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Computation of Turbulence, Transport, and Flows in Large-Scale Magnetically Confined Plasmas.

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Computation of Turbulence, Transport, and Flows in Large-Scale Magnetically Confined Plasmas

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Outline

• Basic Transfer Dynamics
  — low frequency basics, energy transfer in turbulence, equilibrium
  — nonlinear instability and saturation processes

• Gyrofluid Core Turbulence
  — electromagnetic, fully realistic parameters
  — self-generated flow stabilisation and energetics

• Gyrofluid Global Model
  — self consistent evolution of MHD equilibrium
  — core turbulence versus rotation
  — edge turbulence versus MHD instabilities (ELM crash scenario)

• Gyrokinetic Edge Turbulence
  — energy conservation consistency assured
  — trapping effects, saturation, mode structure
Magnetic Confinement

$$J \times B = c \nabla p$$

MHD equilibrium

- strong magnetic field, small gyroradius
- closed magnetic flux surfaces
- --> confined plasma
- however . . . turbulence --> losses

eddies, few gyroradii
Magnetic Field

Tokamak Magnetic Field

axisymmetric MHD equilibrium
toroidal, poloidal components
mainly toroidal
ratio of components --> pitch parameter “q” $\frac{B^\zeta}{B^\theta}$
Low Frequency Drift Motion

- Magnetic field
- Sense of gyration for ions
- Drift of gyrocenters ($v_\perp \ll v_\parallel$)
- V-space details: “gyrokinetic”
- Few moments: “gyrofluid”

General

Low frequencies $\omega \ll \Omega$
Low Pressure (Beta) Dynamics

low “‘beta’”

\[ p << B^2 / 8\pi \]

“flute mode”

vortices/filaments

\[ k_\parallel << k_\perp \]

magnetic field \( B \)

--> strict perpendicular force balance

\[ \nabla(\tilde{\rho} + 4\pi \tilde{B} \tilde{B}) \sim 0 \]

low frequencies

\[ \omega << k_\perp v_A \]

pressure disturbance \( \tilde{p} \)

magnetic disturbance \( \tilde{B} \)

(parallel to \( B \))

\[ \omega \sim k_\parallel v_A \]

--> electromagnetic parallel dynamics
computations: align coordinates to magnetic field (sheared, curved)
(only one contravariant component of B is nonvanishing)
(nonorthogonal, takes advantage of slowly varying B)

ExB Drift at Finite Gyroradius

\[ v_E = \frac{c}{B^2} B \times \nabla \phi \]

\[ k \rho \ll 1 \]

\[ u_E = \frac{c}{B^2} B \times \nabla J_0 \phi \]

\[ k \rho \sim 1 \]
Phase Shifts and Transport

$\nabla p$

p and phi in phase
---> no net transport

phase shift --> net transport
down gradient
---> free energy drive
Role of Parallel Forces on Electrons

Equation of motion for electrons parallel to B

\[
n_{\text{e}} e \left( \frac{1}{c} \dot{A}_\parallel + \nabla_\parallel \phi + \eta_\parallel J_\parallel \right) = \nabla_\parallel p_\text{e} + \text{inertia}
\]

Alfvén (MHD) coupling  
adiabatic (fluid compression) coupling

a “two fluid” effect

Static balance of gradients --> “adiabatic electrons”

general: response of currents to static imbalance

Controls possible phase shifts  
\[ \tilde{p}_\text{e} \quad <--> \quad \tilde{\phi} \]
Drift (Alfven) Wave Dynamics

\[ \nabla \tilde{p} \]

\[ \tilde{\phi} \]

\[ \text{ion current} \]

\[ \text{electron current} \]

\[ \text{sound waves} \]

\[ \rightarrow \text{structure drifts} \]

\[ \rightarrow \tilde{\phi} \text{ coupled to } \tilde{p} \text{ through Alfven dynamics} \]

\[ \rightarrow \tilde{\phi} \text{ continually excites } \tilde{p} \text{ in the gradient} \]

(M Wakatani A Hasegawa Phys Fluids 1984)

(B Scott Plasma Phys Contr Fusion 1997)
Scales of Motion

broad range of both time and space scales — to ion gyroradius

slowest time scale reflect flow/equilibrium component

for equal temperatures, space scale range includes ion gyroradius

high resolution, long runs (> 1000 "gyro–Bohm" times) are necessary

(B Scott Plasma Phys Contr Fusion 2003, 2006)
Numerical Methods

- nonlinearities have the form of brackets

\[ \frac{\partial f}{\partial t} + [\psi, f]_{xy} + \cdots = 0 \quad \text{with} \quad [\psi, f]_{xy} = \frac{\partial \psi}{\partial x} \frac{\partial f}{\partial y} - \frac{\partial f}{\partial x} \frac{\partial \psi}{\partial y} \]

