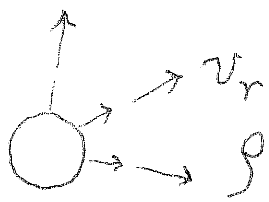
Parker Solar Wind

Parker (ApJ, 123, 664, 1958)

→ Stationary, spherically symmetric



→ Mass conservation: $\boxed{\rho v_r r^2 = -\dot{M}/4\pi}$ (1)

$\dot{M}$ = mass loss rate

→ Radial force balance:

$$\boxed{v_r \frac{d}{dr} v_r = -\frac{1}{\rho}\frac{d}{dr} p + g_r} \quad (2), \qquad g_r = \frac{-GM_\odot}{r^2}$$

→ Constant entropy, $s \sim \ln(P/\rho^\gamma)$, so that $\frac{1}{\rho}\frac{dp}{dr} = \frac{d}{dr} h$, $\quad h = \int \frac{dp}{\rho}\Big|_{s=\text{Const.}}$

→ Equation (2) becomes

$$\frac{d}{dr}\left(\frac{v_r^2}{2} + h - \frac{GM_\odot}{r}\right) = 0$$

Thus $B = \frac{v_r^2}{2} + h - \frac{GM_\odot}{r}$ = Bernoulli's Constant

Simple case: isothermal eqn. of state.

$$p = c_s^2 \rho \,, \quad c_s = \sqrt{\frac{k_B T}{\bar{m}}} = \text{isothermal sound speed} = \text{const.}$$

Then, $h = c_s^2 \ln(\rho)$, and

$$\frac{v_r^2}{2} + c_s^2 \ln\left(\frac{\dot{M}}{4\pi v_r r^2}\right) - \frac{GM_\odot}{r} = \text{Const.}$$

or

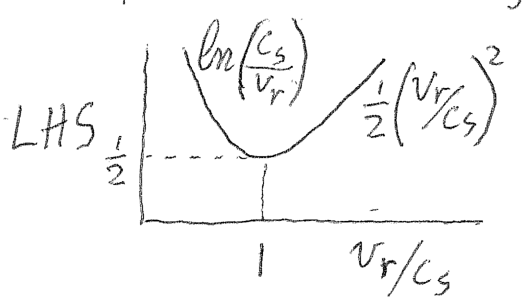
$$\frac{v_r^2}{2} + c_s^2 \ln\left(\frac{c_s}{v_r}\right) = \frac{GM_\odot}{r} + c_s^2 \ln\left(\frac{4\pi r^2 c_s}{\dot{M}}\right) + \text{Const.}$$

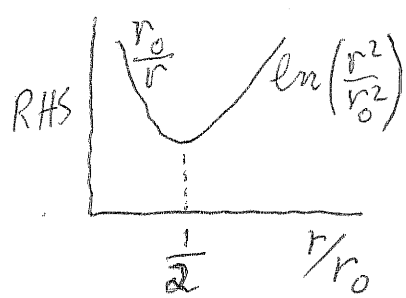
or

$$\frac{(v_r/c_s)^2}{2} + \ln\left(c_s/v_r\right) = \frac{r_0}{r} + \ln\left(\frac{r^2}{r_0^2}\right) + C'$$

where $r_0 \equiv \dfrac{GM_\odot}{c_s^2}$

$$\text{Left-hand-side}\left(\frac{v_r}{c_s}\right) = \text{Right-hand-side}\left(\frac{r}{r_0}\right) + C'$$





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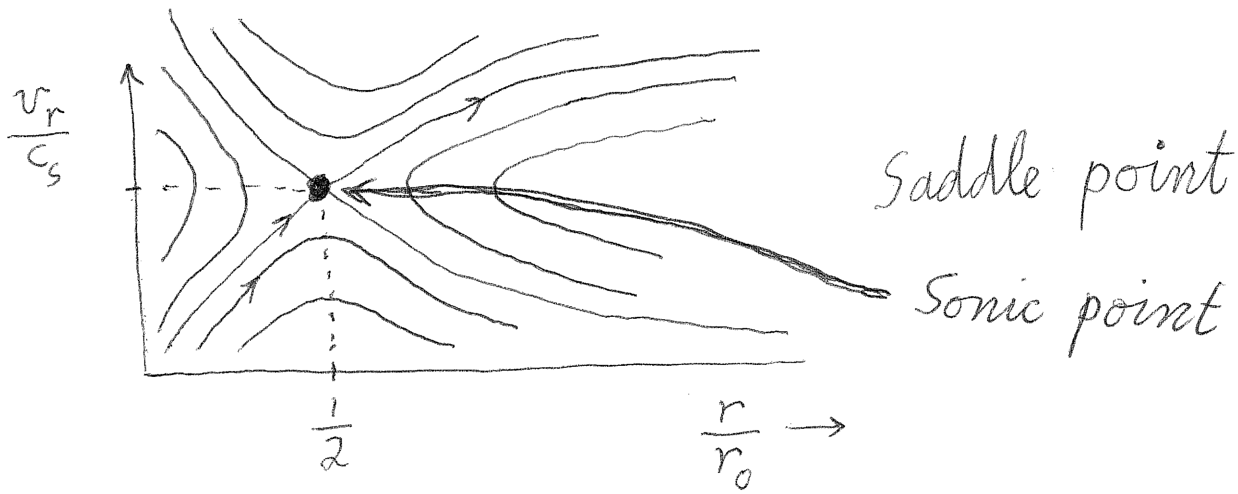

Analogous to deLaval nozzle: