Numerical simulations of MRI in the shearing box aproximation

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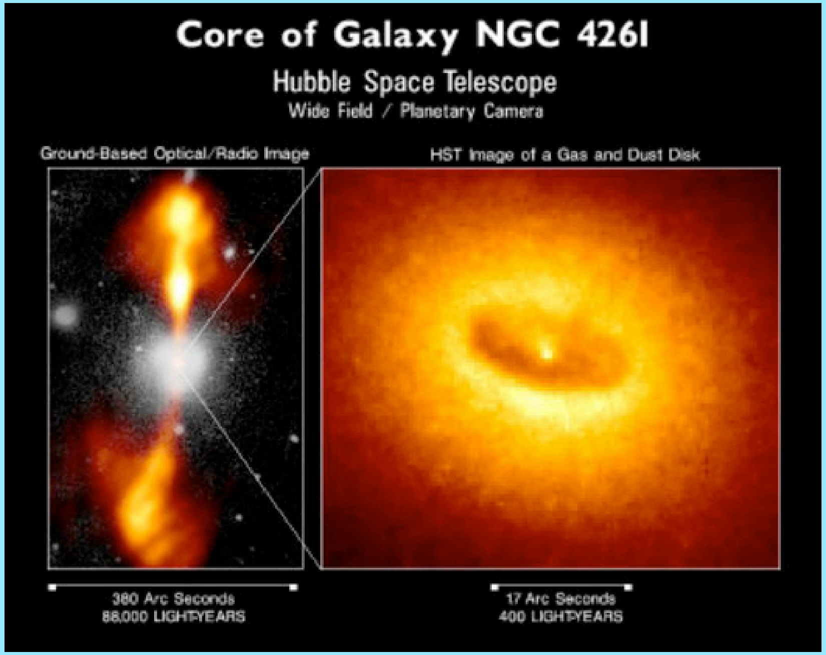
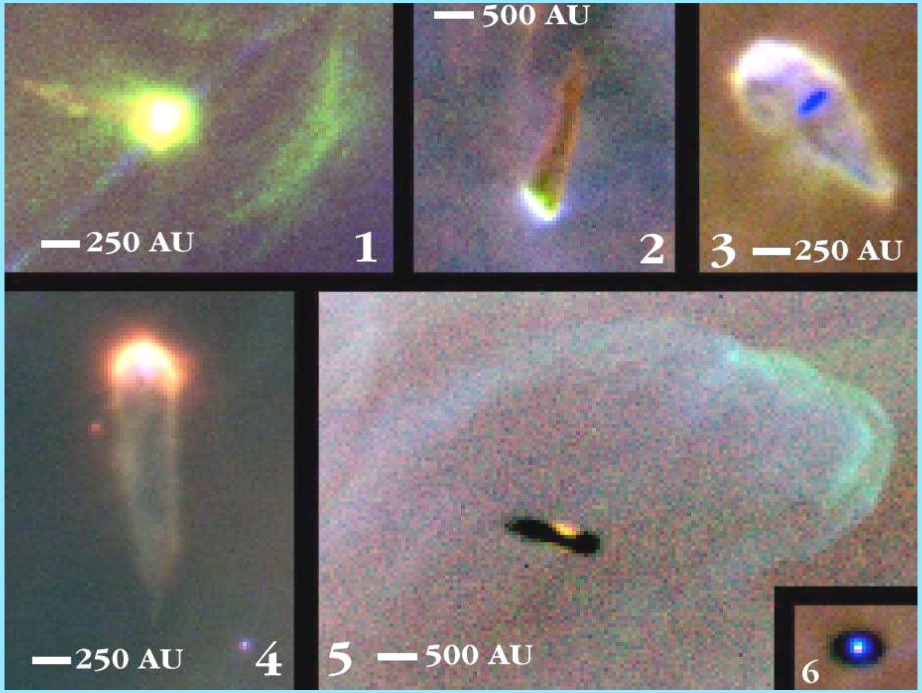
Accretion flows are ubiquitous in astrophysics:
they occur around protostars,
accreting compact objects in binary systems,
supermassive black holes at the cores of galaxies.
Accretion flows are mainly in the form of accretion disk

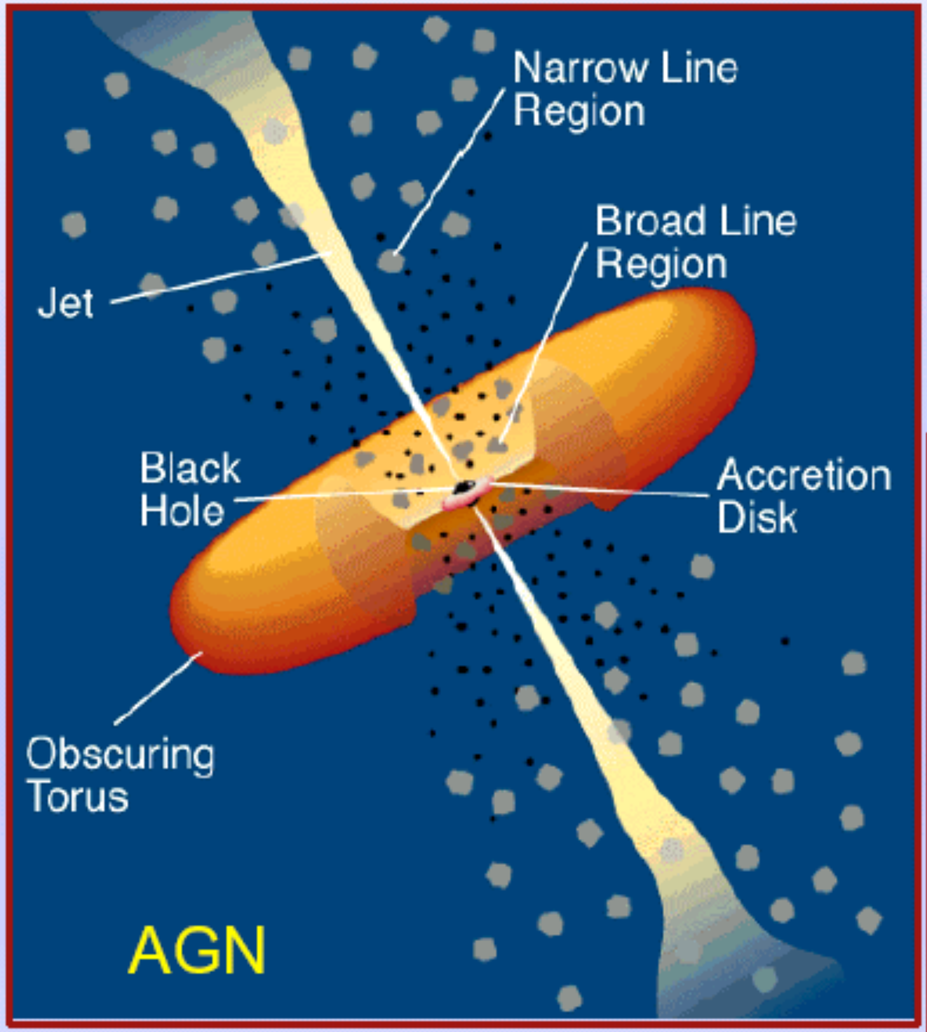
AGN

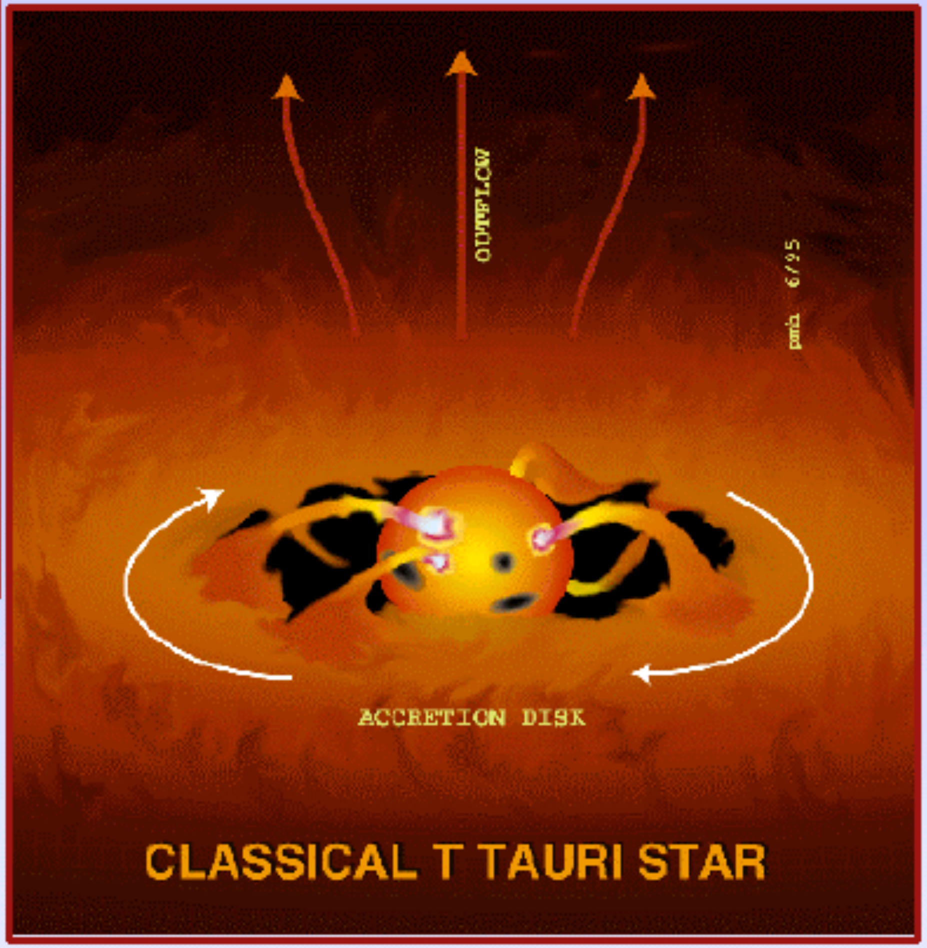




YSO

The jet/disk paradigm





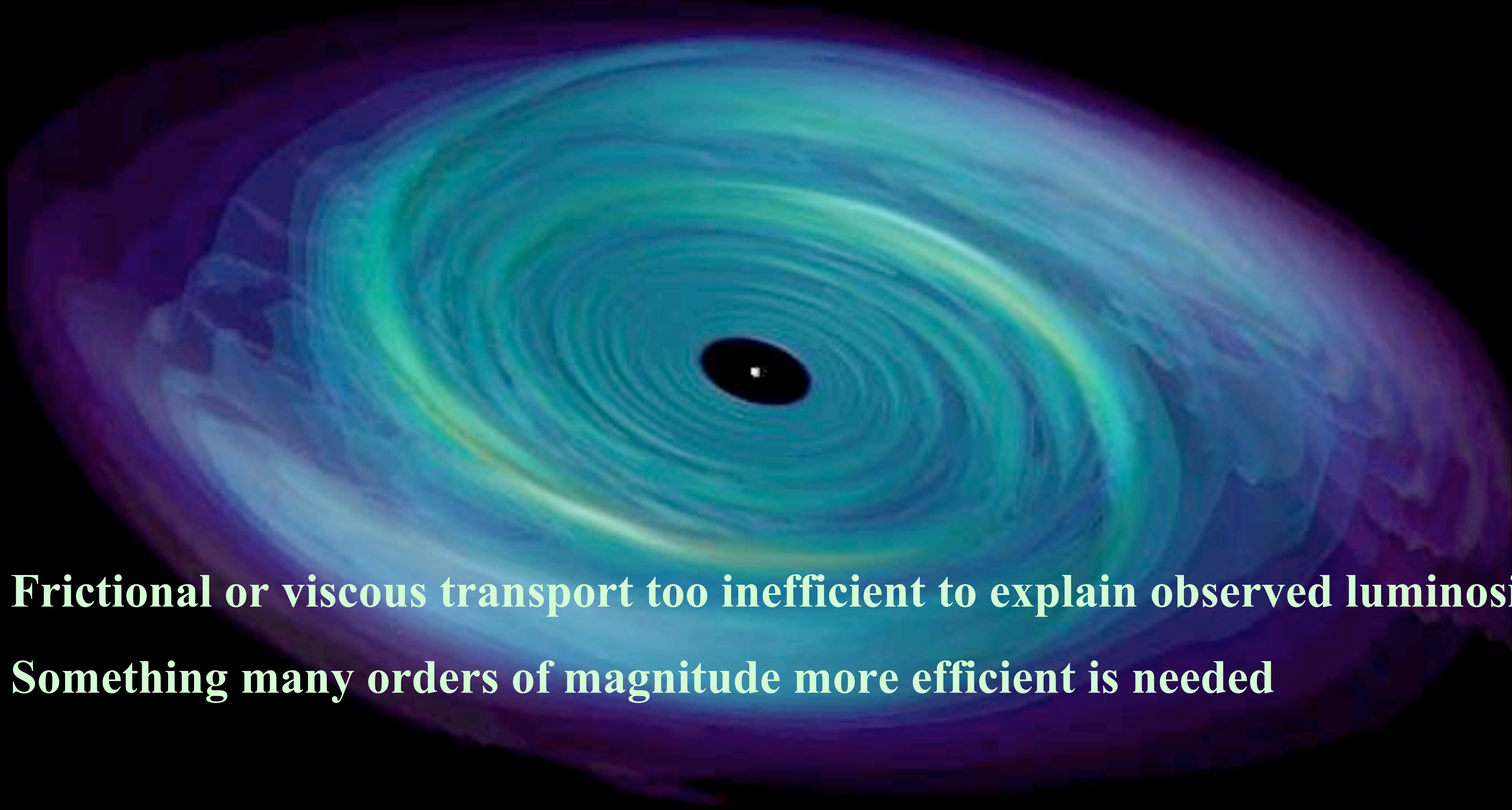
Central black hole or star

Subsonic/supersonic inflow

Supersonic and relativistic outflow

Angular momentum transport

- If angular momentum is conserved matter just orbits the central object
- Accretion rate is determined by the <u>outward</u> transport of angular momentum



- Frictional or viscous transport too inefficient to explain observed luminosities
- Something many orders of magnitude more efficient is needed

Turbulent transport

Shakura & Sunyaev (1973) assumed transport was due to turbulence in the disc. For reasonable transport rates assumption gave

Reasonable disc structures

Reasonable accretion rates

In a turbulent flow effective diffusivity

$$D = UL = \nu\left(\frac{UL}{\nu}\right) = \nu Re$$

characteristic length scale

characteristic velocity

$$D = \alpha c_s h$$

c_s sound speed

h disk thickness

What is the physical origin of the turbulence?

