



*The Abdus Salam
International Centre for Theoretical Physics*



1939-7

**Joint ICTP-IAEA Workshop on Nuclear Structure and Decay Data:
Theory and Evaluation**

28 April - 9 May, 2008

**Theory: b decay in the interacting
boson-fermion model.**

Slobodan BRANT
*Dept. of Physics, Faculty of Science
University of Zagreb
Croatia*

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**β decay in the interacting
boson-fermion model**

OBJECTIVES



To test the nuclear model by analyzing experimental data

Wave functions (two odd-even and one even-even nucleus are involved)

Transition operators

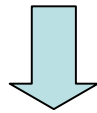


To provide reliable information for astrophysical applications

The process is very sensitive to configuration mixing both in the initial and final states. A detailed knowledge of the wavefunctions is required. Beta decay properties can be calculated by using:

- ✧ Shell model (in light nuclei and in medium-mass and heavy nuclei in the neighborhood of doubly magic nuclei)
- ✧ Other models for medium-mass and heavy nuclei.

Example: Simple pairing theory

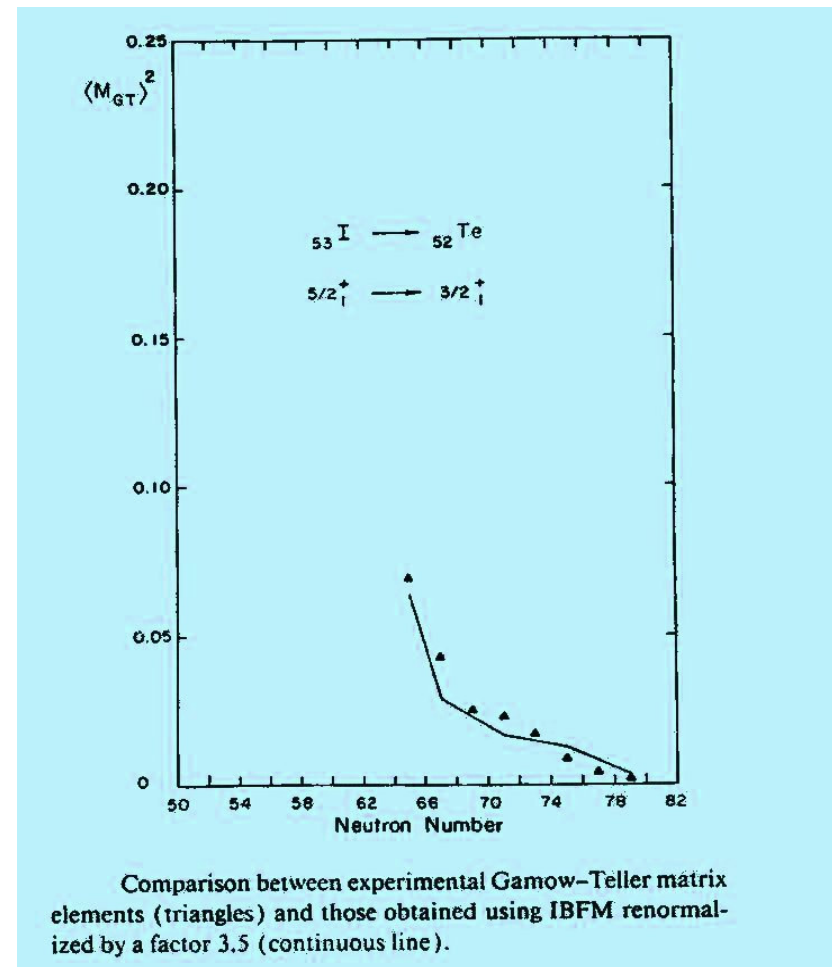
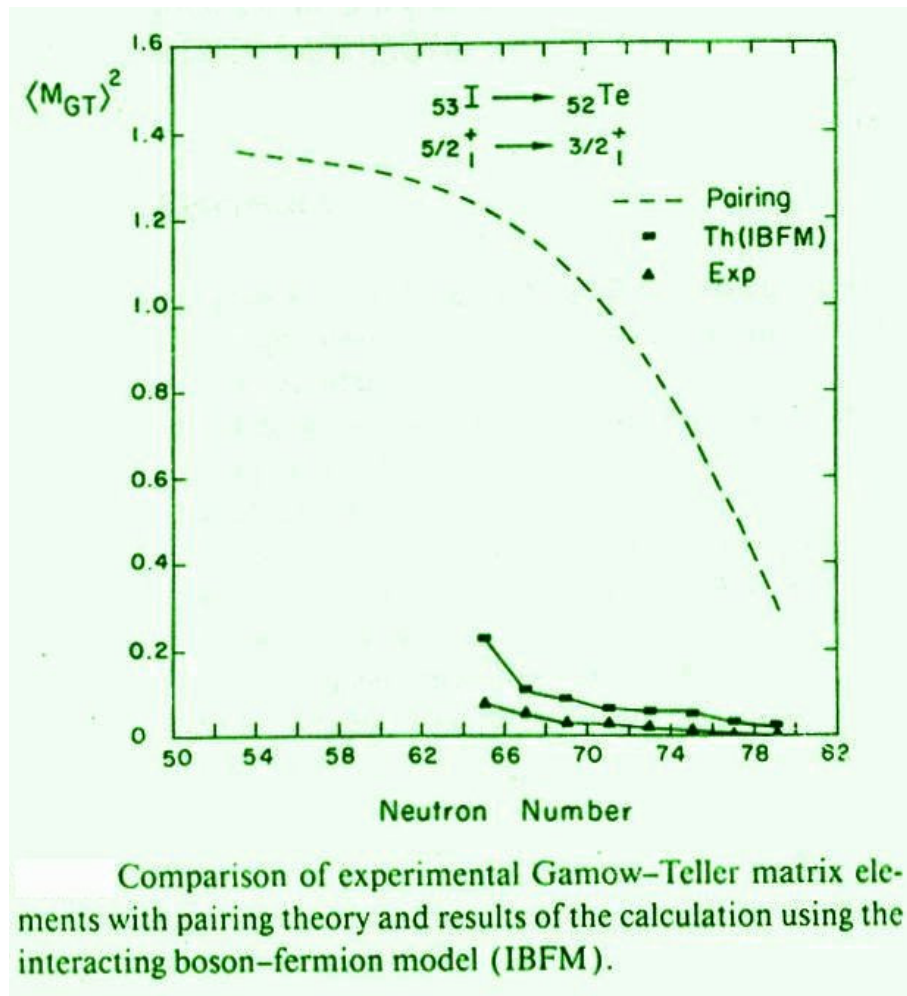


Overestimates the Gamow-Teller strengths by a large factor
(up to a factor 70 !!!!!)

How to account for the large hindrance:

Nuclear deformation
Mixing with 2p-2h states
Mesonic degrees of freedom

In the IBFM there is NO quenching factor (once the wave functions have been calculated, the calculation of beta decay properties is parameter free), or the quenching factor is SMALL.



PROCEDURE

IBM2 calculations of the structure of even-even core nuclei:

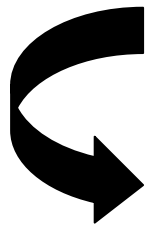
Energy levels and wave functions



Electromagnetic properties
(electric quadrupole and magnetic dipole moments, $B(E2)$ and $B(M1)$ values, branching ratios).



Wave functions



Beta decay calculations:

Matrix elements, $\log ft$ values

IBFM2 calculations of the structure of odd-even parent and daughter nuclei:

Energy levels and wave functions



Electromagnetic properties
(electric quadrupole and magnetic dipole moments, $B(E2)$ and $B(M1)$ values, branching ratios).



