MHD Winds from accretion disks

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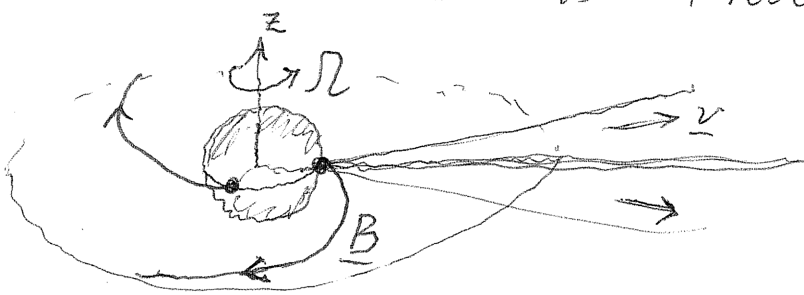
1. MHD winds from rotating stars:
   Weber-Davis model of solar wind (ApJ, 148, p217, 1967)
   Belcher & MacGregor (ApJ, p498, 1976)
2. MHD winds from accretion disks:
   Blandford-Payne model (MNRAS, 199, p883, 1982)
   Lovelace, Berk, & Contopoulos (ApJ, 379, p696, 1991)

Weber-Davis Model:



$$\underline{v} = v_r \hat{r} + v_\phi \hat{\phi}$$

$$\underline{B} = B_r \hat{r} + B_\phi \hat{\phi}$$

spherical coordinates

1. → Conservation of mass: $\boxed{\rho v_r r^2 = \text{Const.}}$ (1)

2. → ideal MHD, perfect conductor, $\underline{E} = -\underline{v} \times \underline{B}/c$.
   In reference frame rotating with star $E_\phi = 0$:

$$\boxed{(v_r B_\phi - v_\phi B_r) r = -\Omega r^2 B_r} \quad (2)$$

3. → Magnetic flux conservation:

$$\boxed{r^2 B_r = \text{const.} = r_0^2 B_{r0}} \quad (3)$$

4. → Angular momentum conservation:

$$\rho \frac{v_r}{r}\frac{d}{dr}(r v_\phi) = \frac{1}{c}(\underline{J}\times\underline{B})_\phi = \frac{1}{4\pi}\left[(\underline{\nabla}\times\underline{B})\times\underline{B}\right]_\phi$$

$$= \frac{B_r}{4\pi r}\frac{d}{dr}(r B_\phi)$$

but $\dfrac{B_r}{4\pi\rho v_r} = \dfrac{B_r r^2}{4\pi \rho v_r r^2} = \text{const.}$ so that

$$\boxed{r v_\phi - \frac{B_r}{4\pi\rho v_r} r B_\phi = \text{const.} \equiv L} \quad (4)$$

L = specific angular momentum.

Now let $\underline{A} = \underline{B}/\sqrt{4\pi\rho}$ = Alfvén velocity

Then (4) becomes $L = r\left(v_\phi - \dfrac{A_r A_\phi}{v_r}\right)$, (5)

and (2) becomes $\dfrac{A_\phi}{A_r} = \dfrac{(v_\phi - \Omega r)}{v_r}$. (6)

Solve (5) and (6) to get:

$$v_\phi = \Omega r \left(\frac{v_r^2 \frac{L}{\Omega r^2} - A_r^2}{v_r^2 - A_r^2} \right) \qquad (7)$$

To avoid a singularity in v_ϕ at the Alfvén point (where $v_r = A_r$) we must have $L = \Omega r_a^2$.

5. $\rightarrow$ Radial component of the Euler eqn.:

$$\Rightarrow \quad \mu = \frac{1}{2}\left(v_r^2 + v_\phi^2\right) + \frac{c_s^2}{\gamma - 1} - \frac{1}{2} V_e^2 - \Omega r \, A_r A_\phi / v_r$$

$= \text{Const.} = \text{Bernoulli's const.}$

Here, $c_s \equiv \sqrt{\frac{\gamma p}{\rho}}$, where $p = \text{Const.}\ \rho^\gamma$

