



The Abdus Salam
International Centre for Theoretical Physics



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**Joint ICTP-IAEA Workshop on Nuclear Structure and Decay Data:
Theory and Evaluation**

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Theory: High spin states in the interacting boson and interaction boson.

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2.

**High spin states
in the interacting boson and
interacting boson-fermion model**

Interacting Boson Model (IBM-1) based models constructed to describe the physics of high-spin states in nuclei ($10 \hbar \leq J \leq 30 \hbar$):

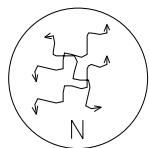
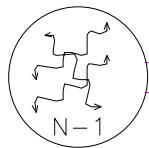
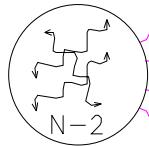
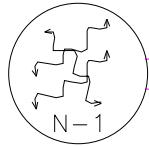
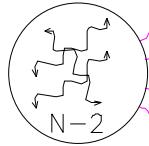
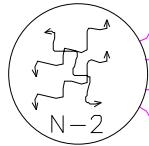
- **Interacting boson plus broken pairs model (IBBPM) for even-even nuclei**
- **Interacting boson fermion plus broken pairs model (IBFBPM) for odd-even nuclei**

In the formulation of these models one has to go beyond the boson approximation and include selected non-collective fermion degrees of freedom. By including part of the original shell-model fermion space through successive breaking of correlated S and D pairs, the IBM can describe the structure of high-spin states.

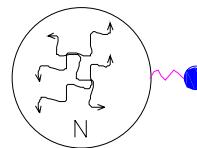
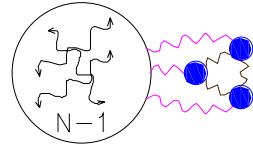
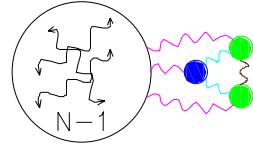
The models are based on the IBM-1; the boson space consists of *s* and *d* bosons, with no distinction between protons and neutrons. To generate high-spin states, the models allow one or two bosons to be destroyed and to form non-collective fermion pairs, represented by two- and four-quasiparticle states which recouple to the boson core. High-spin states are described in terms of broken pairs.

Advantages of using models based on the IBM over more traditional approaches based on the cranking approximations:

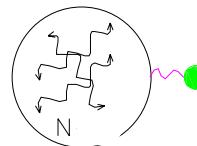
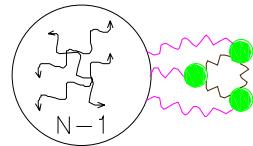
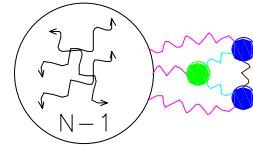
- No assumption has to be made about the geometrical picture of high-spin bands
- The bands result from a consistent calculation of the complete excitation spectrum, which includes also the ground state band
- Polarization effects directly result from the model fermion-boson interactions
- All calculations are performed in the laboratory frame, and therefore the results can be directly compared with experimental data
- This extension of the model is especially relevant for transitional regions, where single-particle excitations and vibrational collectivity are dominant modes, and the traditional cranking approach to high-spin physics is not adequate

 $|N\rangle$  $|N-1\rangle \otimes |\pi_1\pi_2\rangle$  $|N-2\rangle \otimes |\pi_1\pi_2\pi_3\pi_4\rangle$  $|N-1\rangle \otimes |v_1v_2\rangle$  $|N-2\rangle \otimes |v_1v_2v_3v_4\rangle$  $|N-2\rangle \otimes |\pi_1\pi_2\rangle \otimes |v_1v_2\rangle$

a) ODD PROTON

 $|N\rangle \otimes |\pi_1\rangle$  $|N-1\rangle \otimes |\pi_1\pi_2\pi_3\rangle$  $|N-1\rangle \otimes |\pi_1\rangle \otimes |v_1v_2\rangle$

b) ODD NEUTRON

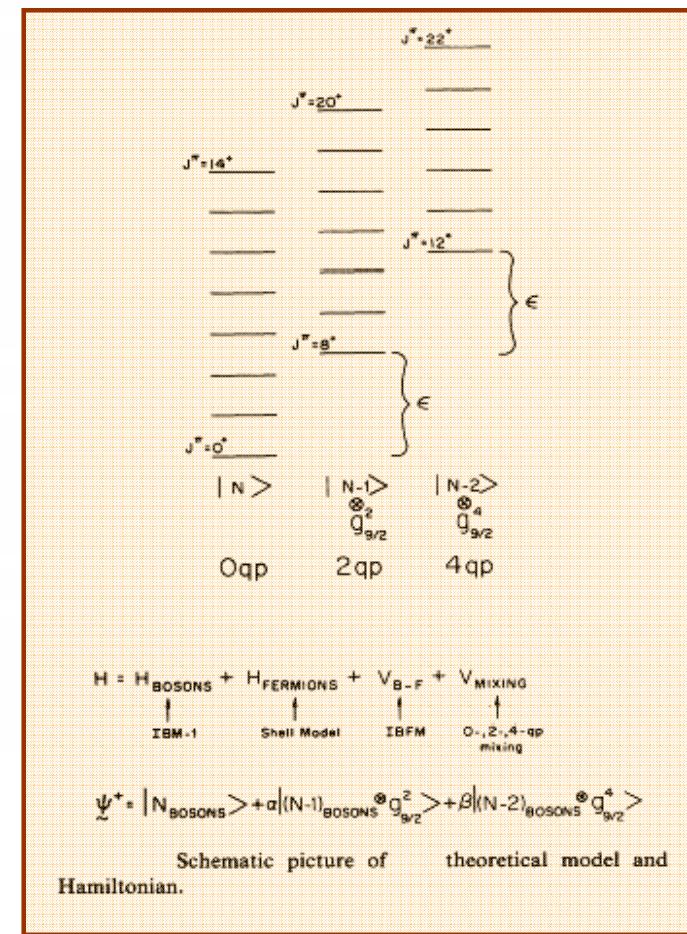
 $|N\rangle \otimes |v_1\rangle$  $|N-1\rangle \otimes |v_1v_2v_3\rangle$  $|N-1\rangle \otimes |v_1\rangle \otimes |\pi_1\pi_2\rangle$

The model space for an even-even nucleus with $2N$ valence nucleons is

$$| N \text{ bosons} \rangle \oplus | (N - 1) \text{ bosons} \otimes 1 \text{ broken pair} \rangle \oplus | (N - 2) \text{ bosons} \otimes 2 \text{ broken pairs} \rangle$$

This means that the fermion basis can contain two-proton, two-neutron, four-proton, four-neutron and two-proton-two-neutron configurations.

For odd- A nuclei, two-broken pair configurations are not included in the model space. They would generate five-quasiparticle configurations resulting in exhaustive numerical calculations. The IBFBPM can describe one- and three-fermion structures. The two fermions in a broken pair can be of the same type as the unpaired fermion, resulting in a space with three identical fermions. If the fermions in the broken pair are different from the unpaired fermion, the fermion basis contains two protons and one neutron or vice versa.



The Interacting boson plus broken pairs model (IBBPM) Hamiltonian for an even-even nucleus:

$$H = H_B + H_{\nu F} + H_{\pi F} + V_{\nu BF} + V_{\pi BF} + V_{\nu}^{mix} + V_{\pi}^{mix} + V_{\nu \pi}$$

The label π stands for protons and ν for neutrons. If broken pairs contain both protons and neutrons, the full model Hamiltonian is used. Otherwise, when broken pairs contain only protons ($\alpha = \pi$) or neutrons ($\alpha = \nu$), the model Hamiltonian is reduced to:

$$H = H_B + H_{\alpha F} + V_{\alpha BF} + V_{\alpha}^{mix}$$

In description of high-spin states in odd-even nuclei we employ the Interacting boson fermion plus broken pairs model (IBFBPM).

- When the two fermions in a broken pair are of the same type as the unpaired fermion, the reduced Hamiltonian is used, where α labels the type of fermion (proton or neutron).
- If the fermions in the broken pair are different from the unpaired fermion, the full Hamiltonian is used, without the pair breaking interaction of the unpaired fermion and with the fermion Hamiltonian of the unpaired fermion containing only single-fermion energies.

H_B is the boson Hamiltonian of IBM-1 describing a system of N interacting bosons (correlated S and D pairs) that approximate the valence nucleon pairs:

$$\begin{aligned}
 H_B &= \varepsilon \hat{N} + \frac{1}{2} v_0 ([d^\dagger \times d^\dagger]_{(0)} \times [\tilde{s} \times \tilde{s}]_{(0)} + h.c.)_{(0)} \\
 &+ \frac{1}{\sqrt{2}} v_2 \left([d^\dagger \times d^\dagger]_{(2)} \times [\tilde{d} \times \tilde{s}]_{(2)} + h.c. \right)_{(0)} \\
 &+ \sum_{L=0,2,4} \frac{1}{2} C_L \sqrt{2L+1} \left([d^\dagger \times d^\dagger]_{(L)} \times [\tilde{d} \times \tilde{d}]_{(L)} \right)_{(0)}
 \end{aligned}$$

$$n_s = N - n_d$$

$H_{\alpha F}$ is the fermion Hamiltonian which contains single-fermion (quasiparticle) energies and fermion-fermion interactions. The quasiparticle energies and occupation probabilities contained in the fermion Hamiltonian and other terms, are obtained in a BCS calculation with some standard set of single fermion energies.

$$H_{\alpha F} = \sum_i \varepsilon_{\alpha_i} a_{\alpha_i}^\dagger \tilde{a}_{\alpha_i} + \frac{1}{4} \sum_{abcd} \sum_{JM} V_{\alpha abcd}^J A_{JM}^\dagger(\alpha_a \alpha_b) A_{JM}(\alpha_c \alpha_d)$$

$$A_{JM}^\dagger(\alpha_a \alpha_b) = \frac{1}{\sqrt{1 + \delta_{ab}}} [a_{\alpha_a}^\dagger \ a_{\alpha_b}^\dagger]^M_J$$

$$V_{\alpha abcd}^J = (u_{\alpha_a} u_{\alpha_b} u_{\alpha_c} u_{\alpha_d} + v_{\alpha_a} v_{\alpha_b} v_{\alpha_c} v_{\alpha_d}) G(\alpha_a \alpha_b \alpha_c \alpha_d J) + 4 v_{\alpha_a} u_{\alpha_b} v_{\alpha_c} u_{\alpha_d} F(\alpha_a \alpha_b \alpha_c \alpha_d J)$$

$V_{\alpha BF}$ is the interaction between the unpaired fermions and the boson core containing the dynamical, exchange and monopole interactions of the IBFM-1:

$$V_{\alpha BF} = V_{\alpha DYN} + V_{\alpha EXC} + V_{\alpha MON}$$

$$V_{\alpha DYN} = \Gamma_0 \sum_{\alpha j_1 \alpha j_2} \sqrt{5} (u_{\alpha j_1} u_{\alpha j_2} - v_{\alpha j_1} v_{\alpha j_2}) \langle \alpha j_1 \| Y_2 \| \alpha j_2 \rangle \left([a_{\alpha j_1}^\dagger \times \tilde{a}_{\alpha j_2}]^{(2)} \times Q_B^{(2)} \right)^{(0)}$$

$Q_B^{(2)}$ is the standard boson quadrupole operator

$$Q_B^{(2)} = [s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} + \chi [d^\dagger \times \tilde{d}]^{(2)}$$

$$V_{\alpha EXC} = \Lambda_0 \sum_{\alpha j_1 \alpha j_2 \alpha j_3} (-2) \sqrt{\frac{5}{2 \alpha j_3 + 1}} (u_{\alpha j_1} v_{\alpha j_3} + v_{\alpha j_1} u_{\alpha j_3}) (u_{\alpha j_2} v_{\alpha j_3} + v_{\alpha j_2} u_{\alpha j_3}) \langle \alpha j_3 \| Y_2 \| \alpha j_1 \rangle \langle \alpha j_3 \| Y_2 \| \alpha j_2 \rangle : \left([a_{\alpha j_1}^\dagger \times \tilde{d}]_{\alpha j_3} \times [\tilde{a}_{\alpha j_2} \times d^\dagger]_{\alpha j_3} \right)^{(0)} :$$

$$V_{\alpha MON} = A_0 \sum_{\alpha j} \sqrt{5} (2\alpha j + 1) \left([a_{\alpha j}^\dagger \times \tilde{a}_{\alpha j}]^{(0)} \times [d^\dagger \times \tilde{d}]^{(0)} \right)^{(0)}$$

The pair breaking interaction V_α^{mix} which mixes states with different number of fermions, conserving the total nucleon number only:

$$V_\alpha^{mix} = -U_0 \left\{ \sum_{\alpha j_1 \alpha j_2} u_{\alpha j_1} u_{\alpha j_2} (u_{\alpha j_1} v_{\alpha j_2} + u_{\alpha j_2} v_{\alpha j_1}) \langle \alpha j_1 \parallel Y_2 \parallel \alpha j_2 \rangle^2 \frac{1}{\sqrt{2 \alpha j_2 + 1}} ([a_{\alpha j_2}^\dagger \times a_{\alpha j_2}^\dagger]^{(0)} \cdot \tilde{s}) + hc \right\}$$

$$-U_2 \left\{ \sum_{\alpha j_1 \alpha j_2} (u_{\alpha j_1} v_{\alpha j_2} + u_{\alpha j_2} v_{\alpha j_1}) \langle \alpha j_1 \parallel Y_2 \parallel \alpha j_2 \rangle ([a_{\alpha j_1}^\dagger \times a_{\alpha j_2}^\dagger]^{(2)} \cdot \tilde{d}) + hc \right\}$$

The proton-neutron interaction is:

$$V_{\nu\pi} = \sum_{\nu\nu'\pi\pi'} \sum_J h_J(\nu\nu'\pi\pi') (u_\nu u_{\nu'} - v_\nu v_{\nu'}) (u_\pi u_{\pi'} - v_\pi v_{\pi'}) \left([a_\nu^\dagger \times \tilde{a}_{\nu'}]^{(J)} \cdot [a_\pi^\dagger \times \tilde{a}_{\pi'}]^{(J)} \right)$$

The coefficients $h_J(\nu\nu'\pi\pi')$ are connected to the two-body matrix elements of the residual proton-neutron interaction by:

$$h_J(\nu\nu'\pi'\pi) = (-)^{j_\nu + j_\pi} \sum_{J'} (-)^{J'} \sqrt{2J' + 1} \langle (j_\nu j_\pi) J' \parallel V(1, 2) \parallel (j_{\nu'} j_{\pi'}) J' \rangle W(j_\nu j_\pi j_{\nu'} j_{\pi'}; J' J)$$

The residual proton-neutron interaction is usually taken in the form:

$$H_\delta = 4\pi V_\delta \delta(\vec{r}_\pi - \vec{r}_\nu) \delta(r_\pi - R_0) \delta(r_\nu - R_0)$$

The strength parameters of the boson-fermion interactions should be those obtained in the analysis of the neighboring nuclei. For example, the boson-fermion strength parameters for the couplings of two and four-proton configurations to the boson core in an even-even nucleus, have to be the same as for coupling of one-proton configurations to the boson core in the neighboring odd-even nucleus. This is the case for spherical, transitional and γ -soft nuclei. However, approaching the rotational SU(3) limit of IBM, the boson-fermion interaction strengths are not identical for an even-even nucleus and its odd-even neighbor. The effective core for configurations based on broken pairs in a deformed nucleus can be somewhat different from the one obtained by a simple decrease of the boson number by one.

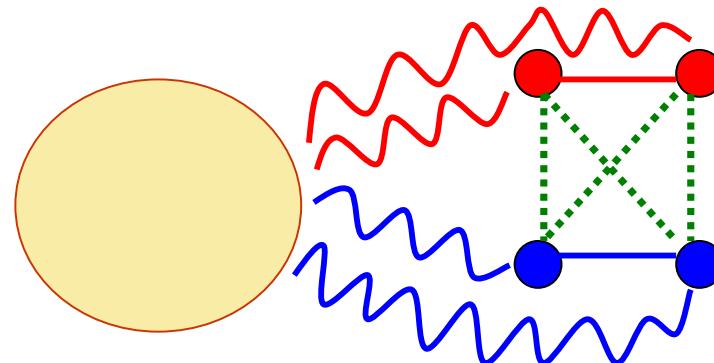
$$T(E2) = \frac{3}{4\pi} e^{\text{vib}} R_0^2 [(d^\dagger \times \vec{s} + s^\dagger \times \vec{d})^{(2)} + \chi(d^\dagger \times \vec{d})^{(2)}] \\ - e \frac{1}{\sqrt{5}} \sum_{j_1 j_2} g_{j_1 j_2} [(u_{j_1} u_{j_2} - v_{j_1} v_{j_2})(a_{j_1}^\dagger \times \vec{a}_{j_2})^{(2)} - \frac{u_{j_1} v_{j_2}}{\sqrt{N}} [(a_{j_1}^+ \times a_{j_2}^+)^{(2)} \times \vec{s}]^{(2)} + \frac{u_{j_2} v_{j_1}}{\sqrt{N}} [(\vec{a}_{j_1} \times \vec{a}_{j_2})^{(2)} \times s^\dagger]^{(2)}]$$

where

$$g_{j_1 j_2} = \langle j_1 | r^2 Y_2 | j_2 \rangle .$$

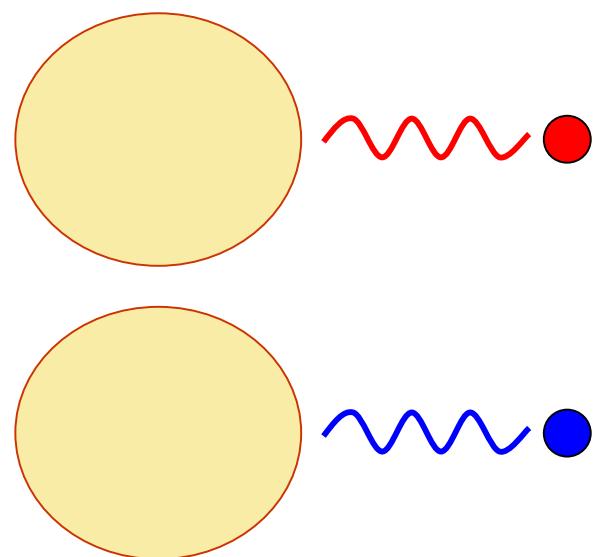
We take $\langle r^2 \rangle = \frac{3}{5} R_0^2$, and $R_0 = 0.12 A^{1/3} \times 10^{-12}$ cm. N is the number of bosons.

$$T(M1) = \sqrt{30/4\pi} g_R (d^\dagger \times \vec{d})^{(1)} \\ - \frac{1}{\sqrt{4\pi}} \sum_{j_1 j_2} [g_1 \langle j_1 | \vec{j} | j_2 \rangle + (g_s - g_1) \langle j_1 | \vec{s} | j_2 \rangle] \\ \times \{(u_{j_1} u_{j_2} + v_{j_1} v_{j_2})(a_{j_1}^\dagger \times \vec{a}_{j_2})^{(1)} - \frac{u_{j_1} v_{j_2}}{\sqrt{N}} [(a_{j_1}^\dagger \times a_{j_2}^\dagger)^{(1)} \times \vec{s}]^{(1)} + \frac{u_{j_2} v_{j_1}}{\sqrt{N}} [(\vec{a}_{j_1} \times \vec{a}_{j_2})^{(1)} \times s^\dagger]^{(1)}\}$$