Hydrodynamic keplerian disks are linearly stable
Rayleigh criterion
Stability for angular momentum growing outward

Finite amplitude instability? Under debate

Numerical simulation show that turbulence is
not sustained, but Reynolds number of simulations
quite low

Princeton experiment

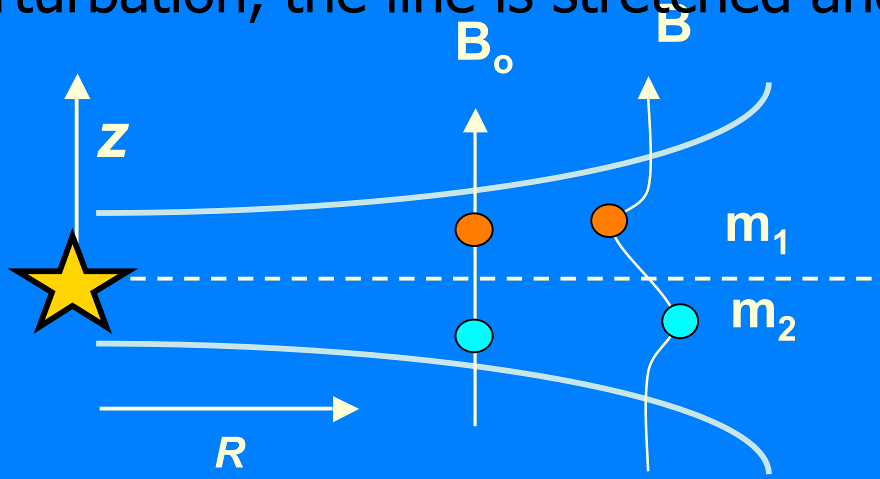
Could be turbulent at high Reynolds numbers
but not efficient in transfering angular momentum
(Lesur & Longaretti 2005)

Magnetic field may change the situation

Magnetorotational instability

Instability first discussed by Velikhov (1959) and Chandrasekhar (1961) and applied in the context of disks by Balbus & Hawley (1991)

- Two fluid elements, are joined by a vertical field line (B_o). The tension in the line is negligible.
- Introducing a perturbation, the line is stretched and develops tension.



- The tension reduces the angular momentum of m_1 and increases that of m_2. This further increases the tension and the process "runs away".

- Stability requirement is

$$(k \cdot v_A)^2 > -\frac{d\Omega^2}{d \ln R}$$

- One can always find a small enough wavenumber k so there will be an instability unless

$$\frac{d\Omega^2}{d \ln R} > 0$$

- Maximum unstable growth rate:

$$|\omega_{max}| = \frac{1}{2}\left|\frac{d\Omega}{d\ln R}\right|$$

- Maximum rate occurs for wavenumbers

$$(k \cdot v_A)^2_{max} = -\left(\frac{1}{4} + \frac{\kappa^2}{16\Omega^2}\right)\frac{d\Omega^2}{d\ln R}$$

- For Keplerian profiles maximum growth rate and wavelengths:

$$|\omega_{max}| = \frac{3}{4}\Omega$$

$$(k \cdot v_A)_{max} = \frac{\sqrt{15}}{4}\Omega$$

Magnetorotational instability (Balbus & Hawley 1991) is thought to be the main process at the base of angular momentum transport in accretion disks.

Simulations are needed for studying the fully developed process.

Local (shearing box) and global (full disk) simulations

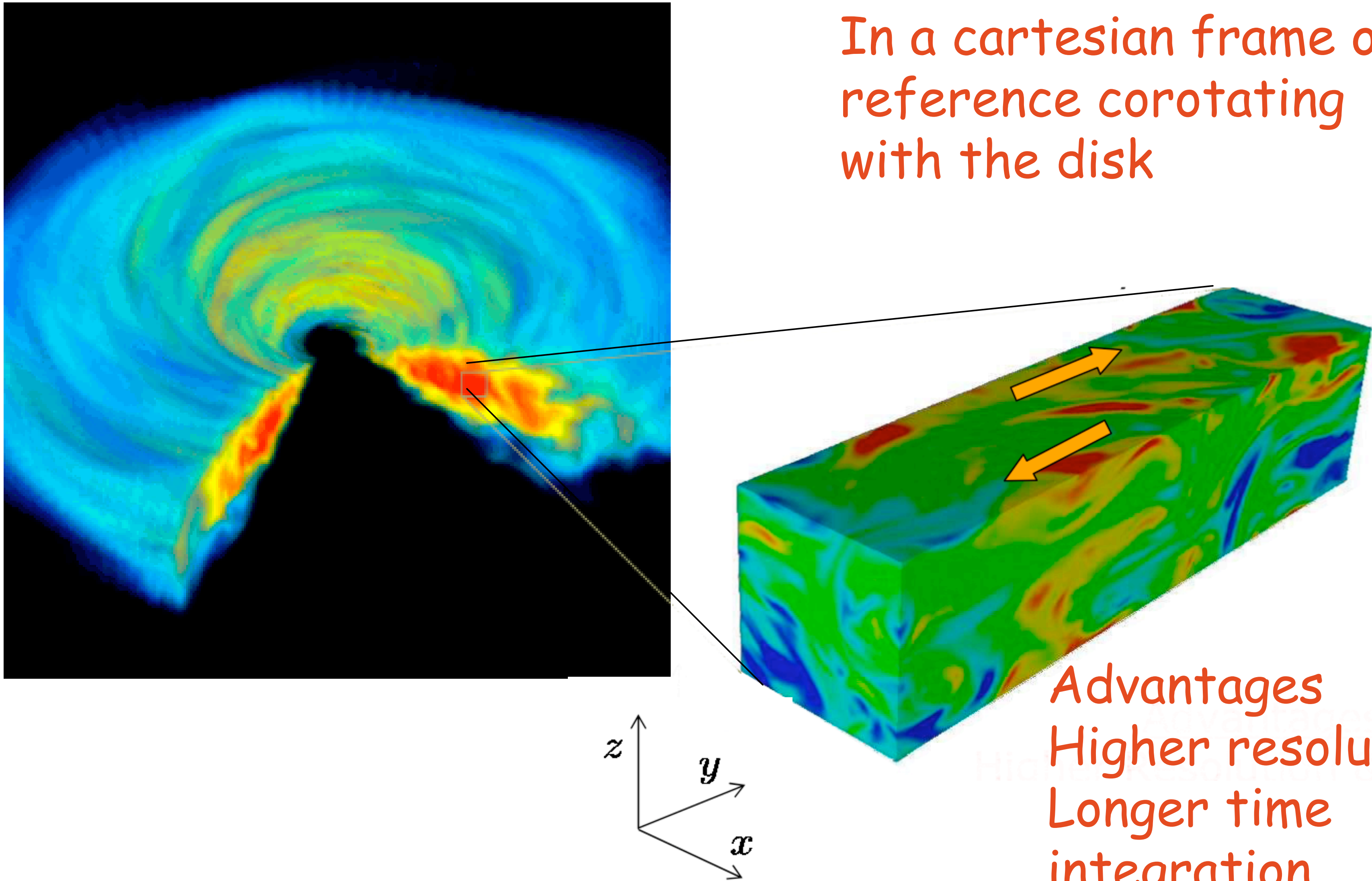
Shearing box simulations: Balbus, Hawley et al, Sano et al, Lesur & Longaretti 2007, Fromang et al 2007

What is the relation between local and global simulations ?

What we learn from local simulations is really meaningful for the full disk ?

Local versus global simulations

In a cartesian frame of reference corotating with the disk



Advantages

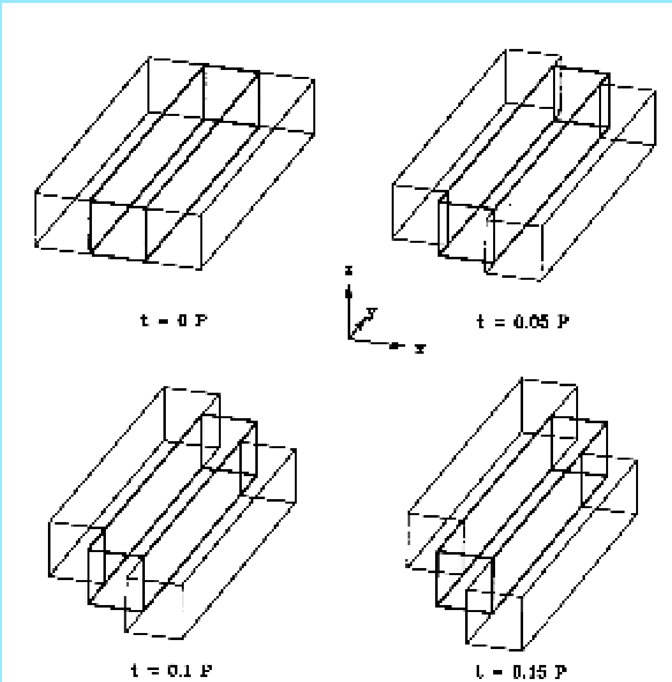
Higher resolution

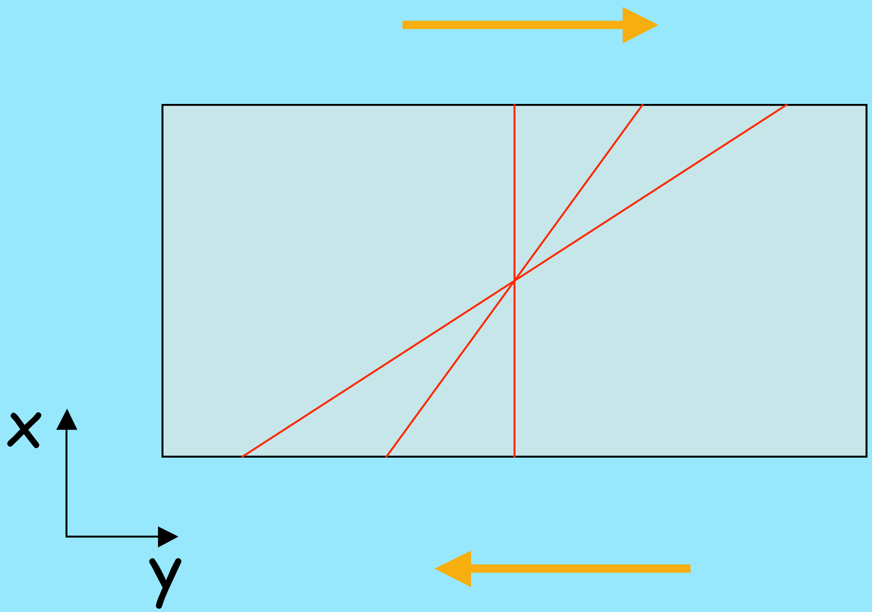
Longer time integration

Shearing Box Simulations are based on a local expansion of the equations of motion.

