The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1953-9

International Workshop on the Frontiers of Modern Plasma Physics

14 - 25 July 2008

Computation of Turbulence, Transport, and Flows in Large-Scale Magnetically Confined Plasmas.

B.D. Scott

Max-Planck Institut fuer Plasmaphysik
Garching Bei Munchen
Germany

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Computation of Turbulence, Transport, and Flows in Large-Scale Magnetically Confined Plasmas

B. Scott

Max Planck Institut für Plasmaphysik
Euratom Association
D-85748 Garching, Germany

ICTP Workshop, Trieste, July 2008

Outline

- **Basic Transfer Dynamics**

  — low frequency basics, energy transfer in turbulence, equilibrium

  — nonlinear instability and saturation processes

- **Gyrofluid Core Turbulence**

  — electromagnetic, fully realistic parameters

  — self-generated flow stabilisation and energetics

- **Gyrofluid Global Model**

  — self consistent evolution of MHD equilibrium

  — core turbulence versus rotation

  — edge turbulence versus MHD instabilities (ELM crash scenario)

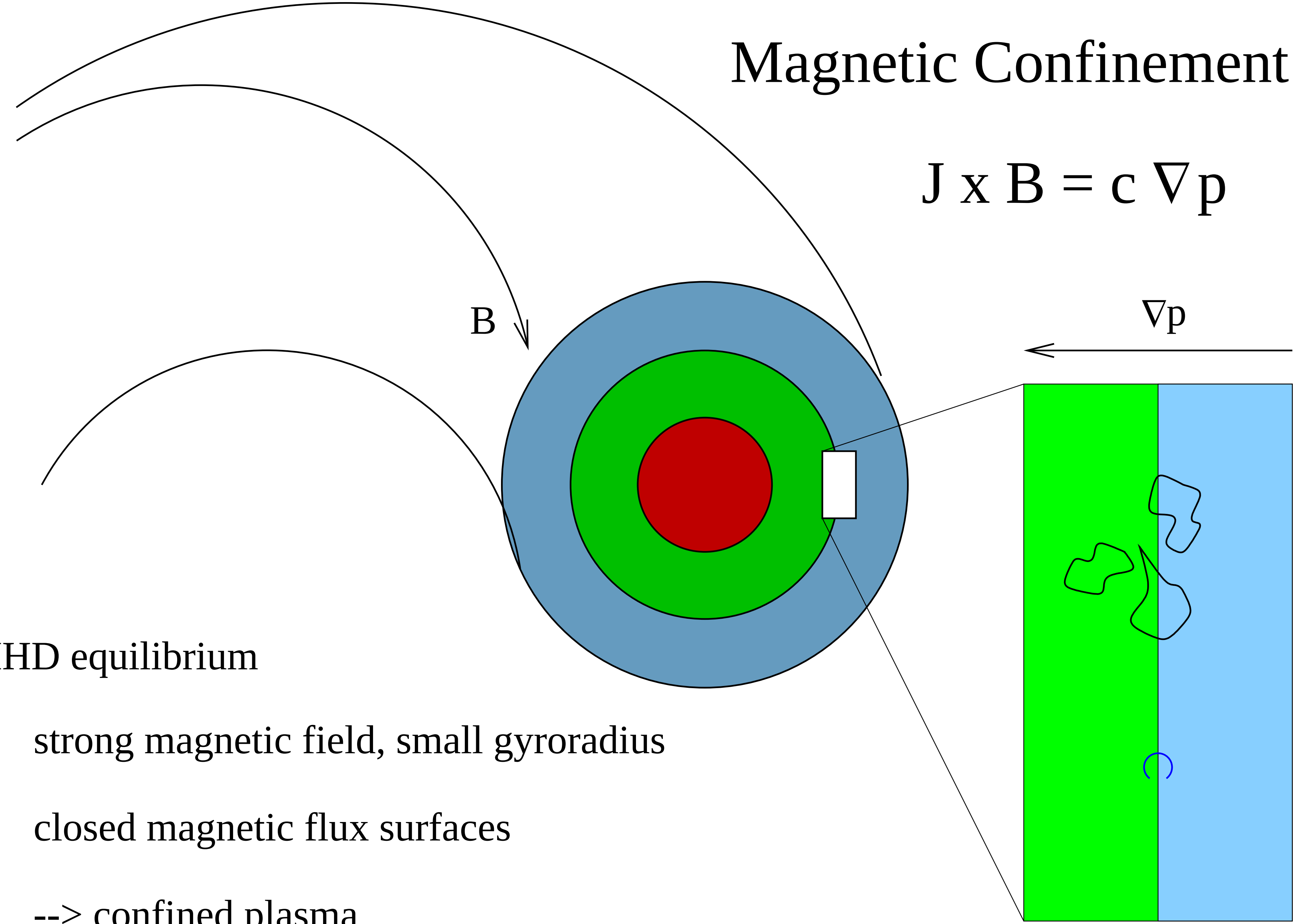
- **Gyrokinetic Edge Turbulence**

  — energy conservation consistency assured

  — trapping effects, saturation, mode structure

Magnetic Confinement

$$\mathbf{J} \times \mathbf{B} = c\, \nabla p$$



MHD equilibrium

strong magnetic field, small gyroradius

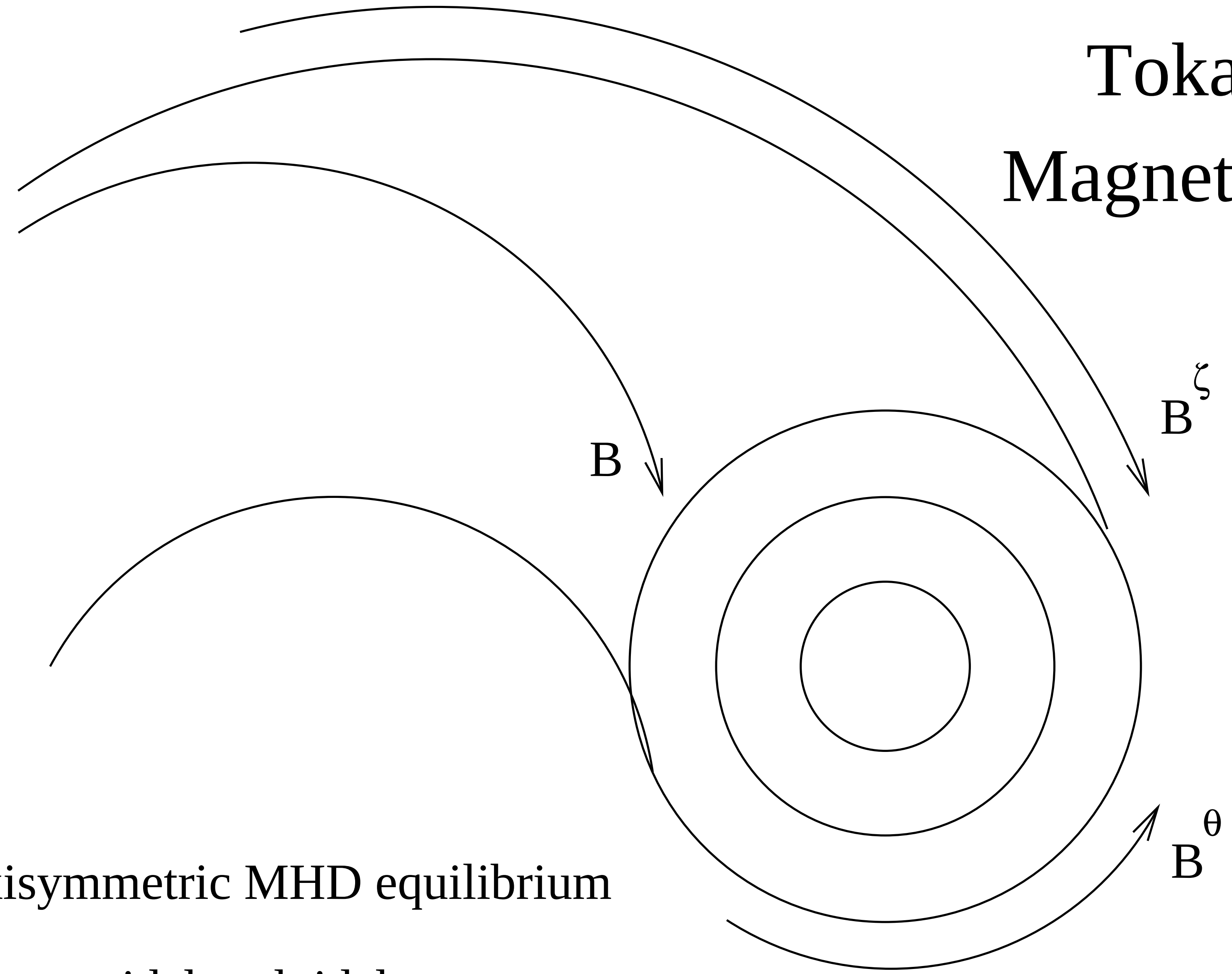
closed magnetic flux surfaces

--> confined plasma

however . . .   turbulence --> losses

eddies, few gyroradii

Tokamak Magnetic Field



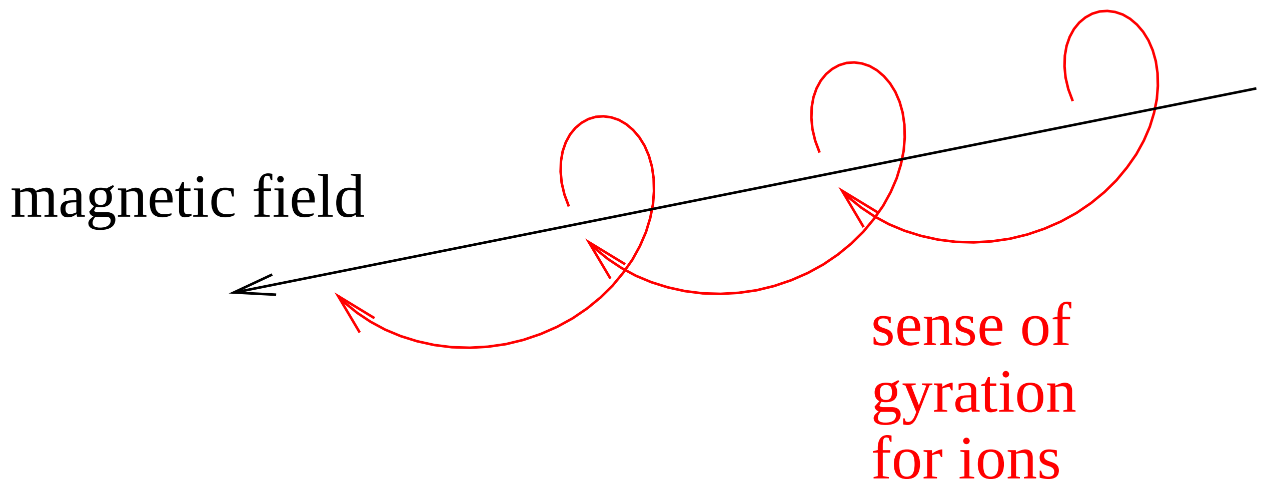
axisymmetric MHD equilibrium

