



**The Abdus Salam  
International Centre for Theoretical Physics**



**1935-11**

## **Spring School on Superstring Theory and Related Topics**

**27 March - 4 April, 2008**

### **Stringy Avatars of Dynamical SUSY Breaking - Lecture 3**

Shamit Kachru

*Department of Physics and SLAC, Stanford University, Stanford CA 94305*

①

## Trieste '08, Lecture III

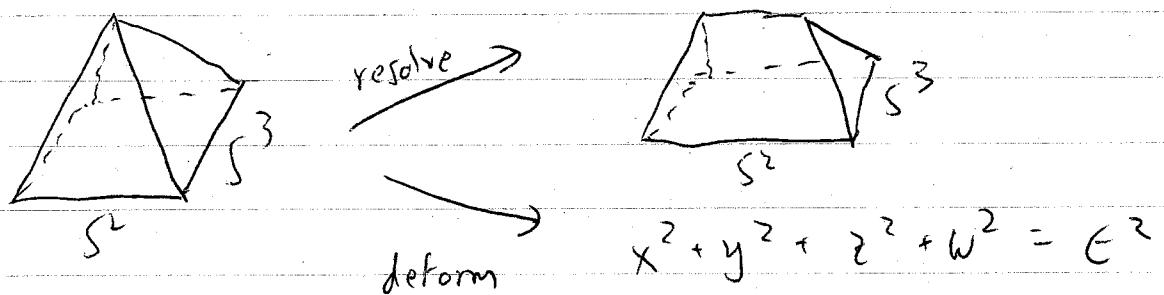
### C. Geometric transitions

[Vafa '00  
Klebanov-Strassler]

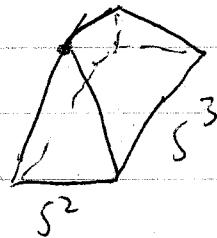
We are going to recast the stringy instanton effects as classical effects in a flux dual [of sum the whole series of instantons]. To do this, we should 1st renew the basic example of a geometric transition.

Consider a conifold geometry

$$\textcircled{X} \quad x^2 + y^2 + z^2 + w^2 = 0$$



$$x^2 + y^2 + z^2 + w^2 = \epsilon^2$$



Wrap  $N$  D5-branes on the  $S^2$  in resolved case

(2)

We can imagine varying the geometry to

collapse the resolving  $\mathbb{P}^1$  & deform the  $S^3$ .

Claim: Get a dual description of D5 gauge theory.

$$D5_s \rightarrow \int_{S^3} F_3^{RR} = N$$

$$\int_{B\text{-cycle}} H_3^{NS} \propto t \quad \left. \begin{array}{l} \text{size of} \\ \text{original} \\ \mathbb{P}^1 \end{array} \right\}$$

Now, the presence of fluxes  $\rightarrow$  a superpotential for (Alabi-Yau moduli):

$$W = \int_X (F_3 - \tau H_3) \wedge \Omega$$

$$= \frac{t}{g_s} S + N S \left[ \log \left( \frac{S}{\Delta^3} \right) - 1 \right]$$

Where we used the periods of the conifold:

(3)

$$\int_A \Omega = S$$

A

$$\int_B \Omega = S \left[ \log \left( \frac{S}{\Delta^3} \right) - 1 \right] + \text{regular} \quad \left. \begin{array}{l} \Delta \\ \text{cutoff} \\ \text{on } B \\ \text{cycle} \end{array} \right\}$$

$$\text{Solving for } S \Rightarrow (S = S^*)$$

$$W|_{S^*} \sim -\Delta^3 \exp \left( -\frac{t}{g_S N} \right) + \dots$$

We will interpret analogues of  $\int$  as giving instanton contributions in our geometries.

## D. SUSY Examples of transitions

### I. Fayet model

Take the special case of

$$UV = \prod_{i=1}^{r+1} (z - z_i(x)) \quad \text{where}$$

$$UV = (z - mx)(z + mx)(z - mx)(z + m(x - 2a))$$

(9)

After blowing up, wrap:

- $M$  D5 branes on  $S_1^2$  at  $z_1(x) = z_2(x)$
  - on  $S_2^2$  at  $z_2(x) = z_3(x)$
  - 1 D5 brane on  $S_3^2$  at  $z_3(x) = z_4(x)$
- [ considering  $U(1)$  instead of O-plane; we'll see you still get a strong instanton effect, cf Petersson arXiv: 0711.1837. Discussion a bit simpler than O-plane case.]

