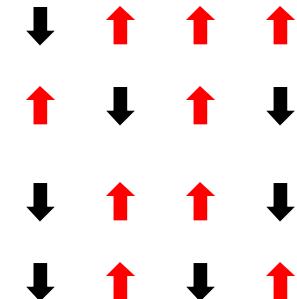


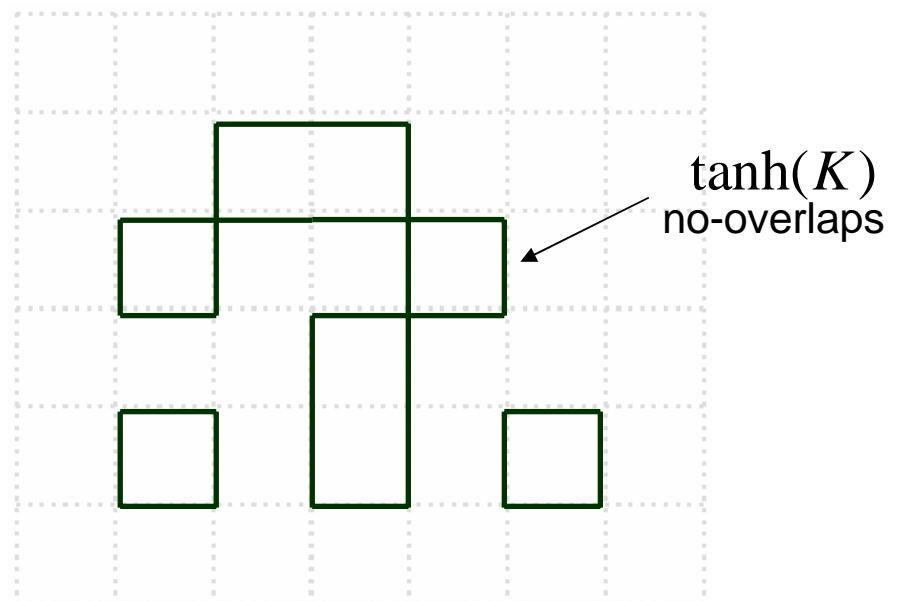
Laboratory: Worm Algorithm for the Ising model:

$$-H/T = K \sum_{\langle ij \rangle} \sigma_i \sigma_j \quad \text{with} \quad \sigma_i = \pm 1 \quad \text{on a square lattice}$$

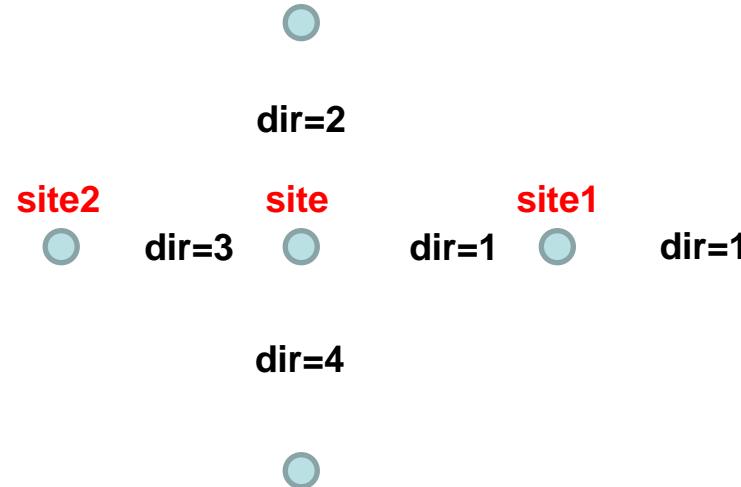
$$Z = \sum_{\{\sigma_i\}} \left(\prod_{\langle ij \rangle} e^{K \sigma_i \sigma_j} \right)$$



$$Z / \cosh^{dN}(K) = \sum_{\{N_b=0,1\}}^{\text{loops}} \left(\prod_{b=\langle ij \rangle} \tanh^{N_b}(K) \right)$$



Link the lattice with periodic boundary conditions



create an array **nn(site,dir)** which returns the nearest neighbor of **site** in the direction **dir**

for example: **site1= nn(site,1)** and **site= nn(site1,1+dim)**

Periodic boundary conditions mean that: **site2= nn(site1,1)** and **site1= nn(site2,1+dim)**

```

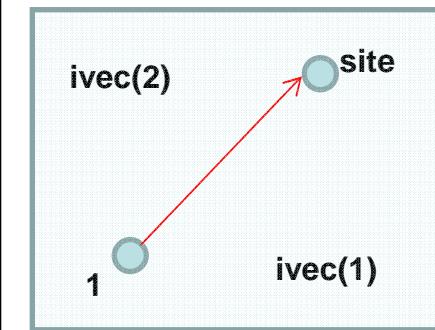
integer, parameter :: L=10, dim=2, N=L**dim, dd=2*dim
double precision, parameter :: JT=0.3d0
double precision :: ratio
integer :: nn(N,dd)
integer :: site,site1
integer :: ivec(dim), i

ratio=tanh(JT)

DO site=1,N
    site1=site-1
    DO i=dim,1,-1
        ivec(i)=site1 / L**(i-1)
        site1=site1-ivec(i)*L**(i-1)
    ENDDO
    DO i=1,dim
        site1=site+L**(i-1)
        IF(ivec(i)==L-1) site1=site1-L**i
        nn(site,i)=site1
        nn(site1,i+dim)=site
    ENDDO
ENDDO

READ*, site, i
Print*, nn(site,i)
END ! compile with ifort