toroidal, poloidal components

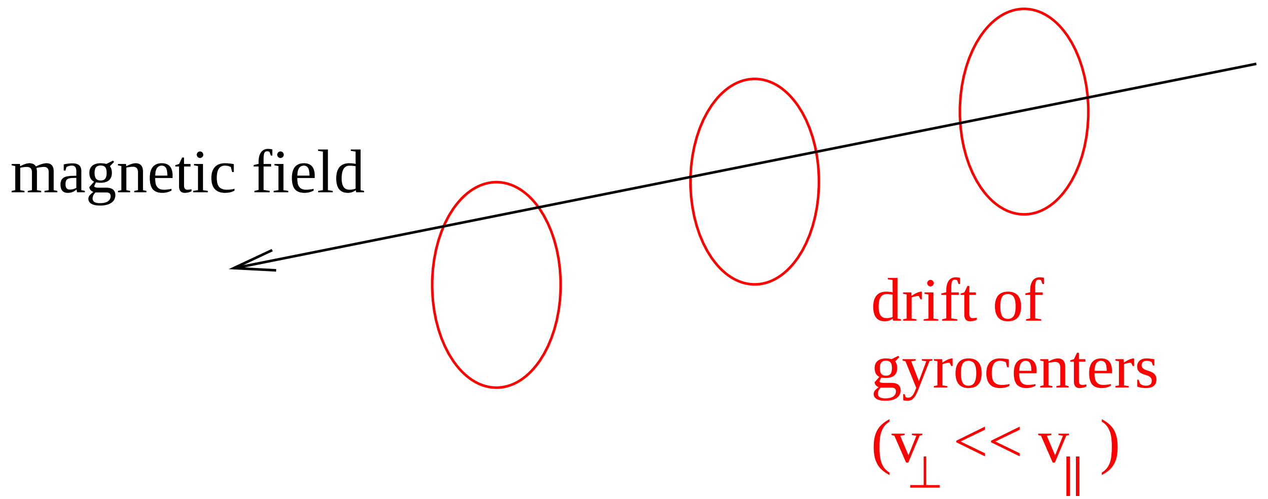
mainly toroidal

ratio of components --> pitch parameter ‘‘q’’ B^ζ / B^θ

Low Frequency Drift Motion



general



low frequencies

$\omega << \Omega$

v-space details: ‘‘gyrokinetic’’

few moments: ‘‘gyrofluid’’

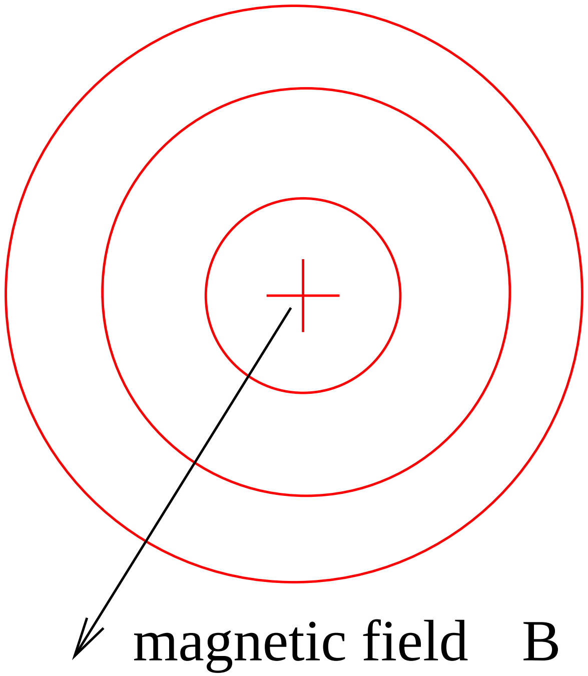
Low Pressure (Beta) Dynamics

low ‘‘beta’’

$p << B^2/8\pi$

low frequencies

$\omega << k_\perp v_A$



pressure disturbance $\tilde{p}$

magnetic disturbance $\tilde{B}$
(parallel to B)

‘‘flute mode’’