Tree-level superpotential?

$$W = \sum_{i=1}^3 W_i(\Phi_i) + Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12} + Q_{23}\Phi_3 Q_{32} - Q_{32}\Phi_2 Q_{23}$$

What are  $W_i(\Phi_i)$ ?

$$W_i = \int dx (\Phi_i(x) - z_{i+}(x))$$

$$\Rightarrow W_1 = 2 \int dx mx = m \Phi_1^2$$

$$W_2 = -m \Phi_2^2$$

(5)

$$W_3 = \int dx \quad 2m(x-a) \sim m(\Phi_3 - a)^2$$

NOTE: Heuristically,  $W_1 \nparallel W_2$  localize brane

1, 2 at same pt on  $x$ -plane  $x=0 \Rightarrow$  get

massless quarks  $Q_{12}, Q_{21}$ ;  $W_3$  localizes singl

D5 at  $x=a \Rightarrow$  get a massive theory the

$[\Phi_3, Q_{23}, Q_{32}$  all heavy].

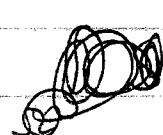
So: Perform geometric transition on node 3!

Geometry after transition:

$$W = (z-mx)(z+mx) \left[ (z-mx)(z+m(x-za)) - S \right]$$

+  $S^3$  has been traded for a 3-cycle  $S^3$

with



$$\int_{S^3} \Omega = S, \text{ with } S = \frac{S}{m}$$

We'll see  $S$  is fixed to be  $\exp(-(-1 + \text{shar}))$ .

⑥

- The 3<sup>rd</sup> DS is now gone, & with it the fields  $\Phi_3, Q_{23}, Q_{32}$ .

- New  $W$ :

$$W_{\text{eff}} = W_1(\bar{\Phi}_1) + \tilde{W}_2(\bar{\Phi}_2, S) + Q_{12} \bar{\Phi}_2 Q_{21} - Q_{21} \bar{\Phi}_1 Q_{12} + W_{\text{flux}}(S)$$

where :

$$W_{\text{flux}}(S) = \frac{t}{g_s} S + S \left[ \log \frac{S}{\Delta^3} - 1 \right]$$

and  $W_2$  has changed to  $\tilde{W}_2$  due to the deformation in the geometry which changes  $Z_3$

To find the new  $Z_3 \rightarrow \tilde{Z}_3(x)$ , we set

$$(z - \tilde{Z}_3(x)) (z - \tilde{Z}_4(x)) = (z - Z_3(x)) (z - Z_4(x)) - S$$

& set  $\tilde{Z}_3$  to be branch that looks asymptotically like  $Z_3(x)$  at large  $x$ .

(7)

Result:

$$\tilde{W}_2(\Phi_2) = \int_{\Delta}^{\Phi_2} (z_2(x) - \tilde{z}_3(x))$$

$$= \int_{\Delta}^{\Phi_2} \left[ -m(x+a) - \sqrt{m^2(x-a)^2 + S} \right] dx$$

Expanding in  $S \rightarrow$  sums up instanton

effects due to Euclidean branes on node 3  
before the transition!

Result:

$$\tilde{W}_2(\Phi_2) = m \operatorname{Tr} \Phi_2^2 - \frac{1}{2} S \operatorname{Tr} \log \frac{a-\Phi_2}{A}$$

+ --

where -- includes terms of  $O(S^2)$  & we dropped  
a constant.

Now, we can solve for the vacuum structure

$S, \Phi_{1,2}$  are expected to be very massive  $\Rightarrow$

(8)

solve for them 1<sup>ST</sup>.

$$F_{\Phi_1} = 2m \Phi_1 - Q_{12} Q_{21}$$

$$F_{\Phi_2} = -2m \Phi_2 + Q_{21} Q_{12} + \frac{S}{2(a-\Phi_2)}$$

$$F_S = \frac{t}{g_s} + \log\left(\frac{S}{\Delta^3}\right) - \frac{1}{2} \text{Tr} \log \frac{(a-\Phi_2)}{\Delta}$$

