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Spring School on Superstring Theory and Related Topics

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Stringy Avatars of Dynamical SUSY Breaking - Lecture 2

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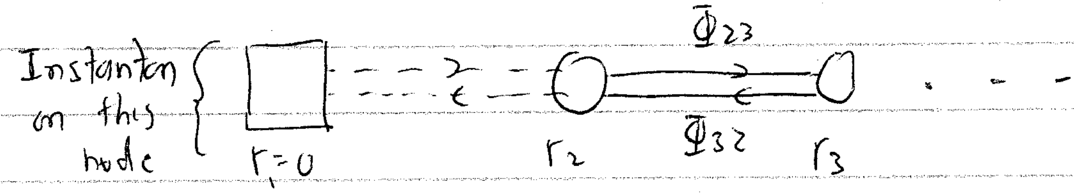
Trieste '08, Lecture II

II. Stringy Instantons

Refs: Blumenhagen, Cvetic, Weigand, Ibanez, Uranga; Florea, SK, McGreevy, Saulina

B. Formalism for rigid instantons

Suppose we're given some quiver:



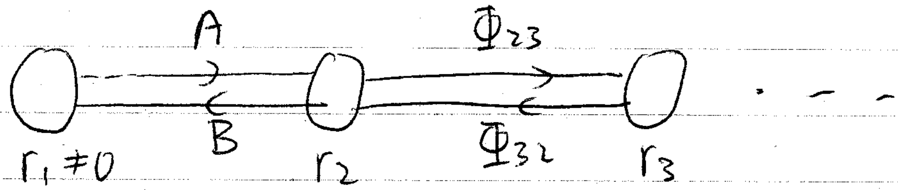
And suppose the Euclidean brane wrapping $\square$ is

- rigid (no moduli)
- half BPS w/ only $\underline{\underline{2}}$ fermion zero modes from the ED - ED sector [$\leftrightarrow$ wraps O-plane]

What effect does it have on spacetime $\mathcal{L}_{eff}$ for other D-branes?

Claim: Answer can be derived given data of the auxiliary gauge theory:

②



Given (in general) some chirals A_i, B_j coming into node 1, and

$$W_{aux} = W_{aux}(A_i, B_j, \Phi) \quad \Big\} \text{ tree level auxiliary thy } W$$

define

combinatorial factor

$$C_{ij\,l_1 \cdots l_n} = d(l) \left. \frac{\partial^{n+2} W_{aux}}{\partial A_i \, \partial B_j \, \partial \Phi_{l_1} \cdots \partial \Phi_{l_n}} \right|_{A,B,\Phi =}$$

Then: instanton in the theory of interest induces

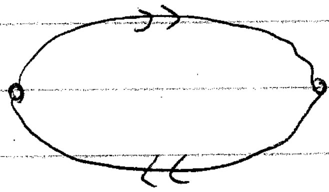
"Area of node instanton wraps"

$$\boxed{\begin{array}{l} \Delta W = e^{-t} \det M(\Phi) \,. \\ \text{where } M(\Phi)_{ij} = \left(\sum_{n=1}^{\infty} C_{ij\,l_1 \cdots l_n} \Phi_{l_1} \cdots \Phi_{l_n} \right) \end{array}}$$

"Proof" in hep-th 0803.2514. (SK + D. Simic)

EXAMPLE:

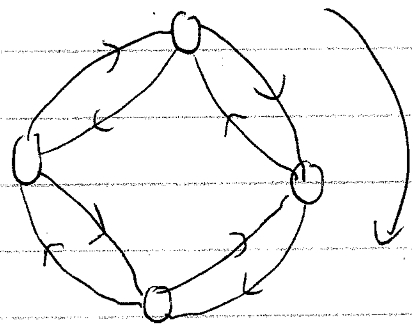
The quiver arising at a conifold singularity is :



$xy = zw$

A simple generalization, orbifolds this to obtain

$$(xy)^n = zw \qquad \text{[Uranga '98]}$$



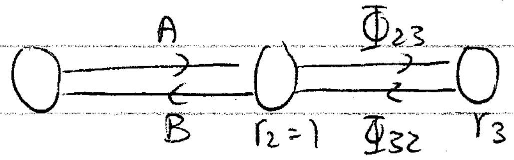
2n nodes connected by bi-funds.

$$W = h \sum_{i=1}^{2n} (-1)^i \, \Phi_{i,i+1} \Phi_{i+1,i+2} \Phi_{i+2,i+1} \Phi_{i+1,i}$$

$\exists$ simple orientifold that projects so

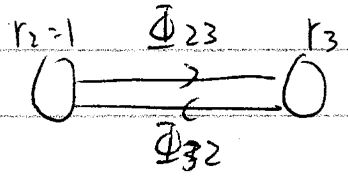
- nodes 1, n+1 $\to$ Sp gauge groups
- others pairwise identified by obvious reflection, $\to$ U(r) groups

E.g.



$$W_{aux} = A\,\Phi_{23}\,\Phi_{32}\,B$$

$\Rightarrow$ if $r_1 = 0$ & we consider the strongly instanton, it takes the $SU(r_3)$ gauge theory w/ one flavor:



& perturbs it by

$$\Delta W = e^{-t_1}\,\Phi_{23}\,\Phi_{32} \quad \Big\} \text{ small mass for flavor}$$

Explanation of Formula:

- In replacing space-filling brane at node 1 w/ instanton, chirals $A, B \Rightarrow$ R sector states α, β stretching from instanton to space-filling branes.

