The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

1935-2

Spring School on Superstring Theory and Related Topics

27 March - 4 April, 2008

Black Holes in Higher Dimensions

Roberto Emparan
ICREA & U. Barcelona

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Black Holes in Higher Dimensions

Roberto Emparan
ICREA & U. Barcelona

ICTP lectures, 31 Mar-2 Apr 2008

- References:
  - RE+Reall: *BH's in Hi-D,* **0801.3471 [hep-th]**
  - Other reviews:
    - Obers: **0802.0519 [hep-th]** (includes KK phases)
    - Kunz et al: **0710.2291 [hep-th]** (w/ charges)
    - Frolov: **0712.4157 [gr-qc]** (symmetries)
    - RE+Reall: **hep-th/0608012** (black rings)

Why higher-dimensional gravity?

Motivations

As applications:

- String / M theory
- Large Extra Dimensions & TeV gravity
- AdS/CFT
- Mathematics: Lorentzian geometry

But, not least, also of intrinsic interest:

- ***D as a* tunable parameter for gravity and black holes**

  What properties of black holes are

  - 'intrinsic' ? → *Laws of bh mechanics…*
  - D-dependent? → *Uniqueness, topology, shape, stability…*

What can a black hole (i.e. *spacetime*) do?

- It’s only recently (~7 yrs ago) that we’ve fully realized how *little* we know about black holes (even classical ones) and their dynamics in D>4

- A better knowledge of them is likely to have a strong impact on *all* the subjects mentioned

- Activity launched initially by two main results:
  - **GL-instability**, its endpoint, and inhomogeneous phases
  - **Black rings**, non-uniqueness, non-spherical topologies

- I’ll mostly focus on simplest set up: vacuum, $R_{\mu\nu} = 0$, asymptotically flat solutions

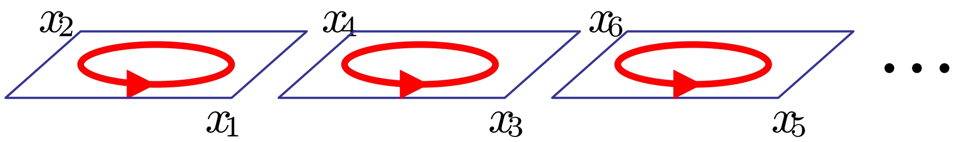
FAQ's

- Why is D>4 richer?

  – More degrees of freedom

  – Rotation:

$$O(D\text{-}1) \supset U(1)^{\lfloor (D\text{-}1)/2 \rfloor}$$



    - more rotation planes
    - gravitational attraction ⇔ centrifugal repulsion

$$-\frac{GM}{r^{D-3}} + \frac{J^2}{M^2 r^2}$$

  – ∃ extended black objects: black p-branes

- Why is D>4 harder?

  – More degrees of freedom

  – Axial symmetries: U(1)'s at asymptotic infinity appear only every 2 more dimensions -- not enough to reduce to 2D σ-model if D>5

Phases of black holes

- Asymptotic conserved charges: M, J_i

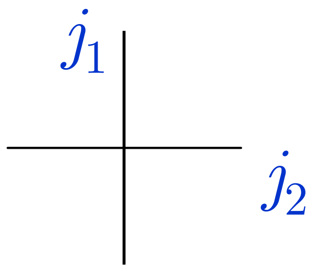
$$\Rightarrow \qquad \mathcal{A}_H(M,\, J_i\,),\; T_H(M,\, J_i\,),\; \dots$$

- To compare solutions we need to fix a common scale
- *Classical* GR doesn't have any intrinsic scale

  → We'll fix the mass M

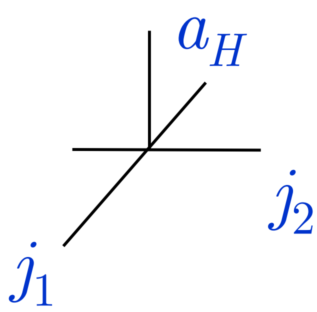
  equivalently factor it out to get dimensionless quantities

$$j_i \propto \frac{J_i}{GM^{\frac{d-2}{d-3}}}$$



black hole solutions will cover a region of this space

$$a_H \propto \frac{\mathcal{A}_H}{(GM)^{\frac{d-2}{d-3}}} \Rightarrow a_H(j_i)$$



a_H gives a surface in this space

Phases of 4D black holes

- Static: Schwarzschild

$$ds^2 = -\left(1-\frac{\mu}{r}\right)dt^2 + \frac{dr^2}{1-\frac{\mu}{r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$\mu = 2GM$$

- Stationary: Kerr

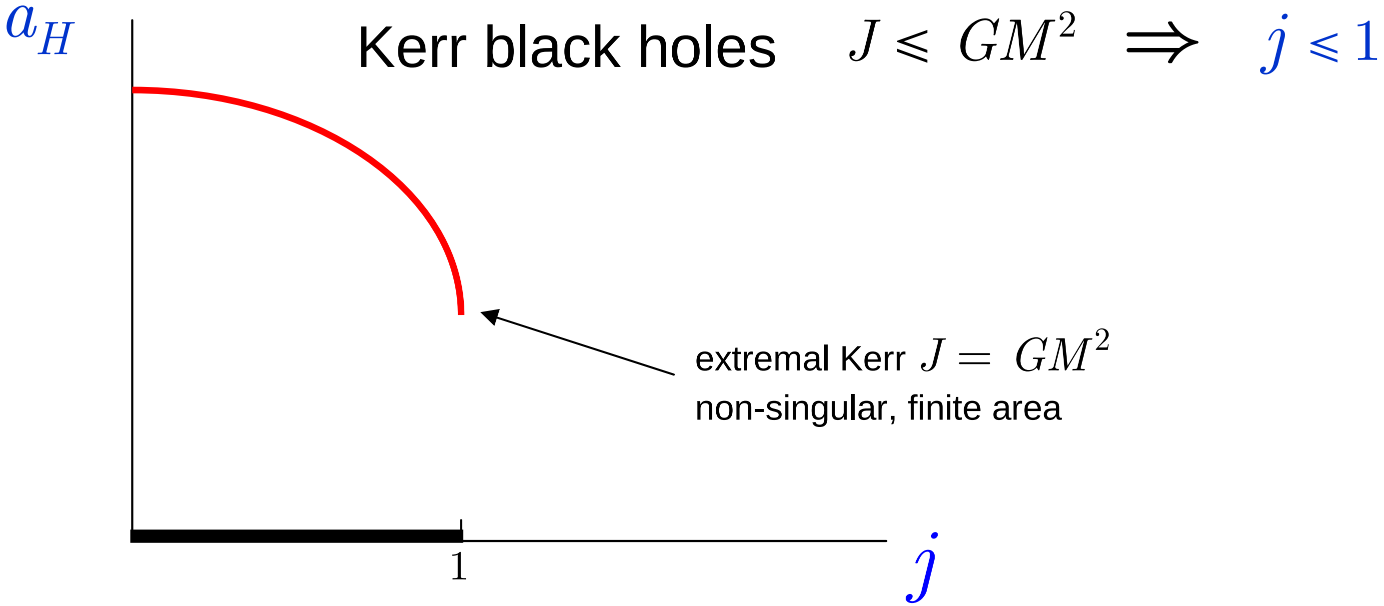
$$ds^2 = -dt^2 + \frac{\mu r}{\Sigma}\left(dt + a\sin^2\theta d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2+a^2)\sin^2\theta d\phi^2$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 - \mu r + a^2\,, \qquad a = \frac{J}{M}$$

Horizon: $\Delta = 0$ $\Rightarrow$ $M \geqslant a$: Upper bound on J for given M

$$J \leqslant GM^2$$

Phases of 4D black holes



Uniqueness theorem: *End of the story*

Multi-bhs not rigorously ruled out, but physically unlikely to be stationary

(eg multi-Kerr can't be balanced)

Black holes in D>4

- Schwarzschild is easy: *Tangherlini 1963*

$$ds^2 = -\left(1 - \frac{\mu}{r^{D-3}}\right) dt^2 + \frac{dr^2}{1 - \mu/r^{D-3}} + r^2 d\Omega_{(D-2)}$$

$$\mu \propto M$$

Black strings & branes

- Not asymp flat bhs, but necessary to understand them…

Take a d-diml black hole
(Ricci-flat)

add p extra dim

black p-brane

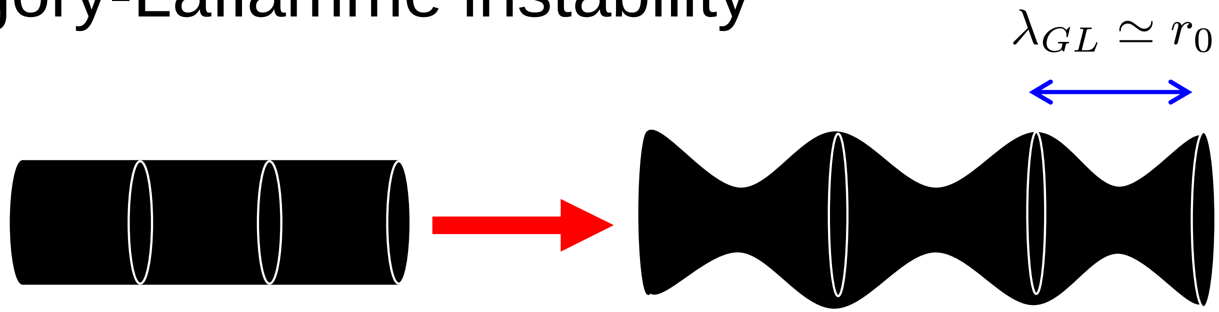
horizon topology
$\mathbb{R}^p$ x $S^{d\text{-}2}$

$$ds^2_{d+p} = ds^2_d(\text{black hole}) + \sum_{i=1}^{p} dx^i dx^i$$

- Solves $R_{\mu\nu} = 0$
- Can boost along x^i to give it momentum

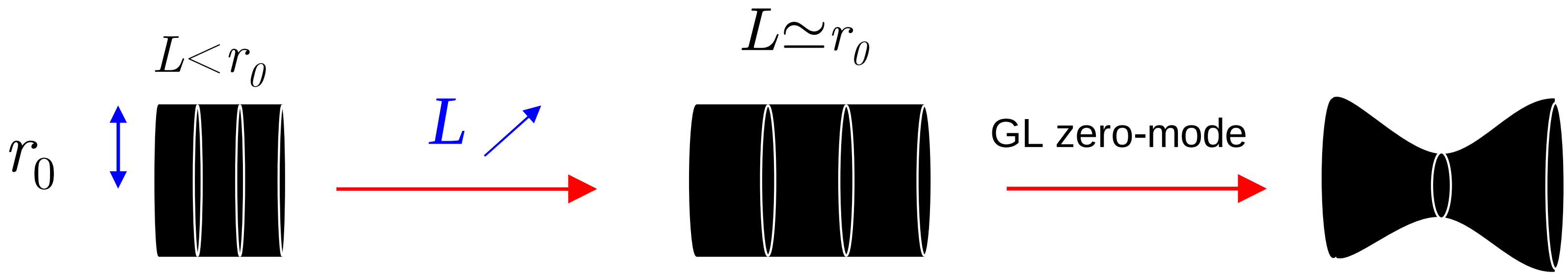
Black strings and branes exhibit

– Gregory-Laflamme instability



– GL zero-frequency mode = static perturbation

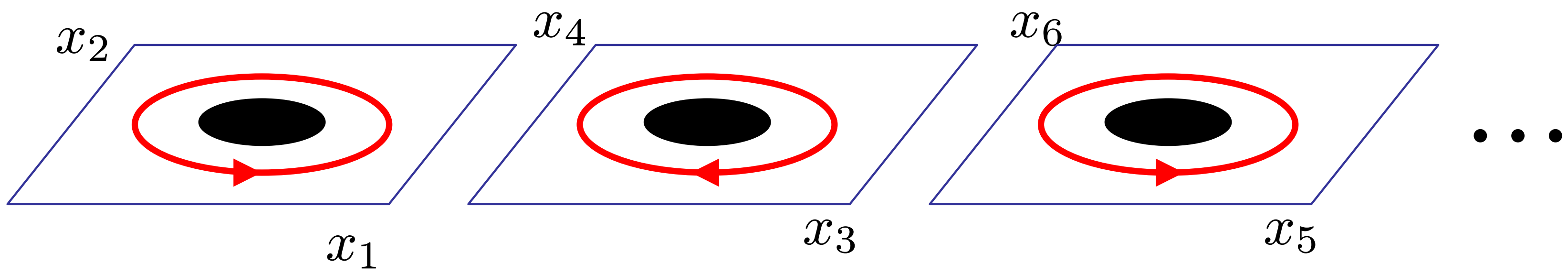
→ branching into static strings w/ non-uniform horizons



branch of static ***lumpy black strings***

Rotation

- *Myers+Perry (1986)*: rotating black hole solutions with angular momentum in an arbitrary number of planes



J_1 J_2 J_3 $\ldots$ $J_{\lfloor (D\text{-}1)/2 \rfloor}$

e.g. $D{=}5{,}6$: $J_1,\ J_2$

$D{=}7{,}8$: $J_1,\ J_2,\ J_3$ etc

- They all have spherical topology S^{D-2}

- Consider a **single spin**:

$$ds^2 = -dt^2 + \frac{\mu}{r^{D-5}\Sigma}\left(dt + a\sin^2\theta\, d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2 + a^2)\sin^2\theta\, d\phi^2$$

$$+r^2\cos^2\theta\, d\Omega^2_{(D-4)}$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}\,,$$

$$\mu \propto M$$

$$a \propto \frac{J}{M}$$

$$\frac{\Delta}{r^2} - 1 = -\frac{\mu}{r^{D-3}} + \frac{a^2}{r^2}$$

gravitational

centrifugal

- Consider a **single spin**:

Horizon: $\Delta = 0$ $$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$

$$\boldsymbol{D=5}: \qquad r_h^2 + a^2 - \mu = 0 \quad \Rightarrow \quad r_h = \sqrt{\mu - a^2}$$

$\Rightarrow \quad a^2 \leqslant \mu \quad \Rightarrow$ upper bound on J for given M

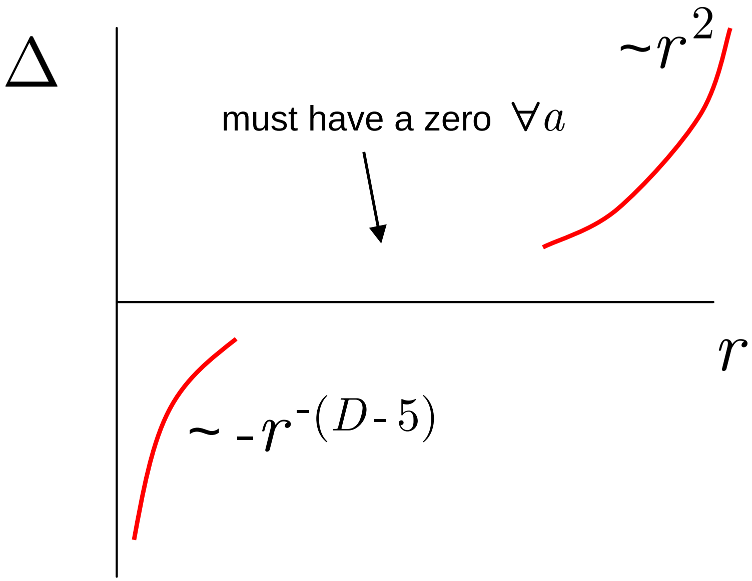
- similar to 4D
- but extremal limit $a^2 = \mu \;\Rightarrow\; r_h = 0$

this is *singular, zero-area*

$D \geqslant 6$:

Horizon: $\Delta = 0$

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$

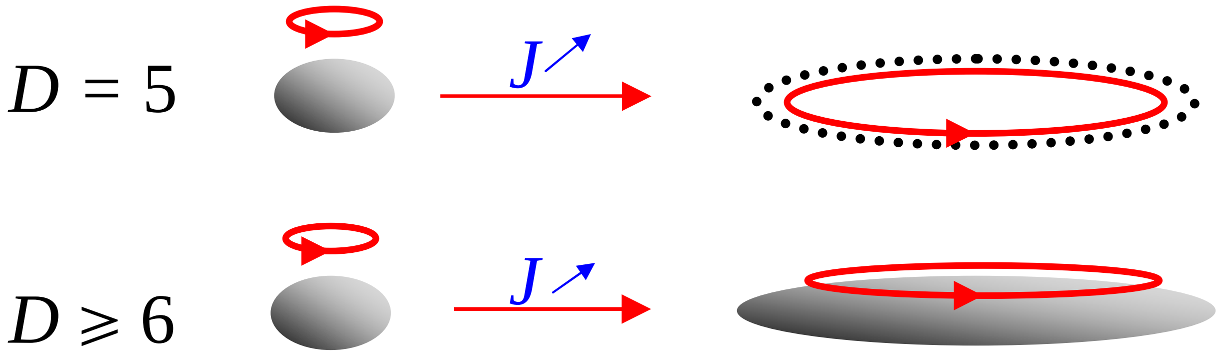


For fixed μ there is an outer event horizon for *any* value of a

$\Rightarrow$ No upper bound on J for given M

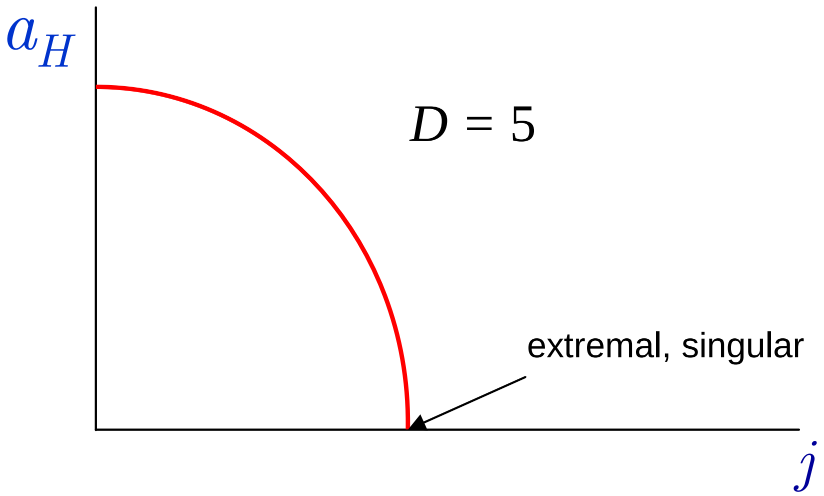
$\Rightarrow \exists$ ***ultra-spinning black holes***

