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**Joint ICTP-IAEA Workshop on Nuclear Structure and Decay Data:
Theory and Evaluation**

28 April - 9 May, 2008

**Data Analyses
(Evaluation of Discrepant Data II)**

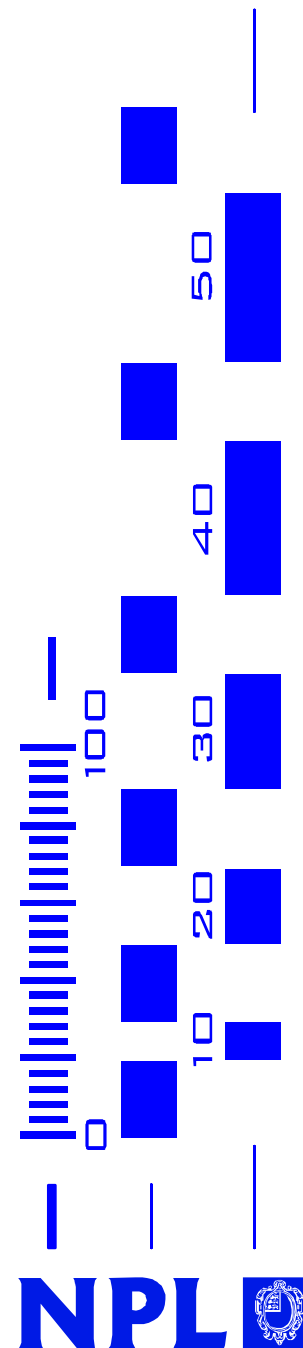
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IAEA Training Workshop
Nuclear Structure and Decay Data

Evaluation of Discrepant Data II

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ICTP May 2008



Evaluation of Discrepant Data

- ◆ Unweighted Mean: 10936 ± 75 days
- ◆ The unweighted mean can be influenced by outliers and has a large uncertainty.
- ◆ Weighted Mean: 10988 ± 3 days
- ◆ The weighted mean has an unrealistically low uncertainty due to the high quoted precision of one or two measurements. The value of 'chi-squared' is very high, indicating inconsistencies in the data.

Evaluation of Discrepant Data

- ◆ LRSW: 10988 ± 33 days
- ◆ The Limitation of Relative Statistical Weights has not increased the uncertainty of any value in the case of Cs-137, but has increased the overall uncertainty to include the most precise value.
- ◆ Median: 10970 ± 23 days
- ◆ The median is not influenced by outliers, nor by particularly precise values. On the other hand it ignores all the uncertainty information supplied with the measurements

Evaluation of Discrepant Data

- ◆ Bootstrap Method 10990 ± 26 days
- ◆ A more robust procedure than the simple median, but does not use quoted uncertainties.
- ◆ Extended Bootstrap 10992 ± 19 days
- ◆ Extends Bootstrap Method to make use of quoted uncertainty data; leads to a smaller final uncertainty.

Evaluation of Discrepant Data

- ◆ There are two other statistical procedures which attempt to:
 - ◆ (i) identify the more discrepant data, and
 - ◆ (ii) decrease the influence of these data by increasing their uncertainties.
- ◆ These are known as the Normalised Residuals Technique and the Rajeval Technique

Evaluation of Discrepant Data

- ◆ Normalised Residuals Technique
- ◆ A normalised residual for each value in a data set is defined as follows:

$$R_i = \sqrt{\frac{w_i W}{(W - w_i)}} \times (x_i - x_w)$$

$$\text{where } x_w = \frac{\sum x_i w_i}{W}; \quad w_i = \frac{1}{\sigma_i^2}; \quad W = \sum w_i$$

Evaluation of Discrepant Data

- ◆ A limiting value, R_0 , of the normalised residual for a set of N values is defined as:

$$R_0 = \sqrt{1.8 \ln N + 2.6} \quad \text{for} \quad 2 \leq N \leq 100$$

- ◆ If any value in the data set has $|R_i| > R_0$, the weight of the value with the largest R_i is reduced until the normalised residual is reduced to R_0 .

Evaluation of Discrepant Data

- ◆ This procedure is repeated until no normalised residual is greater than R_0 .
- ◆ The weighted mean is then re-calculated with the adjusted weights.
- ◆ The results of applying this method to the Cs-137 data is shown on the next slide, which shows only those values whose uncertainties have been adjusted.

