JeTSeT

JET Simulations, **E**xperiments and **T**heories

Modeling MHD jets and outflows*

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MARIE CURIE ACTIONS

www.jetsets.org

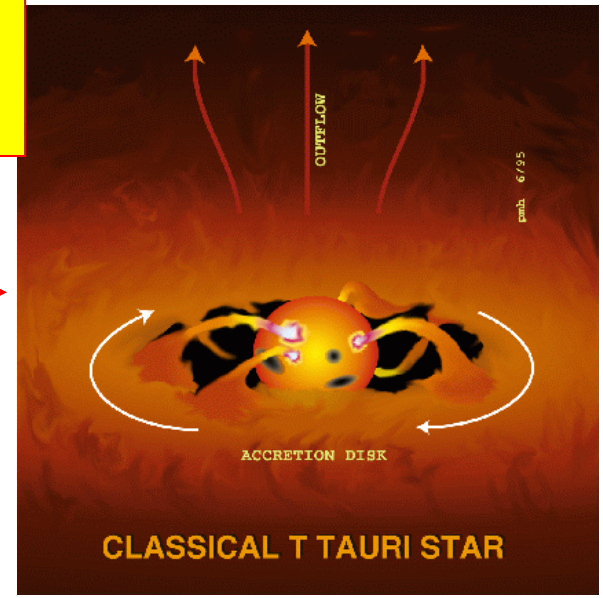
**Various parts of this work have been performed in collaboration with :*

Sergei Bogovalov (Moscow), Christophe Sauty (Paris), Silvano Massaglia, Edo Trussoni and Titos Matsakos, (Torino), Nek Vlahakis (Athens)

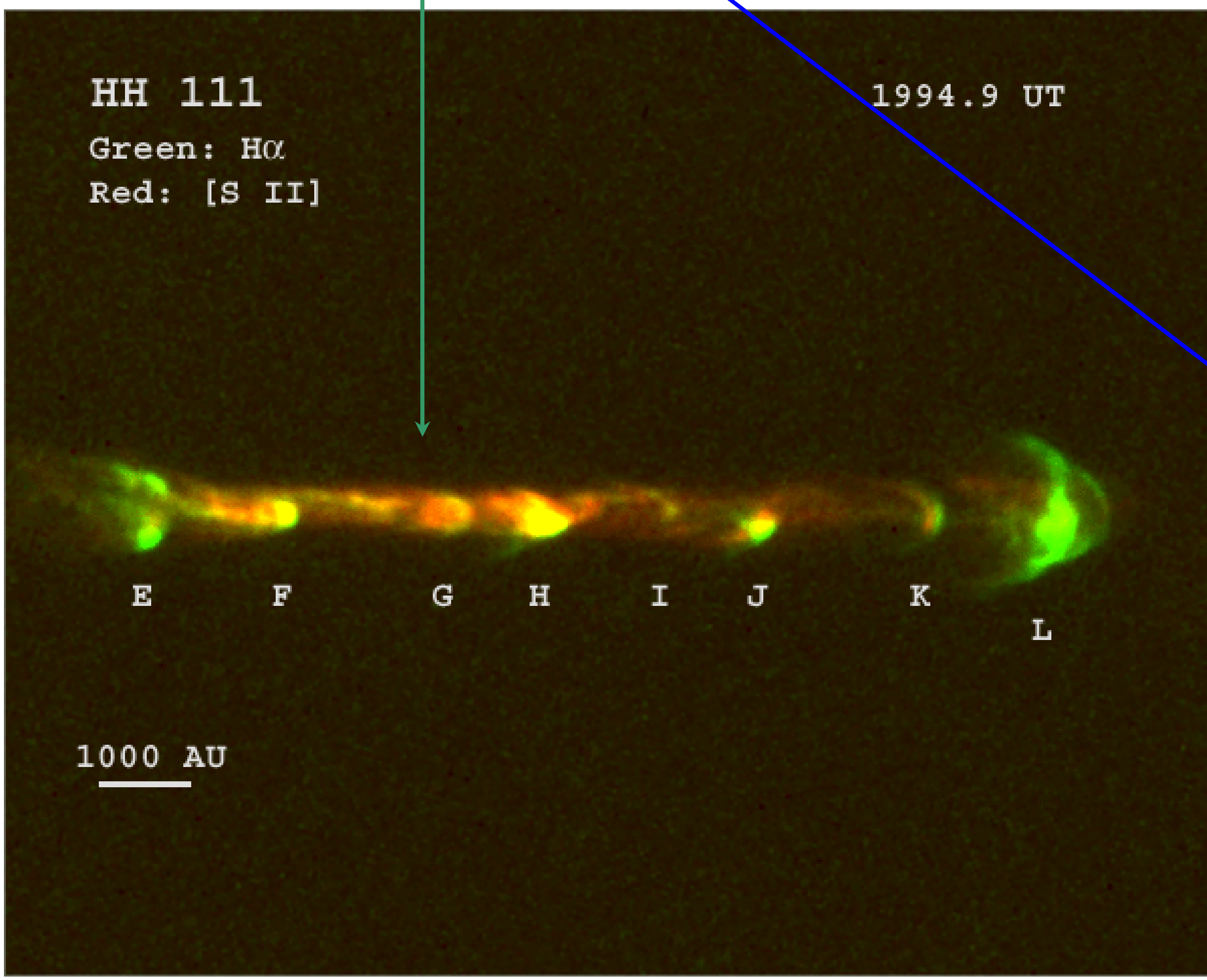
Outline

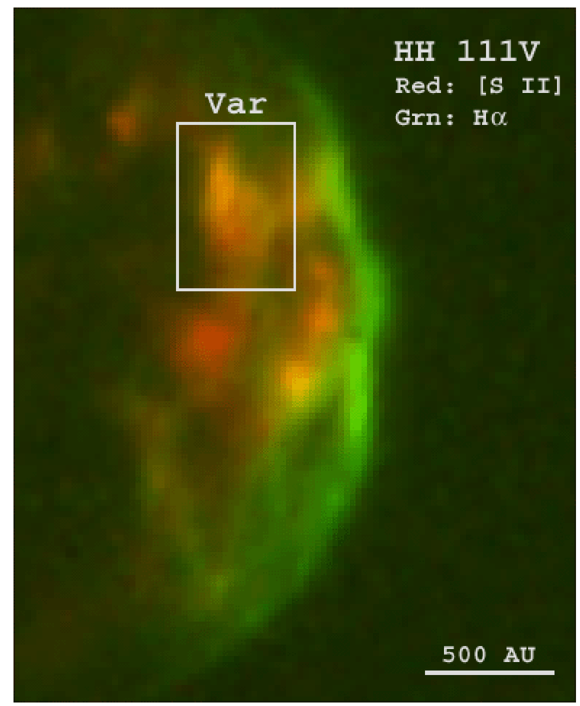
- Observations + Models
- Magnetic self-collimation
- Analytical solutions for 1-component models :
  → Analytical Stellar Outflows (ASO)
  → Analytical Disk Outflows (ADO)
- Numerical solutions for 1-component models :
  Formation of a central jet in efficient magnetic rotators
- Numerical solutions for (non self-similar) 2-component models :
  Collimation of the stellar outflow by a surrounding disk-wind
- Results of numerical solutions for 2-component models using
  as initial conditions a combination of the ASO + ADO solutions
- Conclusions

Observations :Jets from YSO's



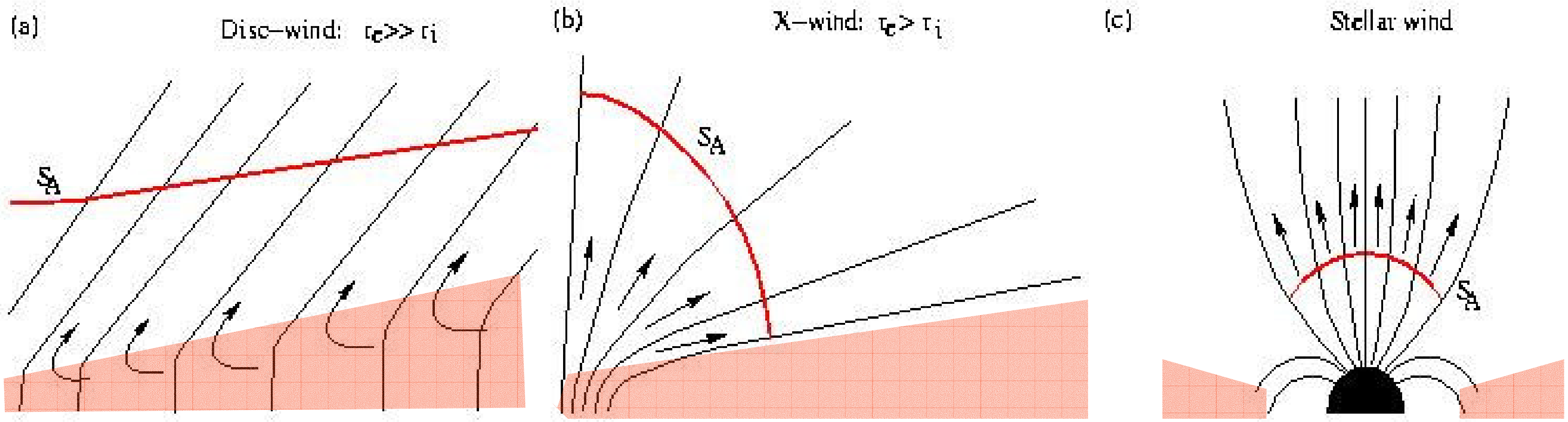
Propagation – bowshocks - source





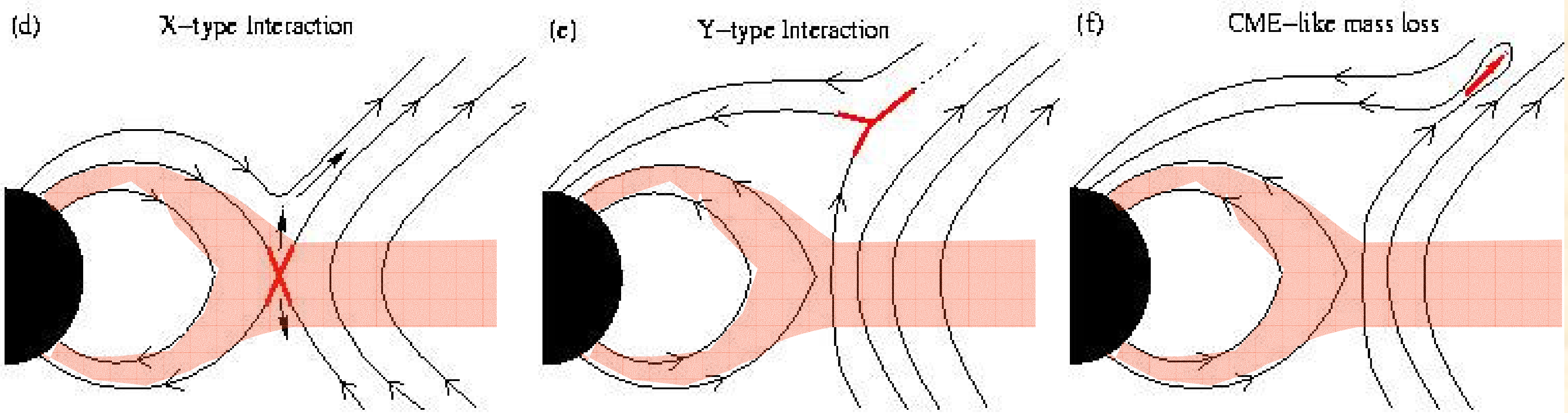
@ Pat Hartigan

The Origin of Jets



Steady ejection sites :

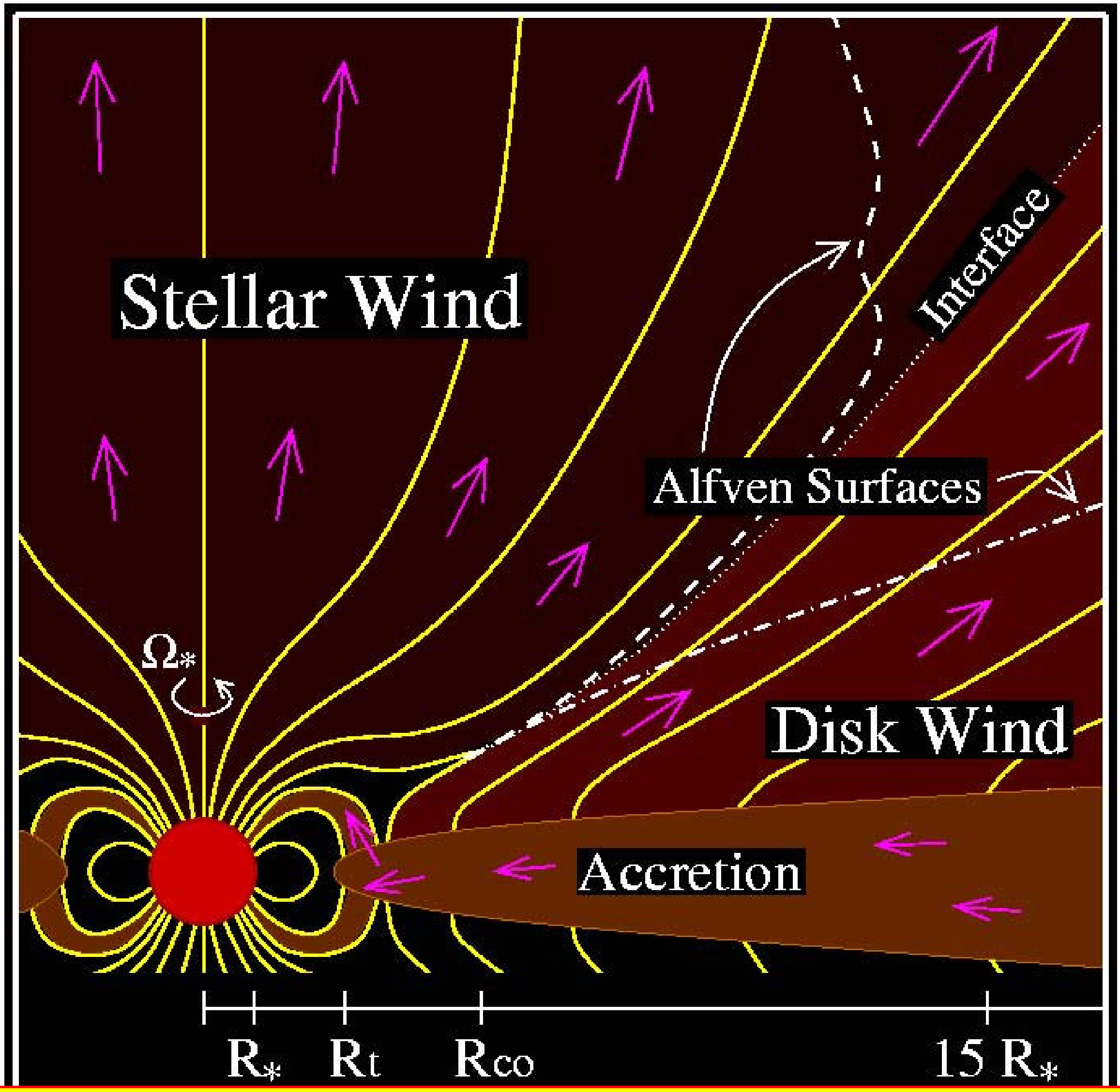
Extended disk-wind or inner disk wind (X-wind), or stellar wind



Unsteady ejection sites:

Re-X wind or CME-type ejections

Stellar wind vs. disk wind in spinning-down p*s



Disk-locking seems problematic to explain the slow rotation of protostars

@ Matt & Pudritz, 2005

Spin-down of protostar via stellar wind :

Torque on star due to accretion of disk matter:

$$\tau_a \equiv \frac{dJ_a}{dt} = \dot{M}_a \sqrt{G M_* R_{trunc}}$$

Torque on star due to angular momentum lost in the wind :

$$\tau_w \equiv \frac{dJ_w}{dt} = -(2/3) \dot{M}_w \Omega_* R_*^2 (r_A / R_*)^2$$

Star rotates as a solid body at a rate which is a fraction f of break-up speed :

$$f = \Omega_* \sqrt{R_*^3 / G M_*}$$

By equating $\tau_w = -\tau_a$, the equilibrium spin rate is,

$$f_{eq} \approx 0.1 \left(\frac{R_{trunc}/R_*}{2} \right)^{1/2} \times \left(\frac{r_A/R_*}{15} \right)^{-2} \times \left(\frac{\dot{M}_w/\dot{M}_a}{0.1} \right)^{-1}$$

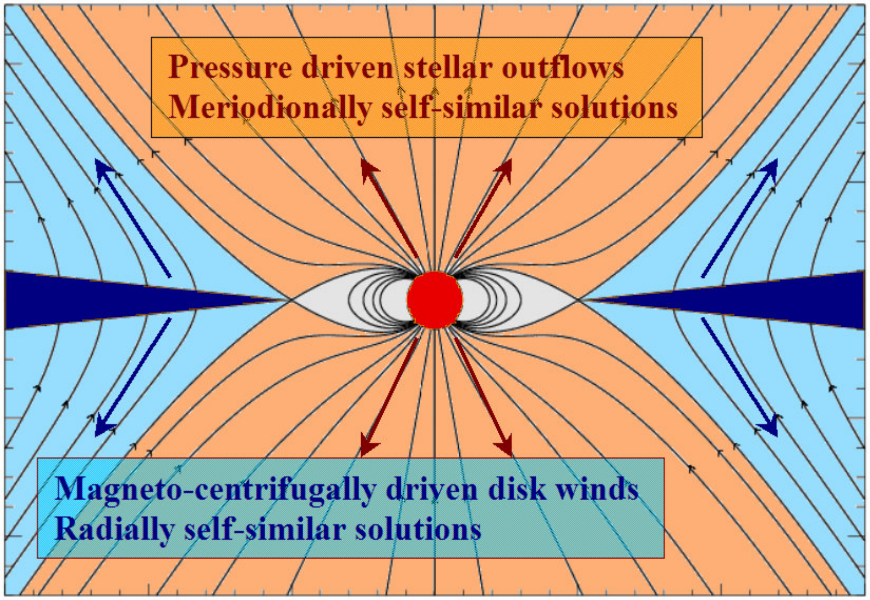
Source : Observations & Models

- **Observations[1,2] : protostellar jet outflows contain two constituents, the dominance of which depends on the intrinsic physical properties of the system (protostellar object and disk)**

  (He line profiles indicate that stellar winds are present in at least 60% of CTTS)
  - Central protostar
  - Surrounding disk