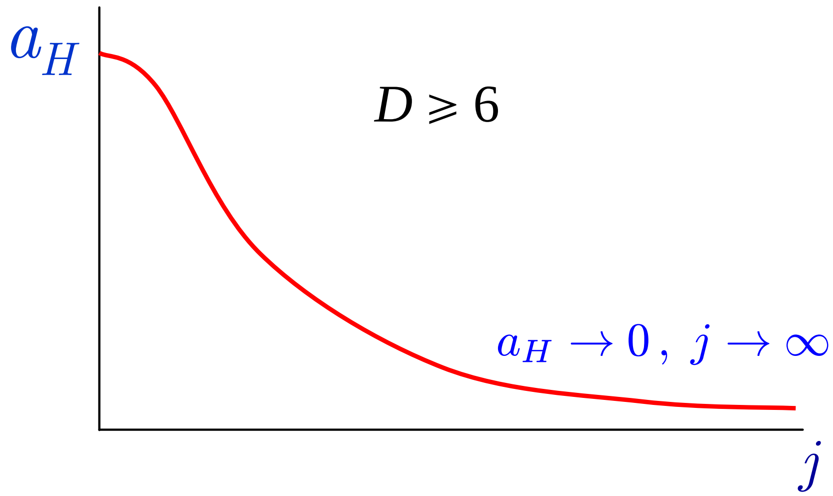
- # Single spin MP black holes:



upper (extremal) limit
on J is singular

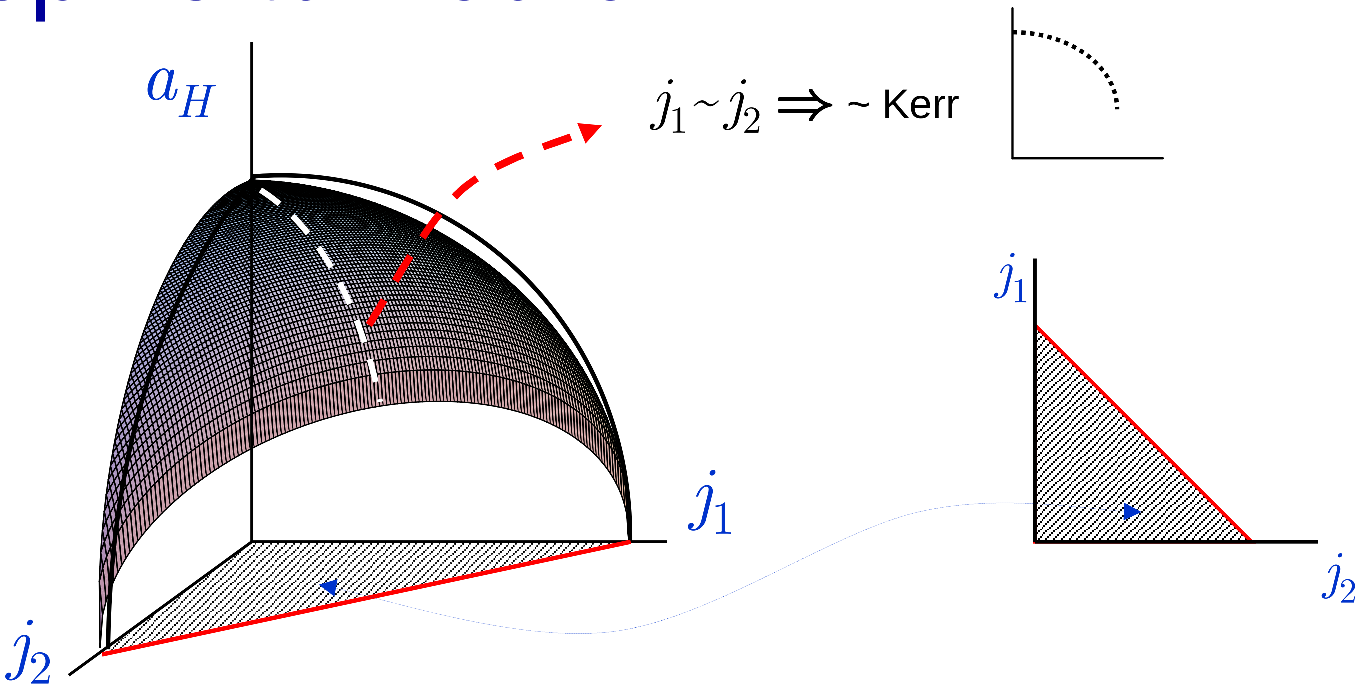
no upper limit on J
for fixed M



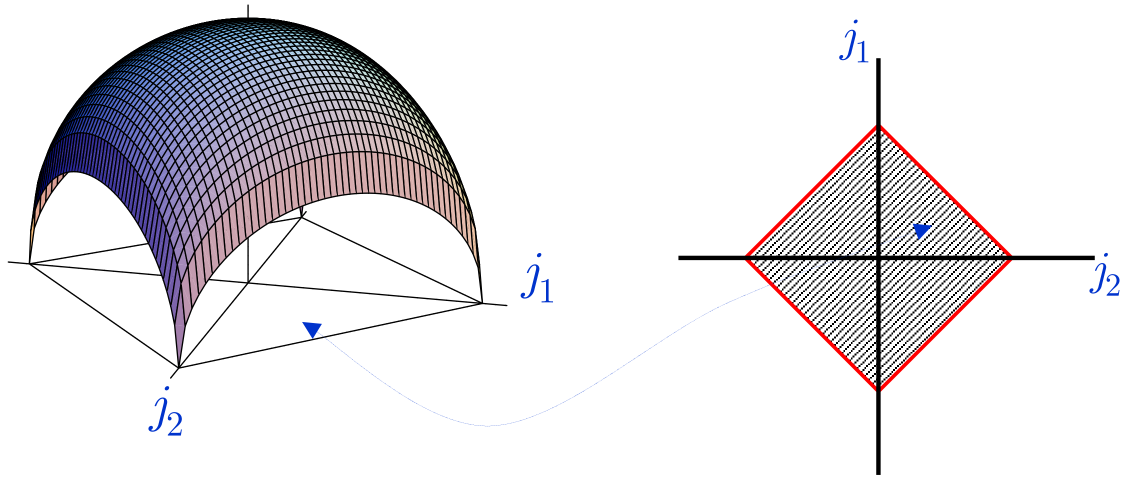


Several spins turned on:

$D=5$



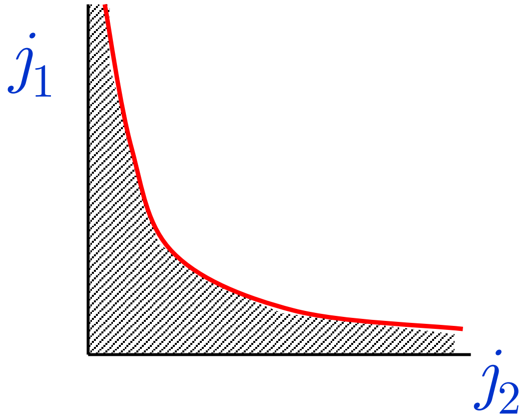
The 5D dome:



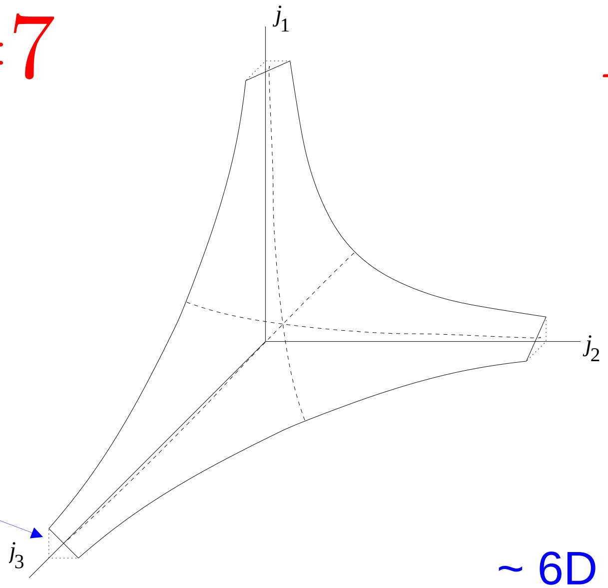
$D \geqslant 6$

- If all $j_1 \sim j_2 \sim \ldots \sim j_{\lfloor (D\text{-}1)/2 \rfloor} \Rightarrow$ ~ Kerr
- $\exists$ ultra-spinning regimes if one (two) j_i are much smaller than the rest

$D=6$

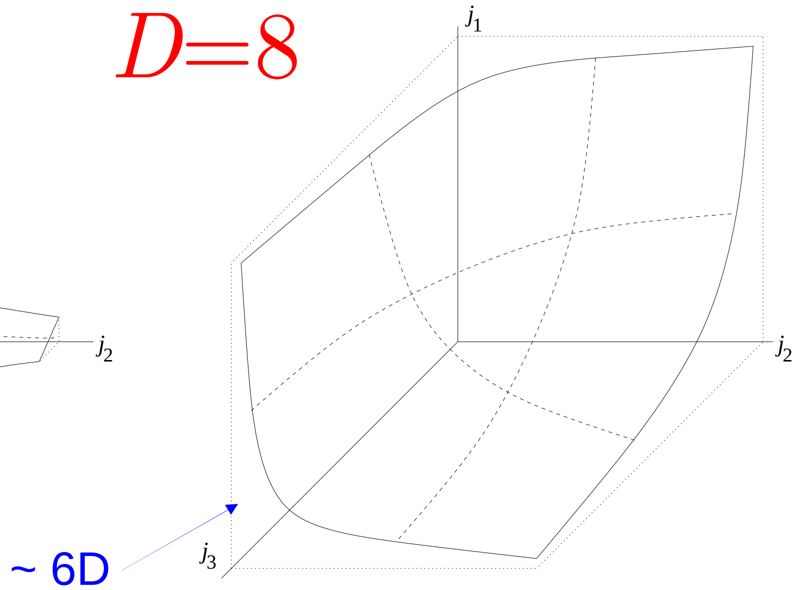


$D=7$



(a)

$D=8$



(b)

Is this all there is in D>4?

Not at all

Combine black branes & rotation:

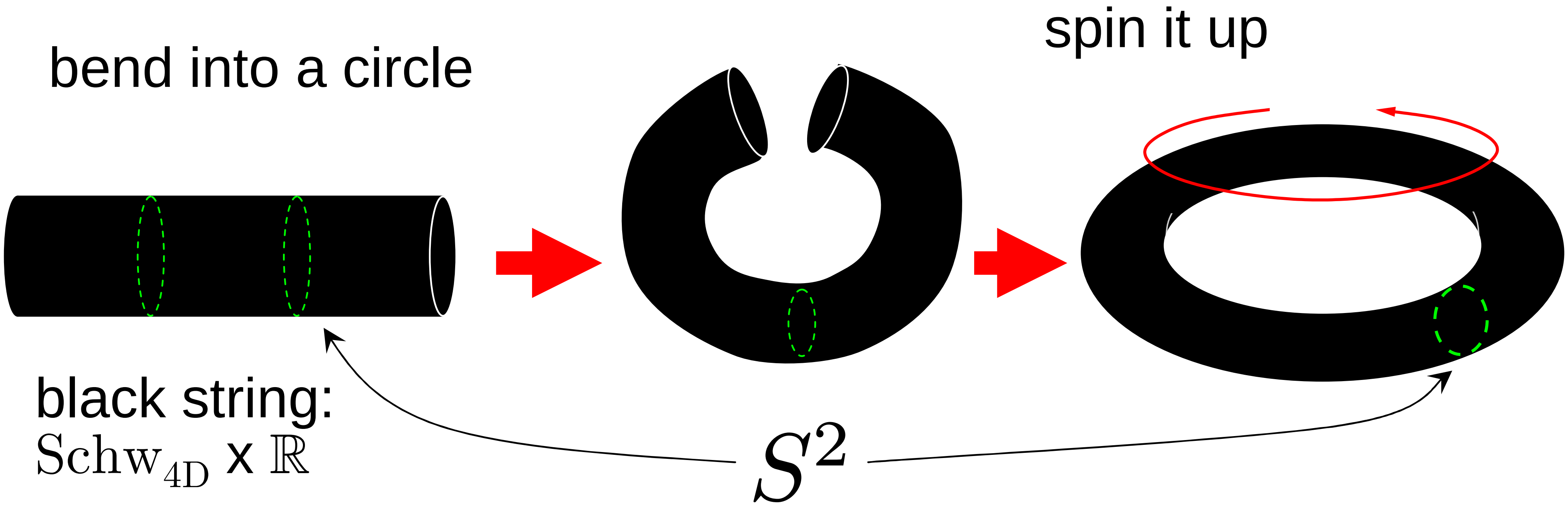
$\Rightarrow$ Black Rings + other blackfolds in $D \geqslant 5$

$\Rightarrow$ *Pinched* black holes in $D \geqslant 6$

D=5

End may be in sight

The forging of the ring (in $D=5$)



Horizon topology S^1x S^2

Exact solution available -- and fairly simple

RE+Reall 2001

Metric

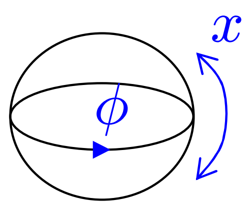
ψ-rotation

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - C\,R\,\frac{1+y}{F(y)}\,d\psi\right)^2$$

$$+\ \frac{R^2}{(x-y)^2}\,F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right]$$

S^1

S^2



$$F(\xi) = 1 + \lambda\xi$$

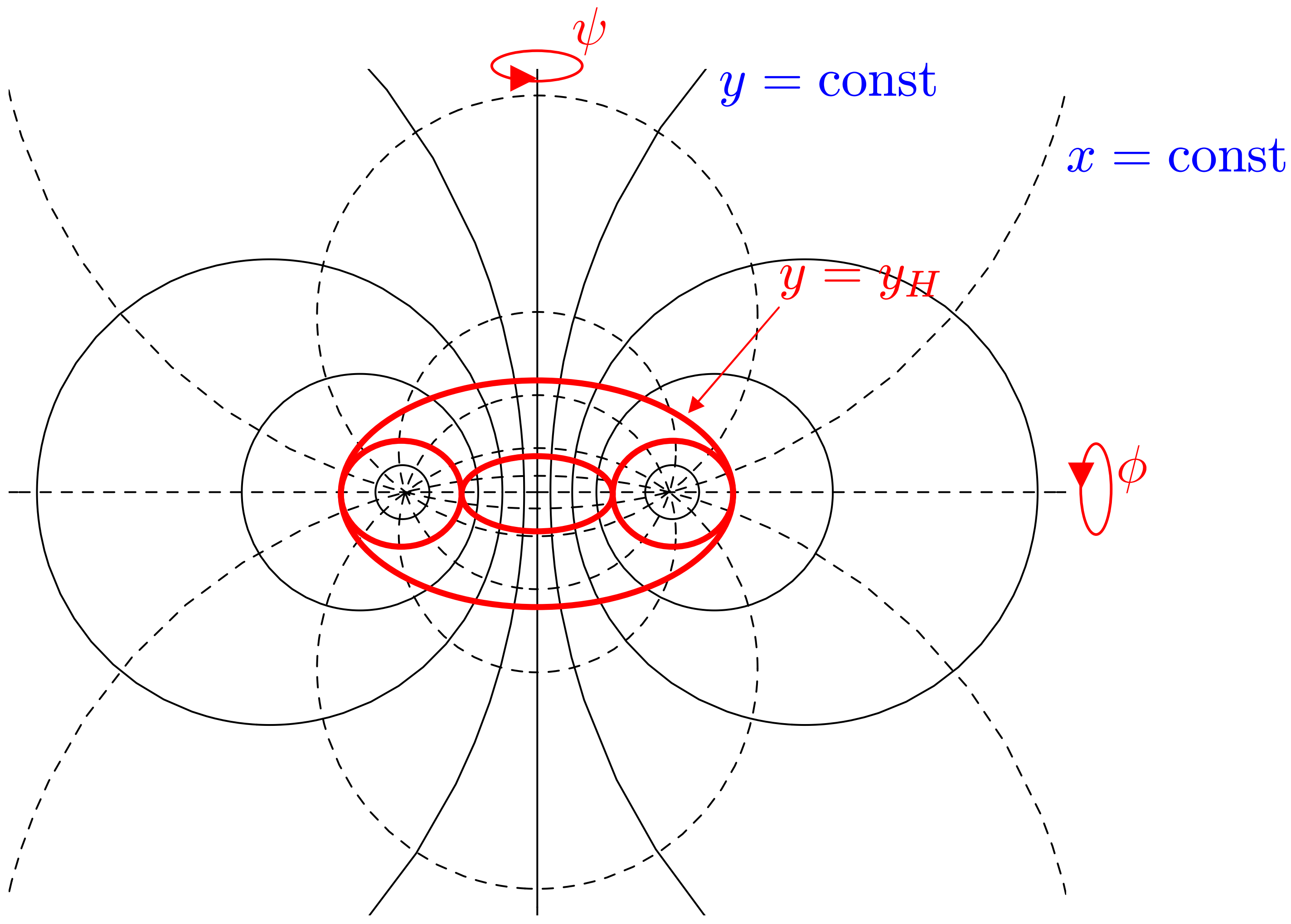
$$G(\xi) = (1-\xi^2)(1+\nu\xi)$$

Parameters $\lambda,\ \nu,\ R$

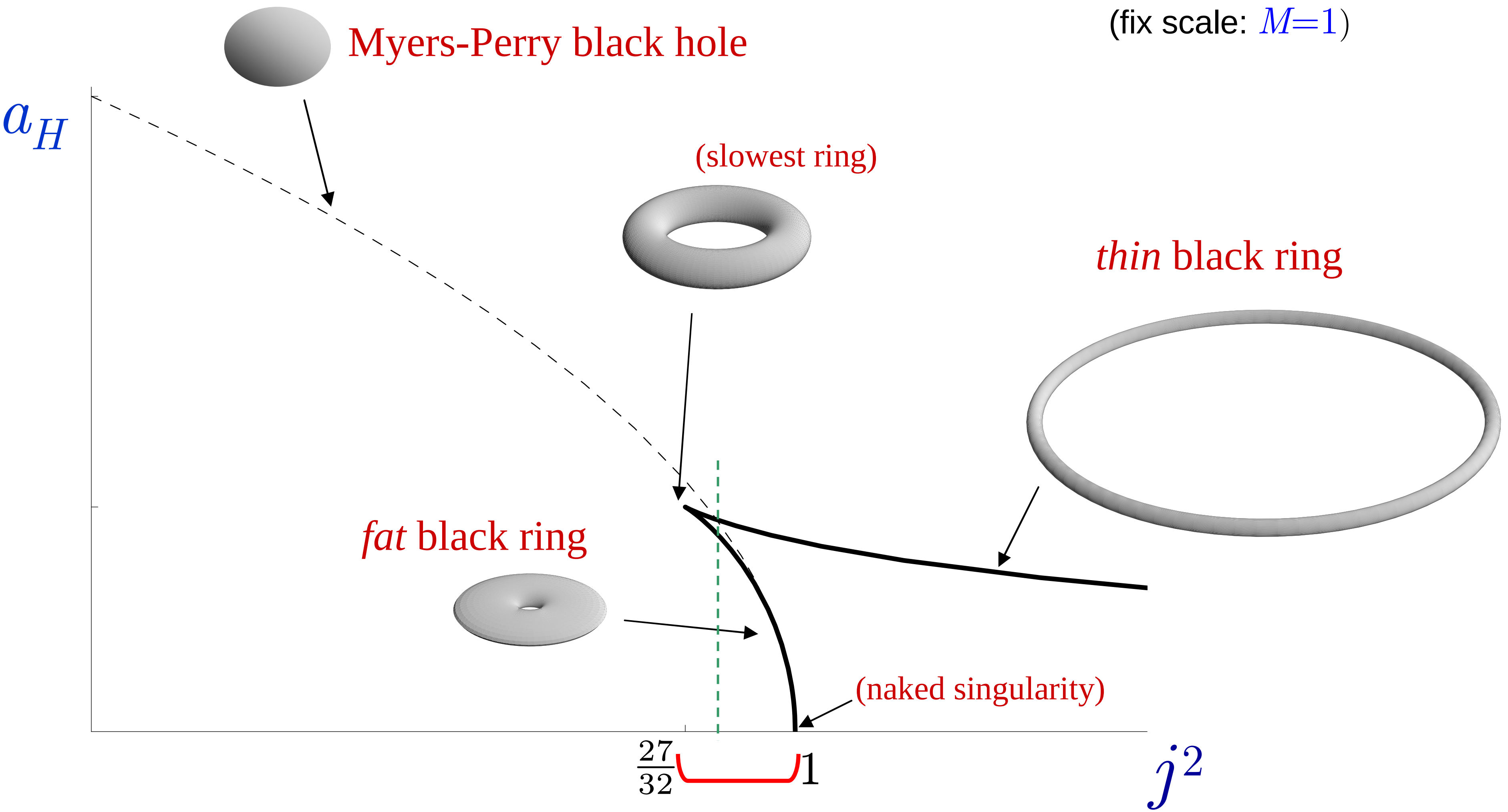
$\nu \sim R_2/R_1 \rightarrow$ shape, $\lambda/\nu \rightarrow$ velocity