Author	Half-life (days)	Original Uncertainty	R_i $R_0 = 2.8$	Adjusted Uncertainty
Wiles 1955	9715	146	- 8.7	453
Gorbics 1963	10840	18	- 8.3	52
Rider 1963	10665	110	- 2.9	114
Lewis 1965	11220	47	4.9	88
Dietz 1973	11020.8	4.1	10.1	18.4
Martin 1980	10967.8	4.5	- 5.4	8.7
Gostely 1992	10940.8	6.9	- 7.4	16.4
Unterweger 2002	11018.3	9.5	3.3	15.5
New Weighted Mean	10985	10		

Rajeval Technique

- ◆ This technique is similar to the normalised residuals technique, in that it inflates the uncertainties of only the more discrepant data, but it uses a different statistical recipe.
- ◆ It also has a preliminary population test which allows it to reject very discrepant data.
- ◆ In general it makes more adjustments than the normalised residuals method, but the outcomes are usually very similar.

Rajeval Technique

- ◆ Initial Population Test:
- ◆ Outliers in the data set are detected by calculating the quantity y_i :

$$y_i = \frac{x_i - x_{ui}}{\sqrt{\sigma_i^2 + \sigma_{ui}^2}}$$

- ◆ Where x_{ui} is the unweighted mean of the whole data set excluding x_i , and σ_{ui} is the standard deviation associated with x_{ui} .

Rajeval Technique

- ◆ The critical value of $|y_i|$ at 5 % significance is 1.96.
- ◆ At this stage only values with $|y_i| > 3 \times 1.96 = 5.88$ are rejected.
- ◆ In the case of the Cs-137 half-life data only the first value, 9715 ± 146 days, is rejected with a value of $|y_i| = 8.61$.

Rajeval Technique

- ◆ In the next stage of the procedure standardised deviates, Z_i , are calculated:

$$Z_i = \frac{x_i - x_w}{\sqrt{\sigma_i^2 - \sigma_w^2}} \quad \text{where} \quad \sigma_w = \sqrt{\frac{1}{W}}$$

Rajeval Technique

- ◆ For each Z_i the probability integral

$$P(Z) = \int_{-\infty}^Z \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2}\right) dt$$

- ◆ is determined.

Rajeval Technique

- ◆ The absolute difference between $P(Z)$ and 0.5 is a measure of the ‘central deviation’ (CD).
- ◆ A critical value of the central deviation (cv) can be determined by the expression:

$$cv = \left[(0.5)^{\frac{N}{N-1}} \right] \quad \text{for } N > 1$$

Rajeval Technique

- ◆ If the central deviation (CD) of any value is greater than the critical value (cv), that value is regarded as discrepant. The uncertainties of the discrepant values are adjusted to

$$\sigma'_i = \sqrt{\sigma_i^2 + \sigma_w^2}$$

Rajeval Technique

- ◆ An iteration procedure is adopted in which σ_w is recalculated each time and added in quadrature to the uncertainties of those values with $CD > cv$.
- ◆ The iteration process is terminated when all $CD < cv$.
- ◆ In the case of the Cs-137 data, one value is rejected by the initial population test and 8 of the remaining 18 values have their uncertainties adjusted as on the next slide:

Author	Half-life (days)	Original Uncertainty	CD cv = 0.480	Adjusted Uncertainty
Gorbics 1963	10840	18	0.500	64
Rider 1963	10665	110	0.498	149
Lewis 1965	11220	47	0.500	121
Dietz 1973	11020.8	4.1	0.500	24
Corbett 1973	11034	29	0.443	31
Houtermans 1980	11009	11	0.473	19
Gostely 1992	10940.8	6.9	0.500	15
Unterweger 2002	11018.3	9.5	0.499	23
New Weighted Mean	10971	4		

Rajeval Technique

- ◆ If the Rajeval Technique table is compared to that for the Normalised Residuals Technique, the differences between them are seen to be:
 - ◆ 1. The Rajeval Technique has rejected the Wiles & Tomlinson value.
 - ◆ 2. In general the Rajeval Technique makes larger adjustments to the uncertainties of discrepant data than does the Normalised Residuals Technique, and has a lower final uncertainty.

Evaluation of Discrepant Data

- ◆ We now have 8 methods of extracting a half-life from the measured data:

Evaluation Method	Half-life (days)	Uncertainty
Unweighted Mean	10936	75
Weighted Mean	10988	3
LRSW	10988	33
Median	10970	23
Normalised Residuals	10985	10
Rajeval	10971	4
Bootstrap	10990	26
Extended Bootstrap	10992	19

Evaluation of Discrepant Data

- ◆ We have already pointed out that the unweighted mean can be influenced by outliers and is, therefore, to be avoided if possible.
- ◆ The weighted mean can be heavily influenced by discrepant data with small quoted uncertainties, and would only be acceptable where the reduced chi-squared is small, i.e. close to unity. This is certainly not the case for Cs-137 with a reduced chi-squared of 18.6.

Evaluation of Discrepant Data

- ◆ The Limitation of Relative Statistical Weights (LRSW), in the case of Cs-137 data, still chooses the weighted mean but inflates its associated uncertainty to cover the most precise value.
- ◆ In this case, therefore, both the LRSW value and its associated uncertainty are heavily influenced by the most precise value of Dietz & Pachucki, which is identified as the most discrepant value in the data set by the Normalised Residuals and Rajeval Techniques.