  [1]Edwards et al. (2006), ApJ, 646,319

  [2]Kwan et al. (2007), ApJ, 657, 897



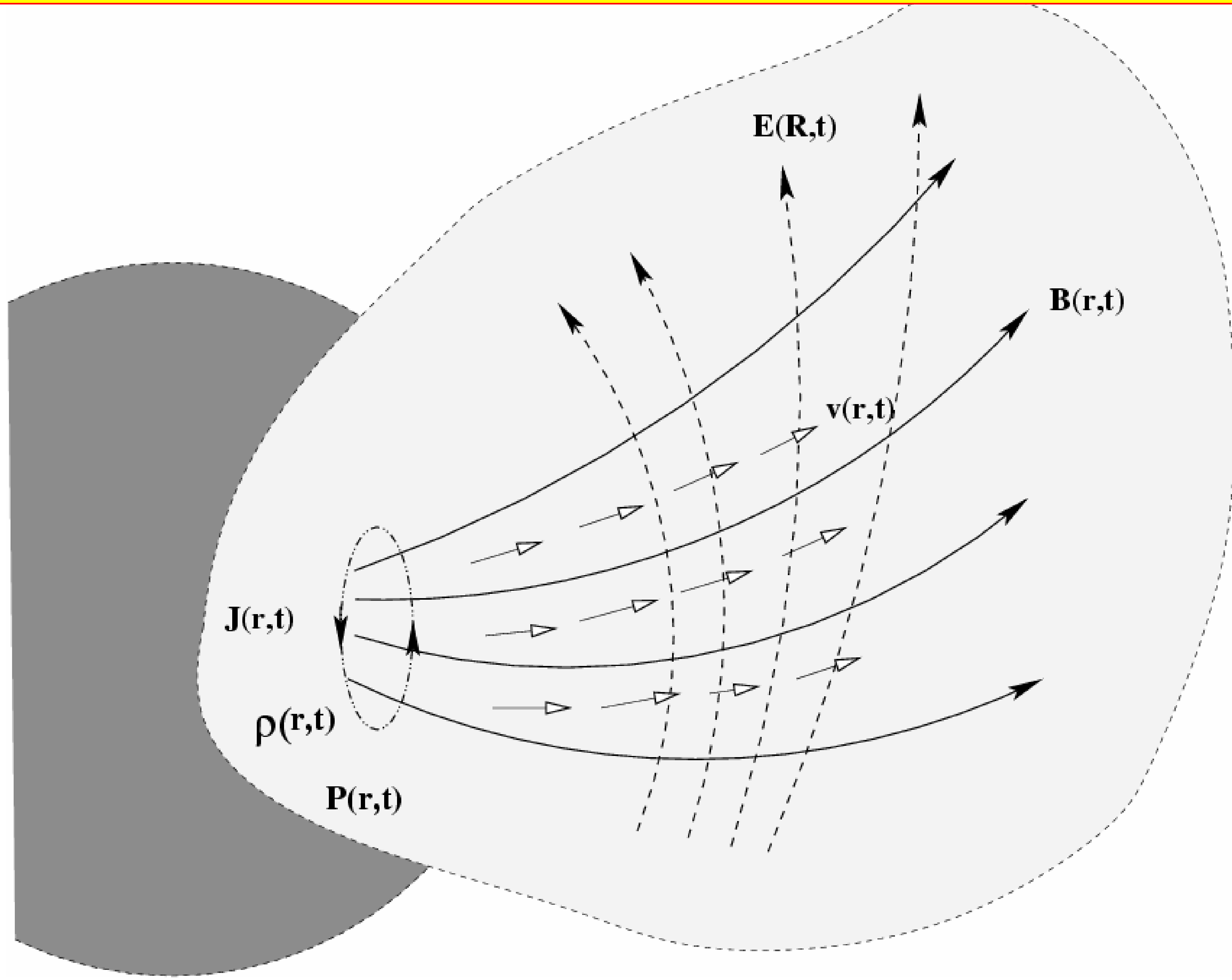
- **Theory[3] : it is argued that jets from YSOs may consist of two components:**
  - Inner pressure driven wind (non-collimated if the star is an inefficient magnetic rotator)
  - an outer magneto-centrifugally driven disk-wind providing most of the high mass loss rate observed.

[3]Bogovalov & Tsinganos (2001), MNRAS, 325,249

Some “representative” studies in the past 50 years :

- Parker’s (1958 - 1960) classical description of the HD Solar Wind
- Weber & Davis (1967) description of an equatorial magnetized stellar wind with slow, Alfven, fast critical pts
- Michel’s (1969) description of relativistic MHD stellar winds
- Blandford & Rees (1974), Lovelace (1976) jets power double radio sources
- Blandford & Payne’s (1982) description of radially self-similar MHD jets -“bead on a rotating wire” analogy
- Sakurai’s (1985) numerical simulation of MHD polytropic winds – first indications of collimation
- Uchida & Shibata’s (1985) simulations of the “uncoilling spring mechanism” for launching MHD jets
- Heyvaerts & Norman’s (1989, 2003) asymptotic analyses of rotating/magnetized outflows
- Sauty & Tsinganos (1994) description of meridionally self-similar MHD jets and criterion for collimation
- Vlahakis & Tsinganos (1998) systematic construction of self-similar models of MHD outflows
- Sauty, Trussoni, Tsinganos series of papers (1994 - 2005) on analytical modelling self-similar MHD outflows
- Ferreira, Casse, Keppens, Romanova (1997, 2002, 2004, 2006) magnetized accretion-ejection structures
- etc, etc.

Illustration of the interaction of magnetized plasmas



Theory : Basic (nonrelativistic) MHD equations

$$\vec{\nabla}\cdot\vec{\mathbf{B}} = 0, \qquad \vec{\nabla}\cdot\vec{\mathbf{E}} = \cancel{4\pi\delta} \approx 0\,,$$

$$\vec{\nabla}\times\vec{\mathbf{E}} = -\frac{1}{c}\frac{\partial\vec{\mathbf{B}}}{\partial t}, \qquad \vec{\mathbf{E}} + \frac{\vec{\mathbf{V}}\times\vec{\mathbf{B}}}{c} = \cancel{\frac{\vec{\mathbf{J}}}{\sigma}} \simeq 0\,, \qquad \underline{\textit{Maxwell}}$$

$$\vec{\nabla}\times\vec{\mathbf{B}} = \frac{4\pi}{c}\vec{\mathbf{J}} + \cancel{\frac{1}{c}\frac{\partial\vec{\mathbf{E}}}{\partial t}} \simeq \frac{4\pi}{c}\vec{\mathbf{J}}\,,$$

. . . .

$$\rho\frac{\partial\vec{\mathbf{V}}}{\partial t} + (\rho\vec{\mathbf{V}}\cdot\vec{\nabla})\vec{\mathbf{V}} = -\vec{\nabla}P + \frac{\vec{\mathbf{J}}\times\vec{\mathbf{B}}}{c} - \rho\vec{\mathbf{G}}\,, \qquad \underline{\textit{Newton}}$$

. . . .

$$\vec{\nabla}\cdot\rho\vec{\mathbf{V}} + \frac{\partial\rho}{\partial t} = 0\,, \qquad \underline{\text{mass conservation}}$$

. . . .

$$\rho\left[P\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho}\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P}{\rho(\Gamma-1)}\right)\right] = \rho\frac{\partial h}{\partial t} - \frac{\partial P}{\partial t} + \rho\vec{\mathbf{V}}\cdot\left[\nabla h - \frac{\nabla P}{\rho}\right] = q\,.$$

$\vec{\mathbf{V}}(x_1, x_2, x_3, t)$: Bulk Flow Speed of Plasma

$\vec{\mathbf{B}}(x_1, x_2, x_3, t)$: Magnetic Field in Plasma

$\vec{\mathbf{J}}(x_1, x_2, x_3, t)$: Electric Current Density in Plasma

$\vec{\mathbf{G}}(x_1, x_2, x_3)$: External (gravitational) Field in Plasma

$\rho(x_1, x_2, x_3, t)$: Plasma Density

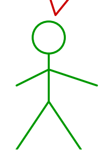
$P(x_1, x_2, x_3, t)$: Plasma Pressure

$h(x_1, x_2, x_3, t)$: Enthalpy $(= \frac{\Gamma}{\Gamma-1}\frac{P}{\rho})$

$q(x_1, x_2, x_3, t)$: Volumetric Rate of Energy Addition in System

<u>Question:</u>

How can we "extract" from the general set of these MHD Eqs. the description of a magnetized outflow (jet or wind) ??



MHD modelling of cosmical outflows

- ***I. Steady models***

Advantages:

- **analytical treatment**
- **parametric study**
- **physical picture**
- **cheap method**

Difficulties:

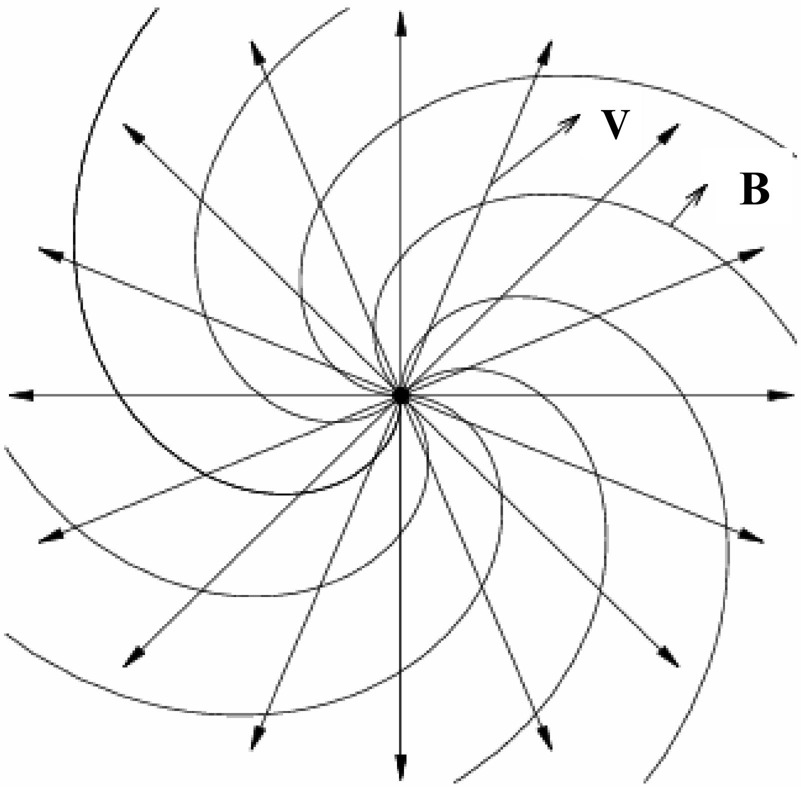
- **Nonlinearity of MHD set !**
- **2-dimensionality - PDEs !**
- **Causality - unknown critical surfaces !**

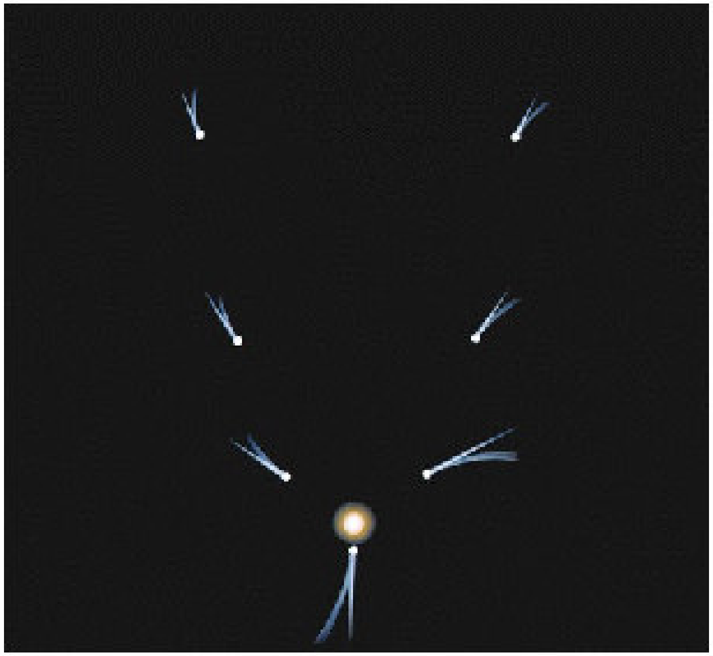
- ***II. Time-dependent models***

- **(No-analytical treatment)**
- **temporal evolution**
- **nonideal MHD effects**
- **(expensive method)**

- **3D MHD code with magnetic flux conservation !**
- **large grid space - large lengths of jets !**
- **correct boundary conds - boundary effects !**

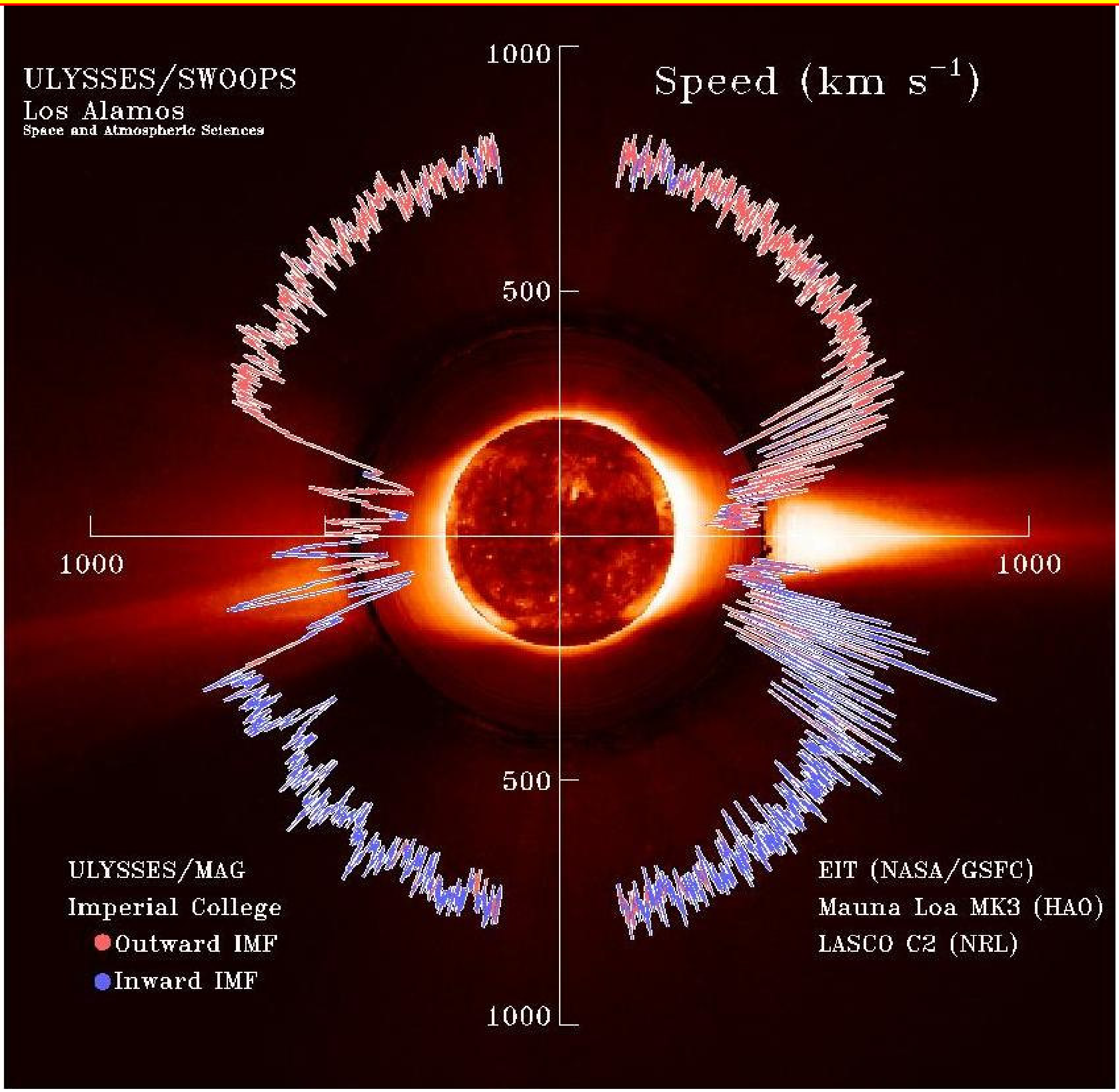
Ia) 1D-HD : The classical Parker wind



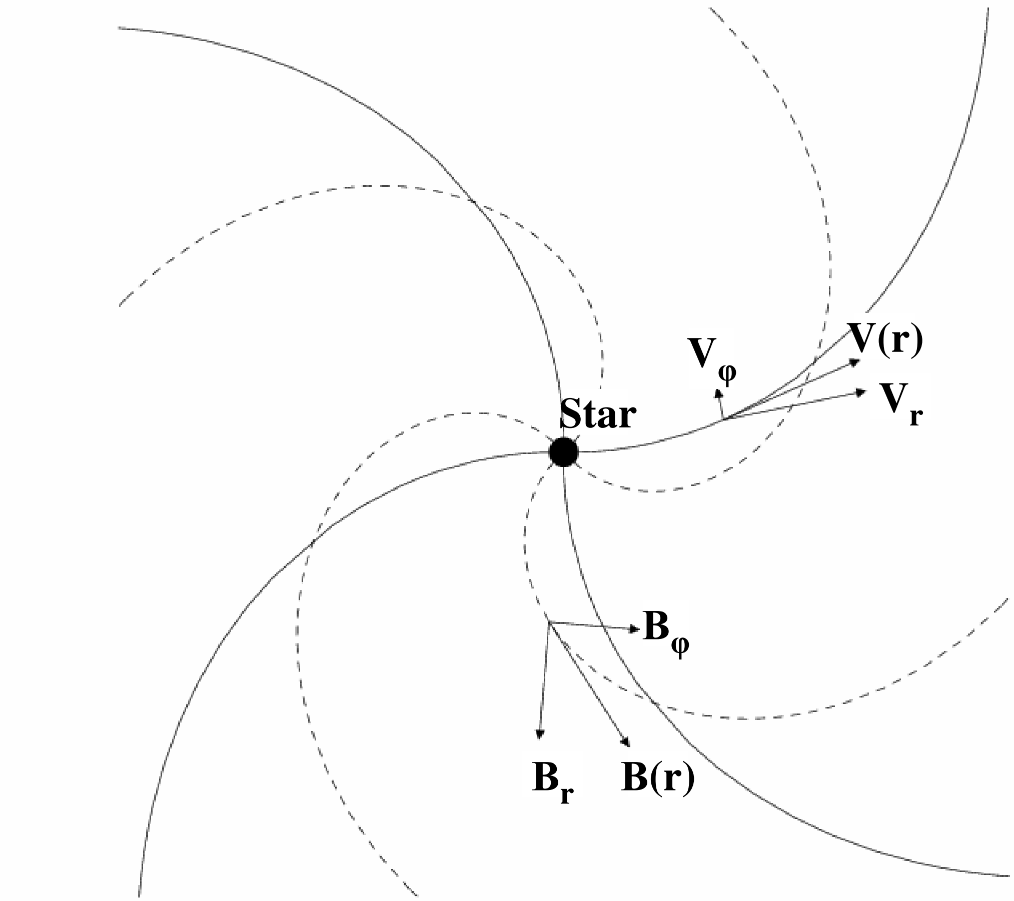


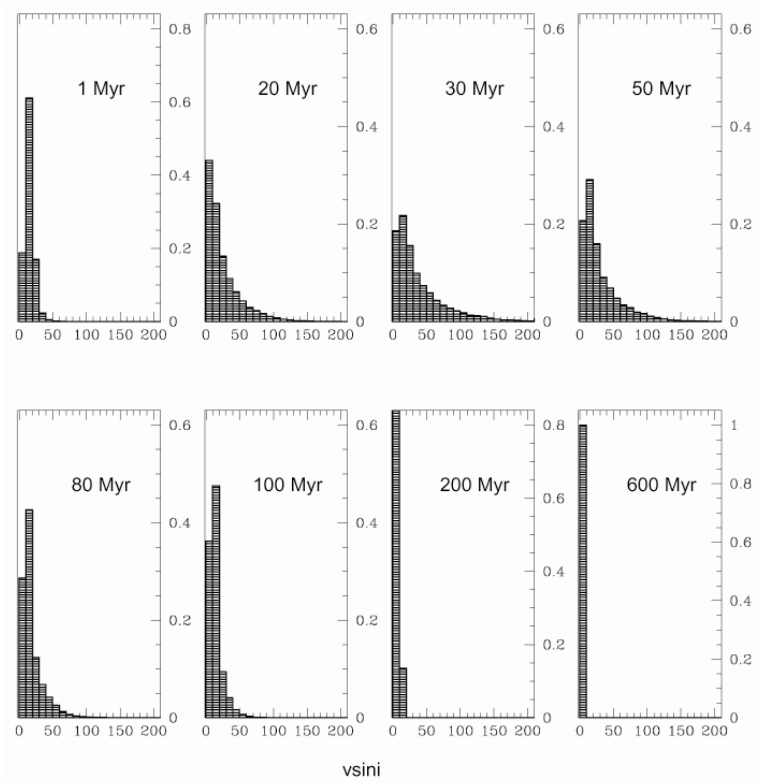
Ulysses *in situ* measurement of solar wind speed

