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Relaxation in plasmas with several types of free energy.

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Relaxation in plasmas with several types of free energy

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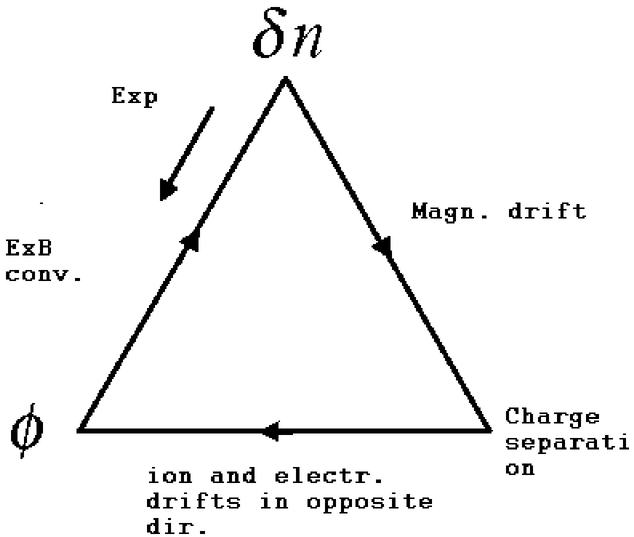
Outline

- Competition between the relaxation of different types of free energy
- Interchange and thermal type instabilities
- Feedback loops and their coupling
- Pinch fluxes
- Radial profiles of transport coefficients
- Fluid closure aspects
- Momentum transport

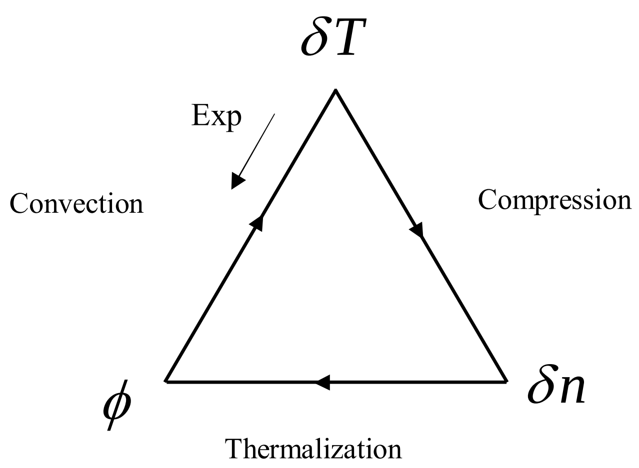
Interchange instability

Involves charge separation, Density gradient drive



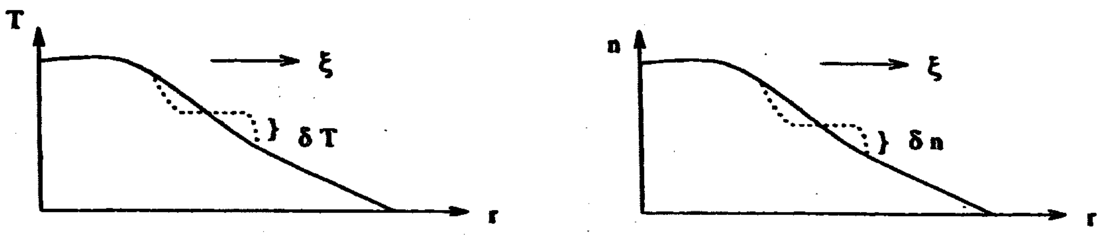
Thermal instability

- Charge separation replaced by compressibility, Temperature gradient drive



Linear perturbations

- The linear perturbations of density and temperature are due to both convection and compression



$$\delta n = -\xi \cdot \nabla n + \lambda_{nT} \xi \cdot \nabla T + \lambda_{Be} \xi \cdot \nabla B$$

$$\delta T_e = -\xi \cdot \nabla T_e + \lambda_{ne} \xi \cdot \nabla n + \lambda_{Be} \xi \cdot \nabla B$$

Critical gradients

- Compression (expansion) clearly counteracts convection. This leads to conditions for positive feedback of the types:

$\eta < \eta_c$ Interchange type

$\eta > \eta_c$ Thermal type

For toroidal modes the condition for thermal instability depends on R and reduces to the form