equilibrium $\rightarrow \lambda(\nu)$

"Ring coordinates" $x,\ y$

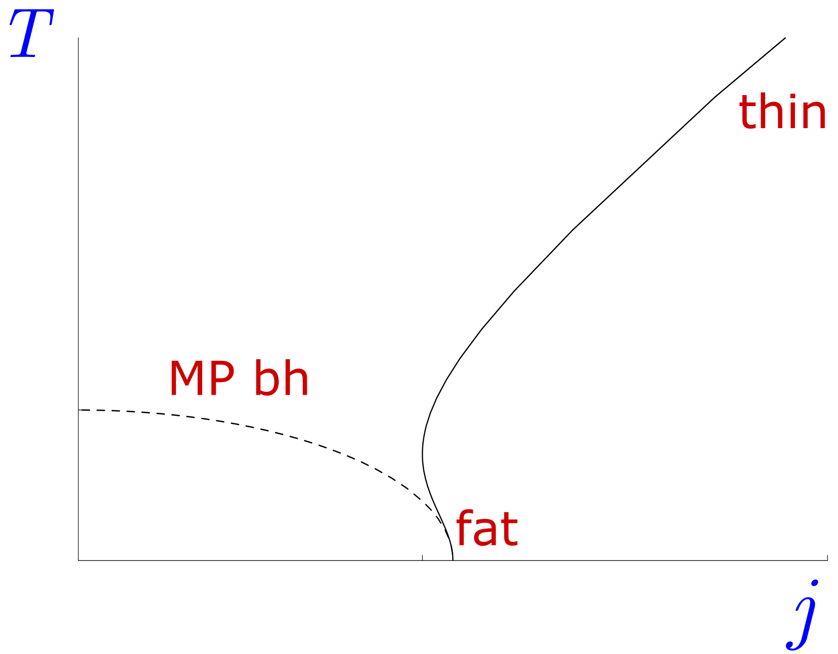


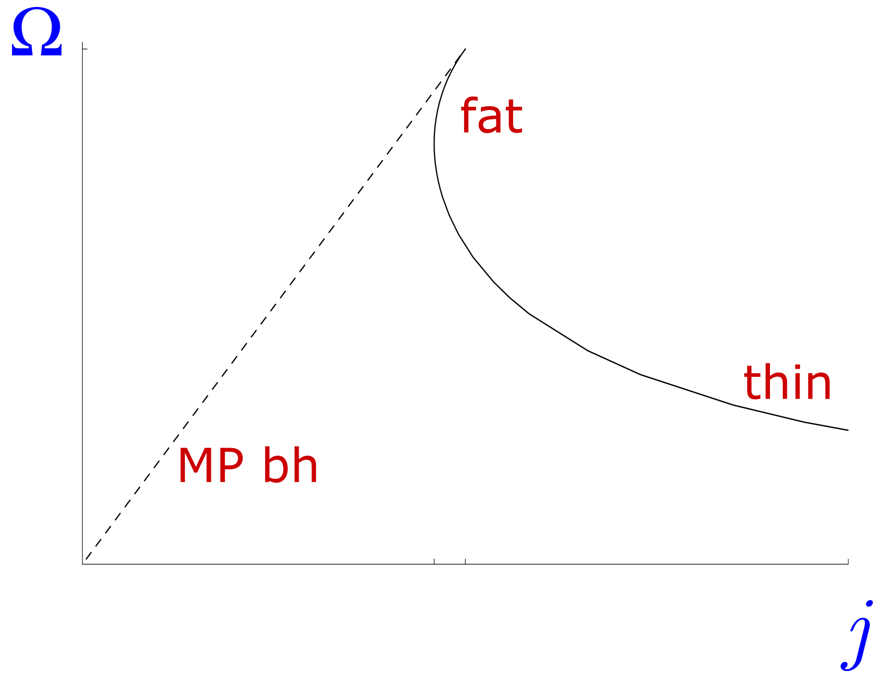
5D: one-black hole phases

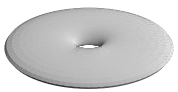


3 different black holes with the same value of ***M,J***

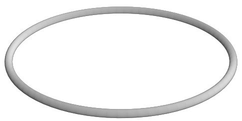
Other properties







fat rings ~ ("drilled-through") MP black holes



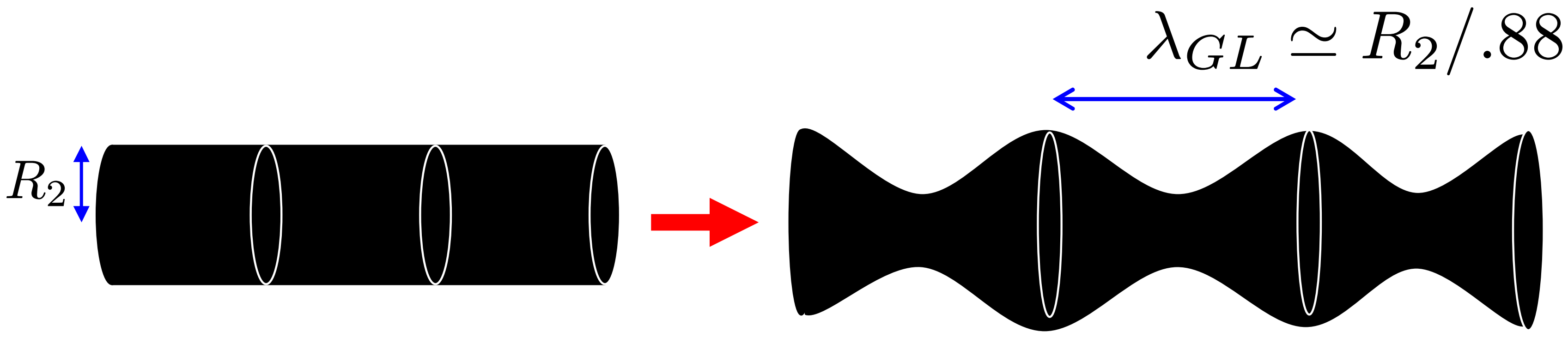
thin rings ~ (circular) black strings

Are they dynamically stable?

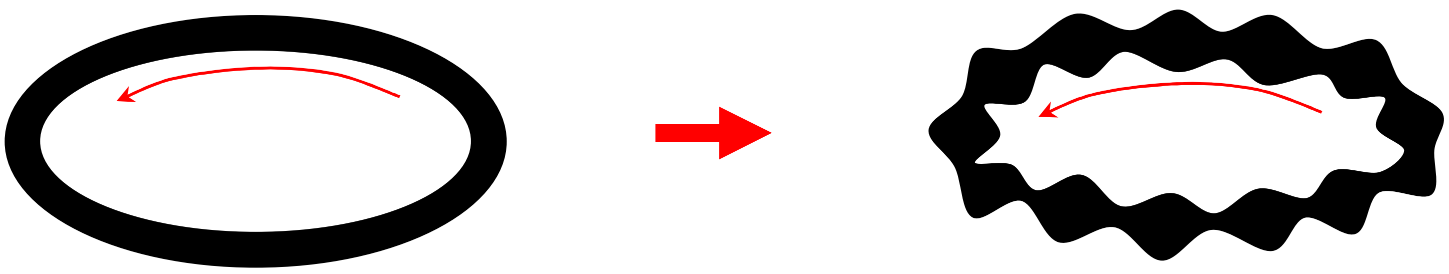
- Gravitational perturbation theory in $D>4$ is largely to be developed yet
  - Many more degrees of freedom
  - No Newman-Penrose formalism developed
  - Black rings don't possess Killing tensors (no separation of variables) -- MP bh's do

- Proceed heuristically
  - Hope to guide future analytical / numerical studies

Recall Gregory-Laflamme instability



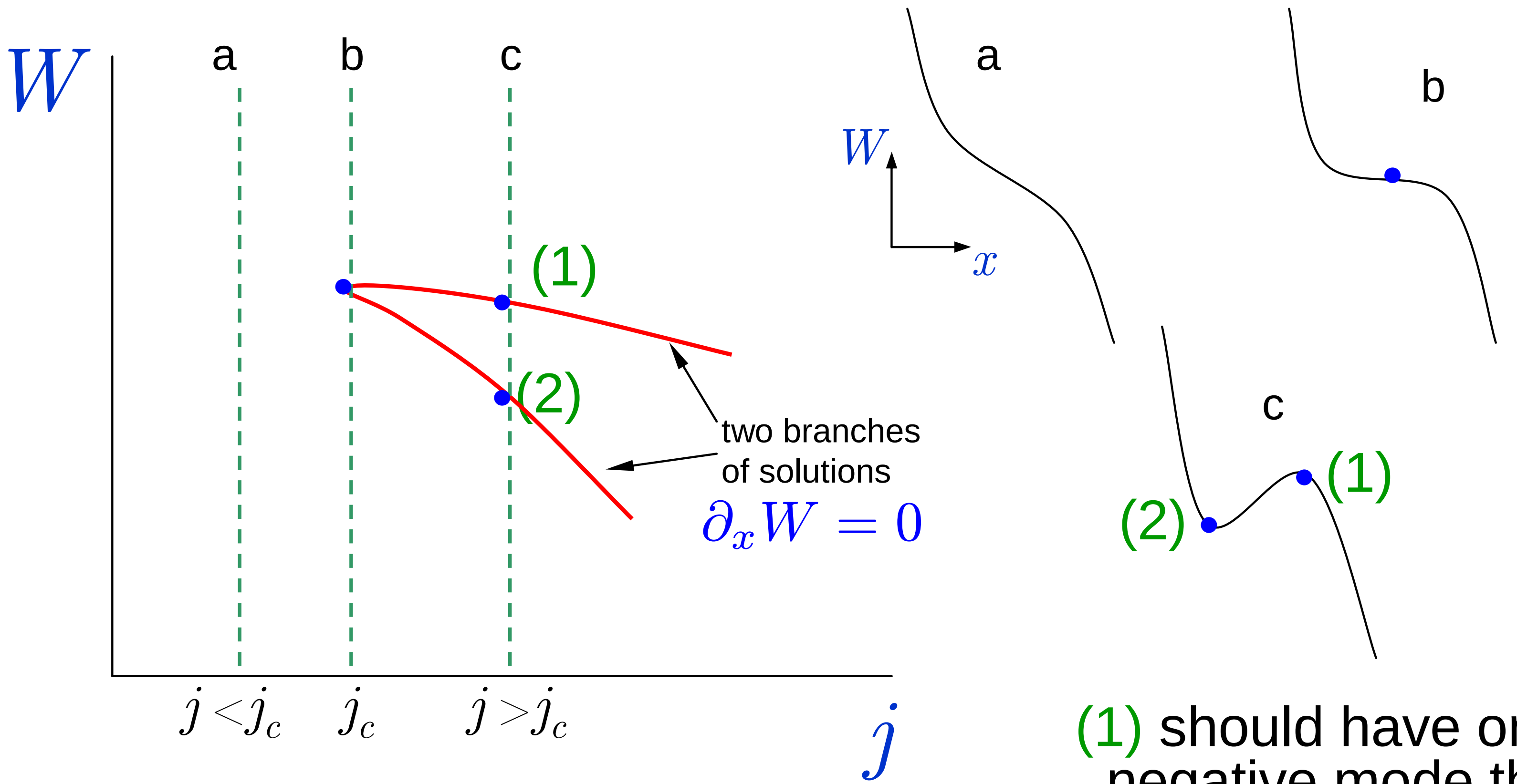
Expect instability for thin enough rings $j \gtrsim O(1)$



Turning-point instability (Poincaré)

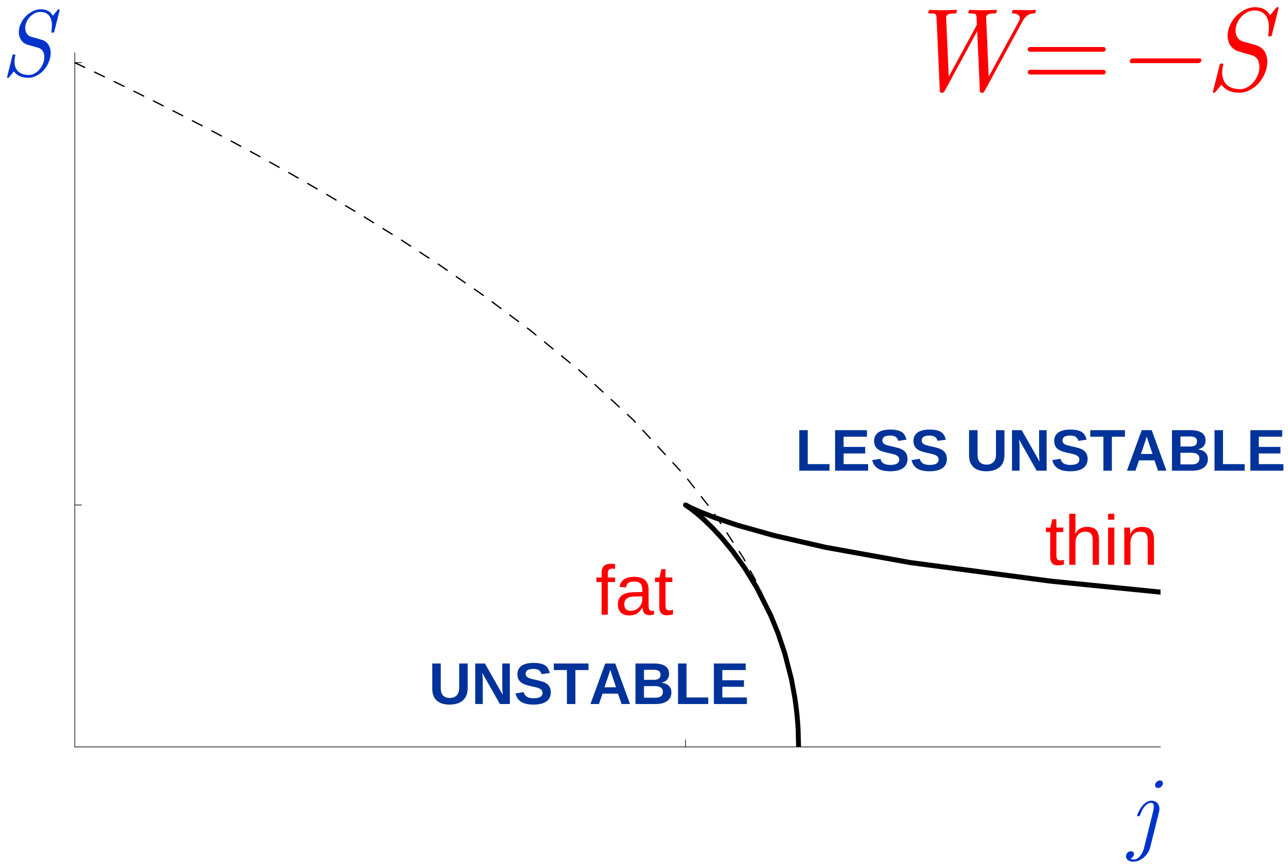
Arcioni+Lozano-Tellechea

- Suppose stable solutions correspond to minima of some potential $W(x;j)$



(1) should have one more negative mode than (2)

Turning-point of black rings

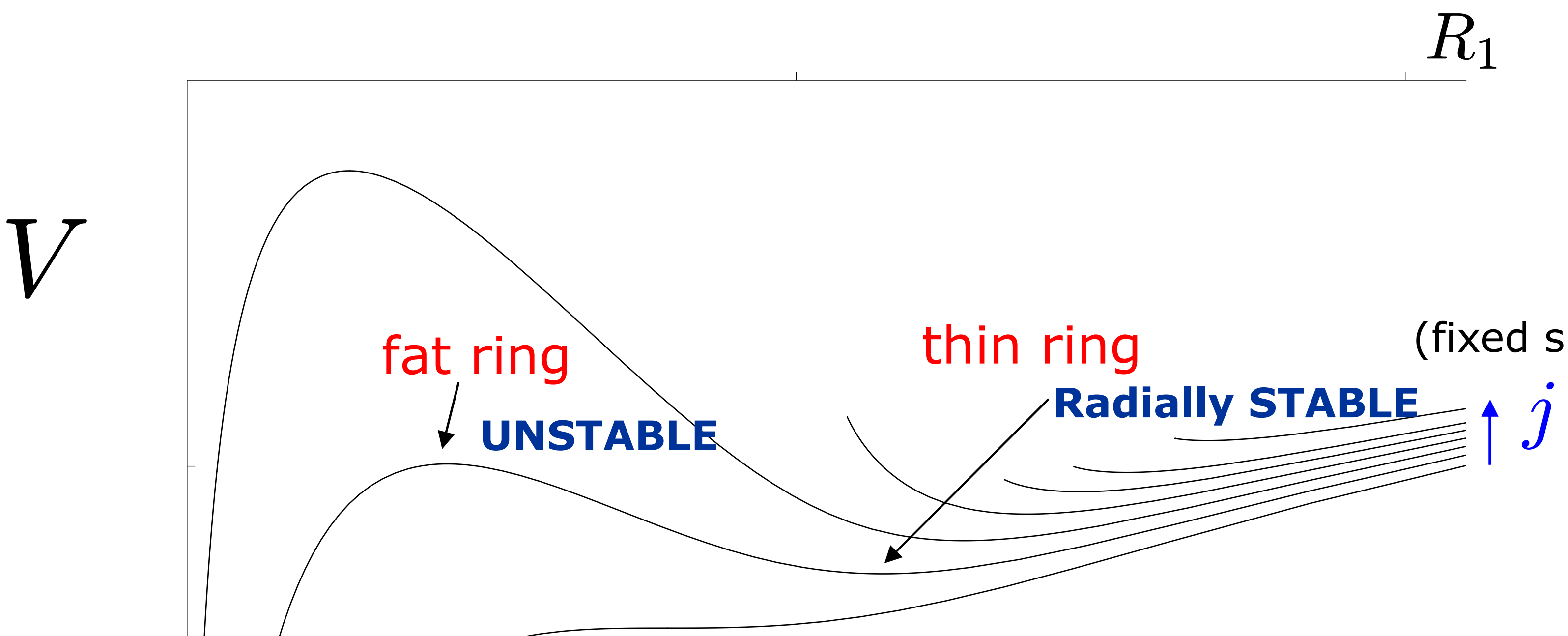


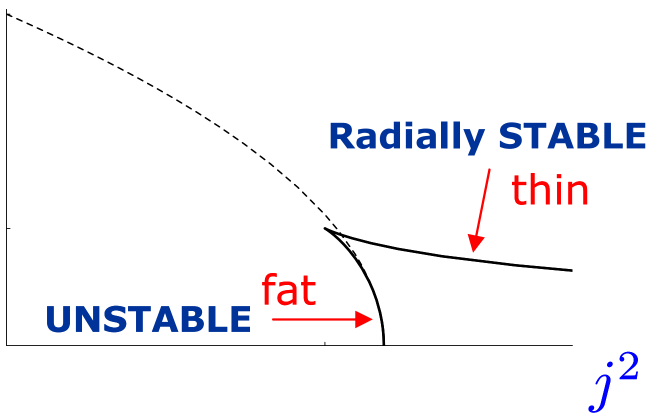
But, what kind of instability is this?