Simulate a small portion of the disk

Assume that the box is surrounded by identical boxes, strictly periodic at t = 0





Shearing sheet equations

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} + 2\boldsymbol{\Omega}\times\mathbf{v} = \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi\rho} - \frac{1}{\rho}\nabla\left(\frac{\mathbf{B}^2}{8\pi} + \mathbf{p}\right) - \nabla\left(2\mathbf{A}\boldsymbol{\Omega}\mathbf{x}^2 + \frac{1}{2}\cancel{\boldsymbol{\Omega}^2\mathbf{z}^2}\right)$$

Coriolis force

Tidal expansion
Effective potential

Shear rate $A = \frac{R}{2}\left(\frac{\partial\Omega(R)}{\partial R}\right)_{R_0}$

$A = -\frac{3}{4}\Omega$ keplerian

$v_{y0} = -2Ax$ Basic velocity profile

Goodman & Xu (1994) showed the existence of exact exponentially growing solutions of shearing sheet incompressible equations (channel solutions)

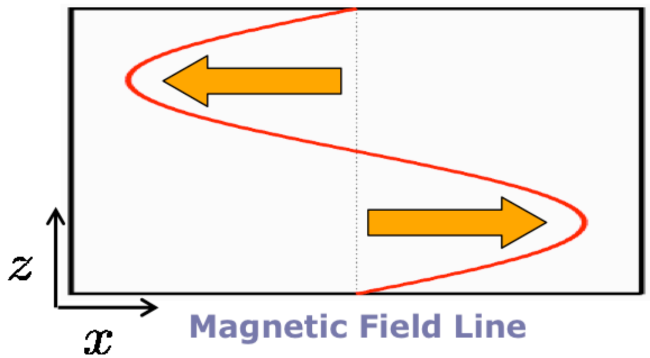
Case with average vertical field

$$(B_x, B_y, B_z) = \epsilon B_0 \exp(st) \cos(Kz)(\sin\gamma, -\cos\gamma, 0) + (0, 0, B_0)$$

$$(v_x, v_y, v_z) = \epsilon v_0 \exp(st) \sin(Kz)(\cos\gamma, \sin\gamma, 0) + (0, 2Ax, 0)$$

γ is the angle between the magnetic field and the y direction

Channel solutions with wavelength equal to the vertical size of the box apeear to be a dominant feature of shearing box MRI simulations (with non zero average vertical field)



Can the channel solution (k=0 in the radial direction) play a role in full disk with proper boundary conditions?

First check how different is the behavior changing the aspect ratio:
radial (x) size / vertical (z) size

Typical value in the simulations aspect ratio = 1
Never investigated other values apart from Winters, Balbus & Hawley 2003 for the case with zero net flux

Compressible, isothermal simulations, no explicit dissipation

PLUTO code (Mignone et al. 2007), for details on the algorithm see the poster

Parameters $\beta = 10^4$

Fastest growing mode has wavelength 1/3 of the vertical size

Short discussion on 2D behavior

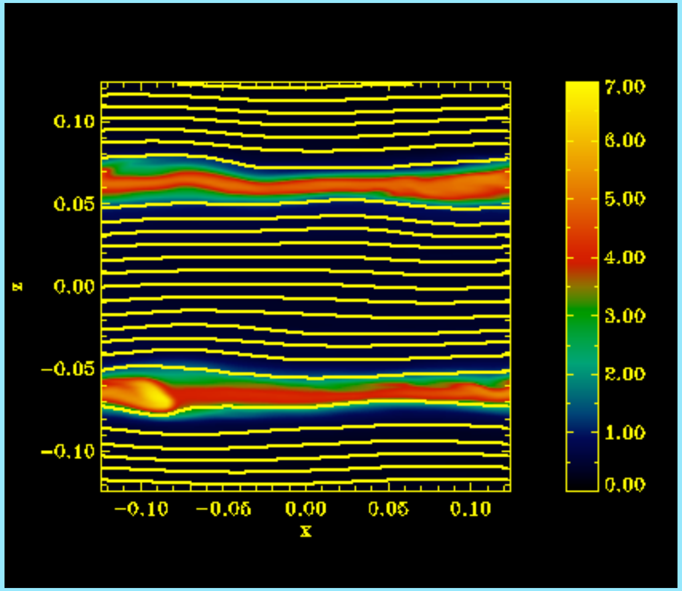
3D results:

Aspect ratio Lx/Lz = 1, 4, 8 32 points vertical

Aspect ratio Lx/Lz = 1, 4 128 points vertical

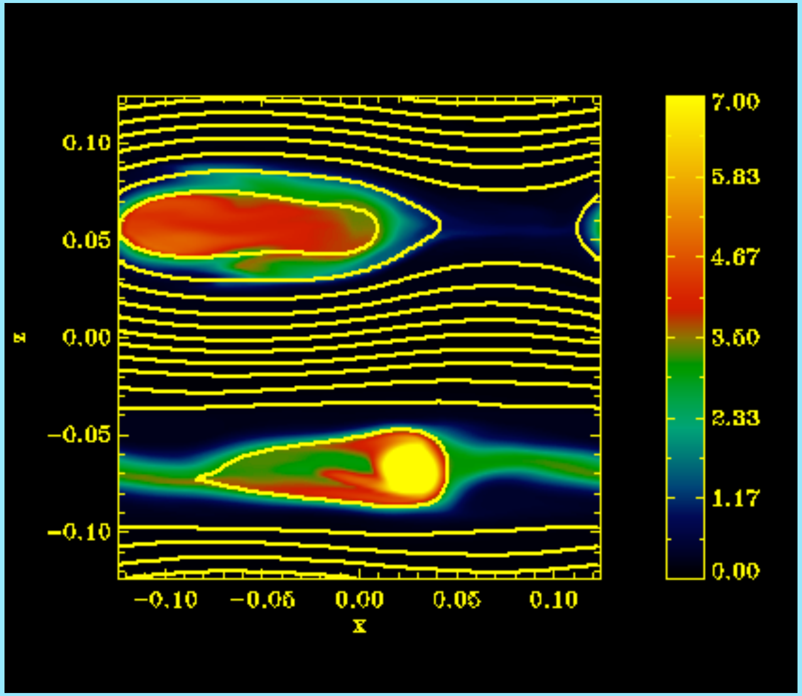
Ly/Lz = 4

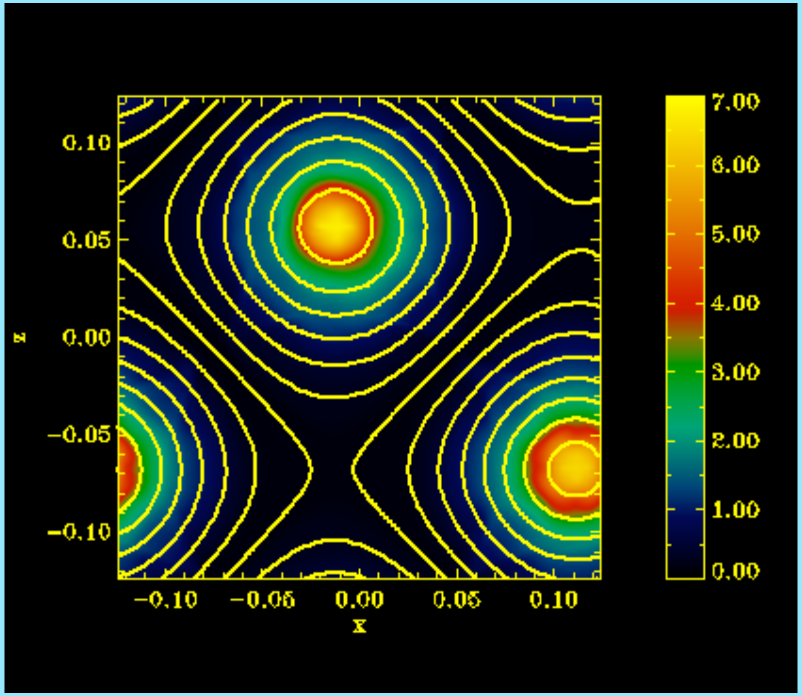
2D Results

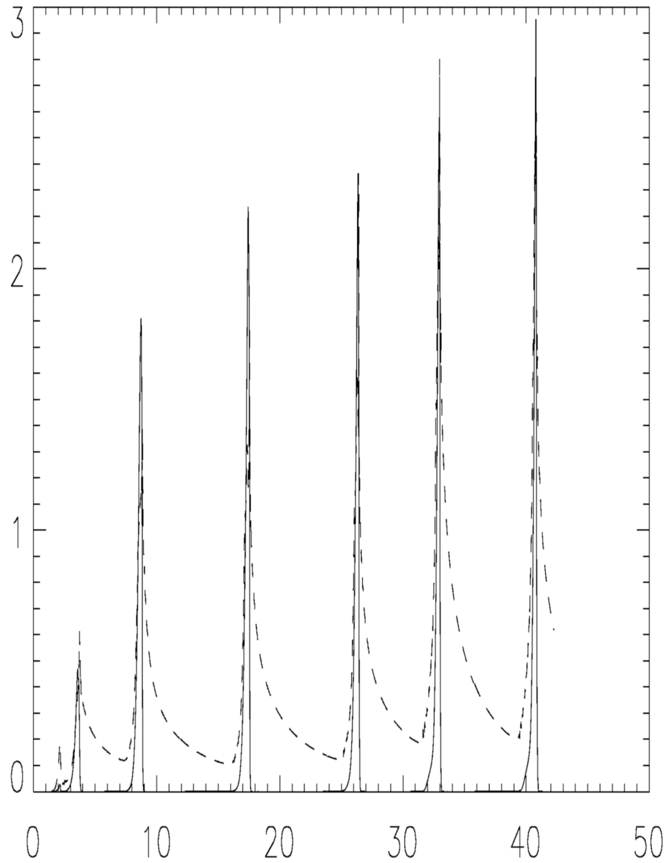


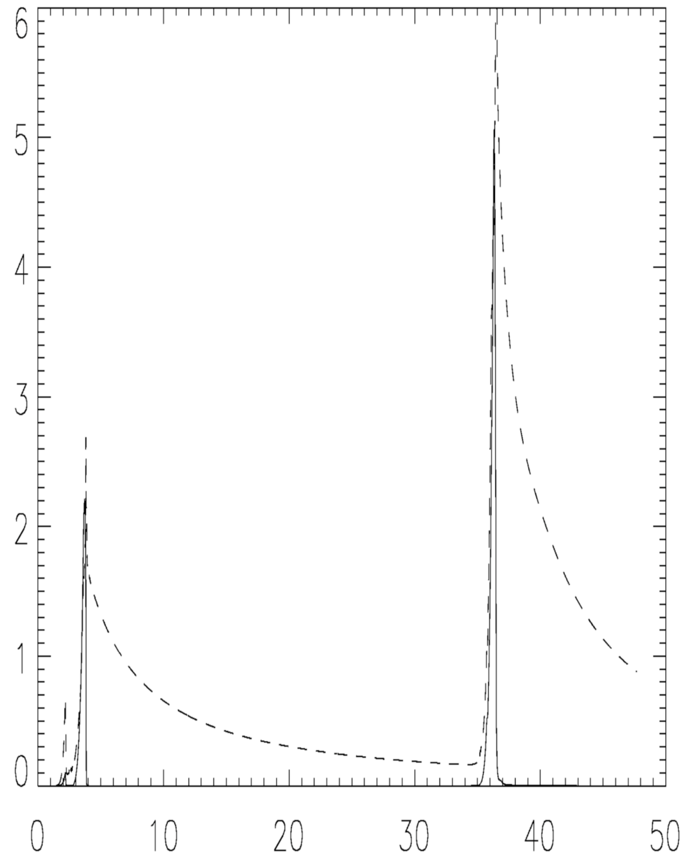
We have only axisymmetric modes

Cyclic behavior: quasi periodic transition between two states



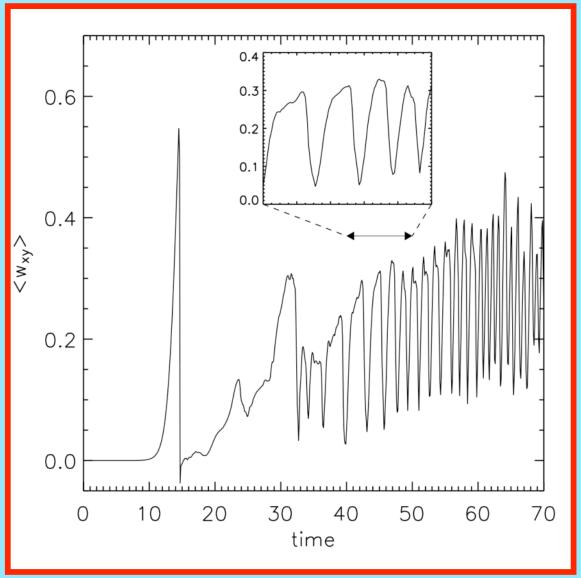


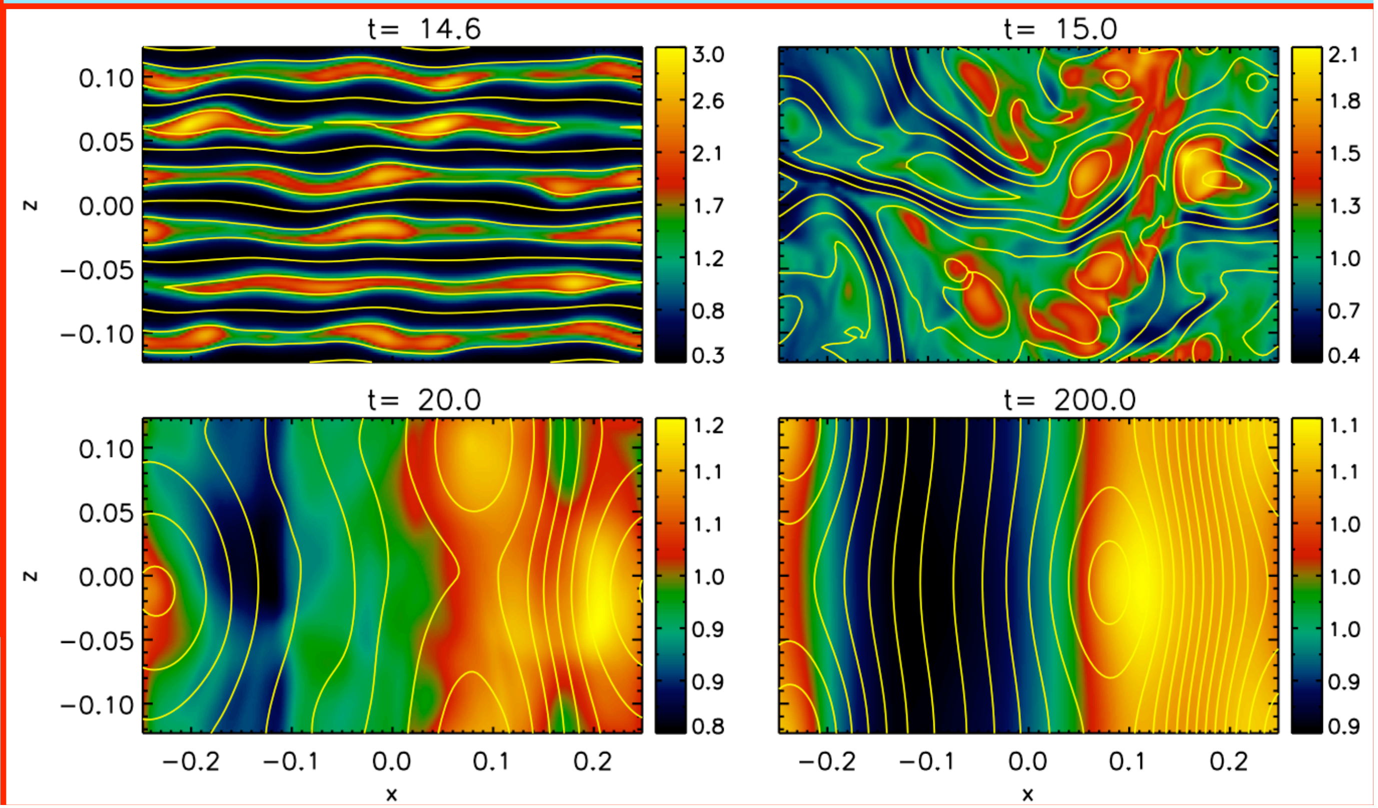


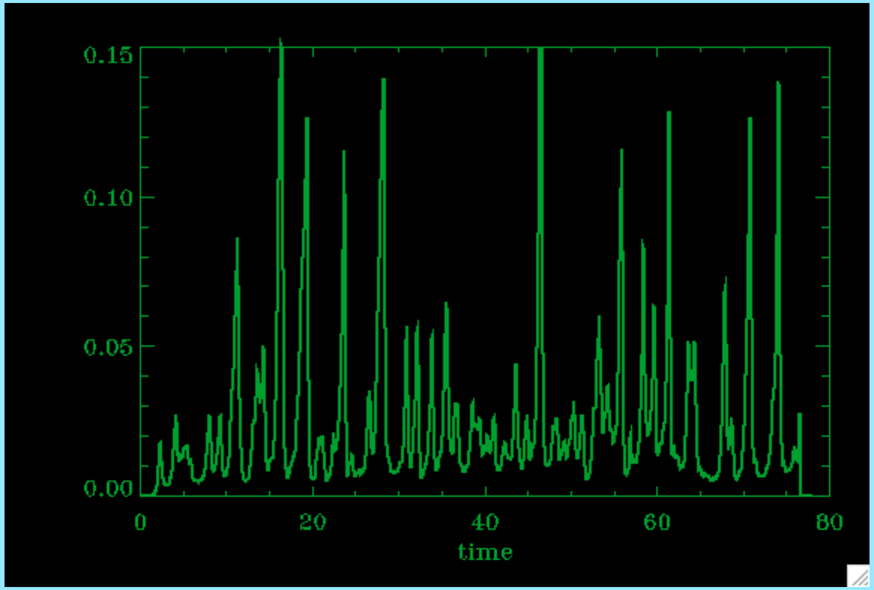


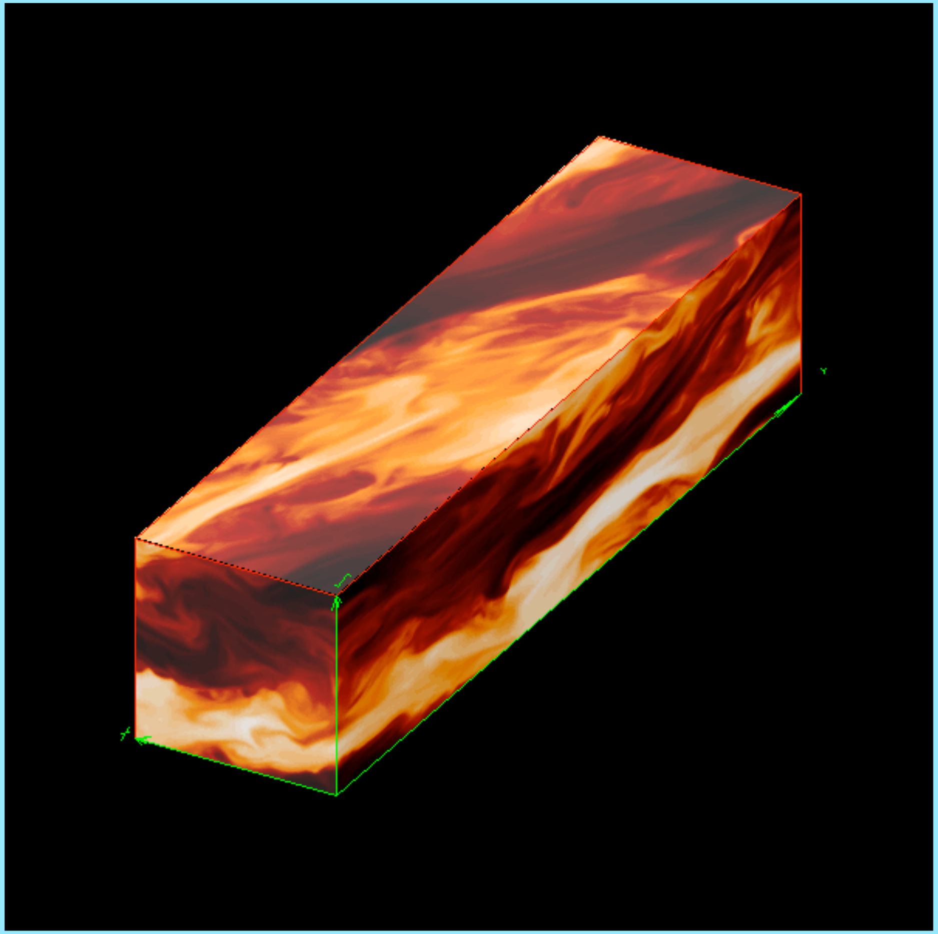
Behavior of average stresses

2D solutions with different aspect ratio show different evolutions







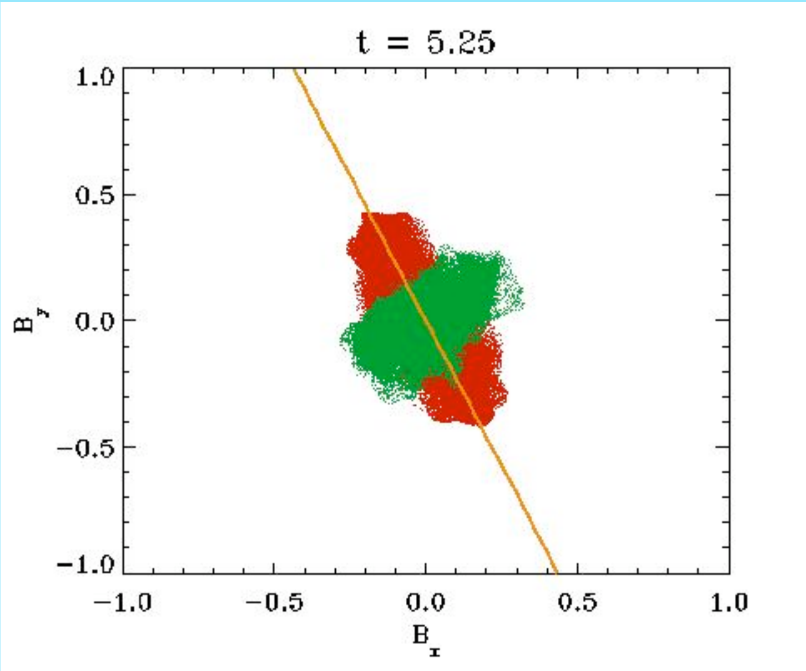


Channel solution
in correspondence
of peaks of
Maxwell stresses

(Sano & Inutsuka 2001)

The first peak in the curve corrspond to a channel Solution with wavelength equal to 1/3 of the z box size (mode ith maximum growth rate). When this is disrupted by secondary instabilities only the 1-channel appears

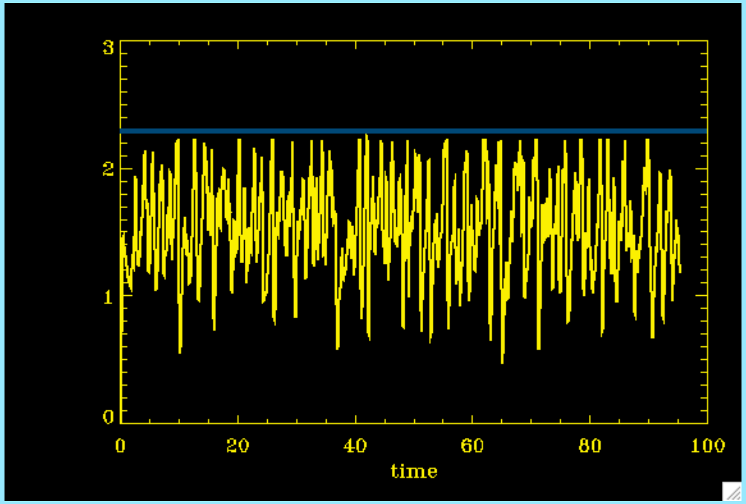
All the peaks (after 1) in the curve of the stresses correspond to a 1-channel solution

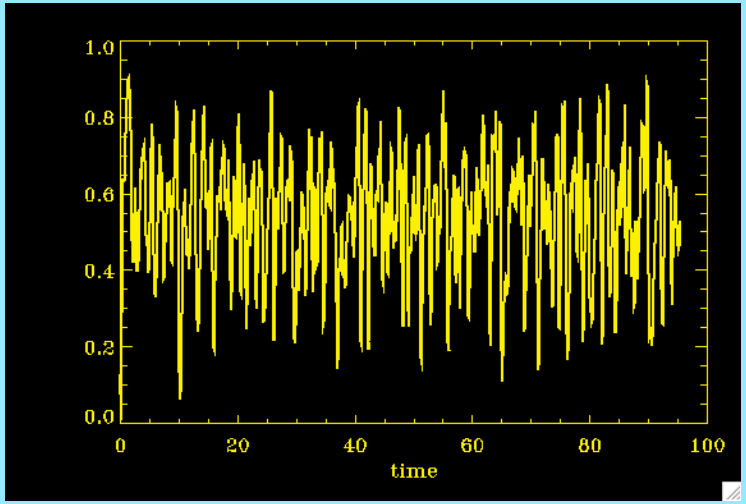


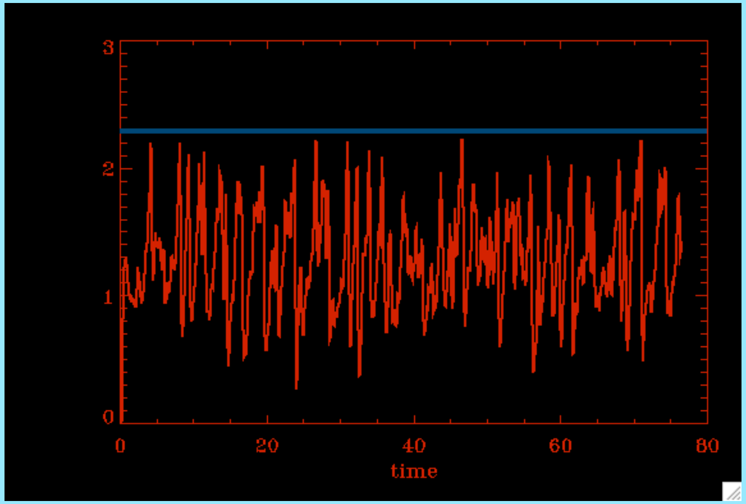
Typical scatter plot of B_y vs B_x for a maximum

