Variance reduction techniques used in BEAMnrc

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How do we get calculational efficiency?

- efficiency of a calculation is given by $$\epsilon = \frac{1}{s^2 T}$$
  - s^2 is an estimate of the variance (σ^2) on a quantity of interest
    - fluence in 1x1 cm^2 grid
    - dose on central axis or profile
  - T is the CPU time for the calculation

$$s^2 \propto \frac{1}{N} \quad T \propto N \Rightarrow \epsilon \text{ is independent of } N$$

Improve the efficiency by decreasing s^2 or T

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Variance estimation

Batch method

-Break cal'n into N batches and determine uncertainty by distribution of results for batches

-large uncertainty in the uncertainty

$$s_{\overline{X}} = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \overline{X})^2}{N(N-1)}}$$

History by history method

batches = #histories

-much better estimate

-algebraically equivalent

$$s_{\overline{X}} = \sqrt{\frac{1}{N-1}\left(\frac{\sum_{i=1}^{N} X_i^2}{N} - \left(\frac{\sum_{i=1}^{N} X_i}{N}\right)^2\right)}$$

-problem: needed to do the sum at the end of every history for a large number of regions=> very slow

Salvat's trick

Salvat recognized that we didn't have to do the sum for every region at the end of each history, just do for each region being affected, and clean up at end.

```
IF(nhist=X_last) THEN
  X_tmp=X_tmp+delta
ELSE
  X=X+X_tmp
  X2=X2+(X_tmp)**2
  X_tmp=delta
  X_last=nhist
ENDIF
```

nhist counts which history we are on

delta is some quantity being scored on this step.

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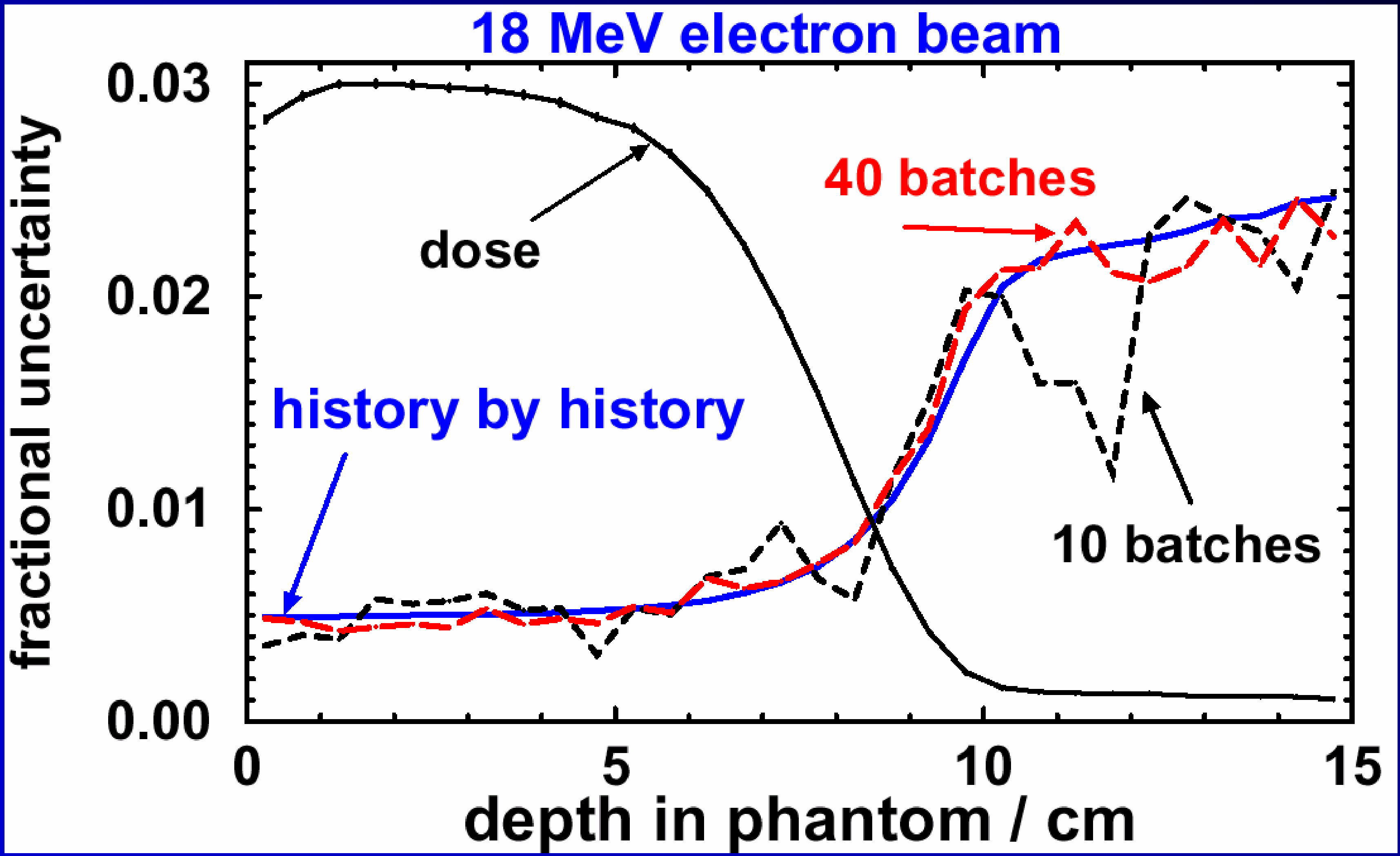
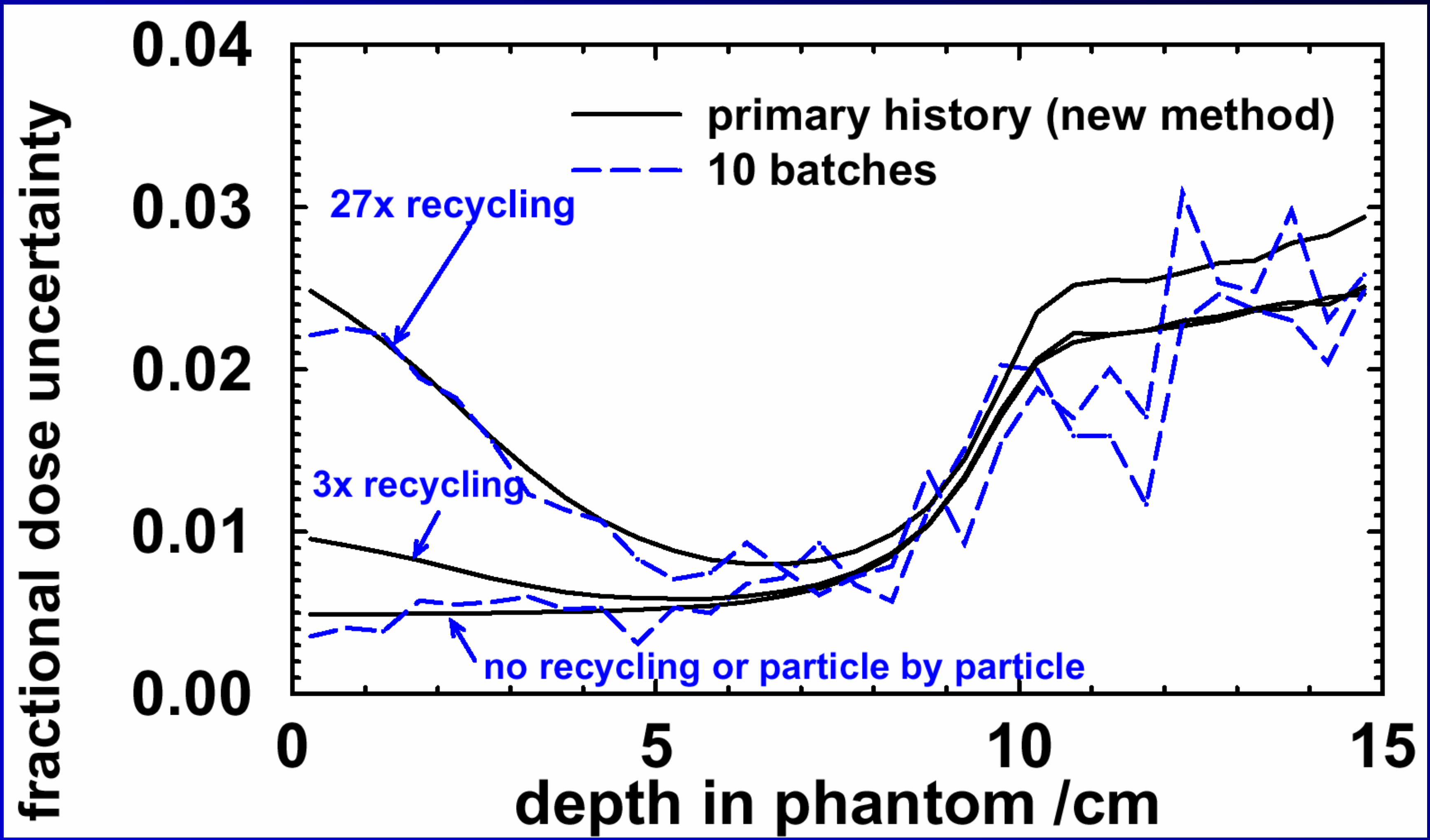
History by history technique *(cont)*