Wave functions



Cs → Xe

A = 125, 127, 129

Soft nuclei close to the O(6) limit → complex wave functions

→ Sensitive test of the model

IBFM2 Hamiltonian



$$H = H^B + H^F + V^{BF}$$

IBM2 Hamiltonian (core nuclei) :

$$\begin{aligned} H^B = & \epsilon_d (n_{d_\nu} + n_{d_\pi}) + \kappa (Q_\nu^B \cdot Q_\pi^B) \\ & + \frac{1}{2} \xi_2 ((d_\nu^\dagger s_\pi^\dagger - d_\pi^\dagger s_\nu^\dagger) \cdot (\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)) + \sum_{K=1,3} \xi_K ([d_\nu^\dagger d_\pi^\dagger]^{(K)} \cdot [\tilde{d}_\pi \tilde{d}_\nu]^{(K)}) \\ & + \frac{1}{2} \sum_{L=0,2,4} c_L^\nu ([d_\nu^\dagger d_\nu^\dagger]^{(k)} \cdot [\tilde{d}_\nu \tilde{d}_\nu]^{(k)}) + \frac{1}{2} \sum_{L=0,2,4} c_L^\pi ([d_\pi^\dagger d_\pi^\dagger]^{(k)} \cdot [\tilde{d}_\pi \tilde{d}_\pi]^{(k)}) \end{aligned}$$

$$H^F = \sum_i \epsilon_i n_i$$

Hamiltonian of the odd fermion

BCS



ϵ_i is the quasi-particle energy of the i th orbital
 n_i is its number operator

Interaction between bosons and the odd fermion :

$$\begin{aligned} V^{BF} = & \sum_{i,j} \Gamma_{ij} \left([a_i^\dagger \tilde{a}_j]^{(2)} \cdot Q_\rho^B \right) \\ & + \sum_{i,j} \Gamma'_{ij} \left([a_i^\dagger \tilde{a}_j]^{(2)} \cdot Q_{\rho'}^B \right) \\ & + \sum A_i n_i n_{d_\rho} + \sum A'_i n_i n_{d_{\rho'}} \\ & + \sum_{i,j} \Lambda_{ki}^j \left\{ : \left[[d_\rho^\dagger \tilde{a}_j]^{(k)} a_i^\dagger s_\rho \right]^{(2)} : \cdot [s_{\rho'}^\dagger \tilde{d}_{\rho'}]^{(2)} \right. \\ & \left. + H.c. \right\} \\ & + B J \cdot L_\rho + B' J \cdot L_{\rho'}. \end{aligned}$$

ρ and ρ' denote π (ν) and ν (π) if the odd fermion is a proton (a neutron).

Orbital dependence of the interaction strengths

$$\Gamma_{i,j} = (u_i u_j - v_i v_j) Q_{i,j} \Gamma$$

$$\Lambda_{k,i}^j = -\beta_{k,i} \beta_{j,k} \left(\frac{10}{N_\rho (2j_k + 1)} \right)^{1/2} \Lambda$$

$$\beta_{i,j} = (u_i v_j + v_i u_j) Q_{i,j}$$

$$Q_{i,j} = \langle l_i, \frac{1}{2}, j_i || Y^{(2)} || l_j, \frac{1}{2}, j_j \rangle$$

**u_j and v_j
from BCS**

Electromagnetic transition operators

$$T^{(E2)} = e_\pi^B Q_\pi^B + e_\nu^B Q_\nu^B + \sum_{i,j} e'_{i,j} [a_i^\dagger \tilde{a}_j]^{(2)}$$

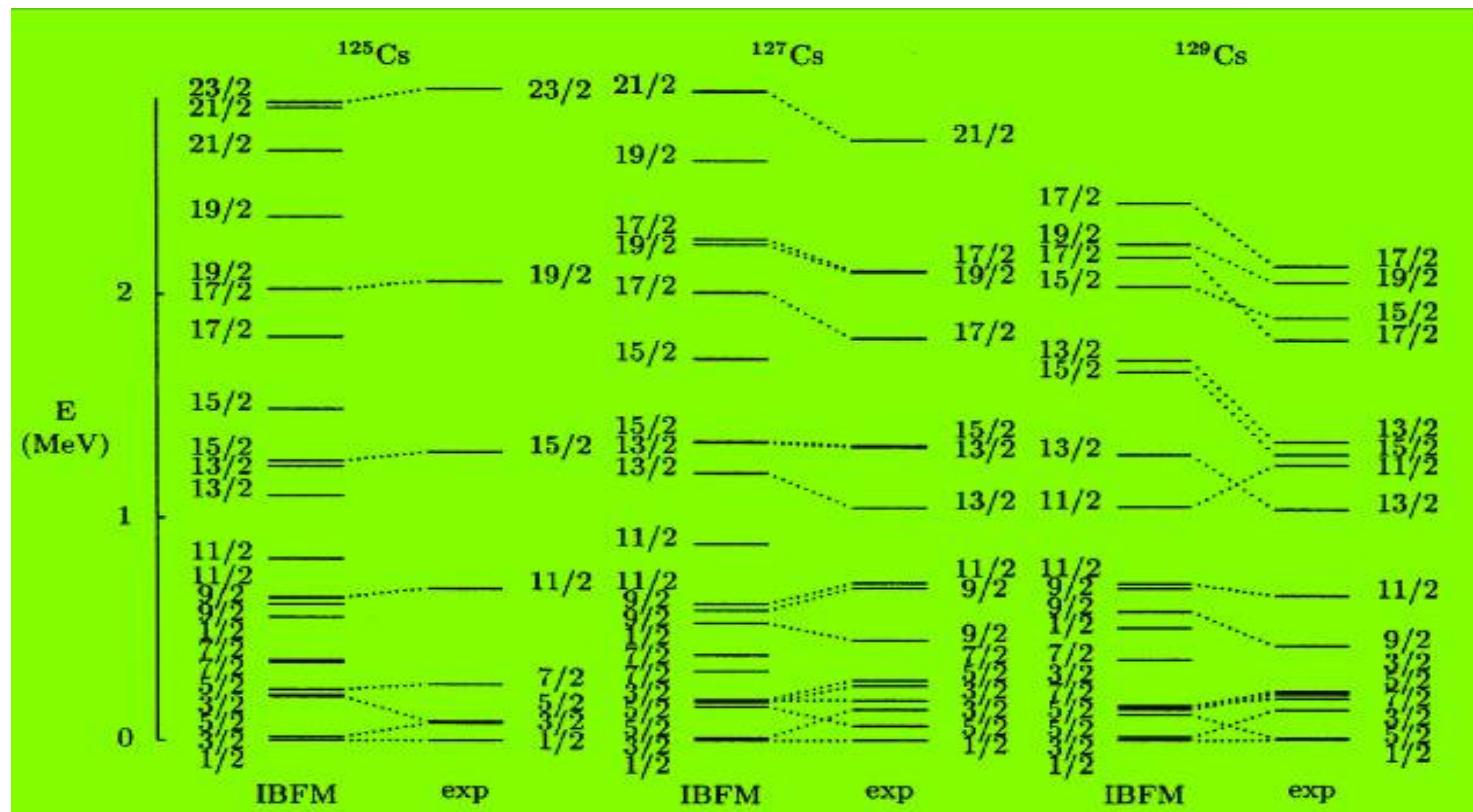
**u_j and v_j
from BCS**

$$e'_{i,j} = -\frac{e_\rho^F}{\sqrt{5}} (u_i u_j - v_i v_j) \langle i || r^2 Y^{(2)} || j \rangle$$

$$T^{(M1)} = \sqrt{\frac{3}{4\pi}} \left(g_\pi^B L_\pi^B + g_\nu^B L_\nu^B + \sum_{i,j} e_{i,j}^{(1)} [a_i^\dagger \tilde{a}_j]^{(1)} \right)$$