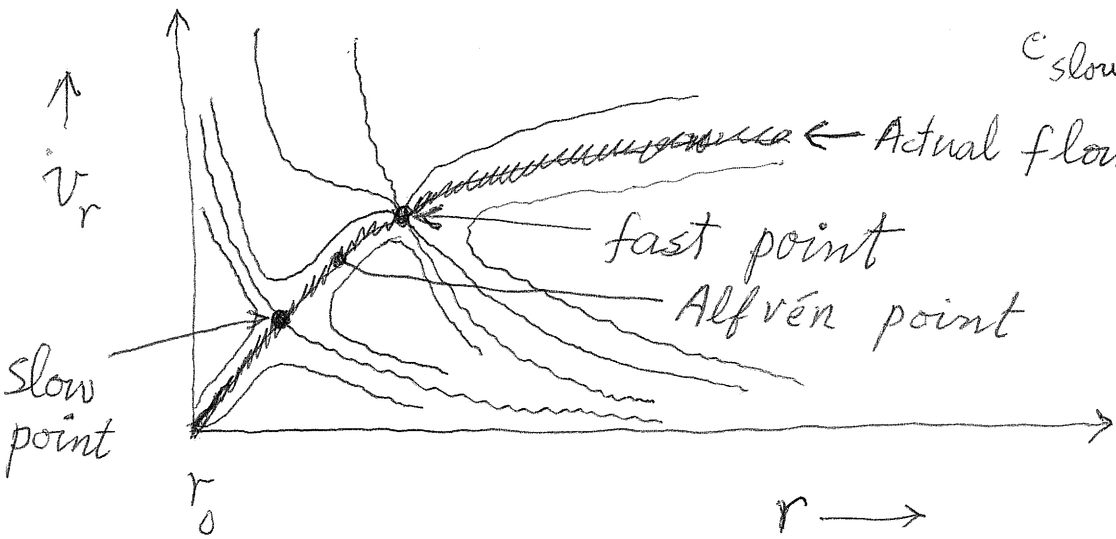
$V_e \equiv \left(\frac{2GM}{r}\right)^{1/2}$ = local escape velocity

Differentiating with respect to r gives

$$\frac{r}{v_r}\frac{dv_r}{dr} = \frac{(v_r^2 - A_r^2)(2c_s^2 + v_\phi^2 - V_e^2/2) + 2v_r v_\phi A_r A_\phi}{(v_r^2 - A_r^2)(v_r^2 - c_s^2) - v_r^2 A_\phi^2}$$

Denominator vanishes at slow and fast magnetosonic points, $v_r = c_{slow}$, c_{fast}.

$c_{slow} < v_A < c_{fast}$



Define $V_M \equiv \left[\dfrac{(\Omega r_0 A_{r0})^2}{v_{r0}} \right]^{1/3}$

$$V_P = \left(\frac{2 c_{s0}^2}{\gamma - 1} + v_{r0}^2 - v_{e0}^2 \right)^{1/2}$$

(Parker wind)

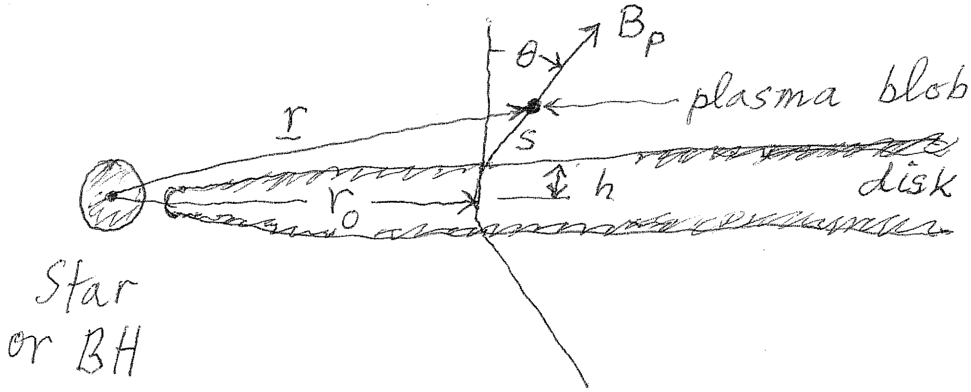
→ Slow magnetic rotator if $V_M \ll V_P$

in which case $v_{r\infty} \approx V_P$

→ Fast magnetic rotator if $V_M \gg V_P$

in which case $v_{r\infty} \approx V_M$

MHD winds from accretion disks



Position of plasma blob: $\underline{r} = (r_0 + s\sin\theta)\hat{r} + s\cos\theta\,\hat{z}$,

where s = distance along field line, and $\tan\theta = \frac{(B_r)_h}{B_z}$.

Effective potential is $U(s) = -\frac{1}{2}\omega_0^2 (r_0 + s\sin\theta)^2 - \frac{GM}{|\underline{r}|}$

Force along field line: $\mathcal{F}_{\parallel} = -\frac{\partial U}{\partial s}$

$$\mathcal{F}_{\parallel} = -(\omega_K^2 - \omega^2) r \frac{(B_r)_h}{|\underline{B}_p|} + \omega_K^2 z (3\tan^2\theta - 1)\frac{B_z}{|B_p|}$$

$$\mathcal{F}_{\parallel} = 0 \quad \text{at} \quad z_{crit} = r\left(1 - \frac{\omega^2}{\omega_K^2}\right)\frac{\beta_r}{3\beta_r^2 - 1}$$

$$\beta_r \equiv \tan\theta \quad \text{and} \quad \omega_K = \left(\frac{GM}{r_0^3}\right)^{1/2}$$

radial equilibrium of disk: $1 - \frac{\omega^2}{\omega_K^2} = \frac{-\frac{\partial p}{\partial r}}{\rho r \omega_K^2} \approx \underbrace{\frac{c_s^2}{v_K^2}}_{\ll 1}$

z_{crit} finite and $O(h)$ if

$$\beta_r = \tan\theta \gtrsim \frac{1}{\sqrt{3}} \longrightarrow \theta \gtrsim 30^\circ$$