Walters et al Med Phys 29(2002) 2745

History by history technique *(cont)*



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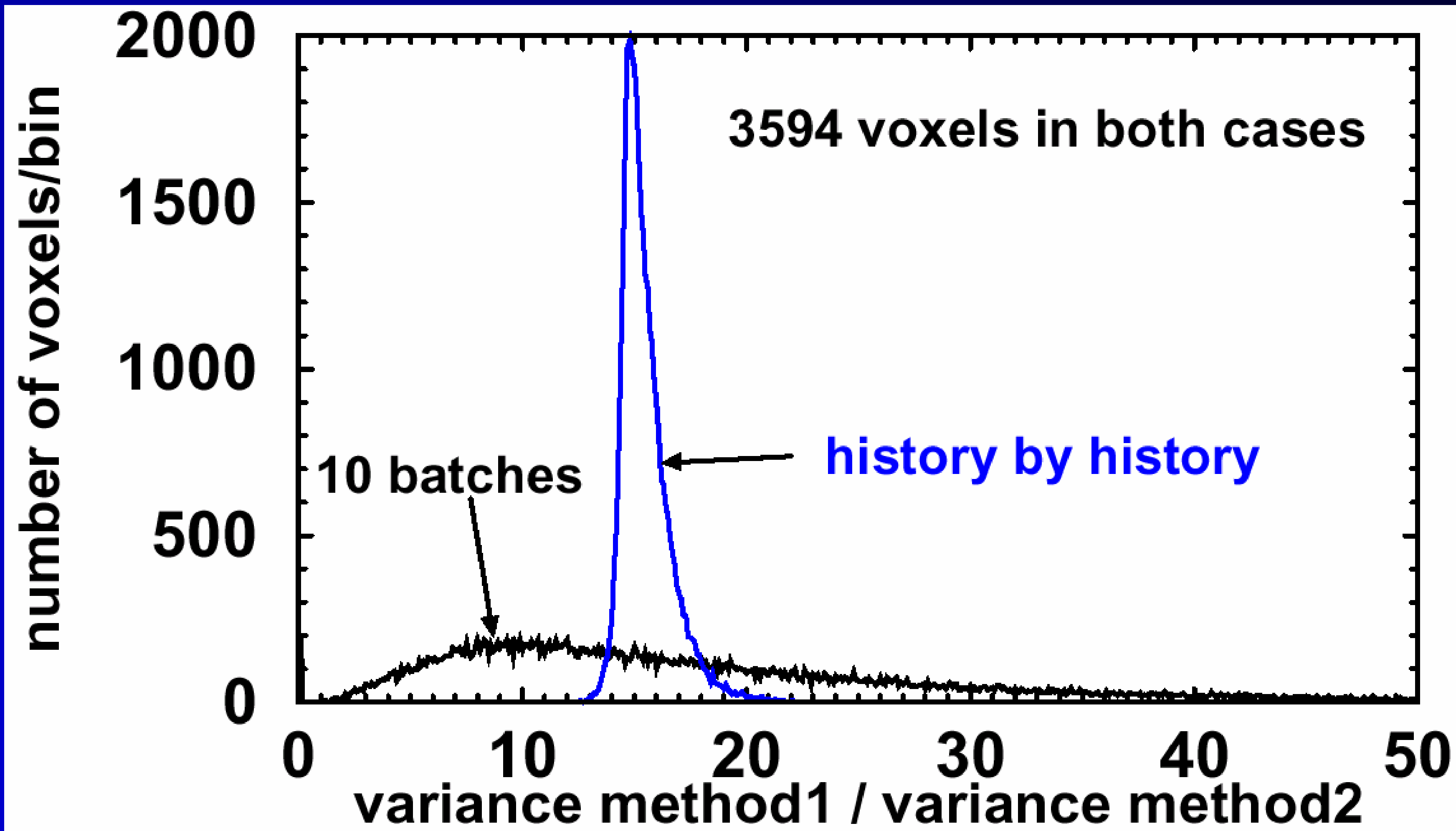
Walters et al Med Phys 29(2002) 2745

Advantage of history by history



Dose in a brachytherapy phantom: from G Yegin

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Condensed history technique

- as electrons slow, they have many interactions
- Berger's grouping into condensed history steps made Monte Carlo transport of electrons feasible.
  - individual scattering events grouped via multiple-scattering theories
  - low-energy-loss events grouped into restricted stopping powers
- this increases efficiency by decreasing T (by a lot)
- modern transport mechanics algorithms are very sophisticated in order to maximize step size while maintaining accuracy (to gain speed).

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Variance reduction techniques (VRTs)

- A VRT is a method which increases the efficiency for some quantity of interest by decreasing s^2 for a given N while not biasing the result.
  - they often increase time per history
  - VRTs may simultaneously make s^2 for some other quantity increase
    - eg pathlength shrinking will improve the efficiency for dose near the surface but decrease the efficiency for dose at depth

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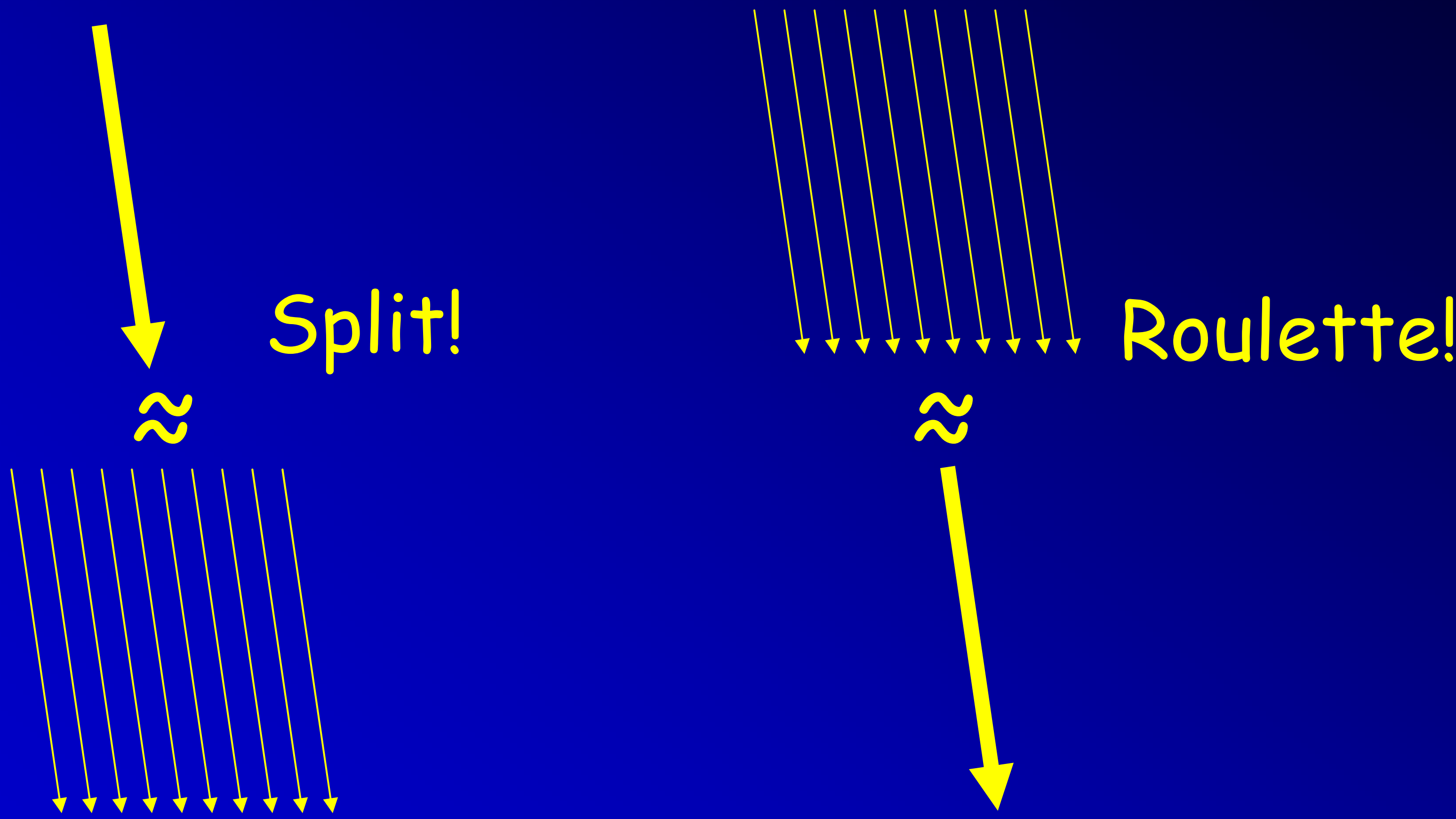
Variance reduction techniques

- For a recent review, see Sheikh-Bagheri et al's 2006 AAPM summer school chapter
  http://www.physics.carleton.ca/~drogers/pubs/papers/SB06.pdf
- examples
  - splitting (brem splitting: UBS, DBS; in-phantom)
  - Russian roulette
  - interaction forcing
  - enhanced cross sections (brem: BCSE)
  - track repetition*
  - STOPS (simultaneous transport of particle sets)*