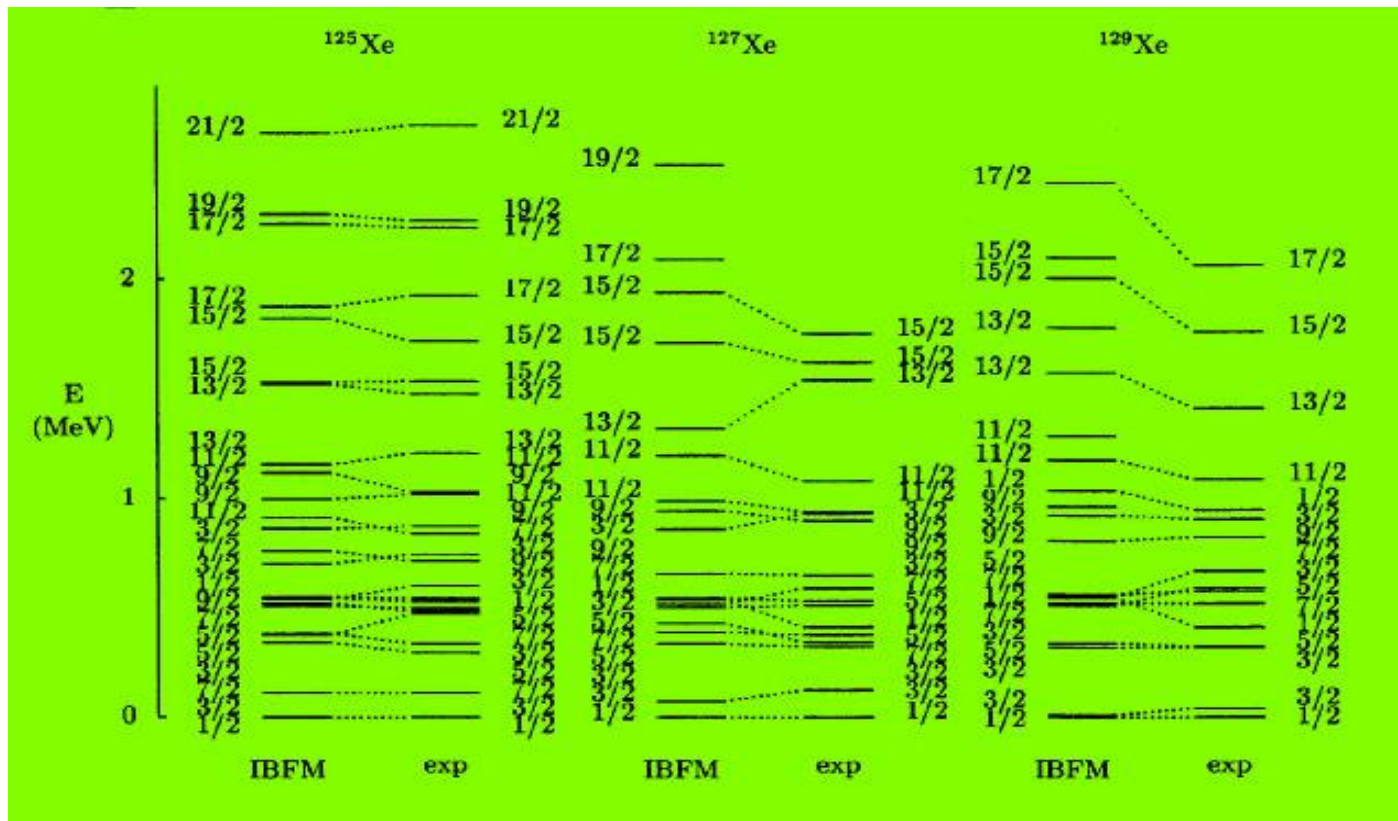
$$e_{i,j}^{(1)} = -\frac{1}{\sqrt{3}} (u_i u_j + v_i v_j) \langle i || g_l \mathbf{l} + g_s \mathbf{s} || j \rangle$$

Cs isotopes (odd proton)



Positive
parity
levels

Xe isotopes (odd neutron)



Positive
parity
levels

The Fermi $\sum_k t^\pm(k)$ and the Gamow-Teller $\sum_k t^\pm(k)\sigma(k)$ transition operators can be expressed in the framework of IBFM2. They can be constructed from the transfer operators.

$$\begin{aligned}
 A_m^{\dagger(j)} &= \zeta_j a_{jm}^\dagger + \sum_{j'} \zeta_{jj'} s^\dagger [\tilde{d} a_{j'}^\dagger]_m^{(j)} \\
 &\quad (\Delta n_j = 1, \Delta N = 0) \\
 B_m^{\dagger(j)} &= \theta_j s^\dagger \tilde{a}_{jm} + \sum_{j'} \theta_{jj'} [d^\dagger \tilde{a}_{j'}]_m^{(j)} \\
 &\quad (\Delta n_j = -1, \Delta N = 1)
 \end{aligned}$$

The former creates a fermion, while the latter annihilates a fermion simultaneously creating a boson. Either operator increases the quantity $n_j + 2N$ by one unit. The conjugate operators are:

$$\begin{aligned}
\tilde{A}_m^{(j)} &= (-1)^{j-m} \left\{ A_{-m}^{\dagger(j)} \right\}^{\dagger} \\
&= \zeta_j^* \tilde{a}_{jm} + \sum_{j'} \zeta_{jj'}^* s [d^{\dagger} \tilde{a}_{j'}]_m^{(j)} \\
&\quad (\Delta n_j = -1, \Delta N = 0)
\end{aligned}$$

The asterisks mean complex conjugate. These operators decrease the quantity $n_j + 2N$ by one unit.

$$\begin{aligned}
\tilde{B}_m^{(j)} &= (-1)^{j-m} \left\{ B_{-m}^{\dagger(j)} \right\}^{\dagger} \\
&= -\theta_j^* s a_{jm}^{\dagger} - \sum_{j'} \theta_{jj'}^* [\tilde{d} a_{j'}^{\dagger}]_m^{(j)} \\
&\quad (\Delta n_j = 1, \Delta N = -1)
\end{aligned}$$

The IBFM image of the Fermi $\sum_k t^{\pm}(k)$ and the Gamow-Teller transition operator $\sum_k t^{\pm}(k) \sigma(k)$

$$\begin{aligned}
O^F &= \sum_j -\sqrt{2j+1} \left[P_{\nu}^{(j)} P_{\pi}^{(j)} \right]^{(0)} \\
O^{GT} &= \sum_{j'j} \eta_{j'j} \left[P_{\nu}^{(j')} P_{\pi}^{(j)} \right]^{(1)}
\end{aligned}$$

$$\eta_{j'j} = -\frac{1}{\sqrt{3}} \langle l' \frac{1}{2}; j' || \sigma || l \frac{1}{2}; j \rangle$$

The transfer operators $P_\rho^{(j)}$ depend on nuclei.
In the present case



$$\begin{aligned} P_\pi^{(j)} &= \tilde{A}_\pi^{(j)} \\ P_\nu^{(j)} &= \tilde{B}_\nu^{(j)} \end{aligned}$$

$$\begin{aligned} \langle M_F \rangle^2 &= \frac{1}{2I_i + 1} | \langle I_f || O^F || I_i \rangle |^2 \\ \langle M_{GT} \rangle^2 &= \frac{1}{2I_i + 1} | \langle I_f || O^{GT} || I_i \rangle |^2 \end{aligned}$$



$$ft = \frac{6163}{\langle M_F \rangle^2 + (G_A/G_V)^2 \langle M_{GT} \rangle^2}$$

in units of second where $(G_A/G_V)^2 = 1.59$

The coefficients η_j , $\eta_{jj'}$, θ_j , $\theta_{jj'}$ appearing in transfer operators

$$\zeta_j = u_j \frac{1}{K'_j}$$

$$\zeta_{jj'} = -v_j \beta_{j'j} \left(\frac{10}{N(2j+1)} \right)^{1/2} \frac{1}{KK'_j}$$

$$\theta_j = \frac{v_j}{\sqrt{N}} \frac{1}{K''_j}$$

$$\theta_{jj'} = u_j \beta_{j'j} \left(\frac{10}{2j+1} \right)^{1/2} \frac{1}{KK''_j}$$