- spatial discretisation:
  centered-diff for linear terms, Arakawa (J Comput Phys 1966) scheme for brackets
  - basic properties of bracket satisfied to machine accuracy

\[ [\psi, f]_{xy} = \frac{1}{3} (J_+ + J_0 + J_\times) \]

- temporal discretisation:
  - both sides expanded \( \Rightarrow \) all mixed terms in Taylor expansion present
  - one evaluation per time step
  - tested on turbulence and coherent vortices (Naulin and Nielsen, SIAM J Math 2003)

\[ \frac{\partial f}{\partial t} = S \quad \text{with} \quad \sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{j \Delta t} = \sum_{j=1}^{3} \beta_j S_j \]
Field Aligned Coordinates

- general axisymmetric Clebsch representation (Dewar/Glasser *Phys Plasmas* 1983)
  - global consistency, shifted metric (B Scott *Phys Plasmas* 1998, 2001)

\[
\mathbf{B} = \nabla \chi \times \nabla y_k \quad y_k = q(\theta - \theta_k) - \zeta - \Delta \alpha_k \quad s = \theta
\]

- Hamada definitions — choose $\Delta \alpha_k(V)$ such that $g_{xy}^k = 0$ at $\theta = \theta_k$

\[
\chi = \chi(V) \quad B^V = B^y_k = 0 \quad B^s = \chi'(V)
\]

- derivative combination in ExB bracket at $\theta = \theta_k$

\[
v_E \cdot \nabla f = \nabla \phi \cdot \hat{\mathbf{F}} \cdot \nabla f \rightarrow F_0^{xs} \frac{\partial \phi}{\partial x} \left( q \frac{\partial f}{\partial y} + \frac{\partial f}{\partial s} \right) - (\leftrightarrow) \equiv [\phi, f]
\]

- all derivatives — tensor transformation rules
  - divergence-free $F_0$ chosen from $\hat{\mathbf{F}} = (c/B^2) \epsilon \cdot \mathbf{B}$, conserves free energy
Nonlinear Saturation

basic feature of any instability — transition to turbulence

linear drive \((n)\) \(\rightarrow\) linear growth

moment of saturation — growth rate \((T)\) drops to zero

saturation maintained — nonlinear transfer to subgrid scale dissipation \((E)\)

transport \((Q)\) overshoots, finds saturated balance

\((B\ Scott\ Phys\ Plasmas\ 6/2005)\)
Nonlinear Cascade in Turbulence

basic statistical character of three wave energy transfer

transfer between wavenumber magnitudes — from $k'$ to $k$

all activity near the $k' = k$ line → cascade character

ExB energy is inverse, while other quantities are direct (to higher $k$)

dominant transfer is through the thermal free energy ($n$), others also active

(S Camargo et al Phys Plasmas 1995, 1996)
Nonlinear Instability

basic feature of drift wave turbulence (edge turbulence test case)

amplitude threshold $\rightarrow$ linear stability

vorticity nonlinearity $\rightarrow$ damped eigenmodes destabilise each other

role of pressure advection nonlinearity $\rightarrow$ saturation

edge turbulence $\rightarrow$ washes out microinstabilities in toroidal magnetic field

Energy Transfer

part of energy theorem governed by vorticity equation

\[-\phi_{-k} \left( \frac{d}{dt} \Omega + v_E \cdot \nabla \Omega + \text{FLR} = \nabla \cdot J || + \nabla \cdot \frac{c}{B^2} B_x \nabla p \right)_k \]

vorticity $\Omega = (n_e - n_i) e$

currents: polarisation parallel diamagnetic

free energy: source in pressure equation, transfer in to vorticity equation

pathways: over parallel dynamics or toroidal compression

between modes within ExB energy —– nonlinear advection

direct, in–context measurement of physical mechanism supporting turbulence

(B Scott Phys Plasmas 2000)
Nonlinear Saturation

basic feature of any instability — transition to turbulence

linear drive (n) —> linear growth

moment of saturation — growth rate (T) drops to zero

saturation maintained — nonlinear transfer to subgrid scale dissipation (E)

transport (Q) overshoots, finds saturated balance

(B Scott Phys Plasmas 6/2005)
Vorticity Energetics -- Transition to Turbulence

turbulence imposes its own mode structure on dynamics

linear interchange mode — balance between diamagnetic/parallel currents

turbulence — emergence of nonlinear ExB vorticity advection

developed turbulence — balance between polarisation/parallel currents

basic mechanism supporting eddies in turbulence differs from linear instability

(B Scott Plasma Phys Contr Fusion 2003)
Energy Transfer: electromagnetic turbulence