[Sauty et al, AA, 432, 687, 2005]:



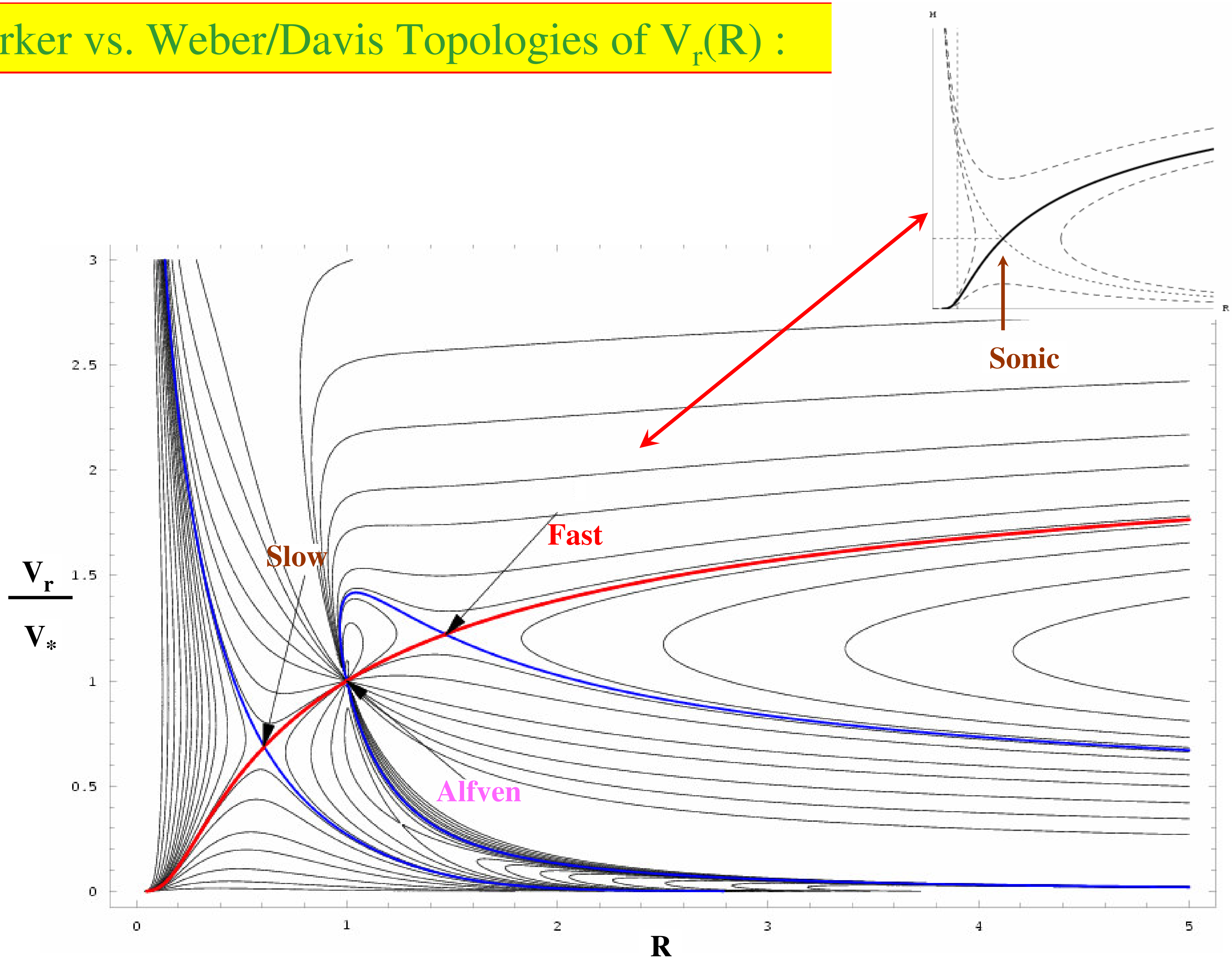
Ib) 1D-MHD: The Weber-Davis magnetized wind





Distribution of Vsini for $1M_o$ stars of different ages
(Bouvier, Forestini & Alain 1997)

Parker vs. Weber/Davis Topologies of $V_r(R)$:



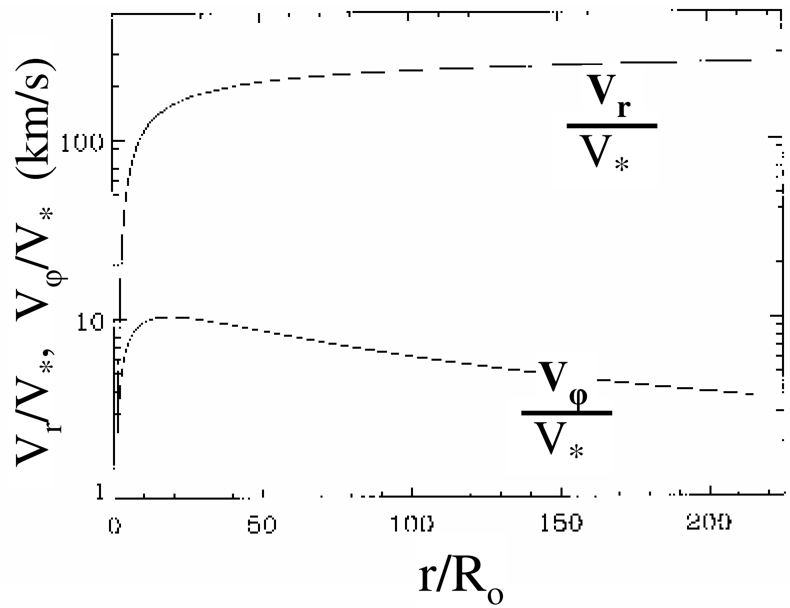
Slow and Fast magnetic rotators :

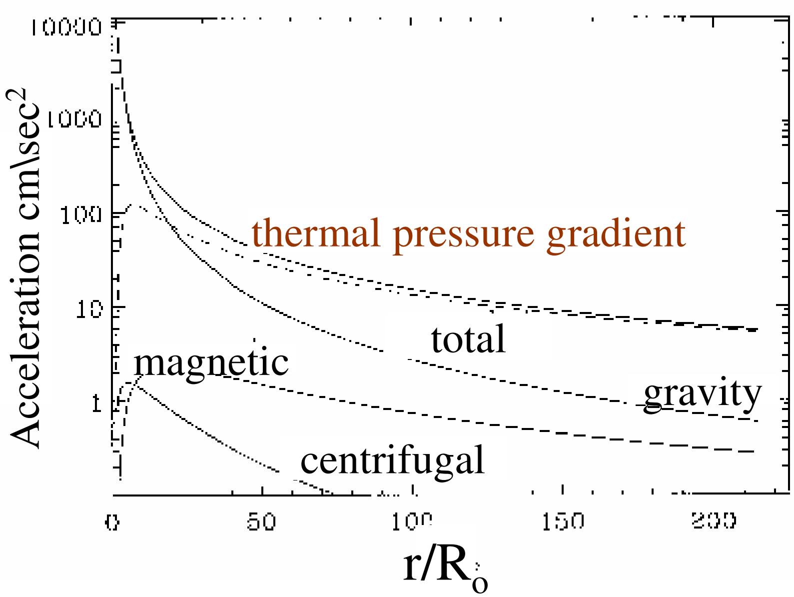
$$\mathcal{E} = \frac{1}{2}V^2 + h - \frac{GM}{r} - \frac{rB_\phi\Omega}{\Psi_A} = \underbrace{\frac{1}{2}V_o^2 + h_o - \frac{GM}{r_o}}_{\mathcal{E}_o} - \underbrace{\frac{r_o B_\phi^o \Omega}{\Psi_A}}_{\Omega L}, \quad \mathcal{E} \simeq \mathcal{E}_o + \Omega L\,,$$

where $\mathcal{E}_o$ is the energy of the thermally driven Parker wind and ΩL the Poynting energy of the magnetic rotator. Depending on which of these two terms dominates we have two possibilities:

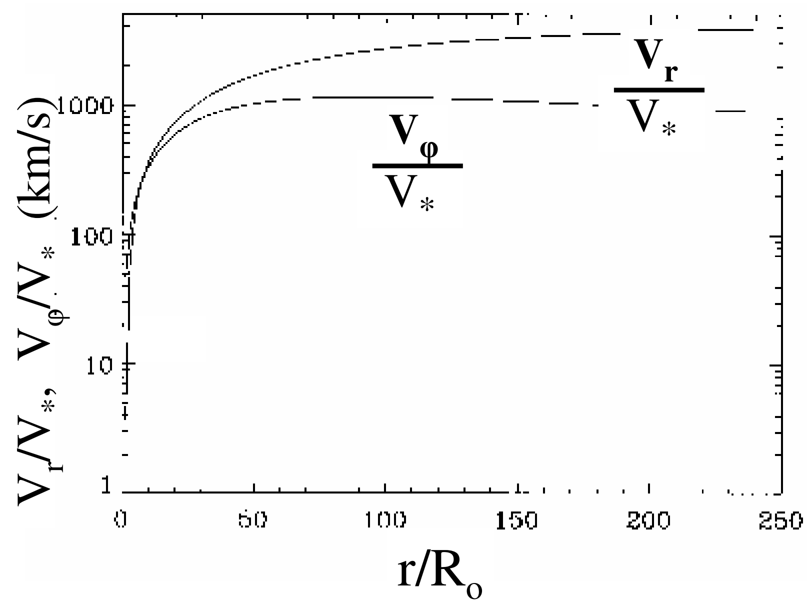
1. $\mathcal{E}_o \gg \Omega L$: Slow magnetic rotator. In this case we have a thermally driven Parker wind
2. $\mathcal{E}_o \ll \Omega L$: Fast magnetic rotator. In this case we have a magnetorotationally driven wind

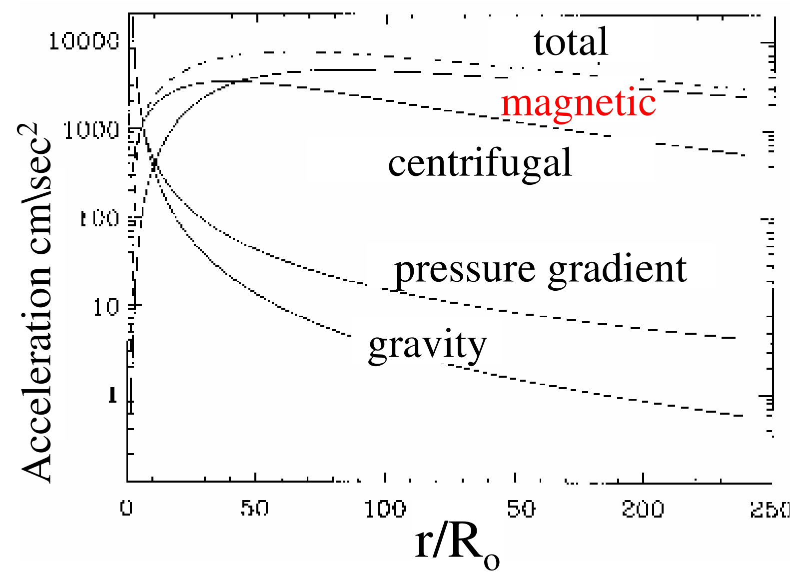
Slow magnetic rotator (our Sun)





Fast magnetic rotator (YSO)





II. 2-D MHD plasma outflows: the issue of collimation

a) Time-independent (steady) outflows i) meridionally selfsimilar

ii) radially selfsimilar

b) Time-dependent plasma outflows

a). 2-D Time-independent (steady) studies - some general conclusions :

- Classes of analytical solutions via a nonlinear separation of the variables (Vlahakis+Tsinganos, 298, 777, 1998)
- Critical points, characteristics and the problem of causality (Tsinganos et al, MNRAS 283, 811, 1996, Vlahakis et al, MNRAS, 318, 417, 2000)
- Classification of observed outflows in terms of efficiency of magnetic rotator (Sauty et al, 348, 327, 1999, Sauty et al, AA, 389, 1068, 2002)
- Topological stability of collimated outflows (Vlahakis + Tsinganos, MNRAS, 292, 591, 1997)
- etc, etc.

Reduced Form of MHD Equations for *Axisymmetric* Plasma States

Magnetic and mass flux functions:

Magnetic Flux: $F = \iint_S \vec{\mathbf{B}}_{\mathrm{P}} \cdot d\vec{\mathbf{S}} = 2\pi A\,, \quad \vec{\mathbf{B}}_{\mathrm{P}} = \vec{\nabla} \times \left(\frac{A}{\varpi}\hat{\varphi}\right)\,,$ ← div.B = 0

Mass Flux: $\dot{M} = \iint_S \rho \vec{\mathbf{V}}_{\mathrm{P}} \cdot d\vec{\mathbf{S}} = \frac{\Psi}{2},\ \rho\vec{\mathbf{V}}_{\mathrm{P}} = \vec{\nabla} \times \left(\frac{\Psi}{4\pi\varpi}\hat{\varphi}\right)$ ← div.ρV$_p$ = 0

$\Longrightarrow$ 4 MHD Integrals:

Streamfunction $\Psi(A)$: $\mathbf{V}_{\mathrm{P}} = \frac{\Psi_A}{4\pi\rho}\mathbf{B}_{\mathrm{P}}\,,$

Angular Momentum L(A) : $L(A) = \varpi\left(V_\varphi - \frac{B_\varphi}{\Psi_A}\right)\,,$

Corotation Frequency $\Omega(A)$: $V_\varphi - \Omega\varpi = \frac{\Psi_A}{4\pi\rho}B_\varphi\,,$

Total Energy $E(A)$: $\frac{1}{2}|\vec{V}|^2 + \frac{\gamma}{\gamma-1}\frac{P}{\rho} + \Phi - \frac{\Omega}{\Psi_A}\varpi B_\varphi = E(A)\,,$

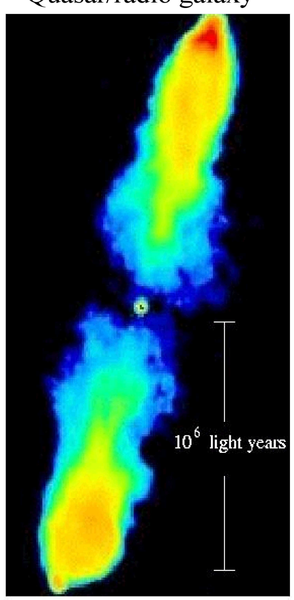
$\Longrightarrow$ Transfield equation for magnetic flux function $A(z,\varphi)$:

[Tsinganos, ApJ, 252, 775, 1982]

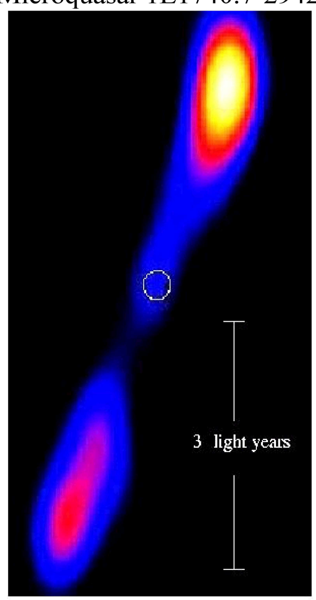
$$\left[1 - \frac{\Psi_A^2}{4\pi\rho}\right]\left[\vec{\nabla}\cdot\left(\frac{\vec{\nabla}A}{\varpi^2}\right)\right] - \Psi_A\left[\frac{\vec{\nabla}A}{\varpi^2}\right]\cdot\left[\vec{\nabla}\left(\frac{\Psi_A}{4\pi\rho}\right)\right] + F(A,\Psi,L,\Omega,\rho) = 0$$

Analytical Solutions: Self-similarity

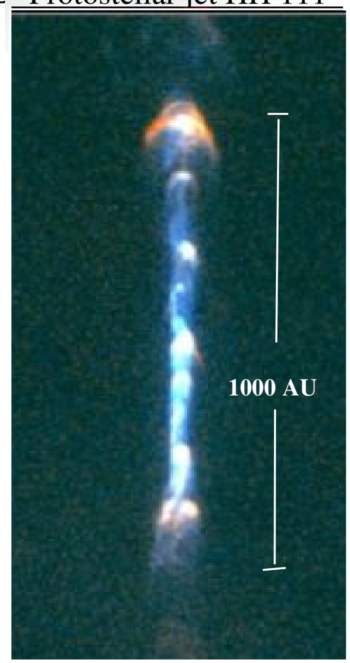
Quasar/radio galaxy



Microquasar 1E1740.7-2942



Protostellar jet HH 111



The problem of a systematic construction of exact models for astrophysical MHD plasma flows

Main assumptions for getting analytical solutions

1. *Ideal* MHD.

2. *Symmetric* outflow configurations, $\partial_3 = 0$, in system (x_1, x_2, x_3) *e.g.*, axisymmetric, or translationally symmetric.

3. *Natural* variables are poloidal Alfvén number and magnetic flux function $(M, A) \Longrightarrow$ switch from (x_1, x_2) to (M, A).