```



Now, we need to initialize the configuration and counters for Z,G,M2,E

```
cd /afs/ictp/home/n/nprokofi/public/  
filename uf.for
```

Compile using

```
ifort uf.for -o Ising
```

Run

```
./Ising
```

Plotting
gnuplot
plot ‘datafile’ u (log(\$1)):(log(\$2))

Now, we need to initialize the configuration and counters for Z,G,M2,E

Configuration is kept in the array **bond(site,dim)**, the sum of all bond numbers is **NB**,
do not forget **Ira** and **Masha** and the distance between them **IM(dim), dist**

And, of course, we need a random number generator.

```
integer :: bond(N,dim), NB, Ira, Masha, IM(dim), dist
double precision :: Z, G(N), M2, E

double precision :: ugen(97) , c7, cd, cm
integer :: ij, kl, j, i97, j97

Z=1.d-15
M2=0.d0
E=0.d0
G=0.d0
bond=0
NB=0
Ira=1
Masha=1
IM=0

ij=4836
kl=2748
call SRANMAR(ij,kl)
```

Collect all definitions
in one place

serve rndm() generator

rndm() initialization

We are all set to do the simulation. Just one update!

```
double precision :: step
```

```
step=0.d0  
DO  
step=step+1  
call UPDATE
```

```
IF(step>N*1000.d0) THEN
```

```
IF(Ira==Masha) THEN  
Z=Z+1.d0  
E=E+NB  
ENDIF
```

```
G(dist)=G(dist)+1.d0  
M2=M2+1
```

```
ENDIF
```

```
IF(MOD(step,1.d7)==0) THEN  
PRINT*, E/Z, M2/Z  
ENDIF
```

```
ENDDO
```

```
contains  
Subroutine UPDATE
```

```
END
```

thermolize!

collect statistics

Z configuration

G configuration; G(0)=Z

print something

SUBROUTINE UPDATE

```
integer :: k, k1, site1, n, nnew
```

```
IF(lra==Masha) THEN  
lra=rndm()*N+1 ; Masha=lra ; ENDIF  
  
k=rndm()*dd+1 ; site1=nn(lra,k)  
  
IF(k>dim) THEN ; k1=k-dim ;  
n=bond(site1,k1)  
ELSE ; n=bond(lra,k) ; ENDIF  
  
IF(n==0 .AND. rndm()>ratio) RETURN  
  
nnew=MOD(n+1,2)  
IF(k>dim) THEN ; bond(site1,k1)=nnew  
IM(k1)=IM(k1)-1 ; IF(IM(k1)<0) IM(k1)=L-1  
ELSE ; bond(lra,k)=nnew  
IM(k)=IM(k)+1 ; IF(IM(k)==L) IM(k)=0  
ENDIF  
lra=site1  
  
dist=1 ;  
DO i=1,dim; dist=dist+IM(i)*L**(i-1); ENDDO  
NB=NB-n+nnew  
  
END SUBROUTINE UPDATE
```

The update itself!

- } If in Z configuration move lra and Masha elsewhere at random direction
- } bond number
- } accept/reject
- } configuration change
- } counters

Formally we are done!

We can now enjoy it and study critical phenomena.

Let's fix $JT=0.44069$ (critical point), and calculate susceptibility (second printout of M^2/Z for different system sizes)

$$L \quad \langle M^2 \rangle$$

- | | |
|------|--------|
| 4. | 12.184 |
| 8. | 41.434 |
| 16. | 139.56 |
| 32. | 470. |
| 64. | 1578 |
| 128. | 5290. |

The data I have

Plot your results as $\log(y)$ vs $\log(x)$ to observe a nice power –law scaling (given by the slope of your nearly straight-line curve).

Prediction of the Onsager solution: $\chi = \langle M^2 \rangle \propto L^{1.75}$

Random number generator is separate!

DOUBLE PRECISION FUNCTION RNDM()

DOUBLE PRECISION, parameter :: r1=1.d-15, r2=1.d-15

DOUBLE PRECISION :: RVAL

RVAL = UGEN(I97) - UGEN(J97)

IF (RVAL < 0.D0) RVAL = RVAL + 1.0

UGEN(I97) = RVAL

I97=I97-1

IF (I97 == 0) I97 =97

J97=J97-1

IF (J97 == 0) J97=97

C7=C7-CD

IF (C7 < 0.D0) C7=C7+CM

RVAL = RVAL - C7

IF (RVAL .LT. 0.D0) RVAL = RVAL + 1.0

RNDM = max(RVAL-r1,r2)

END FUNCTION RNDM

generator itself

SUBROUTINE SRANMAR(IJ,KL)

INTEGER IRAND

INTEGER :: i7, k7, l7, j7, ii7, jj7, m7, ij, kl

DOUBLE PRECISION :: S7, T7

I7 = MOD(IJ/177, 177) + 2

J7 = MOD(IJ, 177) + 2

K7 = MOD(KL/169, 178) + 1

L7 = MOD(KL, 169)

DO ii7 = 1, 97

S7 = 0.D0

T7 = 0.5D0

DO jj7 = 1, 24

M7 = MOD(MOD(I7*j7, 179)*K7, 179)

I7 = J7

J7 = K7

K7 = M7

L7 = MOD(53*L7+1, 169)

IF (MOD(L7*M7, 64) > 32) S7 = S7 + T7

T7 = 0.5D0 * T7

ENDDO

UGEN(ii7) = S7

ENDDO

C7 = 362436.D0 / 16777216.D0

CD = 7654321.D0 / 16777216.D0

CM = 16777213.D0 / 16777216.D0

I97 = 97

J97 = 33

RETURN

END subroutine SRANMAR

call it to initialize rndm()