- Then, in performing $\int_{\text{collective coords}}$ get

$\int d\alpha\, d\beta \quad e^{\alpha \Phi_{23} \Phi_{32} \beta} \Rightarrow \Phi_{23} \Phi_{32}$ operator

perturbing W.

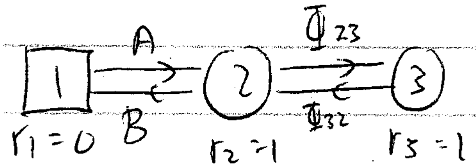
[Can prove $\exists$ no bosonic analogues of α, β for "stringy" instantons -- those on empty nodes]

C. $\cancel{\text{SUSY}}$ from stringy instantons

These effects can lead to DSB in even the simplest gauge theories on D-branes.

1. The Fayet Model

In any of the $n > 2$ models $(xy)^n = zw$, after orientifolding, consider the subquiver:



We just saw that a stringy instanton on node 1 $\Rightarrow$

$$\Delta W = e^{-t_1} \Phi_{23} \Phi_{32} = m \phi_+ \phi_-$$

This is just a $U(1)$ gauge theory with a non-perturbatively small mass.

- Generic $U(1)$ D-term arises from B-field on cycles

$$\Rightarrow \quad D^2 = \left(|\phi_+|^2 - |\phi_-|^2 - r \right)^2$$

This is just the Fayet model, SUSY w/

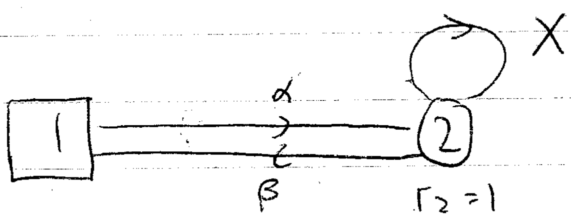
$$F_{\Phi_-} \sim m\sqrt{r} \ll M_p^2$$

due to e^{-t_1} factor.

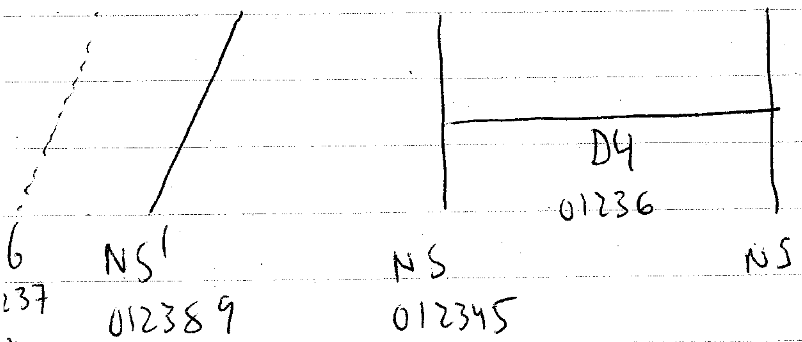
$\boxed{\text{Stringy effect} \rightarrow \text{DSB in an IR free theory.}}$

2. Polonyi Model

Consider the quiver



It can arise from an appropriate IIB singularity or the IIA brane configuration:



237 + 5° in 5189 planes

$$W_{aux} = A X B \implies \text{get}$$

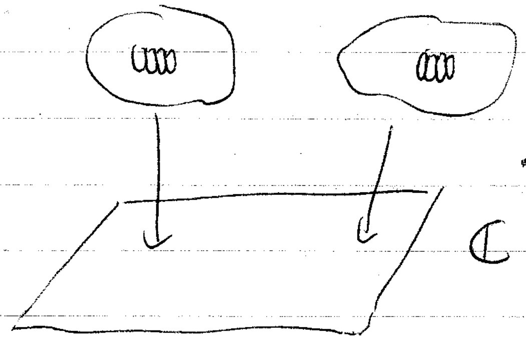
$$\boxed{\Delta W = e^{-t_1} X} \quad \text{from instanton.}$$

This $\Rightarrow$ Polonyi w/ μ^2 naturally $\ll M_P^2$.

III. Geometric transitions & fluxes: Summing the Instantons

A. The relevant geometries

We will consider quiver gauge theories arising from D5-branes wrapping curves in



A_r ADE type singularity

$$uv = \prod_{i=1}^{r+1} (z - z_i(x))$$

Singularities at pts where $u, v = 0$ & $z_i(x) = z_j(x)$ z. There are $\mathbb{P}^1$'s there, which can be blown up. $\to$ r 2-cycle classes

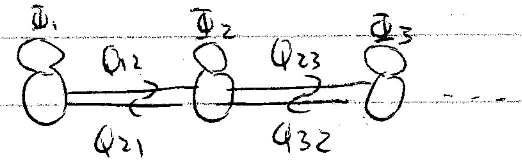
S_i^2 blow ups at $z_i = z_{i+1}$ $i = 1 \cdots r$

We wrap D5s on these.

$S_i^2 \cap S_{i+1}^2 \neq 0 \Rightarrow \exists$ bifundamental of

$SU(r_i) \times SU(r_{i+1})$ $\quad Q_{i,i+1}$ & $Q_{i+1,i}$

Each D5 stack has adjoint $\Phi_i \leftrightarrow$ position of $D5_s$ on X-plane



B_ ~~Superpotential~~ Classical Superpotential

Inherited from $\mathcal{N}=2$ theory that arises for trivial fibration

$$W \supset \mathrm{Tr}\,[\,Q_{i,i+1}\,\Phi_{i+1}\,Q_{i+1,i} - Q_{i,i+1}\,Q_{i+1,i}\,\Phi_i\,]$$

Deformed by $W_i(\Phi_i)$, computed as follows (Witten '97). ← hep-th/9706109

Consider D5s on $\mathbb{R}^4 \times \Sigma$.

SUSY $\longleftrightarrow$ Σ should be a holomorphic curve.

Think of Σ as an abstract surface with a map $\Phi : \Sigma \to X$

λ^a real coords on Σ

$\Phi : \phi^i(\lambda^a)$ } 4d chiral superfields
ϕ^i cplx coords on X, $i = 1, 2, 3$

The superpotential W should

1) be a <u>holomorphic</u> function of ϕ^i

2) have a critical pt precisely if $\Phi(\Sigma)$ is a holomorphic curve in X (unbroken SUSY)

It is easy to guess such a W. Let Ω be the holomorphic 3-form on X. 1st, let Σ be trivial in homology $\to$ $\Sigma = \partial B$, for some 3-manifold B. Then

$$W(\Sigma) = \int_B \Omega$$

$$\delta W = \int_\Sigma \Omega_{ijk}\, \delta\phi^i\, d\phi^j \wedge d\phi^k$$ } variation upon change in Σ

and this W is holomorphic, with critical pts

where $\partial\phi^j \wedge d\phi^k = 0 \Leftrightarrow \Sigma$ a holomorphic curve

Now, suppose Σ is nontrivial in homology (as it is in "real" applications).

- Pick a $\Sigma_0 \subset$ homology class of Σ
- Pick 3-manifold B w/ $\partial B = \Sigma - \Sigma_0$
- Define

$$W(\Sigma) - W(\Sigma_0) = \int_B \Omega$$

This defines W up to an overall additive shift

["Another" constant ambiguity: if $H_3(X, \mathbb{Z})$ is non-trivial, $\exists$ different choices for $[B]$; changing the choice will also shift W by a constant].

EXAMPLE: In our Calabi-Yau

$$uv = F(z, x)$$

cf Aganagic & V. hep-th/0012041 or appendix A of ABK

$$\Omega = \frac{dv \wedge dz \wedge dx}{\frac{dF}{du}} = \frac{dv}{v} \wedge dz \wedge dx$$

For fixed x, z, the CY is $uv = \text{const.}$

(a cylinder).

By shifting phases can make constant $\in \mathbb{R}$;

then writing $u = \tilde{x} + iy$, $v = \tilde{x} - iy$ $\Rightarrow$

get equations for real (x_R, y_R) & imaginary $(y_I, x$

parts of $\tilde{x}$ & y:

a) $\tilde{x}_R^2 + y_R^2 = C + \tilde{x}_I^2 + y_I^2$

b) $\tilde{x}_R \tilde{x}_I = y_R y_I$

a) Gives an S^1 for any value of x_I, y_I

b) restricts allowed $\tilde{x}_I, y_I$ to a 1D curve in

$(\tilde{x}_I, y_I)$ plane $\Rightarrow$ topology is $S^1 \times \mathbb{R}$.

- S^1 size <u>degenerates</u> at $z = z_i(x)$ for any i.

Note that shifting

$$u \to e^{i\theta} u, \quad v \to e^{-i\theta} v$$

cylinder eqn unchanged; simply rotates S^1 factor

So $$\int_{S^1} \Omega = dz \wedge dx \quad \left(\text{up to overall constant}\right)$$

Now, the $\mathbb{P}^1$'s on which we're wrapping D5s are Π of S^1 with an interval in the z dir between some z_i, z_j. For a given $\mathbb{P}^1$ class where S^1 vanishes @ z_i, z_j, get

$$\int_{S^1 \times I_{ij}} \Omega = (z_i(x) - z_j(x))\, dx$$

For the particular S^2_i we introduced, then

$$\boxed{W_i(x) = \int (z_i(x) - z_{i+1}(x))}$$