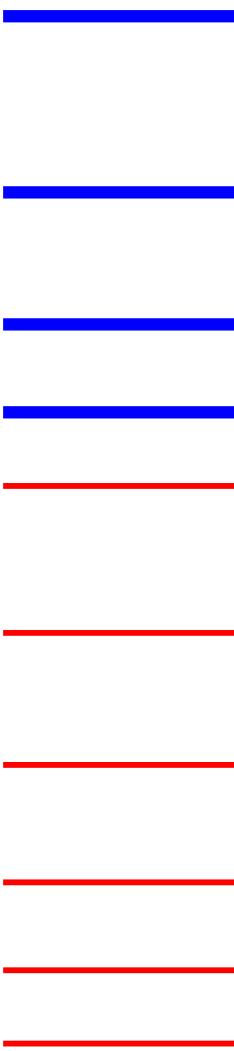


- **proton**
- **neutron**

boson –fermion interactions from odd – A neighbours



Important data for high spin states

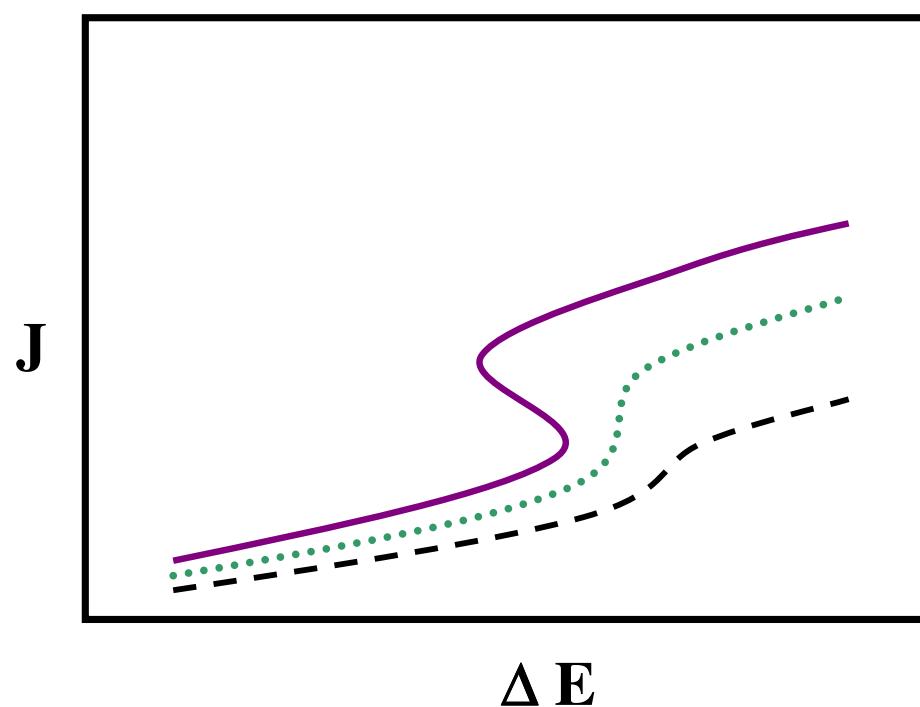


excitation energies

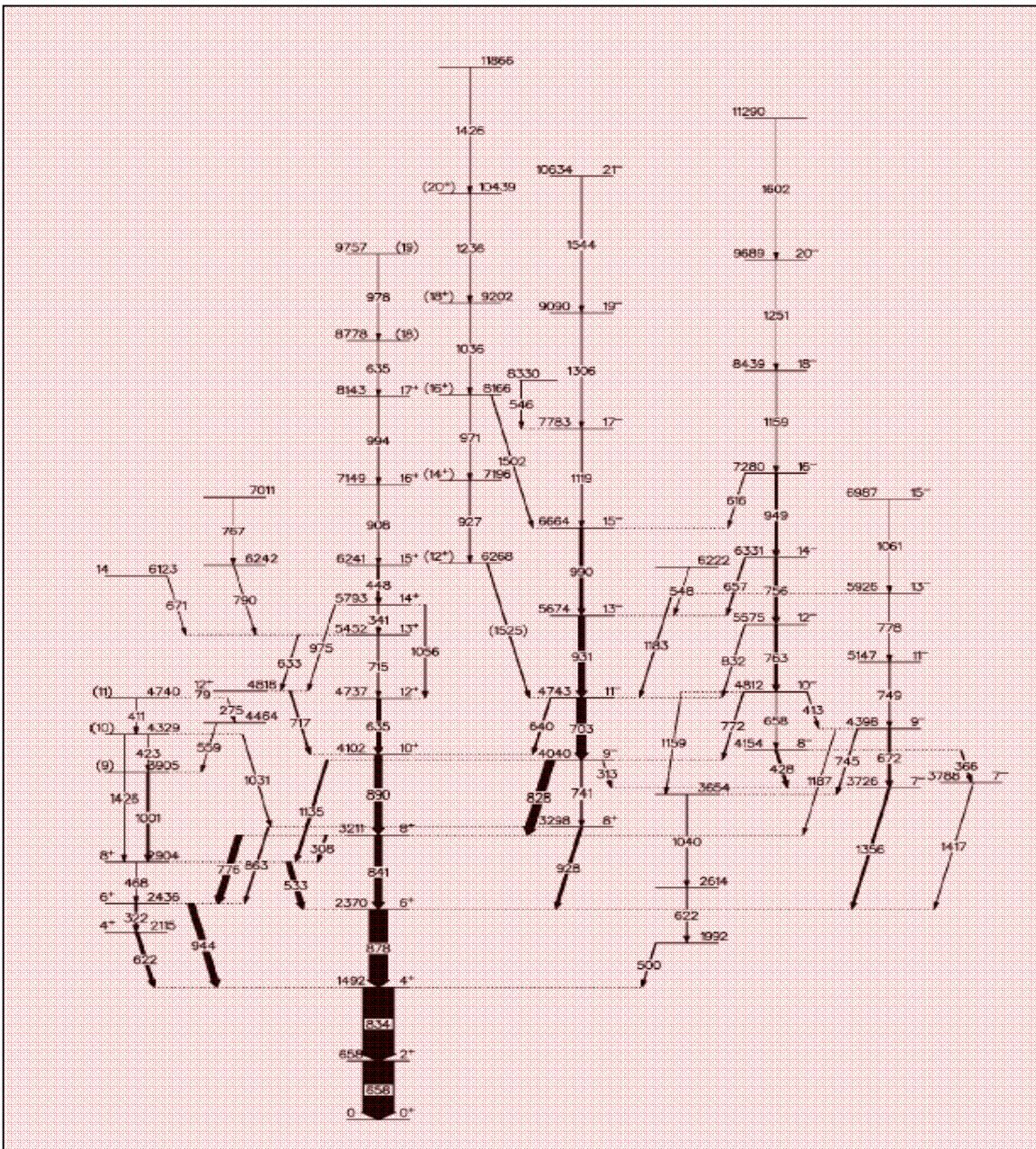
γ branchings

$T_{1/2}$ B(E2) B(M1)

μ (g) for J_C

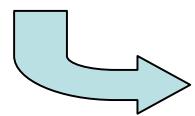


Spherical nuclei

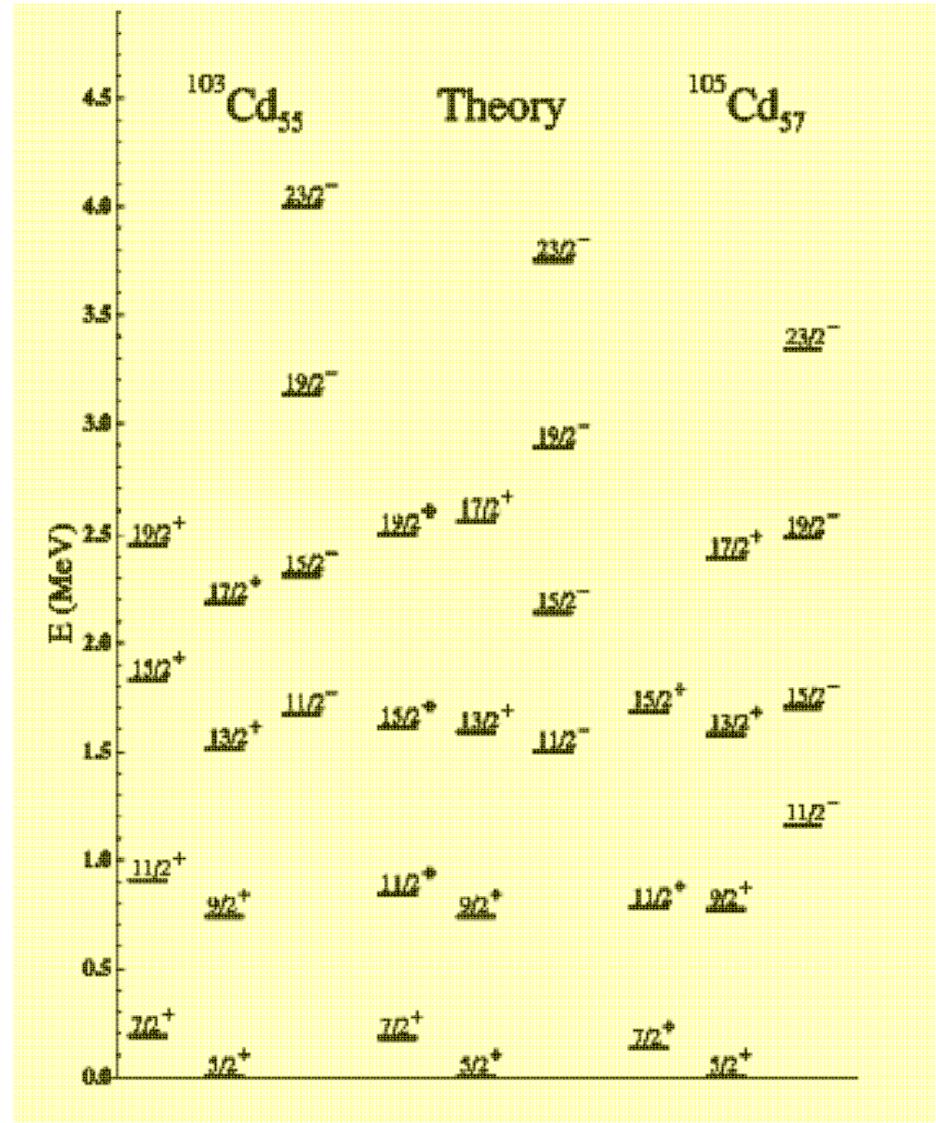


104Cd

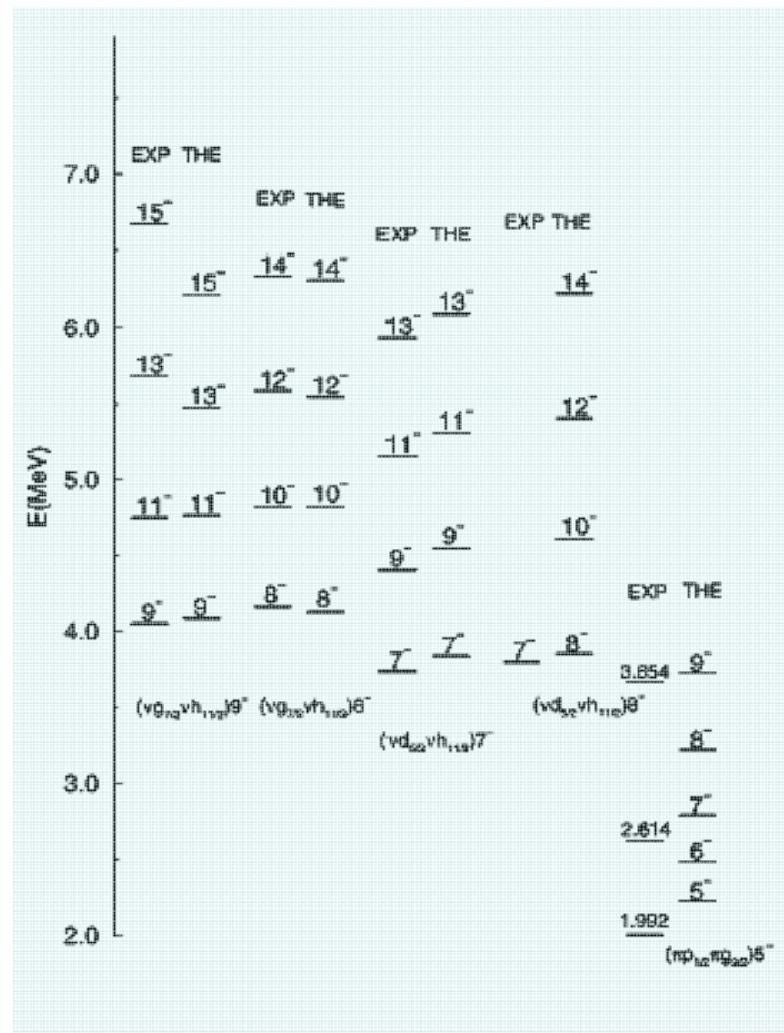
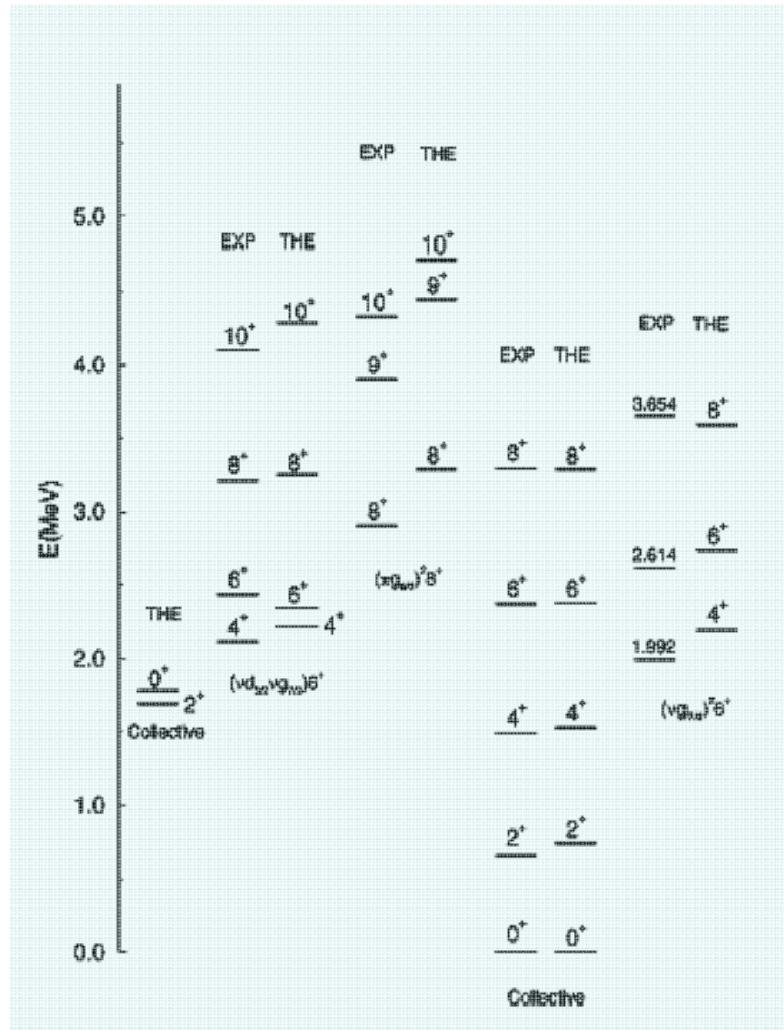
Parameterization for
neutrons from
even-odd Cd nuclei



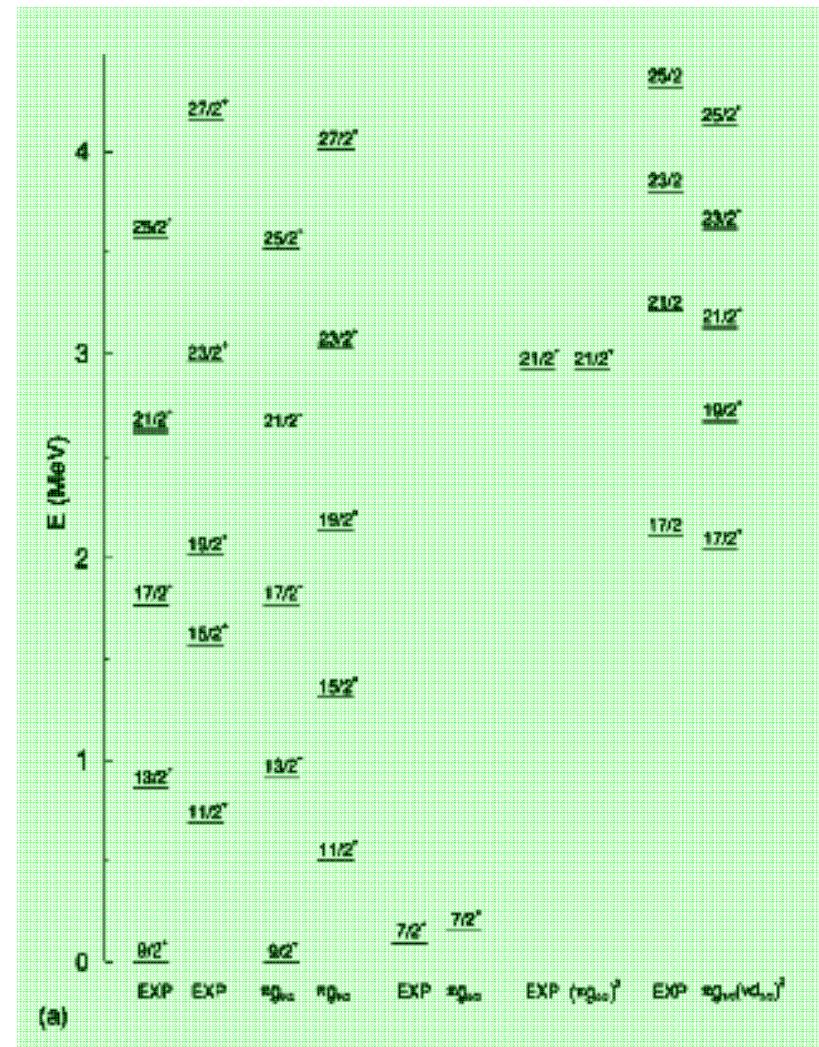
Parameterization for
protons in accordance
with odd-even
In and Ag nuclei



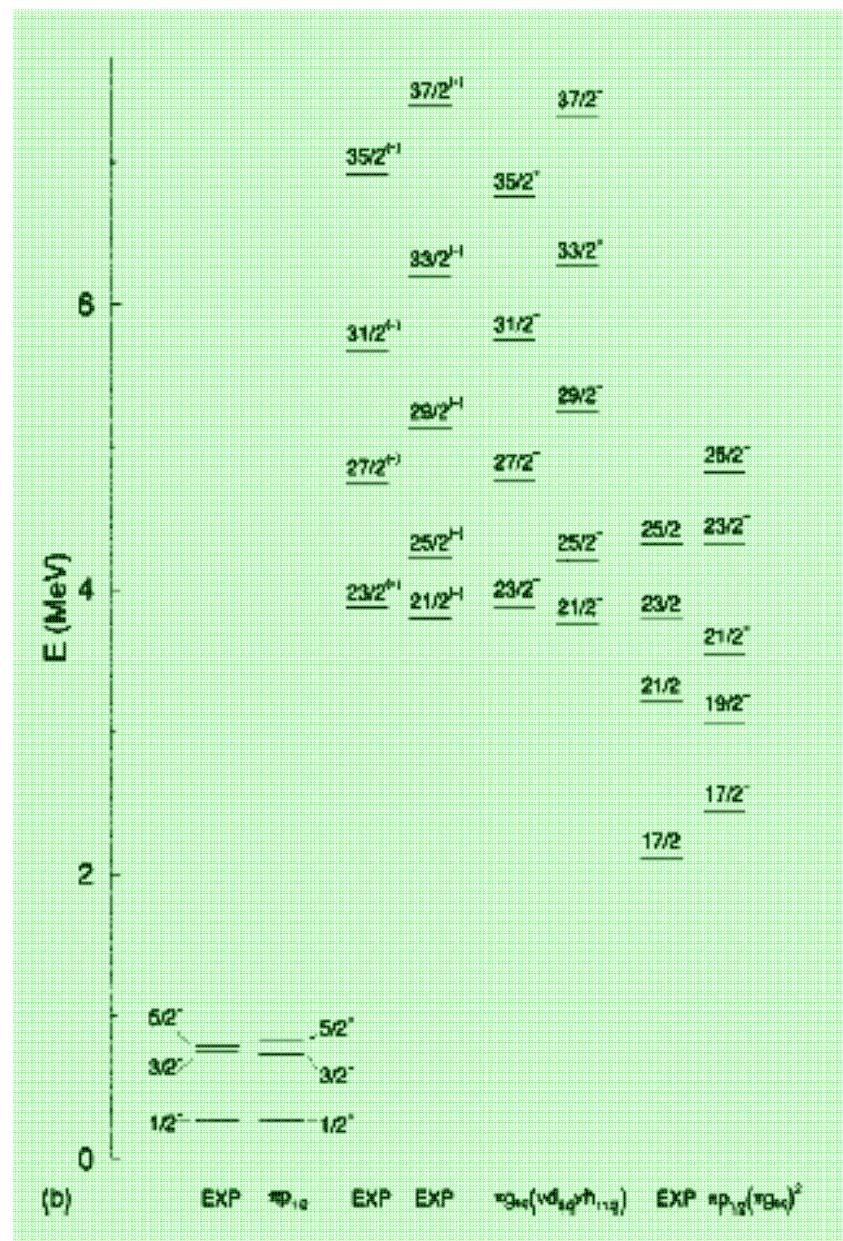
^{104}Cd



¹⁰¹Ag



(a)



(b)

^{101}Ag

E_x (keV)	I^π	Lifetime (ps)				Theory
		DDCM	RDDS	DSA/NGTB	Adopted	
Positive parity						
98	$7/2^+$				425	
687	$11/2^+$	2.7(3)		2.7(3)	2.0	
861	$13/2^+$	11.7(10)		11.7(10)	3.4	
1573	$15/2^+$	2.1(5)		2.1(5)	0.4	
1769	$17/2^+$	1.9(2)		1.9(2)	1.9	
2017	$19/2^+$	9(1)		9(1)	5.0	
2621	$21/2^+$	0.6(1)		0.6(1)	0.4	
2922	$21/2_2^+$				1700 ^a	
					2.8 ^b	
2956	$23/2^+$	1.8(3)		1.8(3)	0.9	
3578	$25/2^+$		<2.0	<2.0	0.4	
4159	$27/2^+$		<2.5	<2.5	0.3	
4572	($29/2^+$)		14(1)		14(1)	
5300	($31/2^+$)			≤ 1.7	≤ 1.7	
Negative parity						
750	$3/2^{(-)}$				10.2	
797	$5/2^{(-)}$				30.5	
3870	$23/2^{(-)}$	11.4(11)		11.4(11)	8.4	
4217	$25/2^{(-)}$	1.1(2)		1.1(2)	1.2	
4749	$27/2^{(-)}$		1.1(1)	1.1(1)	1.1	
5134	$29/2^{(-)}$		0.83(8)	0.83(8)	0.81	
5678	$31/2^{(-)}$		0.41(5)	0.41(5)	0.45	
6197	$33/2^{(-)}$		0.30(4)	0.30(4)	0.34	
6917	$35/2^{(-)}$		0.18(5)	0.18(5)	0.13	
7393	$37/2^{(-)}$		≤ 1.3	≤ 1.3	0.39	
No parity assigned						
2115	17/2		199(7)	199(7)	120	$\pi = +$
3210	21/2	1.2(1)		1.2(1)	1.0	$\pi = -$
3801	23/2				0.5	0.9
4315	25/2				0.5	0.4

^aIf wave function predominantly $\pi^3(g_{9/2})$.