Evaluation of Discrepant Data

- ◆ The median is a more reliable estimator since it is very insensitive to outliers and to discrepant data.
- ◆ However, in not using the experimental uncertainties, it is not making use of all the information available.
- ◆ The Normalised Residuals, Rajeval and Extended Bootstrap techniques have been developed to address the problems of the other techniques and to maximise the use of all the experimental information available.

Evaluation of Discrepant Data

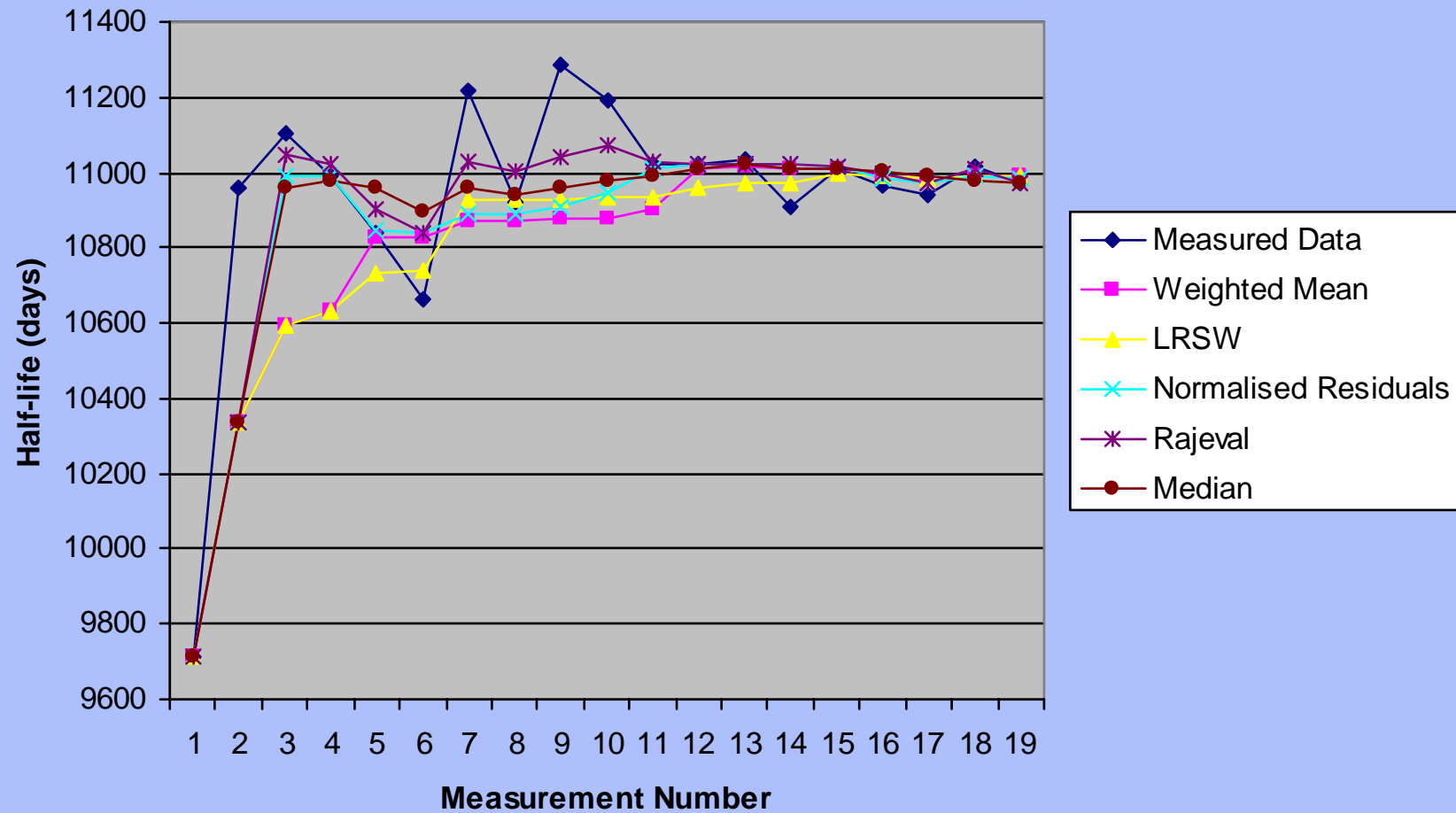
- ◆ The Normalised Residuals and Rajeval techniques use different statistical techniques to reach the same objective: that is to identify discrepant data and to increase the uncertainties of only such data to reduce their influence on the final weighted mean.
- ◆ It can be noted that all the techniques, excepting only the unweighted mean, lead to Cs-137 half-lives in the range 10970 – 10992 days. A value of 10981 ± 11 days covers the results of all the evaluation techniques and could be adopted as the current best estimate.