(S Camargo et al Phys Plasmas 1995 and 1996)
Energy Transfer: equilibrium

\[ \phi \sim \text{ion dissipation} \]

\[ \rho_i \quad \text{transport} \]

\[ \phi \quad \text{Reynolds stress} \]

\[ p_e \quad \text{transport} \]

\[ p_e \quad \text{Alfven couple to pressure} \]

\[ J \quad \text{loop voltage} \]

(B Scott Phys Plasmas 2003)
Suppression of Turbulence by Flows
(Biglari Diamond Terry, Phys Fl B 1991)

eddies tilted into energy–losing relationship to flow vorticity

--> same process as in self generation
Zonal Flow, Toroidal Compression


zonal flow exchanges conservatively with pressure sideband

---→ transfer pathway, equipartition
Energy Transfer: flows and currents

- Ion dissipation
- Diamagnetic compression
- Adiabatic compression
- 2-fluid effects
- Reynolds stress
- MHD effects
- Transport
- Resistivity
- P–S current

eddy Reynolds stress $\rightarrow$ energy transfer from turbulence to flows

turbulence moderately weakened but not suppressed
toroidal compression $\rightarrow$ energy loss channel to pressure, turbulence
entire system in self regulated statistical equilibrium (turb, flows, mag eq)

Nonlinear Threshold Upshift

cyclone ITG, adiabatic electrons, periodic S-alpha, $100 \times 256\rho_s$

growth rate max values for each case, zero point by extrapolation
transport diffusivity curve shows threshold upshift to 6.0 (B Scott PPCF 2006)
captures standard gyrokinetic result (Dimits et al Phys Plasmas 2000)
perturbed equilibrium and ion flow divergence profiles

cyclone ITG, periodic S-alpha, \( R/L_T = 6.91 \), \( \hat{\beta} = 0 \), \( 100 \times 256 \rho_s \)

\[
\phi(x)
\]
\[
\n_i(x)
\]
\[
T_i(x)
\]

\[
\nabla_v \sin \theta(x)
\]
\[
\nabla_{\parallel} \sin \theta(x)
\]
\[
\nabla_v \sin \theta(x)
\]

\( t = 4000 \).
perturbed equilibrium and ion flow divergence profiles

cyclone ITG, periodic S-alpha, \( R/L_T = 4.83 \) \( \beta = 0 \) \( 100 \times 256 \rho_s \)

\( t = 4000. \)
electromagnetic cases — notes

• nominal value of beta

\[ \hat{\beta} = \frac{4\pi p_e}{B^2} \left( \frac{qR}{L_\perp} \right)^2 = 0.465 \]

• very strong “flutter” effects

\[ \nabla_\parallel = b^s \frac{\partial}{\partial s} - \hat{\beta}[A_\parallel, ] - \nabla_\perp^2 A_\parallel = J_\parallel \leftrightarrow \nabla_\parallel (p_e - \phi) \]

• as \( \hat{\beta} \) rises from zero, transport drops
  ○ complete stabilisation for \( \hat{\beta} = 0.465 \) (flows) and 0.52 (flutter)
  ○ onset of kinetic ballooning (no saturation in periodic S-alpha) for \( \hat{\beta} = 0.6 \)

• very important resolution consideration — require \( h_x/h_y = 1/4 \)

\[ \Delta_{rs} = \frac{1}{\hat{s}k_y} \quad \text{hence} \quad h_x < \frac{h_y}{\pi \hat{s}} \]

otherwise short wavelength electron response doesn’t see magnetic shear
Electromagnetic Effect on Transport
cyclone ITG, periodic S-alpha, $100 \times 256\rho_s$

beta stabilises due to both flows and linearly (next slides)gradient destabilisation directly to kinetic ballooning regime (no saturation)
perturbed equilibrium and ion flow divergence profiles

cyclone ITG, periodic S-alpha, \( R/L_T = 6.91 \quad \hat{\beta} = 0.465 \quad 100 \times 256\rho_s \)

\[ t = 4000. \]
time traces, electromagnetic
cyclone ITG, periodic S-alpha, $100 \times 256 \rho_s$

ion heat flux

$\beta = 0.03, 0.2, 0.464, 0.52, 0.6$

$c_s t / L_\perp$
Incorporation of Magnetic Equilibrium

toroidal equilibration current $\leftrightarrow$ Shafranov shift

P–S current equilibrates toroidal diamagnetic compression

Ampere’s Law $\rightarrow$ “Pfirsch–Schlueter magnetic field” $\rightarrow$ toroidal shift

current stays in moment variables, magnetic field in coordinate metric
Global Electromagnetic Gyrofluid (GEM):