Potential for radial deformations

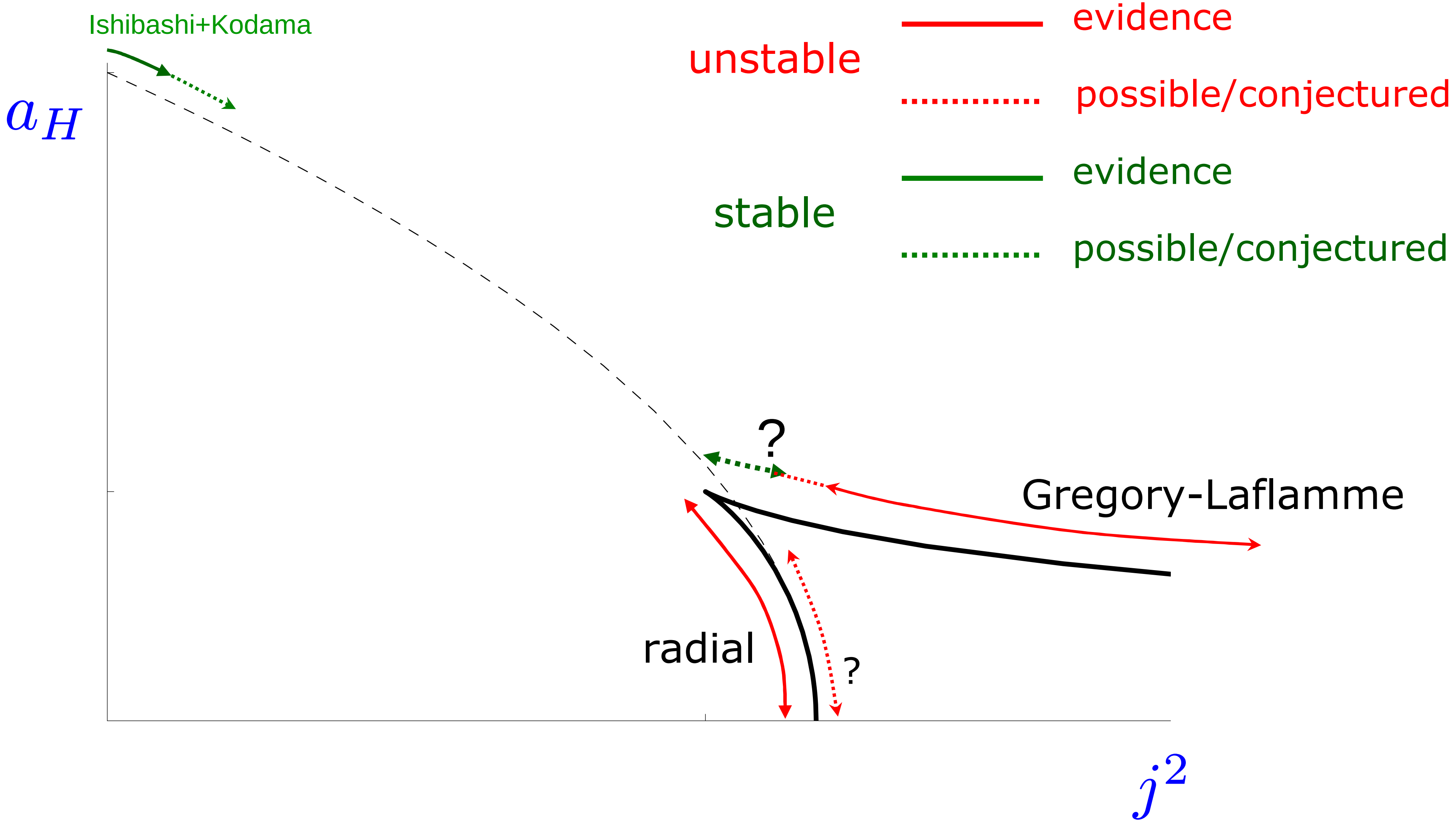
Elvang+RE+Virmani

equilibrium at $V' = 0$





Stability: putting results together



Black Holes in Higher Dimensions (II)

Roberto Emparan
ICREA & U. Barcelona

ICTP lectures, 31 Mar-2 Apr 2008

Constructing solutions

- *Kerr* found his solution using the Newman-Penrose formalism: Einstein eqs+Bianchi ids written out explicitly in a form suited to algebraically special solutions
- Unwieldy in $D>4$: too many eqs and variables
- Impose symmetry: besides stationarity (timelike Killing), require axial symmetry
- In D=4, this allows reduction to an integrable 2D sigma-model

• But, what *axial* symmetry in D>4?

1. $O(D-2)$ symmetry of S^{D-3} spheres rotated around a line axis $dz^2 + dr^2 + r^2 d\Omega^2_{(D-3)}$
   - But the curvature of the S^{D-3} spheres in D> 4 prevents integrability of 2D theory

2. $U(1)^{D-3}$ symmetry of rotations around (spatial) codimension-2 hypersurfaces $dz^2 + dr^2 + \sum_{i=1}^{D-3} r^{\alpha_i} d\phi_i^2$
   - It works! 2D sigma-model is integrable
   - But: only in D=4,5 can have global asymp flatness
   - AFness requires spatial S^{D-2} at infty, ie $O(D-1)$

$$\mathrm{Cartan}[O(D-1)] = U(1)^{\lfloor (D-1)/2 \rfloor}$$

not enough: $\lfloor (D-1)/2 \rfloor < D-3$ if $D>5$

Weyl class: *bubbling* GR

Weyl 1917

- Assume $D\!-\!2$ commuting, non-null, orthogonal Killing vectors $\partial/\partial x^a$ (compatible w/ AFness only in D=4,5)

$$ds^2 = -e^{2U_0(r,z)}dt^2 + \sum_{a=1}^{D-3} e^{2U_a(r,z)}(dx^a)^2 + e^{2\nu(r,z)}(dr^2 + dz^2)$$

$$\sum_{a=0}^{D-3} U_a = \log r$$

RE+Reall

- Einstein eqs → $\left(\partial_r^2 + \frac{1}{r}\partial_r + \partial_z^2\right) U_a = 0$

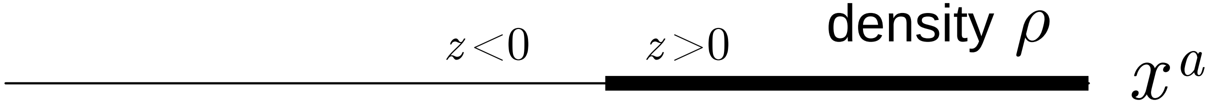
ie *linear* Laplace eq in $ds^2 = dz^2 + dr^2 + r^2 d\phi^2$

Given U_a, non-linear piece $\nu(r,\ z)$ is determined by line integral

- Problem is linear!
- Just specify axisymmetric sources for U_a : "rods"

eg

for each Killing, one set of sources



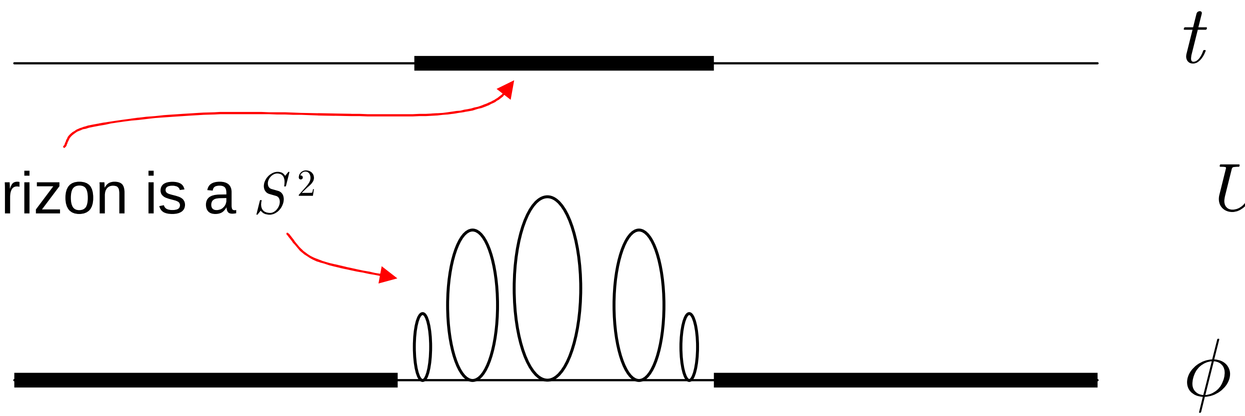
$$\Rightarrow U_a = \rho \log(\sqrt{r^2+z^2} - z)$$

$\left\| \frac{\partial}{\partial x^a} \right\|^2 = e^{2U_a} \to 0$ at rod $r=0,\ z>0$: require smooth circle action

- for density $\rho = 1/2$ then orbits close off smoothly on rod

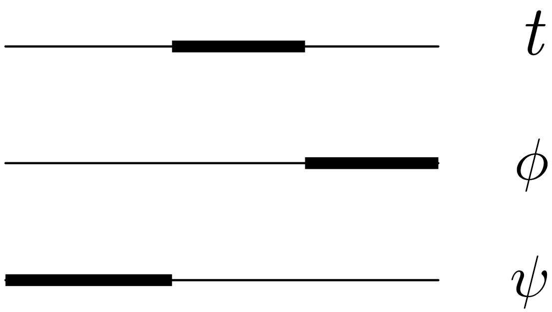
∂_t is null on rod ➔ **HORIZON**

Bubbling Schwarzschild

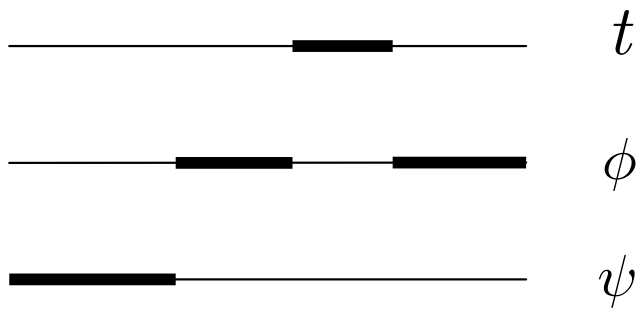


$$U_t + U_\phi = \log r$$

5D Schwarzschild



5D Black ring (static)



Integrability of D=4,5 GR (vacuum)

- Assume D-2 commuting, non-null, *not necessarily orthogonal*, Killing vectors $\partial/\partial x^a$

$$ds^2 = g_{ab}(r,z)\, dx^a dx^b + e^{2\nu(r,z)}\left(dr^2 + dz^2\right)$$

$$\det g_{ab} = -r^2$$

- Einstein eqs

→ $\partial_r U + \partial_z V = 0\,, \qquad U = r(\partial_r g)g^{-1}\,, V = r(\partial_z g)g^{-1}$

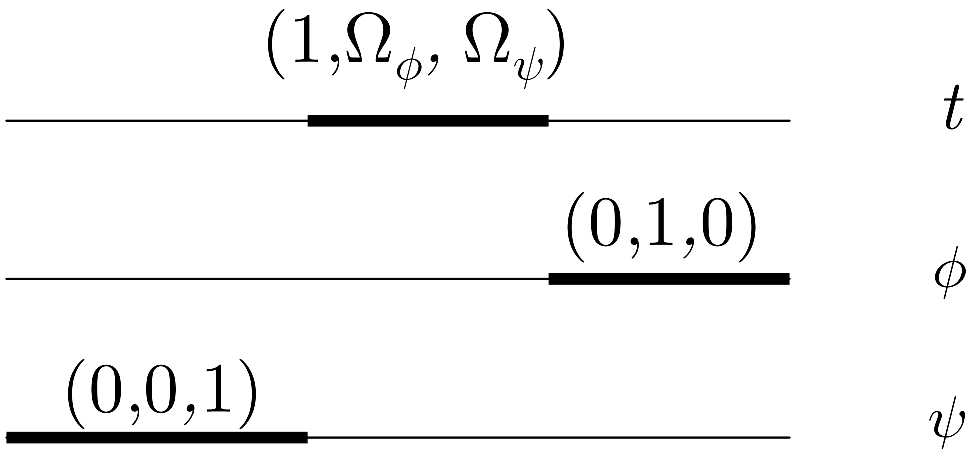
System reduced to 2D non-linear $GL(D-2,\mathbb{R})$ sigma-model

- Rods acquire *orientation* vector v

Harmark

$$g(0,z)v = 0 \text{ on rod } z \in [a,b]$$

5D Myers-Perry

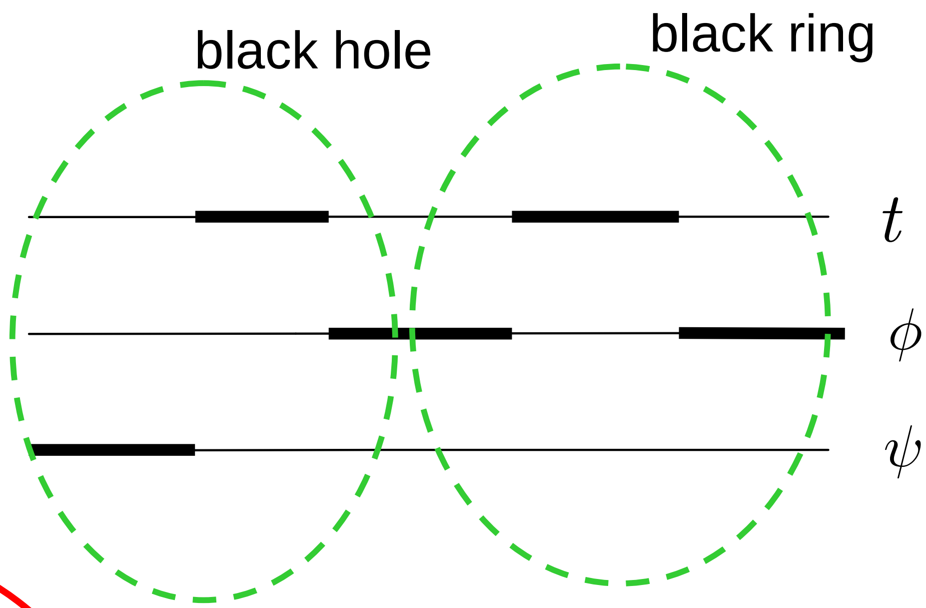


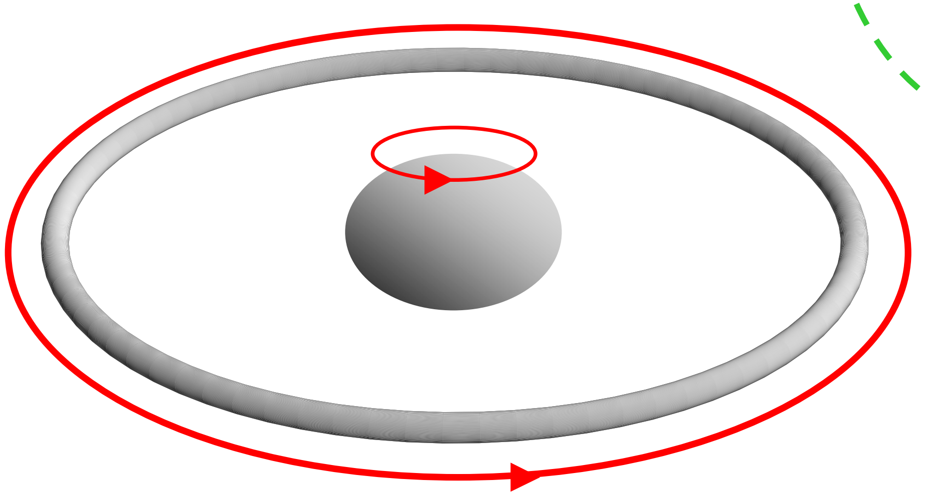
Generating solutions: the method of Belinsky+Zakharov

- Equations are completely integrable: admit Lax pair → inverse scattering, soliton techniques
- Given a 'seed', we can construct new solutions with more rods by 'adding solitons' (~Bäcklund transf)
- Method can be reduced to algebraic procedure
  - Calculationally involved, but straightforward --- can easily implement it in computer
  - There are 'thumb rules' for how to obtain a given solution --- though subtleties often arise

Multi-black holes

- *Black Saturn:*



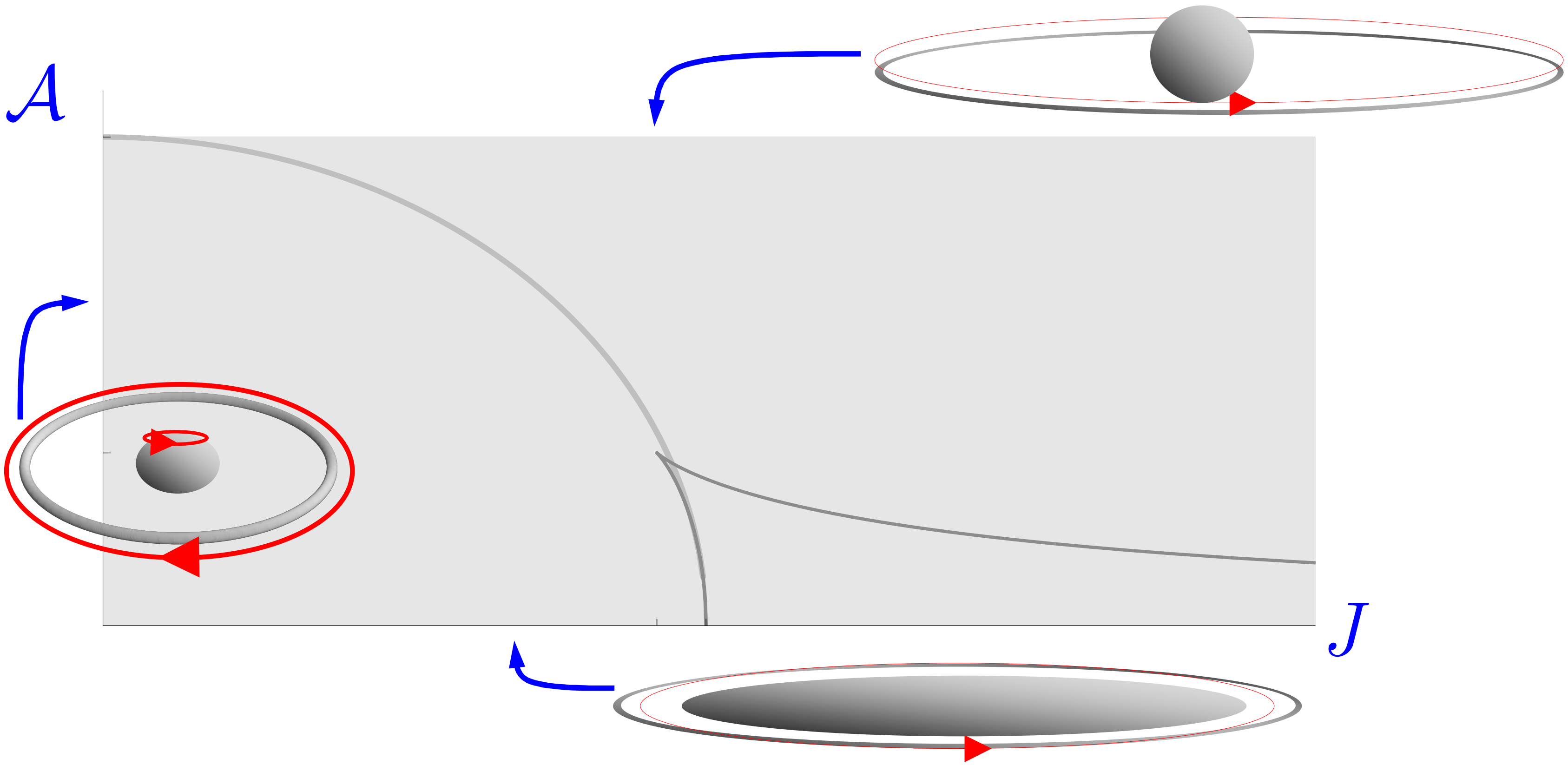


Elvang+Figueras

- Co- & counter-rotating, rotational dragging…

Filling the phase diagram

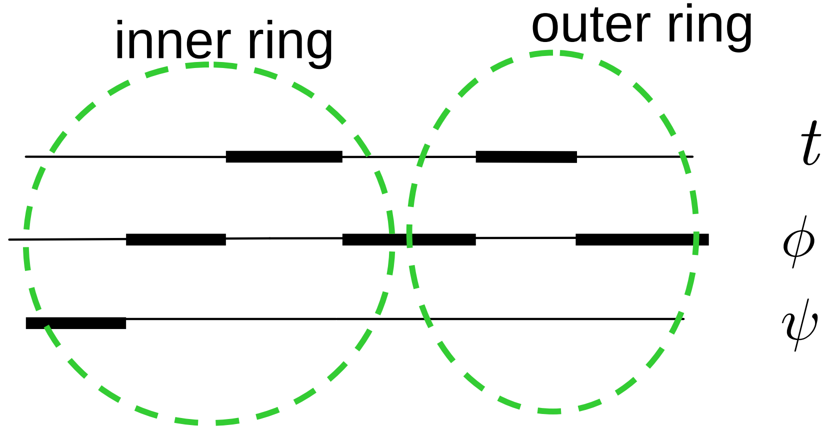
- Black Saturns cover a semi-infinite strip