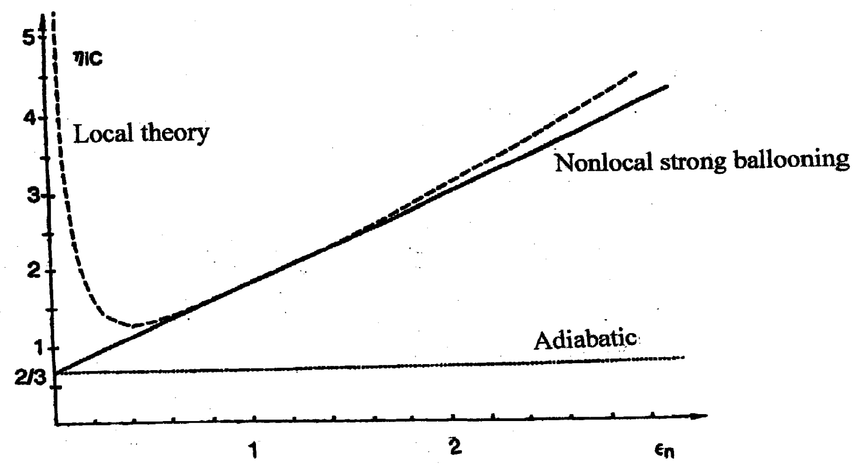
$$R/L_T > (R/L_T)_{crit}$$

for flat density

Critical gradients for the pure ITG

- Including both slab and toroidal drives we have the general stability diagram

Linear threshold



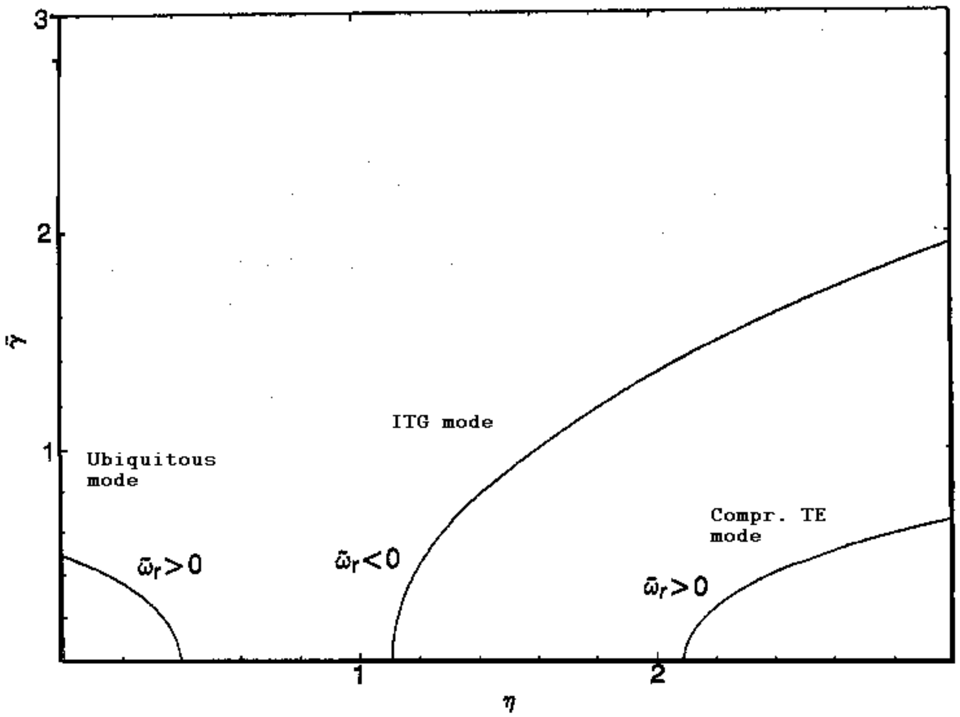
Trapped particle modes - MHD

- We note the fact that trapped particle modes have no parallel motion of trapped particles. This enables charge separation in a similar way as for MHD-modes.

- Compare M.N. Rosenbluth, *Trapped Particle Modes and MHD*, Proc. 1982 ICPP, Göteborg, Sweden, Physica Scripta **T2:1,** 104 (1982)

Growthrates

- The competition between temperature and density relaxations is also shown by the growthrates



- Dependence of growthrates on $\eta_i = \eta_e = \eta$

Transport

- We obtain the fluxes directly by substituting the density and temperature perturbations (1) into the ExB fluxes:

$$\Gamma_n =< \delta n \cdot \mathbf{v}_E >; \Gamma_T =< \delta T \cdot \mathbf{v}_E >$$

- The simple temperature perturbation for Boltzmann electrons is:

$$\frac{\delta T_j}{T_j} = \frac{\omega}{\omega - \frac{5}{3}\omega_{D_j}} \left[\frac{2}{3}\frac{\delta n_j}{n_j} + \frac{\omega_{*e}}{\omega}(\eta_j - \frac{2}{3})\frac{e\phi}{T_e} \right]$$