N is N_π or N_ν , depending on the transfer operator, and K , K'_j , K''_j are determined by

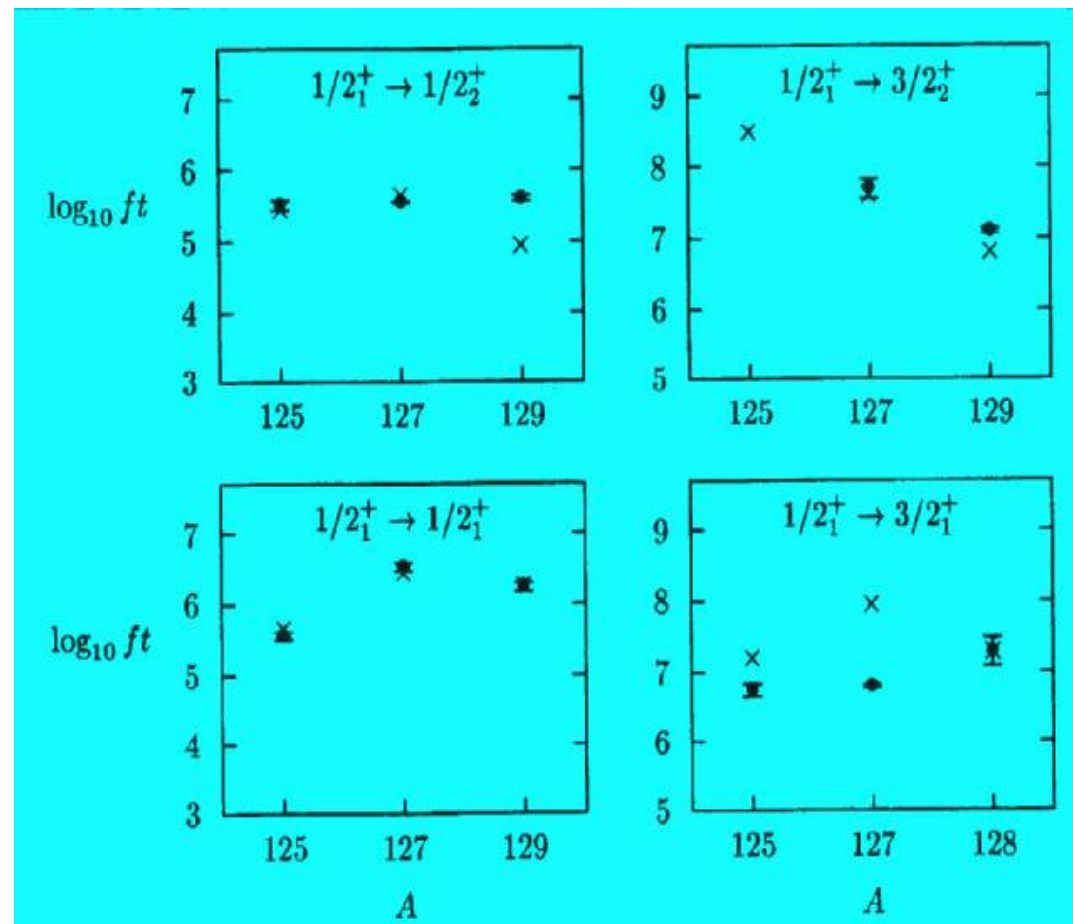
$$K = \left(\sum_{jj'} \beta_{jj'}^2 \right)^{1/2}$$

$$\sum_{\alpha J} \langle \text{odd}; \alpha J || A^{\dagger j} || \text{even}; 0_1^+ \rangle^2 = (2j+1) u_j^2$$

$$\sum_{\alpha J} \langle \text{even}; 0_1^+ || B^{\dagger j} || \text{odd}; \alpha J \rangle^2 = (2j+1) v_j^2$$

When the odd fermion is a hole in respect to the boson core, u_j and v_j have to be interchanged

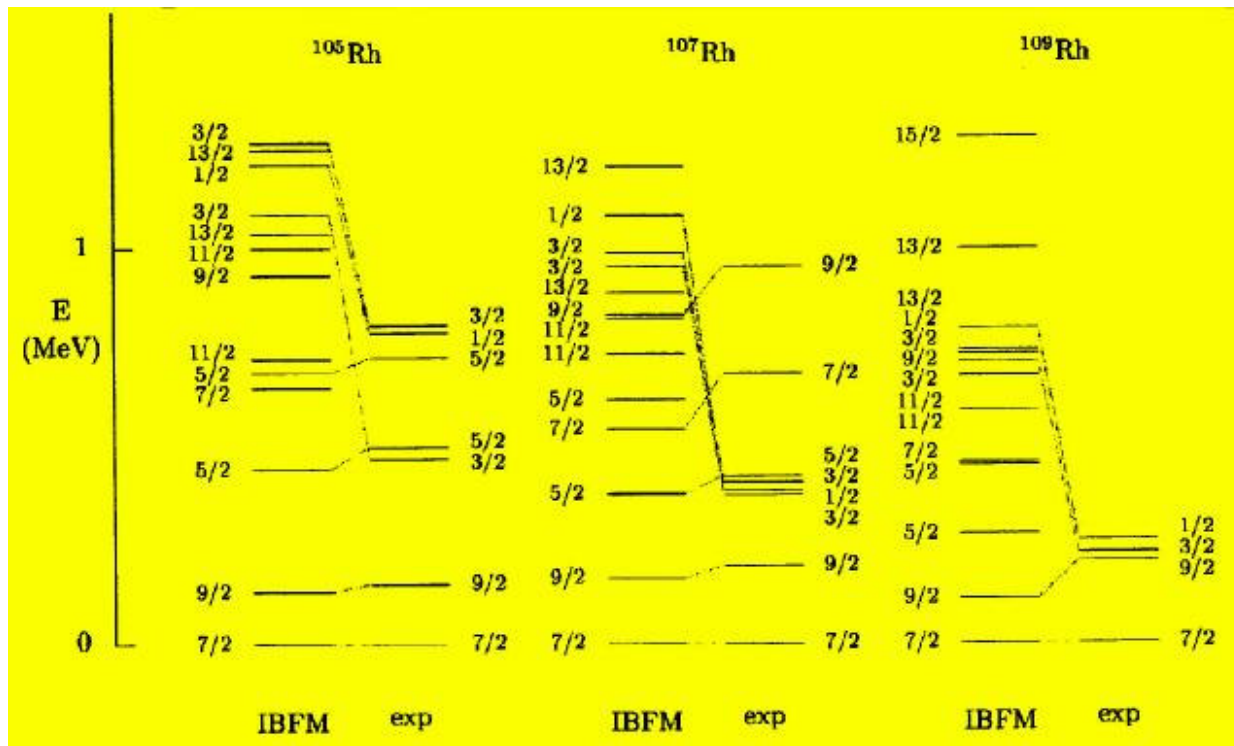
Beta-decay rates from ^ACs to ^AXe shown in terms of $\log_{10} ft$ values. The symbol \bullet with the error bar denotes experimental data, while \times presents the calculated value.