^bIf wave function predominantly $\pi(g_{9/2})$.

Structure of isomers in spherical nuclei

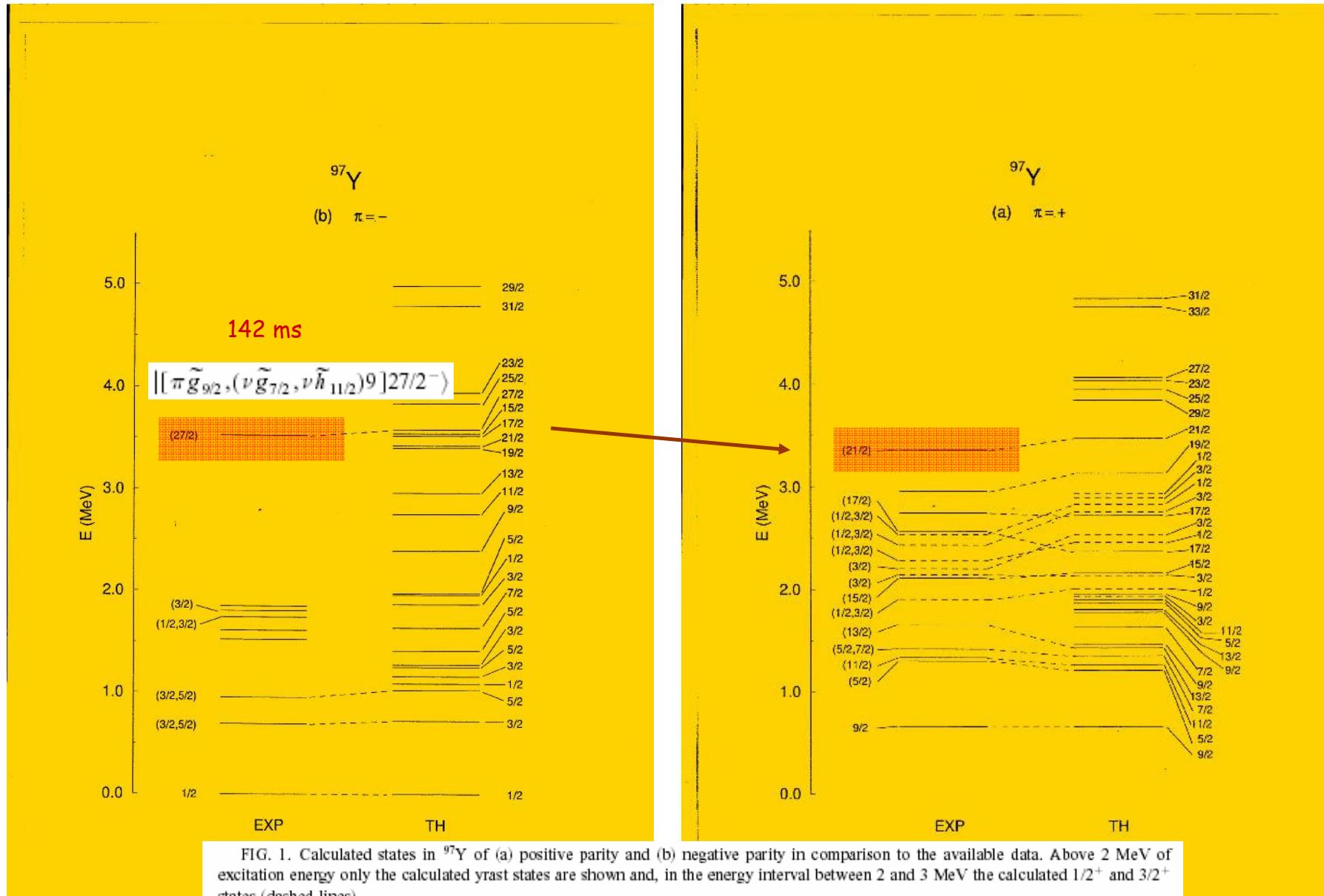
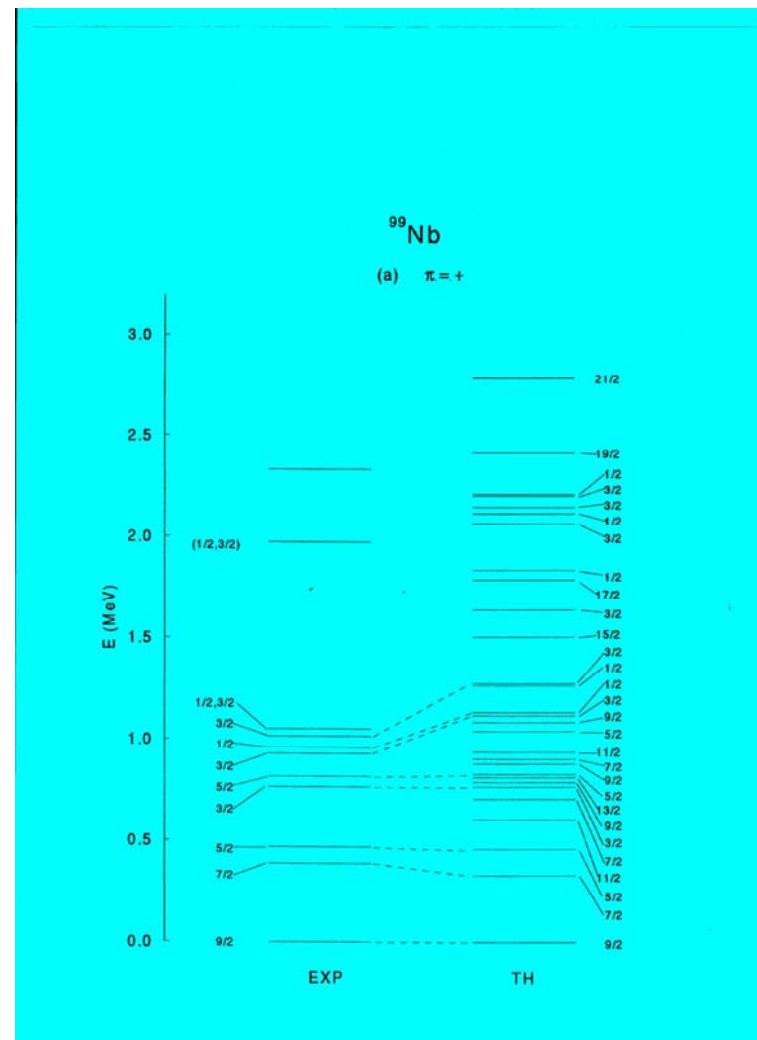
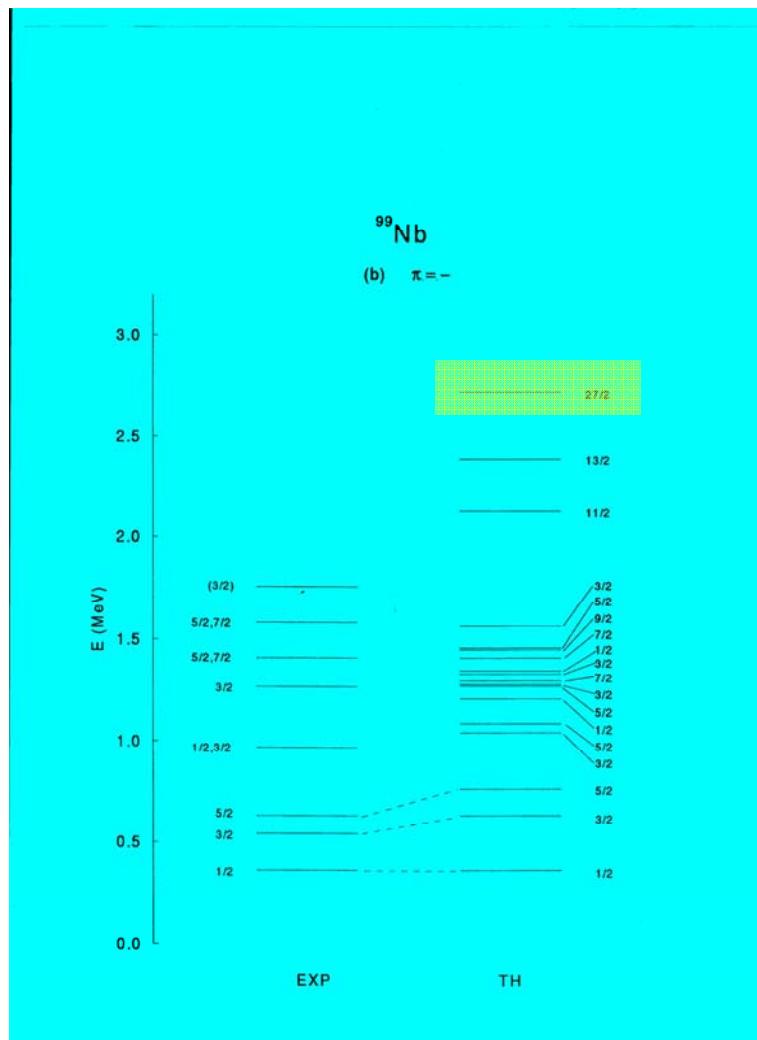
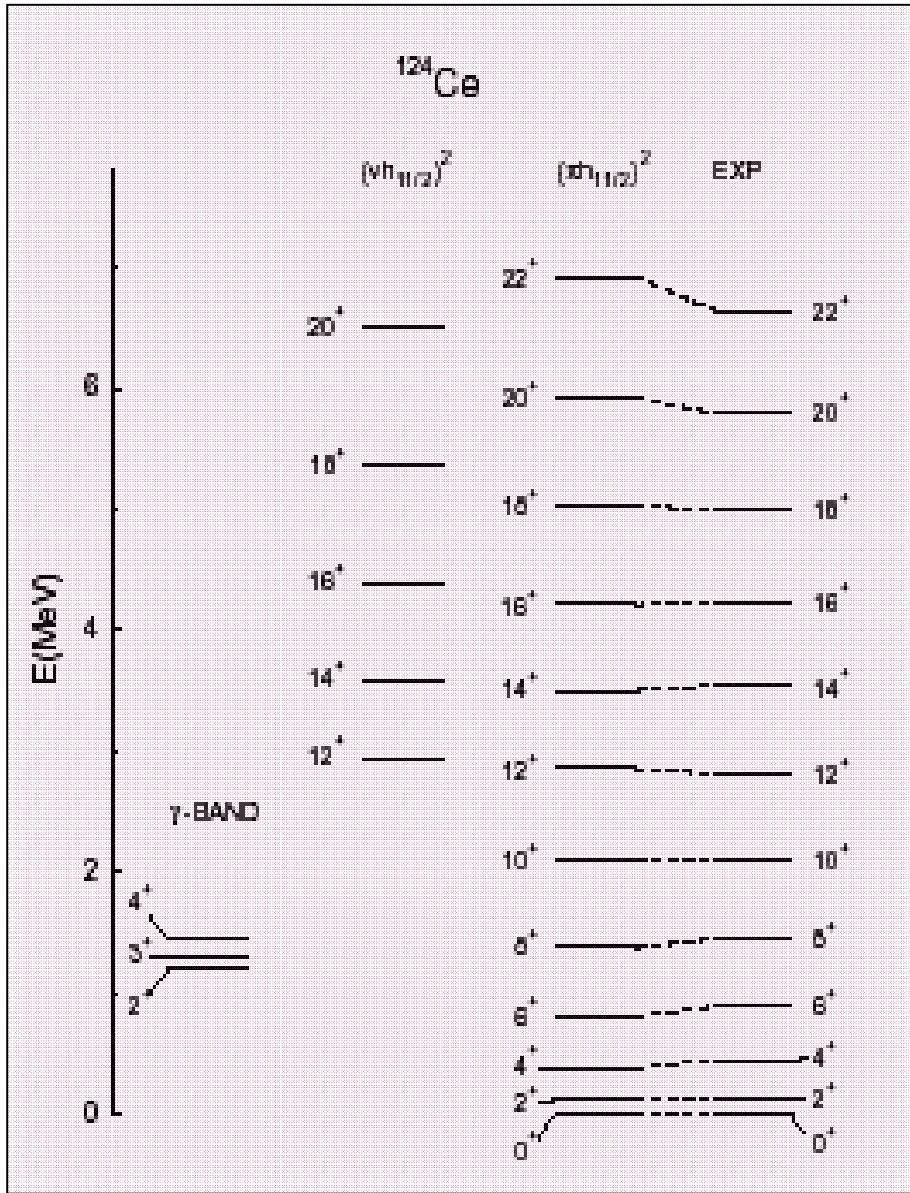


FIG. 1. Calculated states in ^{97}Y of (a) positive parity and (b) negative parity in comparison to the available data. Above 2 MeV of excitation energy only the calculated yrast states are shown and, in the energy interval between 2 and 3 MeV the calculated $1/2^+$ and $3/2^+$ states (dashed lines).



Assuming a possible error of 200 – 300 keV for the predicted energies, a $27/2^-$ isomer with a halflife in the $\mu\text{s} - \text{ms}$ range could be found in ^{99}Nb

Deformed nuclei



This nucleus displays a transitional structure between deformed nuclei (lighter Ce isotopes) described by the SU(3) limit of the IBM, and γ -soft nuclei (heavier Ce isotopes) which correspond to the O(6) limit of the IBM. The SU(3)-O(6) transition can be described by the boson Hamiltonian

$$H_{IBM} = -\frac{\alpha}{10} Q \cdot Q + \frac{\beta}{10} L \cdot L$$

and is determined by the value of the parameter χ in the quadrupole boson operator. The limiting cases are: $\chi = 0$ corresponds to the O(6) limit of the IBM-1, and $\chi = -\frac{\sqrt{7}}{2}$ describes a prolate shape in the SU(3) dynamical symmetry limit.

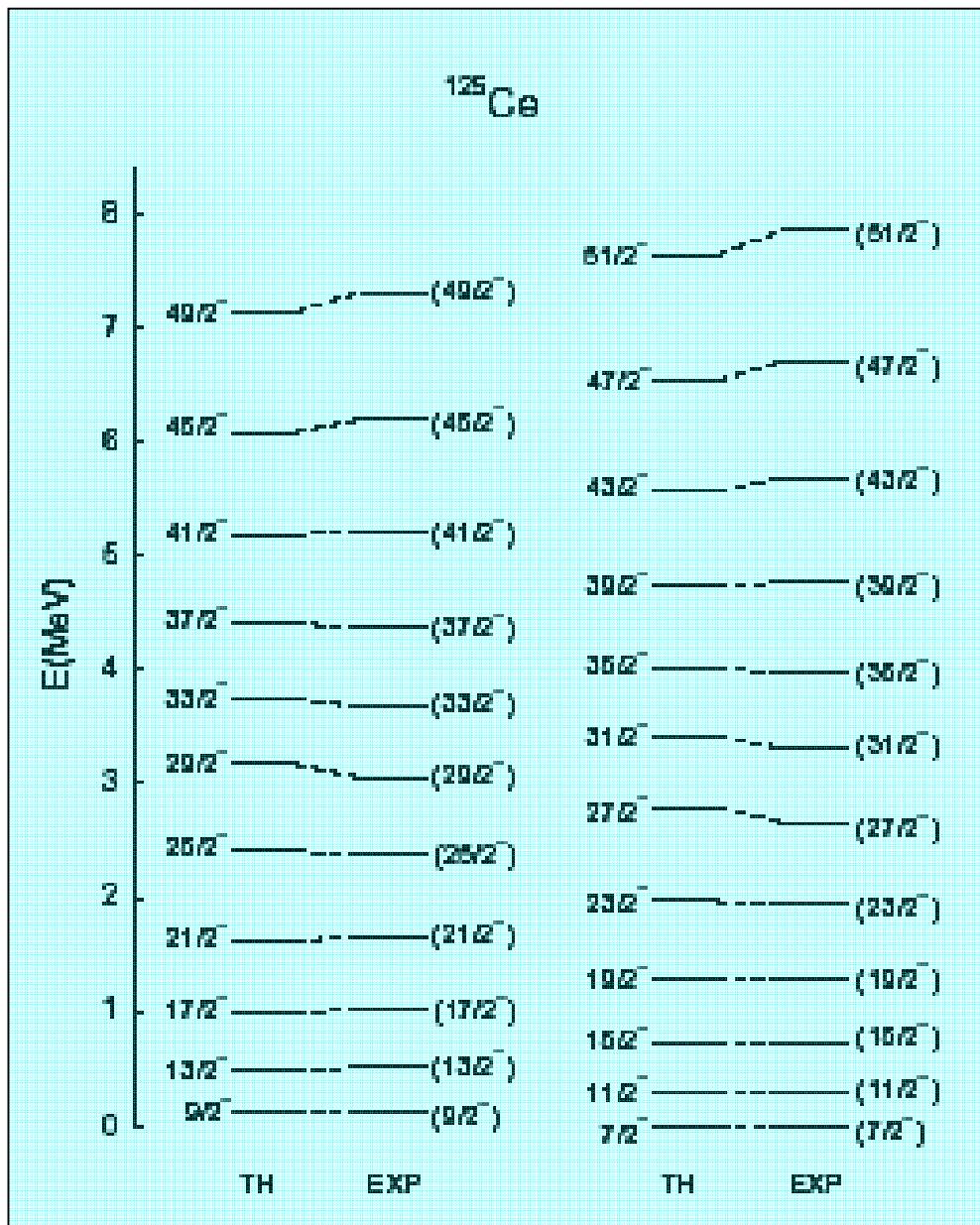
Here: $\alpha = 0.19$ MeV, $\beta = 0.13$ MeV, $\chi = -1.0$ and the boson number $N = 12$.