- turbulence and transport
- (profile + disturbances)
- self consistent magn eq, geometry
  (Pf–Sch currents --- Shafranov shift)

L–Mode Base Case (ASDEX Upgrade generic)
- correct mass ratio, gyroradius
- closed/open flux surfaces, separatrix topology

(B Scott Contrib Plasma Phys 2006)
Global Computation in Divertor Geometry

study of turbulence vs rotation scale separation

\[ n_e(r, \theta) \]

\[ \nabla \cdot \mathbf{v} \sin \theta(r_a) \]

\[ \phi(r_a) \]
Gyro-Bohm Convergence and Large Tokamaks

- global drift parameter $\rho_* = \rho_s/a$

- shape of spectrum
  - follows gyroradius, not profile scale length
  - long wavelength side serving as sink must be wide enough

- transport flux level
  - scales as square of $\rho_*$, converges when spectrum does

- toroidal flow ("neoclassical") equilibrium
  - drifts: forcing scales as $\rho_*$
  - turbulence: forcing scales as square of $\rho_*$
  - flow profile converges when turbulence forcing drops out

- time scale separation
  - both neoclassical and turbulence effects scale as square of $\rho_*$
  - large tokamak regime reached when source or decay effects can be ignored

- large tokamak regime is reached generally for $a/\rho_s = 200$ (AUG size)
  - and for $a/\rho_s = 400$ (JET size) for profiles with structure (e.g., ITBs)
Spectra for Medium to Large Tokamak Cases

- density and vorticity spectra for the three cases

- ion heat source and sink spectra for the three cases
Ion Flow Sideband Divergences — AUG Case

- flow divergence pieces balance closely, slight ZF activity visible
Ion Flow Sideband Divergences — JET Case

- signal of ZF activity now very weak
• signal of ZF activity practically nonexistent, divergences are smaller
Look and Feel of Scale Separation

electromagnetic core cases with $a/\rho_s$ of 50, 100, and 200, non-axisymmetric part

- if you can see the eddies on a global plot they’re too large!
IBM Blowout Studies using GEM
A Kendl and B Scott 2007/8

• main aim: study of ELM blowout
  ◦ actually just a ballooning instability transitioning into turbulence
  ◦ study physical mechanisms and scalings first, then experimental issues

• GEM: electromagnetic 6-moment gyrofluid for both electrons, ions
  (B Scott, Phys Plasmas Oct 2005)

• global geometry, self consistent \( q(r) \) and Shafranov shift from \( J_{||} \)

• Base Case: AUG #17151 H-mode deuterium
  (L Horton et al, Nucl Fusion 2005)

  ◦ main linear mode near toroidal mode 9 – 10
  ◦ violent overshoot, cascade, crash (no nonlinear instability)
  ◦ then segue into remnant turbulence
IBM Blowout

- 20 \( \mu \)sec after apparent quiet
- profile blown away
- finger structure obvious (and trivial)
Blowout Time Traces

- energy, *ca.* 2/3 lost in blowout
- flux, short event (<40 µsec)
- growth rate as per flux, linear then segue into turbulent aftermath
Blowout Spectra

- linear growth (left) and peak-flux (right) phases

- linear growth: peak modes are $n = 9$ and $10$ (MHD: $e\tilde{\phi}/T_e$ largest)

- peak-flux: vorticity already flat to ion gyroradius scale

  crash phase is outside not only MHD but also Braginskii regime
Blowout — Resolution in Drift Angle

• various $N_y$ correspond to max $k_y \rho_s = 0.7 \quad 1.5 \quad 3 \quad 6 \quad (\ > 1 \text{ is required})$
• crossing the threshold ... very high $\beta_e$ saturates earlier
  ◦ note the sound speed normalisation — growth rates are near $\gamma_I$
ELM crash scenario — notes

