbf{B}$ backreacts on $\mathbf{U}$ eventually
- Crucial to understand how dynamos saturate ?

The first "seed" fields in the universe

- **Primordial fields from Early Universe? Uncertain Physics**
  Constrained by observations of CMB, RM

- **Astrophysical Batteries using postive/negative charge asymmetry**

- **Biermann Batteries**: $\mathbf{E}_{Bier} = -\nabla p_e/en_e + ...$

$$(\partial \mathbf{B}/\partial t) = -c\nabla \times \mathbf{E}_{Bier} = -(ck/en_e)\nabla n_e \times \nabla T_e$$

  - **Re-Ionization fronts**: $B < 10^{-19}$ G (Subramanian, Narasimha, Chitre, MN, 1994; Gnedin, Ferrara and Zweibel, ApJ, 2000)
  - **Structure formation Shocks** (Kulsrud et al, 1997)

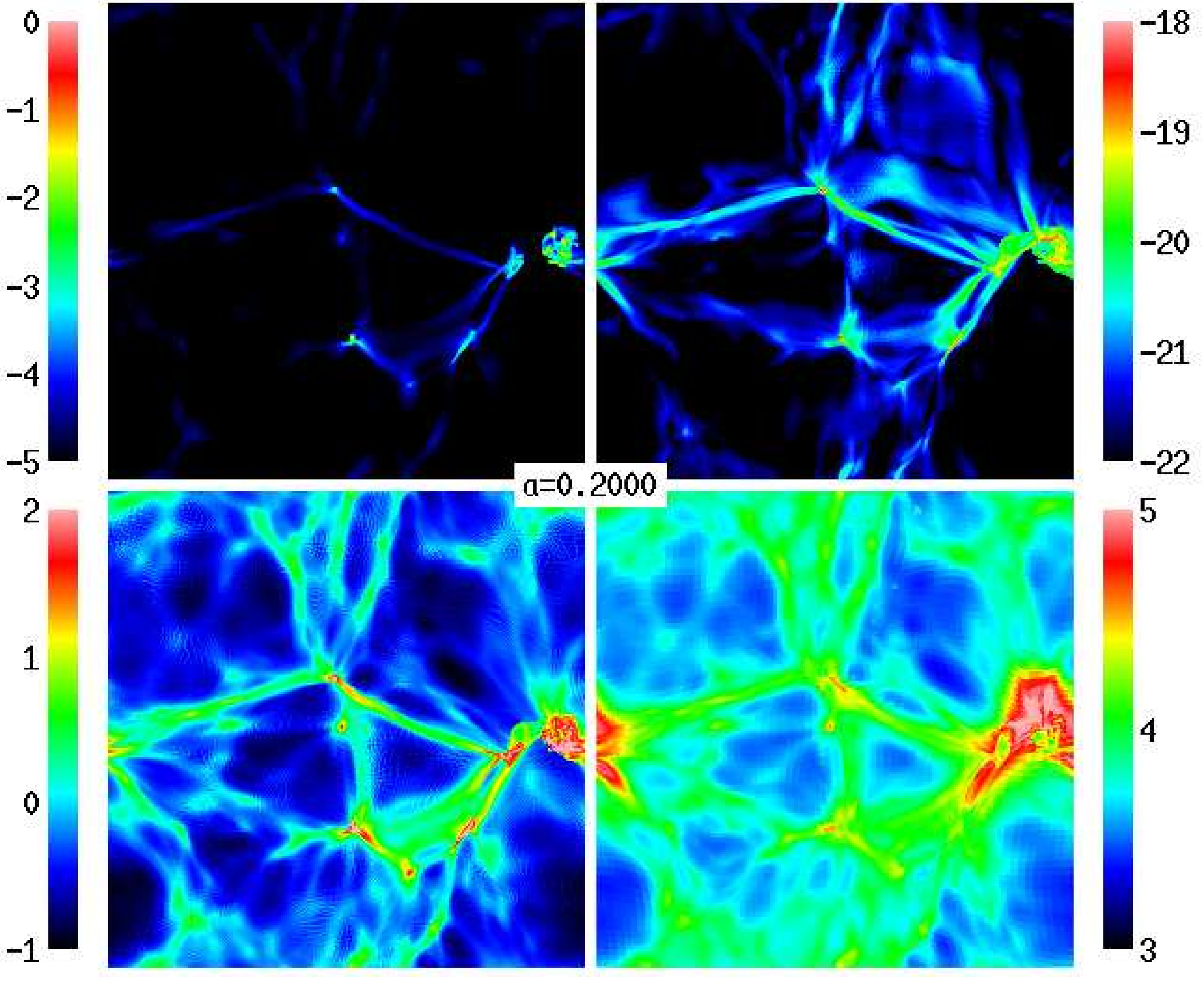
- **During recombination:** $\gamma - e/p$ **scattering asymmetry**
  $B_0 \sim 10^{-30}$ G at MPc (Gopal & Sethi, 2005; Mattarrese et al, 2005);
  $B_0 \sim 10^{-21}$ G at pc (Ichiki et al 2007)

- **Seed fields from first supernovae and AGN outflows**

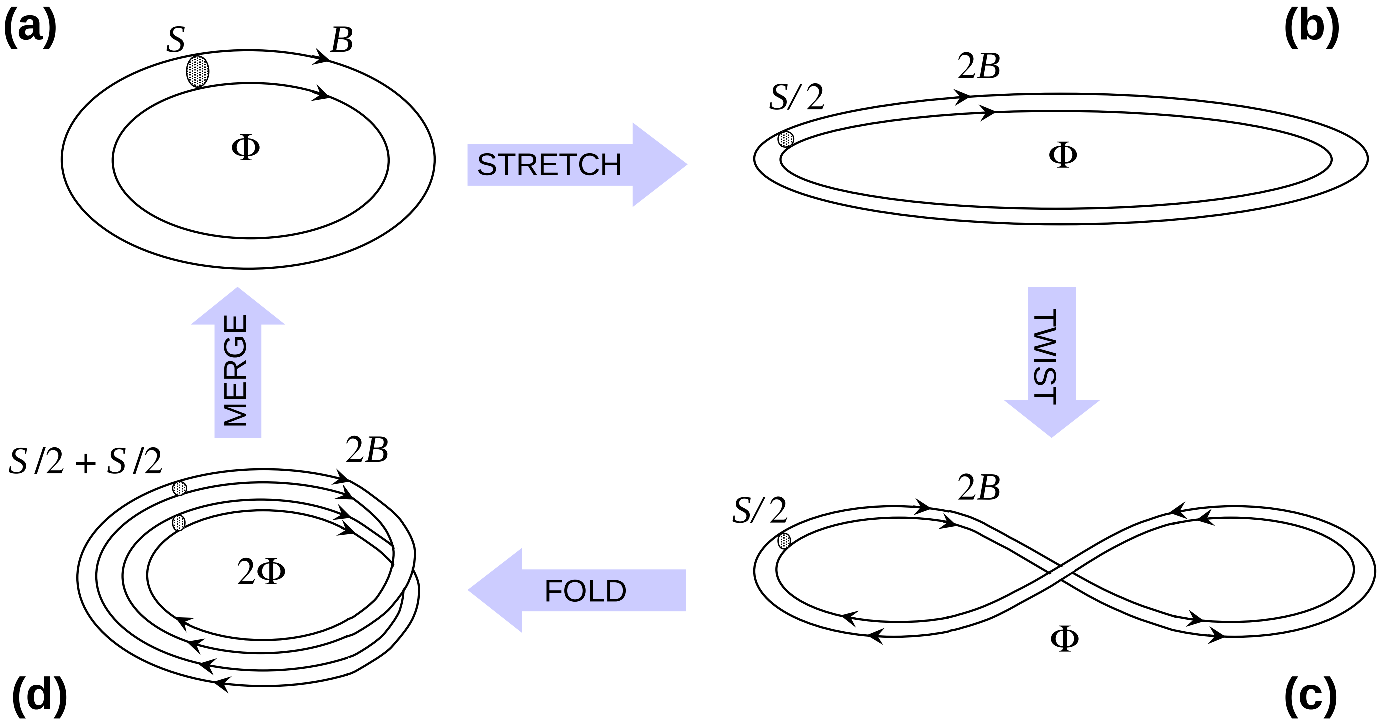
Need Dynamos to explain observed fields and maintain against decay