*last 2 applied in phantom and not discussed here

Splitting, Roulette & particle weight

$1\ w_i = 10\ w_f$ $\quad$ $10\ w_i = 1\ w_f$



from Sheikh-Bagheri's 2006 summer school lecture

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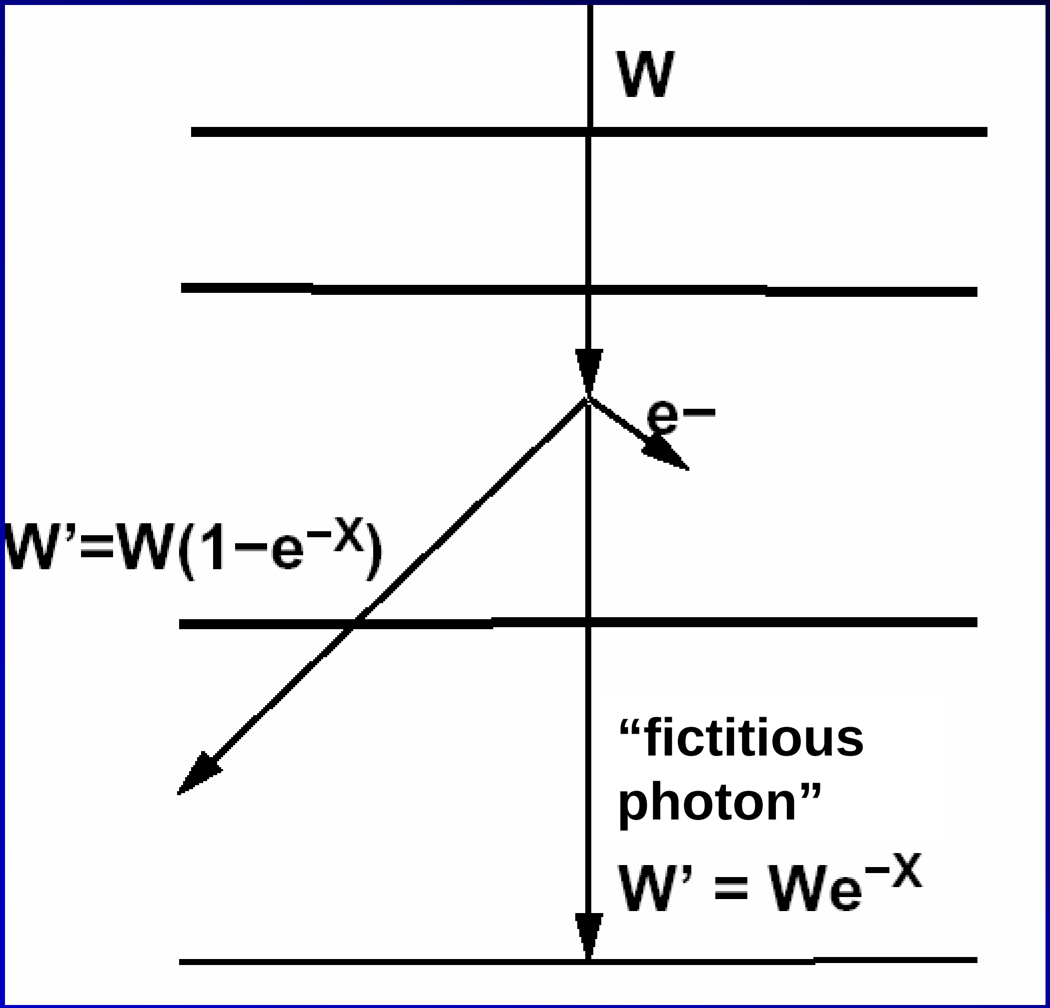
Photon forcing

X = number of mfp in geometry where interaction forced.



Interaction forcing for photons

In normal transport, the number of mean free paths that a photon is to traverse. N_λ is given by:

$$N_\lambda = -ln(1-\zeta) \qquad\qquad 0 \le \zeta < 1$$

If geometry has M_λ mfp in current direction, then:

$$N_\lambda = -ln\left(1 - \zeta\left(1 - e^{-M_\lambda}\right)\right) \qquad\qquad W' = W\left(1 - e^{-M_\lambda}\right)$$

will limit N_λ to a value less than M_λ, ie force it to interact in the medium, but give it a reduced weight.

For small M_λ : $W' = W\left(1 - e^{-M_\lambda}\right) \approx M_\lambda W \qquad \text{for} \;\; M_\lambda << 1$

Interaction forcing for photons

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To force interactions in a region between $M_{\lambda 1}$ and $M_{\lambda 2}$ mfp away:

$$N_\lambda = M_{\lambda 1} - ln\left(1 - \zeta\left(1 - e^{(+M_{\lambda 1} - M_{\lambda 2})}\right)\right)$$

$$W' = W\left(e^{-M_1} - e^{-M_{\lambda 2}}\right)$$

In this case, the fictitious photon is more critical to consider (because there is some geometry on the far side).

Other efficiency-improving techniques

- one can improve the efficiency by decreasing T
  - usually implies an approximation being made
    - must demonstrate the approximation does not lead to significant errors
- Examples
  - range rejection: terminate an e- history if it cannot reach any boundary
    - an approximation since no brem possible
  - higher cutoff energies: terminate tracks sooner
    - an approximation since energy deposited locally
  - both are usually OK (within reason)

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Effect of changing ECUT and AE

AE is the lowest energy secondary electron created
ECUT is the lowest energy electron transported ECUT>= AE

18 MeV electron beam from a Varian accelerator.

Case	AE (MeV)	ECUT (MeV)	cpu s per history	total to file/inc e^-	e^- per 100 inc e^-	γ per 100 inc e^-
1	0.700	0.700	0.0124	0.417	9.15	32.4
5	0.700	0.900	0.0118	0.417	9.09	32.5
6	0.700	1.100	0.0083	0.411	9.13	31.9
7	0.521	0.521	0.0538	0.414	8.79	32.6
10	0.521	0.700	0.0178	0.416	9.09	32.4

BEAM paper, Med Phys 22 (2007) 503-524

Range rejection: effects of threshold

ESAVE is the energy below which range rejection is done.

ESAVE=0 => no range rejection is done

18 MeV electron beam from a Varian accelerator.

Case	AE (MeV)	ECUT (MeV)	ESAVE (MeV)	cpu s per history	total to file/inc e^-	e^- per 100 inc e^-	γ per 100 inc e^-
1	0.700	0.700	5.0	**0.0124**	0.417	9.15	32.4
3	0.700	0.700	2.0	**0.0125**	0.420	9.07	32.8
4	0.700	0.700	0.0	**0.0249**	0.421	9.03	33.0
7	0.521	0.521	5.0	**0.0538**	0.414	8.79	32.6
8	0.521	0.521	2.0	**0.0631**	0.416	8.83	32.7
9	0.521	0.521	0.0	**0.300**	0.421	8.94	33.1

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BEAM paper, Med Phys 22 (2007) 503-524

Problems to overcome

-in photon accelerators, majority of time is spent following electrons

-creating brem photons is a relatively rare event

-most photons are absorbed in the primary collimator

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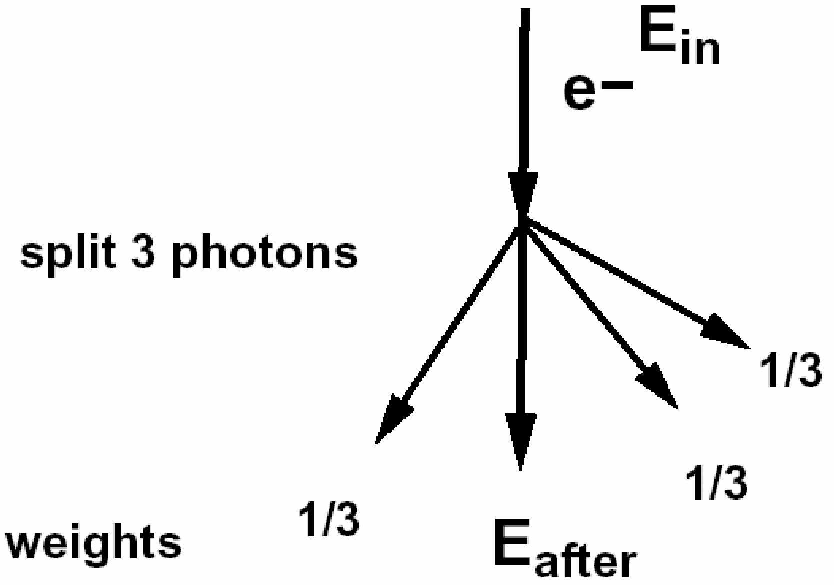
Uniform Brem Splitting

Designed to get over brem creation being rare and getting more photons into the simulation.



Approach A: $E_{after} = E_{in} - E_{gamma1}$

Approach B: $E_{after} = E_{in} - \overline{E}_{gamma}$

B conserves energy on each event

A conserves energy straggling on the electron history

MUST use A - it conserves energy "on average"

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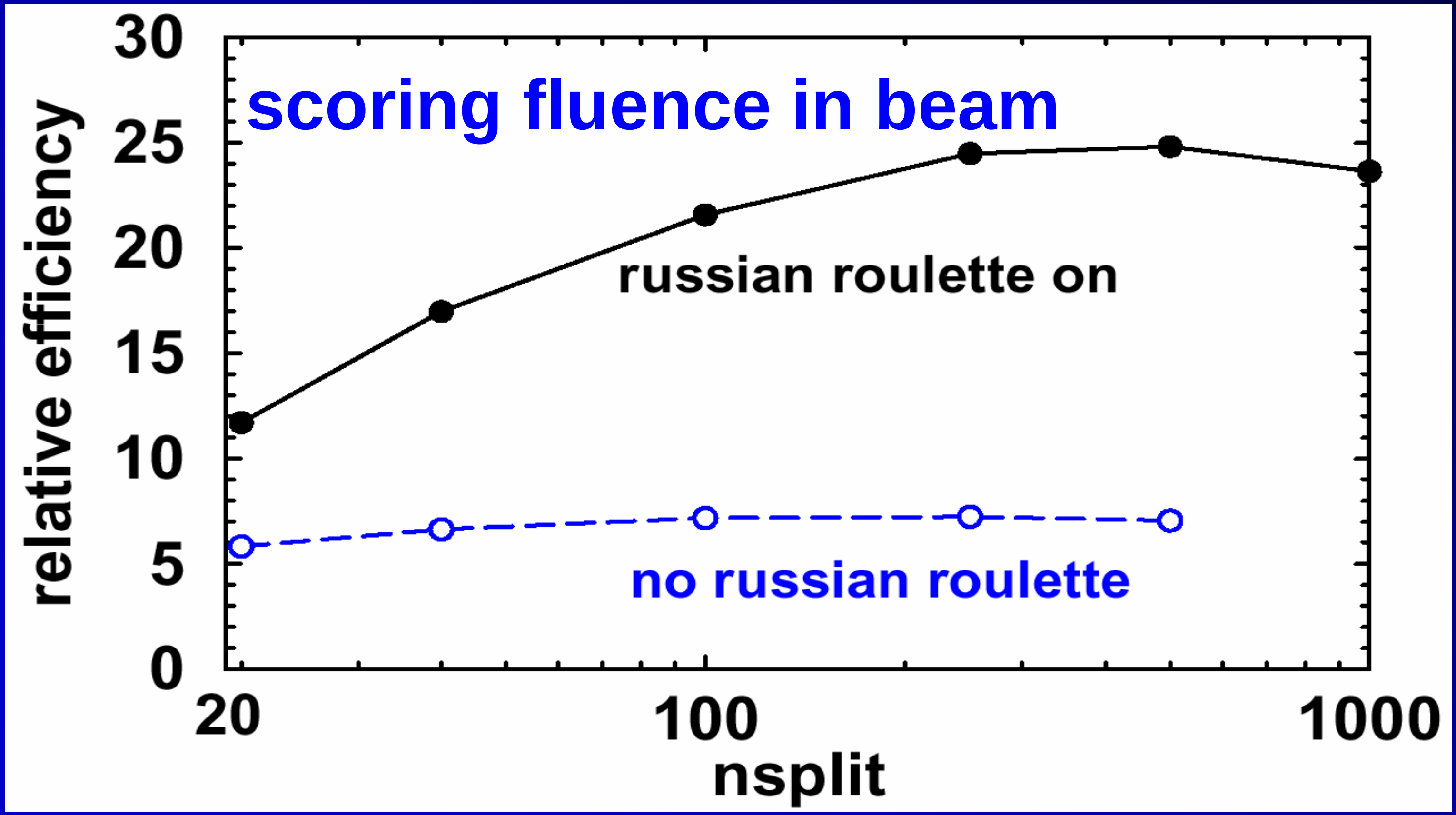
Uniform Brem Splitting