- Here ω_D comes from the diamagnetic heat flow and represents the fluid closure

Transport, cont

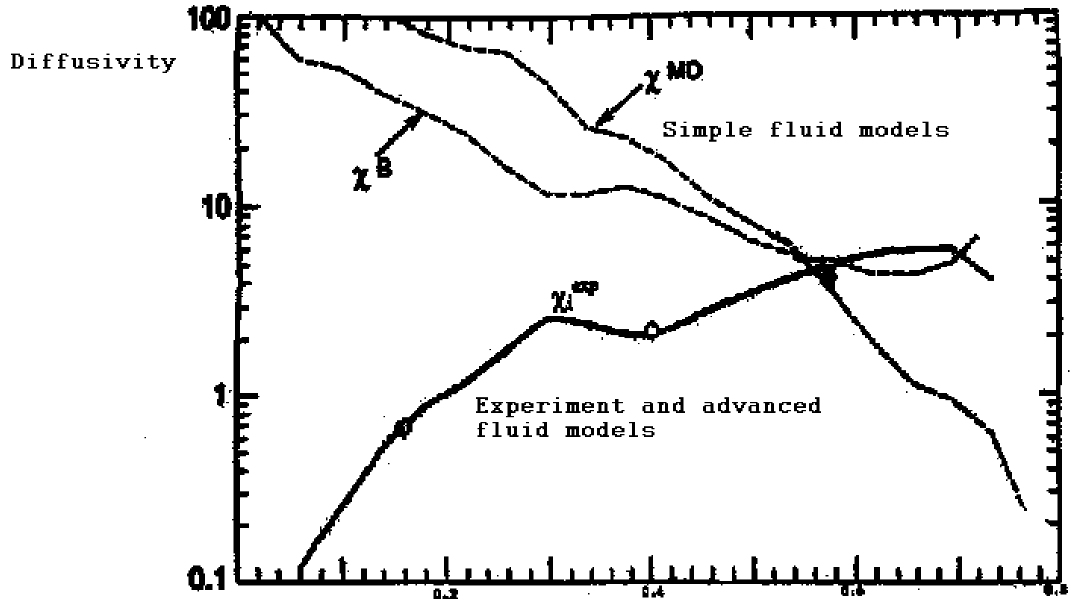
- The ion thermal conductivity for Boltzmann electrons is: $\varepsilon_n = \frac{\omega_D}{\omega_*}$

$$\chi_i = \frac{1}{\eta_i}(\eta_i - \frac{2}{3} - \frac{10}{9\tau}\varepsilon_n)\frac{\gamma^3 / k_r^2}{(\omega_r - \frac{5}{3}\omega_{Di})^2 + \gamma^2}$$

$$\Gamma_i = (\frac{dT_i}{dr} - \frac{2}{3}\frac{T_i}{n}\frac{dn}{dr} - \frac{20}{9\tau}\frac{T_i}{R})\frac{\gamma^3 / k_r^2}{(\omega_r - \frac{5}{3}\omega_{Di})^2 + \gamma^2}$$

Radial variation of χ_i

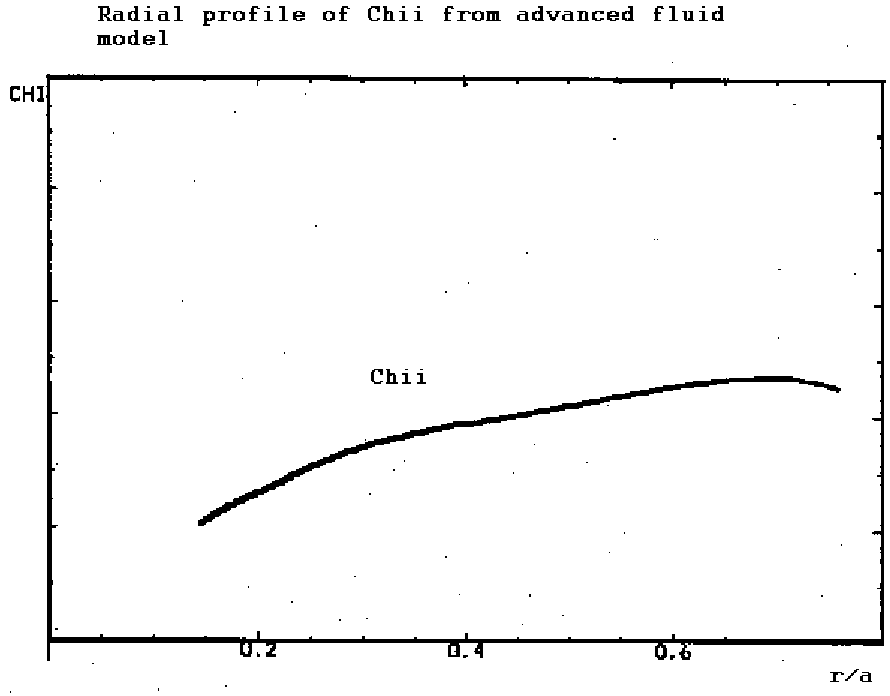
- Simple drift wave models give a decrease of transport with radius due to the Gyro-Bohm condition $\chi \propto T^{1.5}$ S.D. Scott, PRL **64**, 531 (1990).