4. Consider Alfvén number $M(x_1, x_2)$ and cross-section of outflow tube $G(x_1, x_2)$ as functions of a single variable χ:

$$M = M(\chi),\ G = G(\chi)$$

__I.__ In spherical coordinates $(x_1 = r\,, x_2 = \theta\,,)$ this unifying scheme contains two large groups of exact MHD outflow models:

(α) $\quad \chi = \theta \longrightarrow$ *radially* self-similar models with *conical* critical surfaces. Prototype is the Blandford & Payne[1] (1982) model :

$$A(r, \theta) = g(\theta) r^x \text{ and } x = 3/4.$$

(β) $\quad \chi = r \longrightarrow$ *meridionally* self-similar models with *spherical* critical surfaces. Prototype is the Sauty & Tsinganos[2] (1994) model :

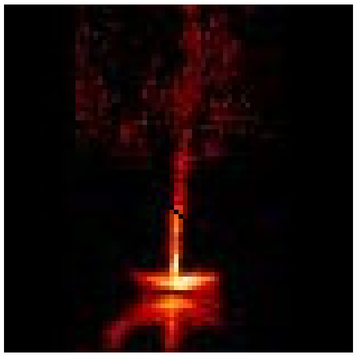
$$A(r, \theta) = f(r) \sin^{2\epsilon}\theta \text{ and } \epsilon{=}1.$$

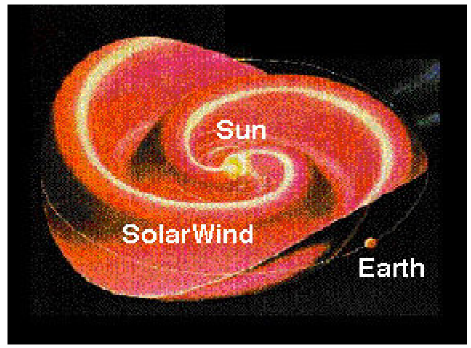
__II.__ In orthogonal coordinates $(x_1 = x\,, x_2 = y\,,)$ this unifying scheme contains the group of *planarly* self-similar MHD models. Prototype is the Petrie et al[3] (2002) model :

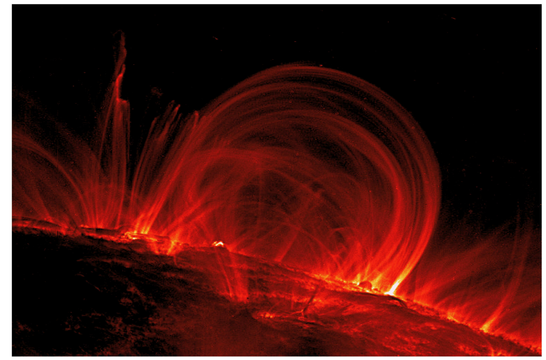
$$A = G(x) e^{-z/H}$$

[†] Vlahakis & Tsinganos 1998, MNRAS, **298**, 777
[1] Blandford & Payne 1982, MNRAS, **199**, 883
[2] Sauty & Tsinganos 1994, A&A, **287**, 893
[3] Petrie, Vlahakis & Tsinganos 2001, A&A, **382**, 1081

The three classes of exact MHD wind/jet models



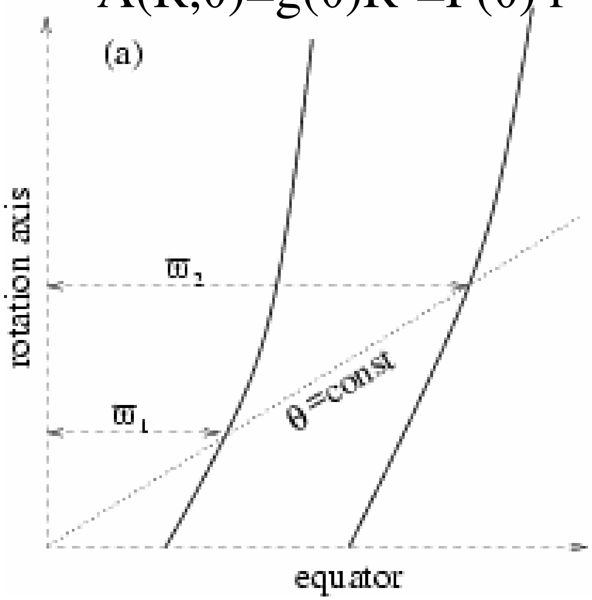




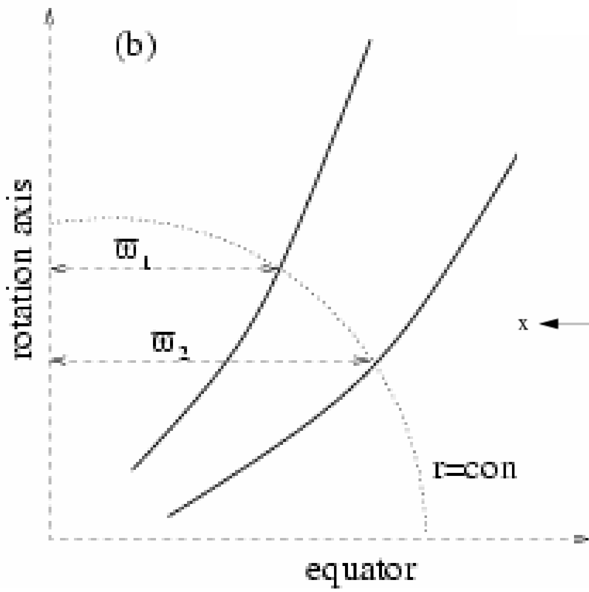
Coronal loop observed with TRACE 26 September 2000

$r = R \sin\theta$

$A(R,\theta) = g(\theta) R^x = F(\theta)\, r^x$

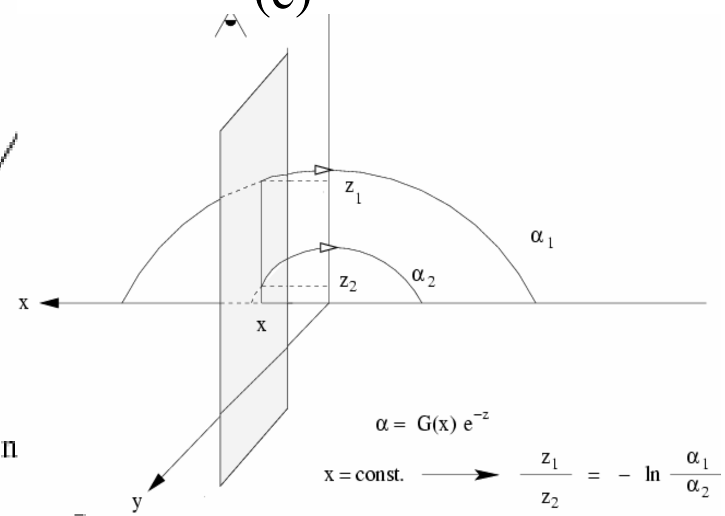


$A(R,\theta) = f(R) \sin^{2\varepsilon}\theta = G(R) r^2$



$A(x,z) = G(x) e^{-z}$

(c)



The problem of a systematic construction of exact models for astrophysical MHD plasma flows

Cases of exact, self-consistent solutions, studied so far :

(i) *Cylindrical* self-similarity, $A = f(\varpi)g(z)$:

Chan & Henriksen, 1980, Ap.J, **241**, 534

(ii) *Radial* self-similarity, $A = f(\theta)r^x$:

(x=3/4: Blandford & Payne, 1982, MNRAS, **199**, 883)

(x$\neq$ 3/4: Contopoulos & Lovelace 1994, ApJ, **429**, 139)

(x$\neq$ 3/4: Vlahakis et al 2000, MNRAS, **318**, 417)

(iii) *Meridional* self-similarity, $A = f(r)\sin^{2\epsilon}\theta$:

($\epsilon \neq 1$: Lima, Priest & Tsinganos, 2001, A&A, **371**, 240)

(ϵ=1: Sauty, Trussoni & Tsinganos 1994, 1997, 1999, 2002, A&A)

(iv) *Planar* self-similarity, $A = f(x)exp(-z)$:

(Petrie, Vlahakis & Tsinganos, 2002, A&A, **382**, 1081)

(v) *General* self-similarity– a unification scheme for all cases :

Vlahakis & Tsinganos, 1998, MNRAS, **298**, 777

In self-similarity, if we know one poloidal streamline we can construct the others. But in order to be able to construct one poloidal streamline, need to calculate $f(\theta)$, or $f(r)$. This is achieved by requiring that the solution pass through appropriate critical points where are found the so-called limiting characteristics, the event horizons of MHD.

Classes of self-similar solutions [Vlahakis+Tsinganos, 298, 777, 1998] :

Meridionally self-similar outflows

Case	$g_1(\alpha)$	$g_2(\alpha)$	$g_3(\alpha)$
(1)	α	$\lambda^2\alpha$	$1+\delta\alpha$
(2)	α	$\xi\alpha+\mu\alpha^\epsilon/\epsilon$	$1+\delta\alpha+\mu\delta_0\alpha^\epsilon$
(3)	α	$\xi\alpha+\mu\alpha\ln\alpha$	$1+\delta\alpha+\mu\delta_0\alpha\ln\alpha$
(4)	$\alpha_0 e^{\frac{\alpha}{\alpha_0}}$	$\lambda e^{\frac{\alpha}{\alpha_0}}$	$1+\delta\alpha e^{\frac{\alpha}{\alpha_0}}+\mu\left(e^{\frac{\alpha}{\alpha_0}}-1\right)$
(5)	$\frac{\alpha_0}{\epsilon}\mid\frac{\alpha}{\alpha_0}-1\mid^{\epsilon-1}\left(\frac{\alpha}{\alpha_0}-1\right)$	$\xi\mid\frac{\alpha}{\alpha_0}-1\mid^\epsilon$	$1+\delta\mid\frac{\alpha}{\alpha_0}-1\mid^\epsilon+\mu\mid\frac{\alpha}{\alpha_0}-1\mid^{\epsilon-1}-\delta-\mu$
(6)	$-\alpha_0\ln\mid\frac{\alpha}{\alpha_0}-1\mid$	$\xi\ln\mid\frac{\alpha}{\alpha_0}-1\mid$	$1+\delta\ln\mid\frac{\alpha}{\alpha_0}-1\mid+\mu\frac{\alpha}{\alpha_0(\alpha-\alpha_0)}$
(7)	$\frac{\alpha}{1-\alpha_0}$	$\mu\ln\frac{\alpha}{\alpha_0}+\xi\alpha$	$1+\delta(\alpha-\alpha_0)+\mu\delta_0\ln\frac{\alpha}{\alpha_0}$
(8)	$\frac{\alpha_0}{\epsilon(1-\alpha_0)}\left(\frac{\alpha}{\alpha_0}\right)^\epsilon$	$\lambda_1\alpha^\epsilon+\lambda_2\alpha^{\epsilon-1}$	$1+\delta_1(\alpha^\epsilon-\alpha_0^\epsilon)+\delta_2\left(\alpha^{\epsilon-1}-\alpha_0^{\epsilon-1}\right)$
(9)	$\frac{\alpha_0}{1-\alpha_0}\ln\frac{\alpha}{\alpha_0}$	$\lambda_1\ln\frac{\alpha}{\alpha_0}+\frac{\lambda_2}{\alpha}$	$1+\delta_1\ln\frac{\alpha}{\alpha_0}+\delta_2\left(\frac{1}{\alpha}-\frac{1}{\alpha_0}\right)$

λ=0, δ=0: Parker wind

Sauty+kT (1984 - 2004)

Lima et al (1986)

Radially self-similar outflows

Case	$q_1(\alpha)$	$q_2(\alpha)$	$q_3(\alpha)$
(1)	$\frac{E_1}{F-2}\alpha^{F-2}$	$\frac{D_1}{F-2}\alpha^{F-2}$	$\frac{C_1}{F-2}\alpha^{F-2}$
(2)	$E_1\ln\alpha$	$D_1\ln\alpha$	$C_1\ln\alpha$
(3)	$E_1\alpha^{x_1}+E_2\alpha^{x_2}$	$D_1\alpha^{x_1}+D_2\alpha^{x_2}$	$C_1\alpha^{x_1}+C_2\alpha^{x_2}$
(4)	$E_1\ln\alpha+E_2\alpha^x$	$D_1\ln\alpha+D_2\alpha^x$	$C_1\ln\alpha+C_2\alpha^x$
(5)	$E_1(\ln\alpha)^2+E_2\ln\alpha$	$D_1(\ln\alpha)^2+D_2\ln\alpha$	$C_1(\ln\alpha)^2+C_2\ln\alpha$
(6)	$E_1\alpha^x\ln\alpha+E_2\alpha^x$	$D_1\alpha^x\ln\alpha+D_2\alpha^x$	$C_1\alpha^x\ln\alpha+C_2\alpha^x$

F=3/4: Blandford & Payne (1982)

The problem of singularities/critical points :

(1) Equation for derivative of poloidal Alfvén number M_a:

$$\frac{\mathrm{d}M_a^2}{\mathrm{d}R} = \frac{N_M(R, F, M_a; \text{parameters})}{D(R, F, M; \text{parameters})},$$

(2) Equation for derivative of thermal pressure P_0:

$$\frac{dP_0}{dR} = \frac{N_P(R, F, M_a; \text{parameters})}{D(R, F, M; \text{parameters})},$$

(3) Equation for derivative of expansion function, or P_1 :

$$\frac{\mathrm{d}F}{\mathrm{d}R} = \frac{N_F(R, F, M_a; \text{parameters})}{D(R, F, M; \text{parameters})},$$

Difficulty: A physically accepted solution is determined by the requirement that it should pass through critical points which are not known *a priori* but are only determined simultaneously with the complete solution !

Singularities (Critical Points) : $\quad N_M = N_F = N_P = D = 0.$

(a) Alfvén transition (star-type singularity): $M_a = 1$

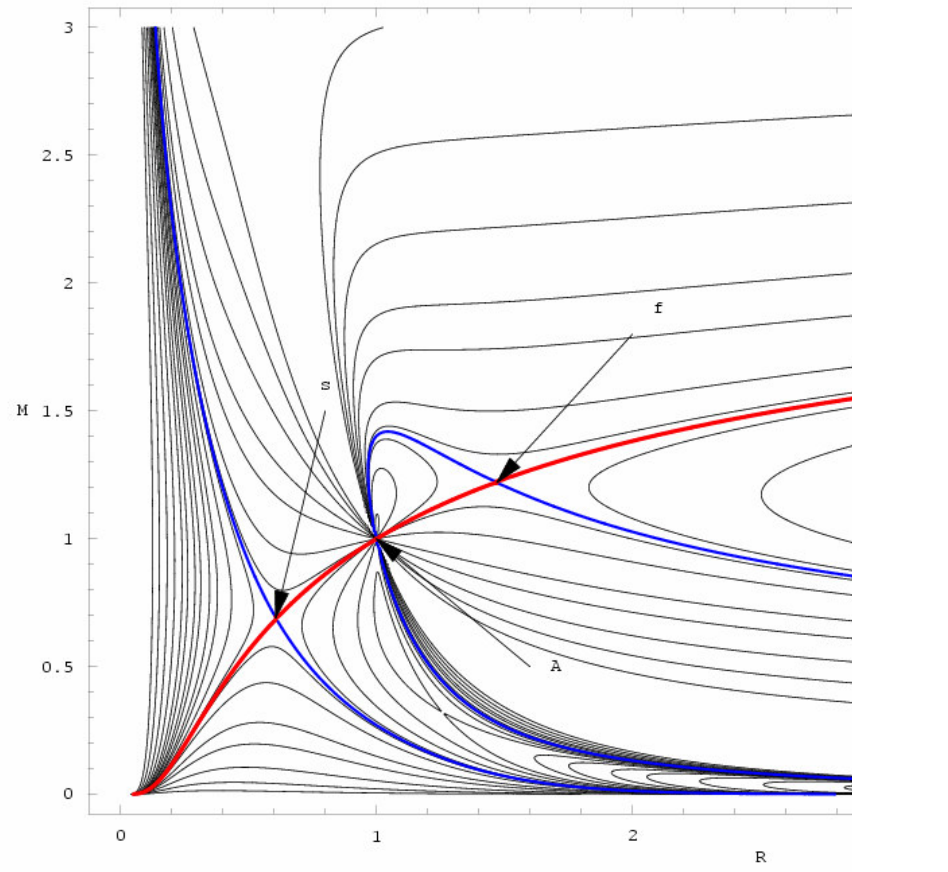
imposes regularity condition $\Longleftrightarrow$ streamlines avoid kink.

(b) X-type MHD singularities selecting physical solution (a proxy for the imposition of physical b.c.'s at r_o and ∞).

$\Longrightarrow$ Obtain unique solution through critical points.

BUT

at which speeds are found these MHD saddle-type critical points ?



Nature of MHD PDE's & correct boundary conditions

1. *Elliptic* PDE's :

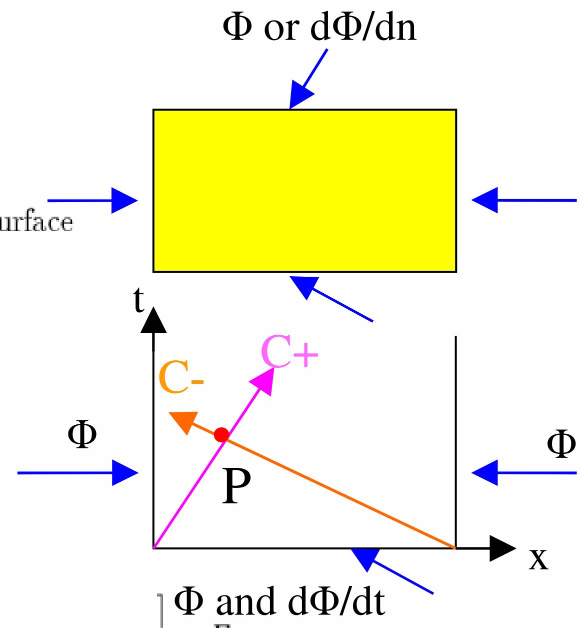
$$\left(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0\right)$$

$\Longrightarrow$ *Dirichlet or Newmann* B.C.'s on a <u>closed</u> surface

2. *Hyperbolic* PDE's :

$$\left(\frac{\partial^2\Phi}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}=0\right)$$

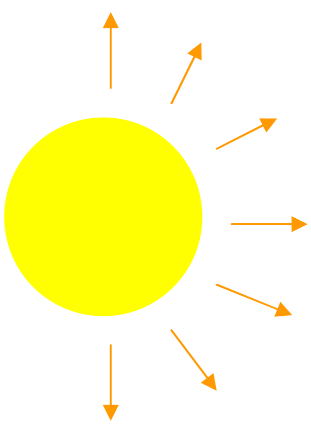
$\Longrightarrow$ *Cauchy* B.C.'s on an <u>open</u> surface.

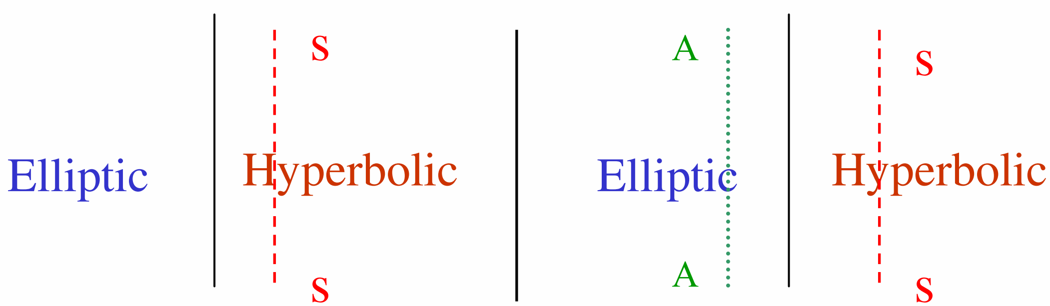


3. *Mixed Elliptic/Hyperbolic* PDE's :

$$\left[\frac{1-M_a^2}{h_3^2}\right]\left[\nabla^2 A-\frac{\vec{\nabla}A\cdot\vec{\nabla}(\vec{\nabla}A)^2}{2(\vec{\nabla}A)^2}\frac{V_p^4}{V_p^4-V_p^2(C_s^2+V_a^2)+C_s^2V_{ap}^2}\right]=F_o$$

Elliptic in domains E_i, hyperbolic in domains H_i, i=1,2, ...