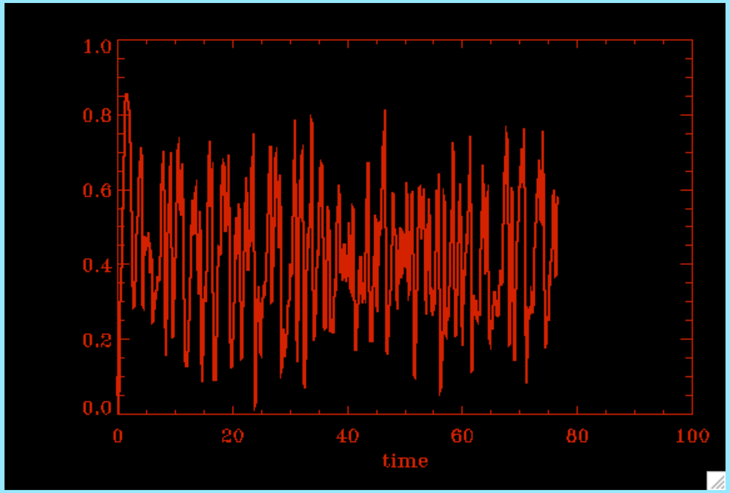
The overplotted line corresponds to the angle of 23° of the analytic solution

We can compute the slope of a linear fit and the corresponding correlation coefficient



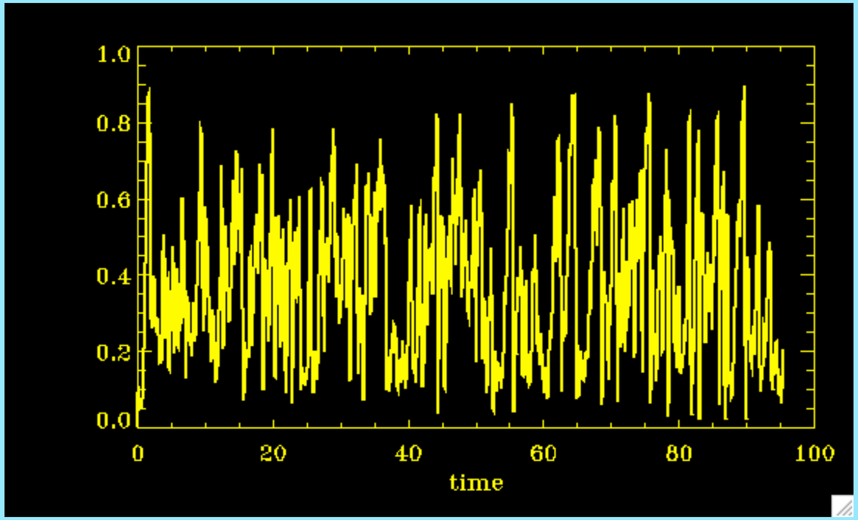




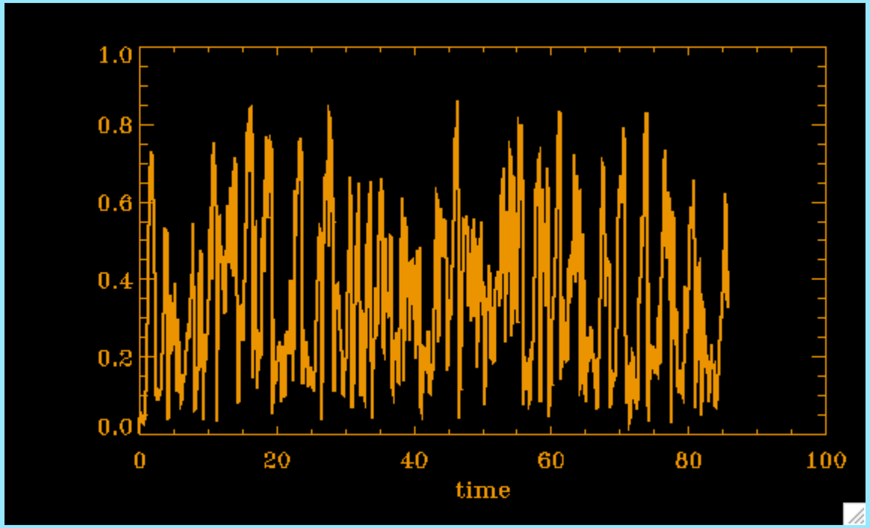


Lesur & Longaretti 2007 propose to use the correlation length in y and z direction to measure importance of channel solution

Correlation length in y direction



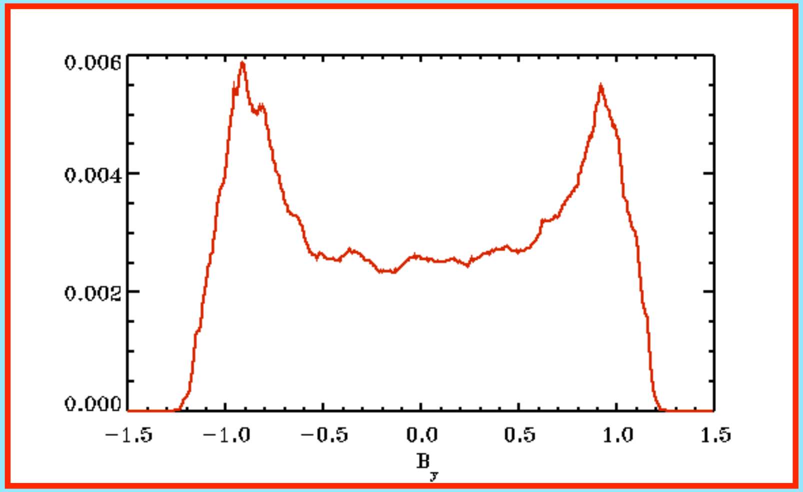
Low resolution



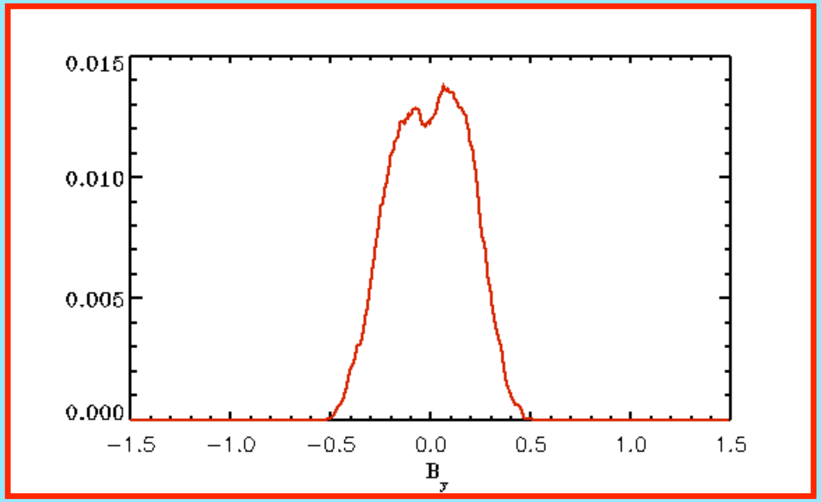
High resolution

If we look at the pdf of the intensity of B_y we have almost gaussian distributions in correspondence of the minima and very distorted distributions in correspondence of the maxima.
The distorted distributions can be connected with the Dominance of a coherent state (1-channel solution)