$$v^2(\pi h_{11/2}) = 0.06$$

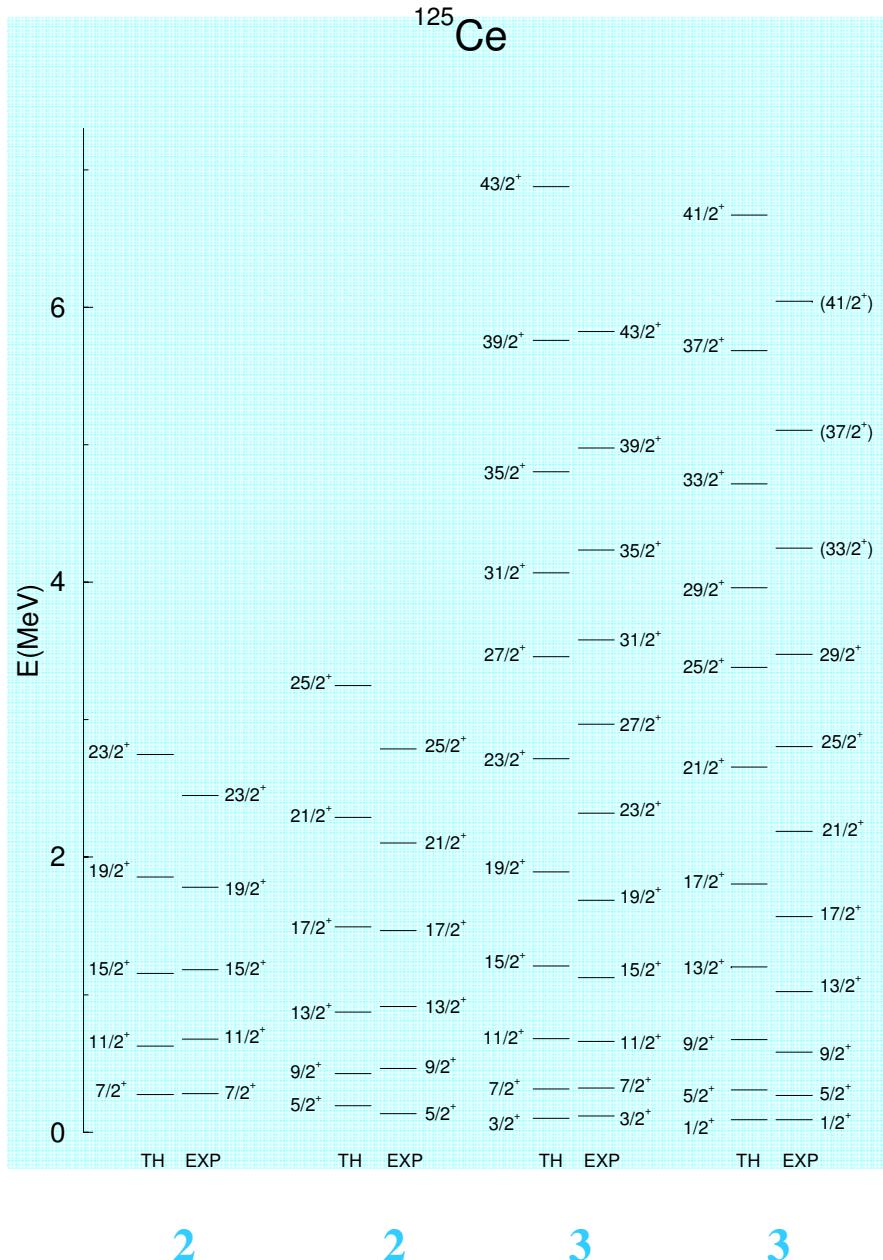
$$\epsilon(\pi h_{11/2}) = 1.70 \text{ MeV}$$

$$v^2(\nu h_{11/2}) = 0.40$$

$$\epsilon(\nu h_{11/2}) = 1.32 \text{ MeV}$$

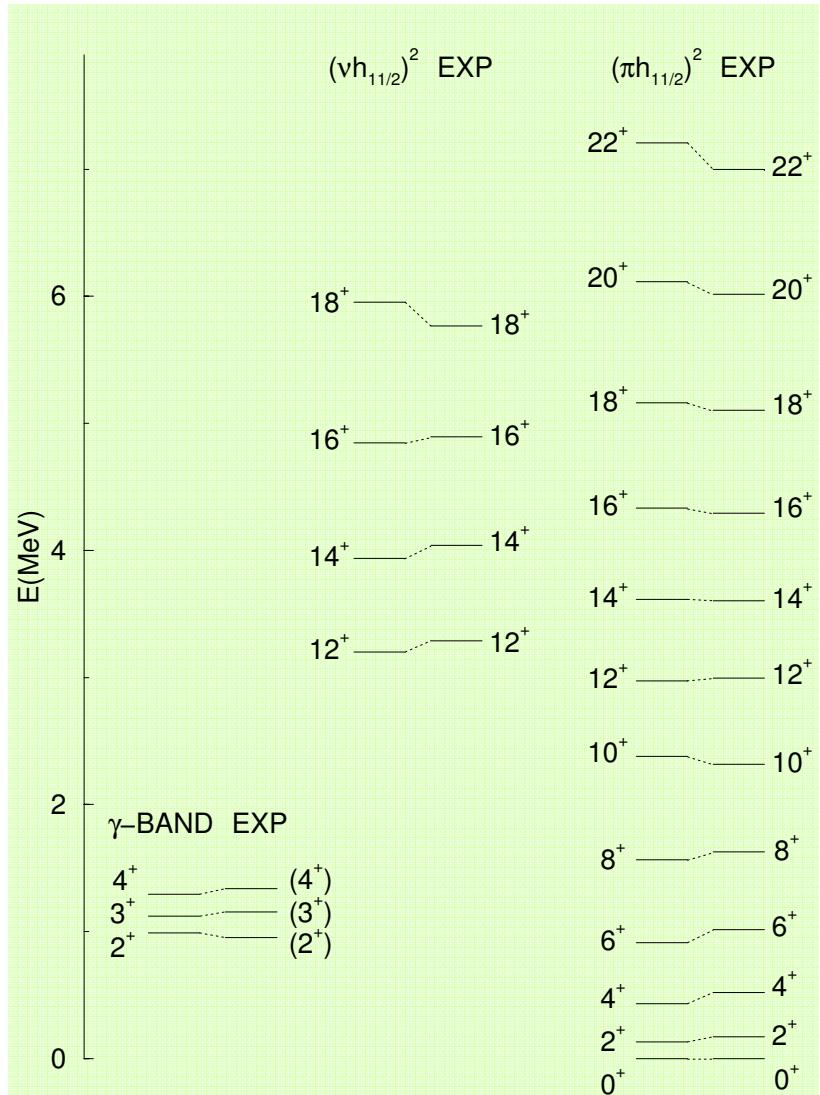


This band is based on the $\nu h_{11/2}$ orbital for the states with $I \leq 27/2^-$, and on the three-fermion configuration $\nu h_{11/2} (\pi h_{11/2})^2$ for $I \geq 29/2^-$. The structure of this band is very simple. The neutron $\nu h_{11/2}$ orbital couples to the yrast sequence of states in the core nucleus ¹²⁴Ce.

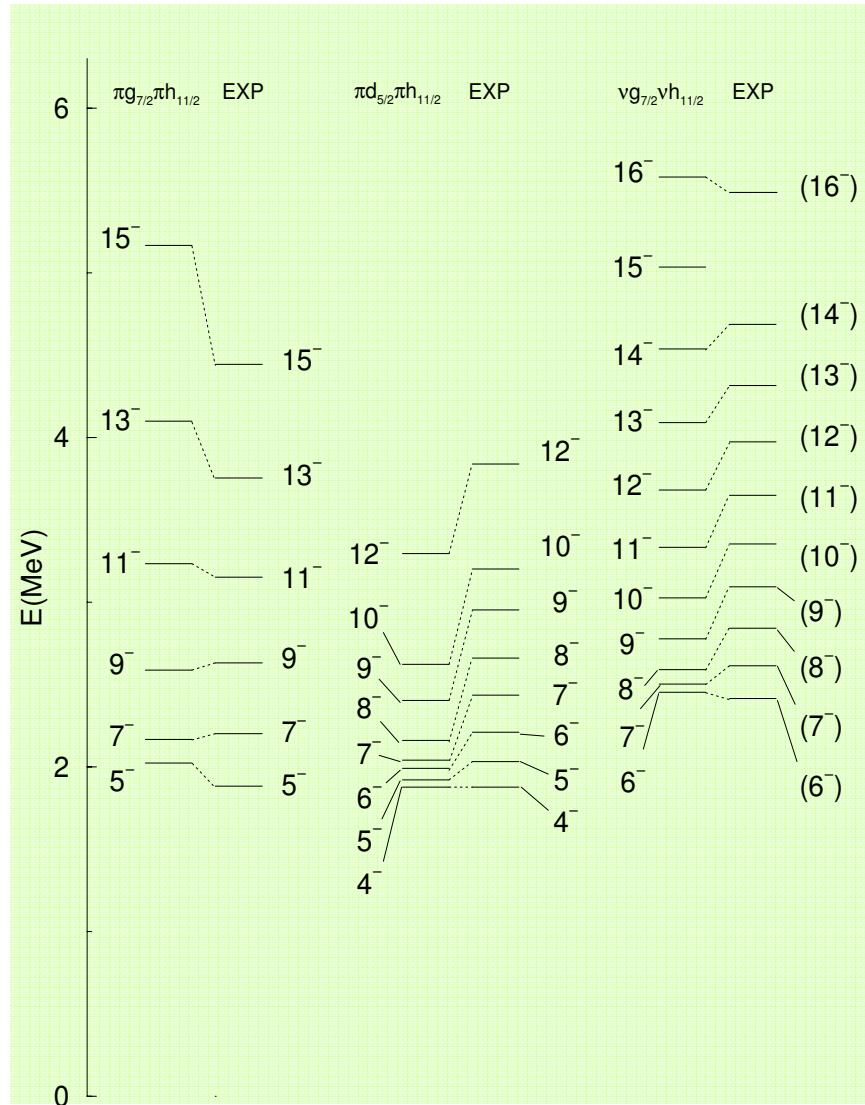


The band 2 is based on the $\nu d_{5/2}$ and $\nu g_{7/2}$ neutron orbitals. The band 3, in addition, contains sizeable components based on the $\nu d_{3/2}$ and $\nu s_{1/2}$ states. While the alignment of a proton pair is not observed in band 2, the states with $I \geq 25/2^+$ of band 3 are based on the one-neutron plus $(\pi h_{11/2})^2$ configuration.

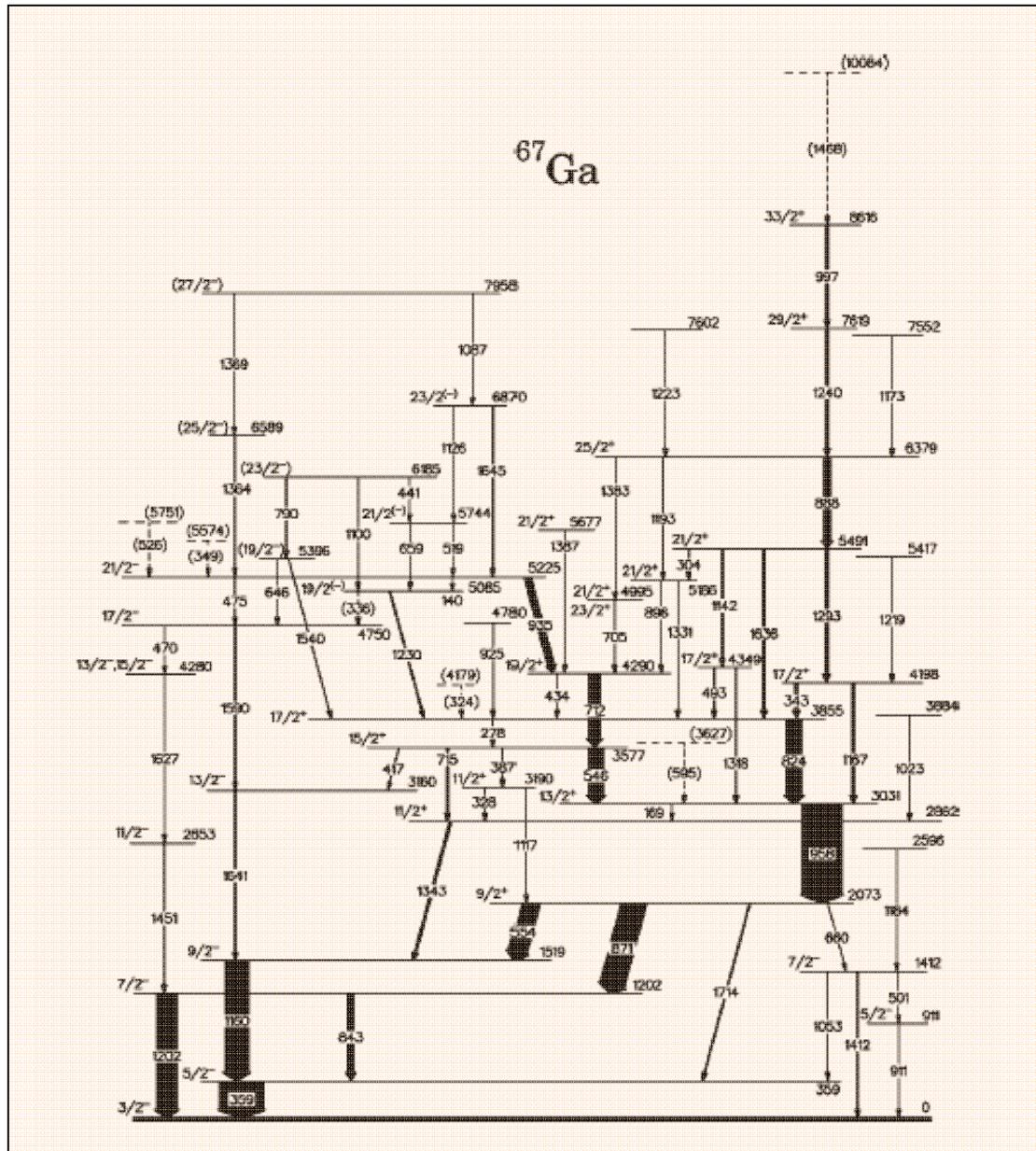
^{126}Ce



^{126}Ce



Transitional nuclei

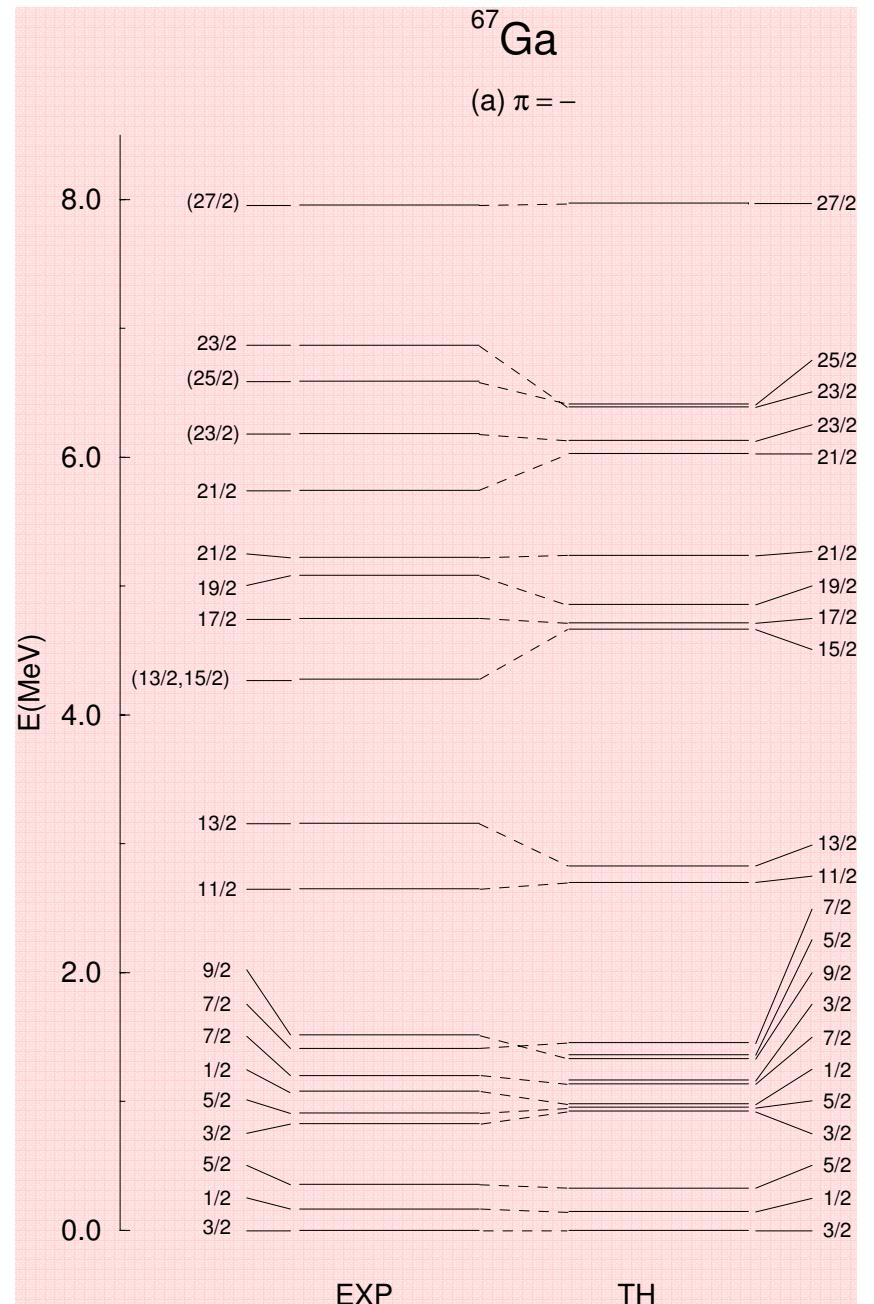


Wave functions

$$\begin{aligned}
|I_k^\pi\rangle = & \sum_{jn_dvR} \xi_{j,n_dvR;I} |\pi\tilde{j}, n_dvR; I\rangle \\
& + \sum_{jj'j''I_{\alpha\alpha}I_{\pi\alpha\alpha}n_dvR} \eta_{jj'j''I_{\alpha\alpha}I_{\pi\alpha\alpha}, n_dvR; I} \\
& \times |[\pi\tilde{j}, (\alpha\tilde{j}', \alpha\tilde{j}'')I_{\alpha\alpha}]I_{\pi\alpha\alpha}, n_dvR; I\rangle
\end{aligned}$$

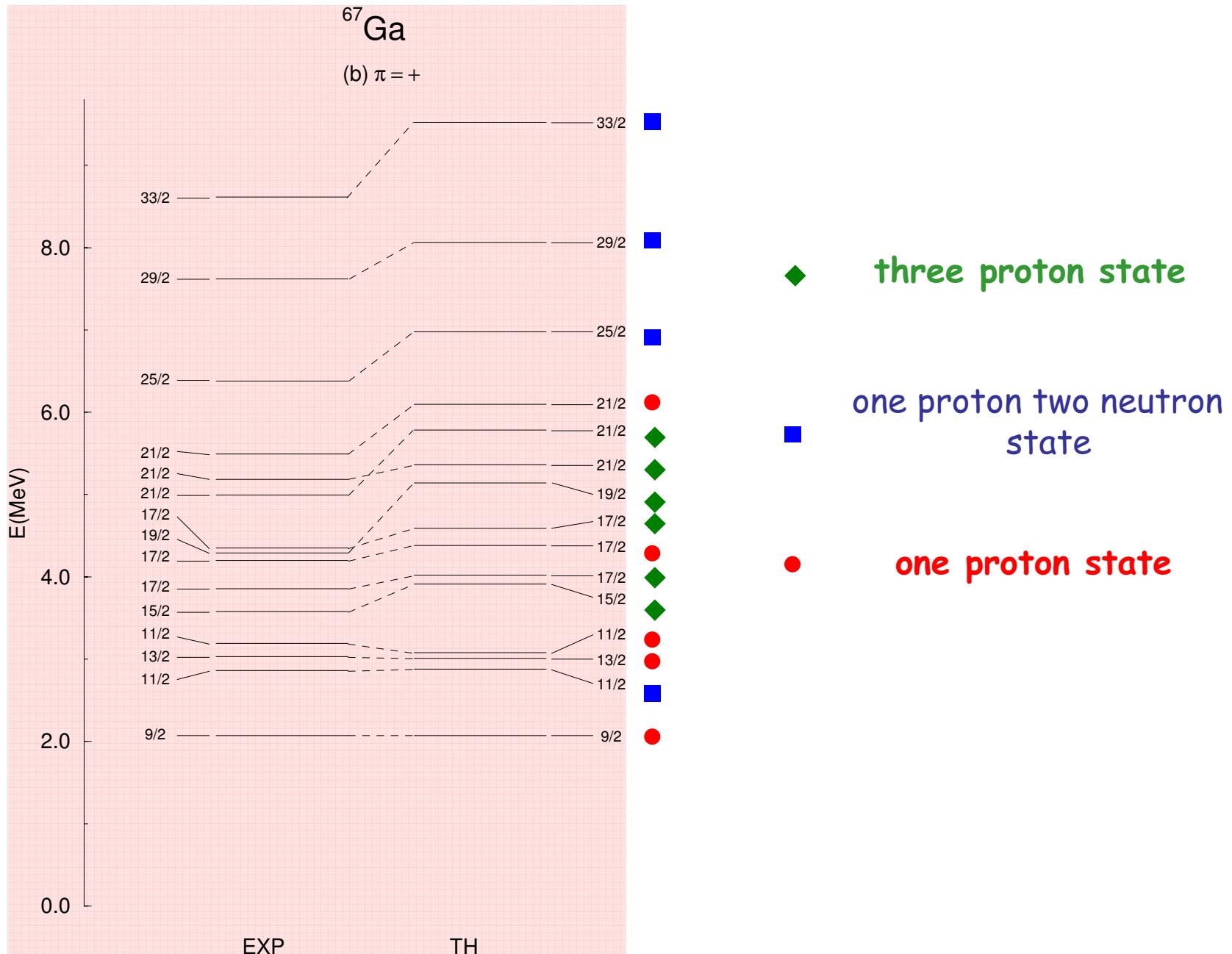
Here $\pi\tilde{j}$ stands for a proton quasiparticle, and $\alpha\tilde{j}', \alpha\tilde{j}''$ for neutron quasiparticles ($\alpha = v$), or proton quasiparticles ($\alpha = \pi$), which are coupled to the angular momentum $I_{\alpha\alpha}$. Angular momenta j and $I_{\alpha\alpha}$ are coupled to the three-quasiparticle angular momentum denoted by $I_{\pi\alpha\alpha}$. In the boson part of the wave function, the n_d -bosons are coupled to the total boson angular momentum R . The additional quantum number v is used to distinguish between the n_d -boson states having the same angular momentum R . We note that the number of s bosons associated with the boson state $|n_dvR\rangle$ is $n_s = N - n_d$, where N is the total number of bosons.