Evaluation of Discrepant Data

- ◆ The adopted half-life of Cs-137 is therefore:

$$\underline{10981 \pm 11 \text{ days}}$$

Cs-137 Half-Life Data Evaluations



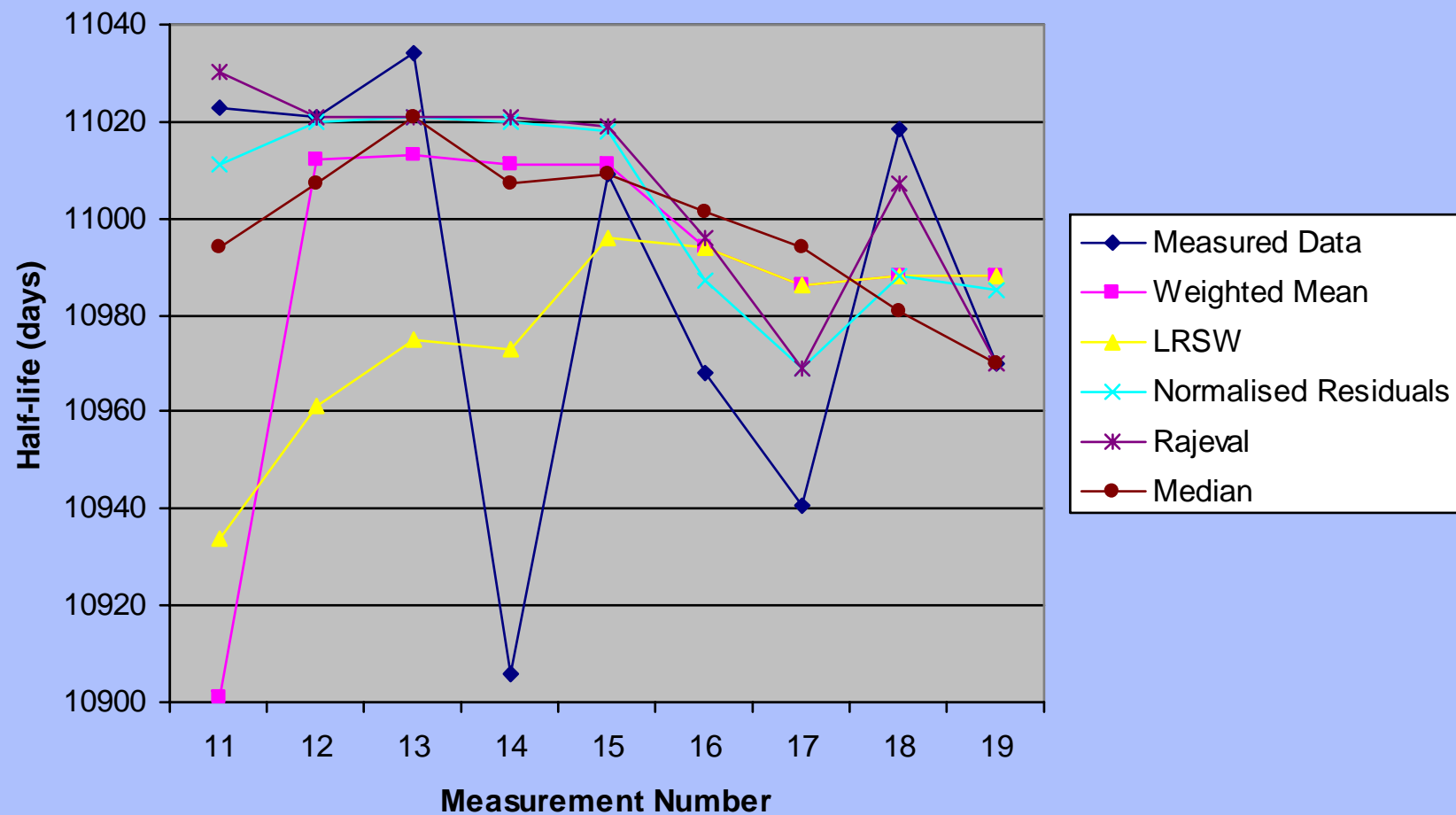
Evaluation of Discrepant Data

- ◆ The previous slide shows how the evaluation techniques behave as each new data point is added to the data set.
- ◆ The left-hand portion of the plot shows that the weighted mean and the LRSW values take much longer to recover from the first, very low and discrepant, value than do the other techniques.

Evaluation of Discrepant Data

- ◆ The next plot shows an expanded version of the second half of the previous plot, showing in more detail how the different techniques behave as the number of data points reaches 19.
- ◆ Taking into account the 19th point the overall spread in the evaluation techniques is only 18 days or 0.16%

Cs-137 data - expanded version of the end of the previous plot





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