Magnetic fields from Reionization

HI, gas density, temperature and B field; Gnedin, Ferrara, Zweibel, 2000, ApJ



Zeldovich STF dynamo

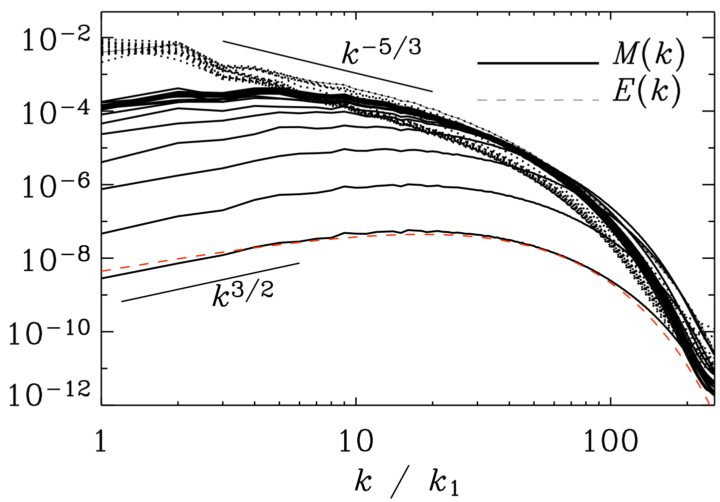


- Field grows by factor 2^N; Energy from motions
- Flux through Eulerian surface grows though nearly frozen!
- Twist and 3d motion required for coherent growth
- Eventual saturation through Lorentz forces.

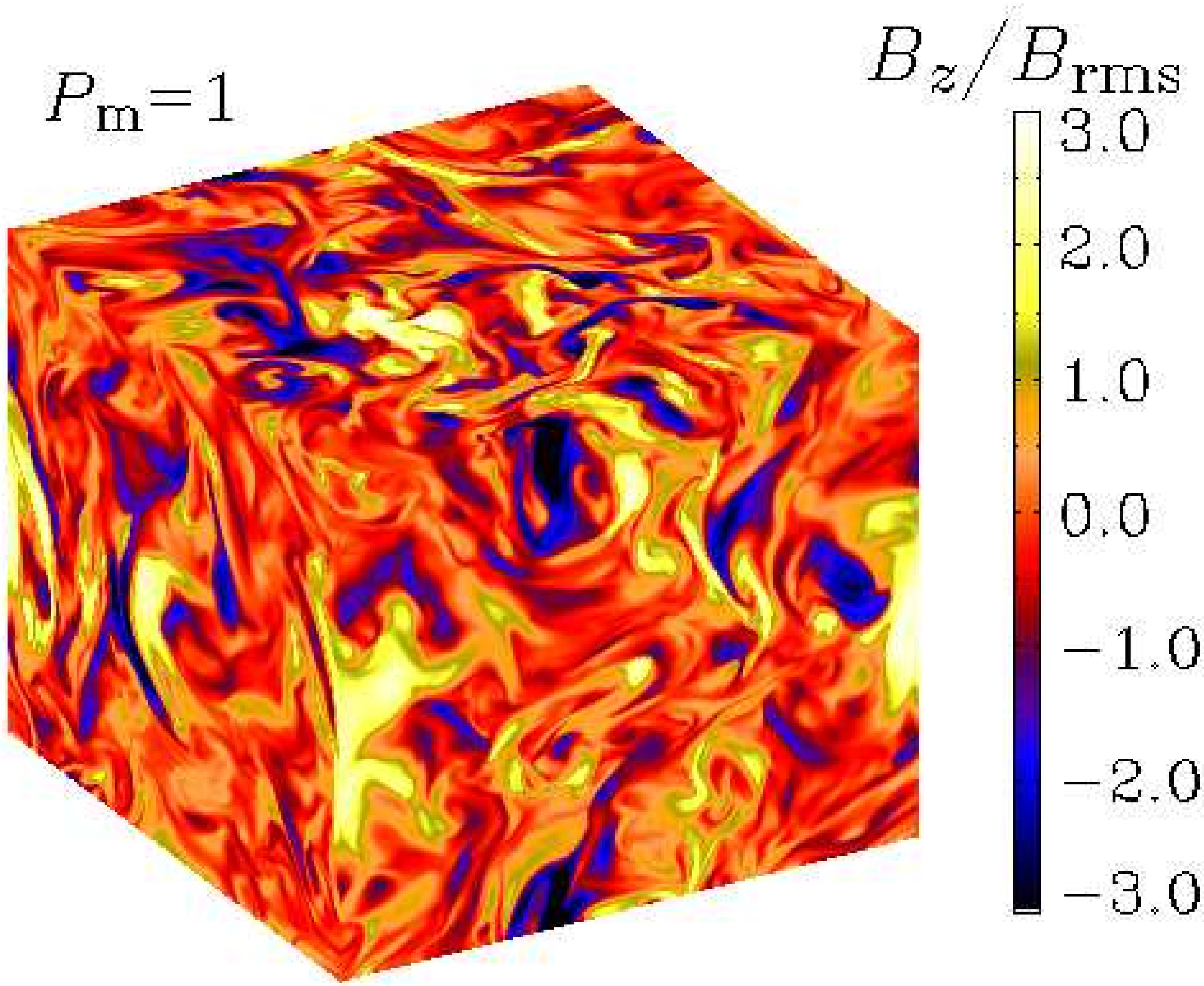
The fluctuation dynamo & Cluster fields

- **Turbulence $\rightarrow$ Random Stretching;**
- **Then Flux freezing $\Rightarrow$ Growth of $\mathbf{B}$**
- $BA =$ **constant and** $\rho AL =$ **constant** $\rightarrow B/\rho \propto L$, **and** $A \propto 1/(\rho L)$
- **Diffusion $\sim$ growth when** $v/L \sim \eta/l_B^2$ **or** $l_B \sim L/R_{\rm M}^{1/2}$
- **Random $\mathbf{B}$ grows if** $R_{\rm M} > R_{crit} \sim 30 - 100$ (Kazantsev 1967)

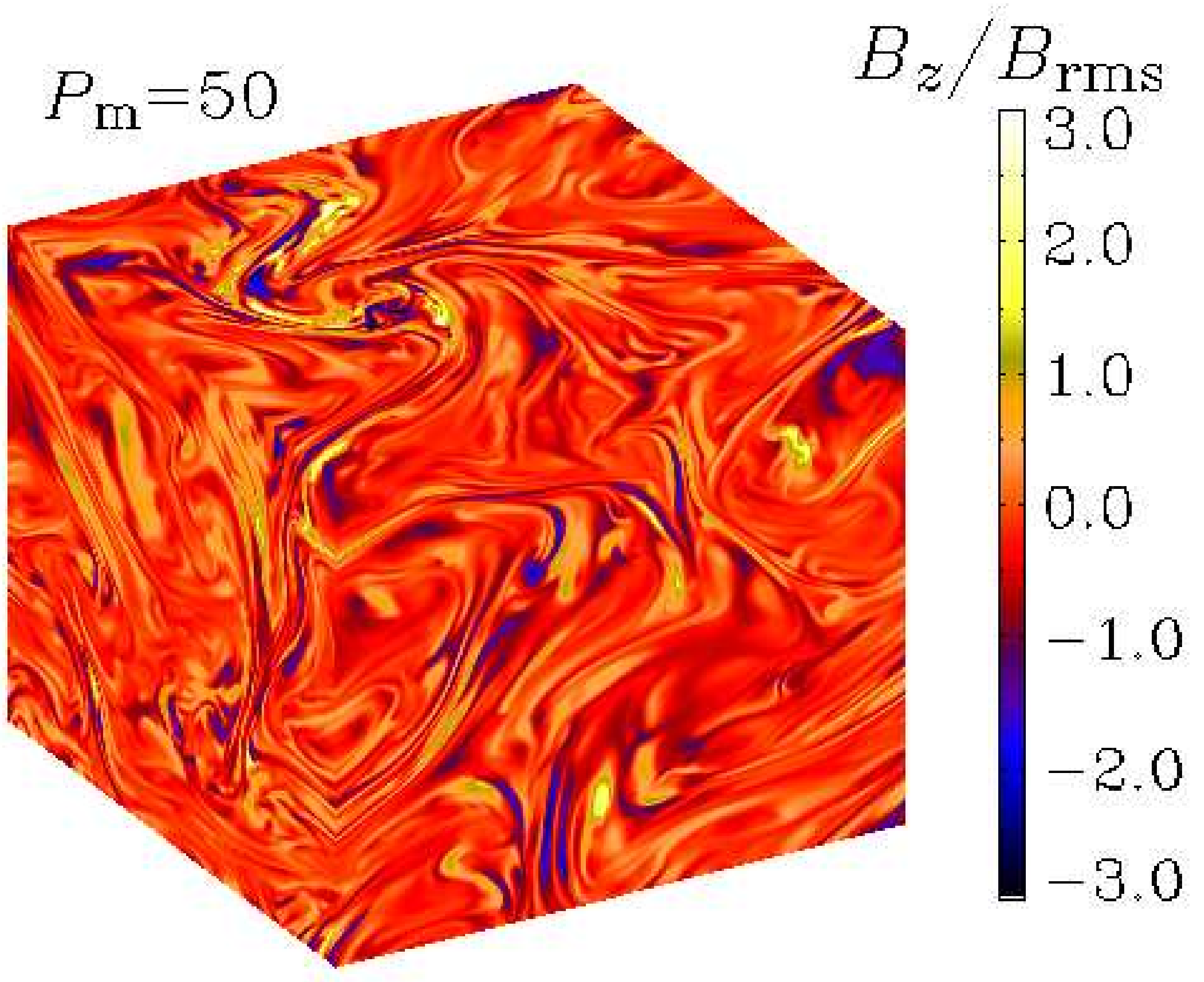
Generated **B** intermittent :Simulations by Axel Brandenburg, 2005



The fluctuation dynamo

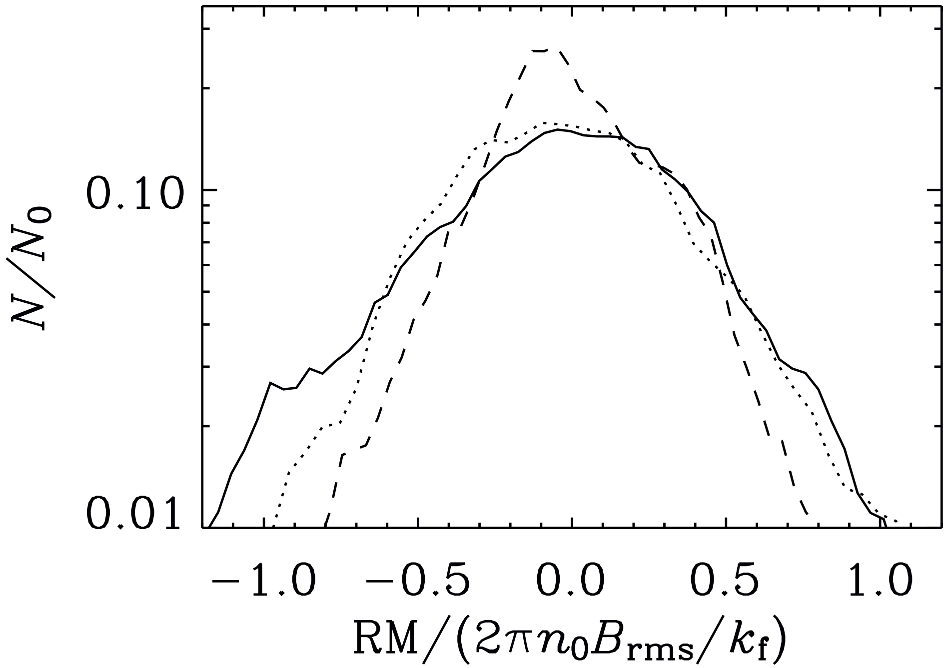


The fluctuation dynamo



Fluctuation dynamo saturation?

- **Renormalized η drives efective $R_{\rm M} \to R_{crit}$, $l_B \sim L/R_{crit}^{1/2}$, Saturated state universal** (Subramanian, PRL, 1999; 2003).
- **Faraday RM Histogram for $P_m = 1, 1/4, 30$; explains cluster RM** (Subramanian, Shukurov, Haugen, MN, 2006) (Ensslin, Vogt, A&A 2006)

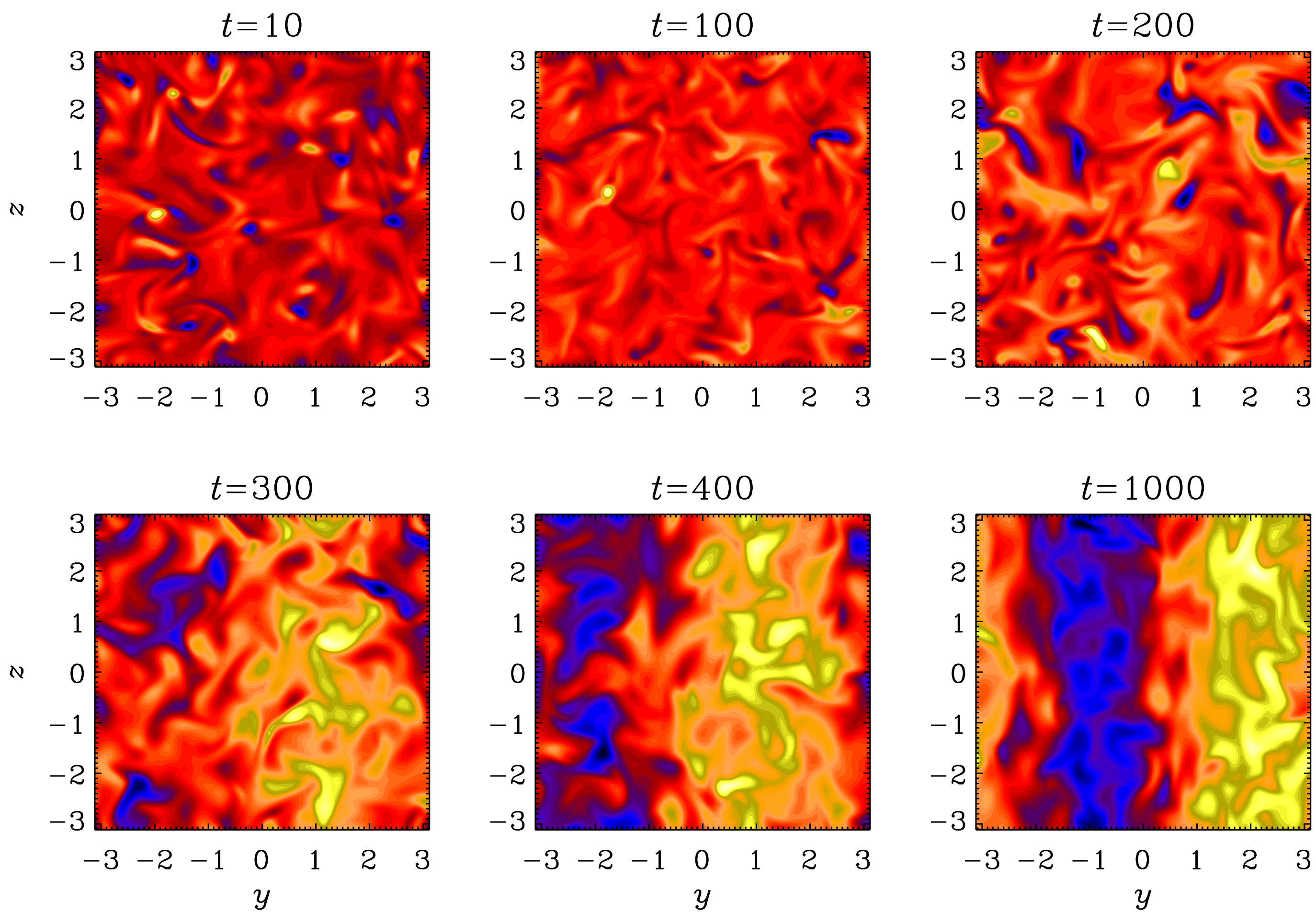


- **Saturation due to Reduced stretching BUT $l_B \sim L/R_{\rm M}^{1/2}$! Plasma effects crucial** (Schekochihin, Cowley et al., ApJ, 2004, 2006)

Important for Cluster/young galaxy Faraday RM/CR confinement

Helically forced turbulent dynamos

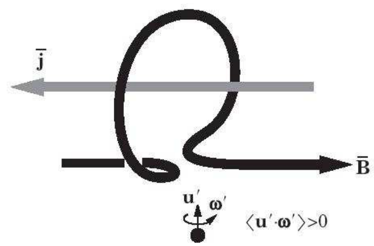
Axel Brandenburg, Ap.J. 550, 824 (2001)



Large scale field grows BUT on resistive time-scales

Mean-Field Dynamo: Galactic

- Galactic Shear generates B_ϕ from B_r
- Supernovae drive HELICAL turbulence (Due to Rotation + Stratification)
- Helical motions generate B_r from B_ϕ