To make it easier to follow the origin of states, for the indexing of the theoretical states we use I_{qp_i} for the quasiparticle+phonon states, I_{bp_i} for proton broken pair states and I_{bn_i} for neutron broken pair states. Here the index i denotes the i th state of the denoted type. In the standard notation I_k , the index k is used as total label obtained from the IBFBPM calculation. The indexing I_k is pointed out only for states where $i \neq k$. Otherwise, the indexes i and k are equal.



one proton two neutron state

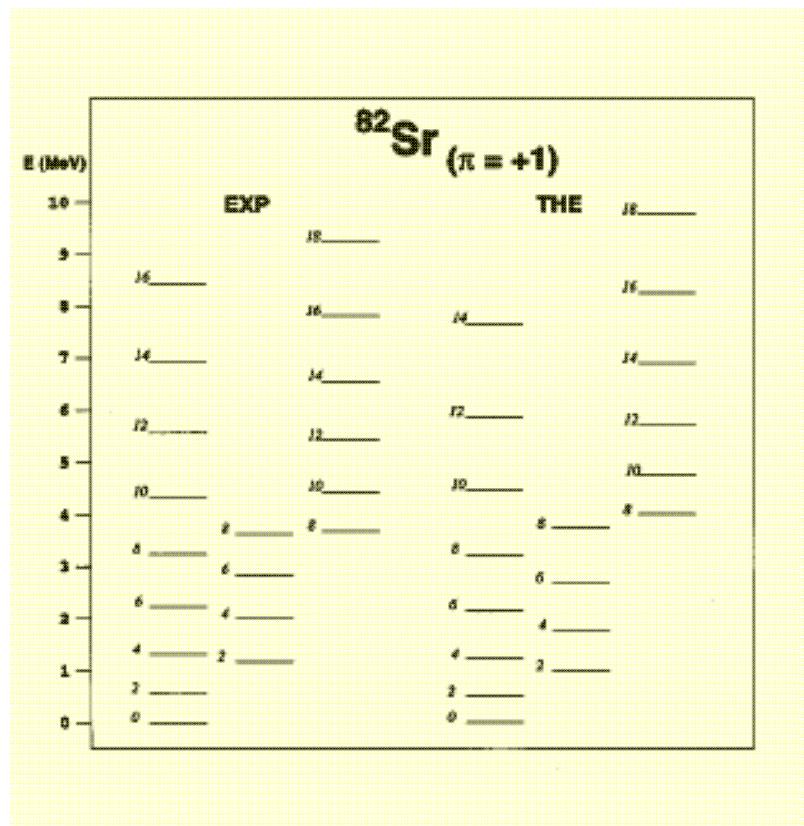
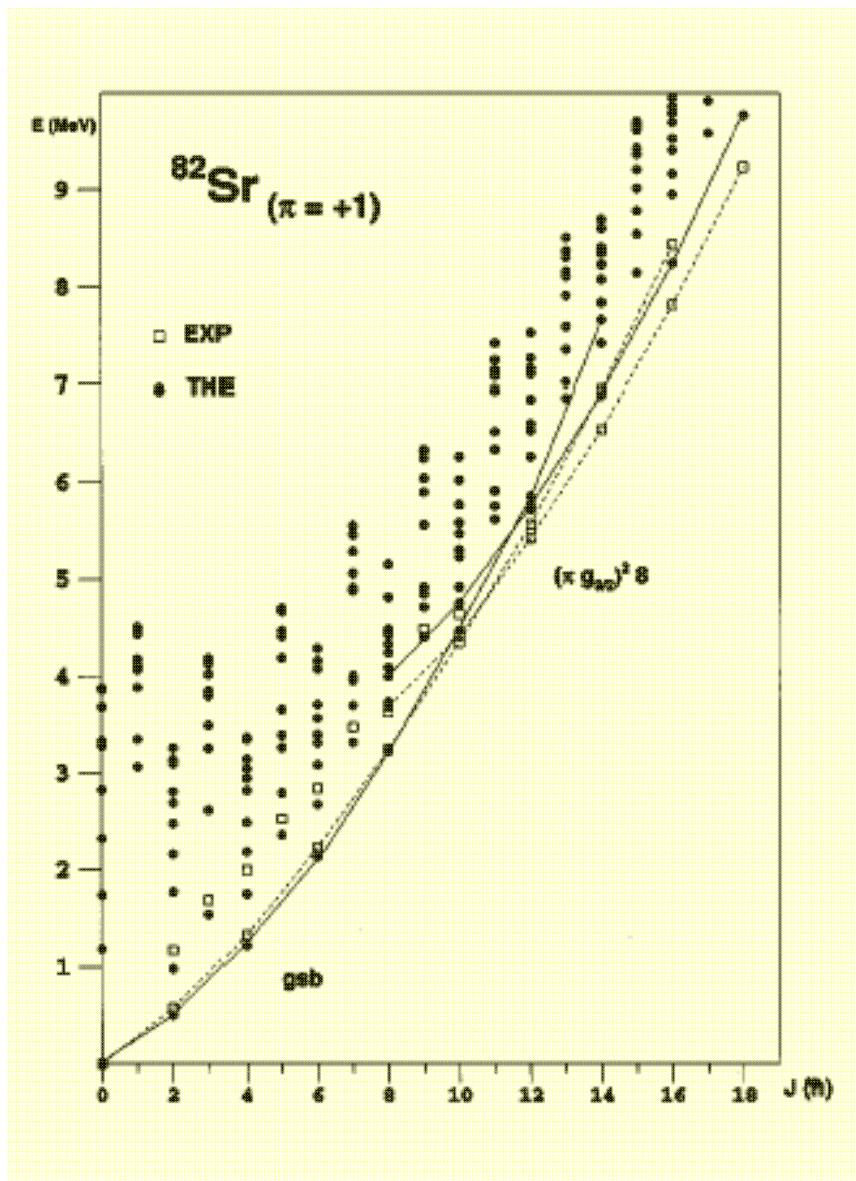
one proton state

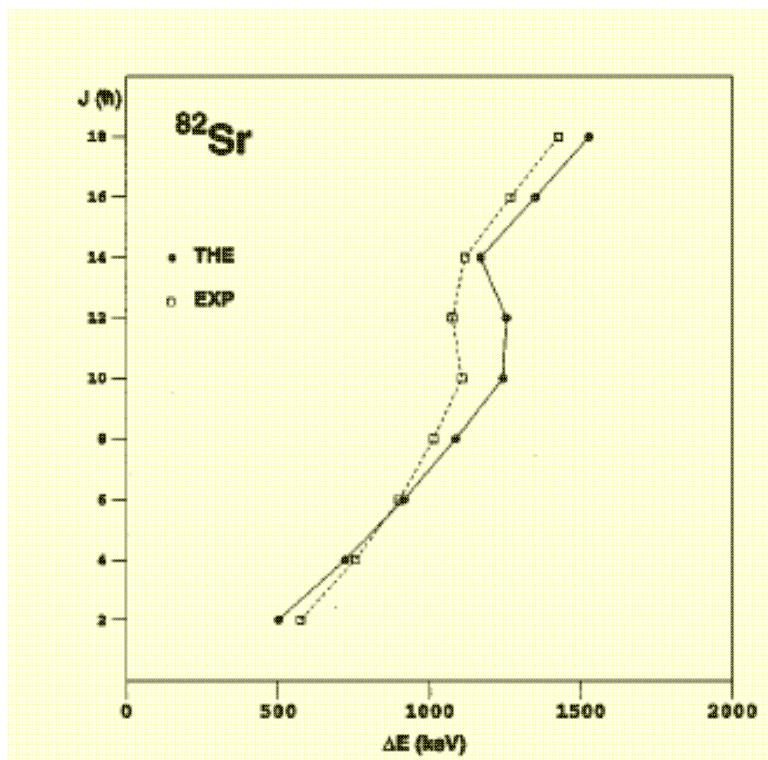


^{67}Ga

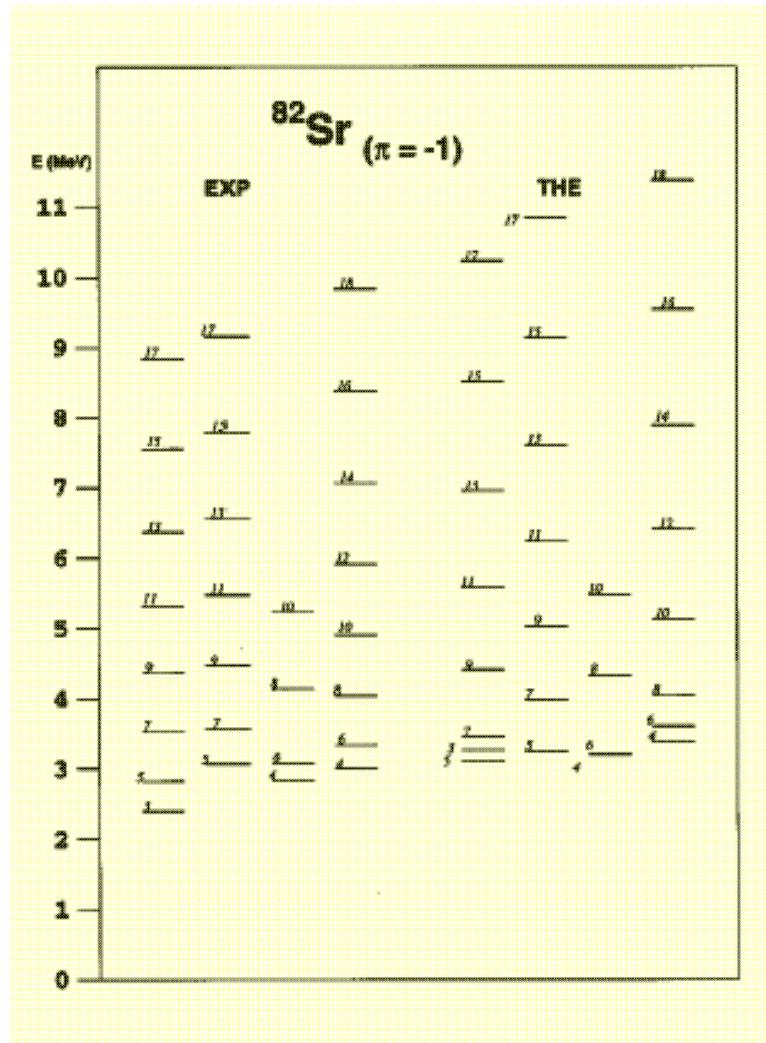
$I_i^\pi \rightarrow I_f^\pi$	$E_i \rightarrow E_f$	$B(E2)(e^2 b^2)$	$B(M1)(\mu_N^2)$	I_γ	
(\hbar)	(\hbar)	Expt. Expt.	IBFBPM	IBFBPM	Expt. IBFBPM
$1/2_{\text{qp}_1}^- \rightarrow 3/2_{\text{qp}_1}^-$	167 → 0	0.013	0.058	100	100
$5/2_{\text{qp}_1}^- \rightarrow 1/2_{\text{qp}_1}^-$	359 → 167	0.027			1.8
→ 0	→ 0	0.003	0.006	100	100
$3/2_{\text{qp}_2}^- \rightarrow 5/2_{\text{qp}_1}^-$	828 → 359	1×10^{-5}	0.013	4	6
→ $1/2_{\text{qp}_1}^-$	→ 167	2×10^{-5}	0.022	10	27
→ $3/2_{\text{qp}_1}^-$	→ 0	0.027	0.028	100	100
$5/2_{\text{qp}_2}^- \rightarrow 3/2_{\text{qp}_2}^-$	911 → 828	0.002	0.306		0.9
→ $5/2_{\text{qp}_1}^-$	→ 359	9×10^{-5}	0.0005	2.2	0.5
→ $1/2_{\text{qp}_1}^-$	→ 167	0.003		2.2	2.0
→ $3/2_{\text{qp}_1}^-$	→ 0	0.029	0.010	100	100
$1/2_{\text{qp}_2}^- \rightarrow 5/2_{\text{qp}_2}^-$	1082 → 911	0.0004			0.0002
→ $3/2_{\text{qp}_2}^-$	→ 828	0.0003	0.533	12	37
→ $5/2_{\text{qp}_1}^-$	→ 359	0.003			1.7
→ $1/2_{\text{qp}_1}^-$	→ 167		0.030	100	100
→ $3/2_{\text{qp}_1}^-$	→ 0	0.021	0.005	33	122
$7/2_{\text{qp}}^- \rightarrow 5/2_{\text{qp}_2}^-$	1202 → 911	0.002	0.131		6
→ $3/2_{\text{qp}_2}^-$	→ 828	0.0004			0.0004
→ $5/2_{\text{qp}_1}^-$	→ 359	0.002	0.012	31	14
→ $3/2_{\text{qp}_1}^-$	→ 0	0.033		100	100
$7/2_{\text{qp}}^- \rightarrow 7/2_{\text{qp}_1}^-$	1412 → 1202	0.002	0.002		0.06
→ $5/2_{\text{qp}_2}^-$	→ 911	0.002	0.011	20	5
→ $3/2_{\text{qp}_2}^-$	→ 828	0.001			0.2
→ $5/2_{\text{qp}_1}^-$	→ 359	0.024	0.008	100	100
→ $3/2_{\text{qp}_1}^-$	→ 0	3×10^{-5}		20	0.3
$9/2_{\text{qp}} \rightarrow 7/2_{\text{qp}_2}^-$	1519 → 1412	0.003	0.018		0.03
→ $7/2_{\text{qp}_1}^-$	→ 1202	0.0007	0.005		0.2
→ $5/2_{\text{qp}_2}^-$	→ 911	0.0003			0.02
→ $5/2_{\text{qp}_1}^-$	→ 359	0.049		100	100
$11/2_{\text{qp}_1} \rightarrow 9/2_{\text{qp}_1}^-$	2653 → 1519	0.001	0.007		5
→ $7/2_{\text{qp}_2}^-$	→ 1412	0.0004			0.4
→ $7/2_{\text{qp}_1}^-$	→ 1202	0.051		100	100
$13/2_{\text{qp}_1} \rightarrow 11/2_{\text{qp}_1}^-$	3160 → 2653	0.0003	0.004		0.1
→ $9/2_{\text{qp}_1}^-$	→ 1519	0.065		100	100
$15/2_{\text{qp}_1}^- \rightarrow 13/2_{\text{qp}_1}^-$	4280 → 3160	0.0009	0.004		1.5
→ $11/2_{\text{qp}_1}^-$	→ 2653	0.056		100	100
$17/2_{\text{qp}_1}^- \rightarrow 15/2_{\text{qp}_1}^-$	4750 → 4280	8×10^{-5}	0.002	12	0.04
→ $13/2_{\text{qp}_1}^-$	→ 3160	0.067		100	100
$21/2_{\text{bn}_1}^- \rightarrow 21/2_{\text{bn}_1}^-$	5744 → 5225	2×10^{-6}	0.003	100	100
→ $19/2_{\text{bn}_1}^-$	→ 5085	0.0003	0.002	24	161
$23/2_{\text{bn}_1}^- \rightarrow 21/2_{\text{bn}_2}^-$	6185 → 5744	0.0004	0.718	30	89
→ $21/2_{\text{bn}_1}^-$	→ 5225	0.0007	0.007		10
→ $19/2_{\text{bn}_1}^-$	→ 5085	0.025		40	40
$25/2_{\text{bn}_1}^- \rightarrow 23/2_{\text{bn}_1}^-$	6589 → 6185	0.0006	0.006		0.4
→ $21/2_{\text{bn}_2}^-$	→ 5744	8×10^{-6}			0.002
→ $21/2_{\text{bn}_1}^-$	→ 5225	0.036		100	100
$23/2_{\text{bn}_2}^- \rightarrow 25/2_{\text{bn}_1}^-$	6870 → 6589	0.007	0.598		20
→ $23/2_{\text{bn}_1}^-$	→ 6185	0.0002	0.0004		0.2
→ $21/2_{\text{bn}_2}^-$	→ 5744	4×10^{-5}	0.003	36	6
→ $21/2_{\text{bn}_1}^-$	→ 5225	0.008	0.0003	100	100
→ $19/2_{\text{bn}_1}^-$	→ 5085	3×10^{-7}			0.005

$I_i^\pi \rightarrow I_f^\pi$	$E_i \rightarrow E_f$	$B(E2)(e^2 b^2)$	$B(M1)(\mu_N^2)$	I_γ	
(\hbar)	(\hbar)	Expt. Expt.	IBFBPM	IBFBPM	Expt. IBFBPM
$27/2_{\text{bn}_2}^- \rightarrow 23/2_{\text{bn}_2}^-$	7958 → 6870	0.041			50
→ $25/2_{\text{bn}_1}^-$	→ 6589	0.005	0.0001	100	100
→ $23/2_{\text{bn}_1}^-$	→ 6185	9×10^{-6}			0.6
$13/2_{\text{qp}_1}^+ \rightarrow 9/2_{\text{qp}_1}^+$	3031 → 2073	0.037			100
$11/2_{\text{qp}_1}^+ \rightarrow 13/2_{\text{qp}_1}^+$	3190 → 3031	0.001	0.119		0.4
→ $9/2_{\text{qp}_1}^+$	→ 2073	0.041	0.058	100	100
$15/2_{\text{bp}_1}^+ \rightarrow 11/2_{\text{qp}_1}^+$	3577 → 3190	0.002			3.4
→ $13/2_{\text{qp}_1}^+$	→ 3031	0.0009	4×10^{-5}	100	100
$17/2_{\text{bp}_1}^+ \rightarrow 15/2_{\text{bp}_1}^+$	3855 → 3577	0.006	0.039	1.2	22
→ $13/2_{\text{qp}_1}^+$	→ 3031	0.015			100
$17/2_{\text{qp}_1}^+ \rightarrow 17/2_{\text{bp}_1}^+$	4198 → 3855	6×10^{-5}	0.095	90	6
→ $15/2_{\text{bp}_1}^+$	→ 3577	0.001	0.028		11
→ $13/2_{\text{qp}_1}^+$	→ 3031	0.043			100
$19/2_{\text{bp}_1}^+ \rightarrow 17/2_{\text{qp}_1}^+$	4290 → 4198	0.002	0.001		0.02
→ $17/2_{\text{bp}_1}^+$	→ 3855	0.004	9×10^{-5}	1.6	1.2
→ $15/2_{\text{bp}_1}^+$	→ 3577	0.029			100
$17/2_{\text{bp}_1}^+ \rightarrow 19/2_{\text{bp}_1}^+$	4349 → 4290	6×10^{-5}	0.004		0.02
→ $17/2_{\text{qp}_1}^+$	→ 4198	3×10^{-5}	0.008		0.8
→ $17/2_{\text{bp}_1}^+$	→ 3855	0.0004	0.001	52	5
→ $15/2_{\text{bp}_1}^+$	→ 3577	0.0001	0.012	< 10	146
→ $13/2_{\text{qp}_1}^+$	→ 3031	0.0014			100
$21/2_{\text{bp}_1}^+ \rightarrow 17/2_{\text{bp}_2}^+$	4995 → 4349	0.035			24
→ $19/2_{\text{bp}_1}^+$	→ 4290	2×10^{-5}	0.033	100	100
→ $17/2_{\text{bp}_1}^+$	→ 4198	0.009			17
→ $17/2_{\text{bp}_1}^+$	→ 3855	0.0002			2.7
$21/2_{\text{bp}_1}^+ \rightarrow 21/2_{\text{bp}_2}^+$	5186 → 4995	0.0002	0.005		0.03
→ $17/2_{\text{bp}_2}^+$	→ 4349	0.0002			0.04
→ $19/2_{\text{bp}_1}^+$	→ 4290	0.003	0.106	31	67
→ $17/2_{\text{bp}_1}^+$	→ 4198	0.0003			0.2
→ $17/2_{\text{bp}_1}^+$	→ 3855	0.040			100
$21/2_{\text{qp}_1}^+ \rightarrow 21/2_{\text{bp}_1}^+$	5491 → 5186	0.0002	0.029	2.1	0.7
→ $21/2_{\text{bp}_2}^+$	→ 4995	1×10^{-5}	0.001		0.1
→ $17/2_{\text{bp}_2}^+$	→ 4349	0.008		27	9
→ $19/2_{\text{bp}_1}^+$	→ 4290	0.0004	0.004		7
→ $17/2_{\text{bp}_1}^+$	→ 4198	0.048		100	100
→ $17/2_{\text{bp}_1}^+$	→ 3855	0.002		67	14