Rh \rightarrow Pd

$A = 105, 107, 109$

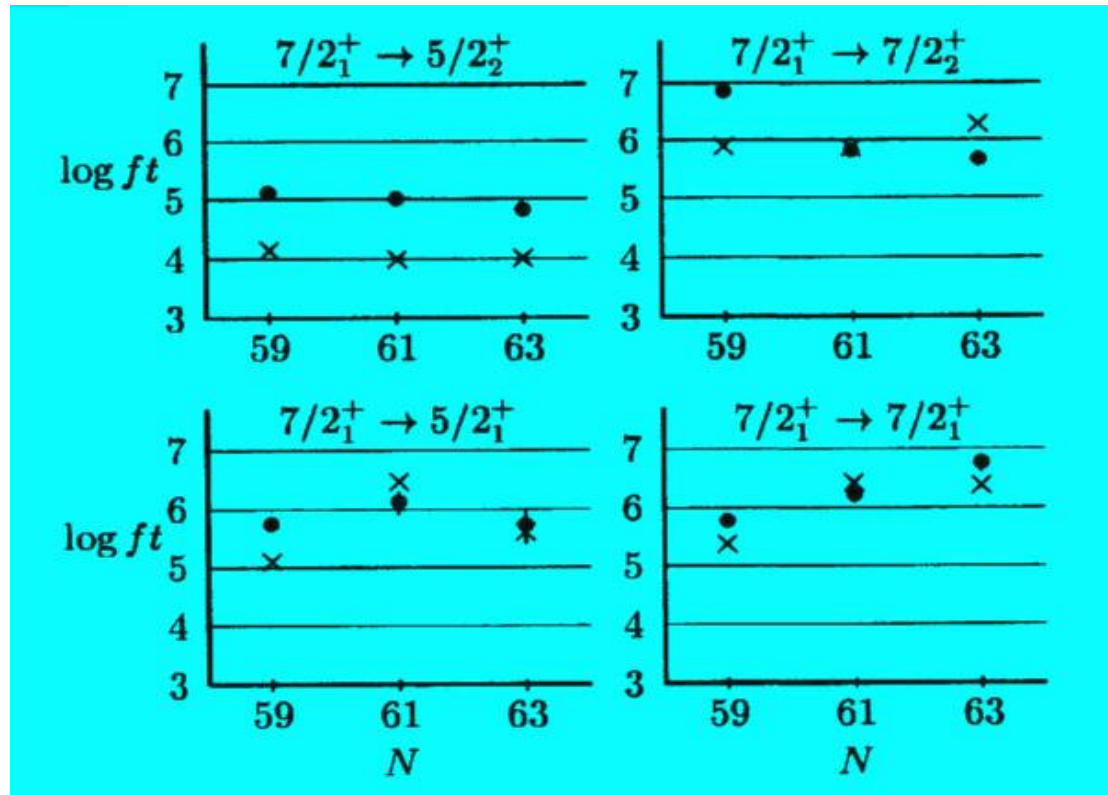
$U(5) \longleftrightarrow O(6)$ nuclei



Positive
parity
levels

$j - 1$
anomaly !!!!

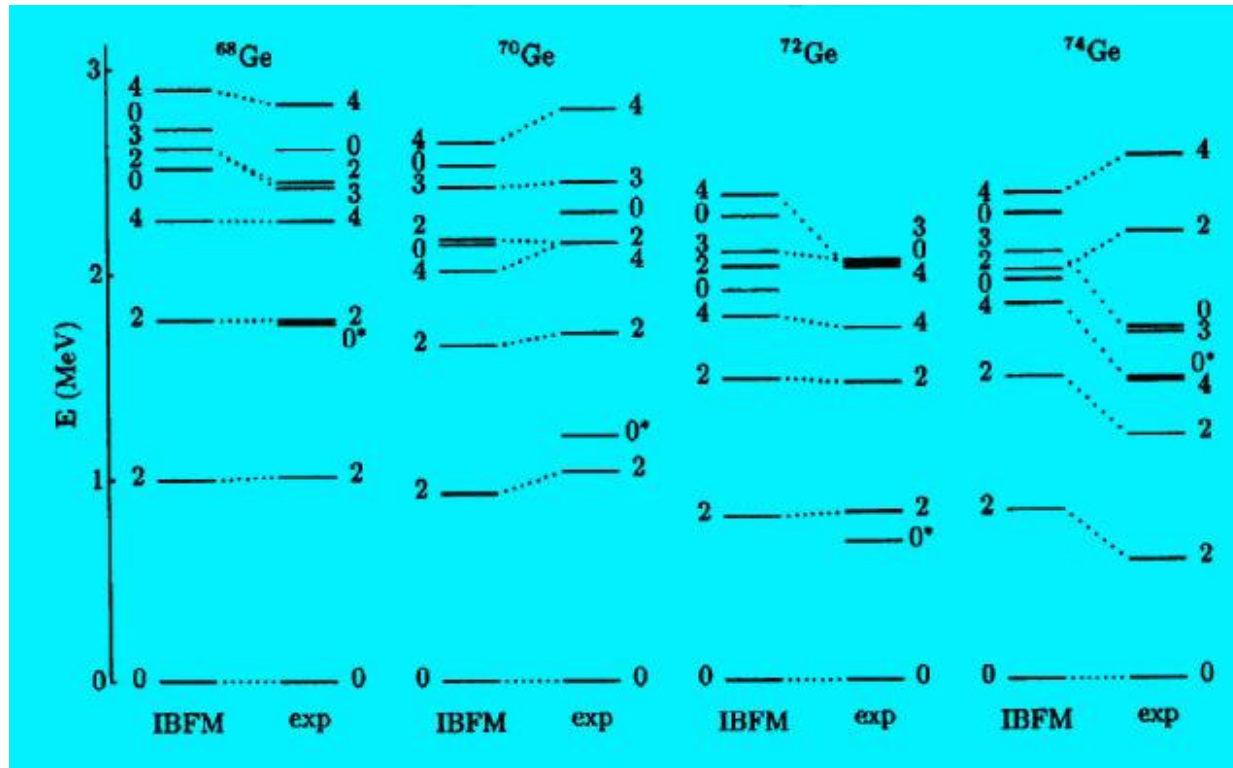
on $g_{9/2}$



$\log_{10} ft$ values in the decays $^{45}\text{Rh}_{N+1} \rightarrow ^{46}\text{Pd}_N$.
The experimental data are presented by • while
the calculated values are shown by ×.