From S.D. Scott et. al. Phys. Rev. Lett. 64, 531 (1990)

Radial Chii profile

- With an ITG model containing the fluid resonance in the energy equation we get a Chii profile which grows with radius



Radial variation of transport

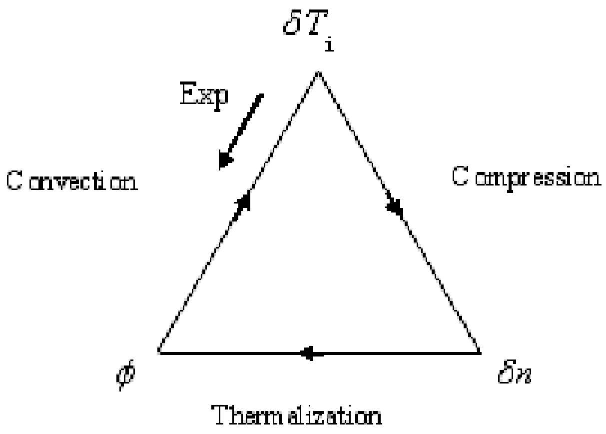
- When the **fluid magnetic drift resonance** is included in the energy equation we get a *coupling between temperature and magnetic field inhomogeneities* which leads to a $\boldsymbol{\chi_i}$ that *grows with radius* in spite of the scaling as $T^{1.5}$!
- Because of this advanced fluid models give about the *right radial profiles* of transport coefficients. This was for a long time considered as a major problem for drift wave models.

- This means that we have a *new regime* of drift wave transport closely coupled to the presence of a *flat density regime near* the axis.

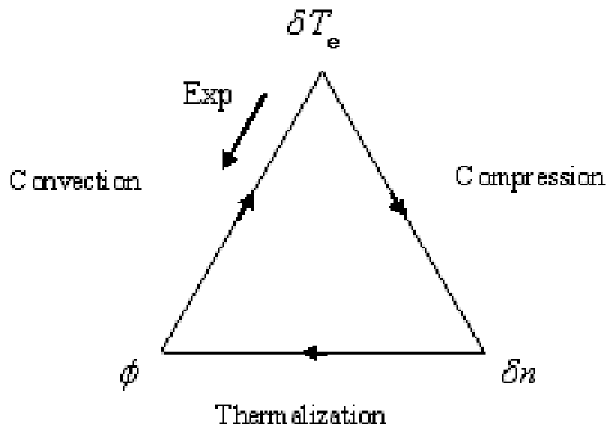
Temperature pinches

- The ultimate result of competition between relaxations of different types of free energy is the occurrence of net inward flows. Consider the simultaneous presence of ion (ITG) and electron (TE) temperature loops

Ion loop



Electron loop



Temperature pinches cont.

- We now consider the case with stable ITG mode, $\eta_i < \eta_{ic}$
- In this case the temperature feedback is in the opposite direction of an initial perturbation. An outward convection leads to a decrease of the temperature and vice versa. This corresponds to a temperature pinch but since the ITG mode is stable the pinch can not occur since there is no drive for it.
- Now include also an unstable TE loop with $\eta_e > \eta_{ec}$
- The ExB drift is the same for ions and electrons so the unstable electron loop will push the stable ion loop through its cycle. This gives an **ion heat pinch!**