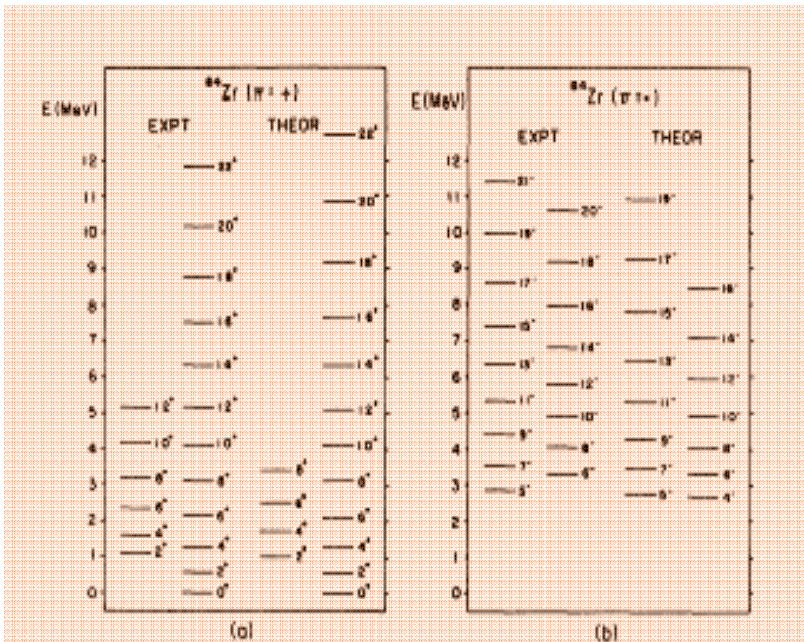
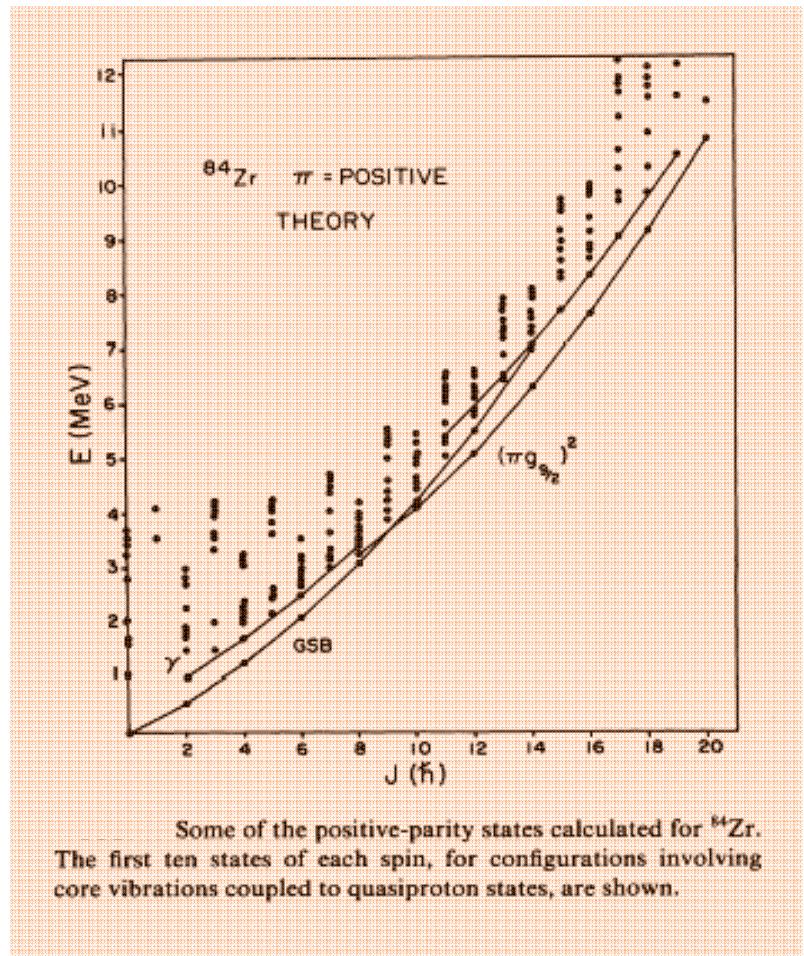


Angular momentum as a function of transition energy
 $\Delta E(J) = E(J) - E(J-2)$, for yrast states in ^{82}Sr .

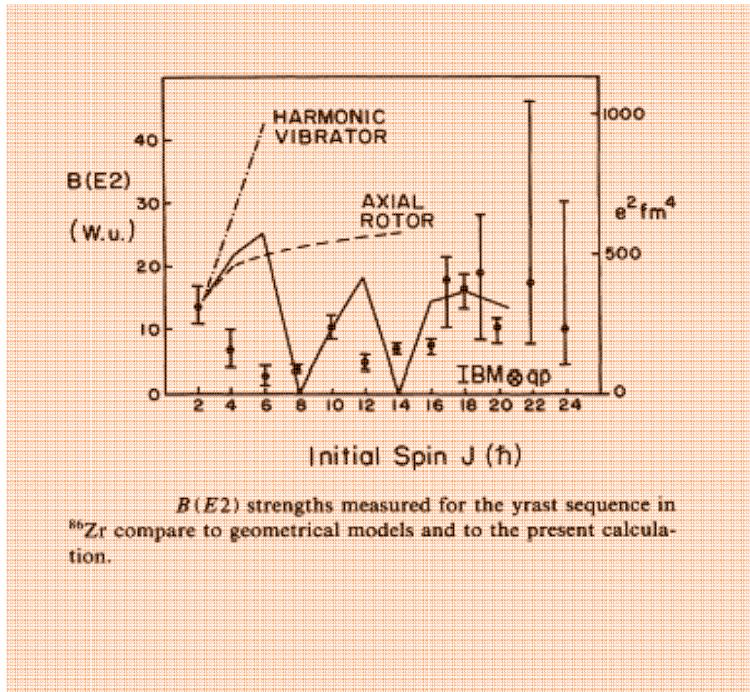


Interactions ?

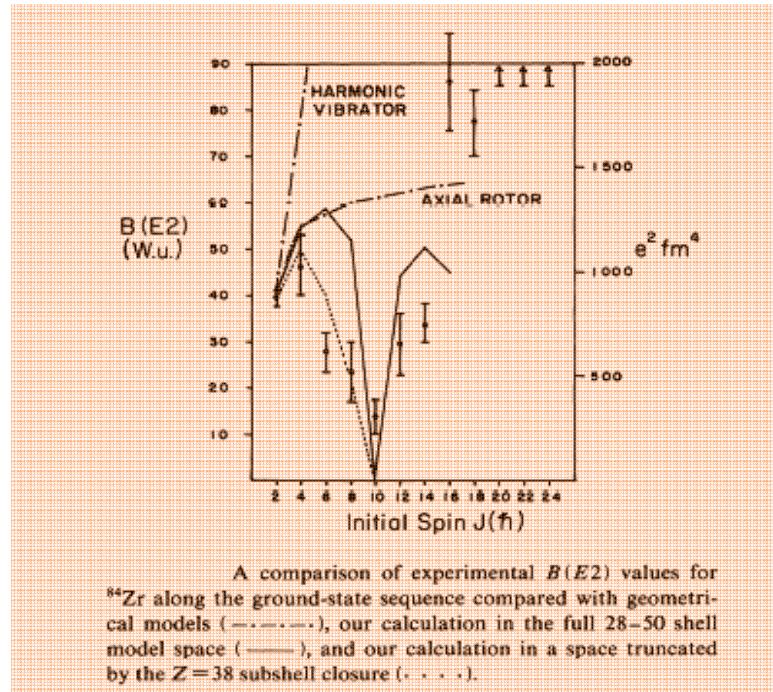
The strength of the exchange interaction is adjusted to reproduce the energy spacings of negative-parity states in ^{82}Sr . It differs considerably from that used for odd-even isotopes. In order to understand the origin of this anomaly, one may consider the coupling of unpaired protons to proton bosons in the ^{82}Sr . To create multiproton states in the even-even nucleus we destroy proton bosons and the effective coupling of the exchange interaction is reduced. In the IBM-2 framework this reduction would be implicit and no adjustment of strength parameters should be needed. However, in our model based on IBM-1, we couple to all the core bosons, irrespective of their nature and the suppression of coupling is greatly diminished. Thus, the need to empirically reduce the strength of coupling parameter. This effect should be especially pronounced near closed shells, and in our case the reduction of the exchange interaction might be due to the subshell closure at $Z=40$.



A detailed comparison of states calculated for ^{84}Zr (a) in the positive-parity sequence (b) in the negative-parity sequence.



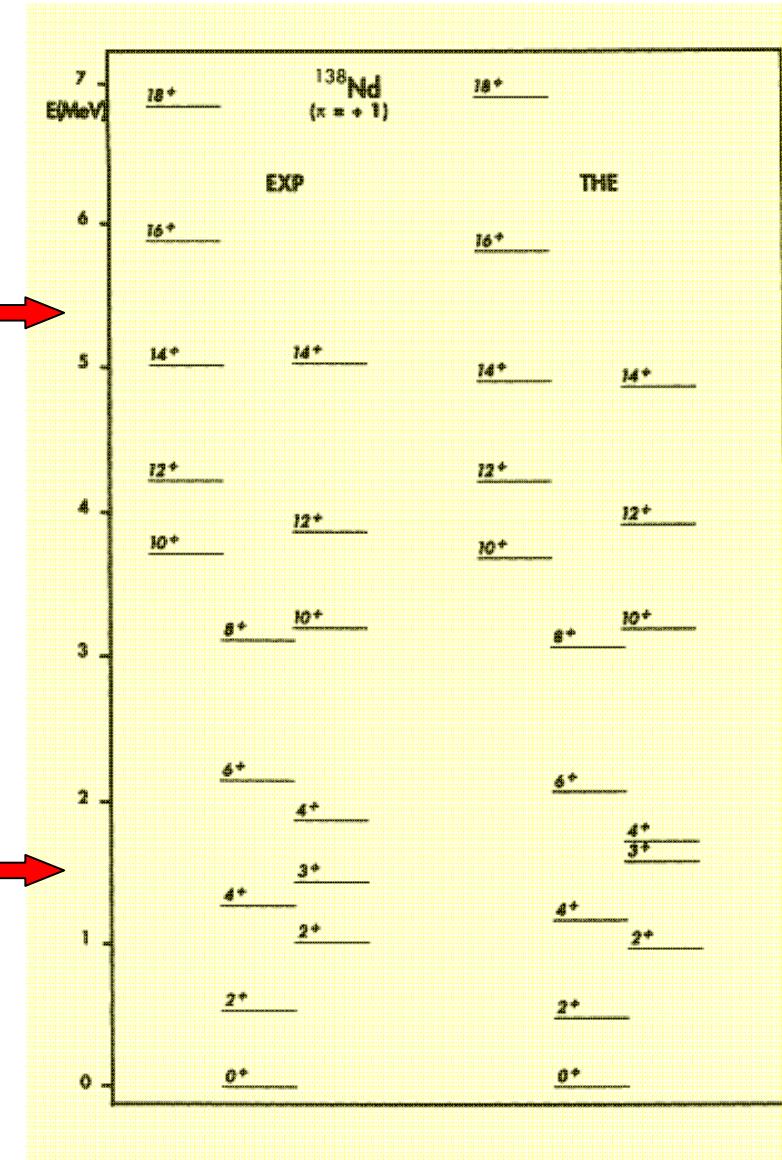
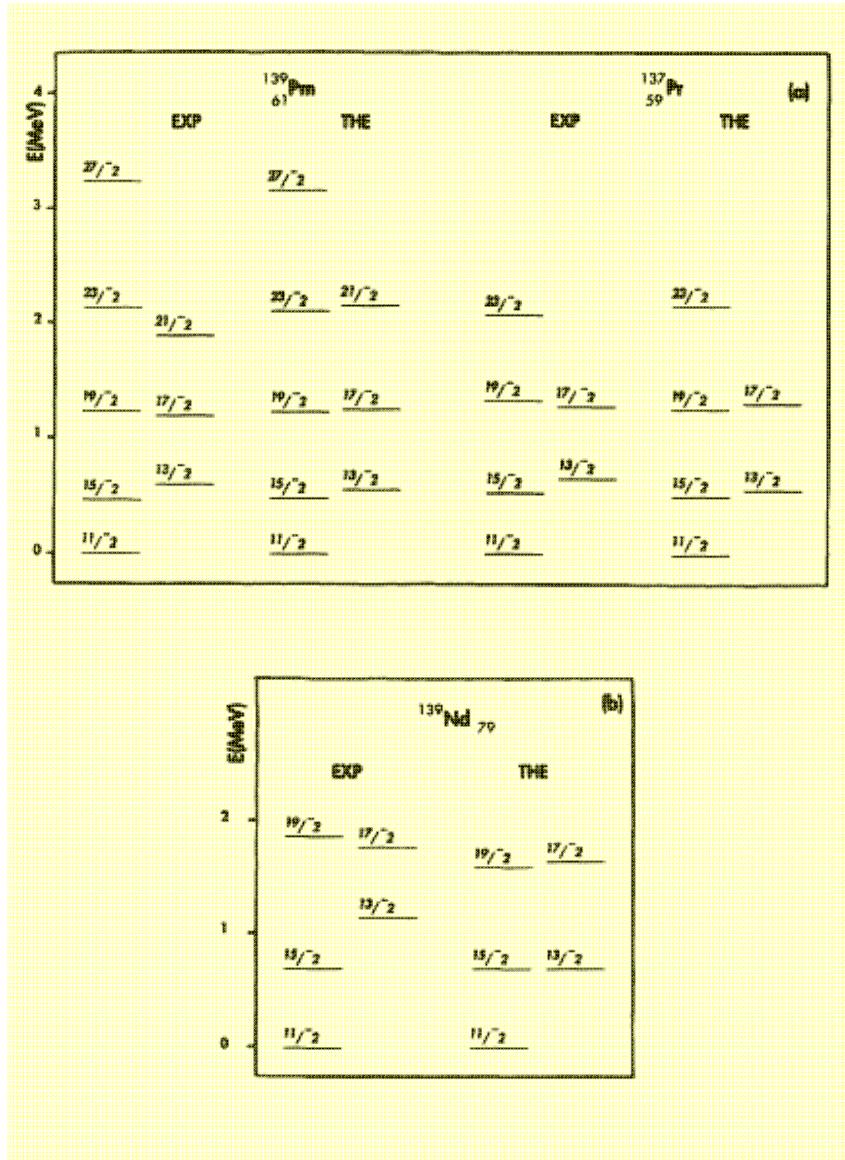
$B(E2)$ strengths measured for the yrast sequence in ^{86}Zr compare to geometrical models and to the present calculation.

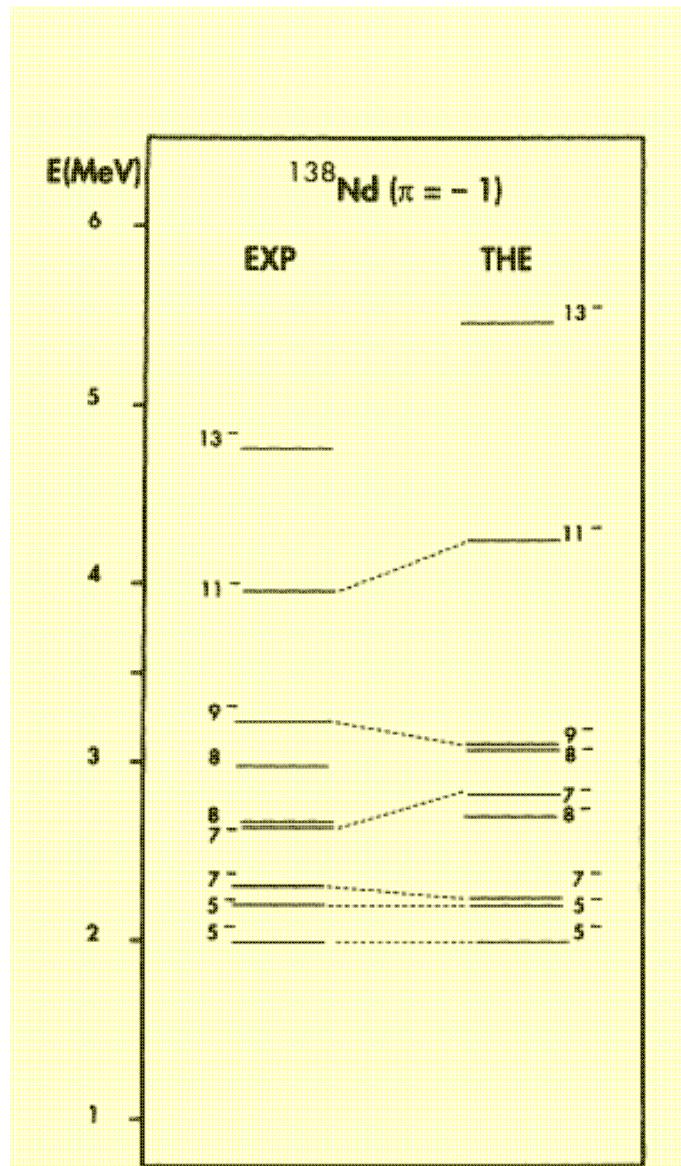
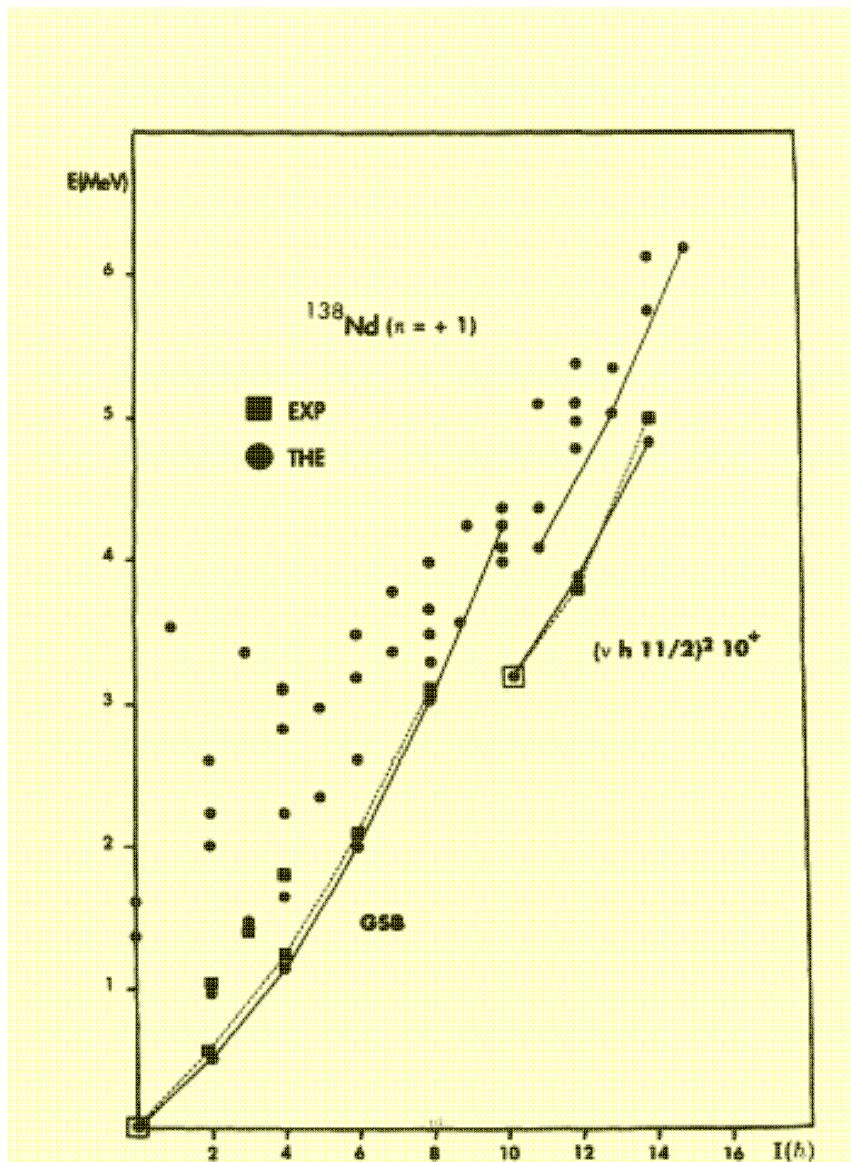


A comparison of experimental $B(E2)$ values for ^{84}Zr along the ground-state sequence compared with geometrical models (----), our calculation in the full 28–50 shell model space (—), and our calculation in a space truncated by the $Z=38$ subshell closure (· · ·).