As \longrightarrow Ge

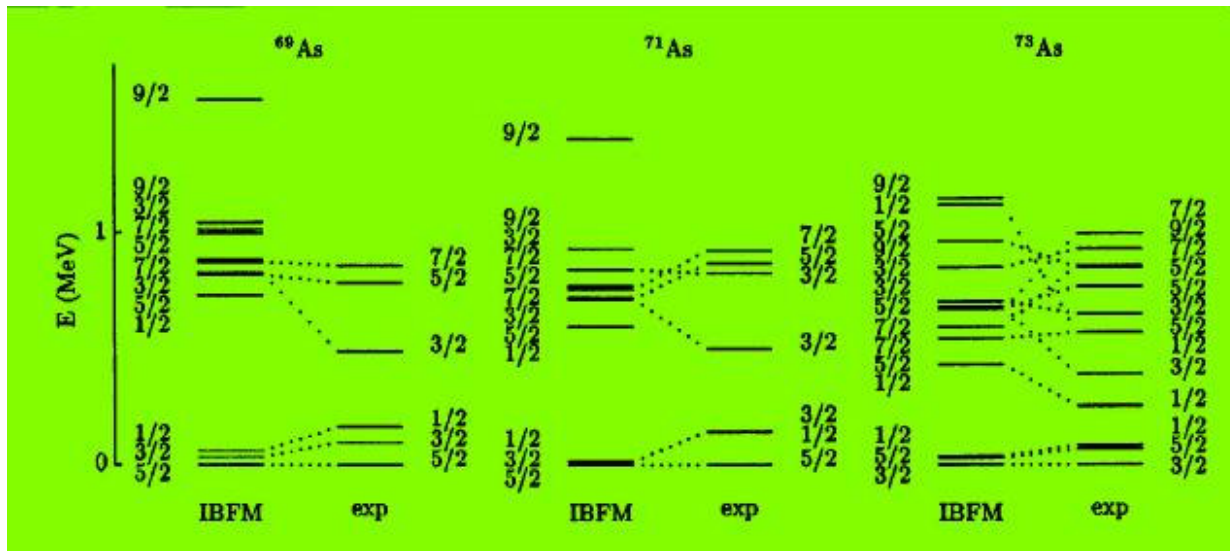
$A = 69, 71, 73$



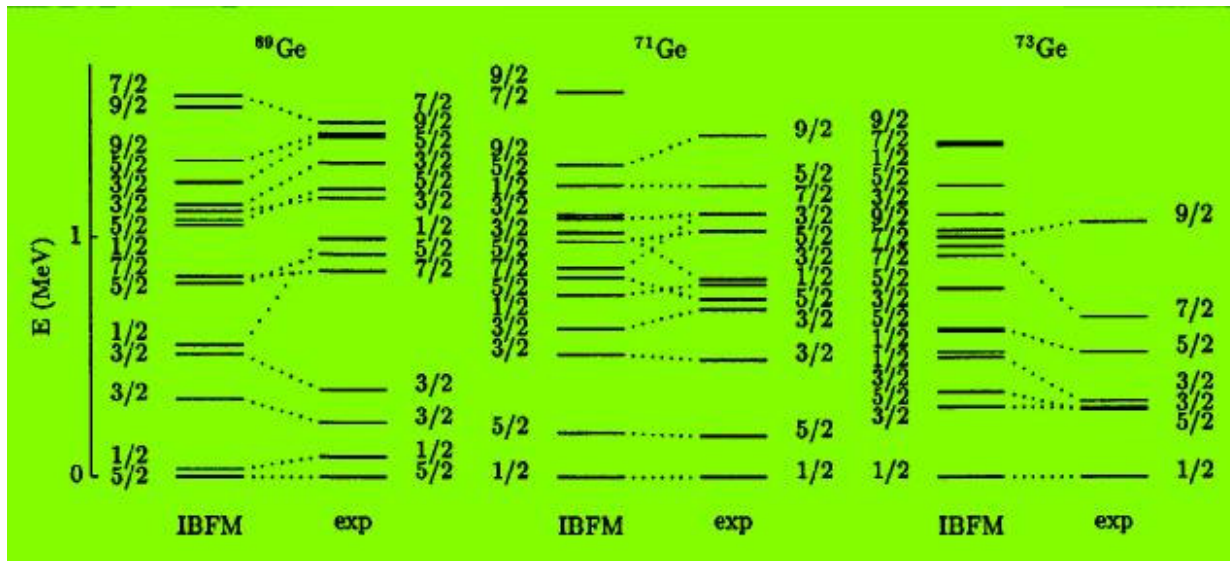
0_2^+ states are intruders !!!!

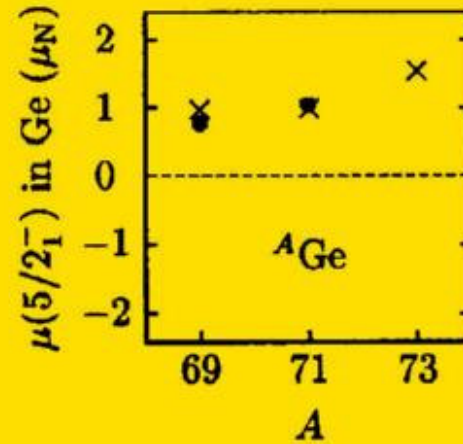
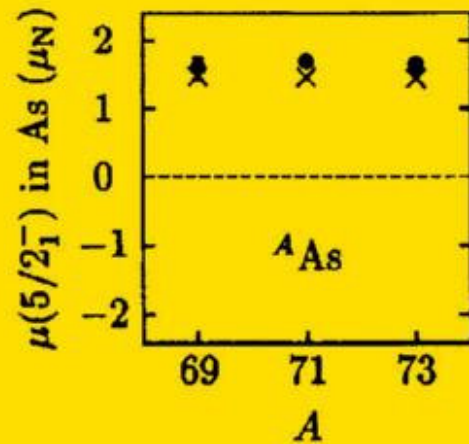
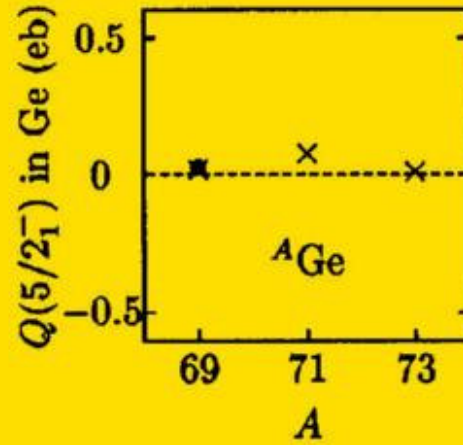
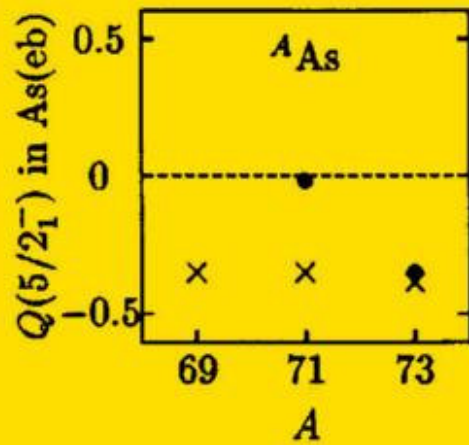
outside the boson space

Consequences ?????



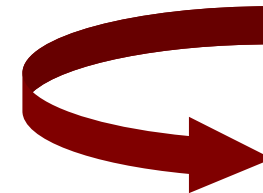
Negative
parity
levels





The exclusion of
intruder components
does not influence
strongly the theoretical
values of static moments

and branching ratios



Branching ratios in ^{71}As

level (MeV)	transition	$I_\gamma(IBM2)$	$I_\gamma(EXP)$
0.143	$1/2_1^- \rightarrow 5/2_1^-$	100	100
0.147	$3/2_1^- \rightarrow 1/2_1^-$	0.0	
	$3/2_1^- \rightarrow 5/2_1^-$	100	100
0.506	$3/2_2^- \rightarrow 3/2_1^-$	100	100 (5)
	$3/2_2^- \rightarrow 1/2_1^-$	7.1	27 (14)
	$3/2_2^- \rightarrow 5/2_1^-$	8.2	
0.829	$3/2_3^- \rightarrow 3/2_2^-$	9.3	
	$3/2_3^- \rightarrow 3/2_1^-$	100	100 (14)
	$3/2_3^- \rightarrow 1/2_1^-$	30.3	9.3 (7)
	$3/2_3^- \rightarrow 5/2_1^-$	29.4	
0.870	$5/2_2^- \rightarrow 3/2_3^-$	0.0	
	$5/2_2^- \rightarrow 3/2_2^-$	28.8	
	$5/2_2^- \rightarrow 3/2_1^-$	36.4	40 (1)
	$5/2_2^- \rightarrow 1/2_1^-$	27.0	1.8 (7)
	$5/2_2^- \rightarrow 5/2_1^-$	100	100.0(7)
0.925	$7/2_1^- \rightarrow 5/2_2^-$	0.0	
	$7/2_1^- \rightarrow 3/2_3^-$	0.0	
	$7/2_1^- \rightarrow 3/2_2^-$	0.0	
	$7/2_1^- \rightarrow 3/2_1^-$	1.1	5.8 (16)
	$7/2_1^- \rightarrow 5/2_1^-$	100	100 (3)