$\Longrightarrow$ B.C.'s on separatrices SS' in hyperbolic domains H_i.

But, these separatrices SS' in domains H_i are not known *a priori* but should be constructed simultaneously with solution.

The problem of causality and limiting characteristics:

The set of steady MHD equations are of mixed elliptic/hyperbolic character.
In hyperbolic regimes exist separatrices separating causally areas which cannot communicate with each other via an MHD signal. [They are the analog of the limiting cycles in Van der Pol's nonlinear differential equation, or, the event horizon in relativity.]

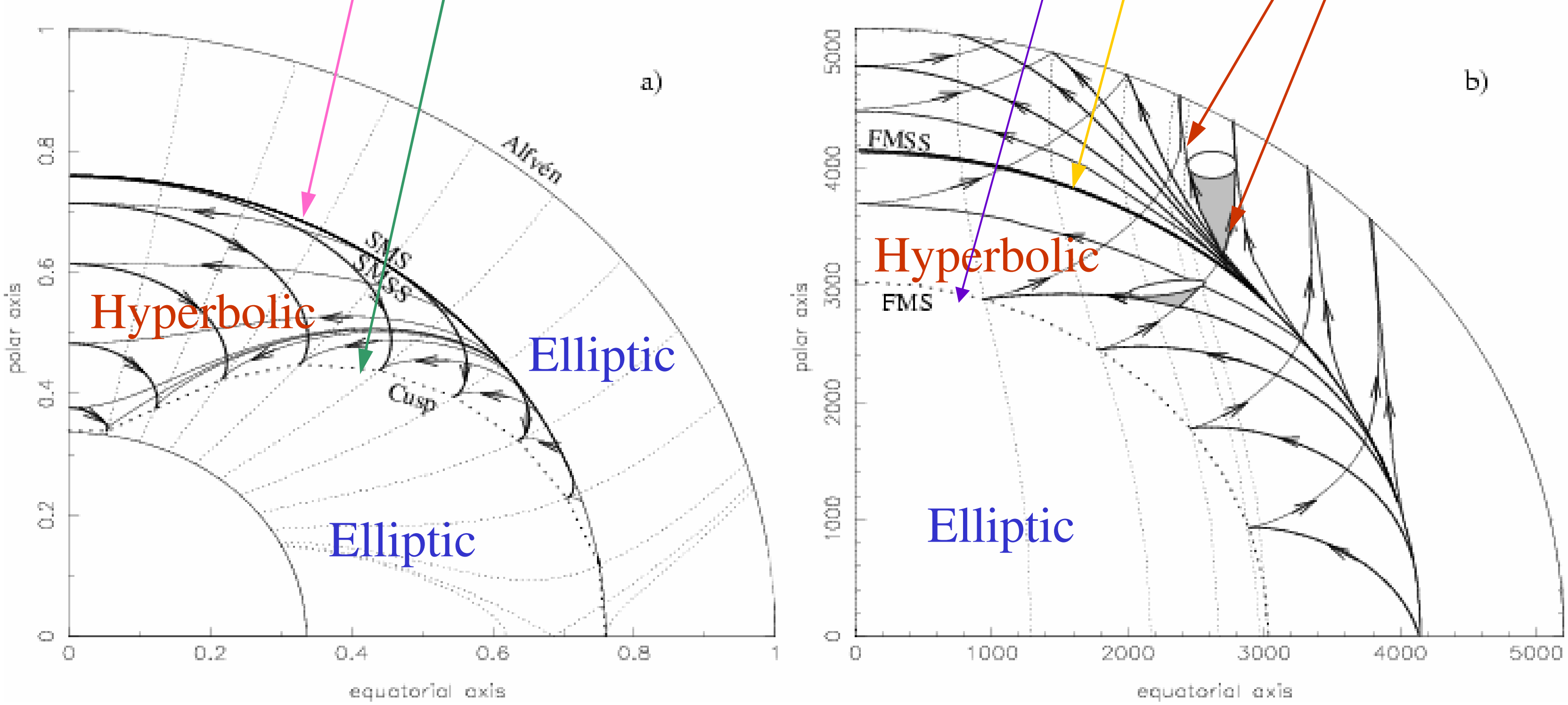
The MHD critical points appear on these separatrices which do not coincide in general with the fast/slow MHD surfaces. To construct a correct solution we need to know the limiting characteristics, but this requires an a priori knowledge of the solution we seek for !

Tsinganos et al, MNRAS, 283, 811, 1996

Plot of the characteristics in the 2nd hyperbolic regime of a meridionally self-similar jet.

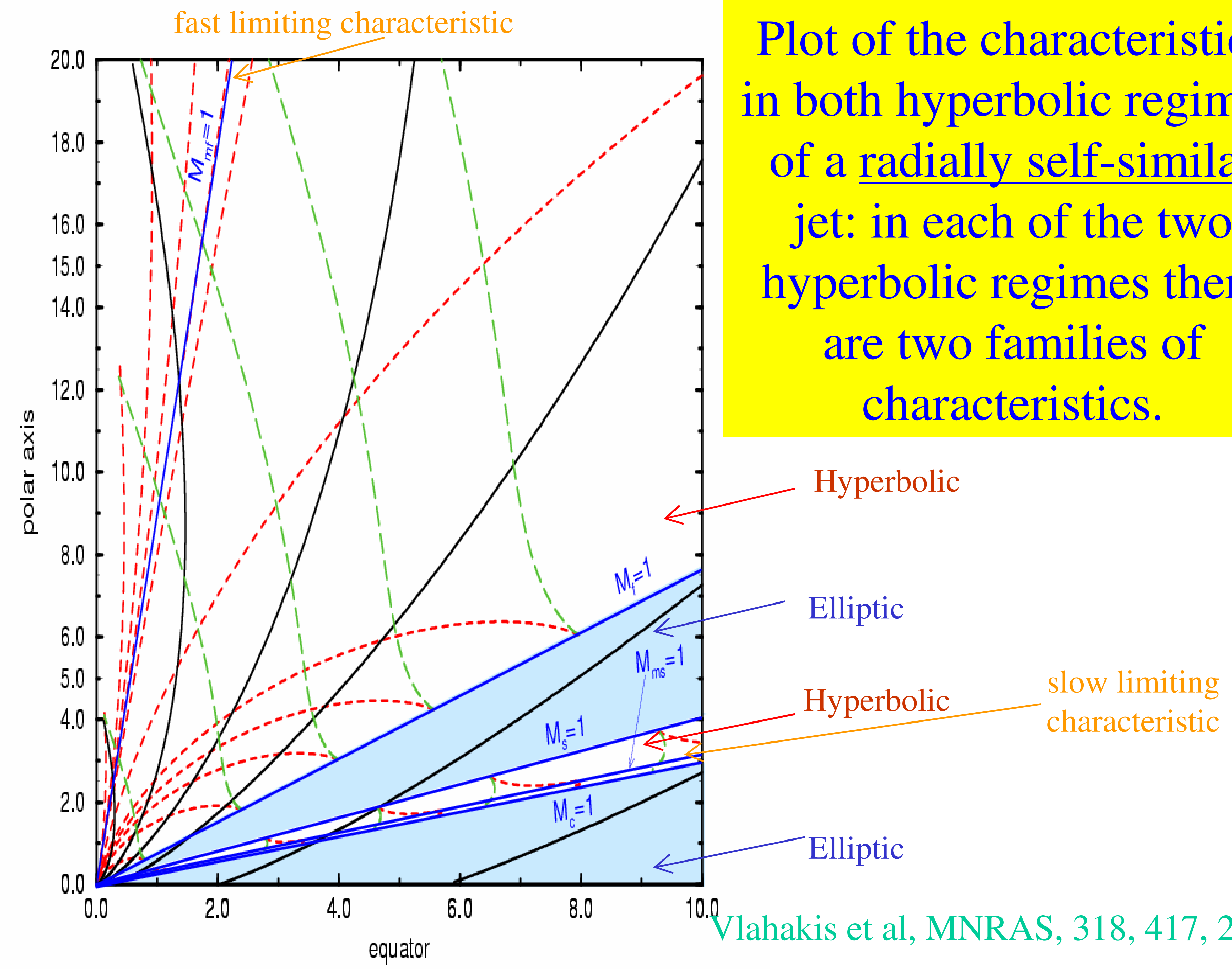
Dotted lines: poloidal magnetic field lines,
Solid lines (characteristics)
Thick-dotted line (cusp surface),
Dot-dashed line (slow-magnetosonic)

in hyperbolic regime there are two families of characteristics – one of them is tangent to the separatrix-limiting characteristic.
Dotted line (fast-magnetosonic)



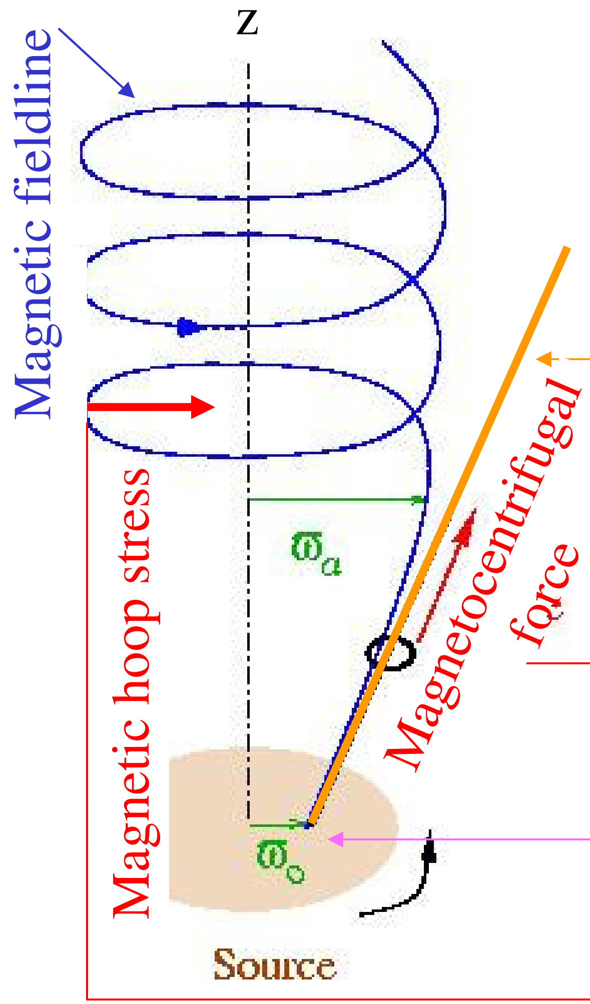
[see Sauty, Trussoni and Tsinganos AA, 421, 797, 2004]

Plot of the characteristics in both hyperbolic regimes of a radially self-similar jet: in each of the two hyperbolic regimes there are two families of characteristics.



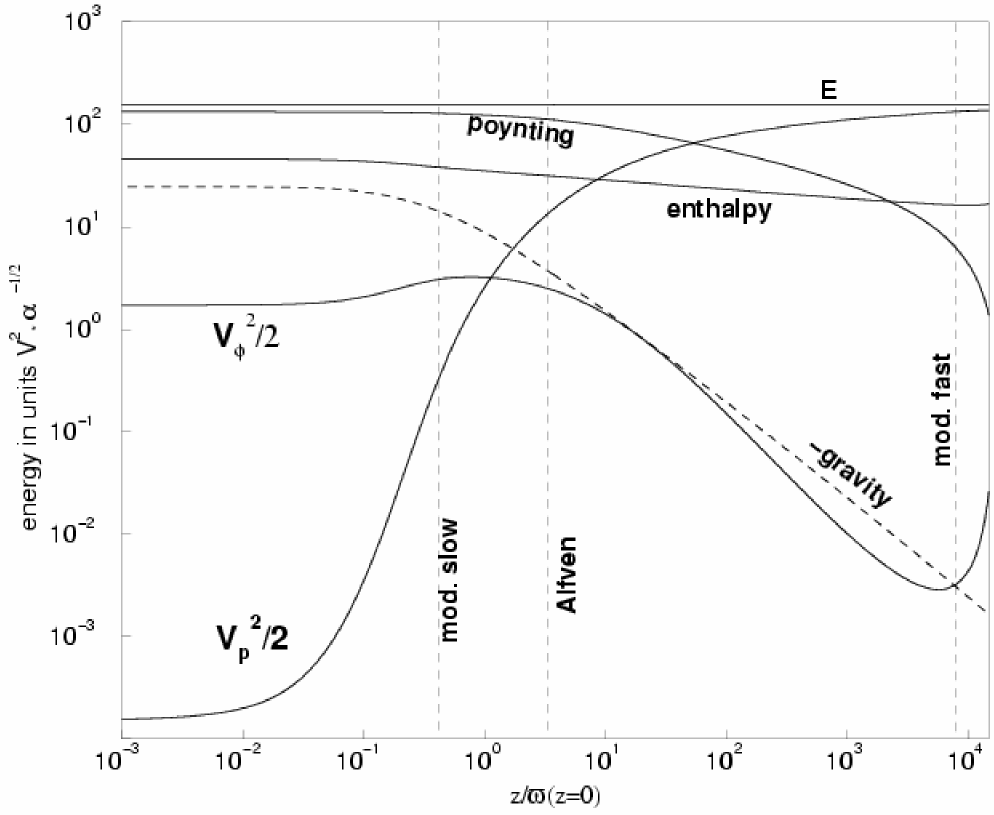
Vlahakis et al, MNRAS, 318, 417, 2000

Basics of jet acceleration and collimation

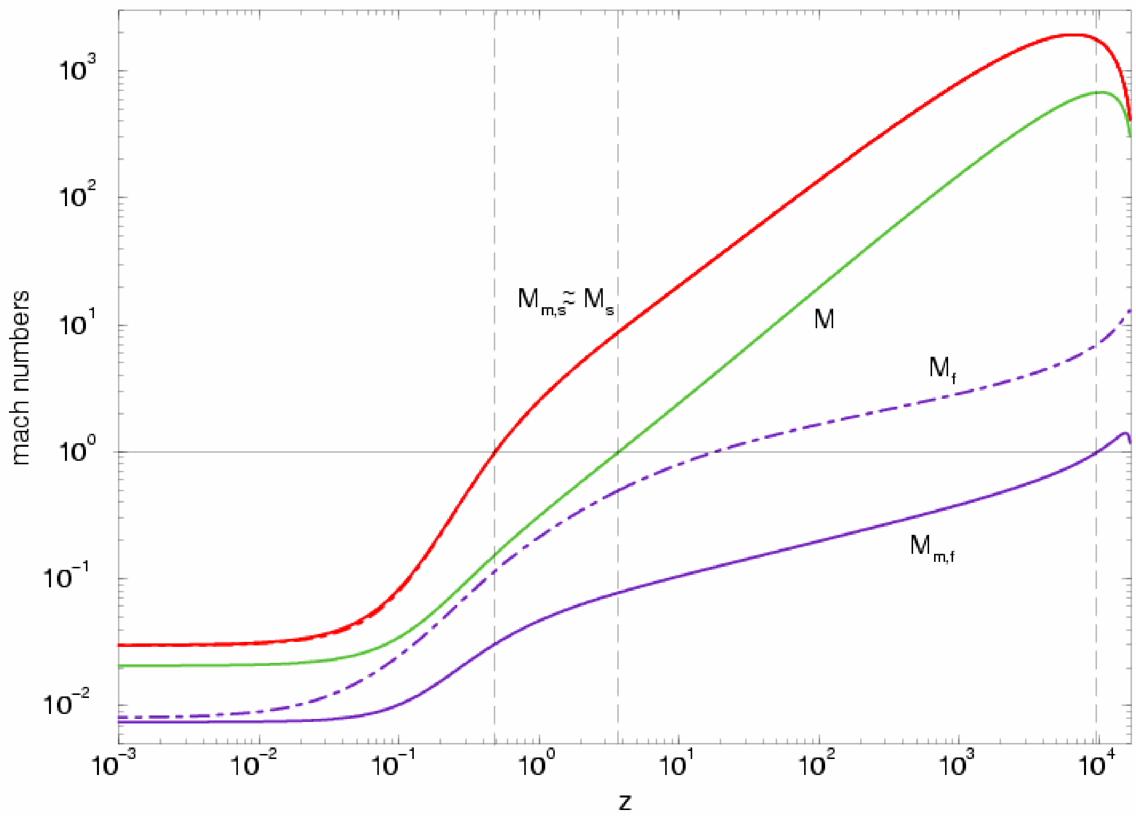


- On the disk, z=0, the rotational kinetic energy exceeds the magnetic enegy →Keplerian rotation of the B-field line rooted at r_o.
- Up to the Alfven distance, the B-field is strong enough → forces the plasma to follow the Keplerian rotation of the roots of the magnetic fieldline. In particular, when the inclination angle is less than 60°, we have the "bead on a rotating wire" magnetocentrifugal acceleration.
- After the Alfven distance, the poloidal B-field energy is weaker than the poloidal kinetic motion → the B-field follows the plasma. The plasma inertia leaves it behind the rotating B-line → creation of strong B_φ
- The created strong B_φ collmates the magnetic field lines towards the z-axis and forms the jet.

Poynting driving in magnetocentrifugal disk-winds:



Mach numbers in such magnetocentrifugal disk-winds:



Removal of disk angular momentum by magnetocentrifugal disk-winds:

A Keplerian disk (Ω_K) accreting at a rate $\dot{M}_a$ needs to get rid of angular momentum in a radius ϖ_o :

$$\dot{J}_a = \frac{1}{2}\Omega_K \varpi_o^2 \dot{M}_a$$

A disk-wind carries away angular momentum with a rate :

$$\dot{J}_w = \Omega_K \varpi_A^2 \dot{M}_w$$

If the disk-wind carries away a fraction f ($0 < f < 1$) of the angular momentum of the accreting matter,

$\dot{J}_w = f \dot{J}_a$, then

$$\frac{\dot{M}_w}{\dot{M}_a} = \frac{f}{2}\frac{\varpi_o^2}{\varpi_A^2}$$

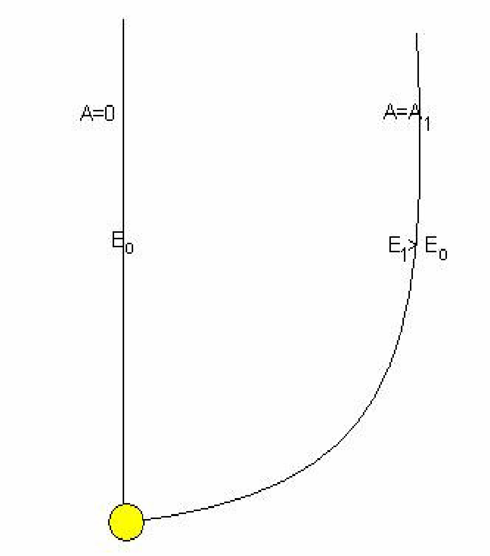
With a magnetic lever arm $\varpi \sim 5\varpi_o$, the disk-wind needs to carry away only a few percent of the accreting mass rate.

