The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1935-7

Spring School on Superstring Theory and Related Topics

27 March - 4 April, 2008

Stringy Avatars of Dynamical SUSY Breaking:
- Geometric Transitions and Dynamical SUSY Breaking
- Simple Stringy Dynamical SUSY Breaking

Shamit Kachru

Department of Physics and SLAC
Stanford University
Stanford, CA 94305
USA

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

SLAC-PUB-12821, SU-ITP-07/15

Geometric Transitions and Dynamical SUSY Breaking

Mina Aganagic,[a] Christopher Beem,[a] and Shamit Kachru[b]

[a] *Department of Physics, University of California, Berkeley, CA 94720*

[b] *Department of Physics and SLAC, Stanford University, Stanford CA 94305*

We show that the physics of D-brane theories that exhibit dynamical SUSY breaking due to stringy instanton effects is well captured by geometric transitions, which recast the non-perturbative superpotential as a classical flux superpotential. This allows for simple engineering of Fayet, Polonyi, O'Raifeartaigh, and other canonical models of supersymmetry breaking in which an exponentially small scale of breaking can be understood either as coming from stringy instantons or as arising from the classical dynamics of fluxes.

September 2007

1. Introduction

It is of significant interest to find simple examples of dynamical supersymmetry breaking in string theory. One class of examples, where stringy D-instanton effects play a starring role, was described in [1]. These models exhibit "retrofitting" of the classic SUSY breaking theories (Fayet, Polonyi and O'Raifeartaigh) [2], without incorporating any nontrivial gauge dynamics. Instead, stringy instantons [3] automatically implement the exponentially small scale of SUSY breaking in theories with only Abelian gauge fields. A related idea using disc instantons instead of D-instantons appears in [4]. These models are simpler in many ways than their existing field theory analogues [5].

In this paper, we show that these results (and many generalizations) admit a clear and computationally powerful understanding using geometric transition techniques [6](see also [7,8]). Such techniques are well known to translate quantum computations of superpotential interactions in non-trivial gauge theories to classical geometric computations of flux-induced superpotentials [9]. They are most powerful when the theories in question exhibit a mass gap. While the classic models we study *do* manifest light degrees of freedom (and hence do not admit a complete description in terms of geometry and fluxes), we find that a mixed description involving small numbers of D-branes in a flux background – which arises after a geometric transition from a system of branes at a singularity – nicely captures the relevant physics of supersymmetry breaking[1]. In the original theory without flux, the SUSY breaking effects are generated by D-instantons either in $U(1)$ gauge factors or on unoccupied, but orientifolded, nodes of the quiver gauge theory (analogous to those studied in [1,15,16]). Both are in some sense "stringy" effects. Simple generalizations involve more familiar transitions on nodes with large N gauge groups.

The geometric transition techniques we apply have two advantages over the description using stringy instantons in a background without fluxes. First, they allow for a classical computation of the relevant superpotential instead of requiring a nontrivial instanton calculation. Second, they incorporate higher order corrections (due to multi-instanton effects in the original description) which had not been previously calculated.

The organization of this paper is as follows. In section 2, we remind the reader of the relevant background about geometric transitions. In section 3, we discuss the geometries we will use to formulate our DSB theories. In sections 4-6, we give elementary examples

[1] For an application of geometric transitions to the study of supersymmetry breaking in the context of brane/anti-brane systems see [10-14].

that yield Fayet, Polonyi, and O'Raifeartaigh models that break SUSY at exponentially low scales. In section 7, we present a single geometry that unifies the three models, reducing to them in various limits. In section 8, we provide a more general, exact analysis of the existence of these kinds of susy-breaking effects. In section 9, we give a few other examples of simple DSB theories (related to recent or well known literature in the area). Finally, in section 10, we extend the technology to orientifold models, in particular recovering models which are closely related to the specific examples of [1].

2. Background: Geometric Transitions

Computing non-perturbative corrections in string theory, even to holomorphic quantities such as a superpotential, is in general very difficult. A surprising recent development [6,17] is that in some cases – namely for massive theories – these non-perturbative effects can be determined by perturbative means in a dual language[2].

Consider, for example, N D5 branes in type IIB string theory wrapping an isolated, rigid $\mathbb{P}^1$ in a local Calabi-Yau manifold. In the presence of D5 branes, D1 brane instantons wrapping the $\mathbb{P}^1$ generate a superpotential for the Kähler moduli[3]. The instanton effects are proportional to

$$\exp\left(-\frac{t}{Ng_s}\right)$$

where $t = \int_{S^2}(B^{NS}+ig_sB^{RR})$. For general N, these D1 brane instantons are gauge theory instantons. More precisely, they are the fractional $U(N)$ instantons of the low energy $\mathcal{N}=1$ $U(N)$ gauge theory on the D5 brane. However, on the basis of zero mode counting, one expects that stringy instanton effects are present even for a single D5 brane.

In the absence of D5 branes, the theory has $\mathcal{N}=2$ supersymmetry, and the Kähler moduli space is unlifted. In that case, the local Calabi-Yau with a rigid $\mathbb{P}^1$ is known to have another phase where the S^2 has shrunk to zero size and has been replaced by a finite S^3. The two branches meet at $t=0$, where there is a singularity at which the D3 branes wrapping the S^3 become massless.

What happens to this phase transition in the presence of D5 branes? Classically, we can still connect the S^2 to the S^3 side by a geometric transition. The only difference is

[2] For a two-dimensional example see [18].

[3] This is a slight misnomer, since t is a parameter, and not a dynamical field for a non-compact Calabi-Yau.

that to account for the D5 brane charge, we need there to be N units of RR flux through the S^3,

$$\int_{S^3} H^{RR} = N.$$

Quantum mechanically the effect is more dramatic. In the presence of D5 branes there is no sharp phase transition at all between the S^2 and the S^3 sides; the interpolation between them is completely *smooth*. As a consequence, the two sides of the transition provide *dual* descriptions of the same physics. Since the theory is massive now, the interpolation occurs by varying the coupling constants of the theory. The fact that the singularity where the S^3 shrinks to zero size is eliminated is consistent with the fact that D3 branes wrapping an S^3 with RR flux through it are infinitely massive. The most direct proof of the absence of a phase transition is in the context of M-theory on a G_2 holonomy manifold [19,20,21]. This is related to the present transition by mirror symmetry and an M-theory lift. In M-theory, the transition is analogous to a perturbative flop transition of type IIA string theory at the conifold, except that in M-theory the classical geometry gets corrected by M2 brane instantons instead of worldsheet instantons [19]. The argument that the two sides are connected smoothly is analogous to Witten's argument for the absence of a sharp phase transition in IIA [22]. In both cases, the presence of instantons is crucial for the singularities in the interior of the classical moduli space to be eliminated.

The fact that the two sides of the transition are connected smoothly implies that the superpotentials have to be the same. The instanton-generated superpotential has a dual description on the S^3 side as a *perturbative* superpotential generated by fluxes. The flux superpotential

$$\mathcal{W} = \int H \wedge \Omega$$

is perturbative, given by

$$\mathcal{W} = \frac{t}{g_s}\, S + N\, \partial_S \mathcal{F}_0 \tag{2.1}$$

where $\mathcal{F}_0(S)$ is the prepotential of the Calabi-Yau, and

$$S = \int_{S^3} \Omega.$$

The first term in (2.1) comes from the running of the gauge coupling t/g_s which implies that there is an H^{NS} flux turned on on the Calabi-Yau through a 3-chain on the S^2 side.

This three-chain becomes the non-compact 3-cycle dual to the S^3 after the transition. Near the conifold

$$\partial_S \mathcal{F}_0 = S\left(\log\left(\frac{S}{\Delta^3}\right) - 1\right) + \ldots$$

where the omitted terms are a model dependent power series in S, and Δ is a high scale at which t is defined. Integrating out S in favor of t, the superpotential $\mathcal{W}$ becomes

$$\mathcal{W}_{inst} = -\Delta^3 exp(-\frac{t}{Ng_s}) + \ldots$$

up to two and higher order instanton terms that depend on the power series in $\mathcal{F}_0(S)$. The duality should persist even in the presence of other branes and fluxes, as long as the S^2 that the branes wrap remains isolated, and the geometry near the branes is unaffected. As we'll discuss in section 10, this can also be extended to D5 branes wrapping $\mathbb{P}^1$'s in Calabi-Yau orientifolds.

3. The Theories

To construct the models in question, we will consider type IIB on non-compact Calabi-Yau 3-folds which are A_r ADE type ALE spaces fibered over the complex plane $\mathbb{C}[x]$. These are described as hypersurfaces in $\mathbb{C}^4$ as follows

$$uv = \prod_{i=1}^{r+1}(z - z_i(x)). \tag{3.1}$$

This geometry is singular at points where $u, v = 0$ and $z_i(x) = z_j(x) = z$. At these points, there are vanishing size $\mathbb{P}^1$'s which can be blown up by deforming the Kähler parameters of the Calabi-Yau. There are r 2-cycle classes, which we will denote

$$S_i^2.$$

These correspond to the blow-ups of the singularities at $z_i = z_{i+1}$, $i = 1, \ldots r$. It is upon these $\mathbb{P}^1$'s that we wrap D5 branes to engineer our gauge theories.

The theory on the branes can be thought of as an $\mathcal{N} = 2$ theory, corresponding to D5 branes wrapping 2-cycles of the ALE space, which is then deformed to an $\mathcal{N} = 1$ theory by superpotentials for the adjoints. For the branes on S_i^2 this superpotential is denoted $\mathcal{W}_i(\Phi_i)$. The adjoint Φ_i describes the positions of the branes in the x-direction, and the superpotential arises because the ALE space is fibered nontrivially over the x plane. The superpotential can be computed by integrating [23,24]

$$\mathcal{W} = \int_{\mathcal{C}} \Omega$$

over a 3-chain with one boundary as the wrapped S^2. In this particular geometry, it takes an extra simple form (the details of the computation appear in appendix A)

$$\mathcal{W}_i(x) = \int (z_i(x) - z_{i+1}(x))dx. \tag{3.2}$$

In addition to the adjoints, for each intersecting pair of two-cycles S_i^2, S_{i+1}^2 there is a bifundamental hypermultiplet at the intersection, consisting of chiral multiplets $Q_{i,i+1}$ and $Q_{i+1,1}$, with a superpotential interaction inherited from the $\mathcal{N}=2$ theory

$$\mathrm{Tr}(Q_{i,i+1}\Phi_{i+1}Q_{i+1,i} - Q_{i,i+1}Q_{i+1,i}\Phi_i).$$

Classically, the vacua of the theory correspond to the different ways of distributing branes on the minimal $\mathbb{P}^1$'s in the geometry [25]. When one of the nodes is massive, the instantons corresponding to D1 branes wrapping the S^2 can be summed up in the dual geometry after a geometric transition. As explained in [1], and as we'll see in the next section, this can trigger supersymmetry breaking in the rest of the system.

As an aside, we note that the systems we are studying are a slight generalization of those described in [15,1]. Those geometries are related to the family of geometries studied here, but correspond to particular points in the parameter space where the adjoint masses have been taken to be large and the branes and/or O-planes have been taken to coincide in the x-plane. In addition, we allow the possibility of $U(1)$ (or in some cases higher rank) gauge groups on the transitioning node, whereas in [15,1] the instanton effects were associated with nodes that were only occupied by O-planes. Nevertheless, we will find the same qualitative physics as in [1] in this broader class of theories.

4. The Fayet Geometry

We now turn to the specific geometry which will engineer the Fayet model at low energies. This will be an A_3 geometry, and (3.1) can be written explicitly as

$$uv = (z - mx)(z + mx)(z - mx)(z + m(x - 2a)). \tag{4.1}$$

After blowing up, we wrap M branes each on S_1^2 at $z_1(x) = z_2(x)$, on S_2^2 at $z_2(x) = z_3(x)$ and one brane on S_3^2 at $z_3(x) = z_4(x)$. The tree-level superpotential (3.2) is now given by

$$\mathcal{W} = \sum_{i=1}^{3} \mathcal{W}_i(\Phi_i) + \mathrm{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \mathrm{Tr}(Q_{23}\Phi_3 Q_{32} - Q_{32}\Phi_2 Q_{23}) \qquad (4.2)$$

where

$$\mathcal{W}_1(\Phi_1) = m\Phi_1^2, \qquad \mathcal{W}_2(\Phi_2) = -m\Phi_2^2, \qquad \mathcal{W}_3(\Phi_3) = m(\Phi_3 - a)^2.$$

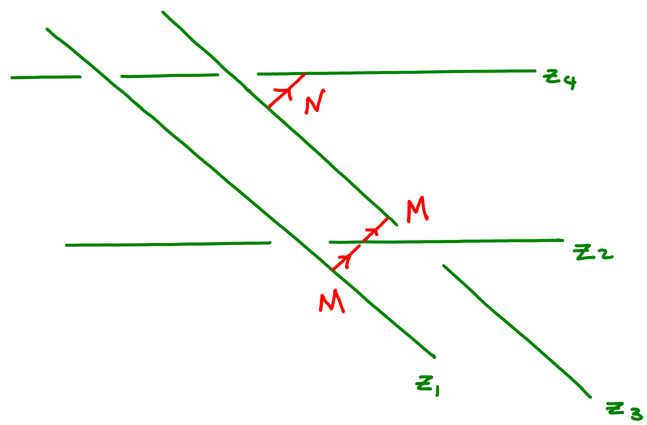

Fig. 1. The A_3 geometry used for retrofitting the Fayet model, *before* the geometric transition. The red lines represent the $\mathbb{P}^1$'s, wrapped by D5 branes. The third node does not intersect the other two and is massive. The geometry after the transition sums up the corresponding instantons. For $N = 1$ branes on S_3^2, the instantons are stringy. For $N > 1$, these are fractional instantons associated with gaugino condensation in the pure $U(N)$ $\mathcal{N} = 1$ gauge theory on that node.

The branes on nodes one and two intersect, since both of the corresponding $\mathbb{P}^1$'s are at $x = 0$. However, the third node, and the single brane on it, is isolated at $x = a$, and the theory living on it is massive. Correspondingly, the the instantons effects due to D-instantons wrapping the third node can be summed up in a dual geometry where we trade S_3^2 for a three-cycle S^3 with one unit of flux through it

$$\int_{S^3} H^{RR} = 1 \ .$$

The geometry after the transition is described by the deformed equation

$$uv = (z - mx)(z + mx)((z - mx)(z + m(x - 2a)) - s) \qquad (4.3)$$

where the size of the S^3

$$\int_{S^3} \Omega = S$$

is given by $S = s/m$. It is fixed to be exponentially small by the flux superpotential, as we shall see shortly. The third brane is gone now, and so are the fields Q_{23}, Q_{32} and Φ_3. The effective superpotential can now be written to leading order in S as

$$\mathcal{W}_{eff} = \mathcal{W}_1(\Phi_1) + \tilde{W}_2(\Phi_2, S) + \mathrm{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \mathcal{W}_{flux}(S).$$

In this geometry, the exact flux superpotential is

$$\mathcal{W}_{flux} = \frac{t}{g_s} S + S\left(\log \frac{S}{\Delta^3} - 1\right)$$

without any polynomial corrections in S. It is crucial here that the superpotential for Φ_2 has changed due to the change in the geometry to $\tilde{\mathcal{W}}_2(\Phi_2)$, where

$$\tilde{\mathcal{W}}_2(x) = \int (z_2(x) - \tilde{z}_3(x))dx,$$

while the superpotential for Φ_1 is unaffected. We have defined

$$(z - \tilde{z}_3(x))(z - \tilde{z}_4(x)) = (z - z_3(x))(z - z_4(x)) - s$$

with $\tilde{z}_3(x)$ being the branch which asymptotically looks like $z_3(x)$ at large values of x. In other words,

$$\tilde{\mathcal{W}}_2(x) = \int_{\Delta}^{x} (-m(x' + a) - \sqrt{m^2(x' - a)^2 + s})dx'.$$

This superpotential sums up the instanton effects due to Euclidean branes wrapping node three.

Before the transition, the vacuum was at $\Phi_2 = 0$. At the end of the day, we expect it to be perturbed by exponentially small terms $\sim S$, so the relevant part of the superpotential is

$$\tilde{W}_2(\Phi_2) = -m\mathrm{Tr}\Phi_2^2 - \frac{1}{2}\, S\, \mathrm{Tr} \log \frac{a - \Phi_2}{\Delta} + \ldots. \tag{4.4}$$

where we've omitted terms of order S^2 and higher and dropped an irrelevant constant. We comment on the form of these corrections in appendix B.

The theories on nodes one and two are asymptotically free. If the fields S and $\Phi_{1,2}$ have very large masses, we can integrate them out and keep only the light degrees of freedom. Keeping only the leading instanton corrections, the relevant F-terms are

$$\begin{aligned} F_{\Phi_1} &= 2m\Phi_1 - Q_{12}Q_{21} \\ F_{\Phi_2} &= -2m\Phi_2 + Q_{21}Q_{12} + \frac{S}{2(a-\Phi_2)} \\ F_S &= t/g_s + \log S/\Delta^3 - \frac{1}{2}\mathrm{Tr}\log(a-\Phi_2)/\Delta \end{aligned} \tag{4.5}$$

Setting these to zero, we obtain

$$S_* = \Delta^3 \exp(-\frac{\tilde{t}}{g_s}) + \ldots$$

where

$$\tilde{t} = t - \frac{1}{2}Mg_s\log(a/\Delta)$$

and

$$\begin{aligned} \Phi_{1,*} &= -\frac{1}{2m}Q_{12}Q_{21} \\ \Phi_{2,*} &= \frac{1}{2m}Q_{21}Q_{12} + \frac{1}{4ma}S_* + \ldots . \end{aligned} \tag{4.6}$$

The omitted terms are higher order in $Q_{21}Q_{12}/ma$ and $\exp(-\frac{\tilde{t}}{g_s})$. The low energy, effective superpotential is

$$\mathcal{W}_{eff} = \frac{1}{m}\mathrm{Tr}(Q_{12}Q_{21}Q_{12}Q_{21}) \; - \; \frac{S_*}{4ma}\mathrm{Tr}Q_{12}Q_{21} + \ldots$$

where we have neglected corrections to the quartic coupling, and the higher order couplings of $Q's$, all of which are exponentially suppressed. As shown in [1], in the presence of a generic FI term for the off-diagonal $U(1)$ under which Q_{12} and Q_{21} are charged,

$$D = Q_{12}Q_{12}^\dagger - Q_{21}^\dagger Q_{21} - r,$$

the exponentially small mass for Q will trigger F-term supersymmetry breaking with an exponentially low scale; we can put $Q_{12,*} = \sqrt{r}$, and then

$$F_{Q_{21}} \sim \frac{\sqrt{r}}{4ma}S_* \, .$$

Geometrically, turning the FI term corresponds to choosing the central charges of the branes on the two nodes to be miss-aligned. Combined with the fact that the nodes one and two have become massive with an exponentially low mass, this provides an extremely simple mechanism of breaking supersymmetry at a low scale. The non-supersymmetric vacuum we found classically is reliable, as long as the scale of supersymmetry breaking is far above the strong coupling scales of the $U(M) \times U(M)$ gauge theory. Had we taken N branes on the massive node instead of one, the story would have been the same, apart from the fact that the flux increases, and correspondingly the vacuum value of S changes to $S_* \sim \Delta^3 \exp(-\frac{\tilde{t}}{N g_s})$. In this case however, the instantons that trigger supersymmetry breaking are the fractional $U(N)$ instantons.

5. The Polonyi Model

In this section we construct the Polonyi model with an exponentially small linear superpotential term for a chiral superfield Φ. This will turn out to be somewhat more subtle, and the existence of the (meta)stable vacuum will depend sensitively on the Kähler potential. We describe specific cases where we know the relevant Kähler potential does yield a stable vacuum in section 7.

Consider an A_2 geometry given by

$$uv = (z - mx)(z - mx)(z + m(x - 2a)) \tag{5.1}$$

which has one D5-brane wrapped on the S_1^2 blowing up $z_1(x) = z_2(x)$, and one D5-brane wrapped on the S_2^2 blowing up $z_2(x) = z_3(x)$. This system has a tree-level superpotential

$$\mathcal{W} = \mathcal{W}_1(\Phi_1) + \mathcal{W}_2(\Phi_2) + Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}. \tag{5.2}$$

where

$$\mathcal{W}_1(\Phi_1) = 0, \qquad \mathcal{W}_2(\Phi_2) = m(\Phi_2 - a)^2$$

This theory has a classical moduli space of vacua parameterized by the expectation value of Φ_1 and where $Q_{12,*} = 0 = Q_{21,*}$, and $\Phi_{2,*} = a$.

At a generic point in the moduli space, away from $\Phi_1 = a$, the theory on the branes wrapping S_2^2 is massive. Then, the instanton effects associated with D1 branes wrapping this node can be summed up by a geometric transition, that replaces S_2^2 by an S^3 with one unit of flux through it. This deforms the Calabi-Yau geometry to

$$uv = (z - mx)((z - mx)(z + m(x - 2a)) - s).$$

which has an S^3 of size

$$\int_{S^3} \Omega = S$$

where $S = s/m$. With this deformation, the superpotential for node 1 is altered as well:

$$\tilde{\mathcal{W}}_1(x) = \int (-m(a-x) + \sqrt{m^2(a-x)^2 + s})dx.$$

The effective superpotential after the transition is simply

$$\mathcal{W}_{eff} = \tilde{W}_1(\Phi_1, S) + \mathcal{W}_{flux}(S)$$

where the flux superpotential has the simple form:

$$\mathcal{W}_{flux}(S) = \frac{t}{g_s}\, S + S(\log S/\Delta^3 - 1)$$

Note that there is no supersymmetric vacuum, since $F_{\Phi_1} \neq 0$ always.

Suppose at a point in the moduli space, centered say at $\Phi_1 = 0$, the Kähler potential takes the form

$$K = |\Phi_1|^2 + c|\Phi_1|^4 + \dots$$

where the higher order terms are suppressed by a characteristic mass scale (which we set to one). Then, provided:

$$|ca^2| \gg 1, \qquad c < 0,$$

the theory has a non-supersymmetric vacuum at

$$\Phi_{1,*} = \frac{1}{ca^*}, \tag{5.3}$$

which breaks SUSY at an exponentially low scale.

This can be seen as follows. Expanded about small Φ_1, the superpotential $\tilde{W}_1$ takes the form

$$\tilde{\mathcal{W}}_1(\Phi_1) = -\frac{S}{2}\log(a - \Phi_1)/\Delta + \dots$$

where the subleading terms are suppressed by additional powers of S, but are otherwise regular at the origin of Φ_1 space. Integrating out S first, by solving its F term constraint, we find

$$S_* = \Delta^3 exp(-\tilde{t}/g_s) + \dots$$

where

$$\tilde{t} = t - \tfrac{1}{2} g_s \log(a/\Delta)$$

and the subleading terms are of order Φ_1/a which will turn out to be small in the vacuum. For large $\tilde{t}$, S is generically very massive, so integrating it out is justified.

The potential for Φ_1 now becomes

$$V_{eff}(\Phi_1) = \frac{1}{1 + c|\Phi_1|^2} \frac{|S_*|^2}{|a - \Phi_1|^2} + \ldots$$

It is easy to see that, up to corrections of order $1/|a^2 c|$ and $S_*/(ma^2)$, this has a non-supersymmetric vacuum at (5.3) where Φ_1 has a mass squared of order

$$-c|\frac{S_*}{a}|^2.$$

This is positive, and the vacuum is (meta)stable, as long as $c < 0$. Note that we could have obtained the Polonyi model as a limit of the Fayet model where we turn on a very large FI term for the off-diagonal gauge group of nodes one and two. In this case, the stability of the Fayet model for a generic (effectively canonical) Kähler potential guarantees that the Polonyi model obtained from it is stable. In fact [1], as we'll review in section 7 , one can show this directly by computing the relevant correction to the Kähler potential, arising from loops of massive gauge bosons.

6. An O'Raifeartaigh model

To represent the third simple classic class of SUSY breaking models, we engineer an O'Raifeartaigh model. Consider the A_3 fibration with

$$z_1(x) = mx,\ z_2(x) = mx,\ z_3(x) = mx,\ z_4(x) = -m(x - 2a)\ . \tag{6.1}$$

The defining equation of the non-compact Calabi-Yau is then

$$uv = (z - mx)(z - mx)(z - mx)(z + m(x - 2a))\ . \tag{6.2}$$

We wrap 1 D5 brane on each of $S^2_{1,2,3}$. The adjoints Φ_1 and Φ_2 are massless, while Φ_3 obtains a mass from its superpotential

$$\mathcal{W}_3(x) = \int (z_3(x) - z_4(x))\ dx \tag{6.3}$$

which gives

$$\mathcal{W}_3(\Phi_3) = m(\Phi_3 - a)^2 \ .$$

Of course, there are also quarks Q_{12}, Q_{21} and Q_{23}, Q_{32}. They couple via the superpotential couplings

$$Q_{12}\Phi_1 Q_{21} - Q_{12}\Phi_2 Q_{21} + Q_{23}\Phi_2 Q_{32} - Q_{23}\Phi_3 Q_{32} \ . \tag{6.4}$$

Because Φ_3 is locked at a, for generic values of Φ_2, Q_{23} and Q_{32} are massive. Then node 3 is entirely massive, and we can perform a geometric transition.

The resulting theory has a new "glueball superfield" S, and effective superpotential

$$\mathcal{W}_{eff} = Q_{12}\Phi_1 Q_{21} - Q_{12}\Phi_2 Q_{21} - \frac{1}{2} S \log(a - \Phi_2)/\Delta + S(\log(S/\Delta^3) - 1) + \frac{t}{g_s} S + \dots \tag{6.5}$$

Integrating out the S field yields (at leading order)

$$S_* = \Delta^3 e^{-\tilde{t}/g_s} \ . \tag{6.6}$$

where

$$\tilde{t} = t + \frac{1}{2} g_s \log(a/\Delta).$$

Plugging this into the superpotential yields:

$$\mathcal{W}_{eff} = Q_{12}\Phi_1 Q_{21} - Q_{12}\Phi_2 Q_{21} - \frac{1}{2} S_* \Phi_2/a \ + \dots \tag{6.7}$$

The omitted terms are suppressed by more powers of Φ_2/a. We recognize (6.7) as the superpotential for an O'Raifeartaigh model, very similar to the one considered in [1]. We see that setting $F_{\Phi_1} = F_{\Phi_2} = 0$ is impossible, so one obtains F-term supersymmetry breaking, with a small scale set by $\Delta e^{-t/3g_s}$.

The stability of the non-supersymmetric vacuum again depends on the form of (technically) irrelevant corrections to the Kähler potential. As in the case of Polonyi model, corrections which yield a stable vacuum can be arranged by embedding the model in a slightly larger theory. We'll turn to this in the next section.

7. A Master Geometry

It is possible to construct one configuration of branes on an A_4 geometry which in appropriate limits can be made to reduce to any of the three simple models discussed in the previous sections. The geometry is described by the defining equation

$$uv = (z - mx)(z - mx)(z + mx)(z - mx)(z + m(x - 2a)) \tag{7.1}$$

which has superpotential given by

$$\mathcal{W}_{master} = \sum_{i=1}^{4} \mathcal{W}_i(\Phi_i) + \sum_{i=1}^{3} \mathrm{Tr}(Q_{i,i+1}\Phi_{i+1}Q_{i+1,i} - Q_{i+1,i}\Phi_i Q_{i,i+1}). \tag{7.2}$$

where we wrap N branes on nodes one, two and three, and a single brane on node four. The superpotentials for the adjoints are given by

$$\mathcal{W}_1(\Phi_1) = 0, \quad \mathcal{W}_2(\Phi_2) = -m\mathrm{Tr}(\Phi_2^2), \quad \mathcal{W}_3(\Phi_3) = m\mathrm{Tr}(\Phi_3^2), \quad \mathcal{W}_4(\Phi_4) = -m(\Phi_4 - a)^2 \ .$$

For simplicity of the discussion, we'll set $N = 1$ in this section. The non-abelian generalization is immediate, since all the nodes are asymptotically free (for large adjoint masses). As long as the scale of supersymmetry breaking driven by the geometric transition is high enough, we can ignore the non-abelian gauge dynamics on the other nodes.

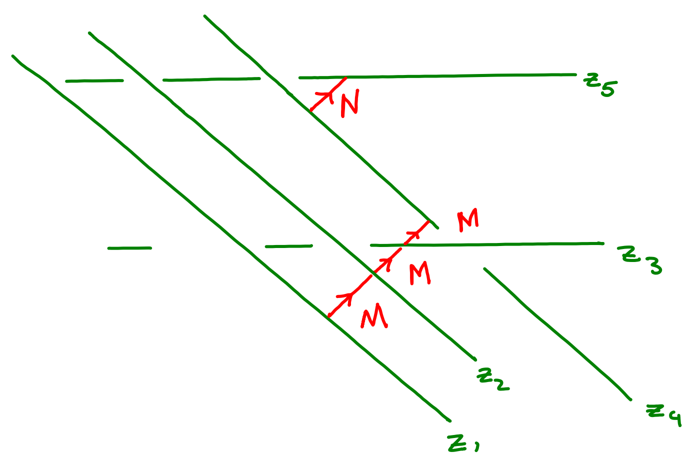

Fig. 2. The master A_4 geometry that gives rise to Fayet, Polonyi and O'Raifeartaigh models by turning on suitable FI terms. The stringy instantons associated with the massive fourth node generate the non-perturbative superpotential that triggers dynamical supersymmetry breaking in the rest of the theory.

The master theory has a metastable non-supersymmetric vacuum for generic, non-zero FI terms. We can recover all three of the models discussed above by introducing large Fayet-Iliopoulos terms for certain pairs of quarks, so we expect that these will have non-supersymmetric vacua as well. This approach to obtaining the canonical models is particularly useful in the case of Polonyi and O'Raifeartaigh models, for which we needed to assume a particular sign for the subleading correction to the Kähler potential. By obtaining the theories from the master theory, we can compute the corrections to the Kähler potential directly and show that they are of the type required to stabilize the susy-breaking vacua.

To see that the master theory has a metastable non-supersymmetric vacuum, we can proceed as in the Fayet model. Node four is massive, and the corresponding non-perturbative superpotential can be computed in the geometry after transition. The effective superpotential after the transition and integrating out the massive adjoints $\Phi_{2,3}$ is then easily seen to be

$$\mathcal{W}_{eff} = Q_{12}Q_{21}\Phi_1 + \frac{S_*}{4ma}(Q_{23}Q_{32} + \ldots)$$

where we have omitted quartic and higher order terms in the Q's which do not affect the status of the vacuum. With generic FI terms setting

$$|Q_{12}|^2 - |Q_{21}|^2 = r_2, \qquad |Q_{23}|^2 - |Q_{32}|^2 = r_3 \ ,$$

this is easily seen to have an isolated vacuum which breaks supersymmetry.

We'll now show that we can recover all of the the three models studied so far in particular regimes of large FI terms.

7.1. O'Raifeartaigh

We can recover the O'Raifeartaigh construction by turning on a large FI term for Q_{23} and Q_{32} – that is, for the U(1) under which these are the only charged quarks. This generates a D-term

$$D_{O'R} = |Q_{23}|^2 - |Q_{32}|^2 - r_3. \tag{7.3}$$

Taking $r_3 >> 0$, this requires that Q_{23} acquire a large expectation value. Additionally there is an F-term for Q_{32}

$$F_{Q_{32}} = Q_{23}(\Phi_3 - \Phi_2) \tag{7.4}$$

which, in light of the D-term constraint, will set Φ_2 equal to Φ_3. The superpotential then becomes just the O'Raifeartaigh superpotential of the previous section (with certain indices renamed),

$$\mathcal{W}_{O'R} = m(\Phi_4 - a)^2 + Q_{12}Q_{21}(\Phi_2 - \Phi_1) + Q_{24}Q_{42}(\Phi_4 - \Phi_2) \ . \tag{7.5}$$

By performing a geometric transition on the massive node, we recover the superpotential (6.5).

7.2. Fayet

Alternatively, we could have turned on a large FI term for Q_{12} and Q_{21}, generating a D-term

$$D_{Fayet} = |Q_{12}|^2 - |Q_{21}|^2 - r_2. \tag{7.6}$$

In conjunction with the F-term for Q_{21}, by the same process as in the O'Raifeartaigh model, Φ_1 is set equal to Φ_2. This time, the remaining superpotential is given by

$$\mathcal{W}_{Fayet} = m\Phi_2^2 - m\Phi_3^2 + m(\Phi_4 - a)^2 + Q_{23}Q_{32}(\Phi_3 - \Phi_2) + \dots \tag{7.7}$$

which is precisely the superpotential associated with the Fayet geometry (4.1). Performing a geometric transition on S_4^2, we recover the Fayet model as discussed in section 4.

7.3. Polonyi

From the Fayet model above, before the geometric transition, we can turn turn on another D-term for the quarks Q_{23} and Q_{32}, which along with the F-term for Q_{32} sets $\Phi_2 = \Phi_3$. The superpotential becomes

$$\mathcal{W} = -m\Phi_3^2 + m(\Phi_4 - a)^2 + Q_{34}Q_{43}(\Phi_4 - \Phi_3)$$

which reproduces the Polonyi model of section 5. Again performing the geometric transition on S_4^2 results in the actual Polonyi model.

7.4. The Kähler potential

The O'Raifeartaigh and Polonyi models have flat directions at tree level. As we discussed for e.g. the Polonyi model, the existence of a stable SUSY-breaking vacuum depends on the sign of the leading, quartic correction to the Kähler potential. When we obtain the model as a suitable limit of our master model as above, we can compute this correction and verify explicitly that the vacuum is stable. Let us go through this in some detail. In fact, for simplicity, let's focus on obtaining a stable Polonyi model as a limit of a Fayet model [1].

After the geometric transition in the Fayet model, the effective theory is characterized by a superpotential

$$\mathcal{W} = \frac{S_*}{ma} Q_{23} Q_{32} + \ldots \tag{7.8}$$

and D-term

$$D = |Q_{32}|^2 - |Q_{23}|^2 - r_3 \ . \tag{7.9}$$

Here r_3 is the FI term for the $U(1)$ under which only Q_{23} and Q_{32} carry a charge. We can expand this theory about the vev $Q_{23} = \sqrt{r_3}$. Renaming

$$X = Q_{32} \ ,$$

the effective theory then has

$$\mathcal{W} = \frac{S_*}{ma} \sqrt{r_3} X \ . \tag{7.10}$$

To find the Kähler potential for X, we should integrate out the massive $U(1)$ gauge multiplet. What happens to the potential contribution from the D-term, (7.9)? As explained in [26], in the theory with the $U(1)$ gauge field, gauge invariance relates D-term and F-term vevs at any critical point of the scalar potential. When one integrates out the $U(1)$ gauge field, there is a universal quartic correction to the Kähler potential which (using the relation) precisely reproduces the potential contribution from the D-term. For the theory in question, the quartic correction to the Kähler potential for X is just

$$\Delta K = -\frac{g_{U(1)}^2}{M_{U(1)}^2} (X^\dagger X)^2 \ . \tag{7.11}$$

Here $M_{U(1)}$ is the mass of the $U(1)$ gauge boson, $M_{U(1)} \sim g_{U(1)} \sqrt{r_3}$. The result is a quartic correction to K

$$\Delta K = -\frac{1}{r_3} (X^\dagger X)^2 \ . \tag{7.12}$$

So in the notation of section 5,

$$c = -\frac{1}{r_3}$$

and the sign $c < 0$ results in a stable vacuum, as expected. Plugging in the F-term $F_X \sim \frac{S_*}{ma}\sqrt{r_3}$, (7.12) gives X a mass

$$m_X \sim \frac{S_*}{ma},$$

in agreement with what it was in the full, Fayet model. Note that while one would obtain other quartic couplings in K after integrating out the $U(1)$ gauge boson, they don't play any role. They involve powers of the heavy field Q_{34}, and since $F_{Q_{34}} \ll F_X$, cross-couplings of the form $Q_{34}^\dagger Q_{34} X^\dagger X$ in K do not correct the estimate for the X mass above appreciably.

8. Generalization

We now present a very general argument for the existence of supersymmetry-breaking effects in a class of stringy quiver gauge theories which includes those just discussed. Suppose we have such an A_r quiver theory in which the last node is isolated and undergoes a transition. Note that this is the case in the master geometry considered in the previous section.

In this case, the transition deforms the geometry to the following:

$$uv = \left(\prod_{i=1}^{r-1}(z - z_i(x))\right)((z - z_r(x))(z - z_{r+1}(x)) - s)$$

where in which case the superpotential for the branes on the second-to-last node becomes

$$\tilde{\mathcal{W}}_{r-1}(\Phi_{r-1}) = \int dx(\tilde{z}_r(x) - z_{r-1}(x))$$

where $\tilde{z}_r(x)$ is the solution to the equation

$$(z - z_r(x))(z - z_{r+1}(x)) = s \tag{8.1}$$

which asymptotically approaches $z_r(x)$. We can re-write the superpotential as a correction to the pre-transition superpotential as

$$\tilde{\mathcal{W}}_{r-1}(\Phi_{r-1}) = \int dx(\tilde{z}_r(x) - z_r(x)) + \mathcal{W}_{r-1}(\Phi_{r-1})$$

and the F-term for Φ_{r-1} and the remaining adjoints are then given by

$$\begin{aligned} F_{\Phi_{r-1}} &= \mathcal{W}'_{r-1}(\Phi_{r-1}) + (\tilde{z}_r(\Phi) - z_r(\Phi)) + Q_{r-1,r}Q_{r,r-1} \\ F_{\Phi_i} &= \mathcal{W}'_i(\Phi_i) + Q_{i-1,i}Q_{i,i-1} - Q_{i,i+1}Q_{i+1,i} \end{aligned} \tag{8.2}$$

which we can combine to obtain the constraint

$$\sum_{i}^{r-1} \mathcal{W}'_i(\Phi_i) = z_r(\Phi_{r-1}) - \tilde{z}_r(\Phi_{r-1}) \tag{8.3}$$

Note that the right hand side here cannot vanish for any value of Φ_{r-1} since $z_r(x)$ can never solve (8.1), the solution to which defines $\tilde{z}_r(x)$

If we now consider turning on generic FI terms for the $U(1)$ gauge groups, the D-term constraints will require that, say, the $Q_{i,i+1}$'s acquire vevs while the $Q_{i+1,i}$'s get fixed at zero. The F-terms for the $Q_{i+1,i}$'s will then in turn require

$$\Phi_i = \Phi_j$$

for all i, j. When the brane superpotentials for the first $r-1$ nodes are of the form

$$\mathcal{W}_i(\Phi_i) = \epsilon_i m\, \Phi_i^2, \qquad i = 1, \dots r-1.$$

where $\epsilon_i = 0 \pm 1$, the left hand side of (8.3) vanishes, while the right hand side is strictly non-zero. It is exponentially small, as long as the last node was isolated

$$\mathcal{W}_r(\Phi_r) = m(\Phi_r - a)^2$$

before the transition. This generically triggers low-scale susy breaking.

In terms of the classic models discussed in this paper, one can immediately see that the susy breaking in the Fayet model and in the master geometry can be explained by the above analysis. In the case of the Polonyi and O'Raifeartaigh models, it is even simpler, since the left hand side of (8.3) vanishes *identically* for those models. One could conduct a similar analysis for configurations with more complicated superpotentials and non-generic F-terms on a case-by-case basis. What we see is that often the susy-breaking effects caused by the geometric transition can be understood at an exact level.

9. SUSY breaking by the rank condition

Here, we exhibit models which break supersymmetry due to the "rank condition." This class of models is very similar to those arising in studies of metastable vacua in SUSY QCD [27]. However, we work directly with the analogue of the magnetic dual variables, and the small scale of SUSY breaking is guaranteed by retrofitting [2].

Consider the A_3 fibration with

$$z_1(x) = mx, \qquad z_2(x) = -mx, \qquad z_3(x) = -mx, \qquad z_4(x) = -m(x-2a) \ . \tag{9.1}$$

Then the defining equation is

$$uv = (z-mx)(z+mx)(z+mx)(z+m(x-2a)) \ . \tag{9.2}$$

We choose to wrap $N_f - N_c$ D5 branes on S^2_1, N_f D5 branes on S^2_2, and a single D5 on S^2_3. The tree level superpotential is

$$\mathcal{W} = \sum_{i=1}^{3} \mathcal{W}_i(\Phi_i) + \sum_{i=1}^{2} (Q_{i,i+1}\Phi_{i+1}Q_{i+1,i} - Q_{i+1,i}\Phi_i Q_{i,i+1}). \tag{9.3}$$

where

$$\mathcal{W}_1(\Phi_1) = m\mathrm{Tr}(\Phi_1)^2, \qquad \mathcal{W}_2(\Phi_2) = 0, \qquad \mathcal{W}_3(\Phi_3) = -m(\Phi_3 - a)^2$$

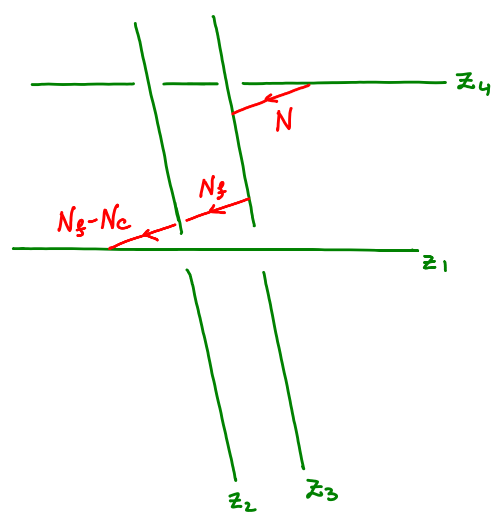

Fig. 3. The (magnetic) A_3 geometry that retrofits the ISS model.

Now, we replace the third ($U(1)$) node with an S^3 with flux, and integrate out Φ_1 trivially (we can take the mass to be very large). The result is:

$$\mathcal{W} = S(\log{(S/Delta^3)} - 1) + \frac{t}{g_s}S - \frac{1}{2}S \ \mathrm{Tr}\log(a - \Phi_2)/\Delta - Q_{12}\Phi_2 Q_{21} \ + \ldots \tag{9.4}$$

where the omitted terms are suppressed by additional powers of S. Integrating out S in a Taylor expansion about $\Phi_2 = 0$, produces a theory with superpotential

$$\mathcal{W} = S_* \mathrm{Tr}\Phi_2/a - \mathrm{Tr}Q_{12}\Phi_2 Q_{21} \ + \dots \tag{9.5}$$

where

$$S_* = \Delta^3 exp(-\tilde{t}/g_s) \ , \tag{9.6}$$

and $\tilde{t} = t - N_f \frac{1}{2} g_s \log(a/\Delta)$. Computing F_{Φ_2}, we see that the contribution from the first term in (9.5) has rank N_f, while the contribution from the second term has maximal rank $N_f - N_c < N_f$. The two cannot cancel and SUSY is broken. However, due to the small coefficient of the $\mathrm{Tr}\Phi_2$ term, the breaking occurs at an exponentially small scale.

This model is very similar to the theories analyzed in [27] (for $N_c+1 \leq N_f < \frac{3}{2}N_c$) and in section 4 of [28]. One difference is that the origin of the small parameter is dynamically explained. The discussion of corrections due to gauging of the $U(N_f)$ factor (which is a global group in [27]) is identical to that in [28] up to a change of notation, and we will not repeat it here. For large a, the higher order corrections to (9.5) (which are suppressed by powers of Φ_2/a) should not destabilize the vacuum at the origin, described in [27,28].

We could also replace the $U(1)$ at node 3 with a $U(N)$ gauge group, still in the same geometry. Then, in (9.4), the coefficient of the $S\log S$ term is changed to N. The only effect, after a geometric transition at node 3, is the replacement of replacement $e^{-t/g_s} \to e^{-t/g_s N}$ in (9.6). This model, where the node upon which we perform the geometric transition has non-Abelian gauge dynamics, is a literal example of the retrofitting constructions of [2]. The Φ_2 field appears in the gauge coupling function of the $U(N)$ gauge group at node 3, because it controls the masses of the quarks Q_{23} and Q_{32} which are charged under $U(N)$. At energies below the quark mass, the $U(N)$ is a pure $\mathcal{N} = 1$ gauge theory and produces a gaugino condensation contribution Λ_N^3 in the superpotential. The standard result for matching the dynamical scale of the low-energy pure $U(N)$ theory to the scale Λ_{N,N_f} of the higher energy theory with N_f quark flavors with mass matrix $\tilde{m}$ is[4]

$$\Lambda_N^{3N} = \Lambda_{N,N_f}^{3N-N_f} \det\tilde{m} \ . \tag{9.7}$$

[4] Here, we are assuming the adjoints are very massive $m \to \infty$ and are just matching the QCD theories with quark flavors.

With the identification of S with the gaugino condensate [6]

$$S \sim tr(W_\alpha^2) = \Lambda_N^3,$$

and identifying the mass matrix $\tilde{m} = a - \Phi_2$, we see that we predict

$$S^N = \Lambda_{N,N_f}^{3N-N_f} \det(a - \Phi_2) \ . \tag{9.8}$$

This is precisely what carefully integrating S out of (9.4) produces, with $\Lambda_{N,N_f}^{3N-N_f} = \Delta^{3N-N_f} e^{-t/g_s}$. So in our model with $N > 1$, the small $\mathrm{Tr}(\Phi_2)$ term in (9.5) can really be thought of as arising from the presence of Φ_2 in the gauge coupling function for the $U(N)$ factor.

10. Orientifold models

In the presence of orientifold 5-planes, we expect D1 brane instantons wrapping 2-cycles that map to themselves to contribute to the superpotential. The D1 brane instanton contributions should again be computable using a geometric transition that shrinks the S^2, and replaces it with an S^3. Geometric transitions with orientifolds have been studied for e.g. in [29,30].

After the transition we generally get 2 different contributions to the superpotential. First, the charge conservation of the D5/O5 brane that disappear after the transition, requires a flux through the S^3 equal to the the amount of brane charge:

$$\mathcal{W}_{flux} = \frac{t}{g_s} S + N_{D5/O5} \partial_S \mathcal{F}_0$$

Second, there can be additional O5 planes that survive as the fixed points of the holomorphic involution after the transition. The O5 planes, just like D5 branes generate a superpotential [31]

$$\mathcal{W}_{O5} = \int_\Sigma \Omega,$$

where the integral is over a three-chain with a boundary on the orientifold plane. The contributions to the superpotential due to O5 planes and RR flux of the orientifold planes are both computed by topological string RP^2 diagrams. The contributions of physical brane charge come from the sphere diagrams.

In this way, geometric transitions can be used to sum up the instanton generated superpotentials in orientifold models. Analogously to our discussion of the previous sections, this can be used for dynamical supersymmetry breaking. We'll discuss in detail the Fayet model below; others can be seen to follow in similar ways.

10.1. The Fayet model

Consider orientifolding the theory from section 3, by combining the worldsheet orientation reversal with an involution I of the Calabi-Yau manifold. For this to preserve the same supersymmetry as the D5 branes, the holomorphic involution I of the Calabi-Yau has to preserve the holomorphic three-form $\Omega = du/udzdx = -dv/vdzdx$.

An example of such an involution is one that takes

$$x \rightarrow -x$$

and

$$u \rightarrow v, \qquad v \rightarrow u$$

A simplest Fayet-type model built on this orientifold is an A_5 geometry that is roughly a doubling of that in section 4:

$$uv = (z - mx)^2 (z + mx)^2 (z - m(x - 2a))(z + m(x - 2a)).$$

We'll blow this up in a sequence:

$$\begin{aligned} &z_1(x) = mx, \qquad z_2(x) = -m(x - 2a), \qquad z_3(x) = mx, \\ &z_4(x) = -mx, \qquad z_5(x) = +m(x + 2a), \qquad z_6(x) = -mx, \end{aligned}$$

It can be shown that the orientifold projection ends up mapping

$$S_i^2 \rightarrow S_{6-i}^2,$$

fixing S_3^2. Consider wrapping M branes on S_i^2 for $i = 1, 2$, and their mirror images, and $2N$ branes on S_3^2. With a particular choice of orientifold projection, the gauge group on the branes is going to be

$$U(M) \times U(M) \times Sp(N)$$

Since the orientifold flips the sign of x, on the fixed node S_3^2 it converts Φ_3 to an adjoint of $Sp(N)$. (Having chosen that the orientifold sends x to minus itself, the action on the rest of the variables is fixed by asking that it preserve the same susy as the D5 branes, *and* that it be a symmetry after blowing up.) In the model at hand, the tree-level superpotential is

$$\mathcal{W} = \sum_{i=1}^{3} \mathcal{W}_i(\Phi_i) + \mathrm{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \mathrm{Tr}(Q_{23}\Phi_3 Q_{32} - Q_{32}\Phi_2 Q_{23}).$$

where

$$\mathcal{W}_1(\Phi_1) = m\mathrm{Tr}(\Phi_1 - a)^2, \qquad \mathcal{W}_2(\Phi_2) = -m\mathrm{Tr}(\Phi_2 - a)^2, \qquad \mathcal{W}_3(\Phi_3) = m\mathrm{Tr}\Phi_3^2.$$

Note that, even though the $\mathbb{P}^1$ is fixed by the orientifold action, it is not fixed point-wise. This means there is no $O5^+$ plane charge on it. Instead, there are two $non-compact$ orientifold 5-planes. This model is T-dual [32] to the O6-plane models of [15].

After the geometric transition that shrinks node three and replaces it with an S^3

$$S_3^2 \rightarrow S^3$$

the geometry becomes:

$$uv = (z - mx)(z + mx)(z - m(x - 2a))^2(z + m(x - 2a))^2((z - mx)(z + mx) - s).$$

where

$$\int_{S^3} \Omega = S$$

with $S = s/m$. Since the orientation reversal acted freely on the S_3^2, there are only N units of D5 flux through the S^3

$$\int_{S^3} H^{RR} = N$$

which gives a superpotential

$$\mathcal{W}_{flux} = \frac{t}{2g_s}\, S + NS(\log\frac{S}{\Delta^3} - 1)$$

the overall factor of $1/2$ comes from the fact that both the charge on the S^2 and its size has been cut in half by the orientifolding. Above, $t = \int_{S_3^2} k + iB^{RR}$ is the combination of Kahler moduli that survives the orientifold projection. In addition, the two non-compact $O5^+$ planes get pushed through the transition. Because the space still needs 2 blowups to be smooth, to give a precise description of the O5 planes would require using a geometry covered with 4 patches. At the end of the day, effectively, the O5 planes correspond to non-compact curves over the two points on the Riemann surface

$$(z - \tilde{z}_3(x))(z - \tilde{z}_4(x)) = ((z - mx)(z + mx) - s) = 0.$$

located at $x = 0$, and the corresponding values of z, $z_\pm(0)$. They generate a superpotential

$$\mathcal{W}_{O5^+} = \int^{z_-(0)} (\tilde{z}_3 - \tilde{z}_4)dx + \int^{z_+(0)} (\tilde{z}_3 - \tilde{z}_4)dx.$$

One can show that the contribution of the O5 planes is

$$\mathcal{W}_{O5^+} = +S(\log\frac{S}{\Delta^3} - 1)$$

The fact that the RP^2 contribution is proportional to that of the sphere is not an accident. It has been shown generally that the contribution of the O5 planes in these classes of models is $\pm\partial_S \mathcal{F}_{S^2}$ [33,30]. This means that the O5 planes and the fluxes add up to

$$N+1$$

units of an "effective" flux on the S^3.

After the transition, the branes on node three have disappeared and with them Φ_3 and Q_{23}, Q_{32}. In addition, the deformation of the geometry induces a deformation of the superpotential for node 2:

$$\tilde{\mathcal{W}}_2(x) = \int (z_2(x) - \tilde{z}_3(x))dx$$

where one picks for $\tilde{z}_3$ the root that asymptotes to $+mx$. This is

$$\tilde{\mathcal{W}}_2(x) = \int (-m(x-2a) - \sqrt{(mx)^2 + s})dx,$$

which, when expanded near the vacuum at $x = a$, gives

$$\tilde{\mathcal{W}}_2(\Phi_2) = -\mathrm{Tr}\, m(\Phi_2 - a)^2 - \tfrac{1}{2} S\, \mathrm{Tr} \log(\Phi_2/\Delta) + \ldots$$

The effective superpotential that sums up the instantons is thus

$$\mathcal{W}_{eff} = \mathcal{W}_1(\Phi_1) + \tilde{W}_2(\Phi_2, S) + \mathrm{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \mathcal{W}_{flux} + \mathcal{W}_{O5}$$

Up to an overall shift of both $\Phi_{1,2}$ by a, this is the same model as in section 3.

We expect a transition here even when $N = 0$, and there are no D5 branes on the S^2. The transition for $Sp(0)$ is analogous to the transition that occurs for a single D-brane on the S^2, and a $U(1)$ gauge theory. In both cases, the smooth joining of the S^2 and the S^3 phases is due to instantons that correct the geometry. In the orientifold case at hand, it is important to note that, while there is no flux through the S^3, the D3 brane wrapping it is absent: the orientifold projection projects out [34] the $\mathcal{N} = 1$ $U(1)$ vector multiplet associated with the S^3, and with it the D3 brane charged under it.

Picking the other orientifold projection, the $Sp(N)$ gauge group gets replaced with an $SO(2N)$ with Φ_3 becoming the corresponding adjoint. In this case, much of the story remains the same, except that the RP^2 contribution becomes

$$\mathcal{W}_{O5^-} = -S(\log \frac{S}{\Delta^3} - 1).$$

This means that the O5$^-$ planes and the fluxes add up to

$$N - 1$$

units of an "effective" flux on the S^3. This is negative or zero for $N \leq 1$. Naively, the negative effective flux breaks supersymmetry after the transition. This is clearly impossible. It has been argued in [30] that the correct interpretation of this is that in fact $SO(2)$, $SO(1)$ and $SO(0)$ cases do not undergo the geometric transition. This has to correspond to the statement that in these cases there are no D1 brane instantons on node three, and that the classical picture is *exact* in these cases. This translates in the statement that in these cases, in

$$\mathcal{W}_{eff} = \mathcal{W}_{eff}|_{S=0}$$

S should not be extremised, but rather set to zero identically in the effective superpotential.

Note that with the SO projection on the space-filling branes, a D-instanton wrapping the same node enjoys an Sp projection. As discussed in [35,15], in this situation direct zero-mode counting also suggests that the instanton should *not* correct the superpotential. There are more than two fermion zero modes coming from the Ramond sector of strings stretching from the instanton to itself. This is in accord with the results of [30]. In contrast, when one has an Sp projection on the space-filling branes, the instanton receives an SO projection, and the instanton with $SO(1)$ worldvolume gauge group has the correct zero mode count to contribute. The presence of the instanton effects when one has this projection (and their absence when one does not), was also confirmed by direct studies of the renormalization group cascade ending in the appropriate geometry in [15].

Acknowledgements

We would like to thank B. Florea, B. Freivogel, T. Grimm, J. McGreevy, D. Poland, N. Saulina, E. Silverstein and C. Vafa for helpful discussions. The research of M.A. and C.B. was supported in part by the UC Berkeley Center for Theoretical Physics. The research of M.A. is also supported by a DOE OJI Award, the Alfred P. Sloan Fellowship, and NSF grant PHY-0457317. S.K. was supported in part by the Stanford Institute for Theoretical Physics, and by NSF grant PHY-0244728 and DOE contract DE-AC003-76SF00515. S.K. acknowledges the kind hospitality of the MIT Center for Theoretical Physics during the completion of this work.

Appendix A. Brane superpotentials

We can compute the superpotential $\mathcal{W}(\Phi)$ as function of the wrapped 2-cycles Σ by using the superpotential [23,24]

$$\mathcal{W} = \int_C \Omega$$

where C is a three-chain with one boundary being Σ and the other being a reference two-cycle Σ_0 in the same homology class. It is easy to show [23] that the critical points of the superpotential are holomorphic curves. We will evaluate it for the geometries at hand. We can write the holomorphic three-form of the non-compact Calabi-Yau in the usual way,

$$\Omega = \frac{dv \wedge dz \wedge dx}{\frac{dF}{du}} = \frac{dv}{v} \wedge dz \wedge dx. \tag{A.1}$$

Now for fixed values of x and z, the equation for the CY threefold becomes $uv = const$, which is the equation for a cylinder. By shifting the definition of u or v by a phase, we can insist that the constant is purely real, and then by writing $u = x + iy$, $v = x - iy$, the equation can be reformulated as two real equations in terms of the real (x_R, y_R) and imaginary (x_I, y_I) parts of x and y.

$$x_R^2 + y_R^2 = C + x_I^2 + y_I^2, \qquad x_R x_I = y_R y_I. \tag{A.2}$$

The first of these can be solved for any given values of x_I and y_I to give an S^1. The second equation restricts the possible values which we choose for x_I and y_I to a one-dimensional curve in the (x_I, y_I) plane, and so we have the topology of $S^1 \times \mathrm{R}$, where the size of the S^1 degenerates at the points where $z = z_i(x)$ for any i. By simultaneously shifting the phases of u and v according to

$$u \to e^{i\theta} u$$
$$v \to e^{-i\theta} v$$

the equation for the cylinder remains unchanged, and we simply rotate about the S^1 factor. We can thus integrate Ω around the circle and obtain

$$\int_{S^1} \Omega = dz \wedge dx$$

up to an overall constant. Now the $\mathbb{P}^1$'s on which we are wrapping the D5 branes are the product of the S^1 just discussed and an interval in the z direction between values where the S^1 fiber degenerates. Thus, for a given $\mathbb{P}^1$ class in which the vanishing S^1 occurs for $z_i(x)$ and $z_j(x)$, we can integrate $dz \wedge dx$ over the interval in the z-plane and obtain

$$\int_{S^1 \times I_{ij}} \Omega = (z_i(x) - z_j(x))dx.$$

The superpotential for the D-branes then becomes a superpotential for the location of the branes on the t-plane. Defining an arbitrary reference point t_*, we then have

$$\mathcal{W}(x) = \int_{t_*}^{t} (z_i(x) - z_j(x))dx \tag{A.3}$$

Of course, the contribution to the superpotential coming from the limit of integration at t_* is just an arbitrary constant and is not physically relevant. Thus we write (A.3) instead as the indefinite integral

$$\mathcal{W}(x) = \int (z_i(x) - z_j(x))dx. \tag{A.4}$$

Appendix B. Multi-instanton contributions

In this appendix we demonstrate the computation of multi-instanton corrections to the superpotential using the Polonyi model of section 5 as an example. All the information about these corrections is contained in the deformed superpotential for Φ,

$$\tilde{\mathcal{W}}(x) = \int \left(m(x-a) - \sqrt{m^2(x-a)^2 + mS} \right) dx \tag{B.1}$$

along with the flux superpotential[5]

$$\mathcal{W}_{flux} = \frac{t}{g_s} S + S\left(\log \frac{S}{\Delta^3} - 1 \right). \tag{B.2}$$

where the scale Δ is determined by the one-loop contributions to the matrix model free energy. The models considered in this paper are particularly convenient since the purely quadratic superpotential for the massive adjoint at the transition node guarantees that the flux superpotential will be exact at one-loop order in the associated matrix model [17].

[5] In the case of the Polonyi model these two terms constitute the entire superpotential. In the more general case, however, there will be more fields with superpotential terms, but it will remain the case that only these two contributions play a role in determining instanton corrections.

Extremizing the flux superpotential and expanding in powers of the instanton action

$$S_{inst} \sim \exp(-t/Ng_s),$$

we can determine multi-instanton contributions to a given superpotential term. Summing up the series contributing to a given Φ^k term in will correspond to computing corrections to a fixed, explicit disc diagram, and so we might expect these series to exhibit some integrality properties.

We first expand the deformed superpotential $\mathcal{W}_1(\Phi)$ as a power series in the glueball superfield S,

$$\tilde{\mathcal{W}}(\Phi) = \int \left(m(x-a) - m(x-a)(1 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n(2n-2)!}{2^{2n-1}(n!)^2} y^n) \right) dx \tag{B.3}$$

where the expansion parameter y can also be expanded as a power series in x,

$$y = \frac{S}{m(x-a)^2} = \frac{S}{ma^2} \left(1 + \sum_{n=1}^{\infty} (n+1)(-1)^n \left(\frac{x}{a}\right)^n \right). \tag{B.4}$$

We can integrate (B.3) term by term to obtain an expansion of the effective superpotential in powers of Φ. However, it will be useful to represent this schematically

$$\mathcal{W}_1(\Phi) = c_1 \mathrm{Tr}\Phi + c_2 \mathrm{Tr}\Phi^2 + \ldots \qquad\qquad c_i = \sum_{n=1}^{\infty} c_i^{(n)} S^n$$

where the coefficients c_i are themselves written as power series in S. Extremizing the superpotential with respect to S gives an equation for the values of S

$$\log \frac{S}{\Delta^3} = -\frac{t}{g_s} - \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} n\ c_i^{(n)} S^{n-1} \mathrm{Tr}\Phi^i \tag{B.5}$$

which can be solved perturbatively in powers of S_{inst}. Re-inserting the resulting values into the original superpotential then allows us to read off the instanton-corrected superpotential of the low energy theory up to any given number of instantons. Below we display the linear and quadratic terms at the three-instanton level.

$$\mathcal{W}_{eff} = \mu \mathrm{Tr}\Phi + m \mathrm{Tr}\Phi^2$$

where

$$\begin{aligned} \mu &= \frac{1}{2}\frac{\Delta^3}{a} e^{-\frac{t}{g_s}} - \frac{1}{8}\frac{\Delta^6}{ma^3} e^{-\frac{2t}{g_s}} + \frac{1}{16}\frac{\Delta^9}{m^2 a^5} e^{-\frac{3t}{g_s}} + \ldots \\ m &= -\frac{7}{8}\frac{\Delta^3}{a^2} e^{-\frac{t}{g_s}} + \frac{11}{16}\frac{\Delta^6}{ma^4} e^{-\frac{2t}{g_s}} + \frac{1}{32}\frac{\Delta^9}{m^2 a^6} e^{-\frac{3t}{g_s}} + \ldots \end{aligned} \tag{B.6}$$

It may be interesting to see if there is some way to relate these to the exact formulae for multicovers derived in the resolution of the singularity in hypermultiplet moduli space when a 2-cycle shrinks in IIB string theory, given (up to mirror symmetry) in [36].

References

[1] O. Aharony, S. Kachru and E. Silverstein, "Simple Stringy Dynamical SUSY Breaking," arXiv:0708.0493 [hep-th].

[2] M. Dine, J. Feng and E. Silverstein, "Retrofitting O'Raifeartaigh models with dynamical scales," arXiv:hep-th/0608159.

[3] R. Blumenhagen, M. Cvetic and T. Weigand, "Spacetime instanton corrections in 4D string vacua: The seesaw mechanism for D-brane models," Nucl. Phys. **B771** (2007) 113, arXiv:hep-th/0609191; M. Haack, D. Krefl, D. Lust, A. Van Proeyen and M. Zagermann, "Gaugino condensates and D-terms for D7-branes," JHEP **0701** (2007) 078, arXiv:hep-th/0609211; L. Ibanez and A. Uranga, "Neutrino majorana masses from string theory instanton effects," JHEP **0703** (2007) 052, arXiv:hep-th/0609213; B. Florea, S. Kachru, J. McGreevy and N. Saulina, "Stringy instantons and quiver gauge theories," JHEP **0705** (2007) 024, arXiv:hep-th/0610003.

[4] J. Kumar, "Dynamical SUSY breaking in intersecting brane models," arXiv:0708.4116.

[5] For excellent reviews of dynamical SUSY breaking in 4d quantum field theory, see: I. Affleck, M. Dine and N. Seiberg, "Dynamical supersymmetry breaking in four-dimensions and its phenomenological implications," Nucl. Phys. **B256** (1985) 557; E. Poppitz and S.P. Trivedi, "Dynamical supersymmetry breaking," Ann. Rev. Nucl. Part. Sci. **48** (1998) 307, arXiv:hep-th/9803107; Y. Shadmi and Y. Shirman, "Dynamical supersymmetry breaking," Rev. Mod. Phys. **72** (2000) 25, arXiv:hep-th/9907225; K. Intriligator and N. Seiberg, "Lectures on supersymmetry breaking," arXiv:hep-th/0702069.

[6] C. Vafa, "Superstrings and topological strings at large N," arXiv:hep-th/0008142.

[7] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities," JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[8] J. M. Maldacena and C. Nunez, "Towards the large N limit of pure N = 1 super Yang Mills," Phys. Rev. Lett. **86**, 588 (2001) [arXiv:hep-th/0008001].

[9] S. Gukov, C. Vafa and E. Witten, "CFTs from Calabi-Yau Fourfolds," arXiv:hep-th/9906070.

[10] M. Aganagic, C. Beem, J. Seo and C. Vafa, "Geometrically Induced Metastability and Holography," [hep-th/0610249].

[11] J. J. Heckman, J. Seo and C. Vafa, "Phase Structure of a Brane/Anti-Brane System at Large N," JHEP **0707**, 073 (2007) [arXiv:hep-th/0702077].

[12] J. J. Heckman and C. Vafa, "Geometrically Induced Phase Transitions at Large N," arXiv:0707.4011 [hep-th].

[13] M. Aganagic, C. Beem and B. Freivogel, "Geometric Metastability, Quivers and Holography," arXiv:0607.0596 [hep-th].

[14] M. R. Douglas, J. Shelton and G. Torroba, "Warping and supersymmetry breaking," arXiv:0704.4001 [hep-th].

[15] O. Aharony and S. Kachru, "Stringy Instantons and Cascading Quivers," arXiv:0707.3126 [hep-th].

[16] R. Argurio, M. Bertolini, S. Franco and S. Kachru, "Metastable vacua and D-branes at the conifold," arXiv:hep-th/0703236.

[17] R. Dijkgraaf and C. Vafa, "A perturbative window into non-perturbative physics," arXiv:hep-th/0208048.

[18] M. Aganagic and C. Vafa, "Perturbative derivation of mirror symmetry," arXiv:hep-th/0209138.

[19] M. Atiyah, J. M. Maldacena and C. Vafa, "An M-theory flop as a large N duality," J. Math. Phys. **42**, 3209 (2001) [arXiv:hep-th/0011256].

[20] M. Aganagic and C. Vafa, "Mirror symmetry and a G(2) flop," JHEP **0305**, 061 (2003) [arXiv:hep-th/0105225].

[21] M. Atiyah and E. Witten, "M-theory dynamics on a manifold of G(2) holonomy," Adv. Theor. Math. Phys. **6**, 1 (2003) [arXiv:hep-th/0107177].

[22] E. Witten, "Phases of N = 2 theories in two dimensions," Nucl. Phys. B **403**, 159 (1993) [arXiv:hep-th/9301042].

[23] E. Witten, "Branes and the Dynamics of QCD," [hep-th/9706109].

[24] M. Aganagic and C. Vafa, "Mirror Symmetry, D-branes and Counting Holomorphic Discs," [hep-th/0012041].

[25] F. Cachazo, S. Katz and C. Vafa, "Geometric transitions and N = 1 quiver theories," arXiv:hep-th/0108120.

[26] N. Arkani-Hamed, M. Dine and S. P. Martin, "Dynamical supersymmetry breaking in models with a Green-Schwarz mechanism," Phys. Lett. B **431**, 329 (1998) [arXiv:hep-ph/9803432].

[27] K. Intriligator, N. Seiberg and D. Shih, "Dynamical SUSY breaking in meta-stable vacua," arXiv:hep-th/0602239.

[28] A. Giveon and D. Kutasov, "Gauge symmetry and supersymmetry breaking from intersecting branes," Nucl. Phys. **B778** (2007) 129 [arXiv:hep-th/0703135].

[29] S. Sinha and C. Vafa, "SO and Sp Chern-Simons at large N," arXiv:hep-th/0012136.

[30] K. Intriligator, P. Kraus, A. V. Ryzhov, M. Shigemori and C. Vafa, "On low rank classical groups in string theory, gauge theory and matrix models," Nucl. Phys. B **682**, 45 (2004) [arXiv:hep-th/0311181].

[31] B. Acharya, M. Aganagic, K. Hori and C. Vafa, "Orientifolds, mirror symmetry and superpotentials," arXiv:hep-th/0202208.

[32] K. Landsteiner, C. I. Lazaroiu and R. Tatar, "(Anti)symmetric matter and superpotentials from IIB orientifolds," JHEP **0311**, 044 (2003) [arXiv:hep-th/0306236].

[33] H. Ita, H. Nieder and Y. Oz, "Perturbative computation of glueball superpotentials for SO(N) and USp(N)," JHEP **0301**, 018 (2003) [arXiv:hep-th/0211261].

[34] I. Brunner and K. Hori, "Orientifolds and mirror symmetry," JHEP **0411**, 005 (2004) [arXiv:hep-th/0303135].

[35] M. Bianchi and E. Kiritsis, "Non-perturbative and flux superpotentials for Type I strings on the Z_3 orbifold, arXiv:hep-th/0702015; R. Argurio, M. Bertolini, G. Ferretti, A. Lerda and C. Petersson, "Stringy instantons at orbifold singularities," arXiv:0704.0262 [hep-th]; M. Bianchi, F. Fucio and J.F. Morales, "D-brane instantons on the T^6/Z_3 orientifold," arXiv:0704.0784 [hep-th]; L. Ibanez, A. Schellekens and A. Uranga, "Instanton induced neutrino majorana masses in CFT orientifolds with MSSM-like spectra," arXiv:0704.1079 [hep-th]; R. Blumenhagen and M. Cvetic, "Lifting D-instanton zero modes by recombination and background fluxes," arXiv:0708.0403 [hep-th].

[36] H. Ooguri and C. Vafa, "Summing up D-instantons," arXiv:hep-th/9608079; N. Seiberg and S. Shenker, "Hypermultiplet moduli space and string compactifications to three-dimensions," arXiv:hep-th/9608086.

SLAC-PUB-12698
SU-ITP-07/11
WIS/12/07-AUG-DPP

Simple Stringy Dynamical SUSY Breaking

Ofer Aharony[1,2], Shamit Kachru[2] and Eva Silverstein[2]

[1]Department of Particle Physics
Weizmann Institute of Science
Rehovot 76100, Israel

[2] Department of Physics and SLAC
Stanford University
Stanford, CA 94305, USA

We present simple string models which dynamically break supersymmetry without non-Abelian gauge dynamics. The Fayet model, the Polonyi model, and the O'Raifeartaigh model each arise from D-branes at a specific type of singularity. D-brane instanton effects generate the requisite exponentially small scale of supersymmetry breaking.

August 2007

1. Introduction

Dynamical supersymmetry breaking (DSB) is a promising candidate solution to the hierarchy problem [1]. Many field theories which dynamically break supersymmetry have been discovered (see [2,3,4,5] for reviews). In each of these examples, non-Abelian gauge dynamics plays a crucial role. In general, the constructions are rather complicated, though they have become simpler over the years [5].

One way to dynamically break supersymmetry (SUSY) in string theory is to embed a non-Abelian gauge theory which dynamically breaks supersymmetry into the low-energy spectrum. Here, we propose an alternative. We find simple D-brane theories which dynamically break supersymmetry after including D-brane instanton effects. The low-energy theories are Fayet, Polonyi or O'Raifeartaigh models. The terms in the superpotential which are responsible for supersymmetry breaking arise due to stringy D-instanton generated perturbations, which have recently been investigated in [6,7,8,9] and many subsequent papers.[1] Non-Abelian gauge dynamics plays no role, and the SUSY breaking "hidden sectors" are extremely modest in size, including a single Abelian gauge field with two charged chiral multiplets or even more minimal field content. One can view our results as indicating that stringy instantons make retrofitting of simple supersymmetry-breaking models [13] a natural feature of D-brane constructions. Because of the importance of the stringy instanton effect, in these models the brane construction plays a more fundamental role than just serving as a way to embed a known low-energy field theory mechanism into string theory.[2]

In §2, we describe the simplest models we have found. All of these models can arise from D-branes at a specific singularity, which can be chosen to be an orientifold of an orbifold of the conifold. In §3, we briefly discuss the prospects for making fully realistic models using our SUSY breaking hidden sectors. The construction of complete models utilizing our SUSY breaking models as hidden sectors is left for future work.

2. Some Simple Models

In this section we present simple D-brane theories where stringy instanton effects yield vacua with exponentially small SUSY breaking scale. The low-energy theories are Fayet, Polonyi and O'Raifeartaigh models.

[1] Early work on similar instanton effects appears in [10,11,12].

[2] Some other papers which study stringy mechanisms to break supersymmetry using systems of branes, anti-branes and fluxes are [14-22].

2.1. The Fayet Model

The Fayet model consists of a $U(1)$ gauge field coupled with strength e to charged chiral multiplets $\Phi_\pm$ with equal and opposite charges and canonical Kähler potential. The superpotential is

$$W = m\Phi_+\Phi_- \ , \tag{2.1}$$

so the F-term equations for supersymmetric vacua require the scalar components to satisfy $\phi_\pm = 0$. The D-term constraints require supersymmetric vacua to satisfy

$$|\phi_+|^2 - |\phi_-|^2 = r, \tag{2.2}$$

where r is the Fayet-Iliopoulos (FI) D-term for the Abelian gauge symmetry.

For generic values of the FI term $r \neq 0$, the F-term equations and (2.2) cannot be simultaneously satisfied. The energy grows without bound at infinity in field space, so this model has a stable ground state which spontaneously breaks supersymmetry. Specifically, for $r \gg m^2/(2e^2)$, the minimum of the scalar potential is at $|\phi_+|^2 = r - m^2/(2e^2) \simeq r$, $\phi_- = 0$, and the breaking of supersymmetry is dominated by the F-term

$$F_{\Phi_-} \simeq m\sqrt{r}. \tag{2.3}$$

We will now exhibit a simple brane realization of this model, with an exponentially small supersymmetry breaking scale obtained by generating m from a stringy instanton effect.

The basic idea is as follows. We can realize the theory described above as a quiver gauge theory, arising at low energies on D-branes probing a non-compact singular Calabi-Yau space in type IIB string theory (or, equivalently, from D-branes stretched between NS-branes in type IIA string theory). The relevant quiver for us is quite simple and could potentially arise from many singularities; it appears below in Figure 1. It has two $U(r)$ nodes of rank $r = 1$ and one $USp(r)$ node of rank $r = 0$. In the geometrical language, the space locally contains two 2-cycles on which space-filling 5-branes (often called "fractional branes") are wrapped, and another 2-cycle $\mathcal{C}$ which is not wrapped by a 5-brane. There are two chiral multiplets arising from open strings between the 5-branes, with charges $(\pm1, \mp1)$ under the $U(1) \times U(1)$ gauge group. The superpotential is zero perturbatively. A Euclidean D1-brane wrapped on $\mathcal{C}$ contributes an instanton effect with precisely the right zero-mode structure to generate the superpotential (2.1); this cannot be interpreted as an ordinary field-theoretic instanton, since there is no field theory associated with this

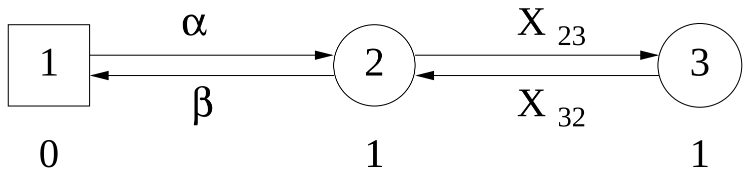

Figure 1: The quiver diagram that leads to the Fayet model. The first, square, node corresponds to a $USp(r_1)$ group, while the circular nodes correspond to $U(r_i)$ groups. For our application we need to have $r_2 = r_3 = 1$, and $r_1 = 0$ (this is the node wrapped by the D-instanton); the bifundamentals connecting node 1 and node 2 are then Ganor strings.

cycle, and no non-Abelian gauge dynamics is required for the effect. m and r are fixed parameters at the level of the non-compact system since they arise from non-normalizable modes.[3]

Concretely, we can obtain the simple subquiver in Figure 1, as well as a generalization relevant for gauge mediation to be discussed in §3, starting from the singular geometries

$$(xy)^n = zw \ . \tag{2.4}$$

These are $\mathbb{Z}_n$ orbifolds of the conifold, studied in [23].[4] The quivers describing the effective gauge theories living on D3 and D5-branes at these singularities have $2n$ $U(r_i)$ nodes with bifundamentals $X_{i,i+1}, X_{i+1,i}$ going each way between adjacent nodes, as in the left-hand side of Figure 2, and with a superpotential

$$W = h \sum_{i=1}^{2n} (-1)^i \ X_{i,i+1} X_{i+1,i+2} X_{i+2,i+1} X_{i+1,i} \ . \tag{2.5}$$

Specific orientifolds of this theory which lead to interesting stringy instanton effects were described in [24,25]. In the case where the quiver nodes are occupied by space-filling wrapped branes, these modify the field content such that nodes 1 and $n+1$ correspond to symplectic gauge groups instead of unitary groups, while the remaining $U(r_i)$ nodes are pairwise identified by the obvious reflection symmetry. The identification of node 1 with itself by the orientifold is important because it reduces the number of fermion zero modes on the Euclidean D1-brane wrapping the corresponding cycle $\mathcal{C}$ to the two that are

[3] In a compact model with finite four dimensional Planck scale, these modes become dynamical. Then, as with all proposals for dynamical supersymmetry breaking in string theory, one must stabilize the closed string moduli which control the scales of the gauge theory.

[4] The quivers we use can probably be obtained from many other singularities as well.

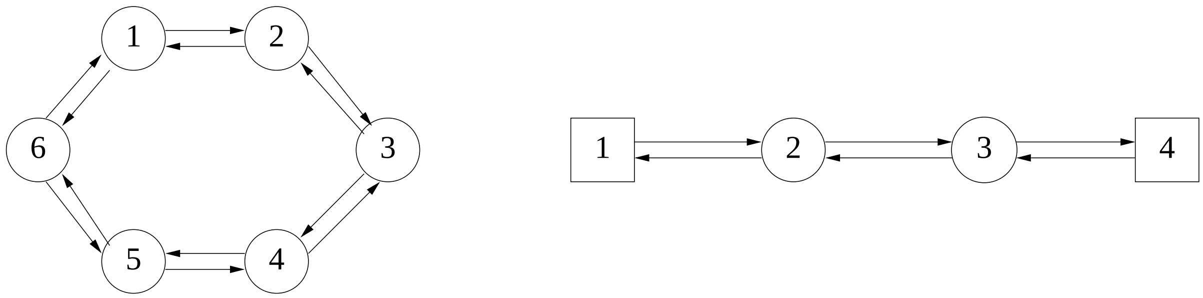

Figure 2: The quiver gauge theories of the orbifolded conifold and of its orientifold for $n = 3$. The circular nodes have $U(r_i)$ gauge groups, and the square nodes have $USp(r_i)$ groups. More generally there are $2n$ nodes before orientifolding and $n + 1$ nodes after orientifolding.

required for a contribution to the space-time superpotential. The T-dual type IIA string description of the branes at this orientifolded orbifolded conifold is shown in Figure 3.

The model we are interested in arises when we have $n \geq 3$, and we have single (space-filling) branes on nodes 2 and 3 ($r_2 = r_3 = 1$), and vanishing occupation numbers elsewhere. The tree-level superpotential (2.5) vanishes in this case. The D-instanton wrapping node 1 has bifundamental fermionic "Ganor strings" α and β stretching to node 2 [12,24] (see Figure 1). These modes have a coupling analogous to (2.5) to the fields X_{23}, X_{32}; performing the path integral over α and β then generates a superpotential [24,26]

$$W = \Lambda_1 X_{23} X_{32} \ , \tag{2.6}$$

where Λ_1 is the instanton action controlled by the size of node 1 in the geometry, and it can naturally be exponentially small.[5]

The sum of the $U(1)$'s associated to nodes 2 and 3 acts trivially on all fields and decouples. The low-energy theory consists of a single $U(1)$ gauge field (the difference of the $U(1)$'s at the two nodes), with X_{23} and X_{32} carrying equal and opposite charges. This $U(1)$ does not decouple at low energies, because its renormalization group running stops below the scale of the mass of the charged fields. Thus, we obtain precisely the Fayet model, with $\Phi_+ = X_{23}$, $\Phi_- = X_{32}$, and with the parameter m of (2.1) having been dynamically generated by a D-instanton. For generic choices of the FI term r (which is a non-normalizable mode in the non-compact geometry), this model breaks supersymmetry at an exponentially low scale $F \sim \Lambda_1 \sqrt{r}$ (2.3). This can be considered a retrofitted Fayet model, in the spirit of [13]. However, no non-Abelian gauge dynamics is invoked in the

[5] For $n = 3$ there would be a similar contribution arising also from node 4.

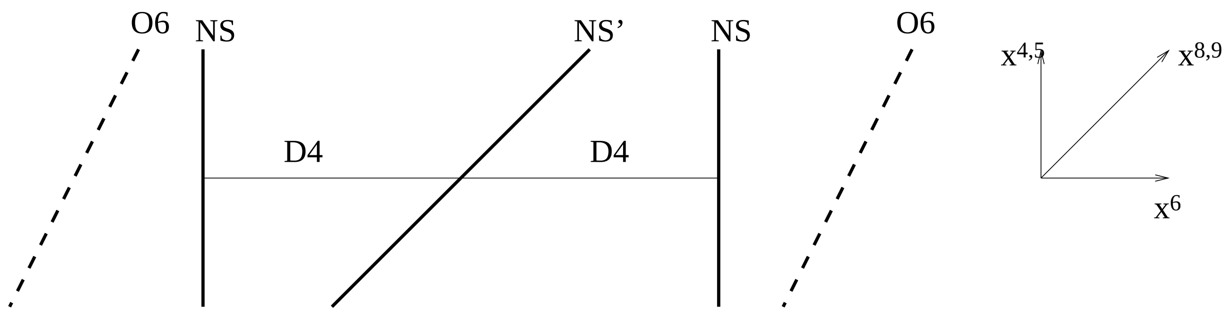

Figure 3: The T-dual type IIA brane configuration for our Fayet model when it is embedded in the $n = 3$ orientifold. NS branes stretch in the 012345 directions, NS' branes in the 012389 directions, and D4 branes stretch in the 01236 directions. The O6 planes extend along the 01237 directions, and lie at a 45 degree angle with respect to the 45 and 89 planes. The x^6 direction is compact and becomes an interval after the orientifolding.

retrofitting; it is automatically implemented by string theory. In the type IIA language of Figure 3, the FI term corresponds to the x^7 position of the NS' between the two D4-branes.

The effective action of the model described above (and of the models we will discuss below) will in general be corrected by higher-dimension operators. These generically shift the location of the vacuum slightly, and can also introduce a supersymmetric vacuum elsewhere in field space, rendering the SUSY breaking vacuum metastable.

2.2. The Polonyi model

An even simpler model of SUSY breaking is the Polonyi model. This is the theory of a single chiral superfield with superpotential

$$W = \mu^2 X \ . \tag{2.7}$$

$F_X = \mu^2$ provides the order parameter of SUSY breaking. At tree level, this model has a flat direction. The existence of a stable non-SUSY vacuum at $X = 0$ depends on the sign of the leading quartic correction to the Kähler potential

$$K = X^\dagger X + \frac{c}{M_*^2}(X^\dagger X)^2 + \cdots . \tag{2.8}$$

M_* denotes the scale of high-energy physics which has been integrated out and corrects K. For one sign of c there is a stable vacuum, and for the other the theory runs away to large values of X. In any given completion of the Polonyi model by a larger field theory or string theory, there will be some corresponding value of c.

In fact, a particularly simple completion manifesting a stable vacuum is provided by the Fayet model discussed above, which reduces to the Polonyi model in a limit. At the level of the field theory model, as r grows large, with $m\sqrt{r} \equiv \mu^2$ fixed, the $U(1)$ under which $\Phi_\pm$ are charged becomes very massive along with Φ_+. The remaining $U(1)$ is free and decouples as before. The low-energy theory therefore reduces to a free $U(1)$ theory with a singlet $X = \Phi_-$ that has mass squared $2m^2 = 2\mu^4/r$ (which goes to zero in our limit) and a linear superpotential $W = \mu^2 X$ as in (2.7). In the string construction realizing this model, we must keep r smaller than the string scale to avoid introducing new degrees of freedom; this still leaves a regime where the low energy effective theory is the Polonyi model with a locally stable minimum.

In the brane construction of Figure 3, turning on a large FI term corresponds to moving an NS' brane far away in the x^7 direction. One could also obtain the Polonyi model directly, with a dynamically generated small scale μ^2, by considering the brane configuration without this NS' brane, such that we have a single D4-brane stretched between two parallel NS5-branes (with orientifolds as in Figure 3). This corresponds to the quiver shown in Figure 4, with $r_2 = 1$ and r_1 vanishing. In the brane language, the field X arises as the translation mode of the D4 along the x^4 and x^5 directions, and the stringy instanton is in this language the Euclidean D0 brane wrapping the interval between the NS 5-brane and the O6-plane. The Ganor strings now have an action of the form $S = \alpha\beta X$, so that this instanton gives precisely a superpotential of the form (2.7).

Instead of moving away an NS' brane along x^7 as described above, one can also obtain this brane configuration from the one in Figure 3 by moving the NS' in the x^6 direction so that it trades places with an NS brane (annihilating the D4-branes ending on it in the process). Such a position-switching process actually happens during the renormalization group cascade [27] which arises for branes with large occupation numbers at the singularities described in the previous subsection. As described in [26], the cascade steps lead to adjoints as in Figure 4, with trilinear couplings of the adjoints to the adjacent bifundamentals replacing (some of) the quartic couplings of (2.5). These trilinear couplings imply that the Ganor strings have the action $S = \alpha\beta X$ as above, which upon performing their path integral leads to the superpotential (2.7). Thus, the quiver of Figure 4 can arise from D-branes at the same singularity described in the previous subsection. This raises the possibility of UV completing the SUSY breaking configuration with a cascading non-Abelian gauge theory. Then, the Polonyi model would arise as the effective low-energy description of the SUSY breaking in much the same way that an O'Raifeartaigh model captures the SUSY breaking vacua of SUSY QCD with slightly massive quark flavors [28]. Of course in the spirit of simplicity and minimality, we are free to consider the final brane configuration of interest (UV completed by string theory) without invoking the RG cascade and the consequent increased complexity of our hidden sector.

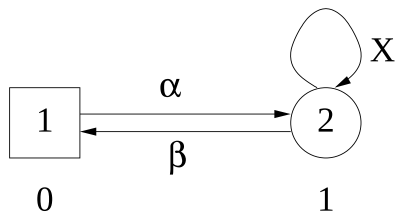

Figure 4: The two-node quiver which gives rise directly to a Polonyi model, after considering the instanton wrapping symplectic node 1. The arrow from node 2 to itself is a chiral superfield in the adjoint representation of $U(r_2)$.

An advantage of obtaining the Polonyi model from a limit of the Fayet model as above, is that (for suitable r) one is certain of the existence of the stable SUSY breaking minimum; this is not clear when we obtain the Polonyi model directly from a brane configuration. It would be interesting to compute the constant c in the latter case, to see if it leads to a stable SUSY breaking vacuum.

2.3. An O'Raifeartaigh model

We obtained the Polonyi model by removing the NS' brane between the two NS 5-branes in our type IIA brane construction of the Fayet model. Now, we can make an O'Raifeartaigh model (retrofitted by a stringy instanton) by inserting another NS-5 brane where the NS' brane originally was. There are then adjoint fields both for node 2 and for node 3, as in Figure 5.[6]

We now have a $U(1)\times U(1)$ gauge group. Let us call the two "adjoints" of $U(1)\times U(1)$ arising at nodes 3 and 2, respectively, X and $\tilde{X}$. In addition, there are bifundamentals $\Phi, \tilde{\Phi}$. The tree-level superpotential is

$$W_{tree} = \tilde{\Phi}\tilde{X}\Phi + \tilde{\Phi}X\Phi \ . \tag{2.9}$$

[6] Again, one can also obtain this configuration by performing several steps in the RG cascade of the theories described in §2.1 [26]. Thus, it corresponds to branes on the same geometrical singularity of §2.1 (with different blow-up parameters).

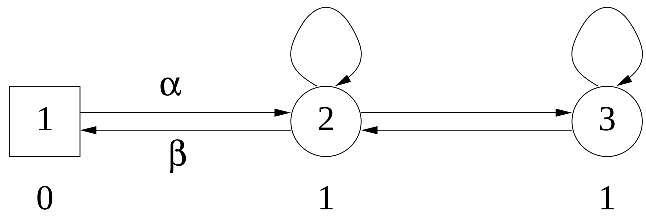

Figure 5: A quiver leading to an O'Raifeartaigh model.

A stringy instanton at node 1 generates a perturbation

$$\delta W = \mu^2 \tilde{X}, \tag{2.10}$$

as in §2.2. The resulting full superpotential is

$$W_{tot} = X\tilde{\Phi}\Phi + \tilde{X}\left(\tilde{\Phi}\Phi + \mu^2\right) \ . \tag{2.11}$$

The X and $\tilde{X}$ F-terms conspire to break supersymmetry. In absence of (2.10), one could solve the D-term constraint $|\phi|^2 - |\tilde{\phi}|^2 = r$ by setting one of $\phi, \tilde{\phi}$ to $\sqrt{r}$ and the other to zero. This would yield a supersymmetric vacuum. The presence of the stringy instanton effect (2.10) instead leads to supersymmetry breaking, with an exponentially small scale set (in the natural regime $r \gg \mu^2$) by μ.

This model has a flat direction at this level of analysis. Lifting the flat direction by "UV completing" the model with a slightly larger quiver, in analogy with what we did for the Polonyi model in §2.2, is one way to potentially stabilize the flat direction.

3. Discussion

For realistic model building, there are various options for communicating supersymmetry breaking to the Standard Model sector. If a Standard Model brane system sits far away from our SUSY-breaking system, we may obtain gravity mediation. We can also generalize the models above in a straightforward way to obtain messengers appropriate for gauge mediation, as follows.[7] Consider (for example) the extension of the brane system of §2.1 depicted in Figure 6, where we now occupy node 4 with a toy "Standard Model."

[7] For a review of gauge mediation, see [29]. For recent attempts to engineer such models using branes, see [30,31,32,33].

This introduces a second set of chiral fields $\eta, \tilde{\eta}$ charged under the new gauge group, and a superpotential of the form

$$W = \Lambda_1 \Phi_+ \Phi_- + \frac{1}{M_*} \eta \tilde{\eta} \Phi_+ \Phi_- + M \eta \tilde{\eta}, \tag{3.1}$$

where the quartic term arises from the superpotential (2.5), and we have included a possible supersymmetric mass term M for $\eta, \tilde{\eta}$. In the supersymmetry breaking vacuum with $\phi_+ \sim \sqrt{r}$ and $F_{\Phi_-} \sim \Lambda_1 \sqrt{r}$, the operator $\Phi_+ \Phi_-$ has zero VEV and an F component of order $\langle \phi_+ \rangle F_{\Phi_-}$. As a result, the superpotential (3.1) is of the form appropriate for gauge mediation with messengers $\eta, \tilde{\eta}$ of mass M, and with an effective SUSY-breaking F-term of order $\langle \phi_+ \rangle F_{\Phi_-}/M_* \sim r \Lambda_1 / M_*$. The quartic term in (3.1) leads to the existence of additional (supersymmetric) vacua far away in field space, but it does not affect the non-supersymmetric vacuum that we are interested in (which is now metastable).

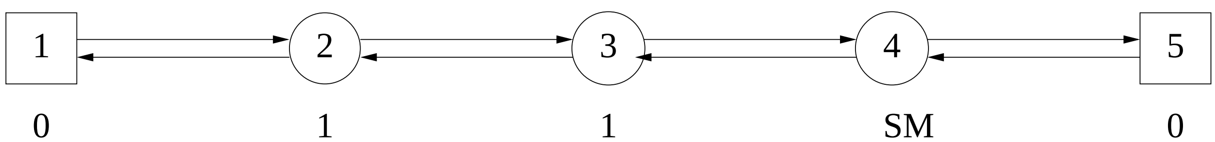

Figure 6: A quiver with a coupling to the "Standard Model" at node 4 and symplectic nodes (with stringy D-instantons) at nodes 1 and 5.

For high-scale gauge mediation, one requires a messenger mass M well below the string scale but much higher than the TeV scale.[8] One possibility for obtaining such a mass is by turning on closed string moduli (blow-up modes), and this then involves a small tune of parameters. If one prefers a dynamical mechanism to obtain M, which is particularly important for lower-scale gauge mediation, one can (as in Figure 6) make node 5 another (unoccupied) symplectic node. Then, if we put a single brane at node 4, we get a mass term for $\eta, \tilde{\eta}$ of magnitude Λ_5 from the stringy instanton at node 5. This provides a tunable messenger mass. Since node 4 must be a $U(1)$ for this to happen, we would need to consider an extension of the Standard Model by this $U(1)$ symmetry, with appropriate charges to get gauge-mediated masses from this setup.

In order to obtain a realistic model, we could investigate the possibility of replacing node 4 above with a full brane realization of the Standard Model (rather than the toy version described above), which is (classically) mutually supersymmetric with our SUSY-breaking sector. In doing so we must require that the new open strings $\eta, \tilde{\eta}$ connecting

[8] This has various phenomenological advantages [34,35].

our SUSY-breaking theory to the Standard Model have the couplings (3.1) (which are the lowest order couplings allowed by the gauge symmetries). Again, one would need to generate messenger masses by an appropriate choice of closed string moduli, or by a dynamical mechanism similar to the one described above. It would be interesting to construct an explicit model of this sort, and to explore to what extent our simple DSB sectors (or obvious analogues) can easily be incorporated in existing semi-realistic brane constructions of the Standard Model (such as [36,37]). It would also be worthwhile to find analogous DSB models in the limits of string theory which more readily admit unification of coupling constants. Another natural generalization may be to apply D-instanton retrofitting to the recently studied O'Raifeartaigh models which spontaneously break R-symmetry [38].

Acknowledgements

This work is supported in part by the Israel-U.S. Binational Science Foundation. OA is supported by a center of excellence supported by the Israel Science Foundation (grant number 1468/06), by the European network HPRN-CT-2000-00122, by a grant from the G.I.F., the German-Israeli Foundation for Scientific Research and Development, and by a grant of DIP (H.52). OA thanks the Isaac Newton Institute for Mathematical Sciences for hospitality during the completion of this work. The research of SK and ES is supported in part by NSF grant PHY-0244728, and in part by the DOE under contract DE-AC03-76SF00515.

References

[1] E. Witten, "Dynamical Breaking Of Supersymmetry," Nucl. Phys. B **188**, 513 (1981).

[2] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking In Four-Dimensions And Its Phenomenological Implications," Nucl. Phys. B **256**, 557 (1985).

[3] E. Poppitz and S. P. Trivedi, "Dynamical supersymmetry breaking," Ann. Rev. Nucl. Part. Sci. **48**, 307 (1998) [arXiv:hep-th/9803107].

[4] Y. Shadmi and Y. Shirman, "Dynamical supersymmetry breaking," Rev. Mod. Phys. **72**, 25 (2000) [arXiv:hep-th/9907225].

[5] K. Intriligator and N. Seiberg, "Lectures on Supersymmetry Breaking," arXiv:hep-ph/0702069.

[6] R. Blumenhagen, M. Cvetic and T. Weigand, "Spacetime instanton corrections in 4D string vacua - the seesaw mechanism for D-brane models," Nucl. Phys. B **771**, 113 (2007) [arXiv:hep-th/0609191].

[7] M. Haack, D. Krefl, D. Lust, A. Van Proeyen and M. Zagermann, "Gaugino condensates and D-terms from D7-branes," JHEP **0701**, 078 (2007) [arXiv:hep-th/0609211].

[8] L. E. Ibanez and A. M. Uranga, "Neutrino Majorana masses from string theory instanton effects," JHEP **0703**, 052 (2007) [arXiv:hep-th/0609213].

[9] B. Florea, S. Kachru, J. McGreevy and N. Saulina, "Stringy instantons and quiver gauge theories," JHEP **0705**, 024 (2007) [arXiv:hep-th/0610003].

[10] K. Becker, M. Becker and A. Strominger, "Five-Branes, Membranes And Nonperturbative String Theory," Nucl. Phys. B **456**, 130 (1995) [arXiv:hep-th/9507158].

[11] E. Witten, "Non-Perturbative Superpotentials In String Theory," Nucl. Phys. B **474**, 343 (1996) [arXiv:hep-th/9604030].

[12] O. J. Ganor, "A note on zeroes of superpotentials in F-theory," Nucl. Phys. B **499**, 55 (1997) [arXiv:hep-th/9612077].

[13] M. Dine, J. L. Feng and E. Silverstein, "Retrofitting O'Raifeartaigh models with dynamical scales," Phys. Rev. D **74**, 095012 (2006) [arXiv:hep-th/0608159].

[14] S. Kachru, J. Pearson and H. L. Verlinde, "Brane/flux annihilation and the string dual of a non-supersymmetric field theory," JHEP **0206**, 021 (2002) [arXiv:hep-th/0112197].

[15] A. Maloney, E. Silverstein and A. Strominger, "De Sitter space in noncritical string theory," arXiv:hep-th/0205316.

[16] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].

[17] A. Saltman and E. Silverstein, "A new handle on de Sitter compactifications," JHEP **0601**, 139 (2006) [arXiv:hep-th/0411271].

[18] M. Aganagic, C. Beem, J. Seo and C. Vafa, "Geometrically induced metastability and holography," arXiv:hep-th/0610249.

[19] H. Verlinde, "On metastable branes and a new type of magnetic monopole," arXiv:hep-th/0611069.

[20] J. J. Heckman, J. Seo and C. Vafa, "Phase Structure of a Brane/Anti-Brane System at Large N," arXiv:hep-th/0702077.

[21] M. R. Douglas, J. Shelton and G. Torroba, "Warping and supersymmetry breaking," arXiv:0704.4001 [hep-th].

[22] J. J. Heckman and C. Vafa, "Geometrically Induced Phase Transitions at Large N," arXiv:0707.4011 [hep-th].

[23] A. M. Uranga, "Brane configurations for branes at conifolds," JHEP **9901**, 022 (1999) [arXiv:hep-th/9811004].

[24] R. Argurio, M. Bertolini, S. Franco and S. Kachru, "Metastable vacua and D-branes at the conifold," JHEP **0706**, 017 (2007) [arXiv:hep-th/0703236].

[25] S. Franco, A. Hanany, D. Krefl, J. Park, A. M. Uranga and D. Vegh, "Dimers and Orientifolds," arXiv:0707.0298 [hep-th].

[26] O. Aharony and S. Kachru, "Stringy Instantons and Cascading Quivers," arXiv:0707.3126 [hep-th].

[27] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and χSB-resolution of naked singularities," JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[28] K. Intriligator, N. Seiberg and D. Shih, "Dynamical SUSY breaking in meta-stable vacua," JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].

[29] G. F. Giudice and R. Rattazzi, "Theories with gauge-mediated supersymmetry breaking," Phys. Rept. **322**, 419 (1999) [arXiv:hep-ph/9801271].

[30] D. E. Diaconescu, B. Florea, S. Kachru and P. Svrcek, "Gauge-mediated supersymmetry breaking in string compactifications," JHEP **0602**, 020 (2006) [arXiv:hep-th/0512170].

[31] I. Garcia-Etxebarria, F. Saad and A. M. Uranga, "Local models of gauge mediated supersymmetry breaking in string theory," JHEP **0608**, 069 (2006) [arXiv:hep-th/0605166].

[32] T. Kawano, H. Ooguri and Y. Ookouchi, "Gauge Mediation in String Theory," arXiv:0704.1085 [hep-th].

[33] A. Amariti, L. Girardello and A. Mariotti, "Meta-stable A_n quiver gauge theories," arXiv:0706.3151 [hep-th].

[34] M. Ibe and R. Kitano, "Sweet Spot Supersymmetry," arXiv:0705.3686 [hep-ph].

[35] J. Feng et al, work in progress.

[36] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, "Toward realistic intersecting D-brane models," Ann. Rev. Nucl. Part. Sci. **55**, 71 (2005) [arXiv:hep-th/0502005].

[37] H. Verlinde and M. Wijnholt, "Building the standard model on a D3-brane," JHEP **0701**, 106 (2007) [arXiv:hep-th/0508089].

[38] D. Shih, "Spontaneous R-symmetry breaking in O'Raifeartaigh models," arXiv:hep-th/0703196.