Branching ratios in ^{69}Ge

level (MeV)	transition	$I_\gamma(IBM2)$	$I_\gamma(EXP)$
0.087	$1/2_1^- \rightarrow 5/2_1^-$	100	100
0.233	$3/2_1^- \rightarrow 1/2_1^-$	43.2	48.3 (13)
	$3/2_1^- \rightarrow 5/2_1^-$	100	100 (3)
0.374	$3/2_2^- \rightarrow 3/2_1^-$	0.7	4.6 (8)
	$3/2_2^- \rightarrow 1/2_1^-$	100	100.0 (15)
	$3/2_2^- \rightarrow 5/2_1^-$	0.1	31.5 (8)
0.862	$7/2_1^- \rightarrow 3/2_2^-$	0.4	0.76 (13)
	$7/2_1^- \rightarrow 3/2_1^-$	0.1	8.4 (21)
	$7/2_1^- \rightarrow 5/2_1^-$	100	100 (3)
0.933	$5/2_2^- \rightarrow 7/2_1^-$	0.0	
	$5/2_2^- \rightarrow 3/2_2^-$	0.5	32 (7)
	$5/2_2^- \rightarrow 3/2_1^-$	16.7	8
	$5/2_2^- \rightarrow 1/2_1^-$	35.5	24 (7)
	$5/2_2^- \rightarrow 5/2_1^-$	100	100 (5)
0.995	$1/2_2^- \rightarrow 5/2_2^-$	0.0	
	$1/2_2^- \rightarrow 3/2_2^-$	7.9	9 (6)
	$1/2_2^- \rightarrow 3/2_1^-$	26.8	41 (9)
	$1/2_2^- \rightarrow 1/2_1^-$	0.7	
	$1/2_2^- \rightarrow 5/2_1^-$	100	100 (21)

$\log_{10} ft$ values for levels in ^{69}Ge .

level	$\log_{10} ft$ (<i>IBFM2</i>)	$\log_{10} ft$ (<i>EXP</i>)
$3/2_1^-$	5.88	6.05 (2)
$3/2_2^-$	7.90	7.21 (5)
$3/2_3^-$	5.07	6.79 (4)
$3/2_4^-$	6.46	6.71 (6)
$3/2_5^-$	6.73	7.02 (6)
$5/2_1^-$	4.26	5.49 (2)
$5/2_2^-$	6.65	6.94 (7)
$5/2_3^-$	5.33	6.65 (5)
$5/2_4^-$	5.49	6.80 (6)
$7/2_1^-$	7.54	6.98 (5)
$7/2_2^-$	6.54	6.81 (5)
$7/2_3^-$	5.96	6.20 (5)

$\log_{10} ft$ values for levels in ^{71}Ge .

level	$\log_{10} ft$ (<i>IBFM2</i>)	$\log_{10} ft$ (<i>EXP</i>)
$3/2_1^-$	6.52	7.19 (1)
$3/2_2^-$	7.79	
$3/2_3^-$	5.73	
$3/2_4^-$	5.21	6.33 (1)
$3/2_5^-$	7.34	6.94 (1)
$5/2_1^-$	4.60	5.85 (1)
$5/2_2^-$	6.08	
$5/2_3^-$	5.63	6.87 (2)
$5/2_4^-$	5.55	6.84 (2)
$7/2_1^-$	7.60	8.79 (25)

The ground states of parent ^{69}As and ^{71}As nuclei are $5/2_1^-$ levels. The hierarchy of values for transitions into different states of each angular momentum is reproduced for ^{69}Ge (except for the transition to the $3/2_3^-$ level that is predicted to have a rather small $\log_{10} ft$ value). The same is true for ^{71}Ge . The theory predicts that the smallest $\log_{10} ft$ value among all $3/2^-$ levels in ^{71}Ge has the $3/2_4^-$ level. This result is in agreement with the experimental data. The only available experimental $\log_{10} ft$ value in ^{73}Ge is for the $1/2_1^-$ level ($\log_{10} ft = 5.4$). The corresponding theoretical value (4.27) is the smallest calculated.

Systematic effect :
For most decays
the calculated values
are smaller than the
experimental values

a) Wave functions ?

If one takes the transition operators without normalization parameters, then the difference between the calculated and experimental values are caused by the transition matrix elements, that in this case have to be overestimated. This may indicate that other components are admixed in the wave functions (for example those involving intruder states), which would decrease the amplitudes of the present IBFM2 components, leading to an increase of the theoretical $\log_{10} ft$ values.



Accurate test of wave functions

b) Transfer operators ?

- ✦ **Normalization factors ?**
- ✦ **Additional terms ?**
- ✦ **Normalization factors + Additional terms ?**

$$A_m^{\dagger(j)} = [\zeta_j a_{jm}^{\dagger} + \sum_{j'} \zeta_{jj'} (s^{\dagger} [\tilde{d} a_{j'}^{\dagger}]_m^{(j)} +)]$$

Overall normalization factor **N ?**

Normalization factor **γ**

- a) Parameter
- b) Microscopic

Additional term(s)

$$\tilde{s} [d^{\dagger} a_{j'}^{\dagger}]_m^{(j)}$$

or

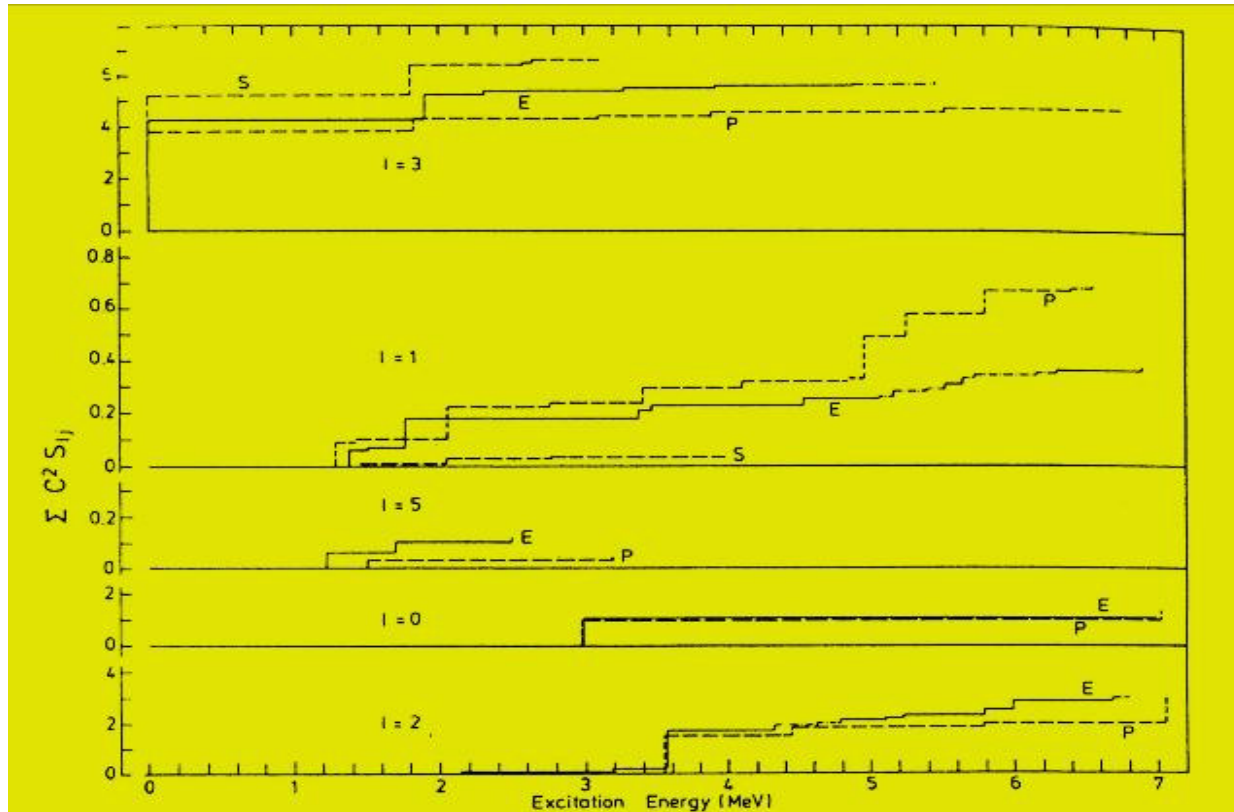
$$\sum_{j',J} \phi_{jj'}^J [(a_{j'}^{\dagger} \times d^{\dagger})^{(J)} \times \tilde{d}]_m^{(j)}$$

or

.....