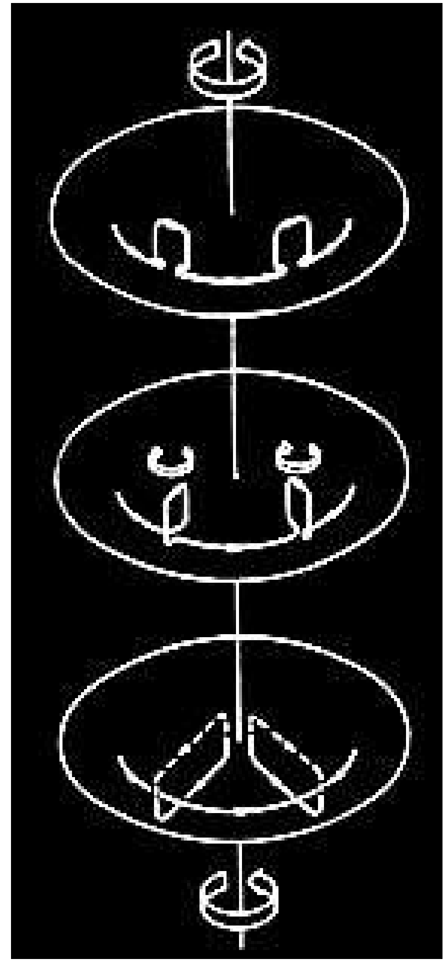
α-effect (Parker, 55)

- Mean field satisfies dynamo equation

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta (\nabla \times \overline{\mathbf{B}}) \right) ;$$

$$\overline{\boldsymbol{\mathcal{E}}} = \overline{\mathbf{u} \times \mathbf{b}} = \alpha \overline{\mathbf{B}} - \eta_{turb} (\nabla \times \overline{\mathbf{B}})$$

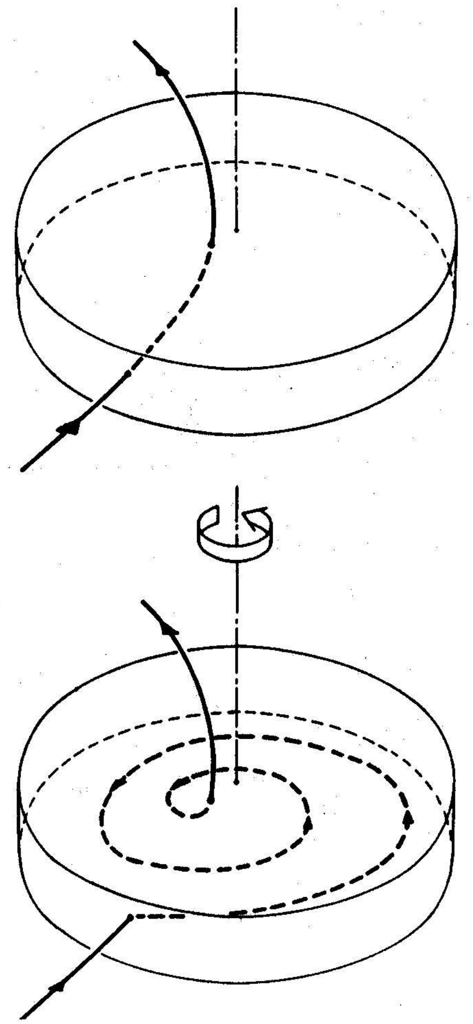
$$\alpha = -\frac{\tau_{corr}}{3} \langle \mathbf{u} \cdot \omega \rangle \quad \eta_{turb} = \frac{\tau_{corr}}{3} \langle \mathbf{u}^2 \rangle$$

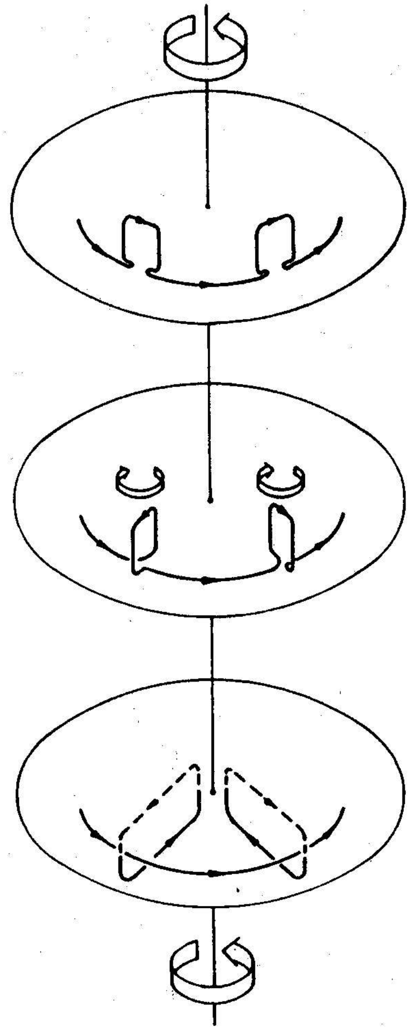


RSS, 1988

- Exponential growth of $\overline{\mathbf{B}}$, $t_{growth} \sim 10^9$ yr

Galactic Shear and α effect

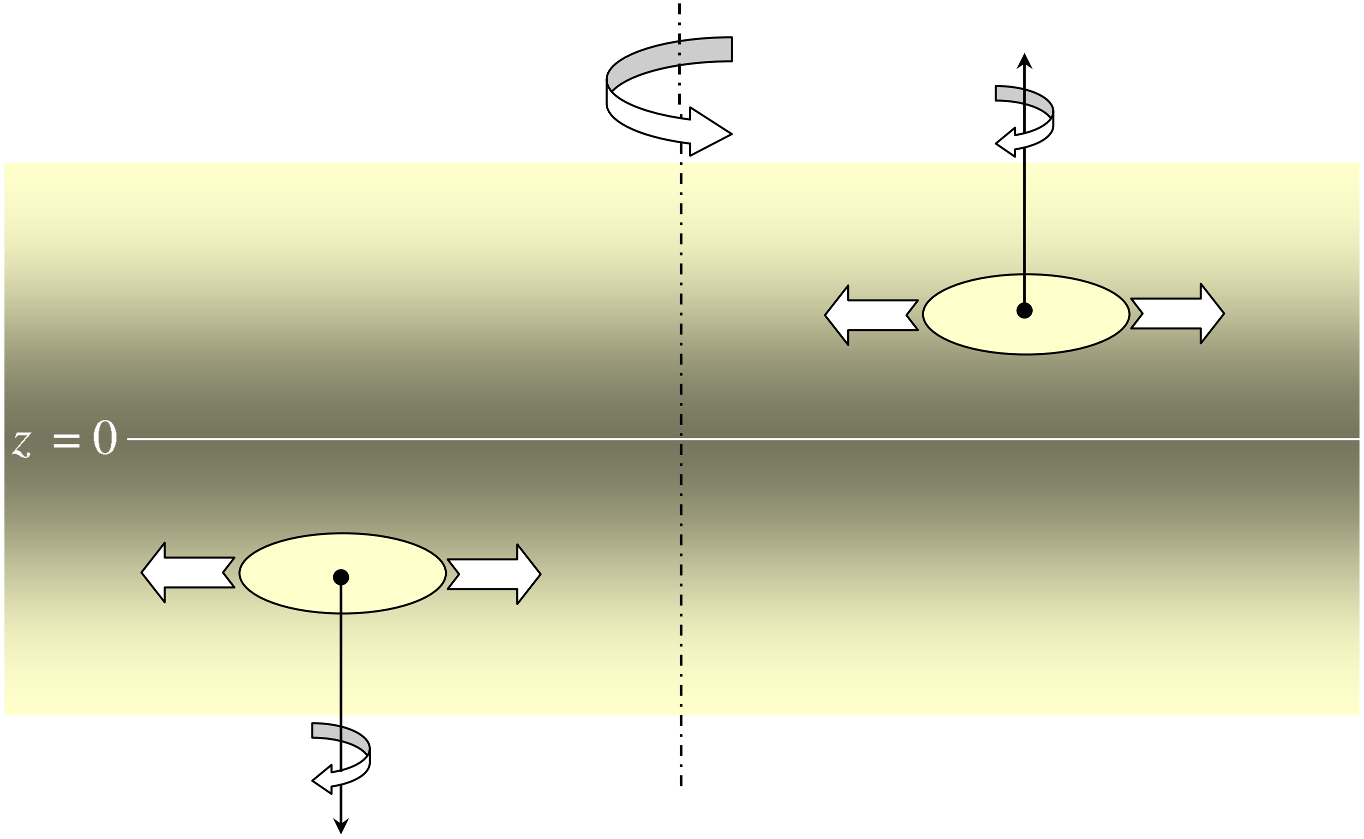




Kinematic Limit?