An energetic criterion for cylindrical collimation:

$$\varepsilon' = \frac{\Delta(\rho E)}{\rho L \Omega}$$

$\Delta f = f$(non polar streamline) - f(polar axis)

- $\varepsilon' < 0$ --> No collimation
- $\varepsilon' > 0$ --> Collimation



$$\varepsilon' = \mu + \varepsilon$$

Efficiency of Pressure Confinement

$$\mu \sim \frac{\Delta P}{P} = \kappa$$

Efficiency of the Magnetic Rotator

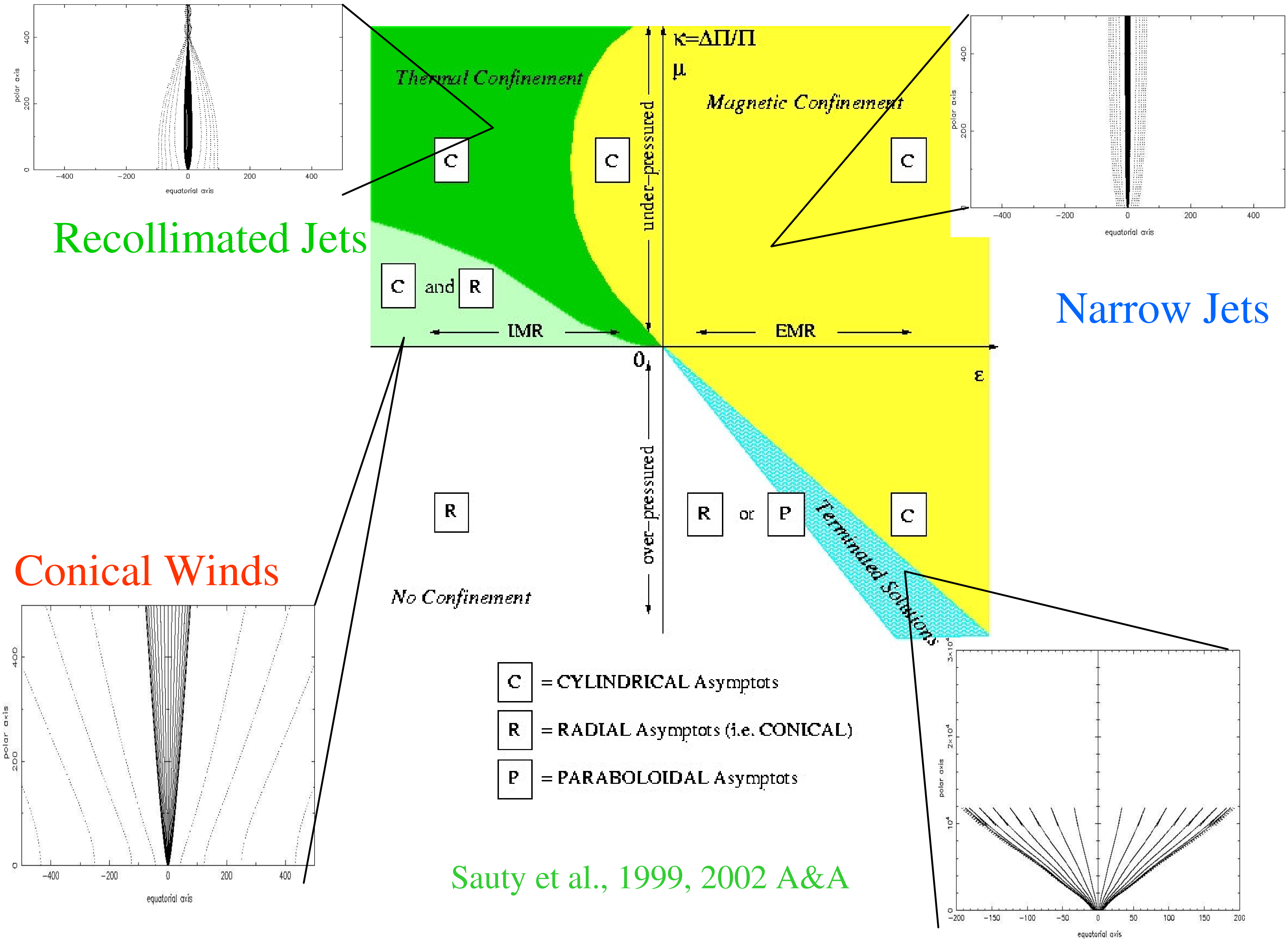
$$\varepsilon = \frac{L\Omega - E_{R,o} + \Delta E_G^*}{L\Omega}$$

where

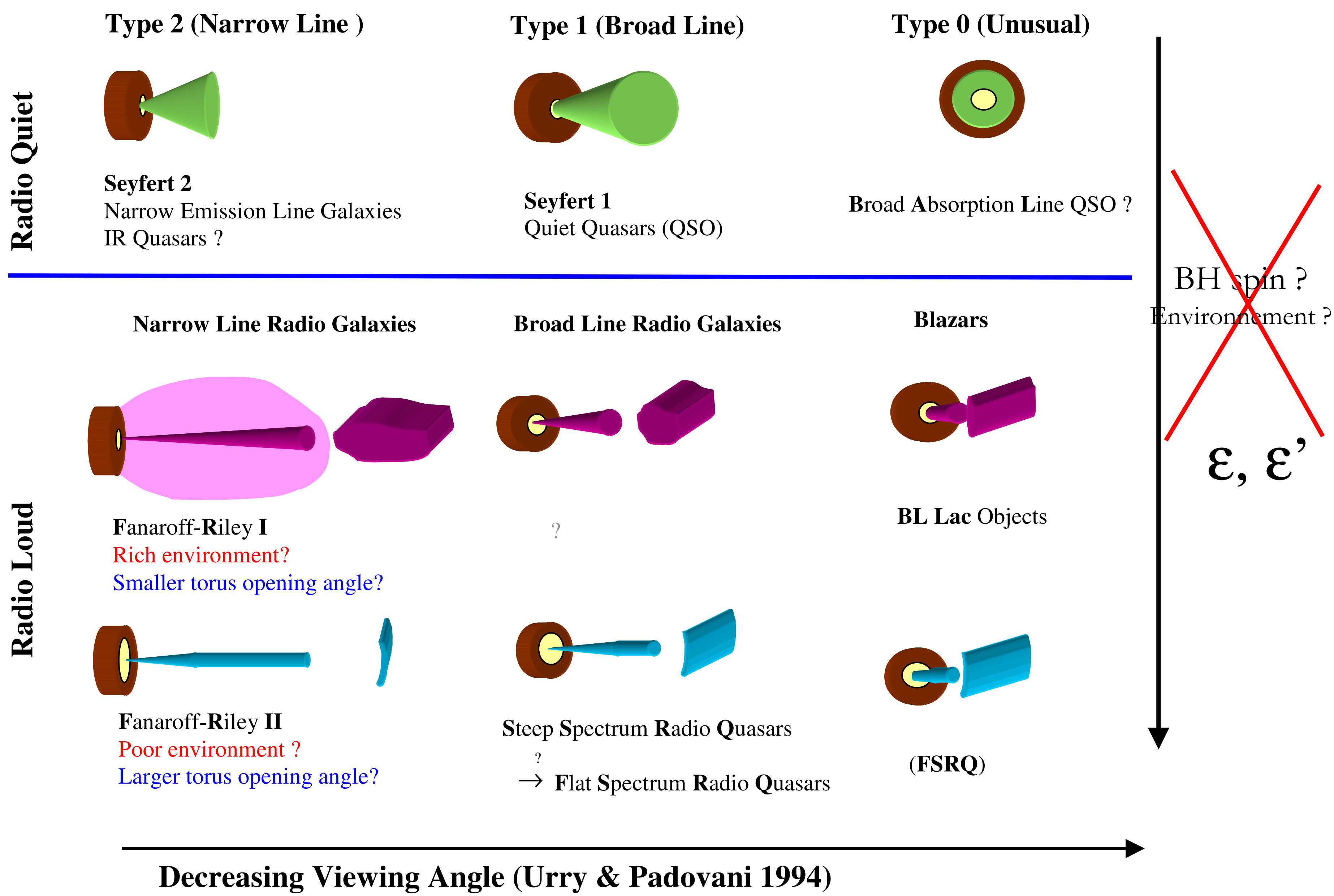
$$\Delta E_G^* = -\frac{GM}{r_0}\left(\frac{-\Delta T}{T_0}\right)$$

- $\varepsilon > 0$ --> Efficient Magnetic Rotator (EMR)
- $\varepsilon < 0$ --> Inefficient Magnetic Rotator (IMR)

A classification of MHD outflows



A classification of AGN jets :



But, how general are all those conclusions of the steady exact MHD modelling ?

III) Time-dependent studies

First, demonstration of the formation of a collimated jet once an outflow along a monopole (radial) magnetic field, starts rotating

(jet formation as seen by a naked eye in video).

For details, see :
Bogovalov + Tsinganos, MNRAS, 305, 211, 1999, MNRAS, 325, 249, 2001, MNRAS, 357, 918, 2005
Tsinganos + Bogovalov, AA, 356, 989, 2000, MNRAS, 337, 553, 2002,

Time-dependent MHD equations

$\mathbf{B} = \mathbf{B}_{\mathrm{p}} + \mathbf{B}_{\varphi}\,,\ \mathbf{B}_{\mathrm{p}} = \nabla \times \dfrac{A(z,\varpi,t)\hat{\varphi}}{\varpi}$ Define poloidal magnetic field in terms of vector potential A_{φ}

$$\frac{\partial A}{\partial t} = -V_{\varpi}\frac{\partial A}{\partial \varpi} - V_z\frac{\partial A}{\partial z},$$

Poloidal component of induction equation

$$\frac{\partial \rho}{\partial t} = -\frac{1}{\varpi}\frac{\partial}{\partial \varpi}(\rho\varpi V_{\varpi}) - \frac{\partial}{\partial z}(\rho V_z),$$

Continuity equation

$$\frac{\partial B_{\varphi}}{\partial t} = \frac{\partial}{\partial z}(V_{\varphi}B_z - V_z B_{\varphi}) - \frac{\partial}{\partial \varpi}(V_{\varpi}B_{\varphi} - V_{\varphi}B_{\varpi}),$$

Azimuthal component of induction equation

$$\frac{\partial V_{\varphi}}{\partial t} = -\frac{V_{\varpi}}{\varpi}\frac{\partial}{\partial \varpi}(\varpi V_{\varphi}) - V_z\frac{\partial V_{\varphi}}{\partial z} + \frac{1}{4\pi\rho}\left(B_{\varpi}\frac{\partial}{\varpi\partial\varpi}(\varpi B_{\varphi}) + B_z\frac{\partial B_{\varphi}}{\partial z}\right),$$

Azimuthal component of momentum equa

$$\frac{\partial V_z}{\partial t} = -V_{\varpi}\frac{\partial V_z}{\partial \varpi} - V_z\frac{\partial V_z}{\partial z} - \frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{GMz}{r^3} - -\frac{1}{8\pi\rho\varpi^2}\frac{\partial}{\partial z}(\varpi B_{\varphi})^2 - \frac{B_{\varpi}}{4\pi\rho}\left(\frac{\partial B_{\varpi}}{\partial z} - \frac{\partial B_z}{\partial \varpi}\right),$$

z- component of momentum equation

$$\frac{\partial V_{\varpi}}{\partial t} = -V_{\varpi}\frac{\partial V_{\varpi}}{\partial \varpi} - V_z\frac{\partial V_{\varpi}}{\partial z} - \frac{1}{\rho}\frac{\partial P}{\partial \varpi} - \frac{GM\varpi}{r^3} - \frac{1}{8\pi\rho\varpi^2}\frac{\partial}{\partial \varpi}(\varpi B_{\varphi})^2 + \frac{V_{\varphi}^2}{\varpi} + \frac{B_z}{4\pi\rho}\left(\frac{\partial B_{\varpi}}{\partial z} - \frac{\partial B_z}{\partial \varpi}\right),$$

Radial component of momentum equ

$(z,\varpi,\varphi) \Longrightarrow$ cylindrical coordinates,
$\rho(z,\varpi,t) \Longrightarrow$ density,
$\vec{V}(z,\varpi,t) \Longrightarrow$ flow speed,
$\vec{B}(z,\varpi,t) \Longrightarrow$ magnetic field,
$A(z,\varpi,t) \Longrightarrow$ poloidal magnetic flux.

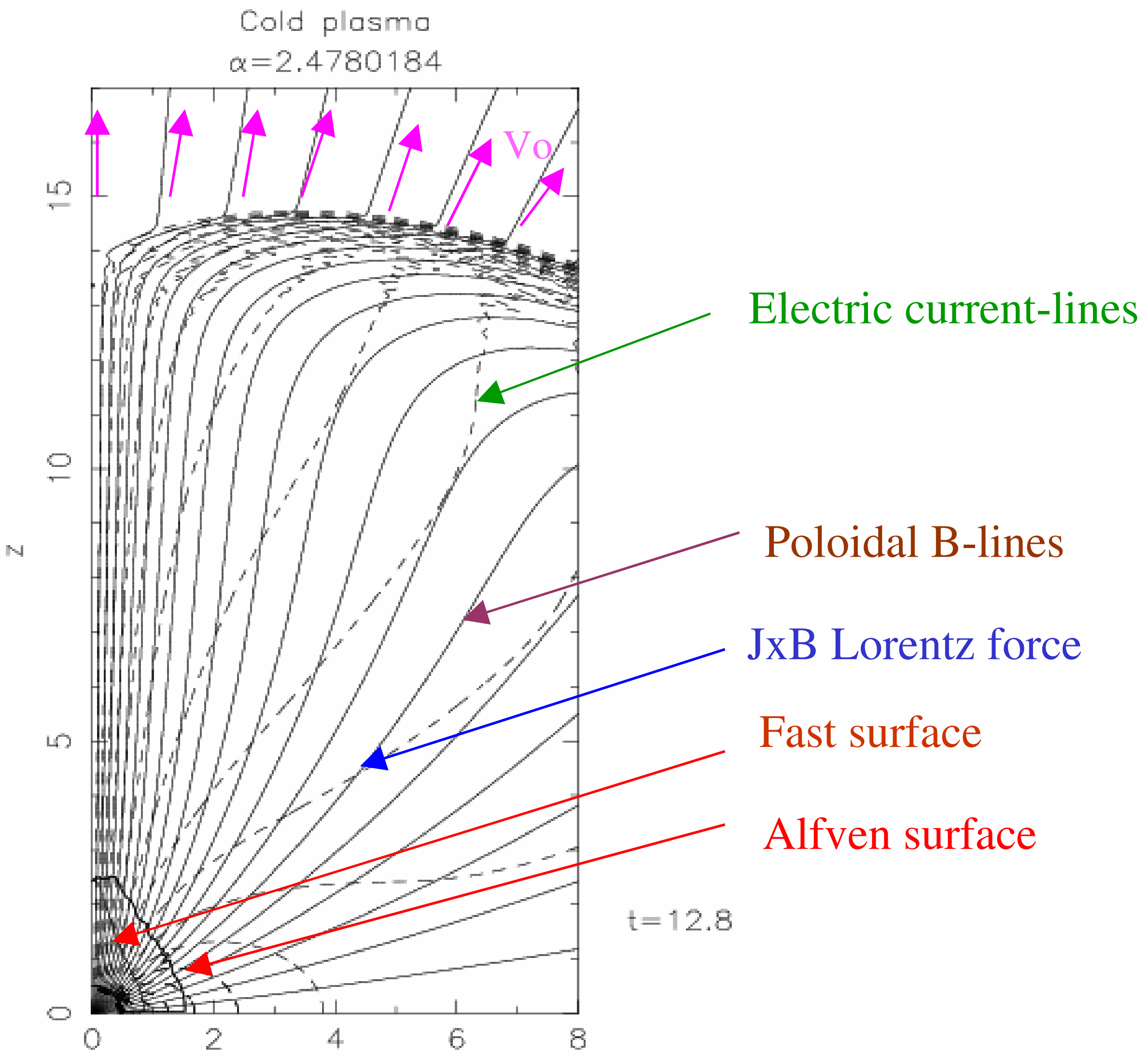
Close system either with polytropic equation of state,
or, by adding an energy equation.

A near zone snapshot on the poloidal plane showing the change of shape of the poloidal magnetic field from an initially uniform with latitude radial monopole (before a stationary state is reached).