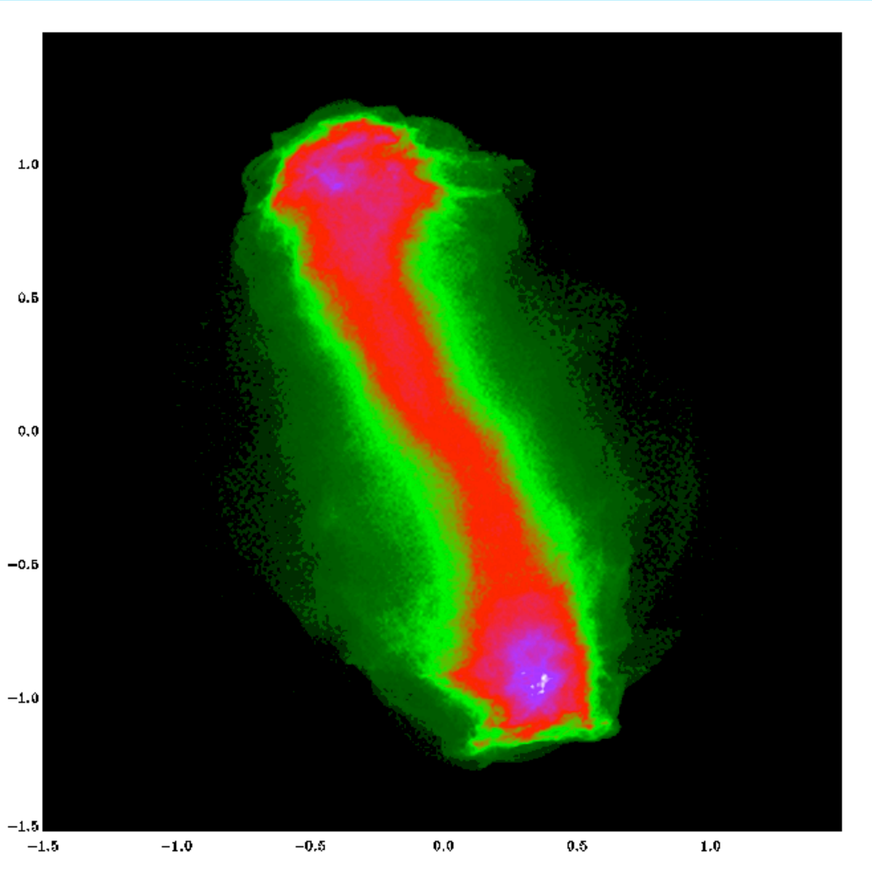
maximum

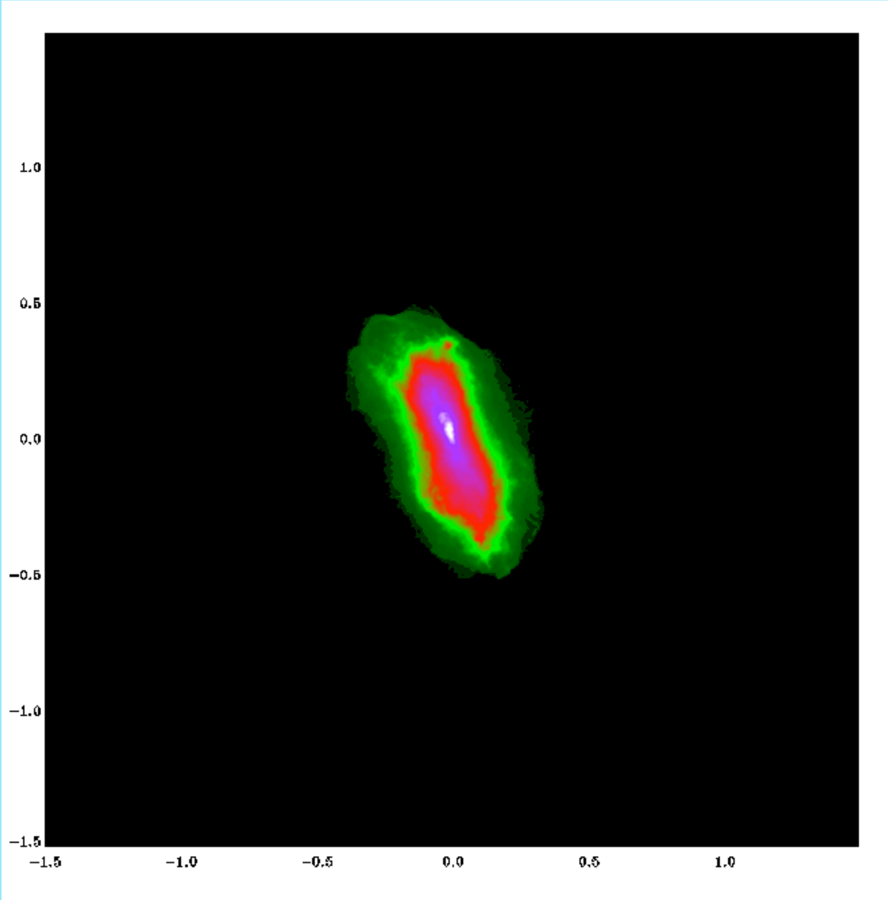


minimum

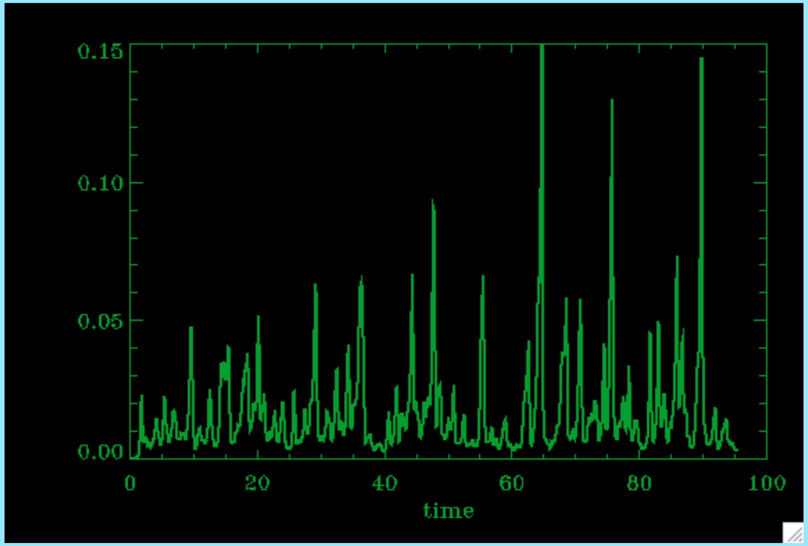


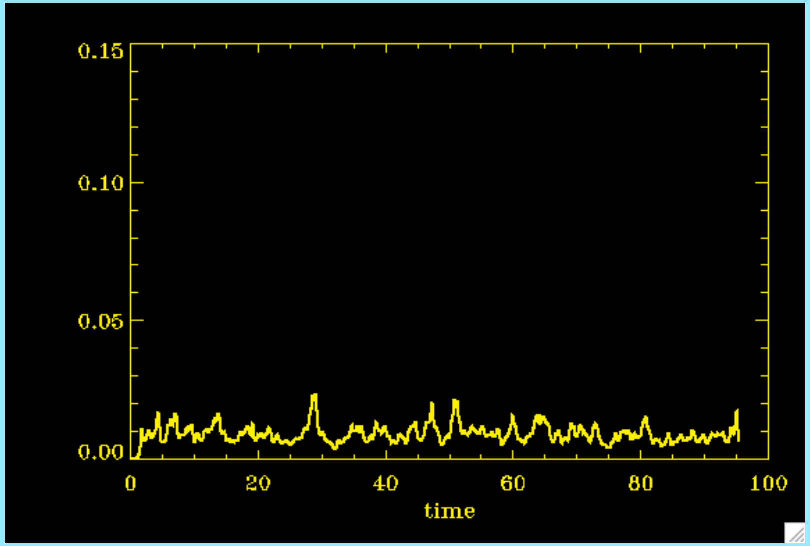
2D distribution function of Bx and By

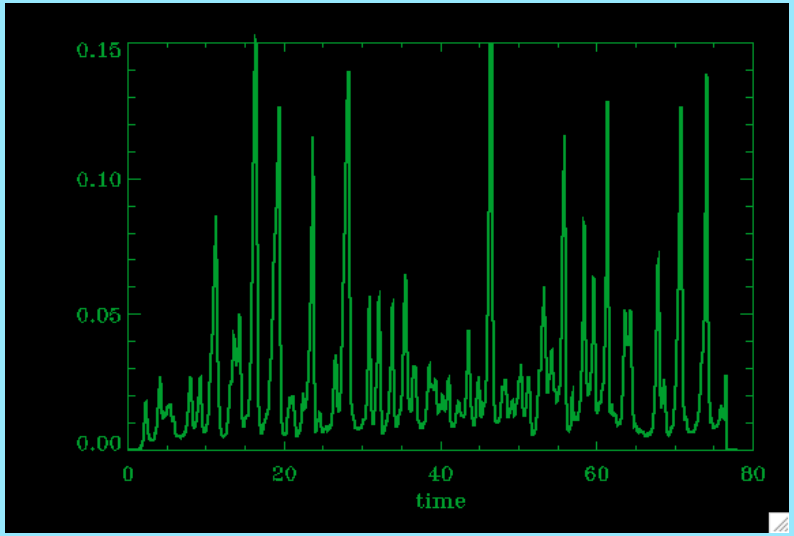


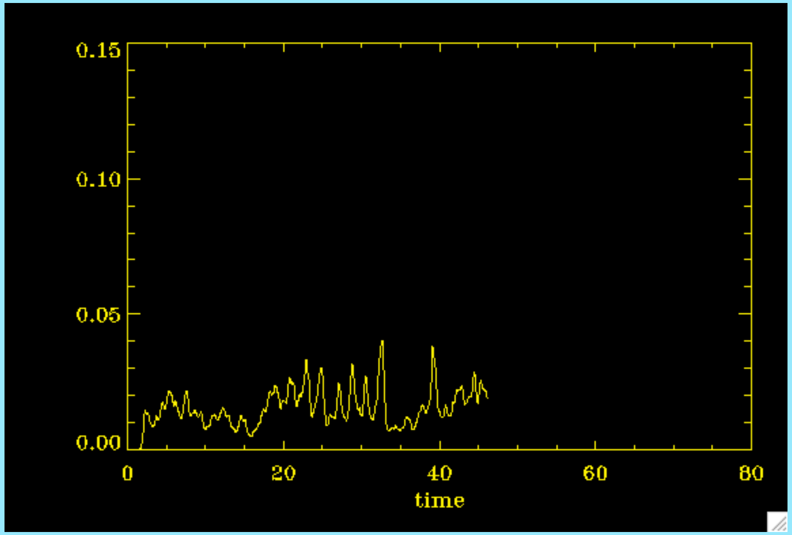


Compare now different aspect ratio

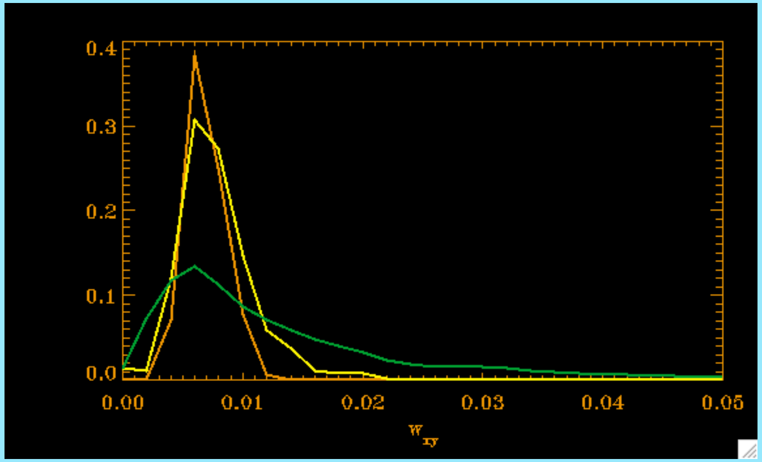


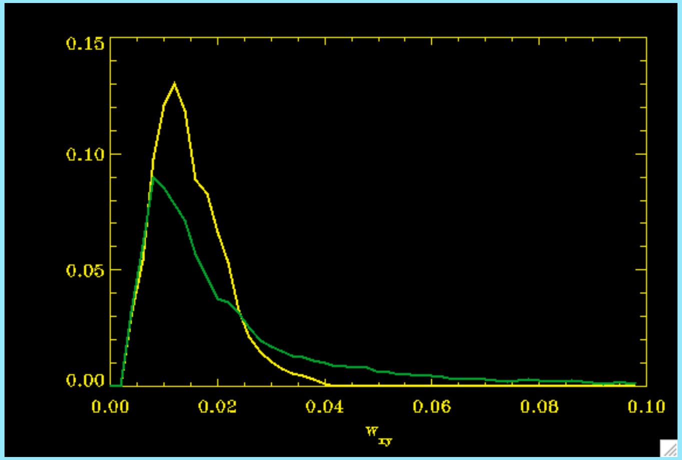






Distribution function of maxwell stresses

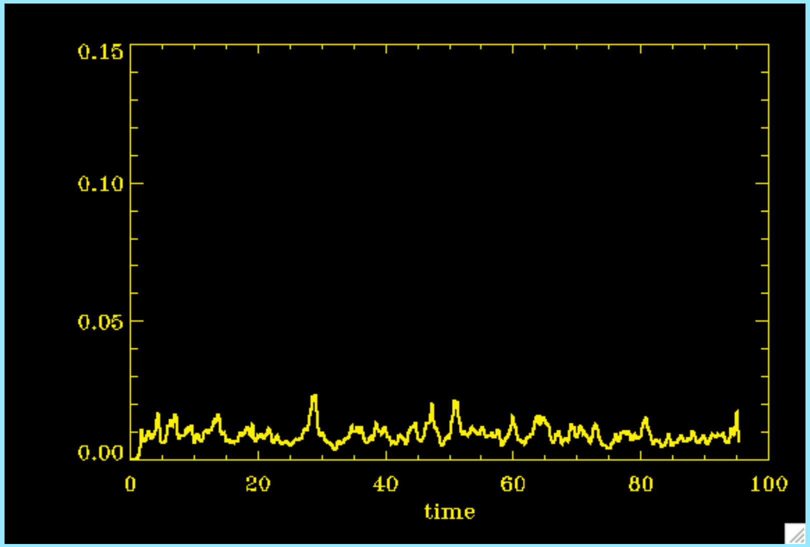


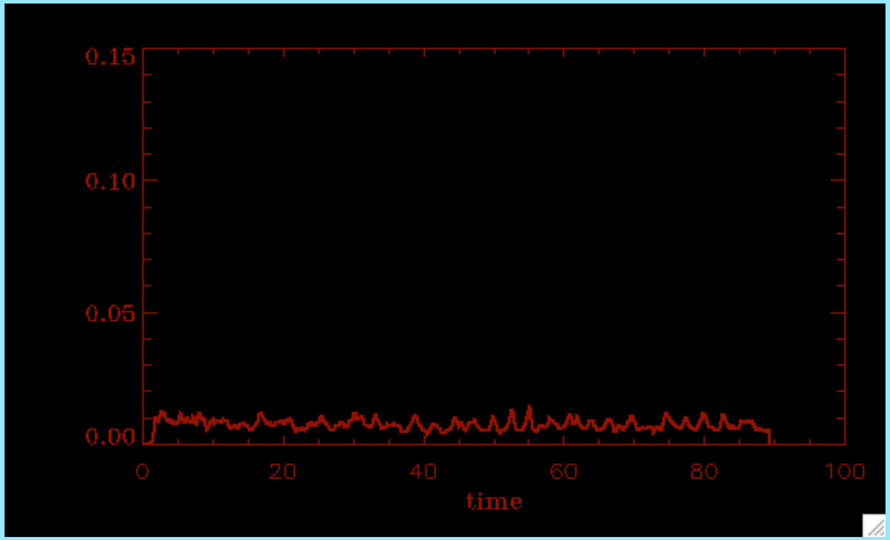


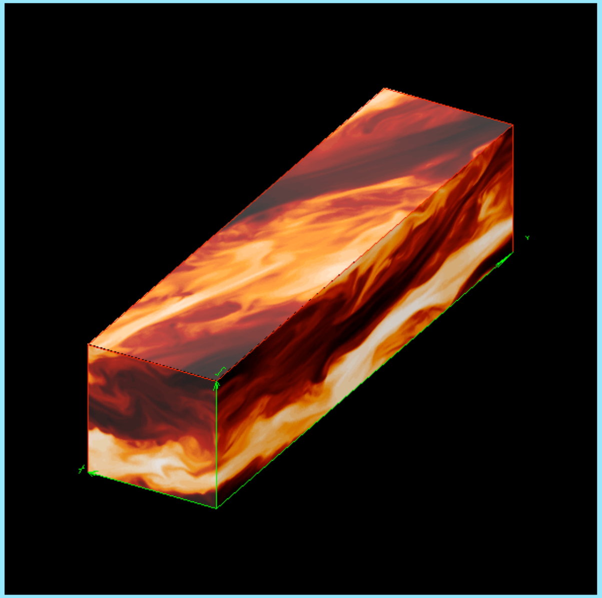
Maxwell stresses vs time

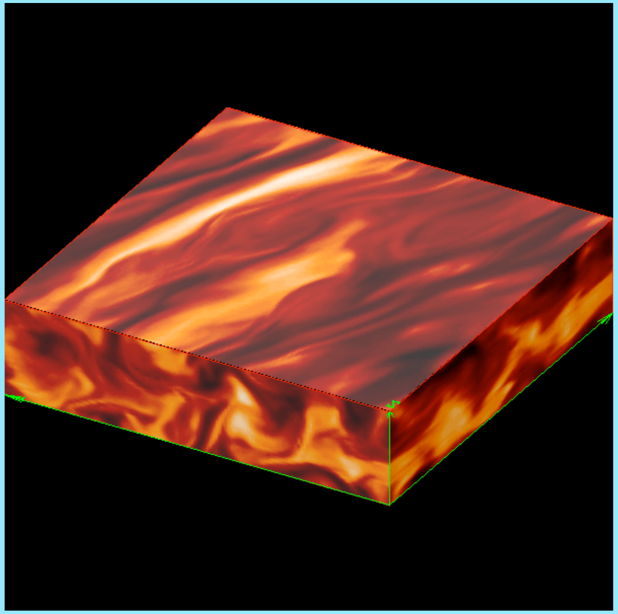
Asp. Ratio = 4

Asp. Ratio = 8

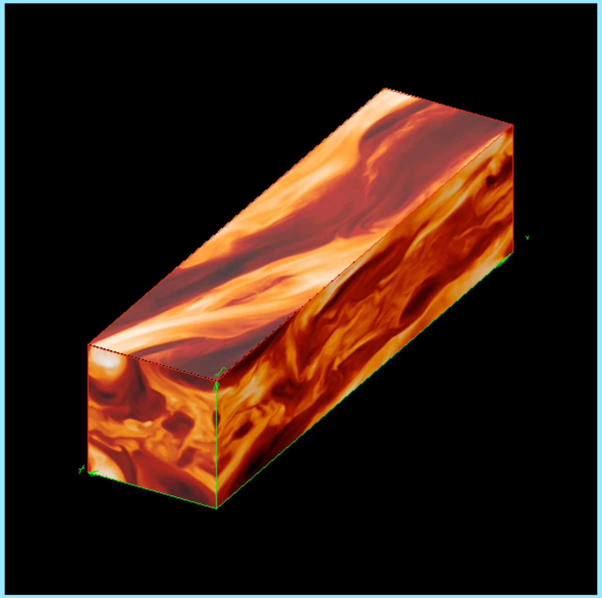


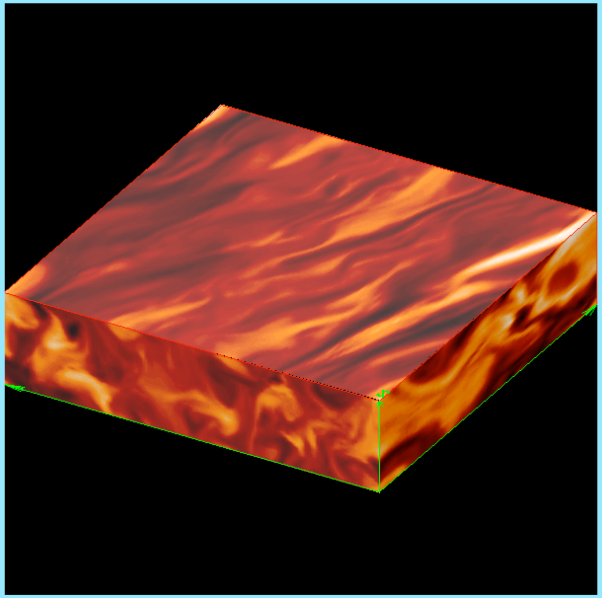






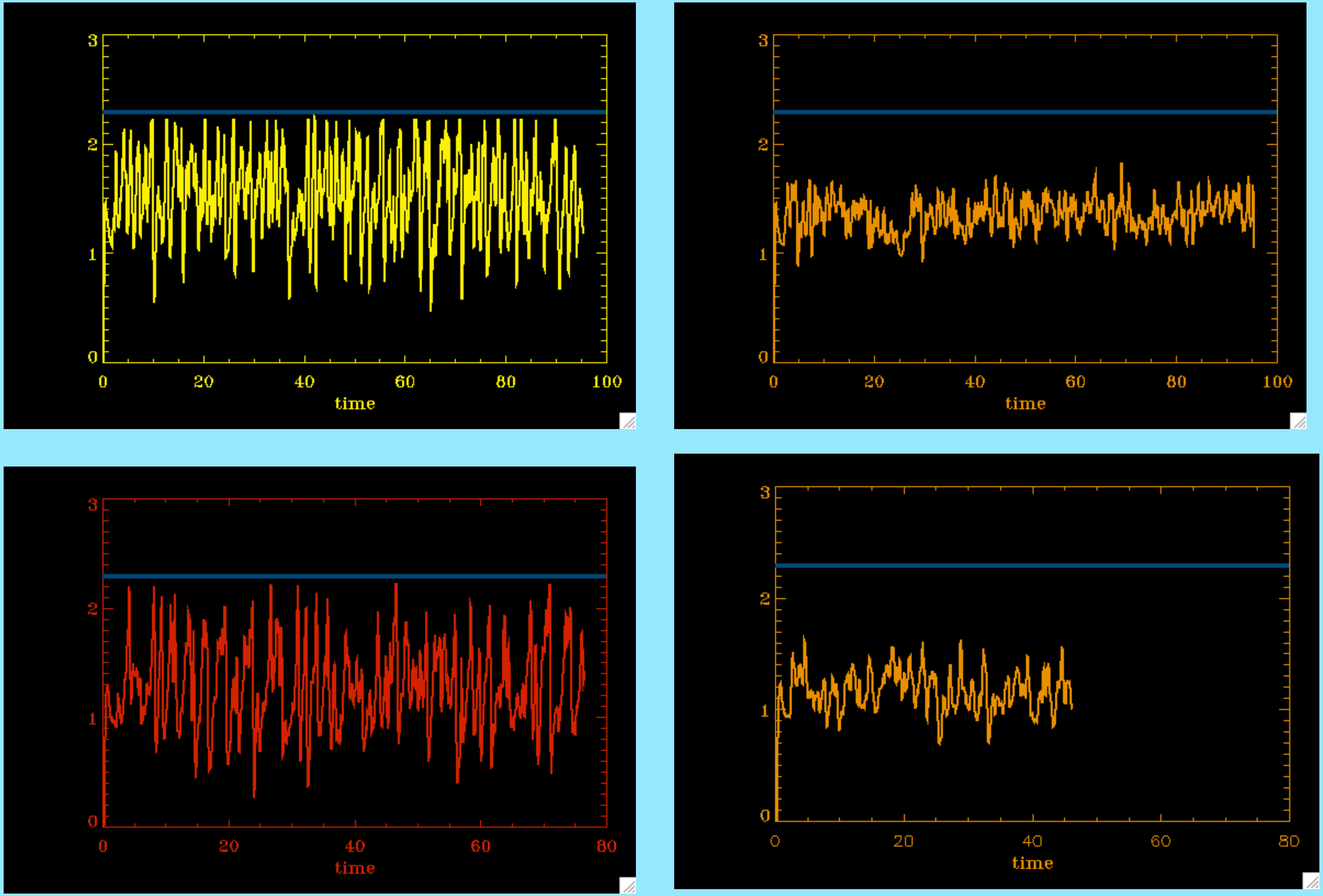
Maximum