All examples are for a 6 MV beam from an Elekta SL25



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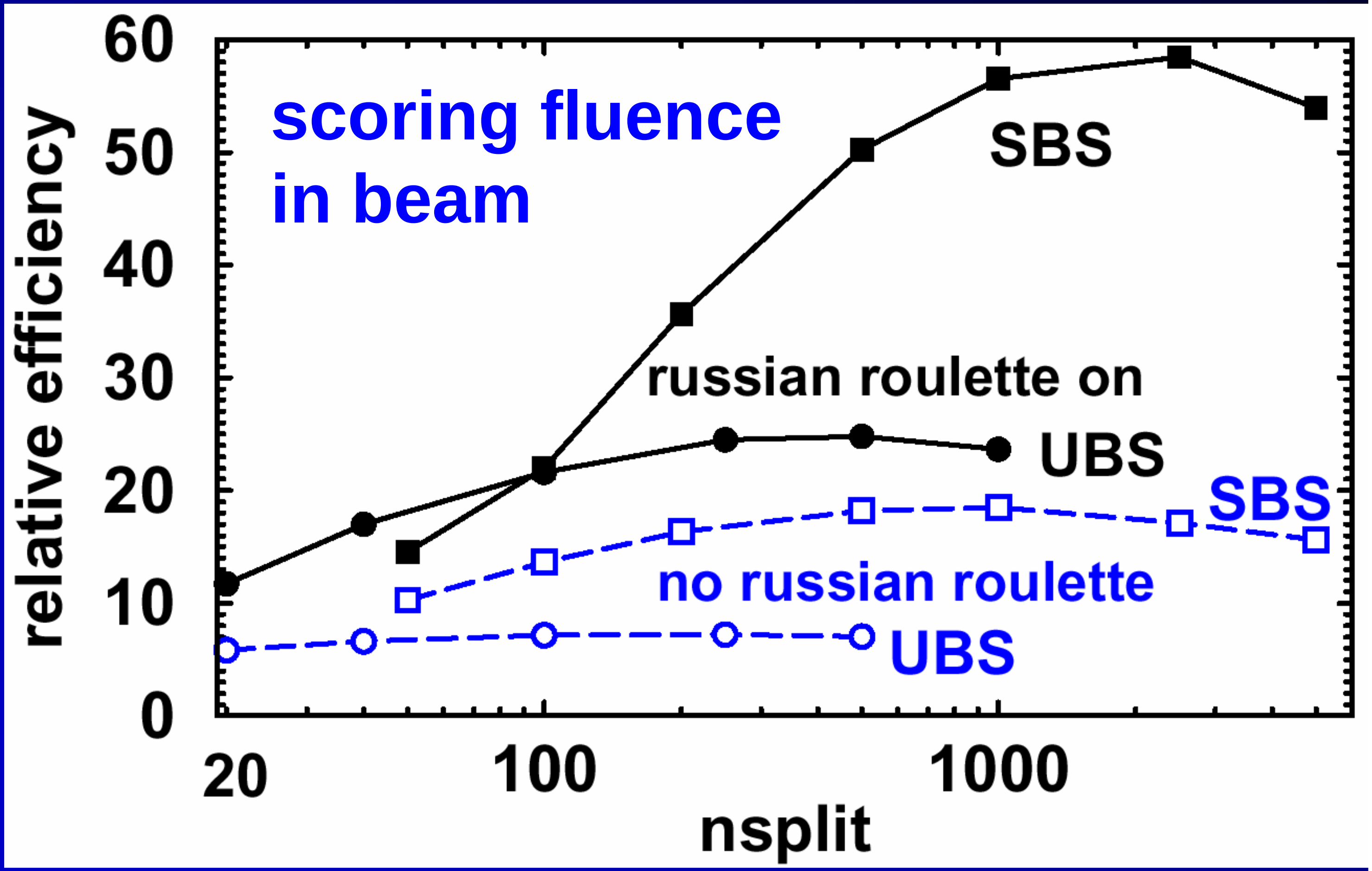
Kawrakow et al Med Phys 31 (2004) 2883-2898

Selective Brem Splitting (SBS)

Introduced by Sheikh-Bagheri.

Preferentially split more often for electrons headed towards region-of-interest



Kawrakow et al Med Phys 31 (2004) 2883-2898

Directional Brem Splitting (DBS)

-goal: all particles in field when reach phase space have same weight

Procedure

i) brem from all fat electrons split nsplit times

ii) if photon aimed at field of interest, keep it, otherwise Russian roulette it:

if it survives, weight is 1 (i.e. fat)

iii) if using only leading term of Koch-Motz angular dist'n for brem: `do_smart_brems` and similar tricks for other interactions

do_smart_brems

-if EGSnrc is using the leading term of the Koch-Motz distribution, then:

do_smart_brems calculates how many of the nsplit brem photons will head to the field and only generates those photons;

+

samples 1 photon from the entire dist'n -if not heading into the field, kept with weight 1.

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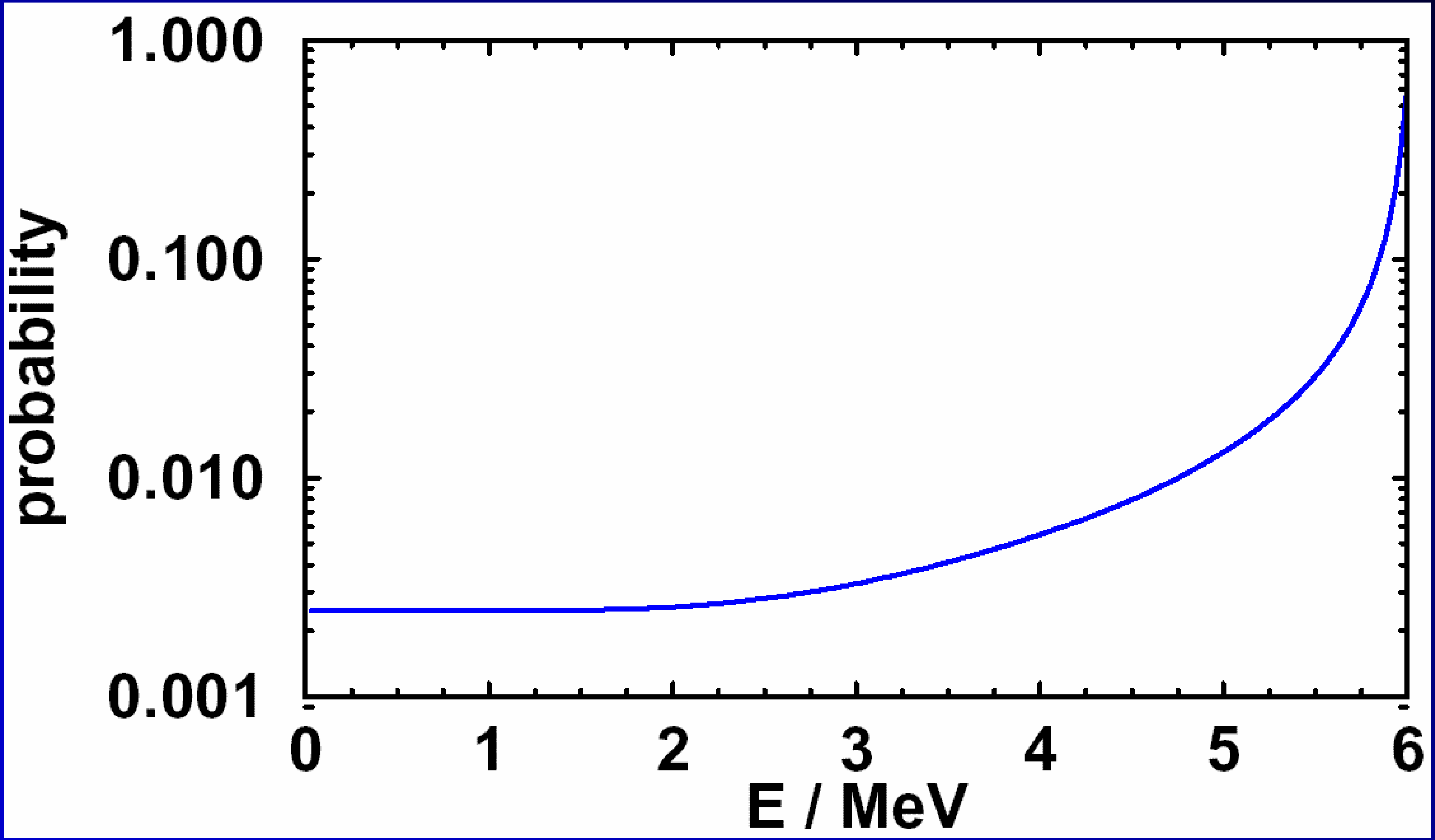
Kawrakow et al Med Phys 31 (2004) 2883-2898

Probability photon heading at field 6 MeV electron in high-Z



Kawrakow, private communication

DBS continued

play similar tricks for other quantities

-e+ annihilation: (`uniform_photons`)

-Compton scattering:
(`do_smart_compton` if Klein Nishina)

-pair production/photo-effect: (Russian roulette **before** sampling)

-fluorescence: (`uniform_photons`)