$\alpha = \Omega R_a / V_o = 2\pi T_{travel} / T_{rotation}$

$\alpha < 1$: Slow rotators

$\alpha > 1$: Fast rotators

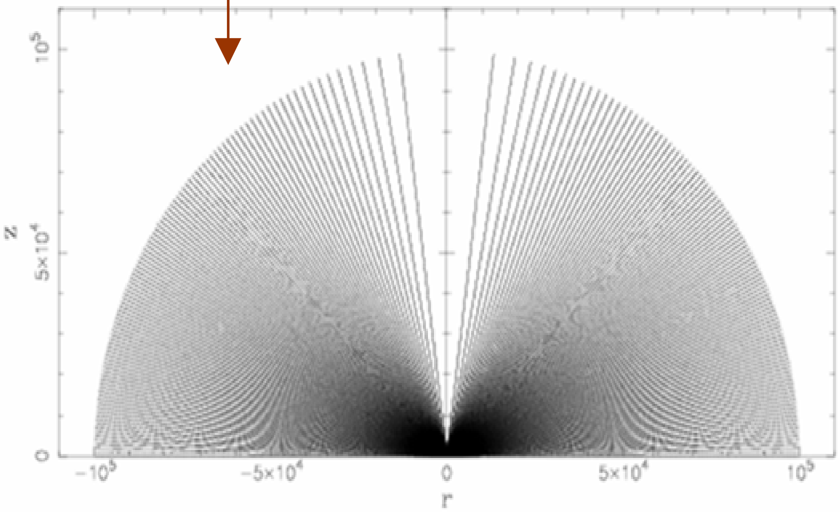


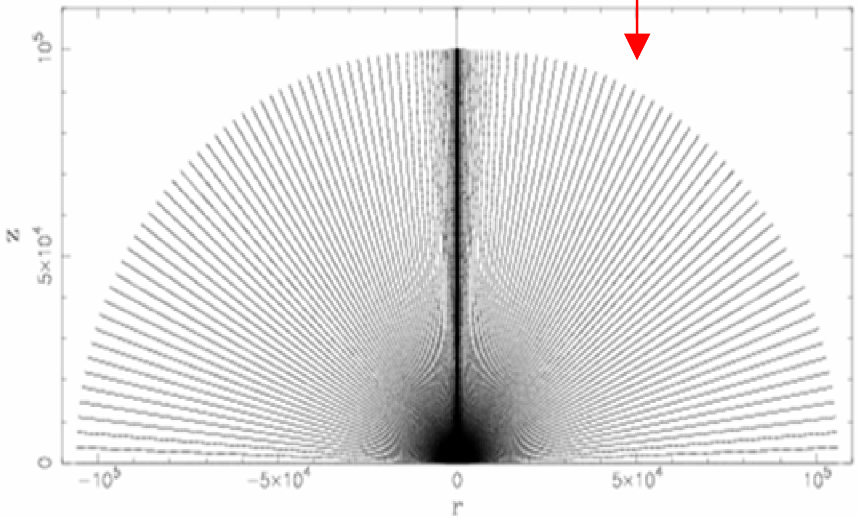
Far Zone :

Poloidal magnetic lines of outflow :

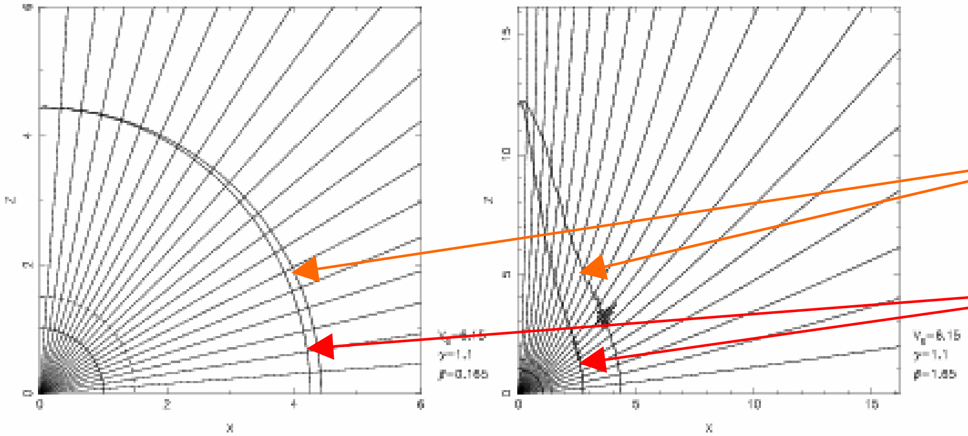
(at intervals of equal magnetic flux)

Before rotation starts – After rotation started





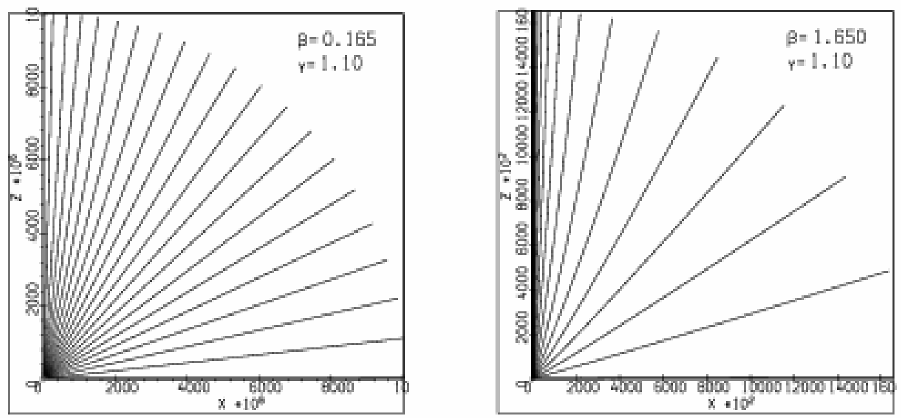
Solution in Near Zone



Fast surface

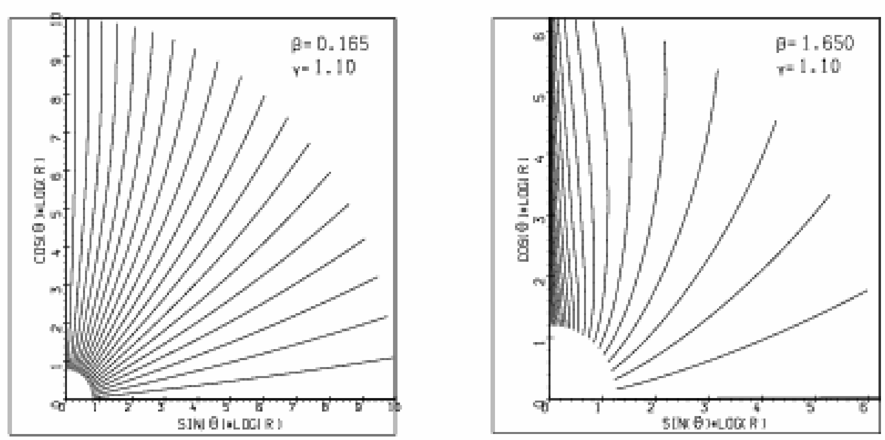
Alfven surface

Weak collimation of outflow from a SLOW magnetic rotator (e.g., our Sun)



Strong collimation of outflow from a FAST magnetic rotator (e.g., a 10 times faster rotating YSO)

Solution in far zone (Logarithmic Plot !)



Solution in far zone (Logarithmic Plot !)

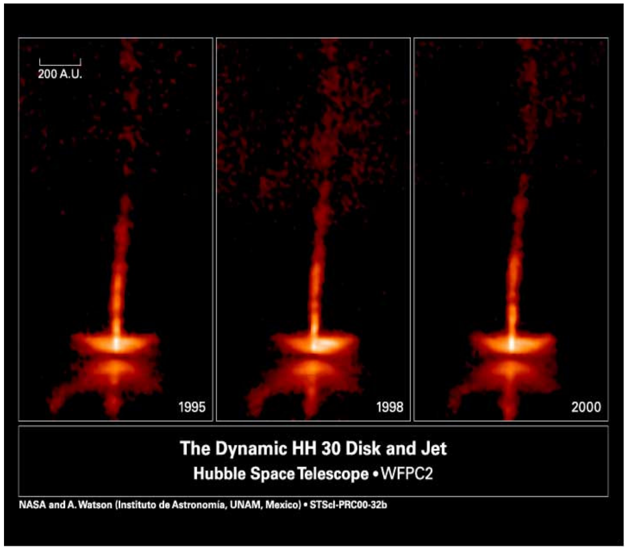
Bogovalov+kT, MNRAS, 305, 211, 1999

Astrophysical implications: an evolutionary scenario

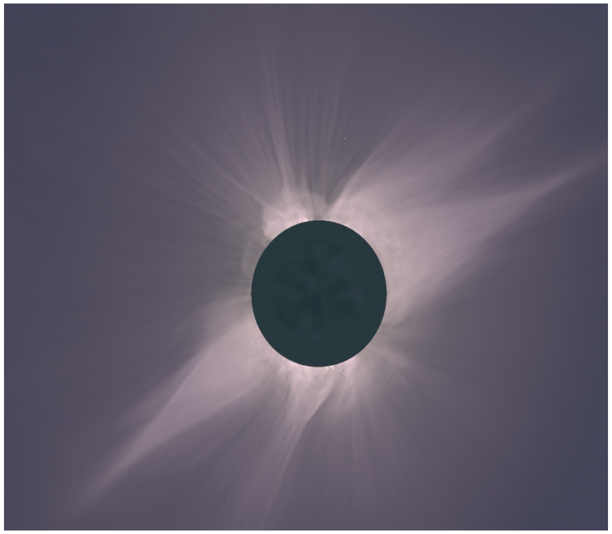
- Start with a non rotating star/disk system having a radial magnetic field.
- As system rotates, poloidal fieldlines focus towards axis by magnetic tension.
- Parameter α = (corotating speed at Alfvén distance)/ (initial flow speed).
- Radius of formed jet $R_{jet} \propto 1/\alpha$.
- Significant flow collimation in fast magnetic rotators with $\alpha > 1$, while very weak collimation in slow magnetic rotators with $\alpha < 1$.
- Reversing time: start with a tightly collimated outflow ($\alpha > 1$) and reduce α to small values, $\alpha \rightarrow 0$:
  $\Longrightarrow$ sequence analoguous to evolution of outflow geometry from a YSO.
- Efficient magnetic rotators have large values of α
  $\Longrightarrow$ highly collimated jets, as observed in association with YSOs.
- As star ages loosing angular momentum it may gradually shift to the stage of a slow magnetic rotator with small α values like our own Sun which produces the almost radial outflow of the solar wind.
- Scenario agrees very well with results of steady (self-similar) modelling where a quantitative *energetic criterion* is given for separating loosely collimated winds associated with efficient magnetic rotators from tightly collimated jets associated with efficient magnetic rotators.
- For large values of α, the magnitude of the flow speed in the jet remains below the fast MHD wave speed everywhere
  $\Longrightarrow$ outflows from classical thin accretion disks may rather be nonstationary and turbulent.

The Dichotomy of Winds and Jets

- Jets = tight collimation

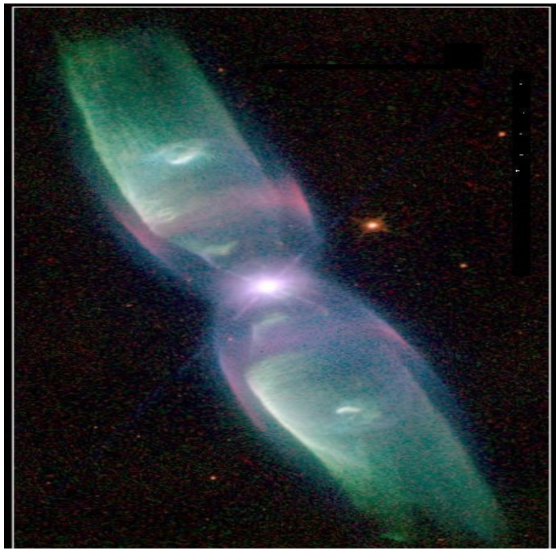


- Winds = no collimation



1. Star-birth (YSO)

2. Solar Wind



Stellar Evolution

3. Star-death (PN)

However, several pbs with 1-component outflows:

Small fraction of mass and magnetic flux is collimated in a single-component outflow.

Weak collimation in case in the central outflow there is no available a strong azimuthal magnetic field, or, the central flow is relativistic (as in AGN jets).

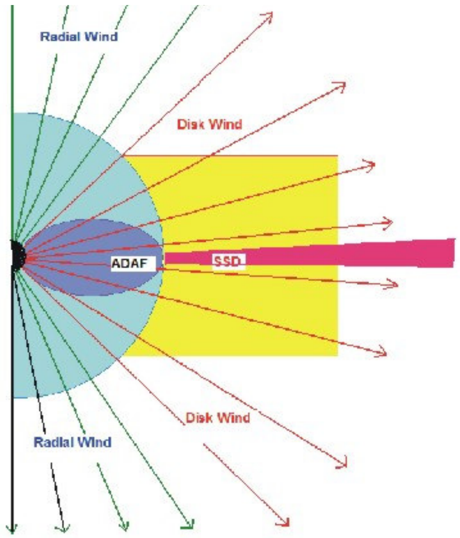
Note also that disk-wind models are "singular" around rotation axis.

A central magnetized wind is needed to slow down the young star

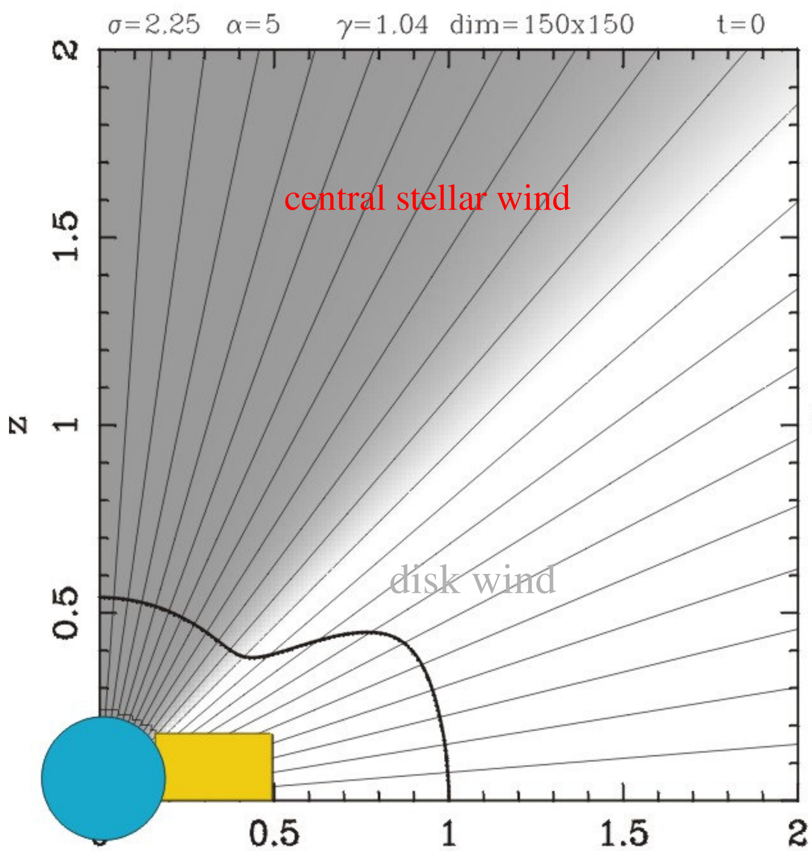
A two-component outflow model with :