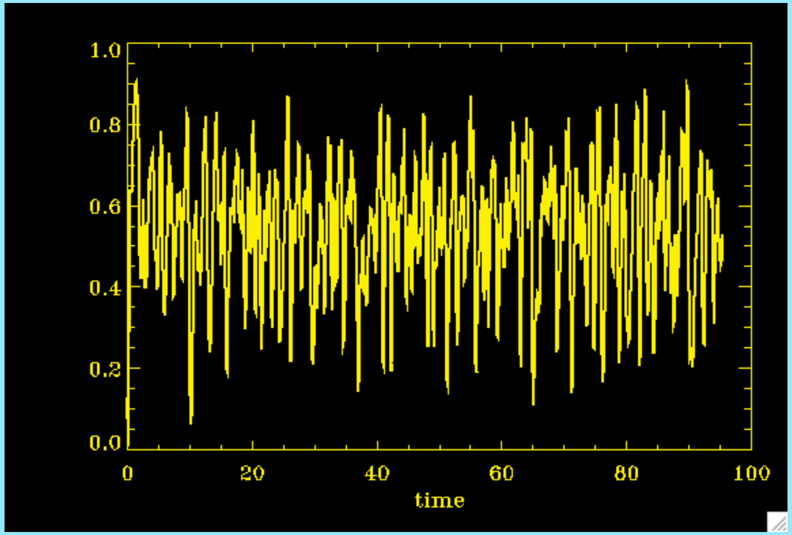


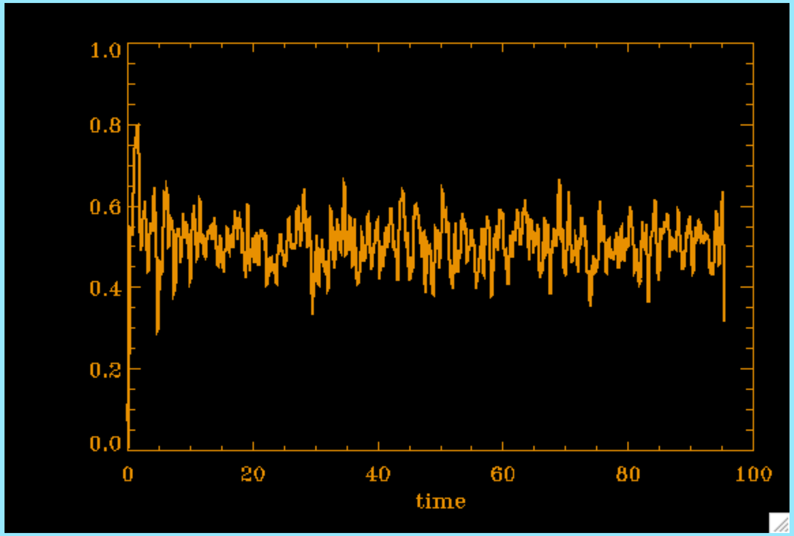
Minimum

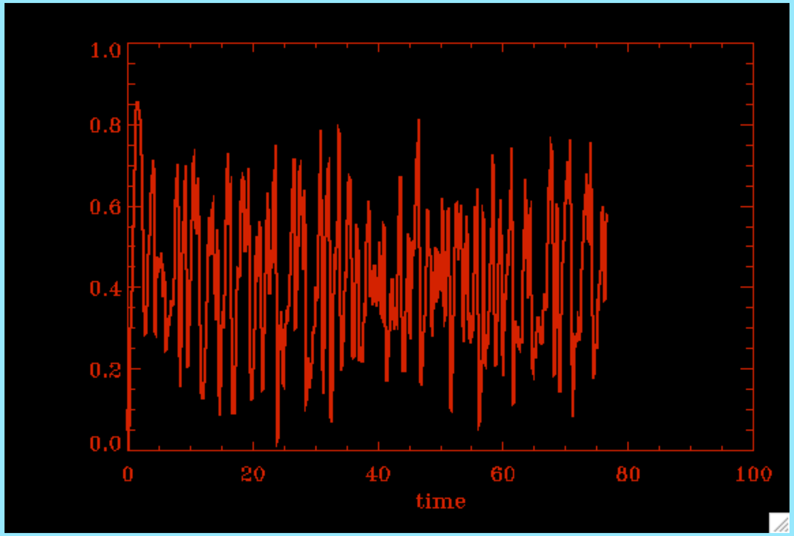
Slope of the correlation

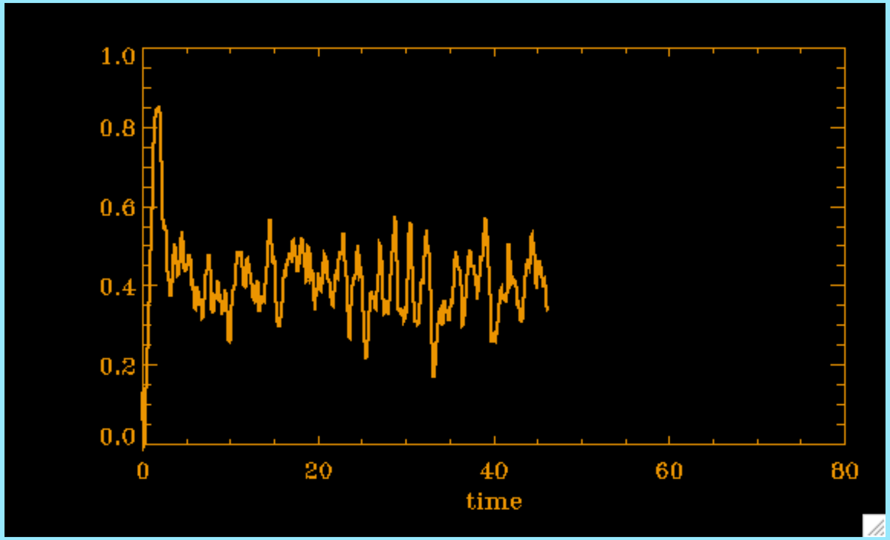


Correlation coefficient

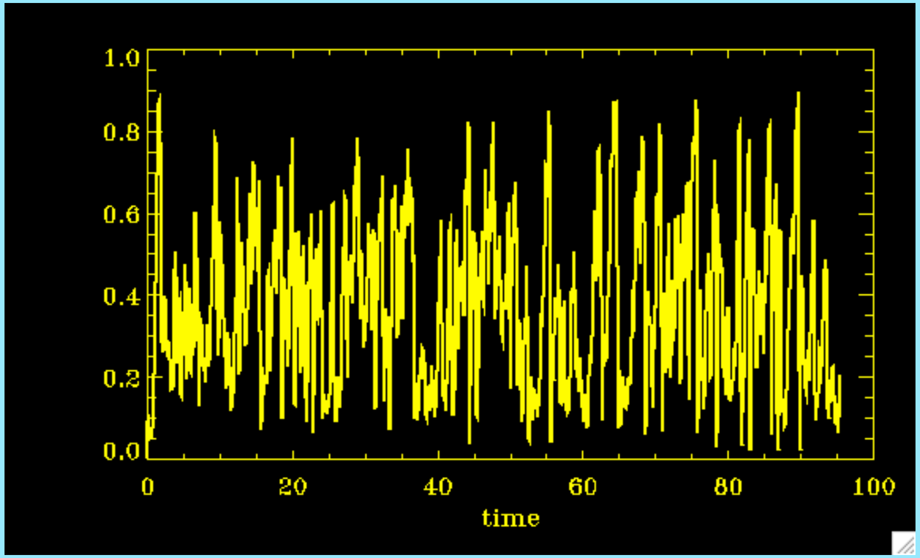




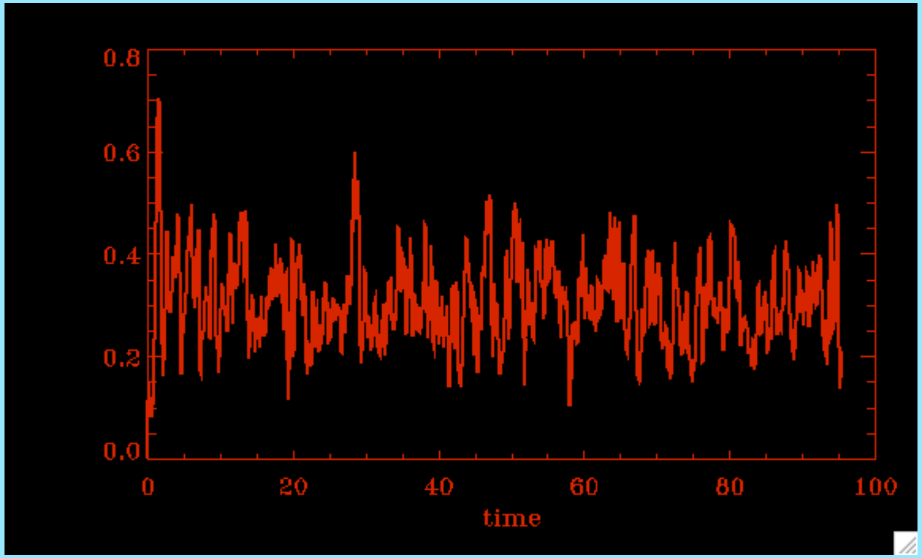




Correlation length in y direction



Aspect ratio 1

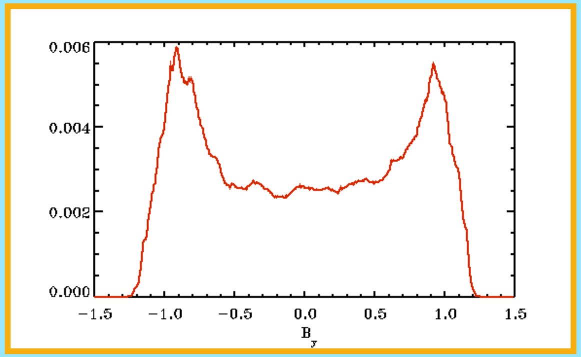


Aspect ratio 4

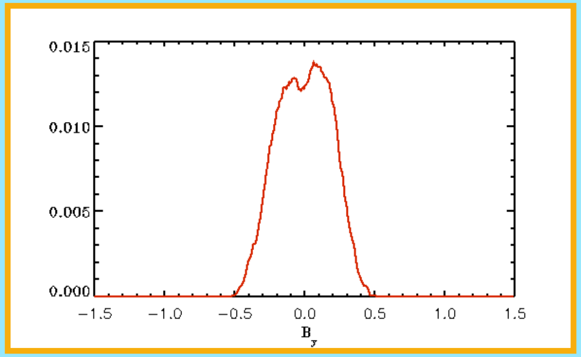
pdf of By

Aspect ratio 1

Maximum

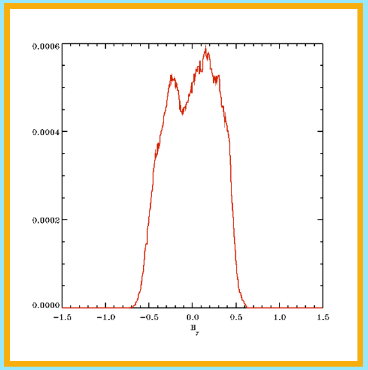


Minimum

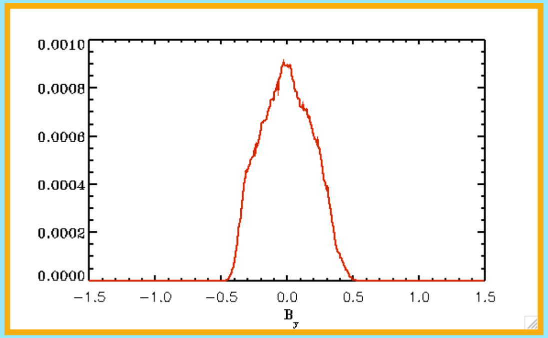


Aspect ratio 4

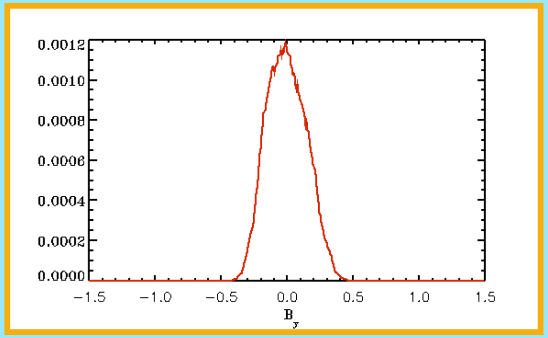
Largest maximum



Typical maximum



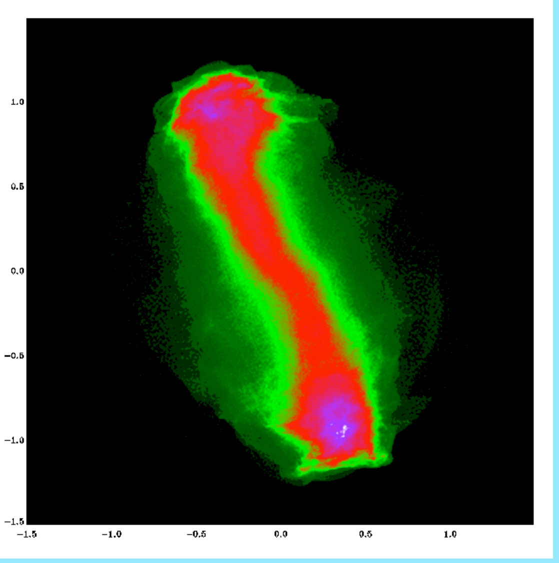
Minimum

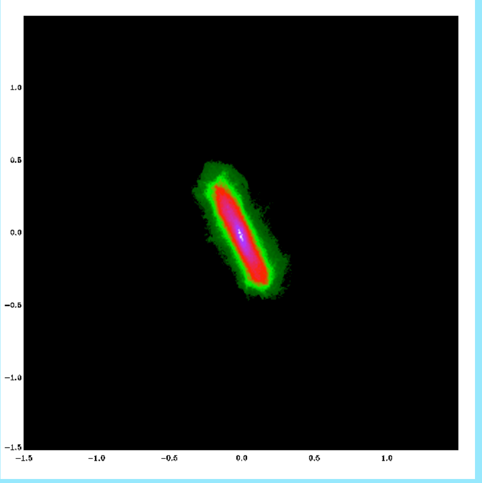


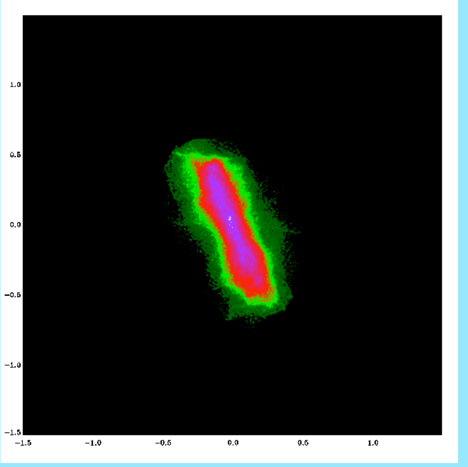
2D distribution Bx By