DBS (cont)

photons

- reaching field have weight 1/nsplit
- outside field are fat

electrons in the field

- usually fat
- a few have weight 1/nsplit from interactions in the air

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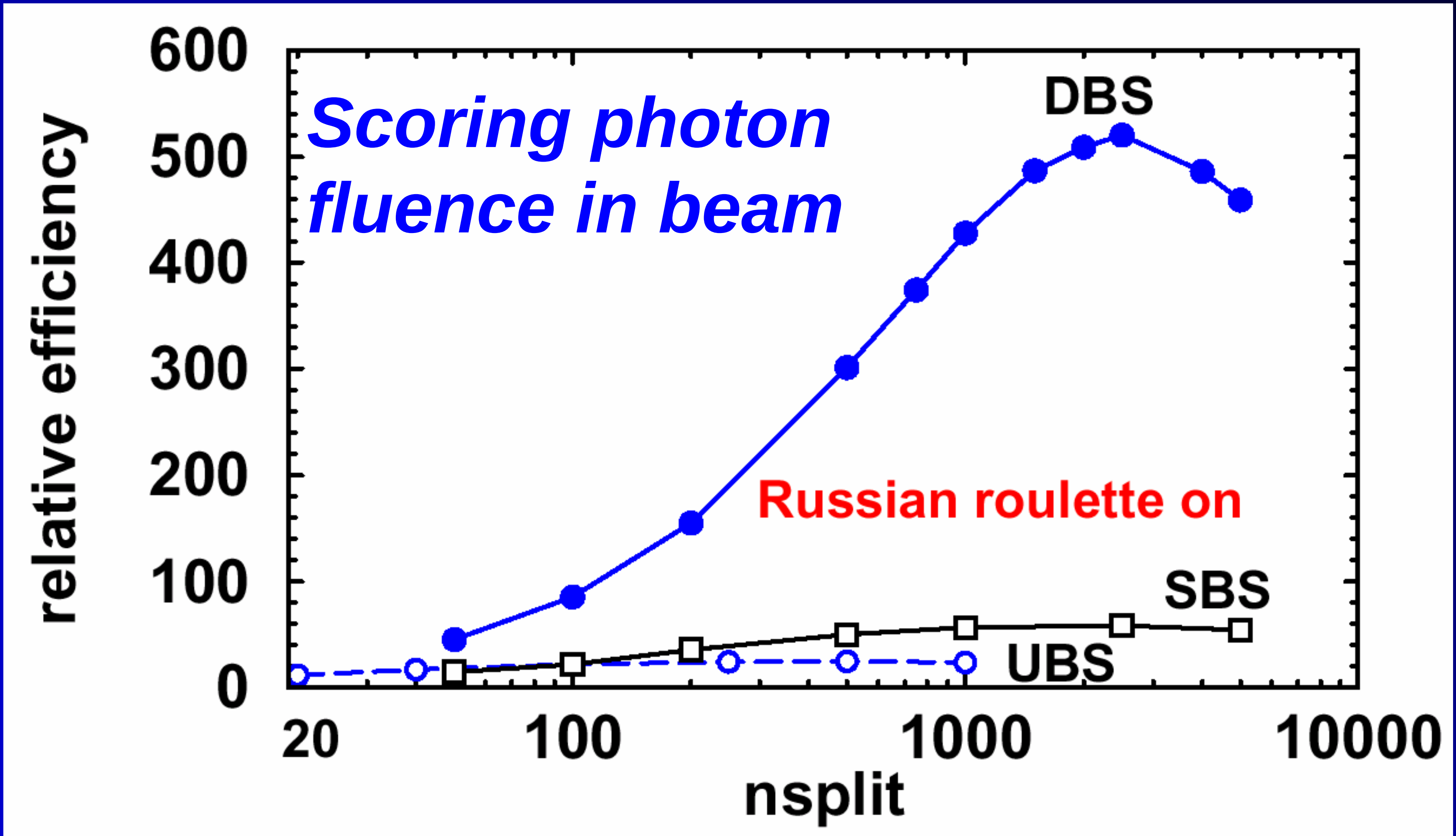
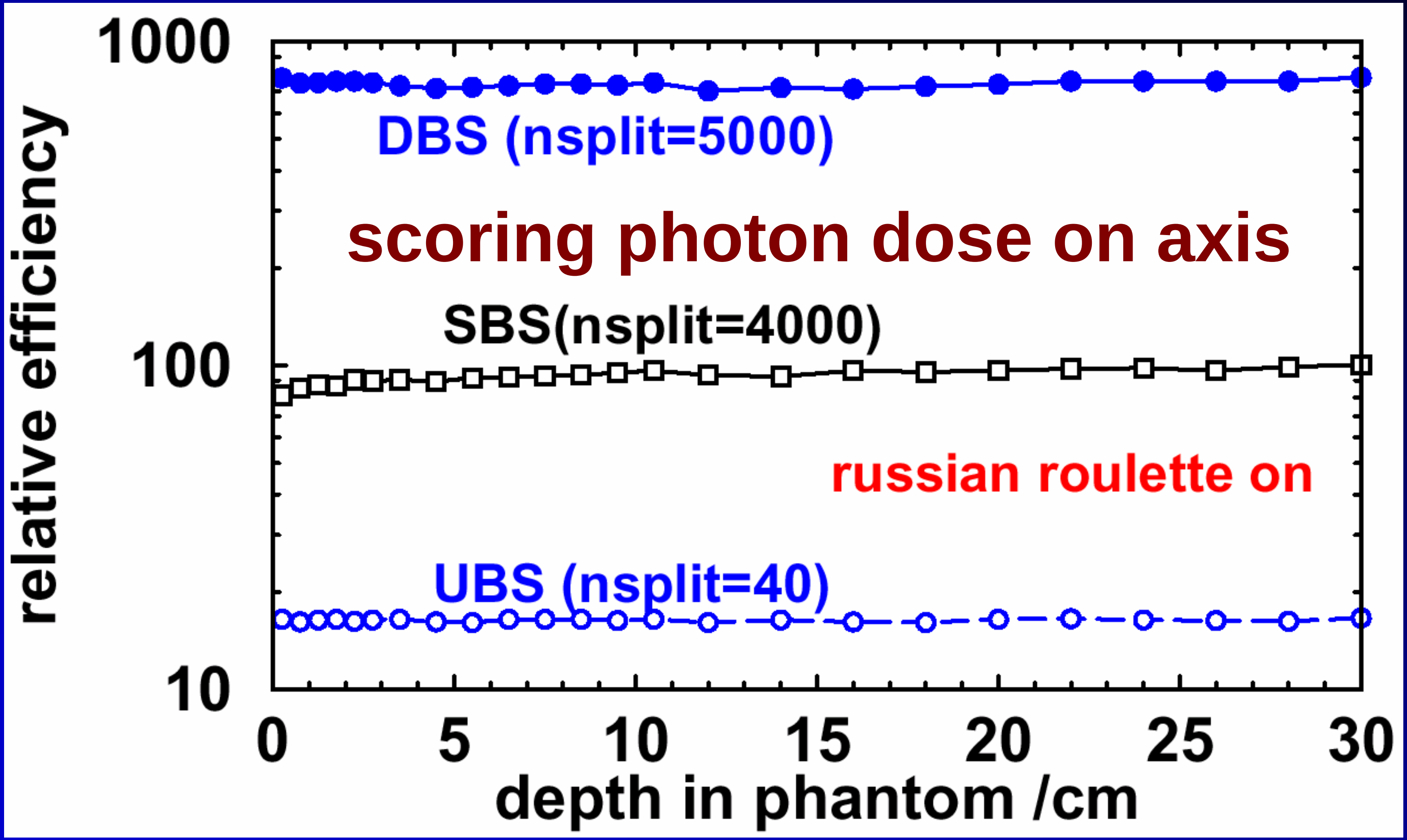
Directional Brem Splitting



Kawrakow et al Med Phys 31 (2004) 2883-2898

Directional Brem Splitting



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Kawrakow et al Med Phys 31 (2004) 2883-2898

Kawrakow's `tricks'

		do_smart_brems	
		ON	OFF
do_smart_compton	ON	1.0	5.3
	OFF	3.3	7.6

No electron splitting.

With electron splitting

		do_smart_brems	
		ON	OFF
do_smart_compton	ON	1.0	2.43
	OFF	1.77	3.2

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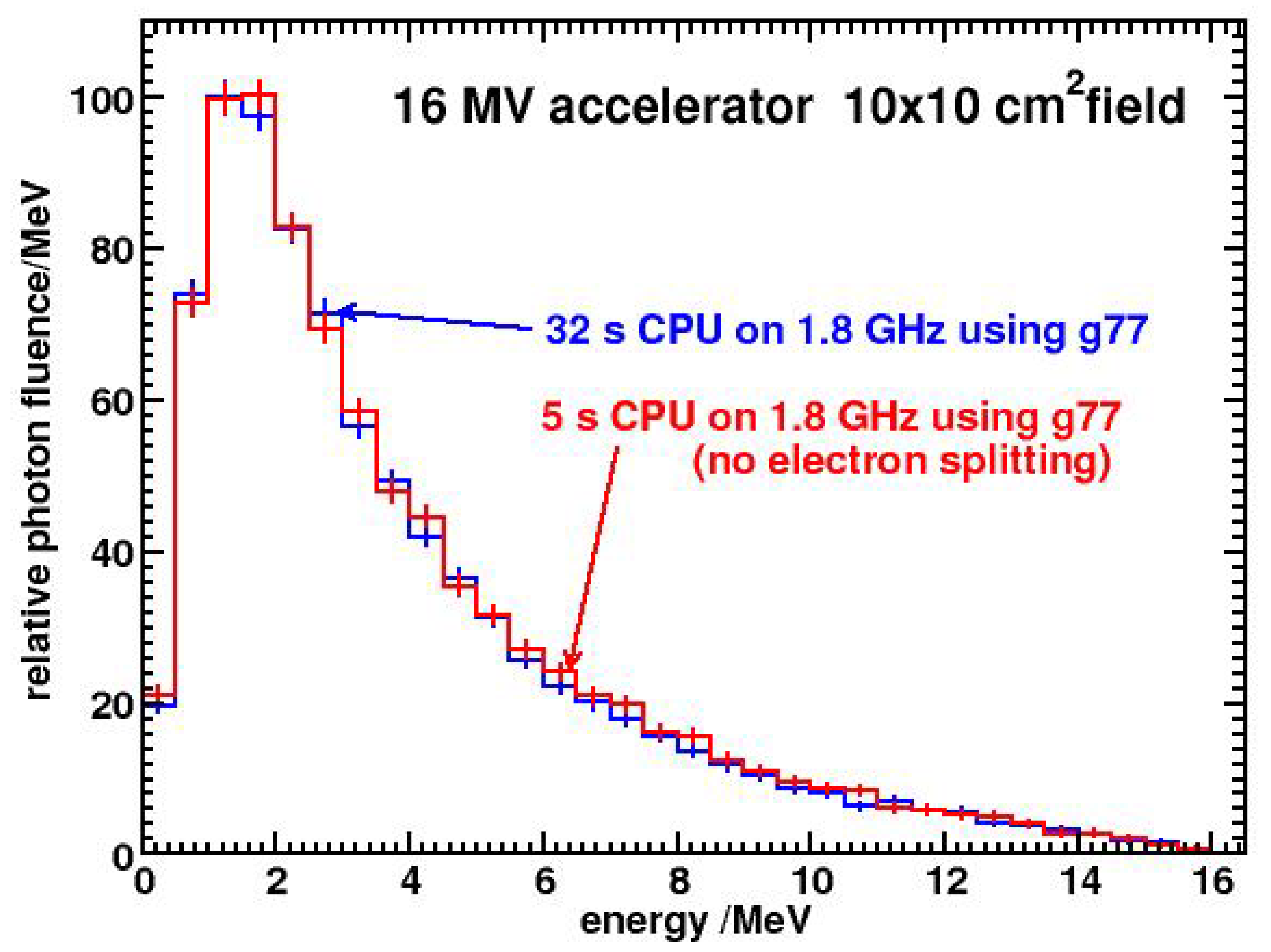
Kawrakow et al Med Phys 31 (2004) 2883-2898

Tx head simulation using BEAMnrc with Directional Bremsstrahlung Splitting



from Sheikh-Bagheri et al. Efficiency Improvement Techniques and Statistical Considerations AAPM Summer School 2006

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Electron problem

-efficiency gain for electrons is only 2

Basis of the solution

-electrons are, almost entirely, from flattening filter and below

-major gains are from treatment of electrons in primary collimator

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Electron solution

introduce 2 planes

-splitting plane: split weight 1 charged particles nsplit times (may distribute symmetrically)

-Russian roulette plane: below this turn off Russian roulette and split all fat photon interactions nsplit times

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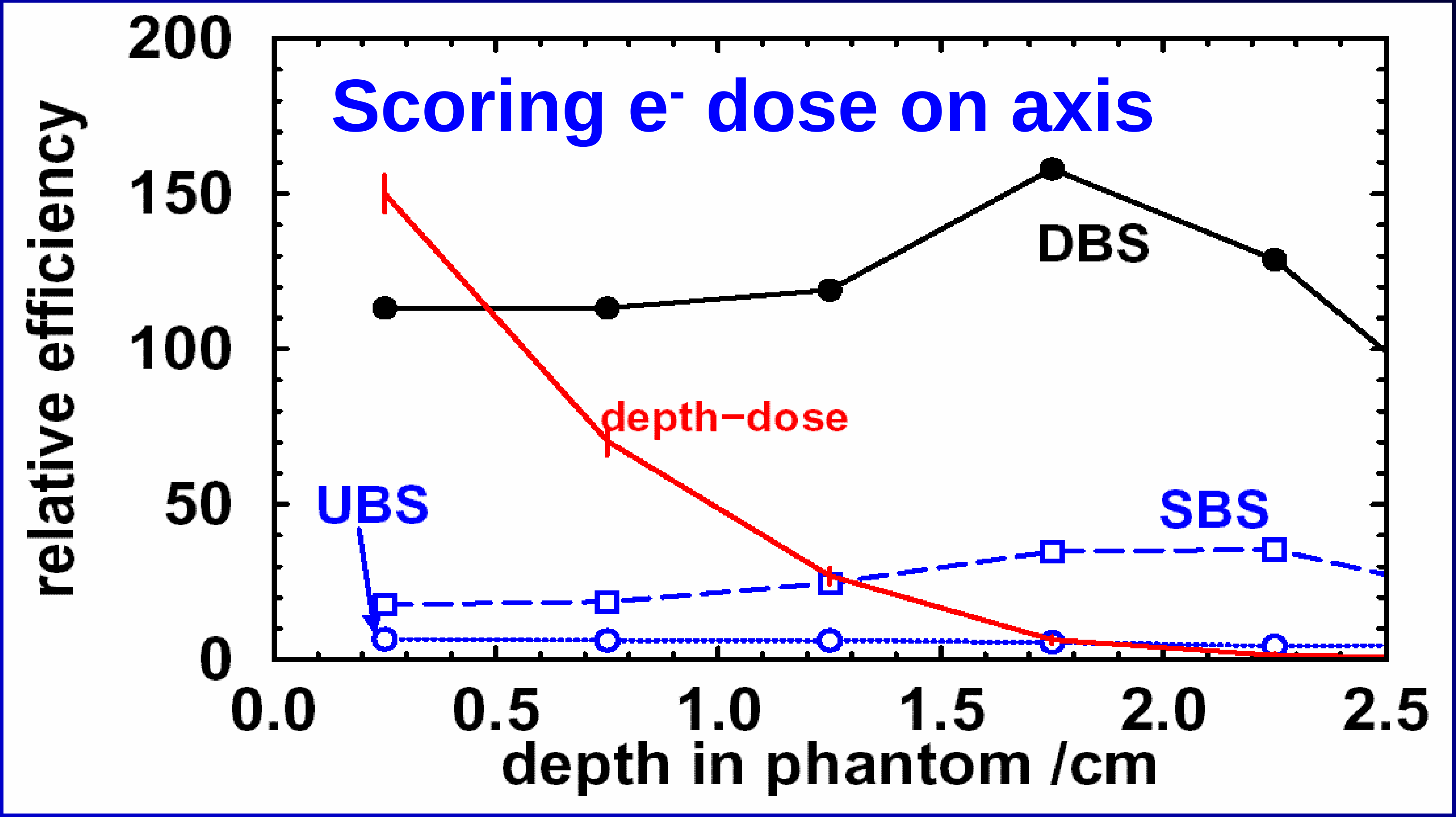
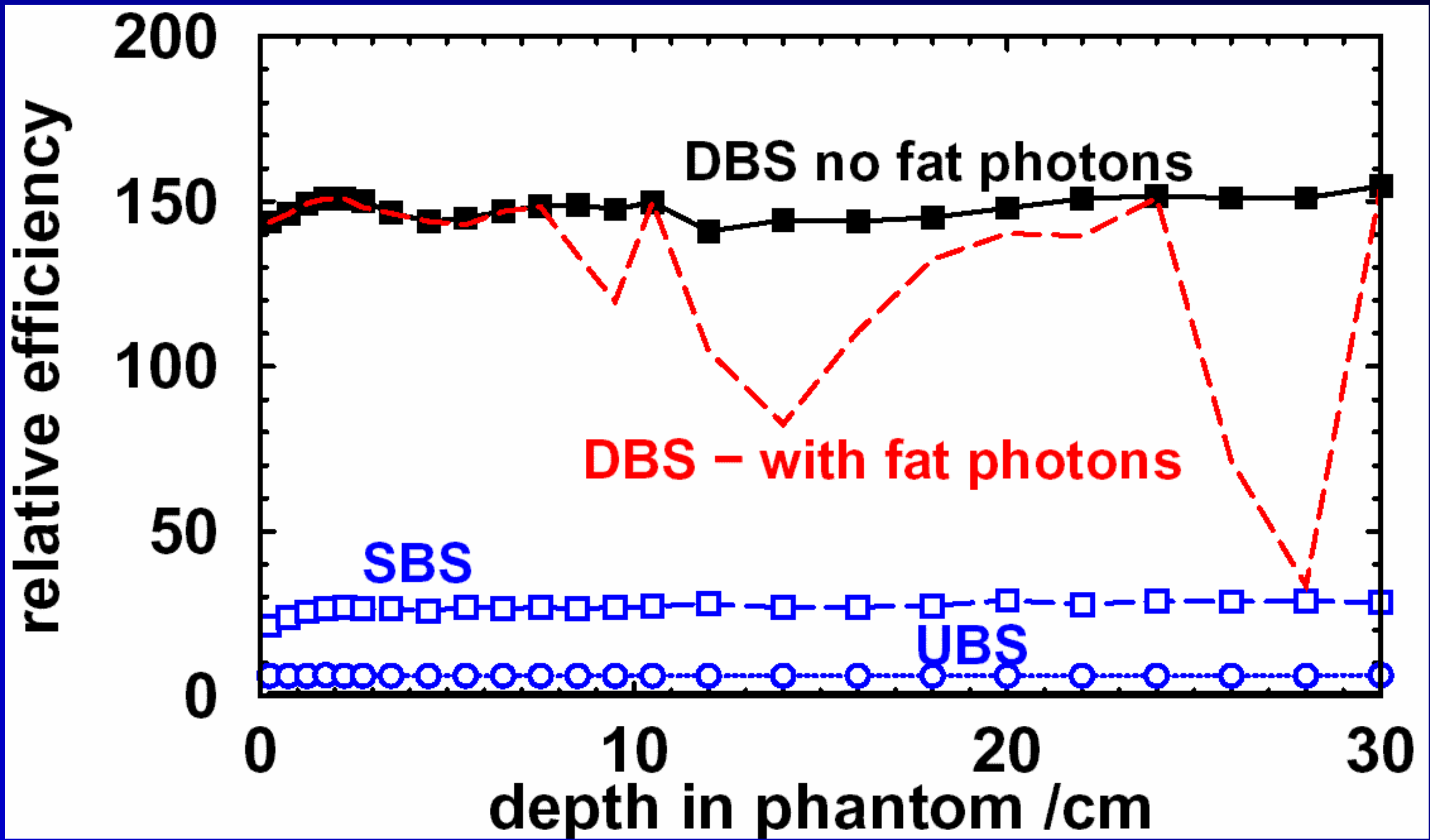
Efficiency increase for e⁻



Kawrakow et al Med Phys 31 (2004) 2883-2898

Efficiency: total dose



Make the region of interest large enough that fat photons do not contribute significant doses, and then ignore them.

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Summary re DBS

DBS, directional brem splitting, improves BEAMnrc's efficiency by a factor of 800 (10 vs SBS) for photon beams (ignore small dose from photons outside field).

For total dose calculations the efficiency improves by factor of 150 (5 vs SBS)

SBS is optimized for greater nsplit than previously realized (5000)

See Kawrakow et al Med Phys 31 (2004) 2883 --2898

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Some remaining issues

- DBS is better when scoring in small regions since as the regions get larger, the correlation between particles from the same brem emission become more important.
- DBS is not as effective for very large fields, since it becomes less selective. It is totally useless in 4π geometries.
- It still requires a brem event to occur, and these are very rare for low energy electrons (eg in x-ray devices).

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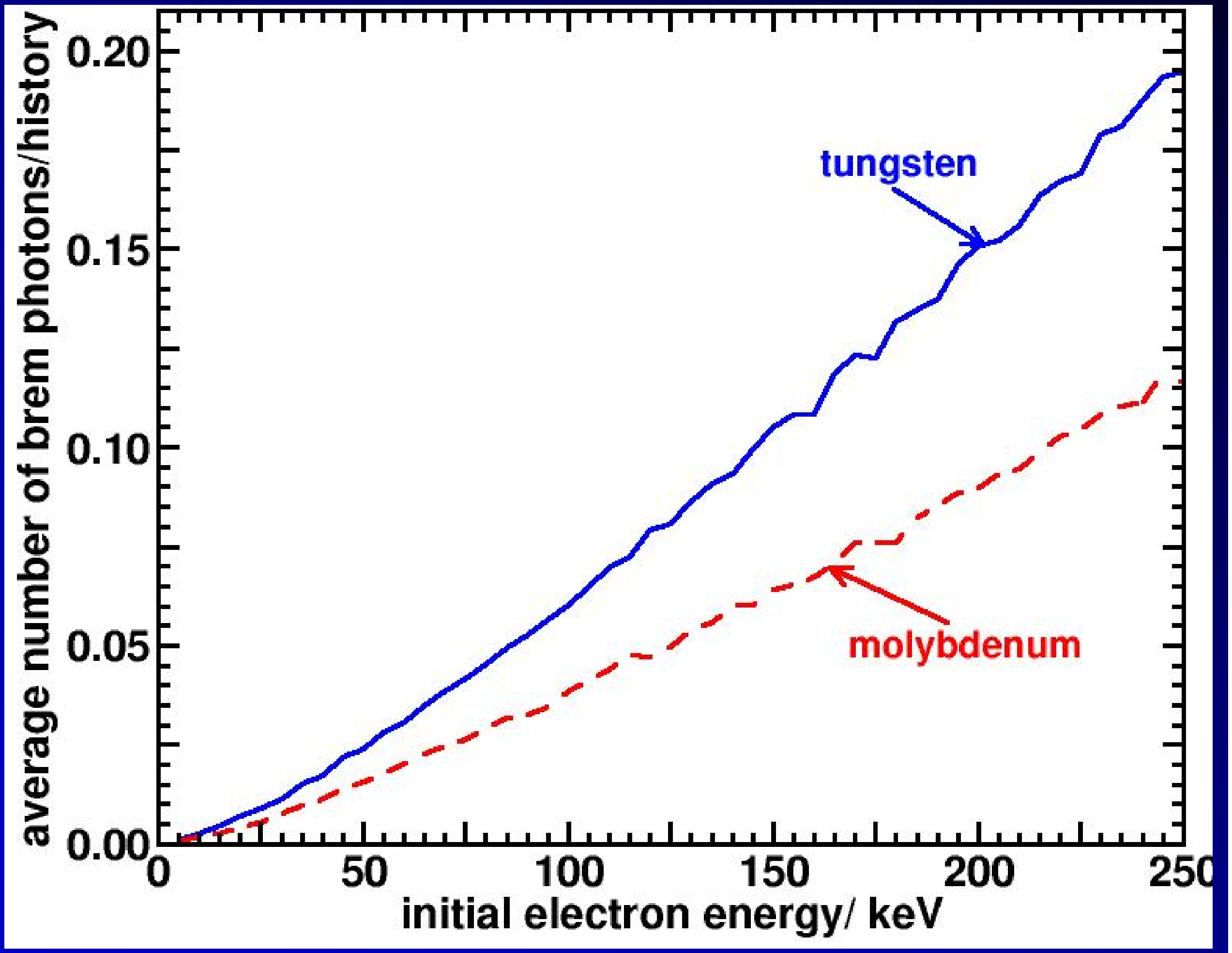
BCSE
brem cross section enhancement

low-energy electrons produce very few brem photons

=> waste time tracking electrons with no effect

Solution: enhance the cross section for brem



Ali & Rogers, Med Phys 34 (2007) 2143-2154

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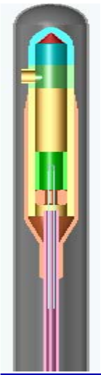
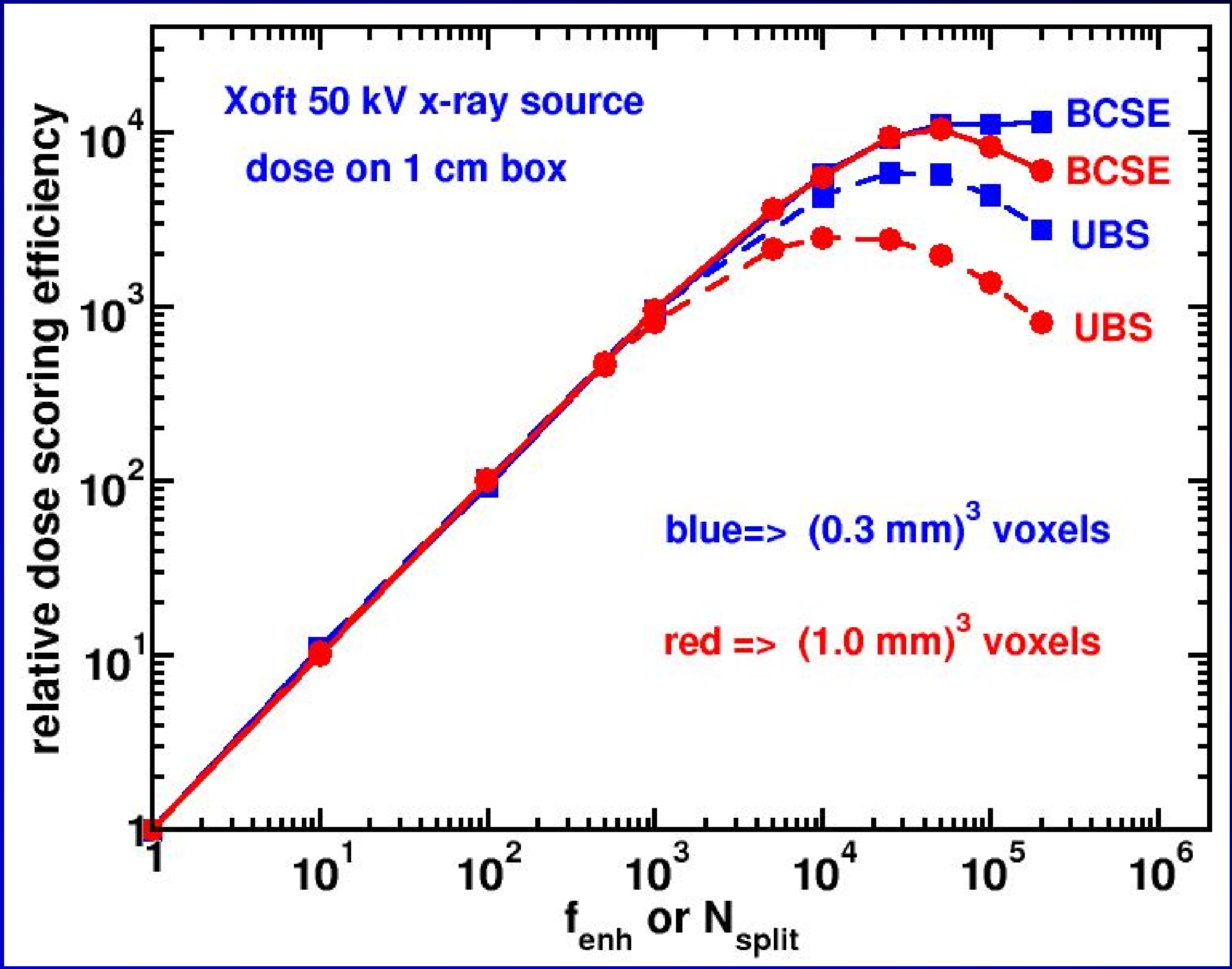


BCSE

UBS = uniform brem splitting

UBS "saturates" sooner because many photons in the same history.