Setting these to 0  $\Rightarrow$

$$S_x = \Delta^3 \exp\left(-\frac{\tilde{t}}{g_s}\right)$$

$$\text{where } \tilde{t} = t - \frac{1}{2} g_s M \log\left(\frac{a}{\Delta}\right) \quad \begin{matrix} 1\text{-loop} \\ \text{renorm of} \\ \text{coupling} \end{matrix}$$

$$\text{and } \Phi_1^+ = -\frac{1}{2m} Q_{12} Q_{21}$$

$$\Phi_2^+ = \frac{1}{2m} Q_{21} Q_{12} + \frac{1}{y_m a} S_x + \dots$$

The omitted terms are higher order in

$$\frac{Q_{12} Q_{21}}{ma} \quad \left\{ e^{-\tilde{t}/g_s} \right. \quad \text{Then,}$$

(9)

irrelevant @ low energy

$$W_{\text{eff}} = \frac{1}{m} \overbrace{\text{Tr} [Q_{12} Q_{21} Q_{12} Q_{21}]} -$$

$$\frac{S^*}{4ma} \text{Tr} Q_{12} Q_{21} + \dots$$

This is Fayet-model w/  $S^* \leftrightarrow$  exponentially

small quark mass. But here, all of the computations are classical:

- $W$  flux deforms geometry (by exp small amount)
- Deformed geometry perturbs  $W_i$ ,  $\rightarrow$  SUSY

A D-term for the off-diagonal U(1) under

which  $Q_{12}, Q_{21}$  charged now  $\Rightarrow$  SUSY.

(can easily also engineer such SUSY models w/

Polongi or rank condition SUSY. Some of the

latter have calculably stable vacua w/ no FI term.

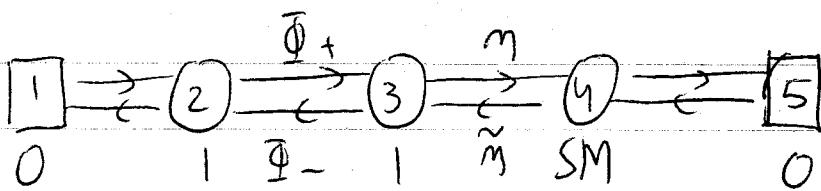
(10)

What's the point?

In addition to finding simple & controllable geometries w/ exponentially small SUSY in string theory, could hope to apply to model building.

For instance, one can try to use similar models to obtain natural gauge-mediated SUSY.

Imagine a simple quiver:



Node 4 should be occupied by a toy "standard model."

We expect:

$$W = \Lambda_1 \bar{\Phi}^+ \Phi^- + \frac{1}{M_4} m \tilde{m} \bar{\Phi}^+ \Phi^- + M \tilde{m} \tilde{m}$$

(11)

where : - quartic arises from tree-level quiver

$W$  as in  $(xy)^n = zw$  geometries

-  $\Lambda_1 \Phi^+ \bar{\Phi}^- \leftrightarrow$  strongy instanton

on node 1

- We included a possible SUSY mass

For  $\tilde{m}, \tilde{m}$  [more on this in a s.

SUSY vacuum:

$$\Phi^+ \sim \sqrt{r} \quad F_{\bar{\Phi}^-} \sim \Lambda_1 \sqrt{r}$$

$$\Phi^+ \bar{\Phi}^- \text{ has } \langle \Phi^+ \bar{\Phi}^- \rangle = 0, \quad F \sim \langle \Phi^+ \rangle F_{\bar{\Phi}^-}$$

so this  $W \Rightarrow$  gauge mediation with

- messenger mass  $M$

$$- \text{effective SUSY } F \sim \frac{\langle \Phi^+ F_{\bar{\Phi}^-} \rangle}{M_*} \sim \frac{r \Lambda_1}{M_*}$$

(12)

IF one wants low scale gauge mediation,

want messenger mass  $\ll M_{\text{strong}}$

(sparticle masses  $\sim \frac{\alpha_F}{M} \text{susy}$ )

One idea: Get  $M$  also from stringy instanton

Eg if you extend the SM by a  $U(1)$ ,

arrows from node 5 to 4 go to the  $U(1)$

Factor, the instanton on node 5  $\rightarrow$

$$\Delta W \sim e^{-\frac{\text{Area(node 5)}}{m^2}} \quad \checkmark$$

So can naturally get light messengers.

I think in general, understanding  
susy string models will be of substantial  
importance in the near future.