a) a central stellar wind

b) a disk-wind



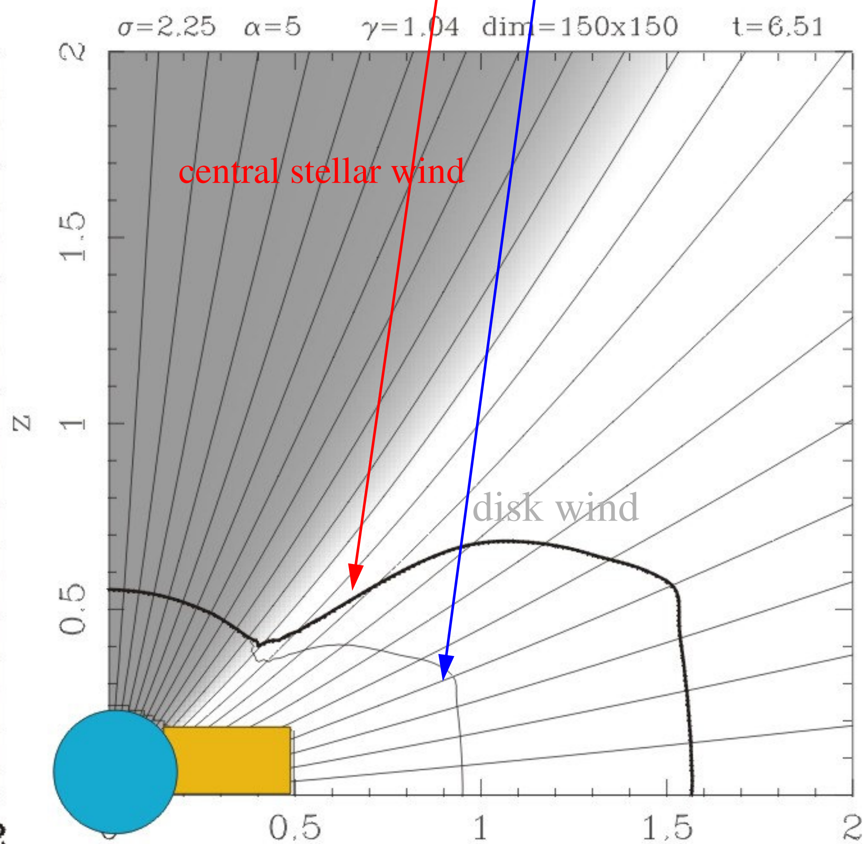
Before rotation starts



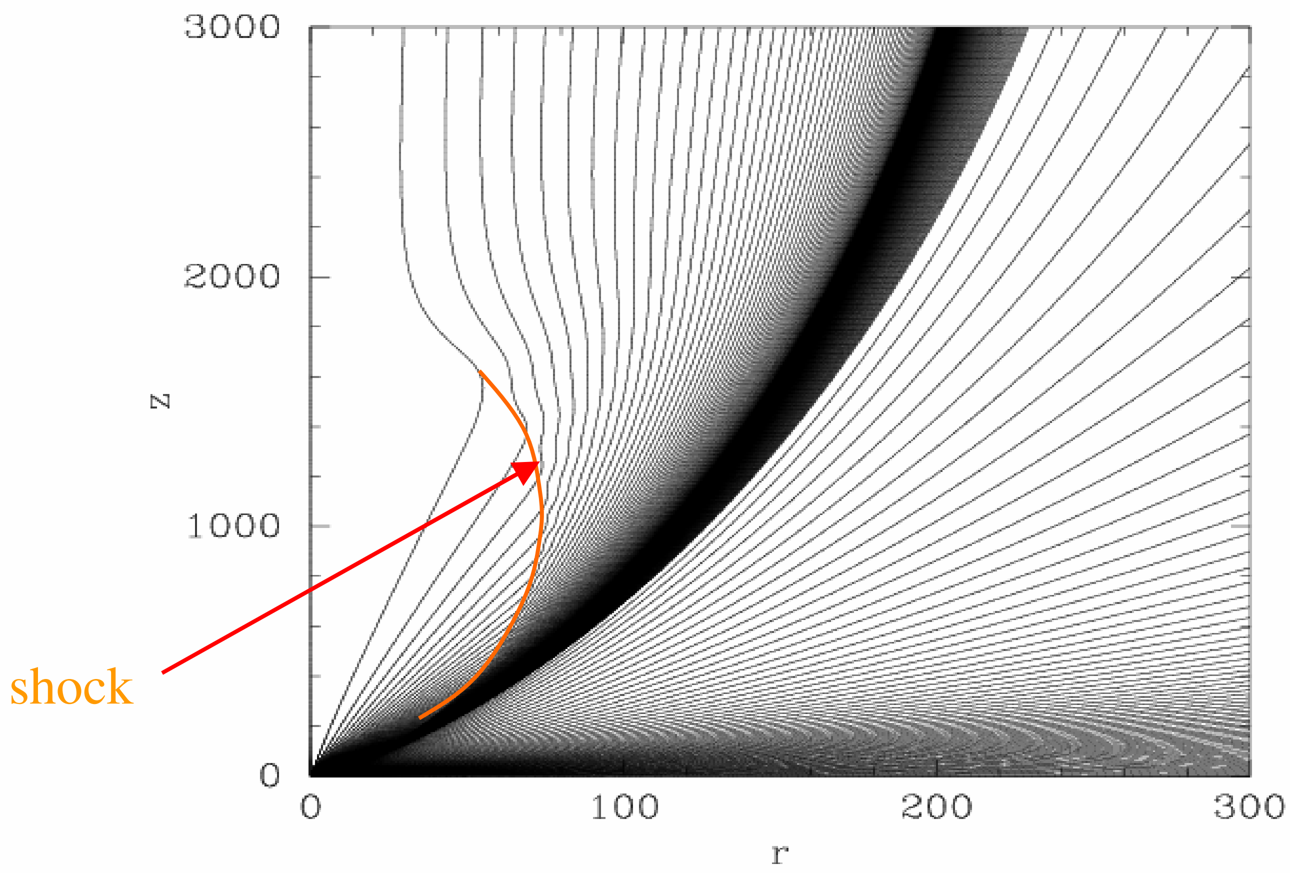
Fast surface

Alfven surface

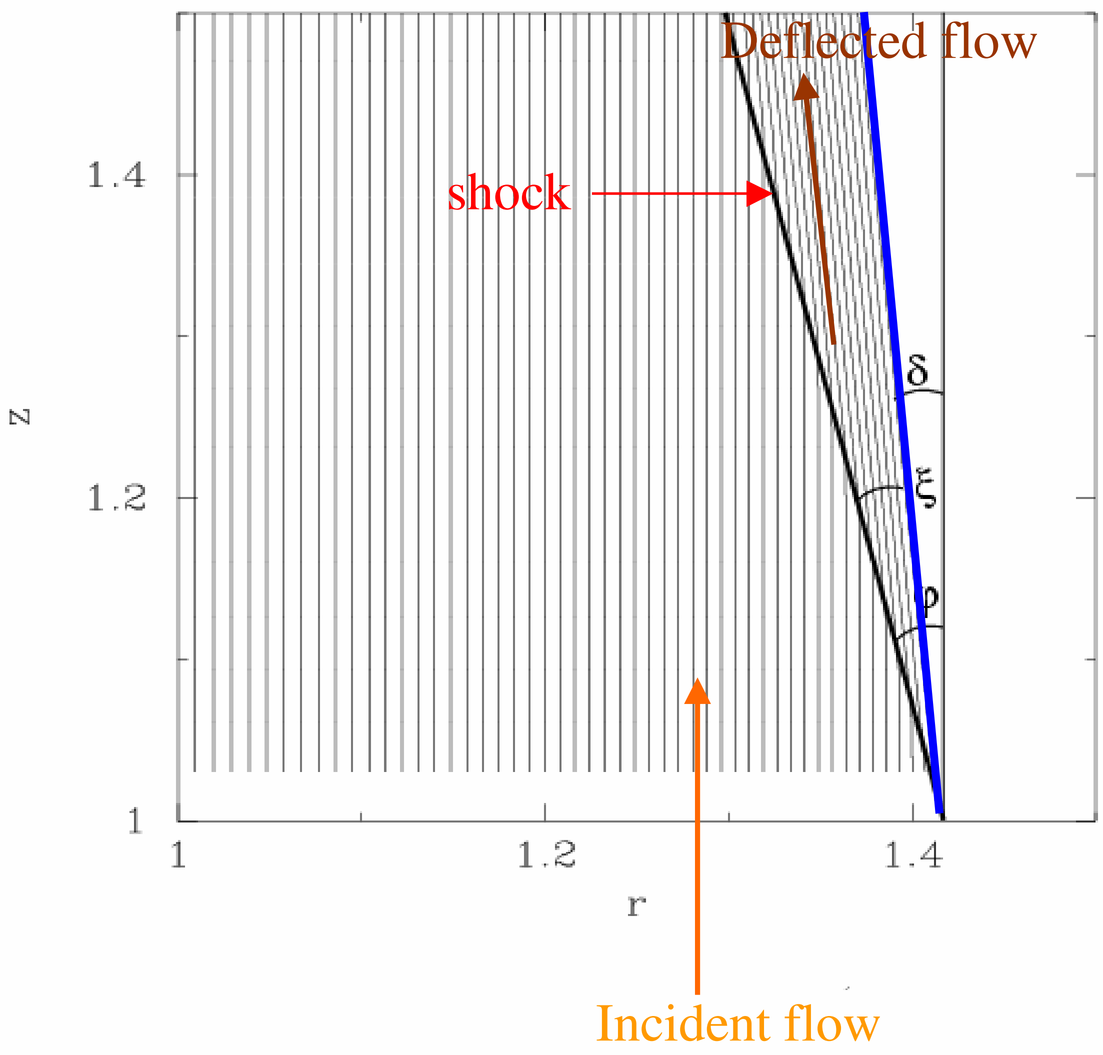
After rotation starts



Collimation of the inner flow with the formation of a shock.

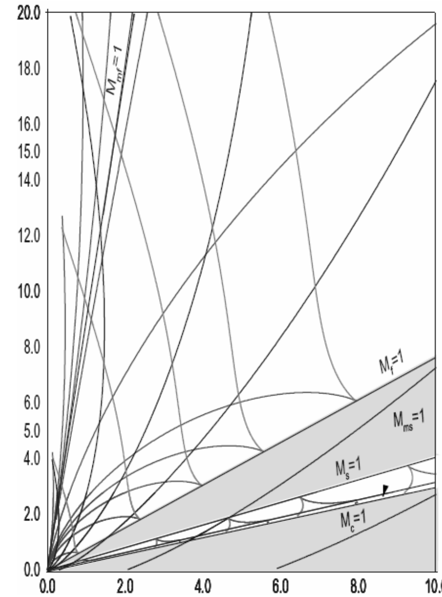


Real-life example: Supersonic flow incident at the vertex of an angle

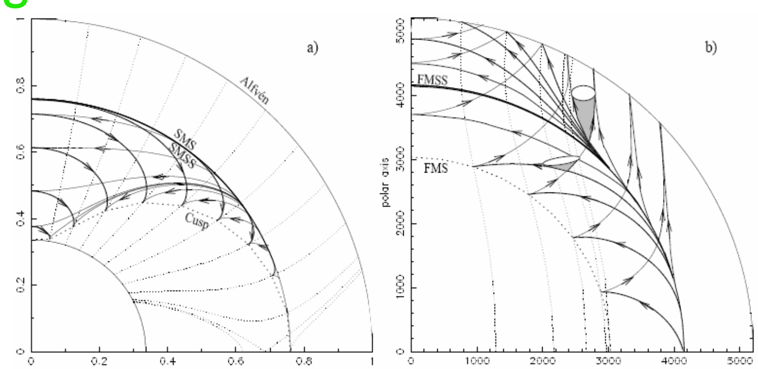


IV). Numerical simulations by using the analytical solutions as initial conditions

- **The Radially Self Similar Models (*r-ss*):**
  - Describe magneto-centrifugally driven disk winds
  - They have conical critical surfaces
  - Allow a constant-γ pressure-density relation
  - However, they are singular at the z-axis

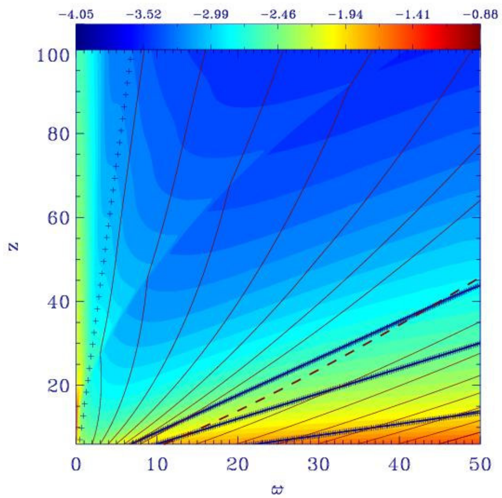


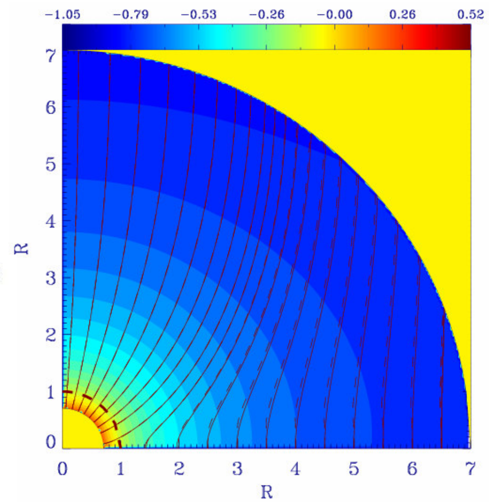
- **The Meridionally Self Similar Models (*θ-ss*):**
  - Describe thermally driven stellar outflows
  - Have spherical critical surfaces
  - Have variable effective polytropic index
  - However, not all magnetic fieldlines are connected to the stellar surface



Mixing

- **The two classes of solutions are complementary:**
  - Axis (stellar outflows) best described by meridionally self-similar solutions
  - Equator (disk winds) best described by radially self-similar solutions

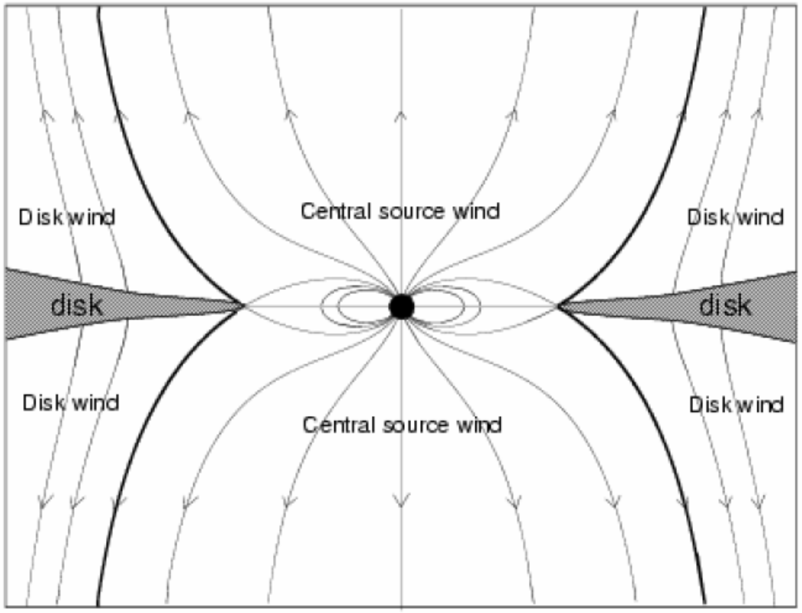




- **The first thing is to show the topological stability of the solutions along with other physical and numerical features [Matsakos et al, 2007, A+A in press].**

Mixing

- **Choice of contribution in the total magnetic field**
- **Choice of a fieldline (α_0)**
- **Choice of the steepness of the transition (d)**



- **Then, initialize each solution with the mixing function:**

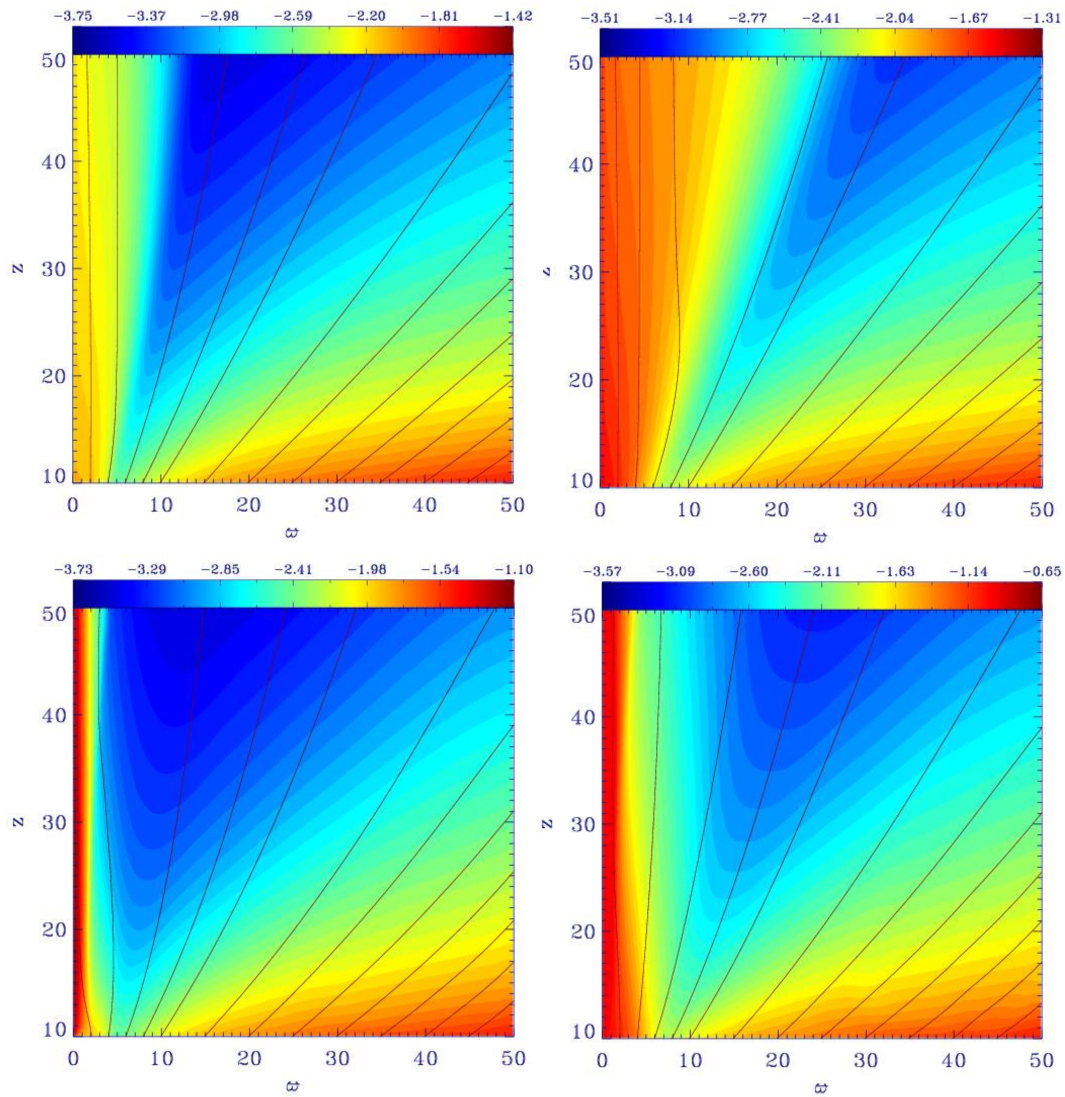
$$V_{2-comp}(\varpi, z) = w_1 V_{\theta ss}(\varpi, z) + w_2 V_{rss}(\varpi, z)$$

$$w_1 = exp\left\{-c\left[\frac{\alpha(\varpi, z)}{\alpha_0}\right]^d\right\} \quad \textbf{and} \quad w_2 = 1.0 - w_1$$

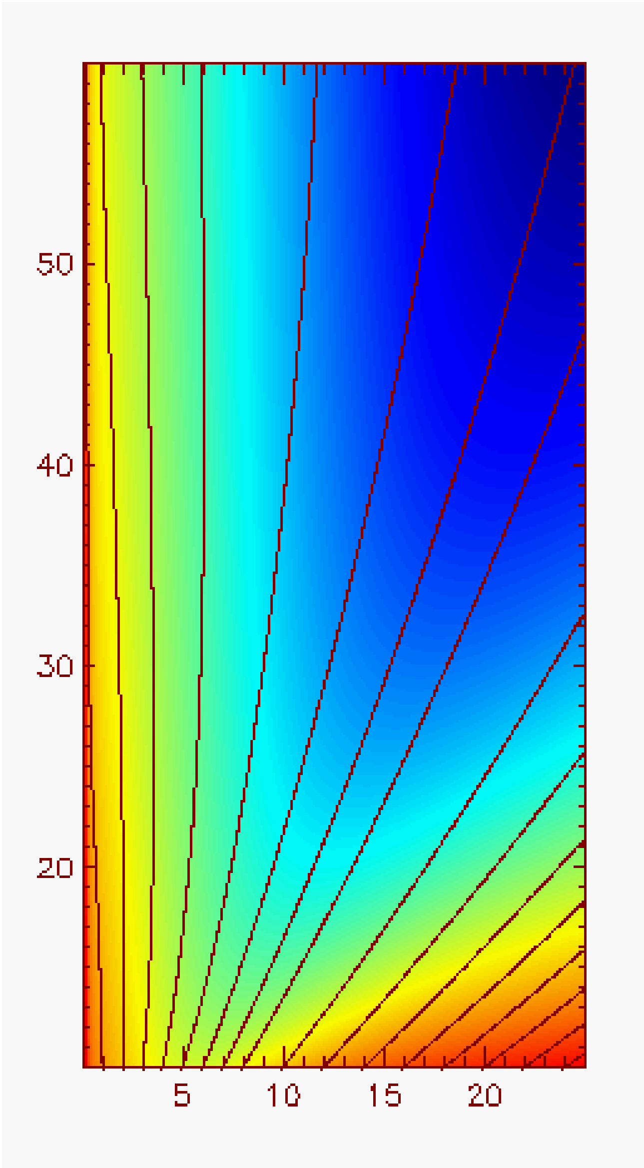
Results

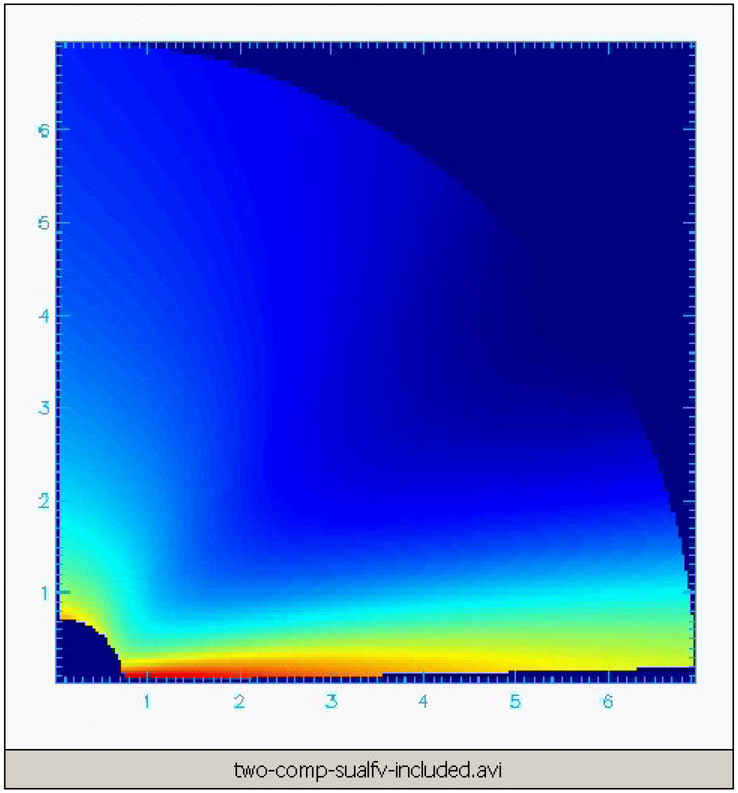
- **Initial setup:**
  - Implementation of both solutions
  - Different contribution of the stellar wind (left and right)

- **Final setup:**
  - Steady-state reached
  - Disk wind does not change
  - Stellar wind is being collimated by the disk component



Results of evolved 2-component





results

- A steady-state is always reached, however at different timescales for each model
- The disk wind remains almost unmodified while effectively collimating the inner stellar outflow
- The final outcome of the simulations stays close to the initial setup, hence retaining the validity of the analytical solutions
- Proper choice of the parameters can explain many of the observed cases of the two-component jets i.e. from the one extreme of stellar dominated ones, up to the other, of the disk-wind being the only contributor

Some conclusions

- Systematic analytical construction of classes of exact MHD models for jets
- Critical role played by of limiting characteristics in seting correct BC's
- Energetic criterion for the transition between winds ← → jets
- Topological stability of most analytical MHD jet models
- Time-dependent simulations support general trends of exact steady studies
- A new 2-component model is needed to describe collimated outflows
- There is yet a new way for producing shocks in jets