Helicity (links) conservation? Supression of Lagrangian Chaos?

Supernovae Drive Helical turbulence



The turbulent $\overline{\mathcal{E}}$

- **Need to find $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ under influence of Lorentz forces**

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{u} \times \overline{\boldsymbol{B}} - \eta \nabla \times \boldsymbol{b}) + \boldsymbol{G}$$

- **Here G is the "pain in neck" term in u and b.**

$$\boldsymbol{G} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b})' = \nabla \times [\boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}]$$

- **Calculating $\overline{\mathcal{E}}$ requires a closure even in the kinematic limit**

The kinematic limit of $\overline{\mathcal{E}}$

- **For short correlation times ($\tau_{\rm cor}$), neglect G, also assume statistical isotropy of the random u :**

$$\overline{\mathcal{E}} = \overline{\boldsymbol{u} \times \int^t dt' (\partial \boldsymbol{b}/\partial \tau)} = \overline{\boldsymbol{u}(t) \times \int^t dt' [-\boldsymbol{u}(t') \cdot \nabla \overline{\boldsymbol{B}} + \overline{\boldsymbol{B}} \cdot \nabla \boldsymbol{u}(t')]}$$

$$\overline{\mathcal{E}}_i = \int_0^t \left[\epsilon_{ijk} \overline{u_j(t) u_{k,p}(t')} \overline{B}_p(t') + \epsilon_{ijp} \overline{u_j(t) u_l(t')} \overline{B}_{p,l}(t') \right] \mathrm{d}t',$$

- **So**: $\overline{\mathcal{E}} = \alpha \overline{\boldsymbol{B}} - \eta_t \nabla \times \overline{\boldsymbol{B}}$, where

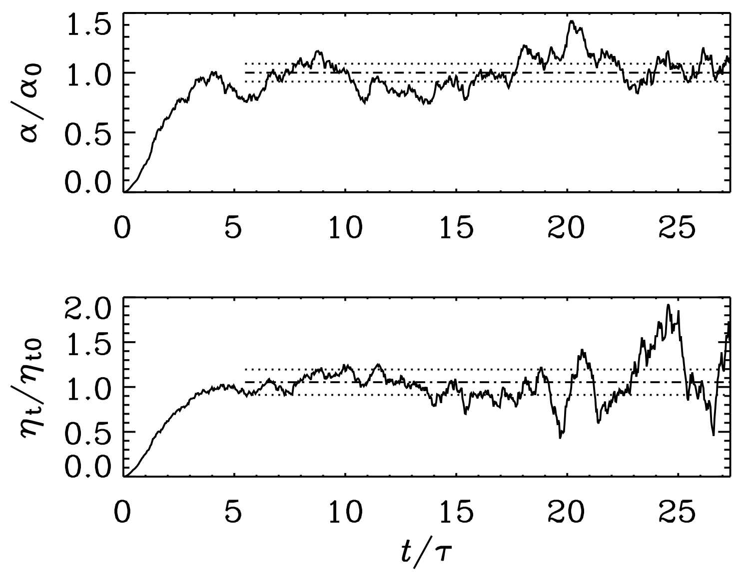
$$\alpha = -\tfrac{1}{3} \int_0^t \overline{\boldsymbol{u}(t) \cdot \boldsymbol{\omega}(t')}\, dt' \approx -\tfrac{1}{3} \tau_{\rm cor} \overline{\boldsymbol{u} \cdot \boldsymbol{\omega}},$$

$$\eta_{\rm t} = \tfrac{1}{3} \int_0^t \overline{\boldsymbol{u}(t) \cdot \boldsymbol{u}(t')}\, dt' \approx \tfrac{1}{3} \tau_{\rm cor} \overline{\boldsymbol{u}^2},$$

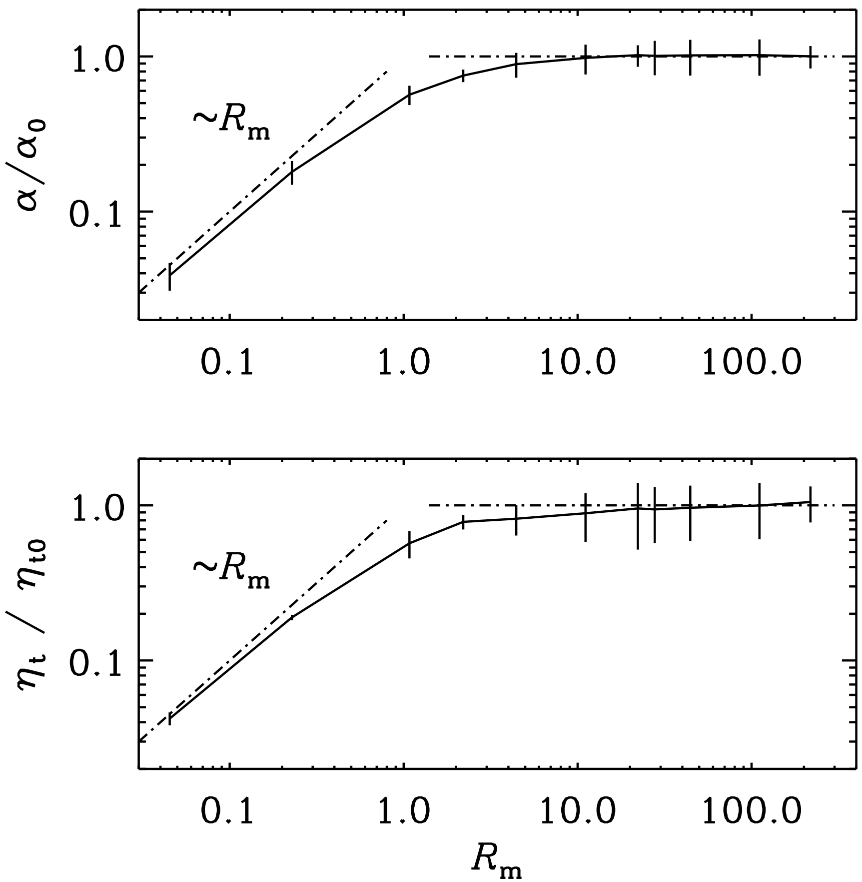
Kinematic α-effect from simulations

Sur, Brandenburg, Subramanian, 2007

Time series for $R_e = 2.2$, $R_m = 220$



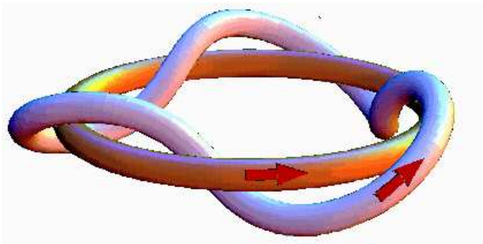
Normalized α, η_t for $R_e = 2.2$



- Measure directly $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ in isotropic turbulence simulations.
- α, η_t as expected. **Independent of R_m,**
  Even in presence of Fluctuation dynamo $\Rightarrow$ Kinematic regime OK?