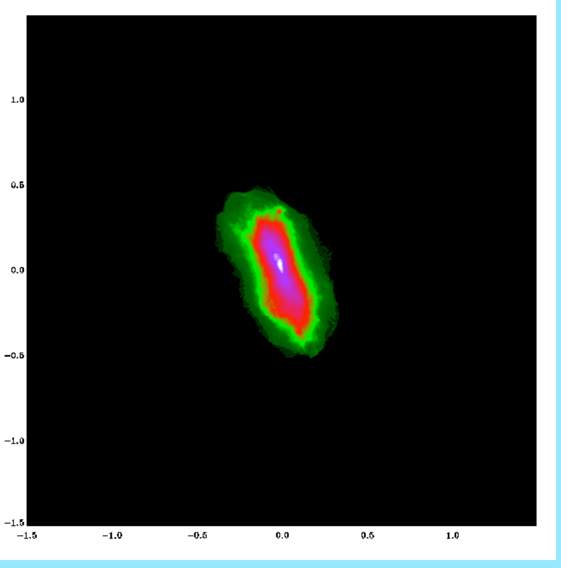
AR = 1

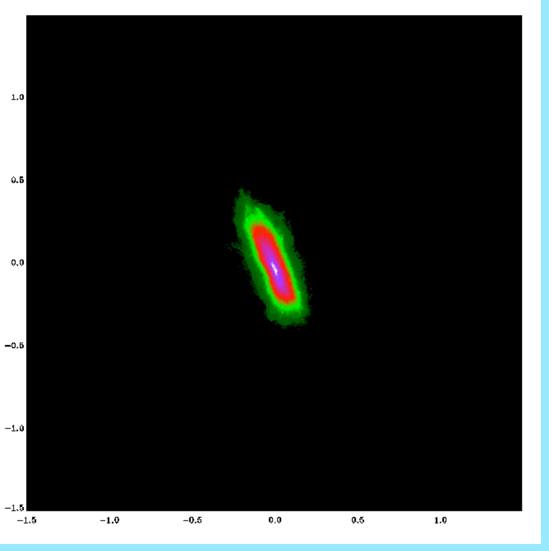
AR = 4











Conclusions

Results for non zero net flux

Behavior of the system depends on the aspect ratio of the box

Aspect ratio 1 all peaks correspond to the formation of a channel solution

Increasing the aspect ratio, the system has more difficulty in forming the channel.

Going from 4 to 8 seems to indicate a tendence towards convergence.

Parasitic instabilities have wavelength larger than the basic channel solution (Goodman & Xu 1994), axisymmetric Instabilties excluded from box with a.r. = 1

Dominance of the channel seems to be an artifact of the way the calculation is done

Shearing box results may be significant for full disk only with large aspect ratio.

Increase in resolution introduces significant changes but not qualitative.

Need for explicit dissipation