Ali & Rogers, Med Phys 34 (2007) 2143-2154

BCSE with uniform brem splitting

4 π geometry

1 cm cube about 50 keV x-ray source

side of cubic voxel (mm)	$(f_{enh}^{opt}, N_{split}^{opt})$	ε_{opt}	R_{BCSE}	R_U
0.3	(500, 200)	13,032	1.13	2.22
1.0	(500, 100)	14,750	1.42	6.43
2.5	(1000, 50)	4,510	1.44	9.81

efficiency gain relative to no VRT

improvement using combination rather than BCSE or UBS alone

43% (21%) improvement over DBS for 6MV(18 MV)

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Implementation

- Scale up the brem. cross section by a factor *f*
- Reduce emitted brem. photon weight to *1/f*
- Randomly decrement charged particle energy once every *f* times of brem. emission
- Get less correlation vs. splitting, but time penalty
- Combine with splitting to get best of both worlds
- Optimization algorithm very user-friendly

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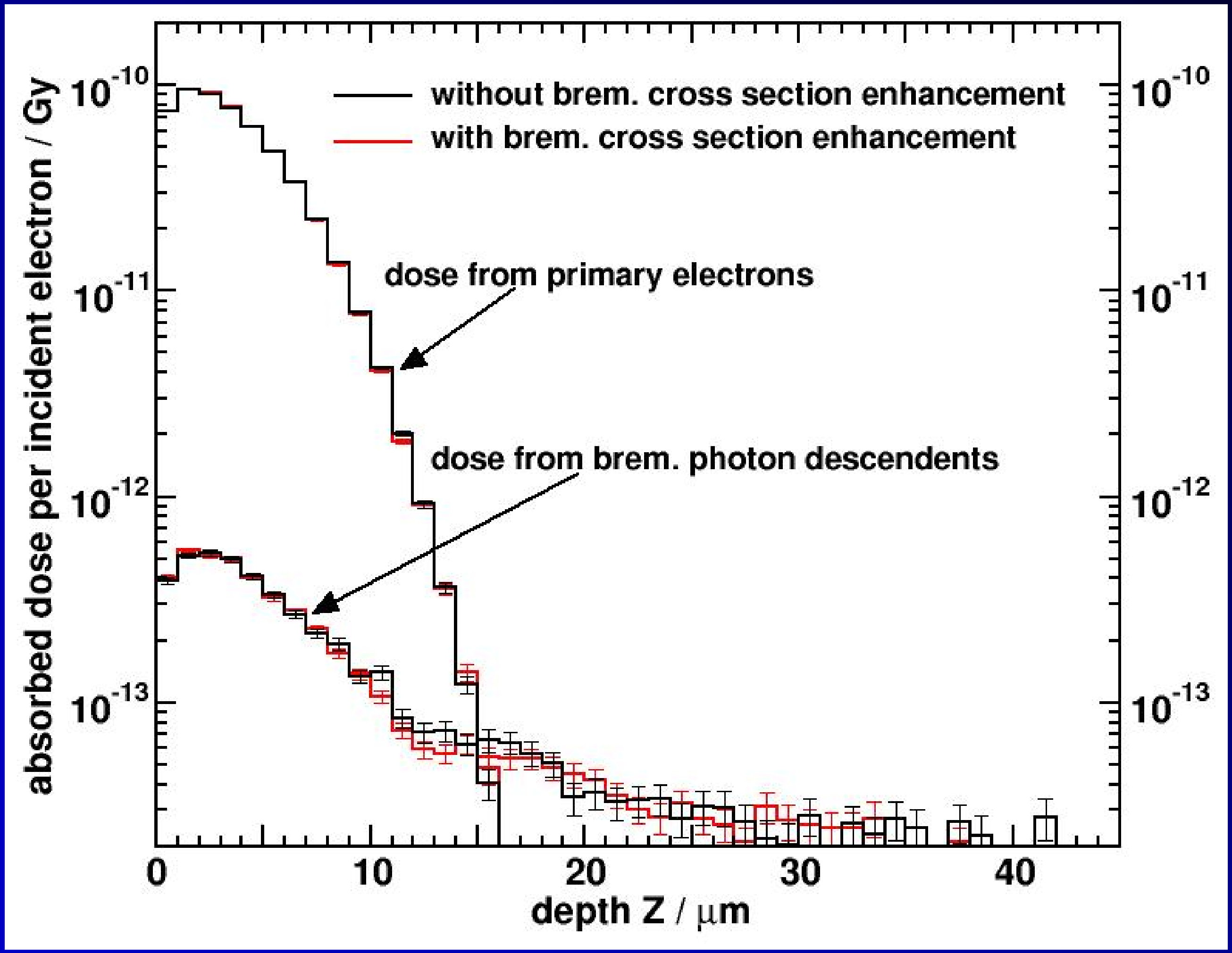
Benchmarking

This is a VRT => no bias

but must prove coding is correct



pencil beam of 130 keV e- on W

Results: directional geometries (diagnostic, mammography & orthovoltage tubes)

field size (cm^2)	scoring zone size (cm^2)	the pair (opt. enh. factor, opt. split. number)	max. eff. from this work / eff. with no variance reduction	max. eff. from this work / best eff. before this work
diagnostic tube, 130 kV DC, 100 cm SSD				
20×20	1×1	(200, 2000)	84,450	1.74
	2×2	(200, 1000)	54,874	2.22
	4×4	(200, 500)	30,181	3.37
mammography tube, 20 kV DC, 65 cm SSD				
18×18	1.5×1.5	(500, 2000)	251,240	4.80
orthovoltage tube, 230 kV DC, 52 cm SSD				
10×10	1×1	(100, 2000)	83,511	2.10

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Results: clinical linear accelerators

- Much more brem. emission than in kV range
- Target self-attenuation & time issues
- Enhance brem. cross section for both e^- & e^+
- Varian 6 & 18 MV linacs simulated
- 10x10 cm^2 @ 100 cm SSD

beam energy (MV)	scoring zone size (cm^2)	the pair (opt. enh. factor, opt. split. number)	max. eff. from this work / eff. with no variance reduction	max. eff. from this work / best eff. before this work
6	1×1	(20, 750)	823	1.43
	10×10	(20, 50)	125	2.62
18	1×1	(20, 100)	83	1.21

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Speed of x-ray tube simulations

Realistic clinical situations, 3.0 GHz Intel WC processor, g77,

2% average uncertainty on fluence, easy optimization

simulation case	SSD (cm)	field size (cm^2)	scoring zone size (cm^2)	$T(2\%)$ (seconds)
130 kV DC diagnostic tube	100	20×20	1×1	100
			2×2	56
			4×4	21
20 kV DC mammography tube	65	18×18	1.5×1.5	43
230 kV DC orthovoltage tube	52	10×10	1×1	37
6 MV Varian accelerator	100	10×10	1×1	20
			10×10	1.4
18 MV Varian accelerator	100	10×10	1×1	56

about

1 minute

Verhaegen *et al*, McGill Univ., PMB **44** (1999) 1767-1789

"Typically, calculation times of *several days* were required on any one machine for simulations of the complete setup"

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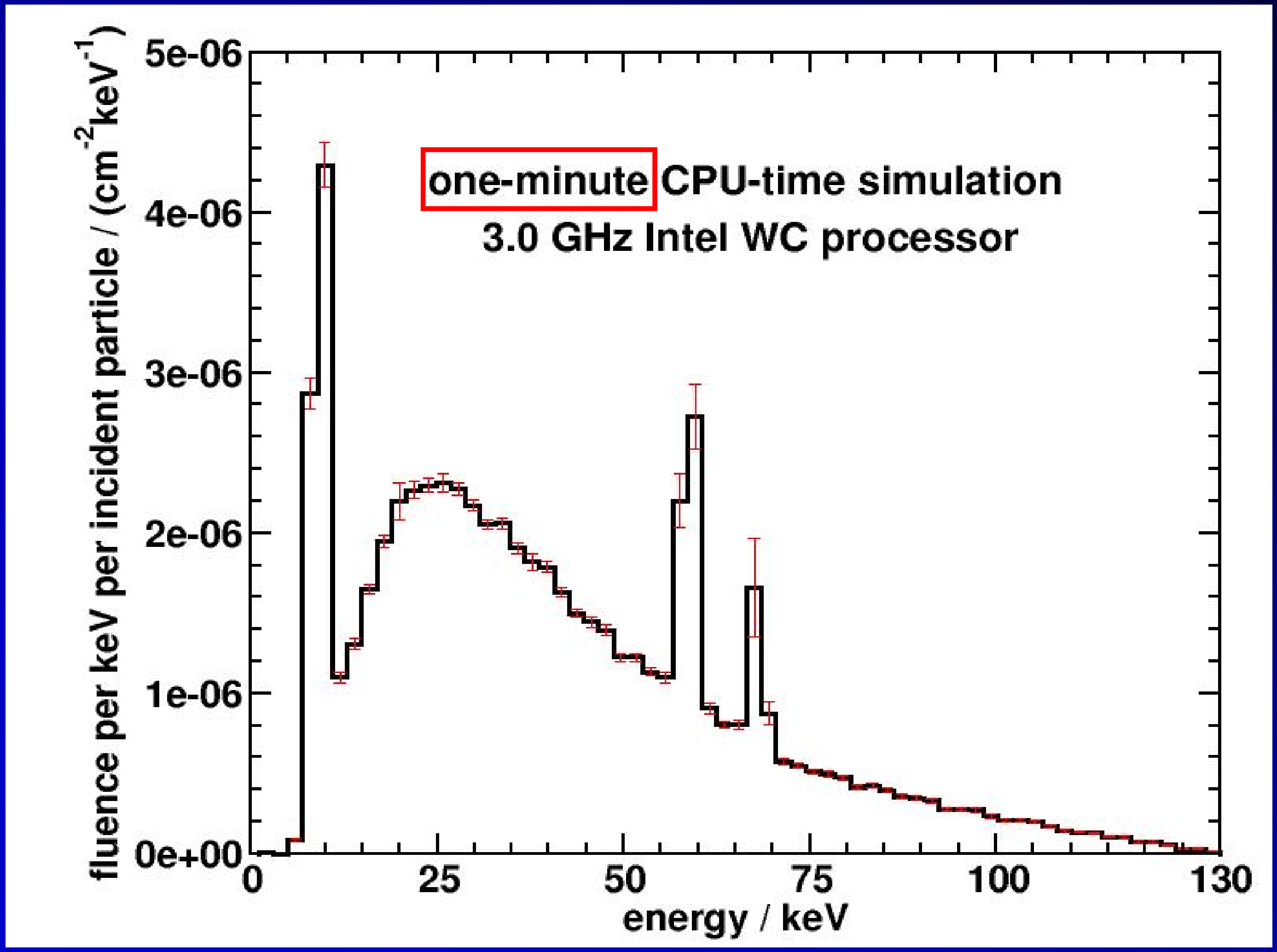
Results: graphical



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Summary

- BEAMnrc system is extremely fast for doing accelerator simulations, especially for photon beams.
- Although EGSnrc is generally 3-5 times faster than GEANT4 or PENELOPE in simple geometries, the major reason for the efficiency increase is the dedicated VRT's as well as several accurate approximations which save considerable time.

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CRSNG

WestGrid

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