$^{58}\text{Ni} (d, ^3\text{He}) ^{57}\text{Co}$ reaction

Normalization factor γ and the term $\tilde{s}[d^\dagger a_{j'}^\dagger]_m^{(j)}$

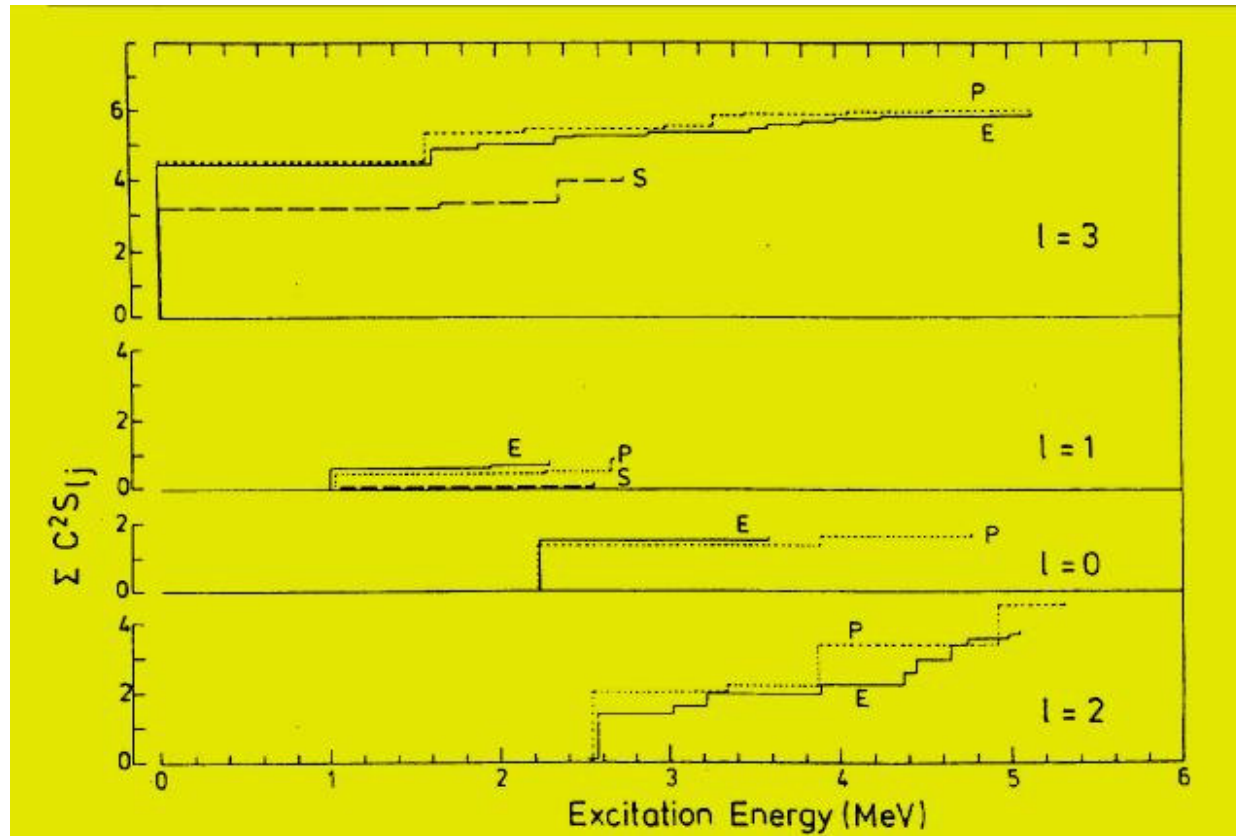


Sums of spectroscopic strengths

E experiment S shell model P IBFM1

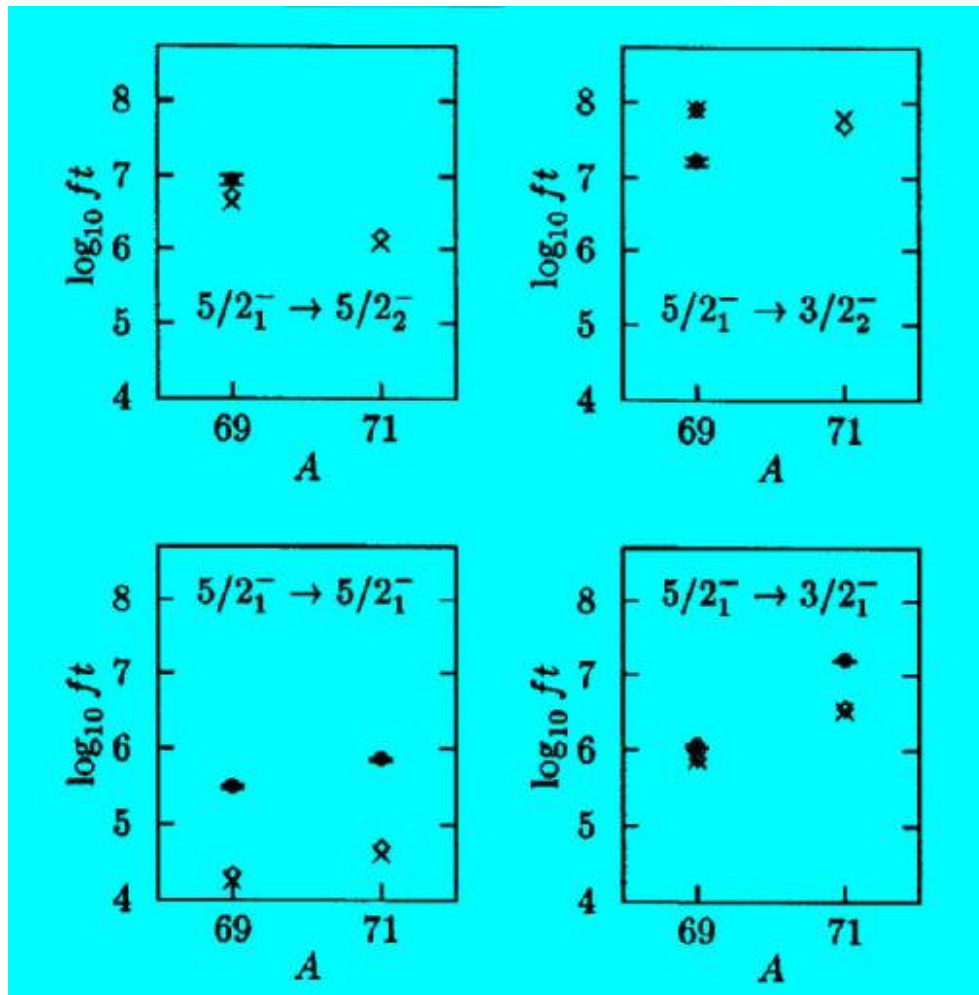
$^{62}\text{Ni} (d, ^3\text{He}) ^{61}\text{Co}$ reaction

Normalization factor γ and the term $\tilde{s}[d^\dagger a_{j'}^\dagger]_m^{(j)}$



Sums of spectroscopic strengths

E experiment S shell model P IBFM1



$\log_{10} ft$ values of the β -decay from the As to the Ge isotopes. The symbol \bullet shows the experimental values with their errors, while the symbol \times shows the results of calculations with the conventional operators. The symbol \diamond shows the results of calculations with the additional d -boson number conserving terms.

The effect of the additional term

$$\sum_{j',J} \phi_{jj'}^J [(a_{j'}^\dagger \times d^\dagger)^{(J)} \times \tilde{d}]_m^{(j)}$$

is small

CONCLUSIONS

The extensions of IBM with fermion degrees of freedom provide a consistent description of nuclear structure phenomena in:

- spherical nuclei
- deformed nuclei
- transitional nuclei

- ☀ The structure results from a consistent calculation that includes interaction strengths obtained in the analysis of neighboring nuclei
- ☀ All calculations are performed in the laboratory frame, and therefore the results can be directly compared with experimental data
- ☀ The models can be related to the shell model
- ☀ The symmetry approach can be applied in special cases
- ☀ There is a strong evidence that collective and single-particle degrees of freedom are closely related