- nonlinear aftermath leaves MHD regime very quickly
  ○ saturates on its own nonlinearly developed ITG turbulence

- nonlinear convergence requires ion gyroradius \((10^{-2} < k_{\perp} \rho_i < 6)\)

- beta dependence: continuous transition MHD ↔ ITG turbulence
  ○ filament size reflects MHD or ITG sides (smaller for ITG)
  ○ strength of overshoot/bursts follows (smaller for ITG)

underlying character of actual burst events in L-Mode and ELMs may be similar (S Zweben et al PPCF 2007)
Comparison -- Fluctuation Statistics

probability distribution of cross phase for each Fourier mode
unified spectrum, phase shifts between 0 and $\pi/4$, in code and TJK experiment
basic signature of drift wave mode structure (parallel current dynamics)

Comparison -- Fluctuation Statistics

wavelet analysis of fluctuation induced transport in code and TJK experiment
both results show same phenomenology: regime break in spectrum
evidence of nonlinear cascade overcoming drive?

(N Mahdizadeh et al Phys Plasmas 2004)
Nonlinear Free Energy Cascade

average transfer

**direct cascade**

\[ \rightarrow \text{nonlinear drive at small scales} \]

\[ \rightarrow \text{passive scalar regime} \]

**frequency/scale correlation**

matches with frequency break

**evidence for onset of**

**passive scalar regime**
Gyrokinetic Edge Turbulence

• “total-f” version (in development)

\[ B^* \frac{\partial f}{\partial t} + \nabla H \cdot \frac{c}{e} \frac{F}{B} \cdot \nabla f + B^* \cdot \left( \frac{\partial H}{\partial p_z} \nabla f - \frac{\partial f}{\partial p_z} \nabla H \right) = C(f) \]

\[ F = \epsilon \cdot B \quad B^* = B - \nabla \cdot \frac{p_z}{e} \frac{c}{B} F \quad B^* = b \cdot B^* \]

• “delta-f” version (from which results shown)

\[ \frac{\partial \tilde{g}}{\partial t} + \frac{cF^{xy}}{eB^2} [\tilde{H}, \tilde{h}]_{xy} + \frac{B^s}{B} [H_0, \tilde{h}]_{zs} + \mathcal{K}(\tilde{h}) = C(\tilde{f}) \]

\[ \tilde{h} = \tilde{f} + eJ_0 \tilde{\phi} \frac{F^M}{T} \quad \tilde{g} = \tilde{f} + e \frac{v_{\parallel}}{c} J_0 \tilde{A}_\parallel \frac{F^M}{T} \quad \mathcal{K} = \nabla \frac{\mu B - mv_{\parallel}^2 \log R}{e} \cdot \frac{c}{e} \frac{F}{B} \nabla \{x,y\} \]

• \( H \) is Hamiltonian, with unperturbed and perturbed parts \( H_0 \) and \( \tilde{H} \)
  ○ \( C \) is collision operator
• shallow rise begins for $\hat{\beta} > 1$
  ○ all ExB transport channels follow each other
  ○ “magnetic flutter” becomes positive for $\hat{\beta}$ in transition to MHD

• trapping in equilibrium enhances transport (long-wave MHD component, see below)
Gyrokinetic and Gyrofluid Transport Compared

nominal and no-trap GK models versus gyrofluid model

- trend in gyrofluid (GEM) very much like gyrokinetic (dFEFI)
  - especially no-trap version, where models agree on upturn position

- exposes the rising beta trend as general
  - mode structure analysis: nonlinear drive of long wavelength MHD component
**Basic Nonlinearity of Edge Turbulence**

gyrokinetic turbulence vs linear growth rates

- exposes the rising beta trend as nonlinear-only in this range
  - long-wave component unimportant in linear stage (low growth rates)
  - ExB energy transfer (see above) gives it extra strength

- transport level determined as much by saturation as by drive
  - self consistency determines overall level
Main Points

basics of energetics a central theme for physical understanding

wide overlap between gyrokinetic and gyrofluid models

  temperature anisotropy and resolution of ion gyroradius are required

coupling of turbulence to flows extends to the magnetic equilibrium

  self consistency: do the magnetic background inside the turbulence model

new physics themes:

  ⚫ global electromagnetic computation

  ⚫⚫⚫ stable reconnection and equilibration currents

    incorporation of trapping effects in fluid codes (may be hopeless)

  ⚫⚫ nonlocal gyrofluid/gyrokinetic models → edge/core transition

    one should expect surprises affecting design of high performance devices