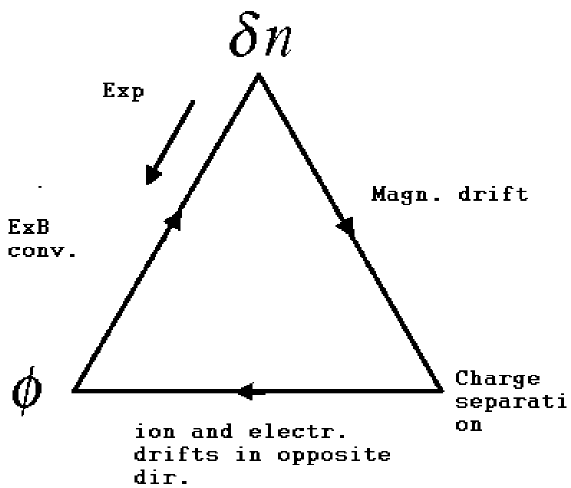
Temperature pinches cont.

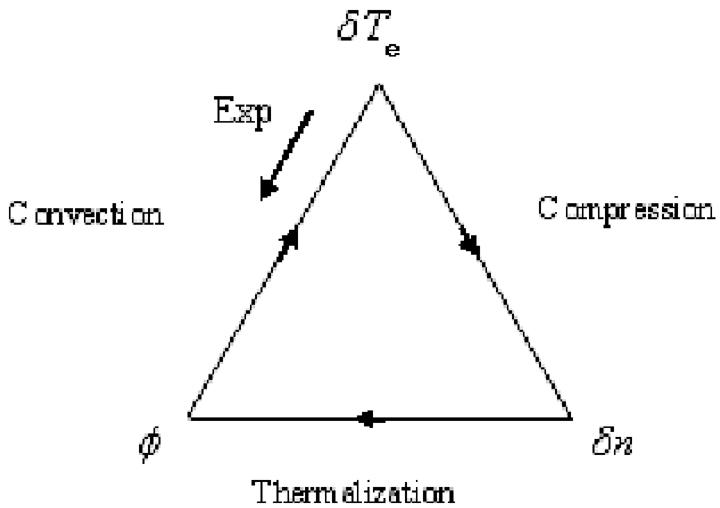
- The ITG and TE loops are cleary interchangable. Thus an ITG mode can also drive an electron termperature pinch if the TE mode is stable and the ITG mode is unstable.
- The unstable thermal instability acts as an *external drive* for the stable loop similar to a compressor for a refridgerator!

Compare J. Weiland, H.Nordman and P. Strand, Recent Research Developments in Physics **6**, 387 (2005), Transworld Research Network, Trivandrum (2005).

Particle pinch

- To get a particle pinch we need a coupling between a Ubiquitous mode and a thermal instability.





Particle pinch

- Just as for the thermal pinch, the outward convection can give a decreased density if expansion (inverted compression) dominates.

  This happens more easily when the background pressure decreases

  due to a strong temperature. Then the Ubiquitous mode is stable but its loop can be driven by a thermal instability. We then get a *particle pinch.*

Indications of particle pinches has for a long time been seen in tokamak experiments

(F. Wagner and U. Stroth, PPCF **35**, 1321 (1993))

Very favourable effects of the particle pinch on ITER performance was obtained in: J. Weiland Proc. EPS conference, Madeira 2001.

Momentum transport

- Nonlinear effects on momentum transport enter in the form of space dependent nonlinear frequency shifts. These come mainly from the nonlinear Reynolds stress but can also be due to the convective nonlinearity in the energy equation

We will here outline the derivation of nonlinear frequency shifts for a simple unmagnetized plasma

Effects of Reynolds stress flow

The Reynolds stress generates a **zonal flow** which enters as a **nonlinear frequency shift.**

A general formulation can be made kinetically

$$i(\mathbf{k}\cdot\mathbf{v}-\omega_k)f_k+\frac{q}{m}\mathbf{E}\cdot\frac{\partial f_0}{\partial\mathbf{v}}=-\frac{q}{m}\sum\mathbf{E}_{k-k'}\cdot\frac{\partial f_{k'}}{\partial\mathbf{v}}e^{i\Delta\omega_{k,k'}t}$$

Substitute the corresponding equation for the component k'into the nonlinear term:

$$i(\mathbf{k}\cdot\mathbf{v}-\omega_k)f_k+\frac{q}{m}\mathbf{E}\cdot\frac{\partial f_0}{\partial\mathbf{v}}=-(\frac{q}{m})^2\sum\mathbf{E}_{k-k'}\cdot\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{E}_{k''}\frac{\partial}{\partial v}f_{k'''}}{\omega_{k'}-\mathbf{k}'\cdot\mathbf{v}}e^{i\Delta\omega_{k,k'}t}$$

Zonal flows

Keeping only phase averaged parts (c.f. Smolyakov, Diamond, Medvedev PoP **7,** 3987 (2000); Kaw, Singh, Diamond, PPCF **44**, 51 (2002)) we obtain:

$$i(\mathbf{k}\cdot\mathbf{v}-\omega_k)f_k+i\sum\alpha_l\left|E_l\right|^2 f_k=-\frac{q}{m}\mathbf{E}\cdot\frac{\partial f_0}{\partial\mathbf{v}}$$

Where α_l describes the medium and can contain phase space derivatives. From this equation we can derive suitable fluid equations including the momentum transport equation. We note that the sum of turbulent intesities is of the same form as sums giving transport coefficients which have previously been expressed in terms of one amplitude at the correlation lenght. Thus we can now generalize the quasilinear equations to include nonlinear frequency shifts and their radial space variations, i.e. Zonal flows.

Zonal flows

- We know that turbulence in tokamaks can be expanded in the perturbations. Thus phase averaging is allowed in quasistationary turbulence.
- However a completely quasilinear description sometimes has limited validity. Nonlinear frequency shifts are important for describing generation of flows and, in fact, also for detuning kinetic resonances!
- Since the nonlinear frequency shifts have components with rapid space variation along radius they generate flowshear i.e. **Zonal Flows.**
- Zonal flows are obtained within a **phase averaged** description and can accordingly be included in **transport codes** for a given turbulent spectrum.
- We have previously found that drift wave turbulence can be well described by turbulence at one mode number at the inverse correlation length.

New model for transport of toroidal momentum

Toroidal momentum transport can be approximately described by the equation of parallel momentum. Since the parallel motion is the same for fluid and guiding centre drifts we must have the same equation in both descriptions

$$m\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)v_{\parallel} = e\left[E_{\parallel} + (\vec{v}x\vec{B})_{\parallel}\right] - \frac{1}{n}\widehat{\mathbf{e}}_{\parallel}\cdot\nabla P - \frac{1}{n}\widehat{\mathbf{e}}_{\parallel}\cdot\nabla\cdot\Pi$$

Here Π is the stress tensor. It has till now mainly been used to describe gyroviscous contributions to the perpendicular drifts but enters also for the parallel motion. A general feature is that the stress tensor cancels convective diamagnetic effects. (J.J. Ramos, Phys. Plasmas **12**, 112301-1 (2005)).

As it turns out it also introduces toroidal curvature contributions (D. Strintzi, A.G. Peeters and J. Weiland, Accepted, Phys. Plasmas). In particular we will get a **convective magnetic drift** with the coefficient 2.

Effects of magnetic curvature on the parallel motion

Convective magnetic drift effects enter in a natural way in the gyrokinetic equation (Freeman and Chen Phys. Fluids **25** 502 (1982), Hahm, Phys. Fluids **31**, 2670 (1988)). Convective magnetic drifts in the parallel momentum equation have for a long time been included in gyrofluid equations (see e.g. Waltz, Dominguez and Hammett Phys Fluids **B4** 3138 (1992).) These effects enter as:

$$m\left[\frac{\partial}{\partial t} + (\mathbf{v}_E + 2\mathbf{v}_D)\cdot\nabla\right]v_{\|} = e\left[E_{\|} + (\vec{v}x\vec{B})_{\|}\right] - \frac{1}{n}\hat{\mathbf{e}}_{\|}\cdot\nabla P \qquad \text{(i)}$$

Now using: $v_{\|} = V_{\|0} + \tilde{v}_{\|}$

$$\left|\hat{\phi}\right| = \frac{\gamma}{k_x c_s k_y \rho_s} \qquad \hat{\phi} = \frac{e\phi}{T_e}$$

and linearizing we obtain the diagonal diffusivity

Diagonal transport elements

$$\chi_\phi = \frac{\gamma^3/k^2}{(\omega_r - 2\omega_{Di})^2 + \gamma^2}$$

Here the Doppler shift comes from the stress tensor in a fluid description
The diagonal ion thermal conductivity has a similar form:

$$\chi_i = \frac{\gamma^3/k^2}{(\omega_r - \frac{5}{3}\omega_{Di})^2 + \gamma^2}$$

However here the Doppler shift comes from the closure term (Diamagnetic heat flow)

Convective part of toroidal momentum transport

We will here follow Hahm et. al. (Phys Plas. **14** (2007)). The main convective fluxes are of a non-thermodynamic Turbulent Equipartition (TEP) type and of thermoelectric (TERM) type. Unfolding the continuity equation from these parts, assuming temperature isotropy we obtain:

$$\Gamma_{tep} = V_{\|0} \operatorname{Re}\left\{ \frac{1}{2} \frac{\omega_{Di}}{\omega - 2\omega_{Di}} \hat{\phi} \mathbf{v}_{Er}{}^{*} \right\}$$

$$\Gamma_{term} = V_{\|0} \operatorname{Re}\left\{ \frac{\omega_{Di}}{\omega - 2\omega_{Di}} \delta T \mathbf{v}_{Er}{}^{*} \right\}$$

Here also the nonlinear decorrelation has been omitted. It will however enter in the final calculation of transport where we make the replacement $\omega \rightarrow \omega - i\omega_{ExB}$

This is the Waltz condition. Note that the stabilization by flowshear was pointed out already in 1966 (B. Lehnert, Phys. Fluids **9**, 1367 (1966)).

Prandtl number

- The ratio χ_ϕ / χ_i is generally called the Prandtl number.

Since the diagonal elements are almost the same, the Prandtl number will mainly be determined by **convective fluxes**

As pointed out, also the main convective parts come from magnetic curvature effects. T.S. Hahm et. al. Phys. Plasmas **14**, 072302-1 (2007) A.G. Peeters and C. Angioni and D. Strinzi, Phys Rev. Lett. **98** 265003 (2007).

Transport of poloidal momentum

Poloidal momentum transport can be formulated in a straight forward way from the Reynolds stress containing ExB and diamagnetic parts (Verified by gyrokinetic derivation J. Weiland, A. Eriksson, H. Nordman, and A. Zagorodny, PPCF **49**, A45 (2007))

We write the poloidal momentum equation as:

$$m\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)v_\theta = e(E_\theta + (\vec{v}\times\vec{B})_\theta) - \frac{1}{n}\frac{\partial P}{r\partial\theta}$$

$$\frac{\partial}{\partial t}V_\theta + \frac{\partial}{\partial r}(v_r v_\theta) = S_v \qquad \Gamma_\theta = v_{Er}v_\theta = -D_m\frac{\partial V_\theta}{\partial r}$$

Formulation cont.

Using $\Gamma_\theta = \langle v_{Er} v_\theta \rangle$ $\quad v_\theta = v_{E\theta} + v_{*\theta} + v_{conv-\theta}$

$$v_{Er} = -ik_\theta D_B \hat{\phi} \qquad v_{E\theta} = ik_r D_B \hat{\phi} \qquad v_{*\theta} = -iD_B k_r \frac{\delta P}{P} \qquad v_{conv-\theta} = -\xi_E \cdot \nabla V_\theta$$

where we used $D_B = \rho_s c_s$ $\quad \hat{\phi} = \frac{e\phi}{T_e}$ and ξ_E is the ExB displacement

$\mathbf{v}_{conv}$ gives the diagonal element

The convective Reynolds stress flux can then be written as:

$$\Gamma_p = -D_B{}^2 k_r k_\theta \frac{1}{2} \hat{\phi}^* \left[\hat{\phi} + \frac{1}{\tau} \hat{P}_i \right] + c.c$$

Diagonal and convective parts

- Since the convective velocity perturbation is

$$f = \frac{k_y}{\omega} D_B \widehat{\phi} \frac{dF_0}{dx} \qquad v_{conv-\theta} = \frac{k_y}{\omega} D_B \widehat{\phi} \frac{dV_\theta}{dx}$$

we get a phase shift due to the growthrate. The saturation level is:

$$\widehat{\phi} = \frac{\gamma}{k_\theta \rho_s k_x c_s}$$

Thus the diagonal diffusivity will be **cubic** in the growthrate

Transport coefficients

- Thus we get the diagonal element

$$\chi_\theta = \frac{\gamma^3 / k_x^{\ 2}}{\omega_r^{\ 2} + \gamma^2}$$

However since the ExB and diamagnetic velocity components are independent of frequency the convective flux is only **square** in the growthrate

$$\Gamma_p = -D_B^{\ 2} k_r k_\theta \frac{1}{2} \widehat{\phi}^* \left[\widehat{\phi} + \frac{1}{\tau} \widehat{P}_i \right] + c.c$$

Transport of poloidal momentum

- As can be seen from the formula. The ExB part of the Reynolds stress tends to be inward (always inward for harmonic variation), thus generating rotation. The diamagnetic part is usually somewhat larger and may also be inward. However, with fixed coefficients (fixed density and temperature) the total momentum is conserved. Thus the rotation will be in different directions at different radii. This, of cource, gives a sheared rotation!

Simulations (J. Weiland et al, International Conference on New Energy Sources October 22-25 2007, Tbilisi Georgia 2007) of JET shots with spontaneous poloidal rotation (K. Crombe et. al. PRL **95**, 155003 (2005)) has given poloidal rotation of the right order of magnitude.

Proceedure of simulations

- We simulate toroidal and/or poloidal fluid momentum. These are then entered into the radial force balance equation. Note that no additional fudge factor has been introduced for momentum transport!

$$E_r = B_\theta V_\phi - B_\phi V_\theta + \frac{1}{eZn}\frac{\partial P}{\partial r}$$

Shearing rate

$$\omega_{ExB} = \frac{r}{q}\frac{\partial}{\partial r}\left[qV_{E\theta} / r\right]$$

Waltz rule

$$\omega \rightarrow \omega - i\omega_{ExB}$$

Fluid closure aspects

- A single wave on a Maxwellian background distribution is Landaudamped since it represents the only deviation from thermodynamic equilibrium
- Quasilinear kinetic descriptions can not account for relaxation in velocity space of the main distribution
- In an inhomogeneous plasma we have the growthrate of the Universal drift instability:

$$\gamma = \left(\frac{\pi}{2}\right)^{1/2} \omega_{*e} \frac{\omega - \omega_{*e}}{k_{||} v_{te}} e^{-\omega^2/(k_{||} v_{te})}$$

- We notice that the inhomogeneity in space can give an instability which depends also on the real eigenfrequency
- This growthrate is sensitive to *nonlinear frequency shifts*!

Fluid closure aspects cont.

- In three wave interaction [1](I. Holod, J. Weiland and A. Zagorodny, *Nonlinear Fluid Closure; Three mode slab ion temperature gradient problem with diffusion*, Phys. Plasmas **9**, 1217 (2002)) the *nonlinear frequency shift* tends to average out the Landau damping. This leads towards a *reactive system*.

Pinch fluxes are *reversible* phenomena. Thus they are generally weaker in irreversible systems. This applies in particular to the particle pinch in quasilinear kinetic descriptions (C. Angioni, A.G. Peeters, F. Jenko and T. Dannert, Phys. Plasmas **12,** 112310 (2005))

Particle trapping is another strongly nonlinear effect which tends to average out kinetic resonances, thus strengthening pinches

1. Using a Mattor-Parker type system (Mattor, Parker PRL **79**, 3419 (1997)

Fluid closure aspects cont.

- *Further results and discussions on fluid closure can be found in*:
- J. Weiland, A. Eriksson, H. Nordman and A. Zagorodny, Plasma Phys. Control. Fusion **49**, A45 (2007).
- J. Weiland, A. Zagorodny and V. Zasenko, *On advanced fluid modelling of drift wave turbulence*, Recent Research Trends in Plasma Physics p 209-234, Transworld Research Network, Trivandrum, India 2007
- J. Weiland, A. Zagorodny and V. Zasenko, *Fluid modelling for times larger than the confinementtime in bounded systems*, Miniworkshop on Stochastic Processes and Transport, Chalmers University of Technology, May 29 2008.
- http://www.chalmers.se:80/rss/EN/research/research-groups/transport-theory/research/miniworkshop-on

Conclusions

We have here given several examples of physics processes that depend on simultaneously having several types of free energy in a system.

The coupling between two different free energies works as adding an external dive like the compressor in a refridgerator.

As it turns out, toroidal effects in a sufficiently accurate description (including the perpendicular fluid resonance) **dramatically** increase the importance of the coupling between different types of relaxation. The fluid resonance in the energy equation is important i.e. the fluid closure is important.

Momentum is not conserved for self-consistent simulations

However, strong flowshear layers can be formed with momentum in different directions without violating momentum conservation.

Conclusions cont.

Gyrofluid curvature effects in the parallel momentum equation can be recovered by stress tensor calculations.

Magnetic curvature effects are very important for the toroidal momentum. They increase both the diagonal (outward) flux and the convective (inward) flux.

Particle and momentum pinches are potentially very important for the performance of ITER.

A peaked density profile could easily give ignition for ITER.

Since no external momentum input is required there should be possibilities for transport barriers in ITER. The triggering is still not resolved and this is a major question for future work.

Conclusions, cont

Toroidal effects are very important for both pinch fluxes and fluid closure.

Pinch fluxes are reversible and are strongest when velocity space is treated in a self-consistent way.