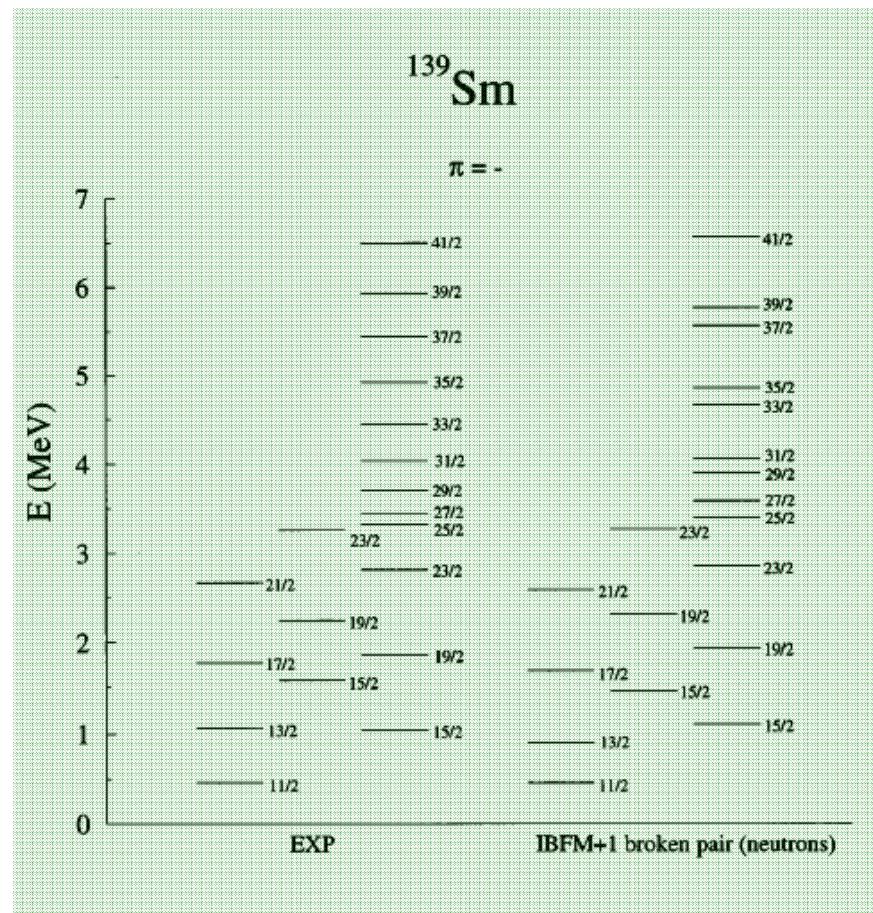
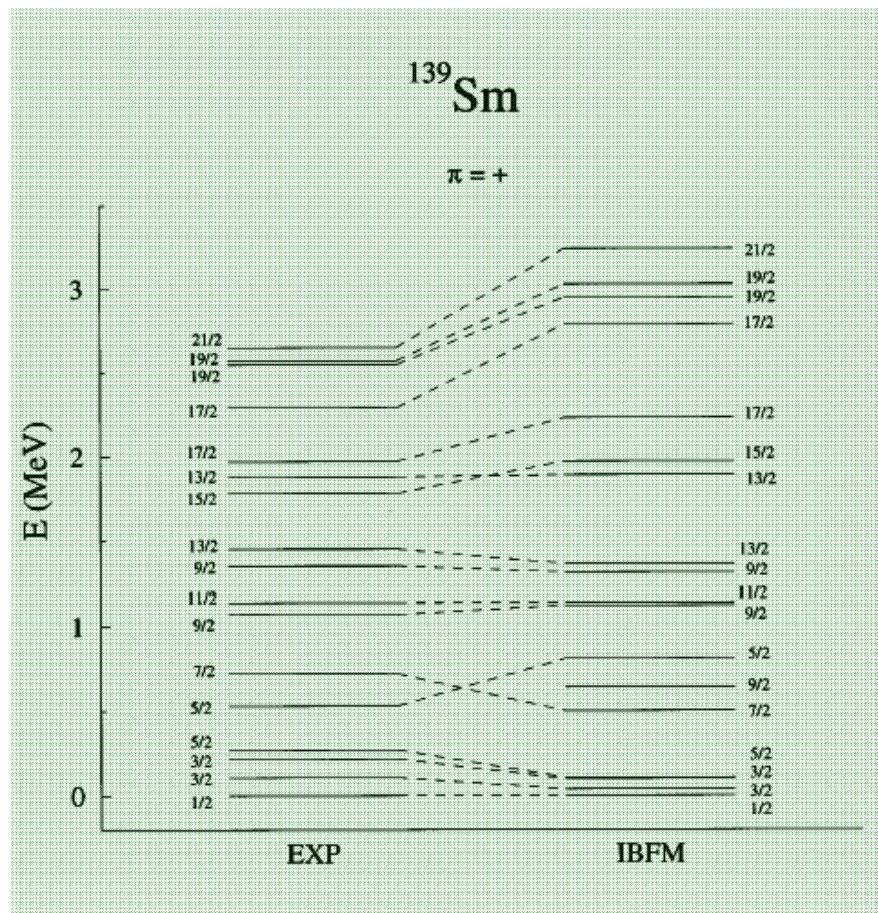
The pair breaking interaction V^{mix} , which mixes states with different number of fermions, and conserves only the total number of valence nucleons, in general does not induce sufficient mixing as can be deduced, for example, from observed transition strengths. It is the lowest order contribution to a pair-breaking interaction. Since the interaction contains only fermion operators of rank 0 and 2, it cannot connect in first order the ground state band with two-fermion states of higher fermion angular momenta. In order to enhance the mixing, interactions that contain fermion operators of higher rank could be included in the model Hamiltonian. However, such an interaction would also require higher order boson operators, with parameters that cannot be determined from available experimental data, or from the intrinsic structure of the model.

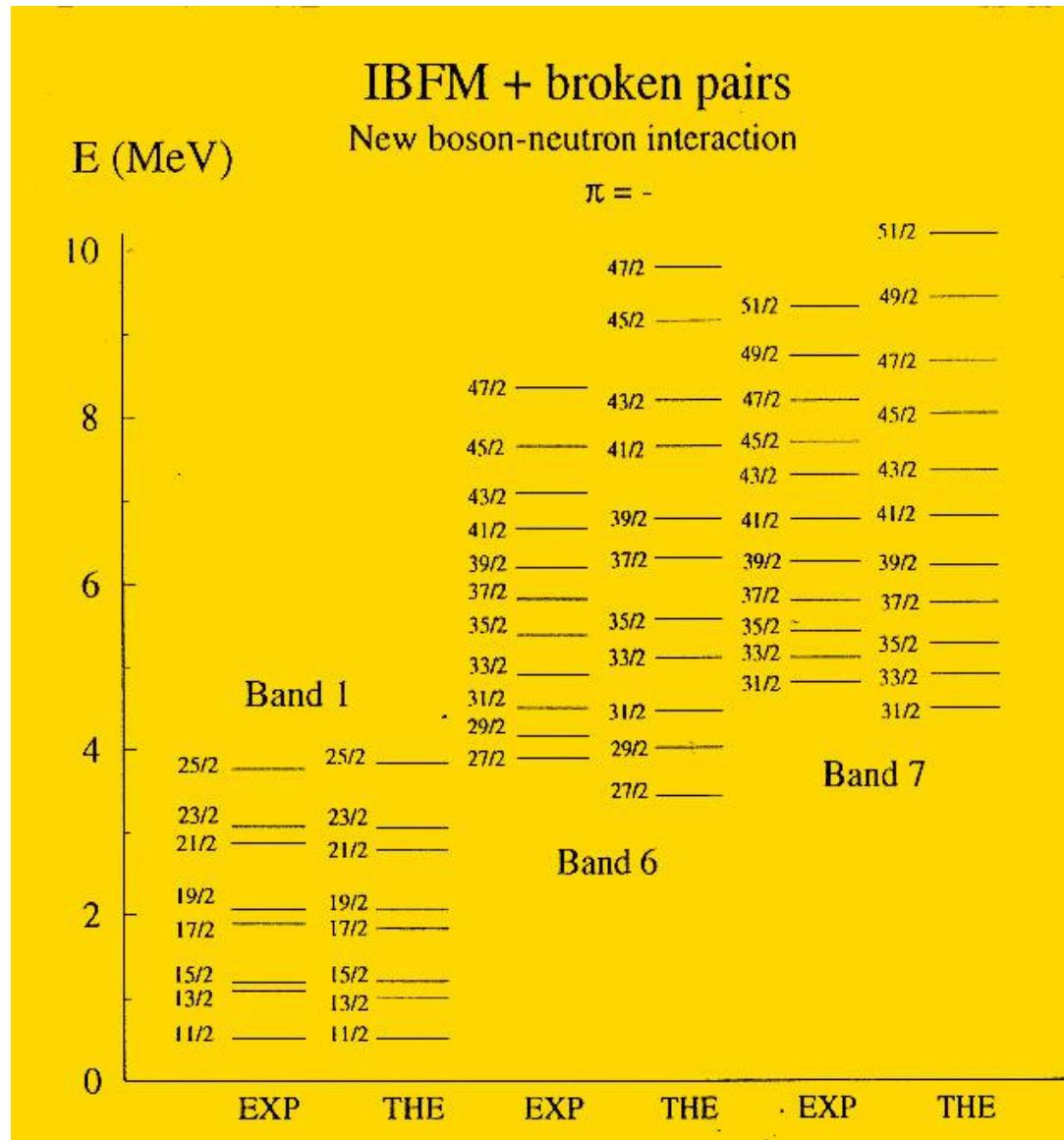
^{138}Nd





h11/2 d3/2 neutron pair



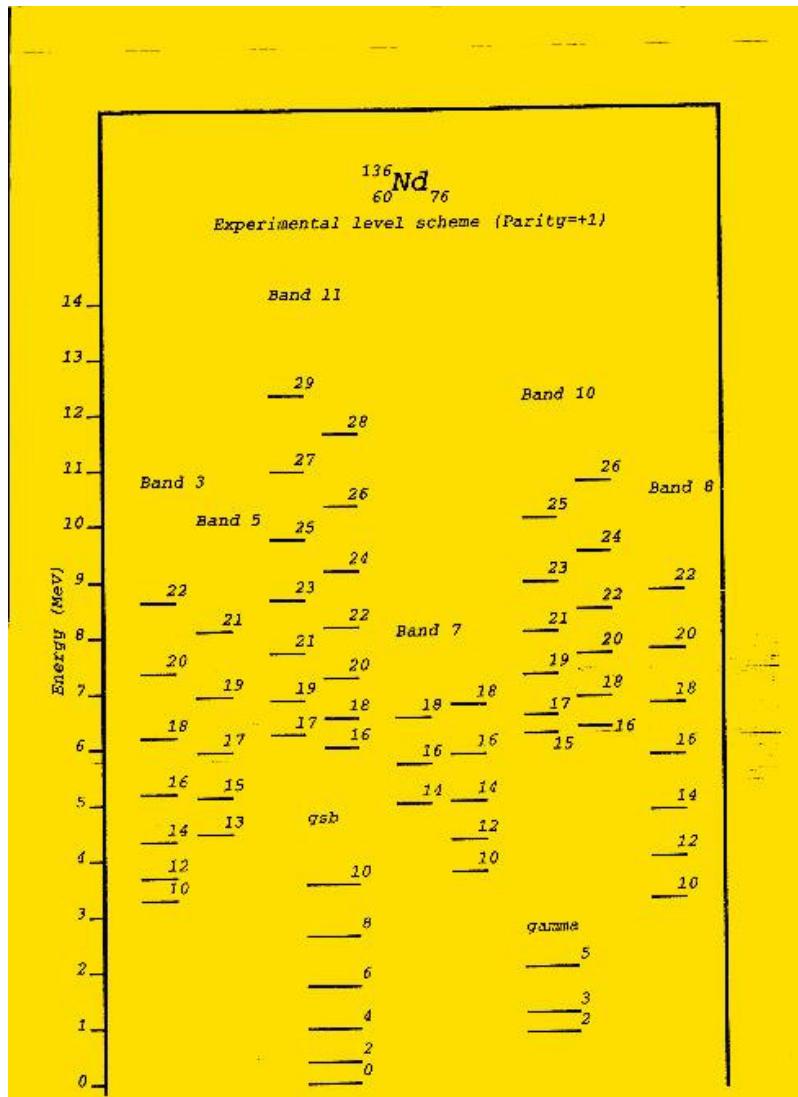


$^{137}\text{Nd}_{77}$

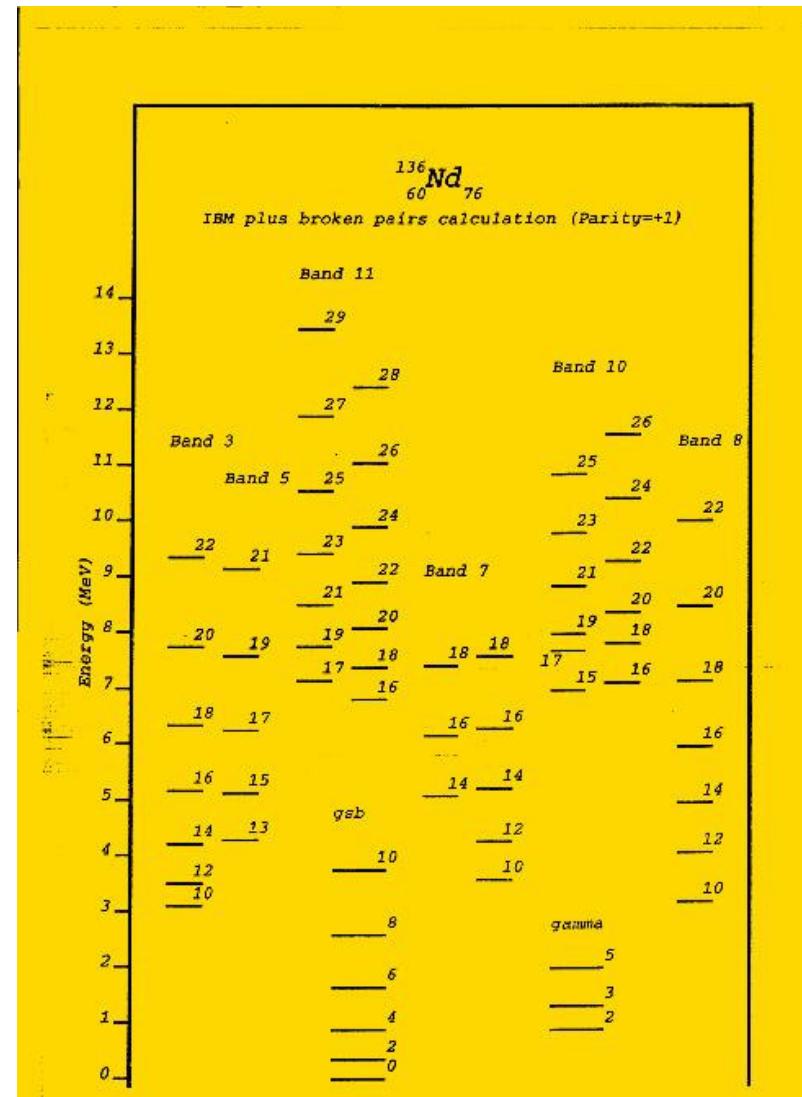
Band 1 ($\nu \text{ h}11/2$)

Band 6 ($\nu \text{ h}11/2)^3$

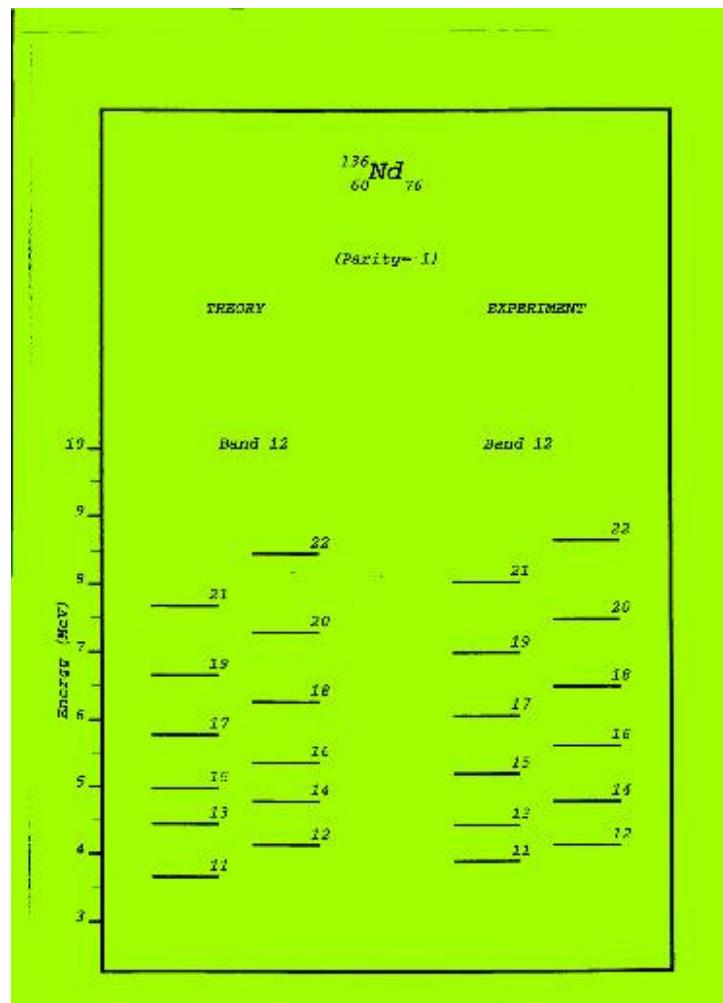
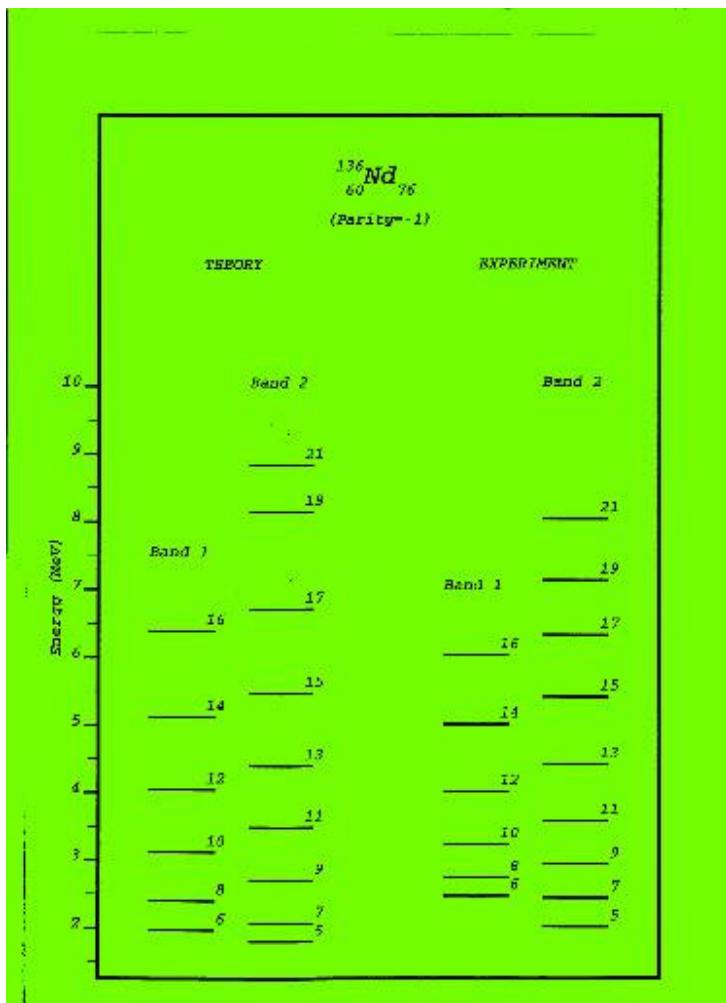
Band 7 ($\nu \text{ h}11/2$) ($\pi \text{ h}11/2)^2$