Magnetic Helicity

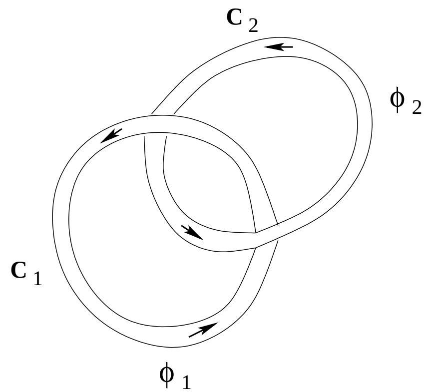
- **Magnetic Helicity** $H = \int_V \mathbf{A} \cdot \mathbf{B}\, dV$, $\quad \nabla \times \boldsymbol{A} = \boldsymbol{B}$
  $\mathbf{A}$ is vector potential, V is **closed** volume
- **Measures links and twists in** B



- H **invariant under gauge transformation for closed fields**

$$H' = \int_V \boldsymbol{A}' \cdot \boldsymbol{B}'\, \mathrm{d}V = H + \int_V \nabla\Lambda \cdot \boldsymbol{B}\, \mathrm{d}V = H + \oint_{\partial V} \Lambda \boldsymbol{B} \cdot \hat{\boldsymbol{n}} \mathrm{d}S = H,$$

- $H = \Phi_1 \oint_{C_1} \boldsymbol{A} \cdot \mathrm{dl} + \Phi_2 \oint_{C_2} \boldsymbol{A} \cdot \mathrm{dl}, = 2\Phi_1\Phi_2$



Magnetic helicity evolution

- **Using Faraday's law and** $(\partial \boldsymbol{A}/\partial t) = -c(\boldsymbol{E} + \nabla\phi)$

$$\frac{1}{c}\frac{\partial}{\partial t}(\boldsymbol{A}\cdot\boldsymbol{B}) = (-\boldsymbol{E} - \nabla\phi)\cdot\boldsymbol{B} + \boldsymbol{A}\cdot(-\nabla\times\boldsymbol{E})$$
$$= -2\boldsymbol{E}\cdot\boldsymbol{B} + \nabla\cdot(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B}).$$

- **Use Ohm's Law**: $\boldsymbol{E} = -(\boldsymbol{U}\times\boldsymbol{B})/c + (4\pi/c^2)\eta\boldsymbol{J}$

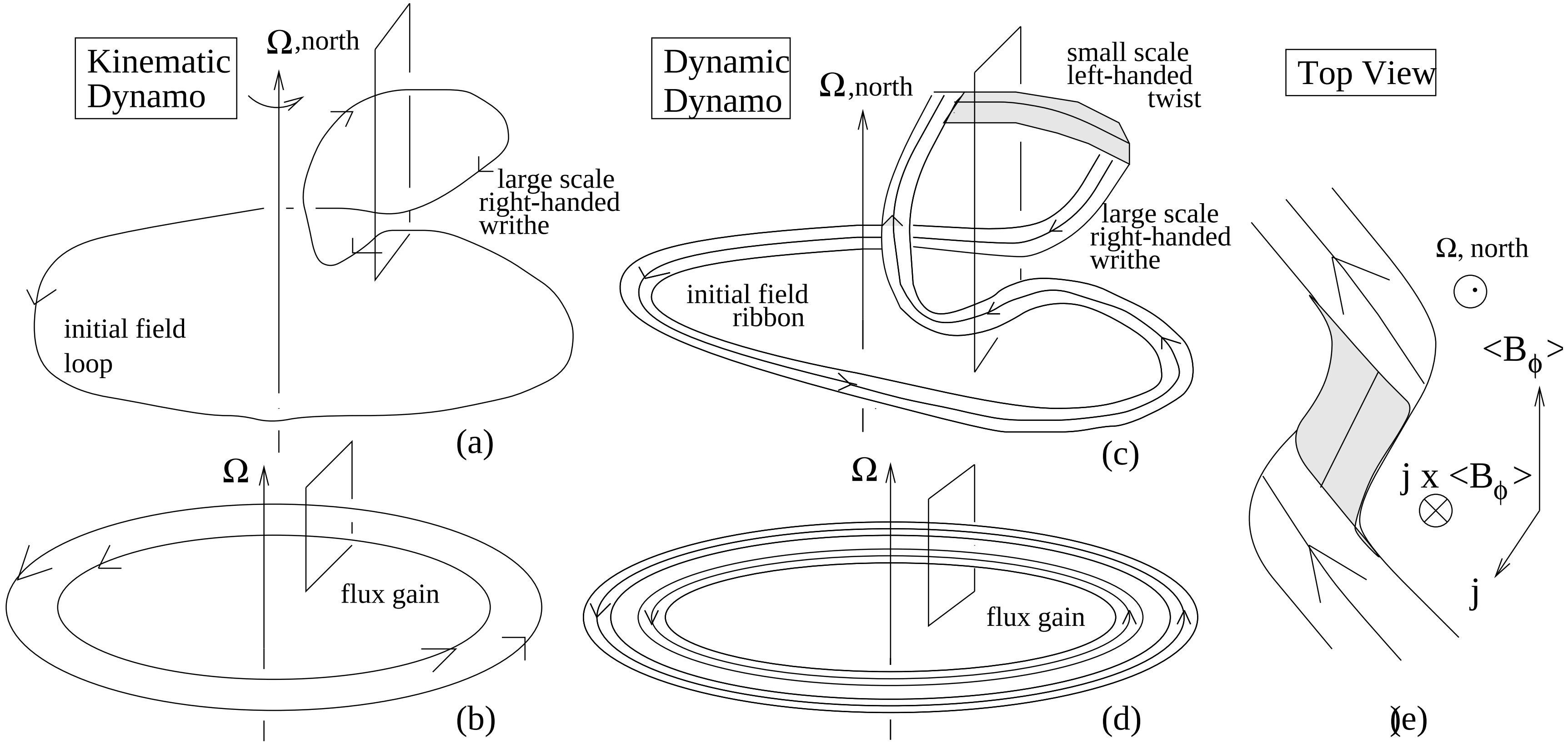
$$\frac{\mathrm{d}H}{\mathrm{d}t} = -2\int_V \boldsymbol{E}\cdot\boldsymbol{B}\mathrm{d}V + \oint_{\partial V}(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B})\cdot\hat{\boldsymbol{n}}\mathrm{d}S$$
$$= -2\eta\int_V (\frac{4\pi}{c})\boldsymbol{J}\cdot\boldsymbol{B}\,\mathrm{d}V$$

Helicity Conservation

- **Helicity evolution** $$\frac{dH}{dt} = -2\eta \int_V dV \, \frac{4\pi}{c}\mathbf{J}\cdot\mathbf{B}.$$
- **For energy** $$\frac{dE_B}{dt} = -\eta \int_V dV \, \frac{4\pi}{c^2}\mathbf{J}^2 - \int_V dV \, \mathbf{U}\cdot\frac{(\mathbf{J}\times\mathbf{B})}{c}$$
- **As $\eta \to 0$, $dE_B/dt \to$ constant with $|\mathbf{J}| \propto \eta^{-1/2}$, $B \propto \eta^0$.**
- **BUT $dH/dt \to 0$!**
- **Helicity is nearly conserved even when energy dissipated**

How does the galactic mean field helicity arise?

Helicity conservation and α-effect



Blackman,Brandenburg 2003

- $\overline{\mathcal{E}}$ **transfers helicity: Oppositely signed WRITHE AND TWIST Helicities**
- **Lorentz force of small-scale twist Helicity grows to cancel kinetic** α

IUCAA

IUCAA Linking number is

$$(\mathbf{4} \times \mathbf{1}) + (\mathbf{4} \times -\mathbf{1}) = \mathbf{0}!!$$

Helicity and catastrophic α quenching?

- $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ **transfers helicity between $\overline{\mathrm{B}}$ and b fields**
- **Now ohms law for mean field is:** $\overline{\boldsymbol{E}} = \overline{\boldsymbol{J}}/\sigma - (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}})/c - \overline{\boldsymbol{\mathcal{E}}}/c$

$$\frac{d}{dt}\langle \overline{\mathbf{A}} \cdot \overline{\mathbf{B}} \rangle = -2\langle \overline{\boldsymbol{E}} \cdot \overline{\boldsymbol{B}} \rangle = 2\langle \overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} \rangle - 2\eta \frac{4\pi}{c} \langle \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} \rangle$$

- **Subtracting from total helicity equation**

$$\frac{d}{dt}\langle \mathbf{a} \cdot \mathbf{b} \rangle = -2\langle \overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} \rangle - 2\eta \frac{4\pi}{c} \langle \mathbf{j} \cdot \mathbf{b} \rangle$$

- **Stationary limit**: $\langle \overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} \rangle = -2\eta \frac{4\pi}{c} \langle \mathbf{j} \cdot \mathbf{b} \rangle \to 0$ as $\eta \to 0$
- **Catastrophic R_{M} dependent quenching** of $\overline{\mathcal{E}}$??