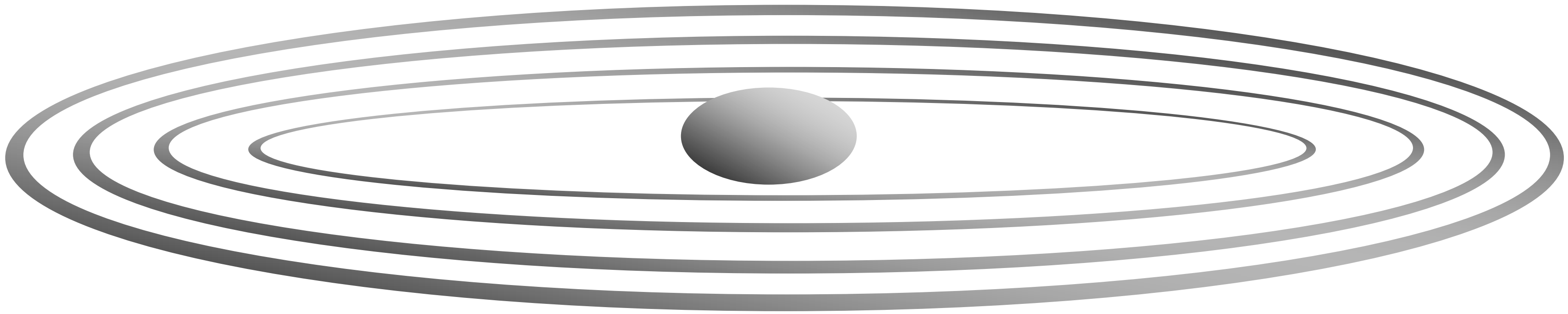
Multi-rings are also possible

- Di-rings explicitly constructed

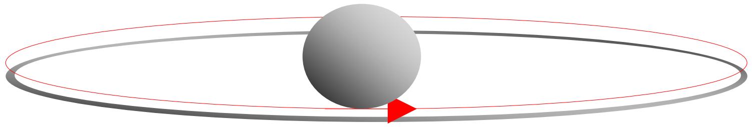
Iguchi+Mishima
Evslin+Krishnan



- Systematic, increasingly messy construction, with arbitrary number of rings



Thermodynamical equilibrium

 is not in thermo-equil

$$T_r \gg T_h\,, \quad \Omega_r \neq \Omega_h$$

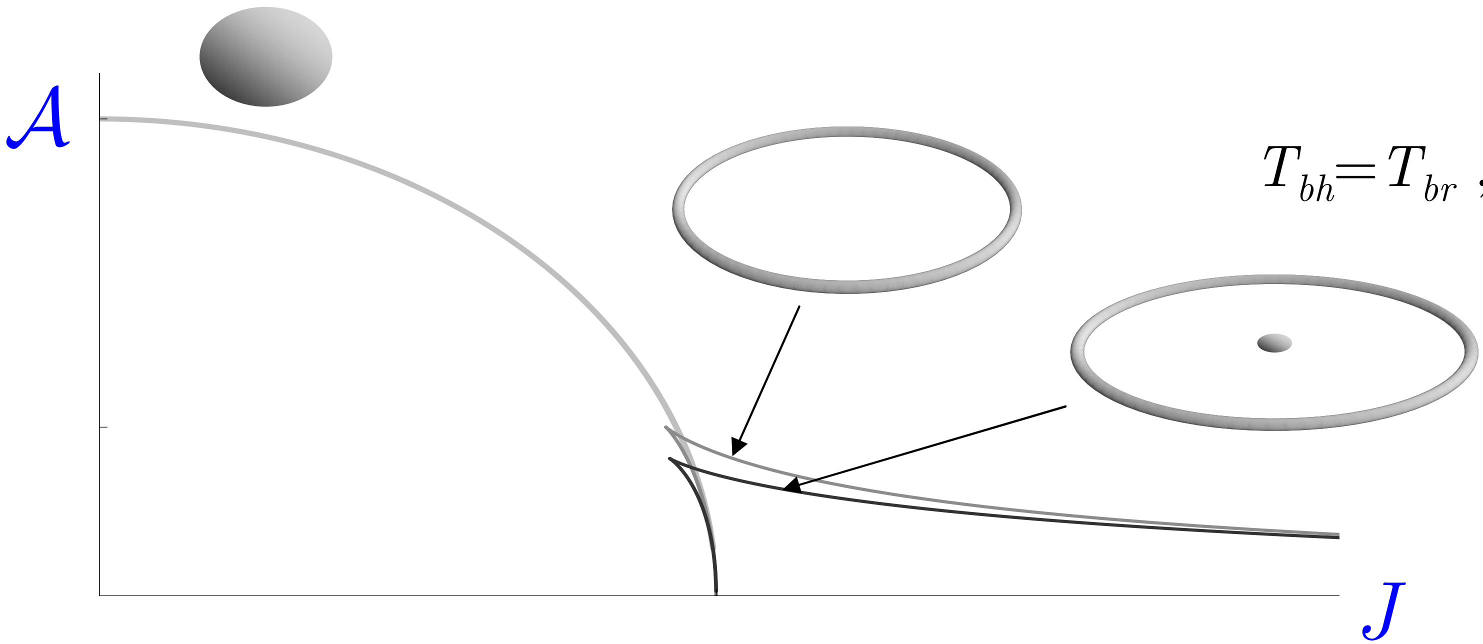
- Beware: bh thermodynamics makes sense only with Hawking radiation
- Radiation is in equilibrium only if

$$T_i = T_j\,,\ \Omega_i = \Omega_j$$

$\Rightarrow$ continuous degeneracies removed

- Multi-rings unlikely (*should* check di-rings!)

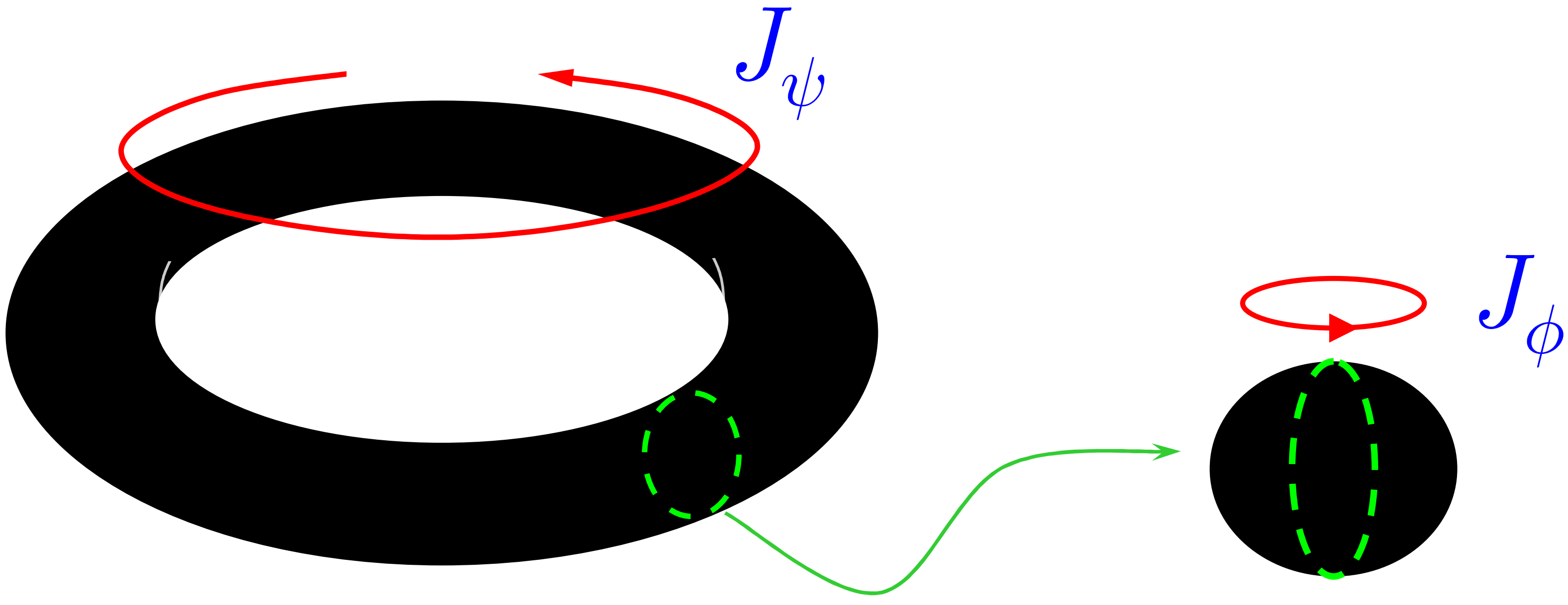
5D phases in thermal equilibrium



$$T_{bh} = T_{br} \ , \quad \Omega_{bh} = \Omega_{br}$$

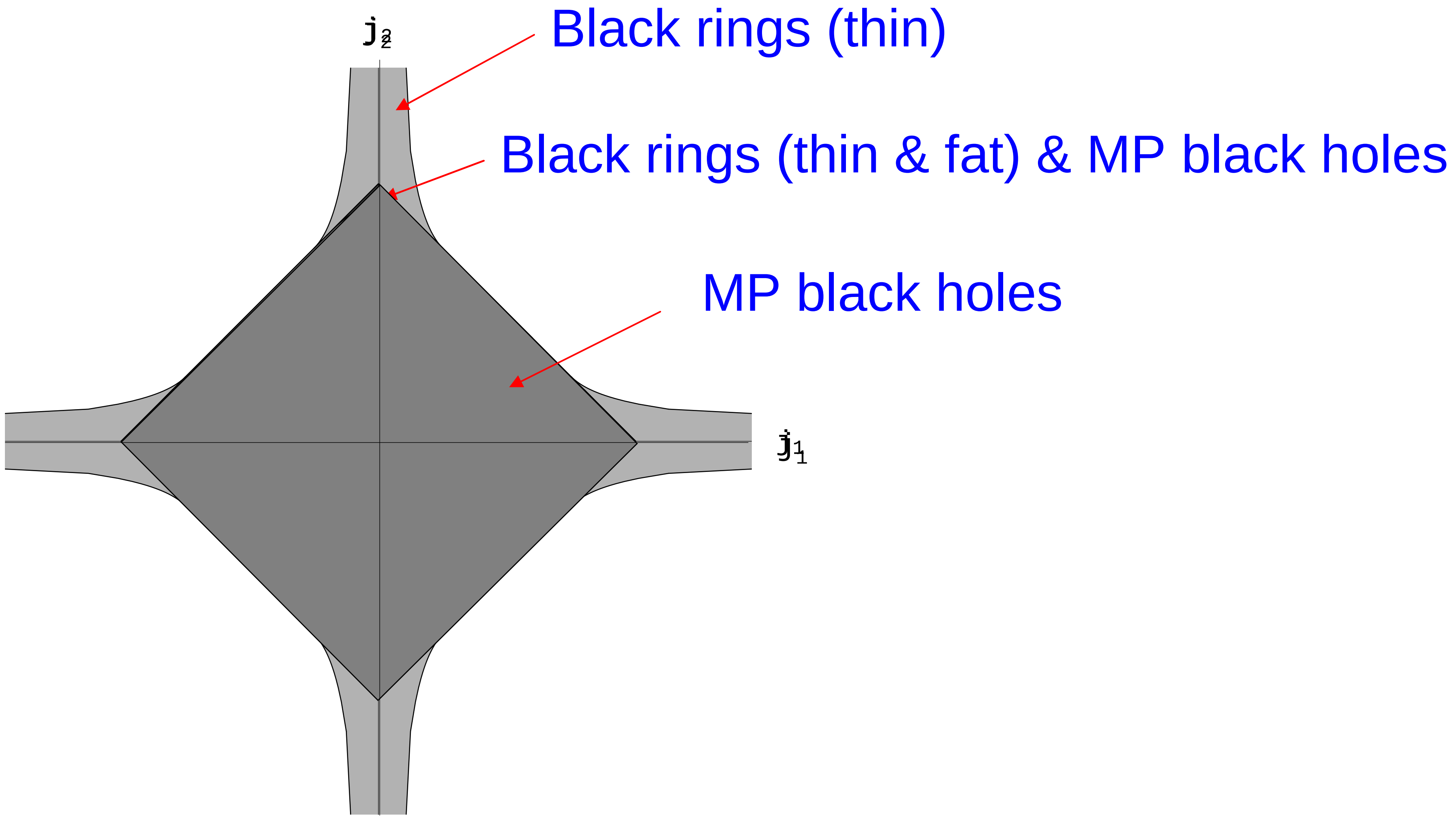
is there anything else?

Black rings w/ two spins

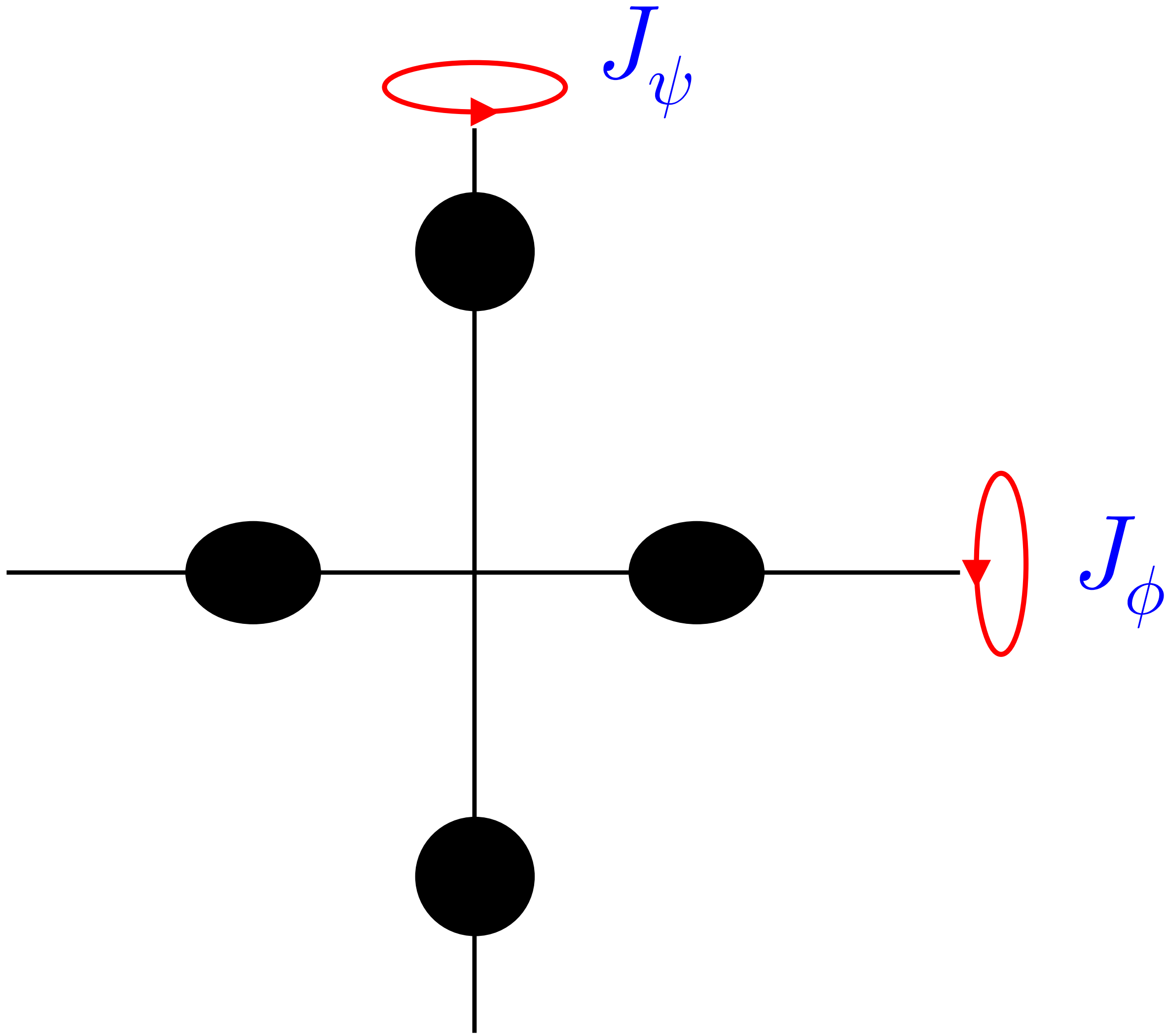


Pomeransky+Sen'kov

5D phase space w/ two spins



Bicycling black rings



Izumi
Elvang+Rodríguez

Towards a complete classification of 5D black holes

- Topology: S^3, S^1x S^2 *Galloway+Schoen*
- If $\mathbb{R}_t$ x $U(1)_\phi$x $U(1)_\psi$ then
  - complete integrability *Pomeransky*
  - "uniqueness" (M,J + rod structure) *Hollands+Yazadjiev*
- Rigidity: stationarity $\Rightarrow$ one axial $U(1)$, but not (yet?) necessarily two

Hollands et al

THIS IS THE MAIN OPEN PROBLEM!

- Essentially all bh solutions *may* have been found:

MP, black rings, multi-bhs (saturns & multi-rings)

(including also with two spins)

(bubbly black holes?)