Bands 3, 5, 7 $(\pi h11/2)^2$
 Band 8 $(\nu h11/2)^2$

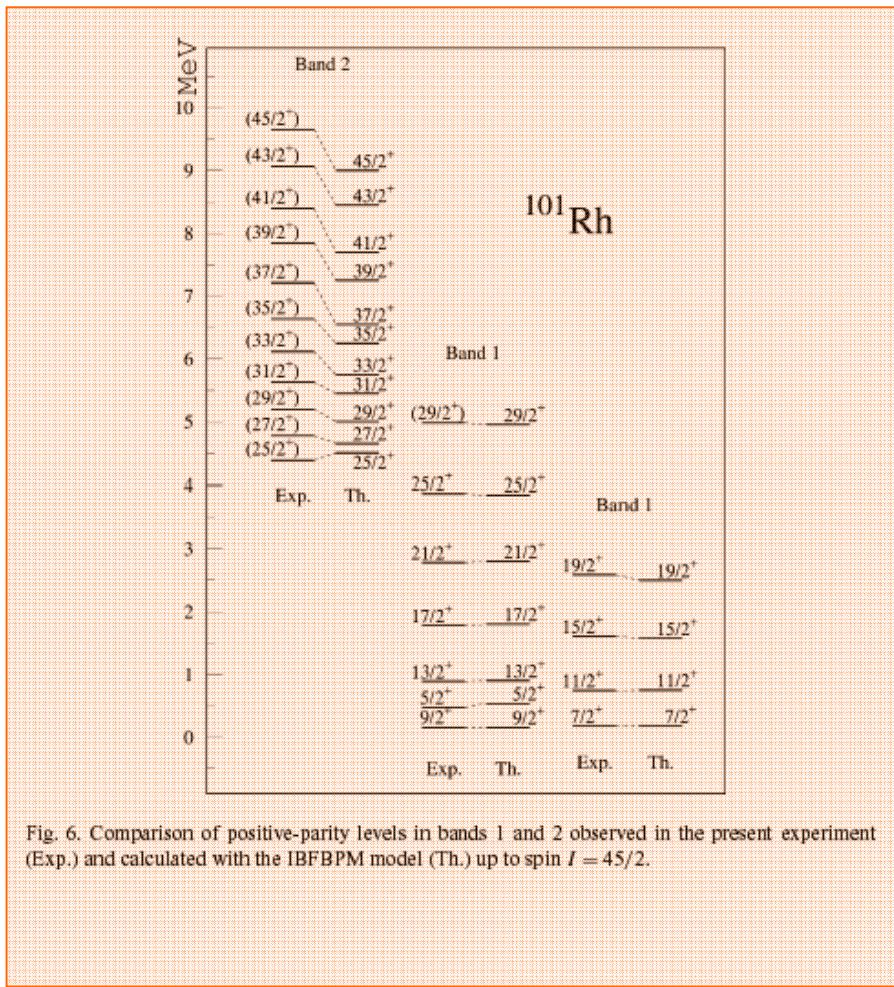
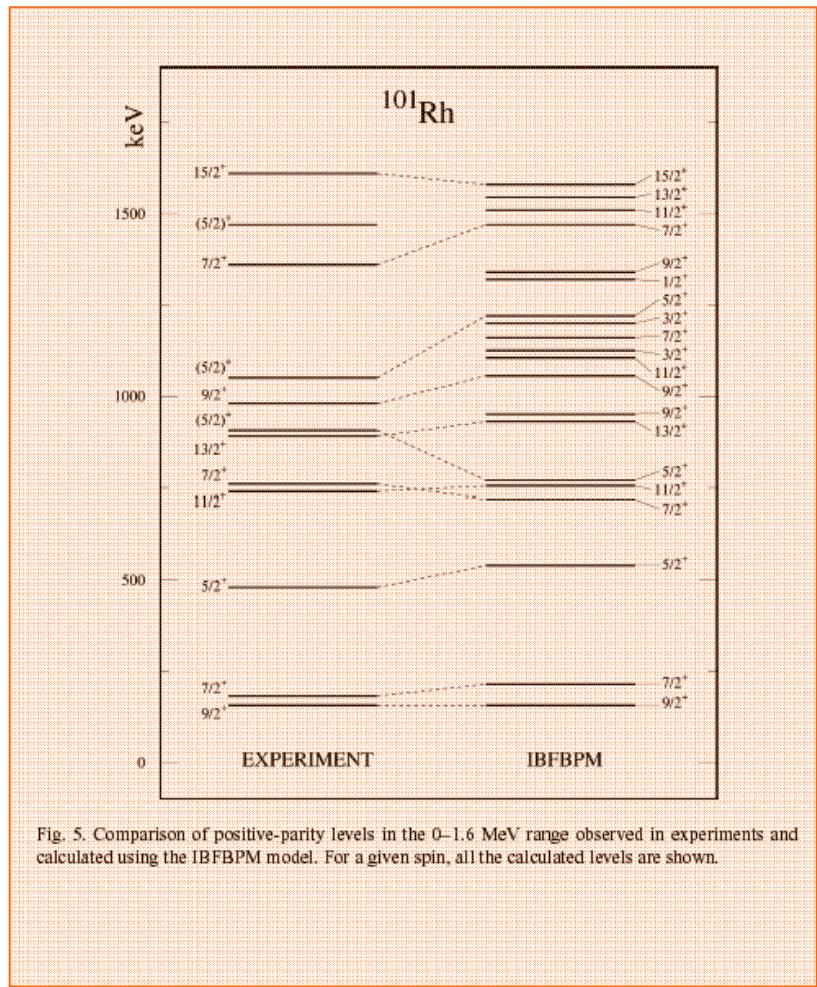


Bands 10, 11 $(\pi h11/2)^2 (\nu h11/2)^2$!!!



Bands 1, 2 (ν d3/2 ν h11/2)

Band 12 (ν d3/2 (ν h11/2)³)



Band 1 $\pi \text{ g}9/2$

Band 2 $\pi \text{ g}9/2 \ (\nu \text{ h}11/2)^2$

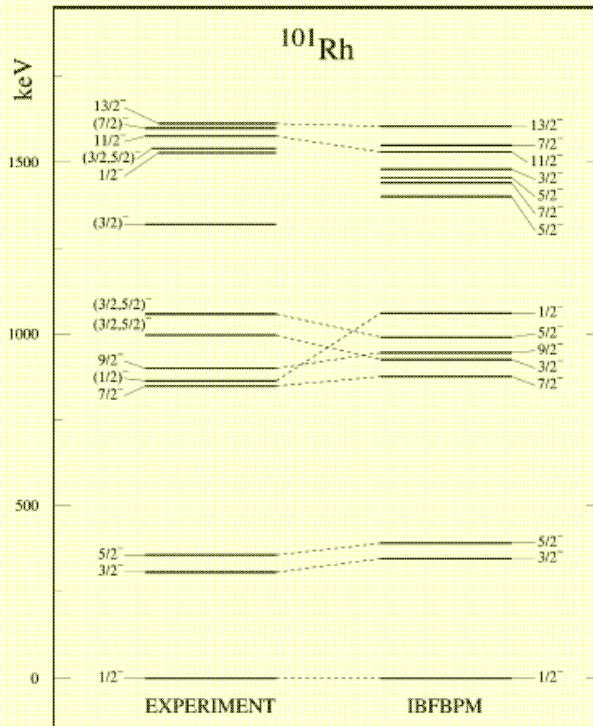


Fig. 8. Comparison of negative-parity levels in the 0–1.6 MeV range observed in experiments and calculated using the IBFBPM model. For a given spin, all the calculated levels are shown.

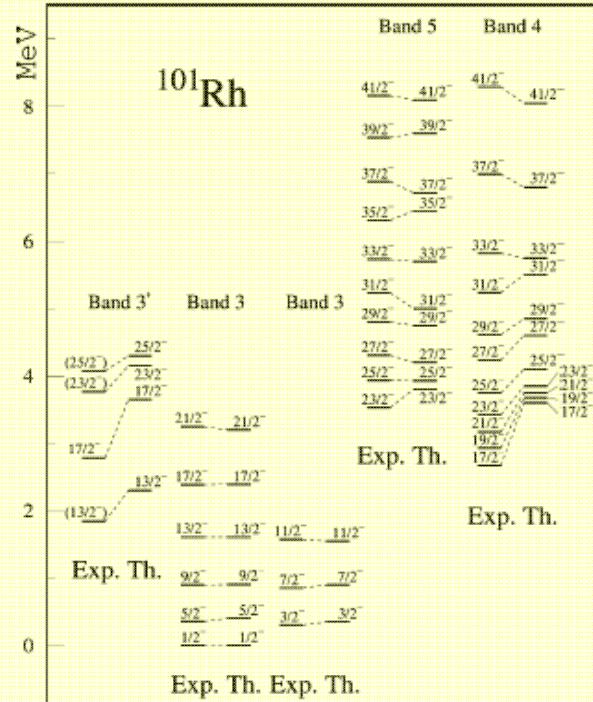
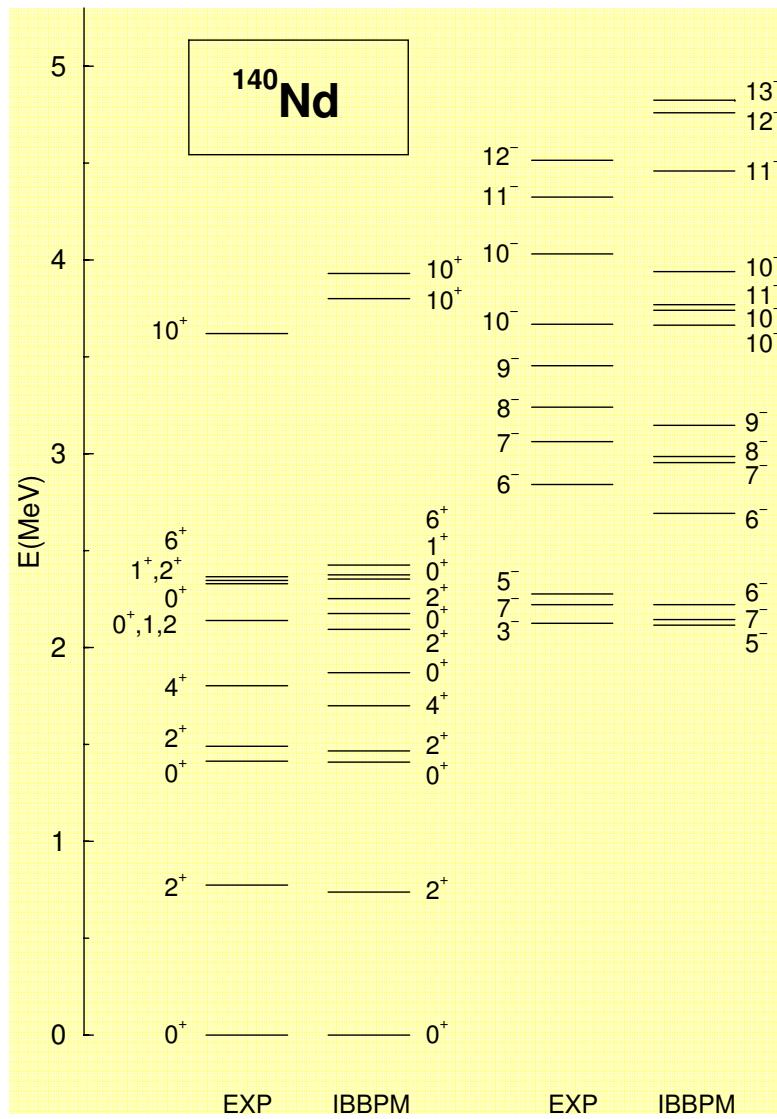
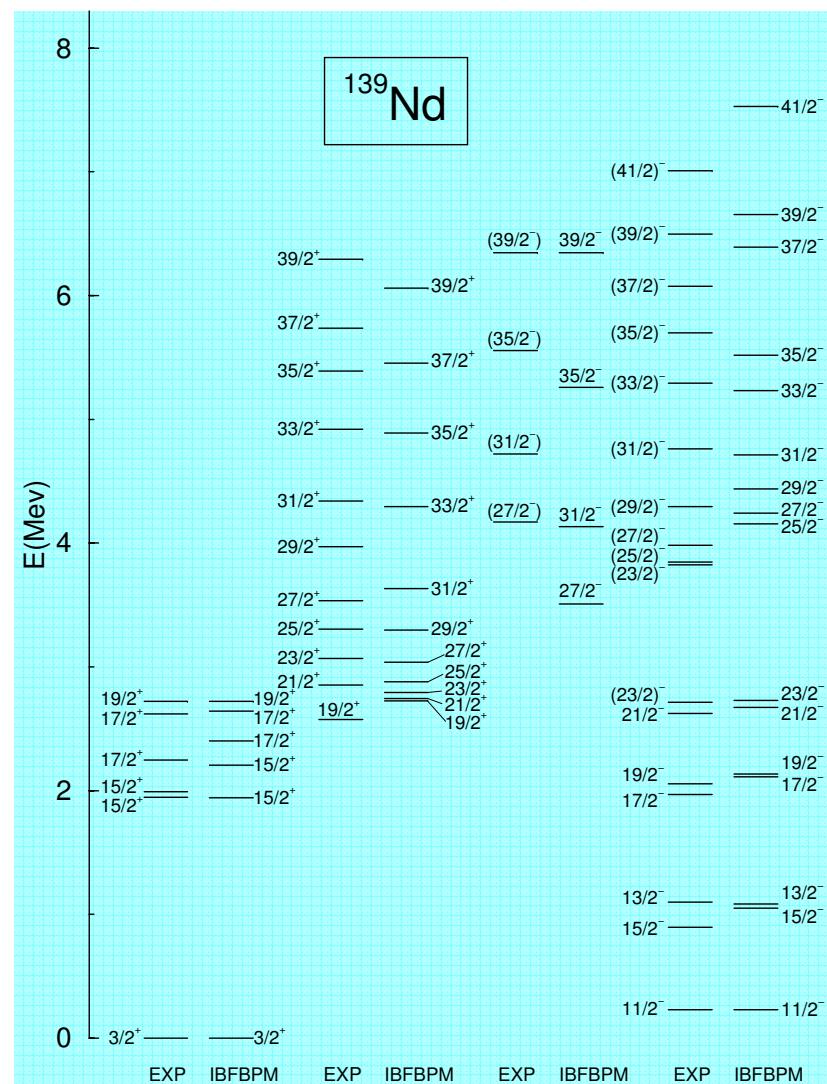
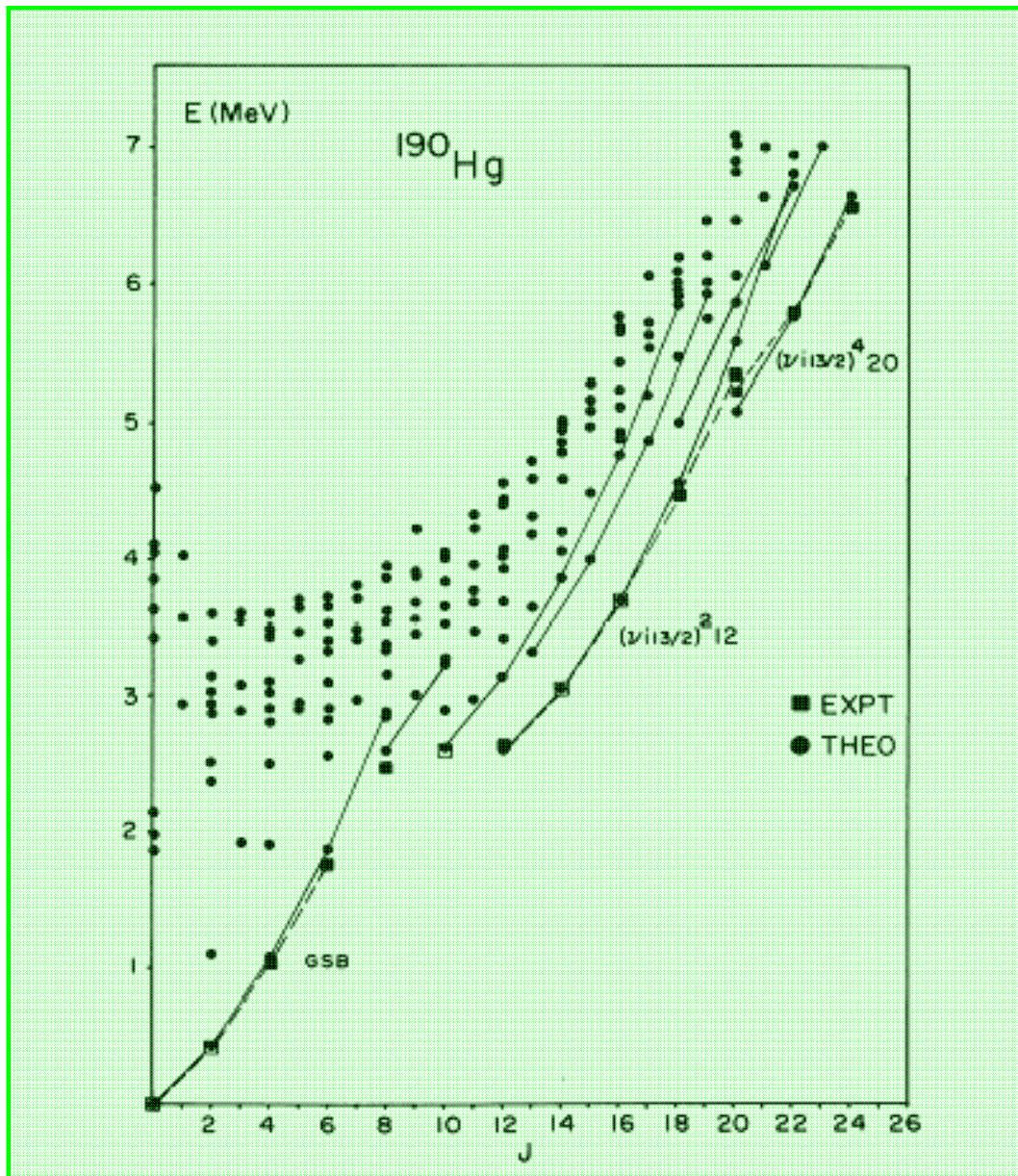


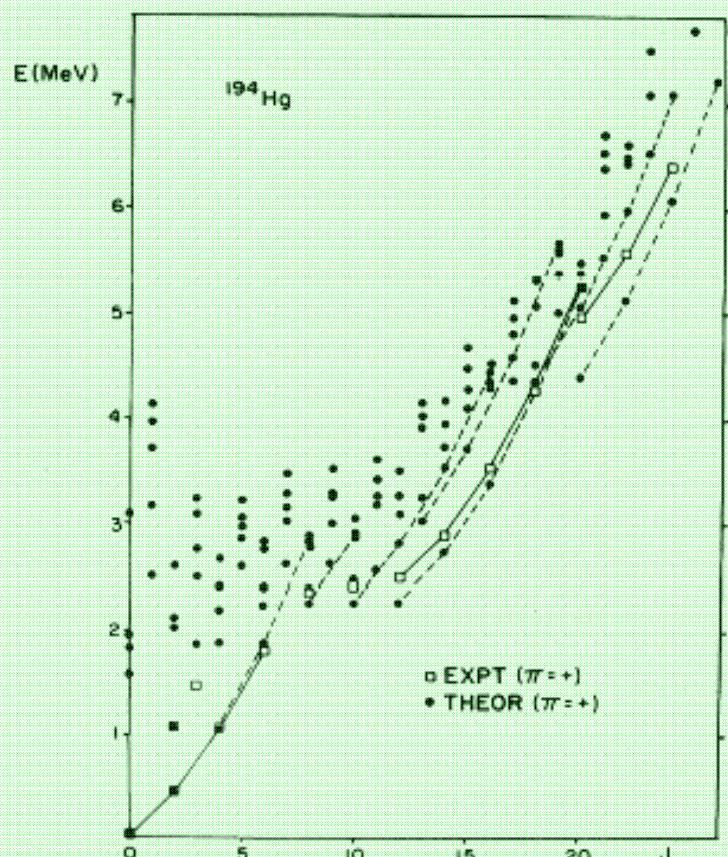
Fig. 9. Comparison of negative-parity levels in bands 3, 4, 5 observed in the present experiment (Exp.) and calculated with the IBFBPM model (Th.) up to spin $I = 41/2$.

Band 3 $\pi \text{ p}1/2$
Bands 4, 5 $(\pi \text{ g}9/2) (\nu \text{ d}5/2 \nu \text{ h}11/2)$ or
 $(\pi \text{ g}9/2) (\nu \text{ g}7/2 \nu \text{ h}11/2)$

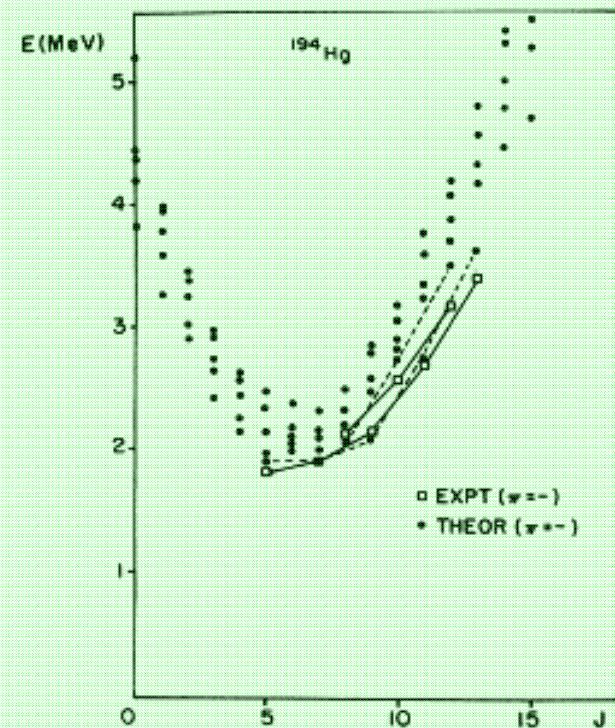








Comparison between calculated (0, 2, and 4qp) (circles) and experimental (squares) positive-parity states in ^{194}Hg . Only the first five calculated levels of each angular momentum J are shown. Sets of states with similar structure (bands) are joined together: dashed lines (calculated bands), solid lines (experimental bands).



Comparison between calculated (0 and 2qp) (circles) and experimental (squares) negative-parity states in ^{194}Hg . Only the first five calculated levels of each angular momentum J are shown. Sets of states with similar structure (bands) are joined together a