Large scale dynamos need helicity fluxes

The turbulent $\overline{\mathcal{E}}$

- **Need to find $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ under influence of Lorentz forces**

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{u} \times \overline{\boldsymbol{B}} - \eta \nabla \times \boldsymbol{b}) + \boldsymbol{G}$$

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} &= -\frac{1}{\rho}\boldsymbol{\nabla}\left(p + \frac{1}{\mu_0}\overline{\boldsymbol{B}} \cdot \boldsymbol{b}\right) + \nu \boldsymbol{\nabla}^2 \boldsymbol{u} \\ &+ \frac{1}{\rho}\left[(\overline{\boldsymbol{B}} \cdot \boldsymbol{\nabla})\boldsymbol{b} + (\boldsymbol{b} \cdot \boldsymbol{\nabla})\overline{\boldsymbol{B}}\right] + \mathbf{f} + \mathrm{T}. \end{aligned} \quad (1)$$

- **Here G and T are the "pain in neck" nonlinear terms in u and b.**

$$\boldsymbol{G} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b})' = \nabla \times [\boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}]$$

$$\mathrm{T} = -(\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u})' - \frac{1}{\mu_0 \rho}\left[(\boldsymbol{b} \cdot \boldsymbol{\nabla} \boldsymbol{b})' - \tfrac{1}{2}\boldsymbol{\nabla}(\boldsymbol{b}^2)'\right]$$

- **Calculating $\overline{\mathcal{E}}$ requires a closure even in the kinematic limit**

The "Minimal τ approximation" closure

- **Closure for the triple correlations arising in $\overline{\mathcal{E}}$**
  (Blackman, Field, 2002; Radler, Kleeorin, Rogachevski, 2003; KS/AB 2005)

$$\frac{\partial \overline{\boldsymbol{\mathcal{E}}}}{\partial t} = \overline{\boldsymbol{u} \times \dot{\boldsymbol{b}}} + \overline{\dot{\boldsymbol{u}} \times \boldsymbol{b}}, = \overline{\boldsymbol{u} \times [-\boldsymbol{u} \cdot \nabla \overline{\boldsymbol{B}} + \overline{\boldsymbol{B}} \cdot \nabla \boldsymbol{u} + \boldsymbol{G}]}$$
$$+ \overline{[\frac{\boldsymbol{b} \cdot \nabla \overline{\boldsymbol{B}} - \boldsymbol{\nabla} p + \overline{\boldsymbol{B}} \cdot \nabla \boldsymbol{b}}{\rho} + \mathbf{f} + \mathrm{T}] \times \boldsymbol{b}}$$
$$= \tilde{\alpha}\, \overline{\boldsymbol{B}} - \tilde{\eta}_{\mathrm{t}} \, (\nabla \times \overline{\boldsymbol{B}}) + 0 + \mathbf{N}$$

- $\tilde{\alpha} = -\frac{1}{3} \left(\overline{\boldsymbol{\omega} \cdot \boldsymbol{u}} - (4\pi / c\rho) \overline{\boldsymbol{j} \cdot \boldsymbol{b}} \right) \qquad \tilde{\eta}_{\mathrm{t}} = \frac{1}{3} \overline{\boldsymbol{u}^2}.$

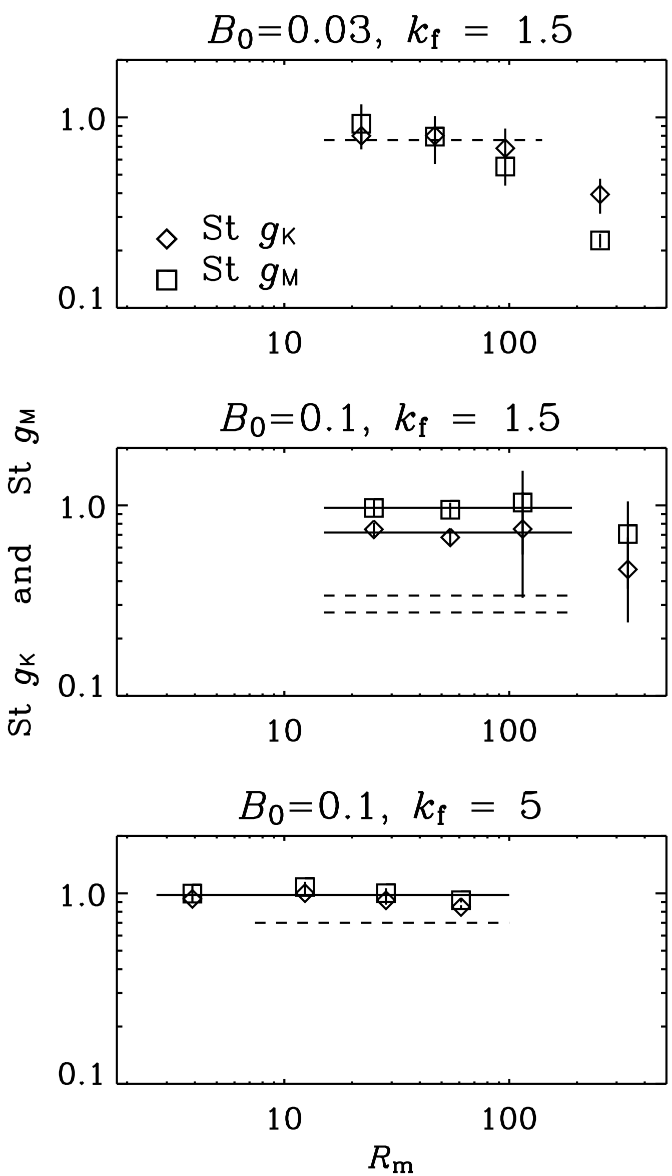
- Closure hypothesis; triple correlations provide damping proportional to $\overline{\mathcal{E}}$ over a relaxation time τ: $\quad \mathbf{N} = -\overline{\mathcal{E}}/\tau$

- **In steady state** $\overline{\mathcal{E}} = \tau \tilde{\alpha}\, \overline{\boldsymbol{B}} - \tau \tilde{\eta}_{\mathrm{t}} \, (\nabla \times \overline{\boldsymbol{B}})$

α-effect gets renormalized by a term proportional to $j \cdot b$
(Also in Pouquet et al., 75 EDQNM closure)

Testing MTA in a box

Brandenburg, KS, 2005, 2007



- $\alpha = \tau(g_K\tilde{\alpha}_K + g_M\tilde{\alpha}_M)$
- $St = \tau u_{rms} k_f$
- τ is positive and ~ 1 in natural units!

Nonlinear saturation of helical dynamos

- $\overline{\mathcal{E}}$ transfers helicity between small-large scales
- Small scale current helicity grows to cancel kinetic α
- Nonlinear $\alpha = -(\tau/3)\langle \mathbf{u}\cdot\omega\rangle + (\tau/3\rho)(4\pi/c)\langle \mathbf{j}\cdot\mathbf{b}\rangle \to 0?$
- CATASTROPHIC QUENCHING OF DYNAMO?
- Need to get rid of small scale helicity, by Helicity fluxes?
  (Blackman & Field; Kleeorin et al).
  But what is gauge invariant helicity density and flux?
- Small scale helicity density h is density of correlated b field links (Subramanian & Brandenburg, ApJ Lett., 2006)

$$\partial h/\partial t + \nabla\cdot\mathbf{F} = -2\overline{\mathcal{E}}\cdot\overline{\mathbf{B}} - 2\eta(4\pi/c)\overline{\mathbf{j}\cdot\mathbf{b}}$$

Large scale dynamos need helicity fluxes

A gauge-invariant helicity density

- **Gauss's linking formula for magnetic helicity**

$$h_{\rm G} = \frac{1}{4\pi}\int\int \boldsymbol{b}(\boldsymbol{x})\cdot\left[\boldsymbol{b}(\boldsymbol{y})\times\frac{\boldsymbol{x}-\boldsymbol{y}}{|\boldsymbol{x}-\boldsymbol{y}|^3}\right]\mathrm{d}^3x\,\mathrm{d}^3y = \int \boldsymbol{a}\cdot\boldsymbol{b}\,\mathrm{d}^3x$$

- **Here:** $\boldsymbol{a} = (1/4\pi)\int \boldsymbol{b}(\boldsymbol{y})\times(\boldsymbol{x}-\boldsymbol{y})/(|\boldsymbol{x}-\boldsymbol{y}|^3)\,\mathrm{d}^3y$
- $\boldsymbol{a}$ **vector potential in "Coulmb gauge"**: $\boldsymbol{\nabla}\times\boldsymbol{a} = \boldsymbol{b}$, and $\boldsymbol{\nabla}\cdot\boldsymbol{a} = 0$.
- Let $\overline{b_i(\boldsymbol{x},t)b_j(\boldsymbol{y},t)} = M_{ij}(\boldsymbol{r},\boldsymbol{R})$, with $\boldsymbol{r} = \boldsymbol{x}-\boldsymbol{y}$, $\boldsymbol{R} = (\boldsymbol{x}+\boldsymbol{y})/2$
- Correlation scale $l \ll L \ll R_s$ the system scale (Formally let $L\to\infty$)
- **The ensemble average helicity is:** $\overline{h}_{\rm G} = \int \mathrm{d}^3R\, h(\boldsymbol{R})$

$$h(\boldsymbol{R}) = \frac{1}{4\pi}\int_{L^3}\mathrm{d}^3r\;\epsilon_{ijk}M_{ij}(\boldsymbol{r},\boldsymbol{R})\,\frac{r_k}{r^3},$$