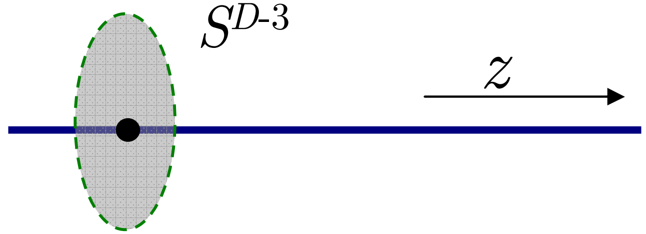
Charges and dipoles

- Conserved gauge charge for asymp flat solns can only be electric charge w.r.t. a 2-form field strength $F_{\mu\nu}$
  (or, dually, magnetic charge w.r.t. D-2-form)
  $$Q = \int_{S^{D-2}} *F_{(2)}$$
- But higher p-forms can also be excited – although they have no net charge associated
- Simplest: **black rings as dipoles** of $H_{\mu\nu\rho}$

$$H_{(3)} = dB_{(2)}$$

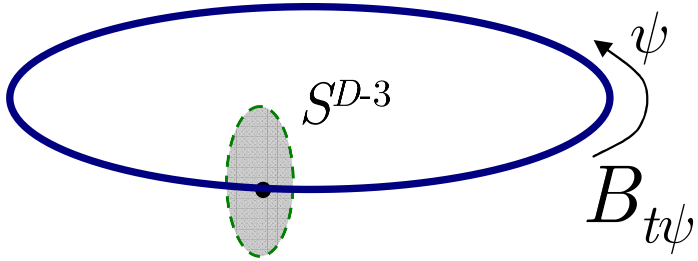
String:

$$q \propto \int_{S^{D-3}} *H_{(3)}$$



B_{tz}

Ring:



Now q is **not** conserved: can shrink ring to zero

Simplest set up: minimal 5D sugra

Einstein+Maxwell+Chern-Simons

$$I = \frac{1}{16\pi G}\int\left(R*1 - 2F\wedge *F - \frac{8}{3\sqrt{3}}F\wedge F\wedge A\right)$$

- Ring couples electrically to F (charge Q) and magnetically to its magnetic dual 3-form $*F$ (dipole q)

$$Q = \int_{S^3} *F_{(2)} \qquad q = \int_{S^2} F_{(2)}$$

- Exact solutions available
  - non-susy, with dipole, and with or without charge

    (charge+rotation $\Rightarrow$ dipole)
  - supersymmetric (w/ charge and dipole)

Stability of rings w/ charges and dipoles

- Charge increases stability
- Supersymmetric black rings expected to be linearly stable
- Near-susy are expected stable too
- Dipole rings (w/out conserved charges):
  - *fat* rings radially unstable / *thin* rings radially stable
  - GL instability expected to switch-off close to extremality (even if not close to susy)

  → larger stability window than for neutral rings

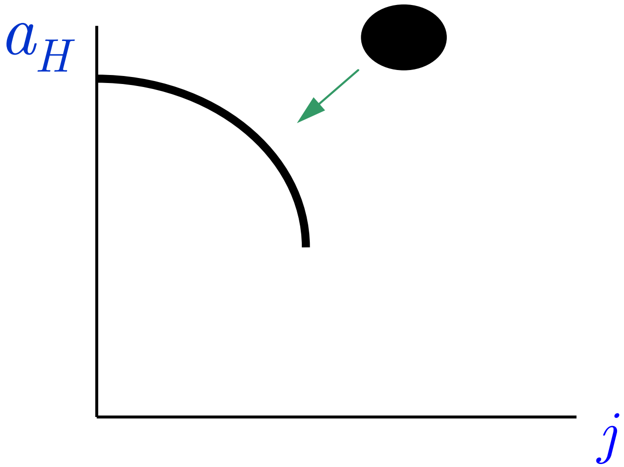
Black Holes in Higher Dimensions (III)

Roberto Emparan
ICREA & U. Barcelona

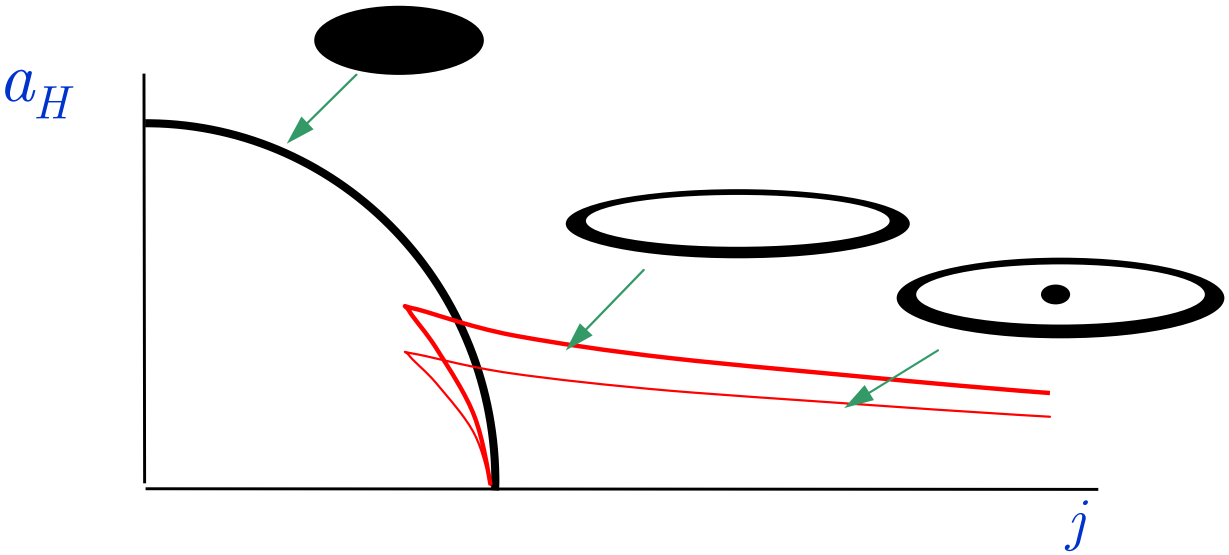
ICTP lectures, 31 Mar-2 Apr 2008

The plot thickens…

$D=4$

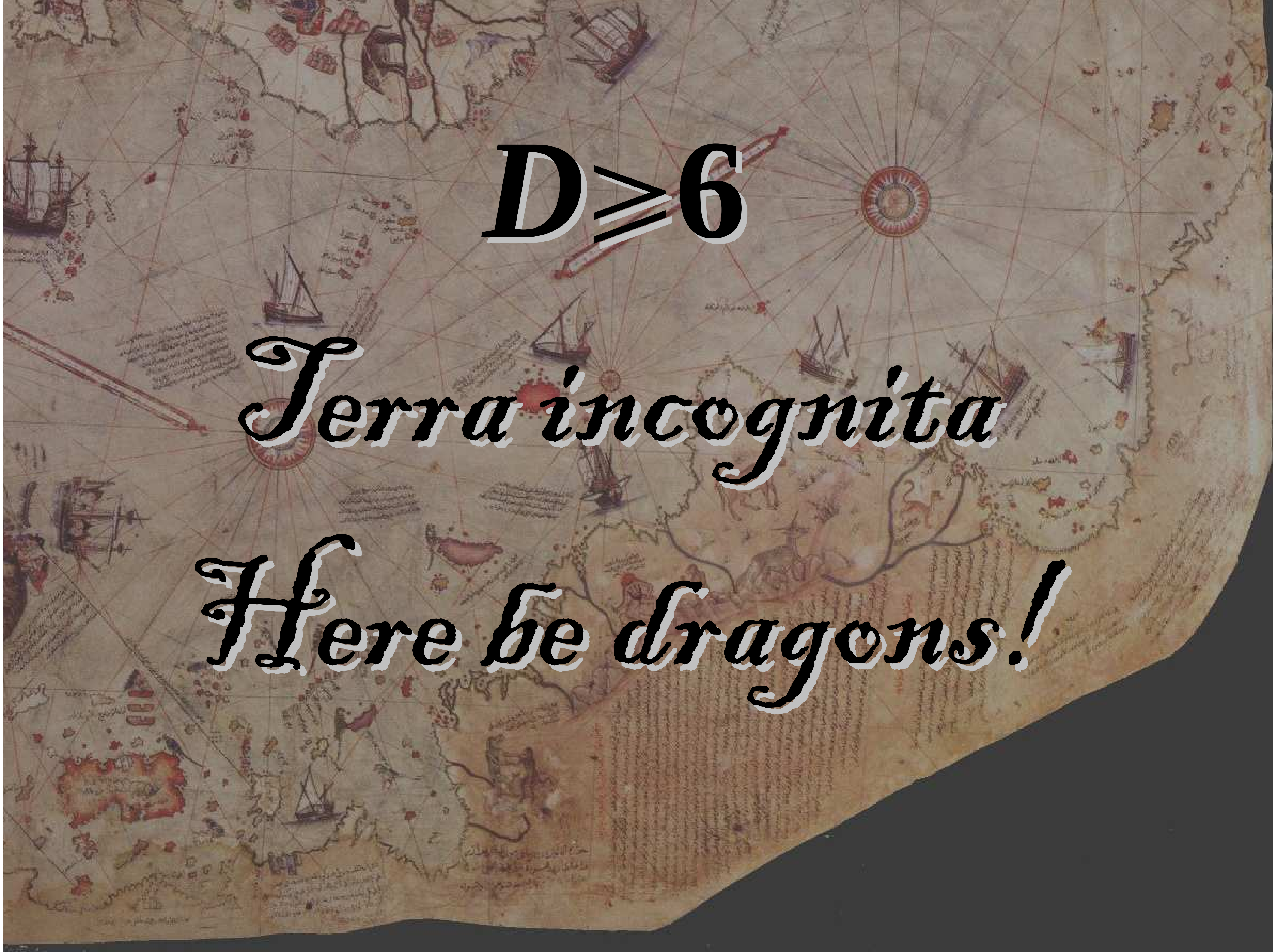


$D=5$



D=5

End may be in sight



$D \geqslant 6$

Terra incognita

Here be dragons!

Road blocks in $D \geqslant 6$

No known solution-construction techniques

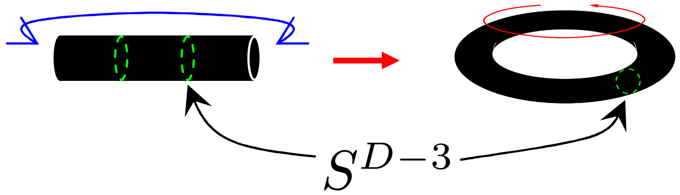
- Newman-Penrose formalism: unwieldy in $D > 4$
- Integrability of Weyl class w/ $\mathbb{R} \times U(1)^{D\text{-}3}$ symm:
  for AF black holes, this only helps in $D = 4,\ 5$
- Kerr-Schild class: $g_{\mu\nu} = \eta_{\mu\nu} + 2H(x) k_\mu k_\nu$
  MP black holes are K-S, but black rings are not

So, *very limited* success in extending 4D approaches

$\Rightarrow$ Need new ideas

- more qualitative & approximate methods (physics-guided)
- may guide later numerical attacks

Thin black rings in $D>5$

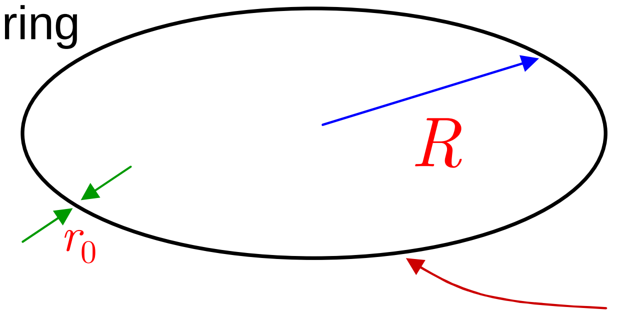
- Heuristic:  seems plausible

- Thin black rings ≃ circular boosted black strings
- Equilibrium can be analyzed w/in linearized gravity:
  - balance between tension and centrifugal repulsion; gravitational self-attraction is subdominant

Matched asymptotic expansion

Harmark
Kol
RE et al

very thin ring

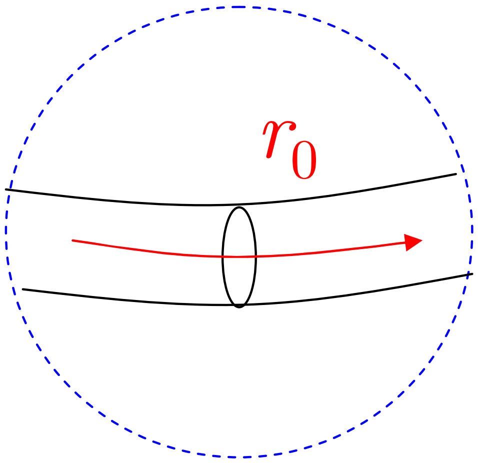


equivalent delta-source

1- linearized soln around flat space

$$\frac{r_0}{r} \ll 1$$

2- perturbations of a boosted black string

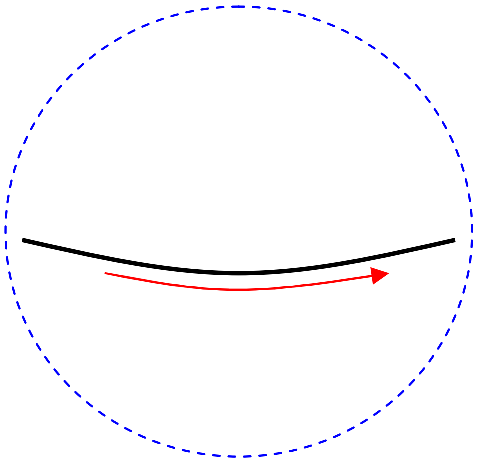


$$\frac{r}{R} \ll 1$$

need bdry conditions to
fix integration constants

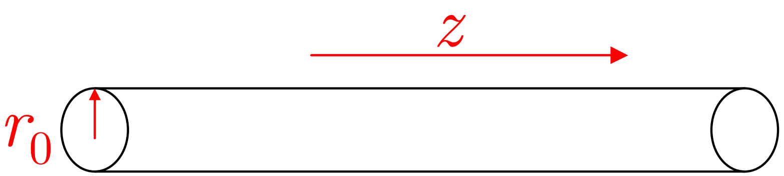
Match in overlap zone

$$r_0 \ll r \ll R$$



Thin black rings from black strings

Boosted black string:



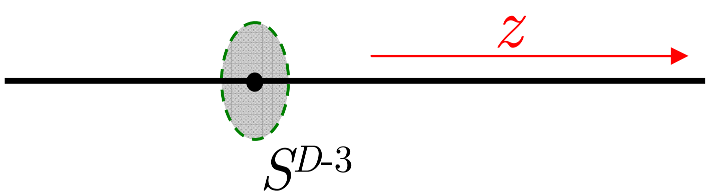
boost $\sigma \to$

stress tensor (ADM):

$$
\begin{aligned}
T_{tt} &\propto r_0^{D-4}\left[(D-4)\cosh^2\sigma + 1\right] \\
T_{tz} &\propto r_0^{D-4}(D-4)\cosh\sigma\sinh\sigma \\
T_{zz} &\propto r_0^{D-4}\left[(D-4)\sinh^2\sigma - 1\right]
\end{aligned}
$$

Equivalent delta-source for thin rings, $r_0 \ll r$

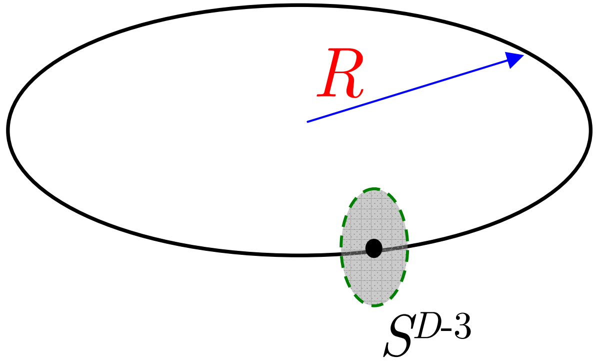
$$T^{(\delta)}_{\mu\nu} = T^{ADM}_{\mu\nu}\,\delta(r)$$



$$
\begin{aligned}
M &= L\int_{S^{D-3}} T^{(\delta)}_{tt} \\
P &= L\int_{S^{D-3}} T^{(\delta)}_{tz} \\
\mathcal{A} &= L\int_{S^{D-3}} \sqrt{g_{hor}}
\end{aligned}
$$

Thin black rings from black strings

→ Thin boosted black string along a circle



delta-source $\quad T^{(\delta)}_{\mu\nu} = T^{ADM}_{\mu\nu}\,\delta(r)$

$$
\begin{aligned}
M &= 2\pi R \int_{S^{D-3}} T^{(\delta)}_{tt} \\
J &= 2\pi R^2 \int_{S^{D-3}} T^{(\delta)}_{tz} \\
\mathcal{A} &= 2\pi R \int_{S^{D-3}} \sqrt{g_{hor}}
\end{aligned}
\quad \Rightarrow \mathcal{A}(M,J,R)
$$

But: what fixes boost σ?

I.e.: how is R fixed in terms of M, J ?