$h(\boldsymbol{R})$: **Gauge invariant helicity density of the random small scale field b.**

A local helicity conservation equation

- **The helicity density conservation equation** :
$$\partial h/\partial t + \nabla \cdot \mathbf{F} = -2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} - 2\eta(4\pi/c)\overline{\mathbf{j} \cdot \mathbf{b}}$$
- **Helicity flux:** $F_i = F_i^{\rm VC} + F_i^{\rm A} + F_i^{\rm bulk} + F_i^{\rm triple}$.
- **Now in stationary limit;** $\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} = -\frac{1}{2}\boldsymbol{\nabla} \cdot \mathbf{F} - \eta\overline{\boldsymbol{j} \cdot \boldsymbol{b}}$
NO catastrophic quenching in presence of helicity flux $\mathbf{F}$**!**
- **Simplest flux due to advection**: $\mathbf{F}^{\rm bulk} = h\overline{\boldsymbol{U}}$
- **Other fluxes driven by** $u - b$ **correlations and anisotropy**:
$F_i^{\rm VC} = 2\epsilon_{qlm}\overline{B}_l(\boldsymbol{R}) \int {\rm i}k_q\chi_{mi}k^{-2}{\rm d}^3k$: **(Vishniac and Cho, 2001)**
$F_i^{\rm A} = -\epsilon_{qlm}\overline{B}_l(\boldsymbol{R}) \int {\rm i}k_i\chi_{mq}k^{-2}{\rm d}^3k$: **(Kleeorin et al 2000)** .
- **Here** $\chi_{jk}(\boldsymbol{k},\boldsymbol{R}) = \int \overline{\hat{u_j}(\boldsymbol{k}+\frac{1}{2}\boldsymbol{K})\hat{b_k}(-\boldsymbol{k}+\frac{1}{2}\boldsymbol{K})}\,{\rm e}^{i\boldsymbol{K}\cdot\boldsymbol{R}}\,{\rm d}^3K$
- **Use helicity conservation equation to derive equation for** $\boldsymbol{j} \cdot \boldsymbol{b}$ **part of** α**-effect**

Dynamical quenching model for MFD

- **Solve dynamo equation with local helicity conservation**

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \alpha \overline{\boldsymbol{B}} - (\eta + \eta_t)(\nabla \times \overline{\boldsymbol{B}})\right), \quad \alpha = (\alpha_{\rm K} + \alpha_{\rm m})$$

- $\alpha_{\rm m} = (\tau/3\rho)(4\pi/c)\langle \mathbf{j} \cdot \mathbf{b} \rangle \simeq (\tau/3\rho) k_0^2\, h$
- **Helicity conservation with flux becomes**, $R_m = \eta_t/\eta$, $B_{\rm eq}^2 = \rho \overline{\boldsymbol{u}^2}$

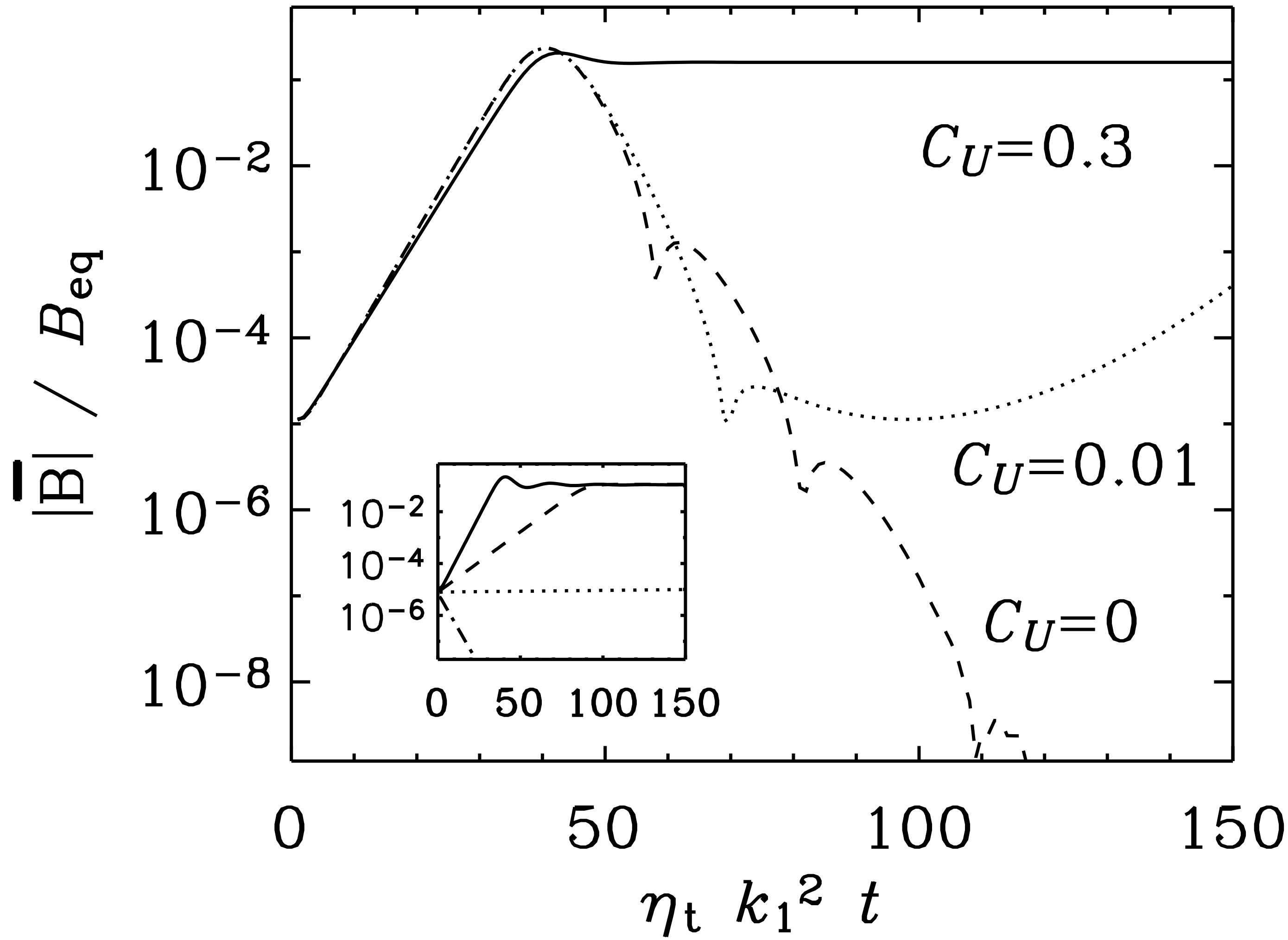
$$\frac{\partial \alpha_{\rm m}}{\partial t} = -2\eta_{\rm t} k_0^2 \left(\frac{(\alpha_{\rm K} + \alpha_{\rm m})\overline{\boldsymbol{B}}^2 - \eta_t (\nabla \times \overline{\boldsymbol{B}}) \cdot \overline{\boldsymbol{B}} + \frac{1}{2} \boldsymbol{\nabla} \cdot \mathbf{F}}{B_{\rm eq}^2} + \frac{\alpha_{\rm m}}{R_m} \right)$$

- **For** $\dot{\alpha}_{\rm m} = 0$, $\mathbf{F} = 0$, **"old algebraic quenching"**

$$\alpha = \frac{\alpha_{\rm K} + \eta_t R_m \nabla \times \overline{\boldsymbol{B}} \cdot \overline{\boldsymbol{B}}}{1 + R_m \overline{\boldsymbol{B}}^2 / B_{\rm eq}^2}$$

Effect of advective helicity flux

Shukurov, Sokoloff, Subramanian, Brandenburg, AA Lett., 2006



Mean field dynamos work with helicity fluxes?

Questions

- When do the first fields arise?
- How do they evolve with redshift in galaxies and the IGM?
- Do ellipticals host coherent fields?
- When is the IGM significantly polluted with magnetic fields?
- Dynamos required to amplify/maintain fields.
- Fluctuation dynamo saturation?
- For mean field dynamos: α, η_t at large R_m?
- How do MFD's saturate; helicity fluxes?
- Is an Early universe field needed? Is it inevitable?

SKA will be crucial to probe the Magnetic Universe.