Curving a black string: equilibrium condition

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \Rightarrow \qquad \frac{T_{zz}}{R} = 0$$

$$T_{zz} \propto \quad r_0^{D-4}\left[(D-4)\sinh^2\sigma - 1\right]$$

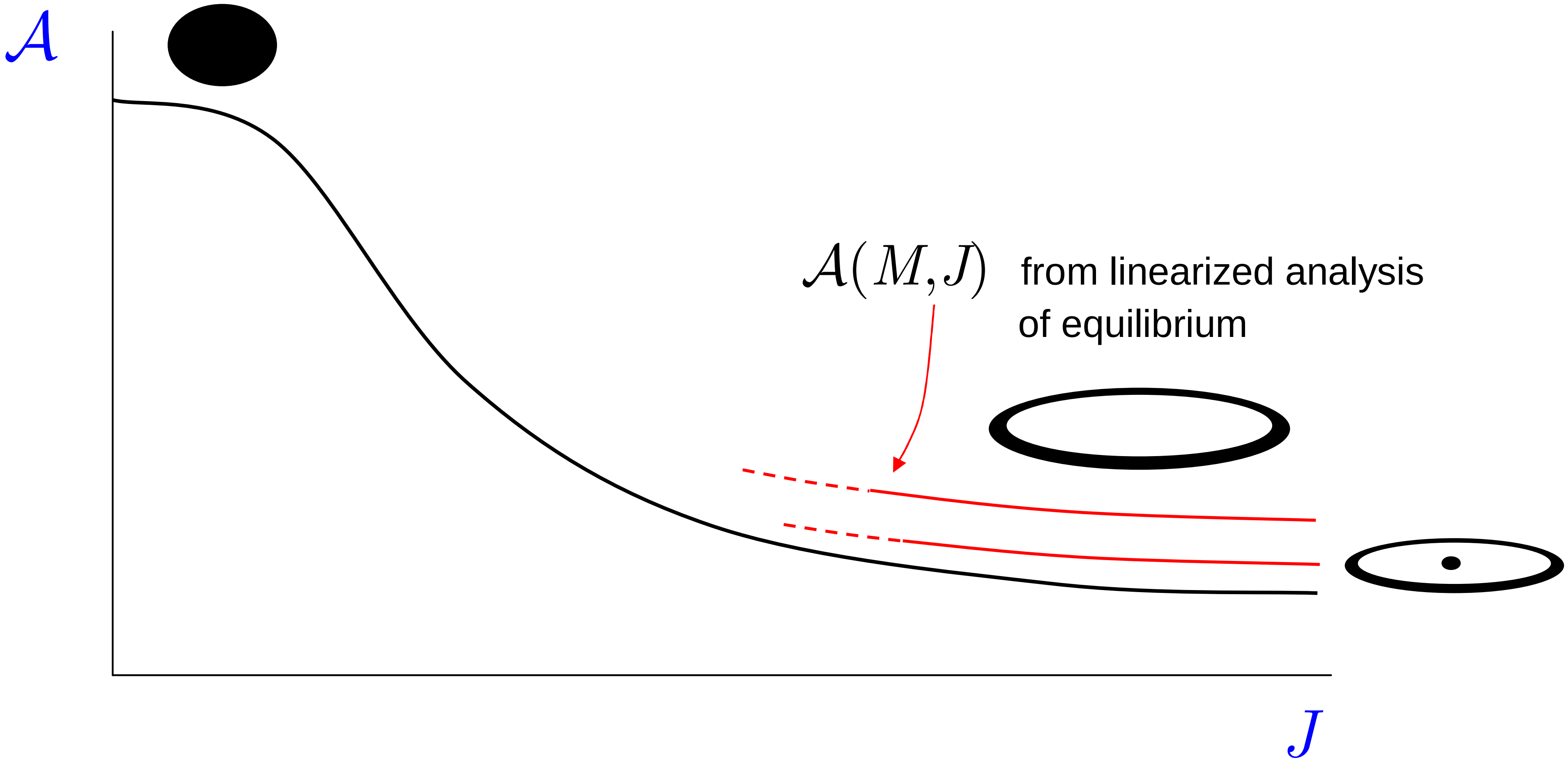
$$\Rightarrow \sinh^2\sigma = \frac{1}{D-4}$$

$$\Rightarrow R = \frac{D-2}{\sqrt{D-3}}\frac{J}{M}$$

← equilibrium condition

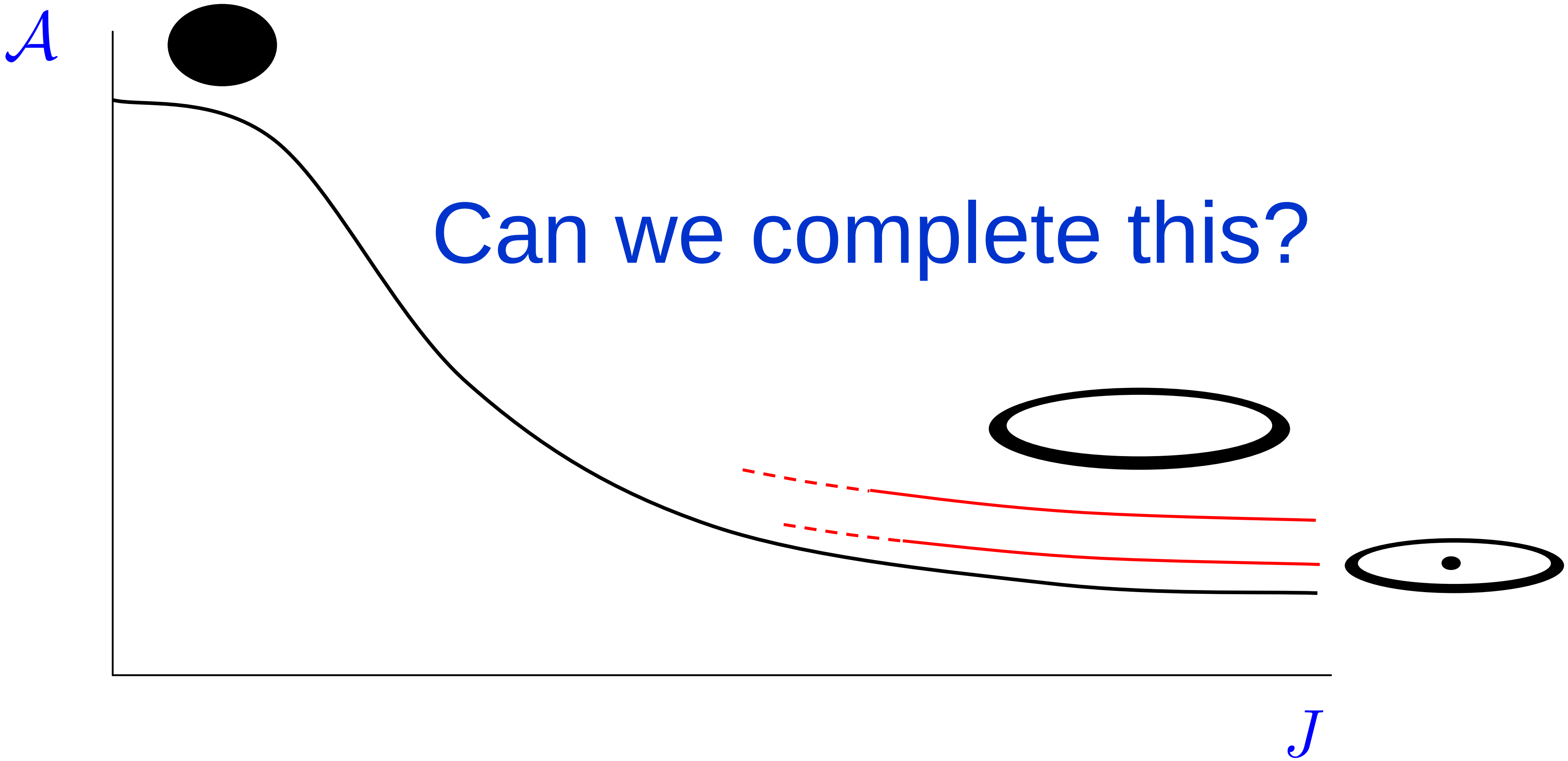
$D \geqslant 6$ phase diagram

(fix M)



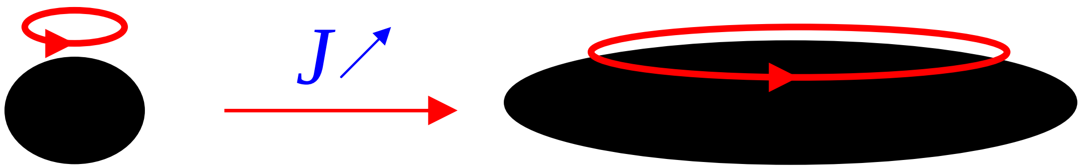
➔ Black rings dominate the entropy at large J

$D \geqslant 6$ phase diagram



Pinched (lumpy) black holes in $D \geqslant 6$

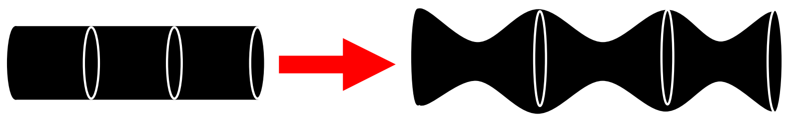
Ultraspinning regime in $D \geqslant 6$



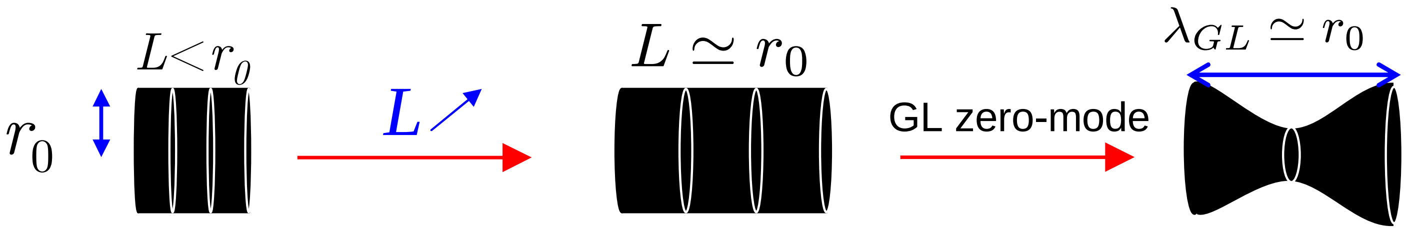
$\Rightarrow$ ***black membrane*** along rotation plane

Recall black branes exhibit

– Gregory-Laflamme instability



– GL zero-mode → branching into non-uniform horizons

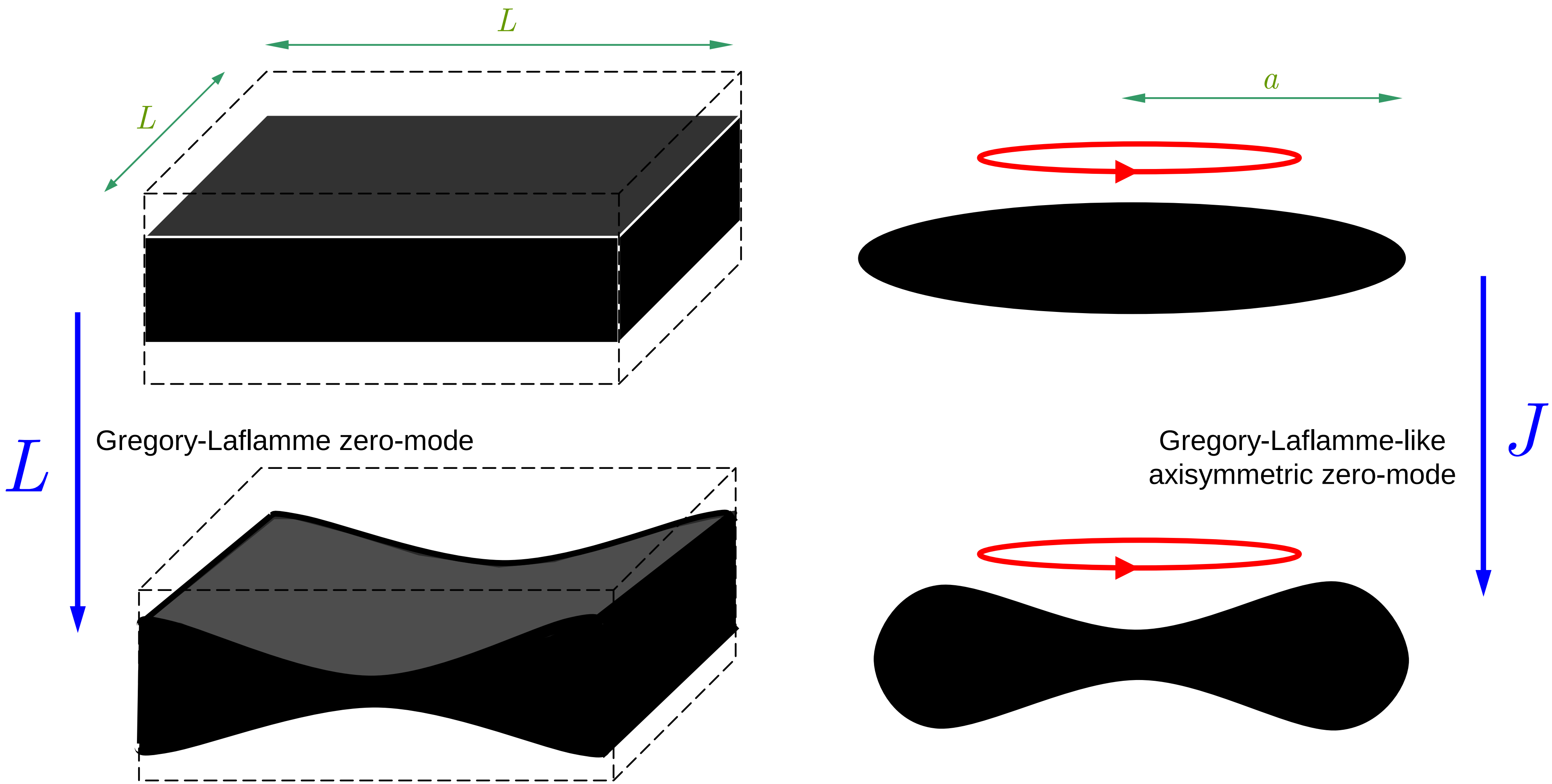


Gubser, Wiseman

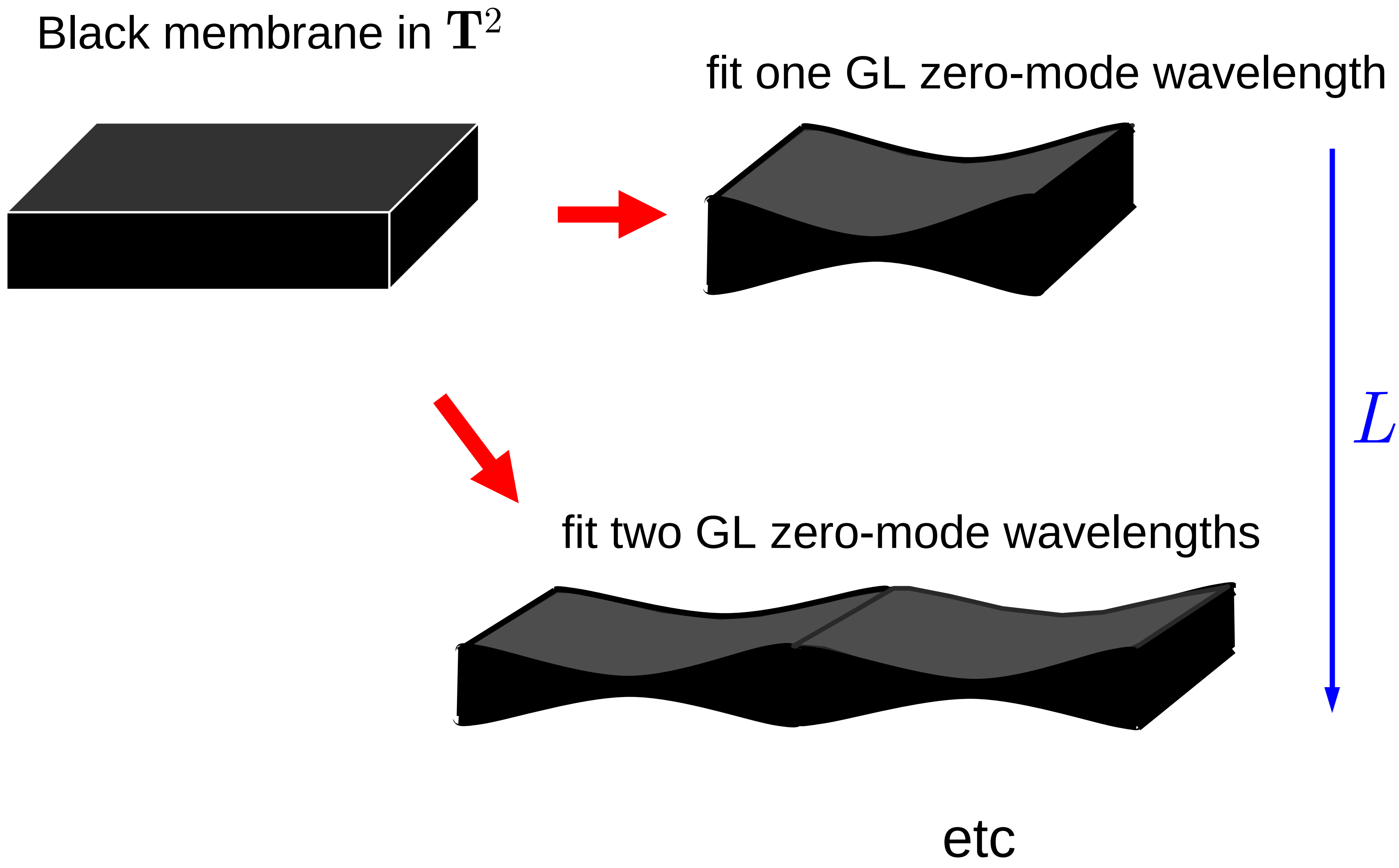
Pinched (lumpy) black holes in $D\geqslant 6$

RE+Myers

Ultra-spinning = membrane-like

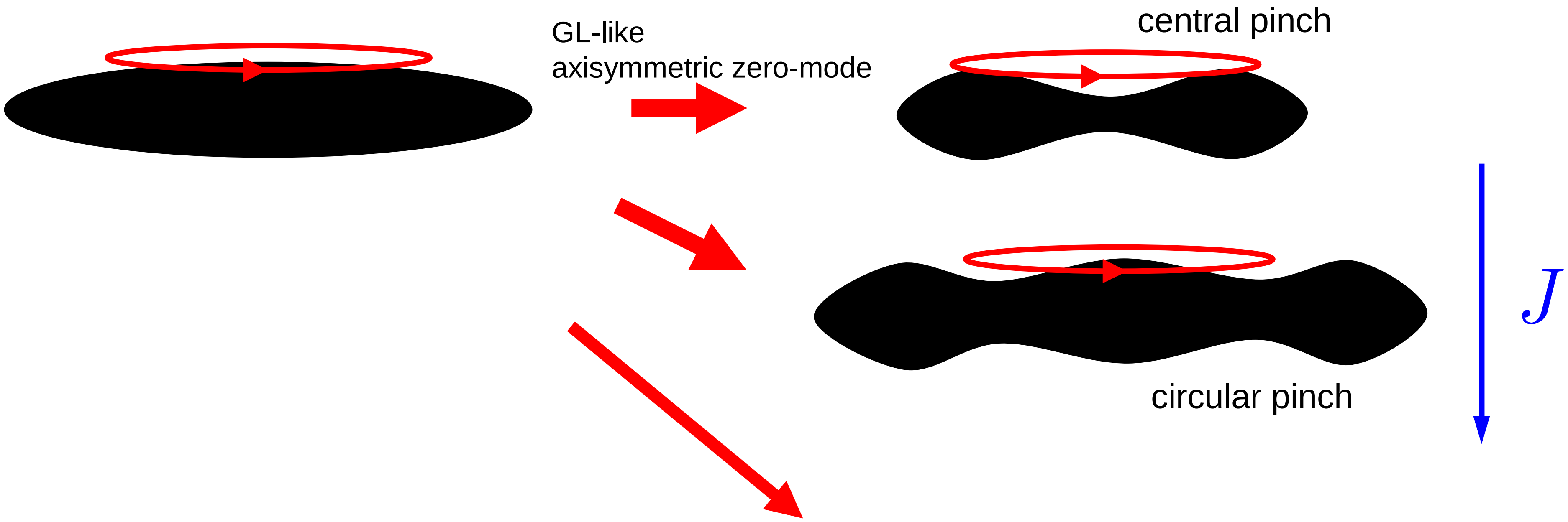


Replicas: multiple pinches



(fixed mass)

Multiply pinched black holes from axisymmetric zero-modes:



…more axisymmetric pinches

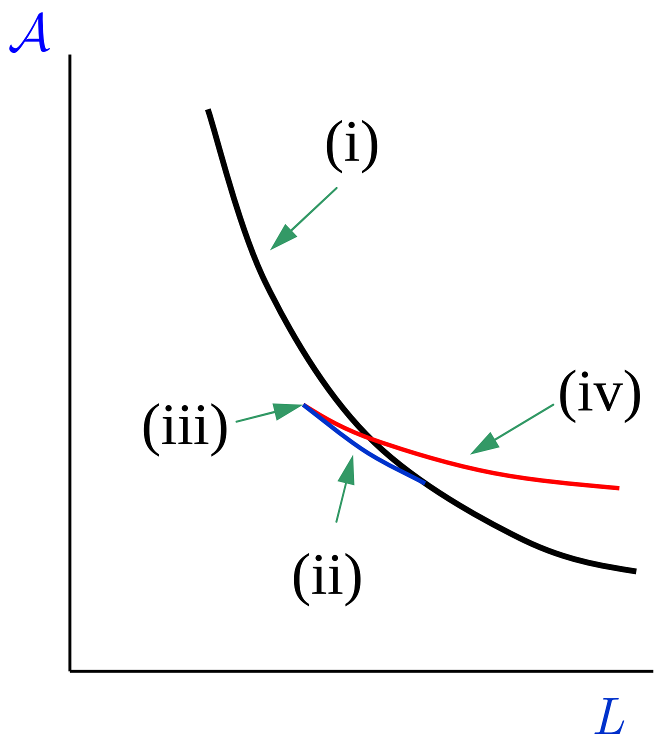
- Not yet found --- presumably numerically or approximately
- *Pinched plasma balls* found by *Lahiri+Minwalla*:

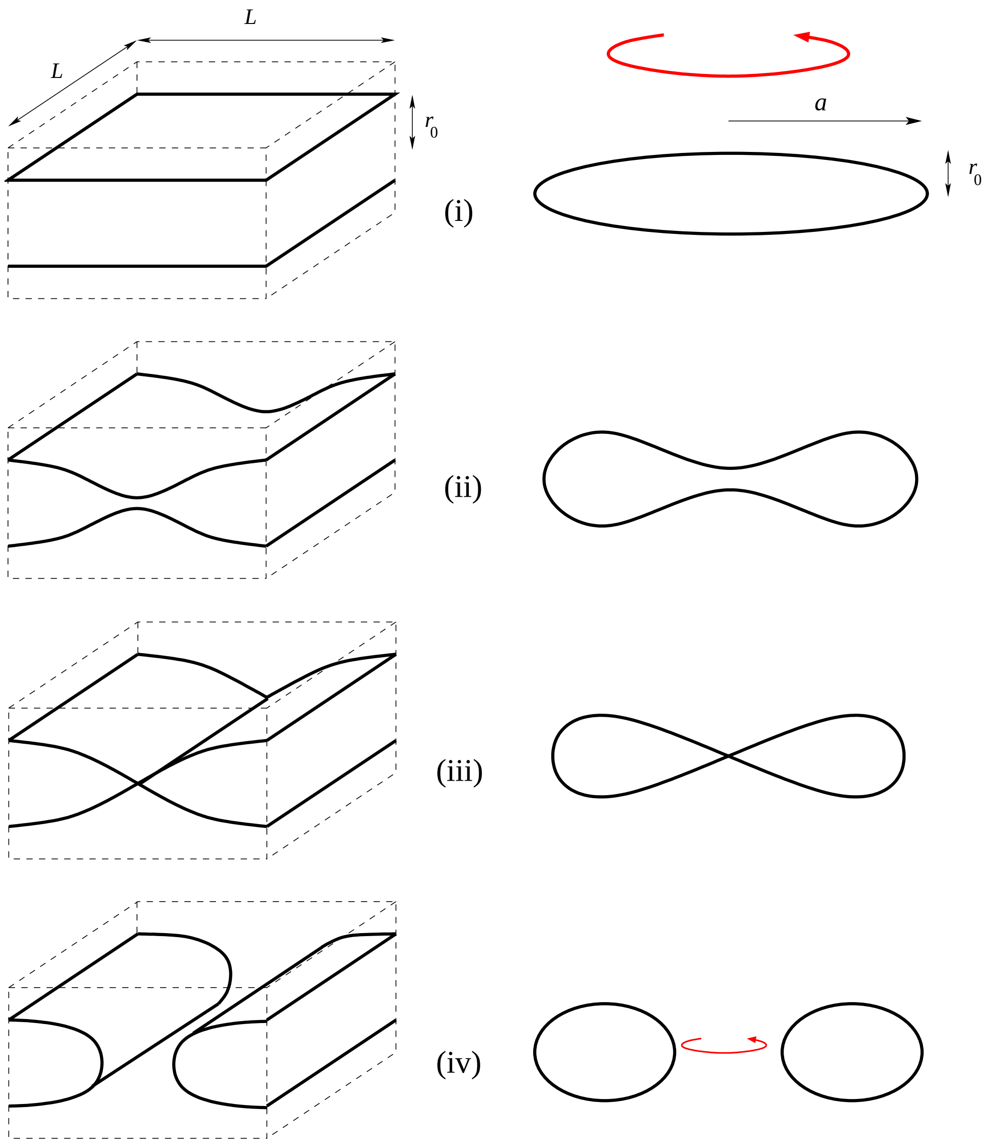
  dual to (large) pinched black holes in AdS

Black membrane ⇔ Rot Black Hole

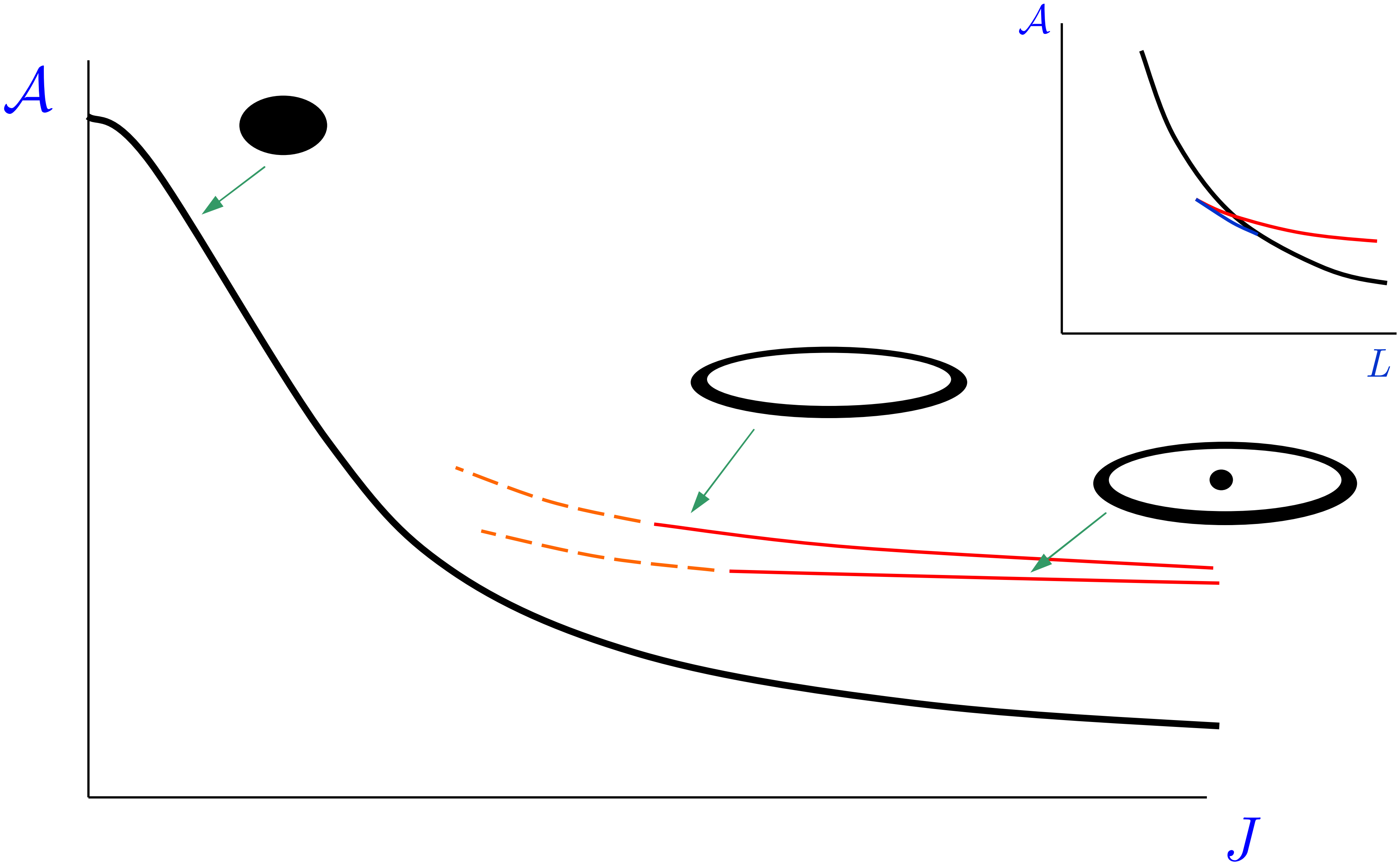
Black membrane in $\mathbf{T}^2$

(fixed mass)

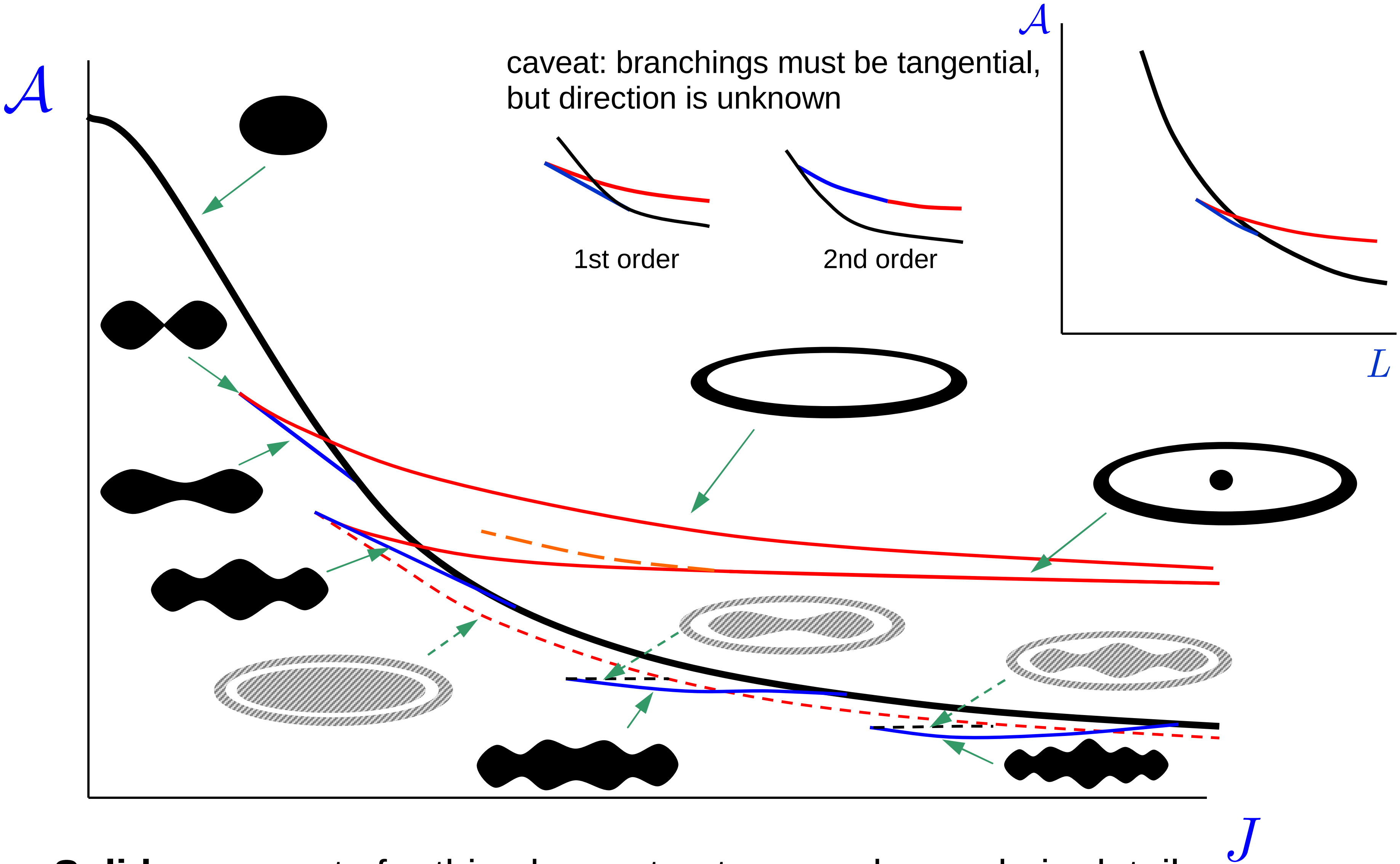




$D \geqslant 6$ phase diagram: a proposal



$D \geqslant 6$ phase diagram: a proposal



Solid: arguments for this phase structure can be made in detail

Grayshade: less tight arguments, but still plausible

General results
&
Open problems

Stability (dynamical, linear)

- Gravitational perturbations:
  1. Decouple eqns: find master eqn (if possible, 2nd order)
  2. Separate variables
- Stability of Schwarzschild proved *Ishibashi+Kodama*
- Myers-Perry:
  - Odd dimensions, equal spins→ only r-dependence

    no sign of instability (not unexpected ~ Kerr) *Kunduri et al* *Murata+Soda*
  - Expect GL-like instability in ultraspinning regimes
- Poorly understood in general
  - identify ultraspinning (GL-like) & turning-point instabilities

Horizon topology

- Hawking's 4D theorem relies on Gauss-Bonnet thm:

$$\int_H R^{(2)} > 0 \Rightarrow H = S^2$$

- $D=5$: Galloway+Schoen: +ve Yamabe $R^{(D\text{-}2)} > 0$ → S^3, S^1x S^2
- $D=6$: Helfgott et al: S^4, S^2x S^2, S^1x S^3, Σ_gx S^2

  so far: S^4 exactly (MP, but possibly others too)

  S^1x S^3, $\mathbf{T}^2$x S^2 approximately
- $D>6$: essentially unknown

  so far: $S^{D\text{-}2}$ exactly (MP, but possibly others too)

  S^1x $S^{D\text{-}3}$, $\mathbf{T}^p$x S^q $(p \leqslant q+1)$ approximately

Uniqueness & Classification

- Schwarschild$_D$ is unique among ***static*** AF black holes

  Gibbons+Ida+Shiromizu

  (proof extends to charged bhs)

  (Note that $\exists$ *non-static* solutions with *zero angular momentum*, eg black saturns)

- ➔ **STATIC** classification solved

Uniqueness & Classification

- **STATIONARY** bh's must admit one spacelike Killing that generates rotations

- But there may be as many as $\lfloor (D-1)/2 \rfloor$ such Killings

- Are there solutions with less than this symmetry?

  Where? How?

- Also: Tools to classify *pinched bh's* still to be developed

1. What is the simplest and most convenient set of parameters that fully specify a bh?
   - In 5D: M, J, + "rod structure": more physical parametrization? Higher D??

2. How many bh's with given charges are relevant to a given physical situation?
   - Conserved charges + additional conditions:
     - Horizon topology alone is not enough
     - Dynamic linear stability (not an issue in 4D classification) may be (just may be) enough
     - But stability does not *per se* rule out a solution – must compare timescales
     - Dipoles introduce more non-uniqueness and enhance stability

Laws of black hole mechanics

- Generally valid indep of dimension
- Dipoles introduce additional terms:

$$dM = \frac{\kappa}{8\pi} d\mathcal{A}_H + \Omega dJ + \Phi dQ + \phi dq$$

RE
Copsey+Horowitz

even if dipoles are *not* conserved charges

can't define globally the dipole potential ϕ

→ extra surface term

Multi-black hole mechanics

- Each connected component of the horizon H_i is generated by a different Killing vector

$$k_{(i)} = \partial_t + \Omega_i \partial_\psi$$

➔ $$M = \frac{3}{2} \sum_i \left(\frac{\kappa_i}{8\pi G} \mathcal{A}_i + \Omega_i J_i \right)$$ (Smarr)

➔ $$\delta M = \sum_i \left(\frac{\kappa_i}{8\pi G} \, \delta \mathcal{A}_i + \Omega_i \, \delta J_i \right)$$ **First Law**

Hawking radiation

- Technical analysis complicated, but physics should remain the same: bh's emit radiation at temperature $T=\kappa/2\pi$ and "chemical potentials" Ω, Φ …

- Multi-bhs will emit multiple components – thermal only of all T_i, Ω_i etc are equal

- Euclidean thermodynamics: much like in 4D
  - real Euclidean sections may not exist
  - convenient to work with complex sections that have real actions

Conclusion: *More is different*

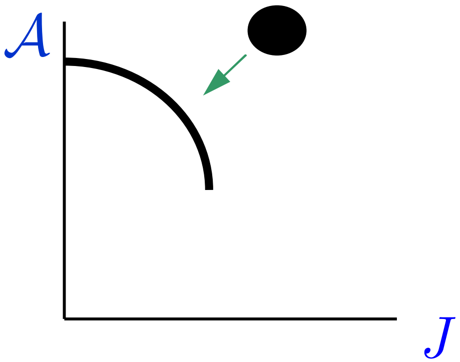
Vacuum gravity $R_{\mu\nu} = 0$ in

- $D=3$ has no black holes
  - GM is dimensionless ➔ can't construct a length scale

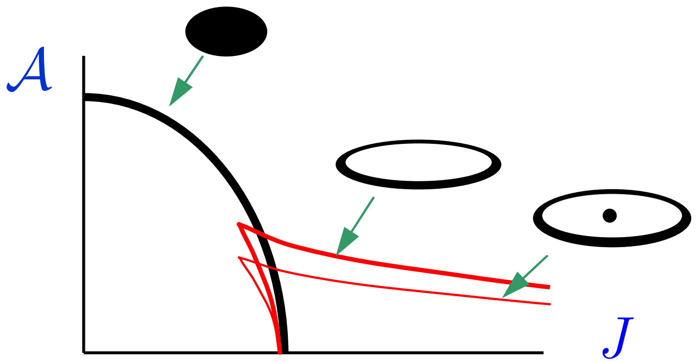
    (Λ, or h, provide length scale)
- $D=4$ has **one** black hole
  - but no 3D bh ➔ no 4D black strings → no 4D black rings
- $D=5$ has **three** black holes (two topologies); black strings → black rings, infinitely many multi-bhs…
- $D\geqslant 6$ seem to have **infinitely many** black holes (many topologies, lumpy horizons…); black branes→ rings, toroids…, infinitely many multi-bhs…

Conclusion: *More is different*

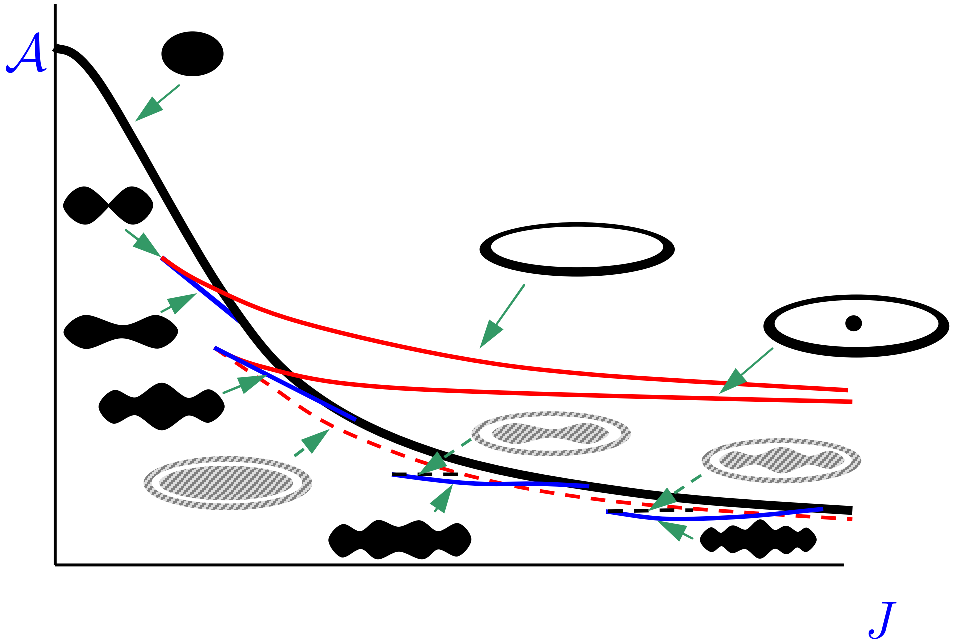
$D{=}4$



$D{=}5$



$D{\geqslant}6$



…we've just begun