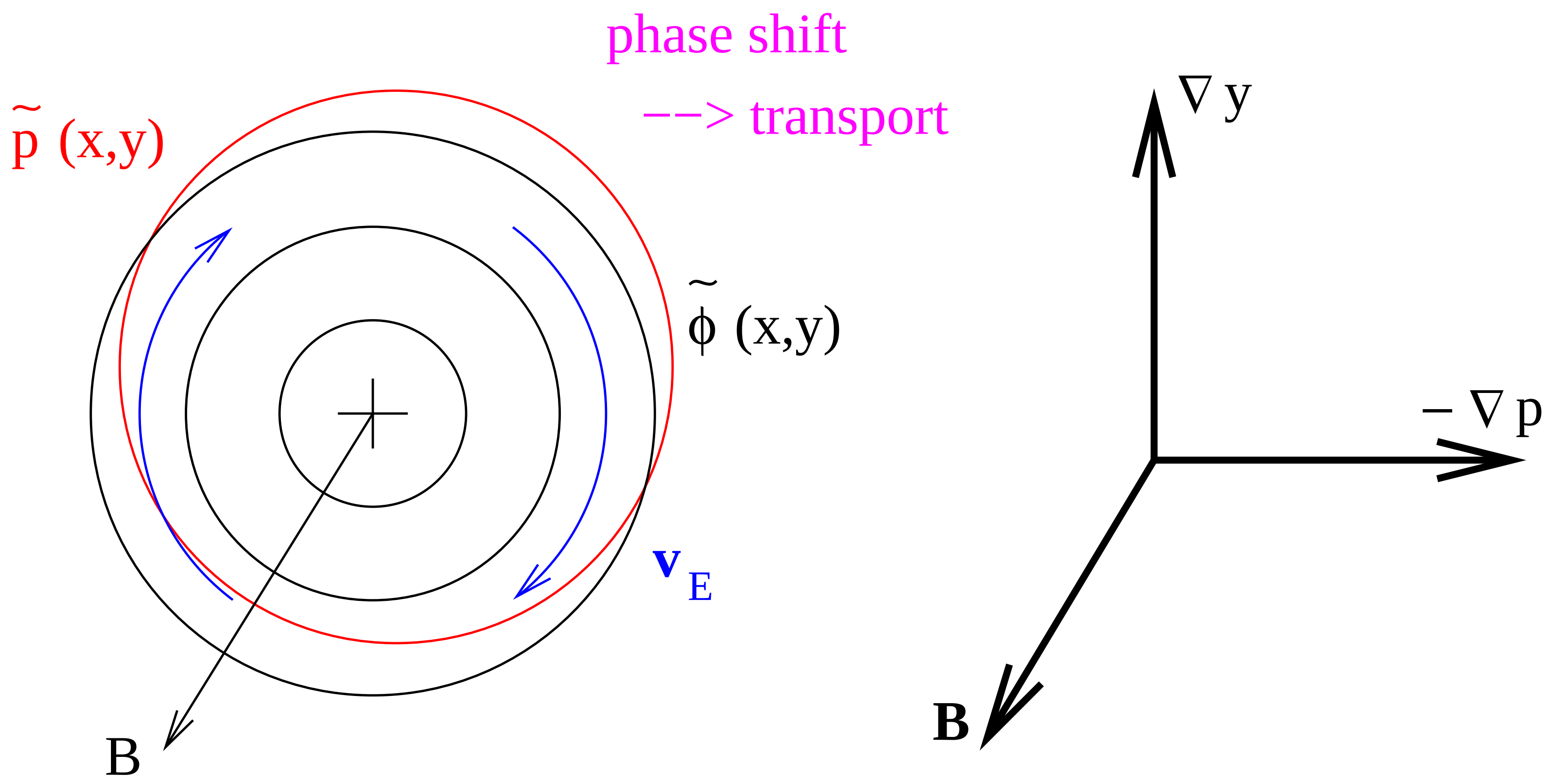
vortices/filaments

$k_{||} << k_\perp$

--> strict perpendicular force balance $\nabla(\tilde{p} + 4\pi\, B\tilde{B}) \sim 0$

$\omega \sim k_{||} v_A$ --> electromagnetic parallel dynamics

Sense of Coordinate Geometry



computations: align coordinates to magnetic field (sheared, curved)

(only one contravariant component of B is nonvanishing)

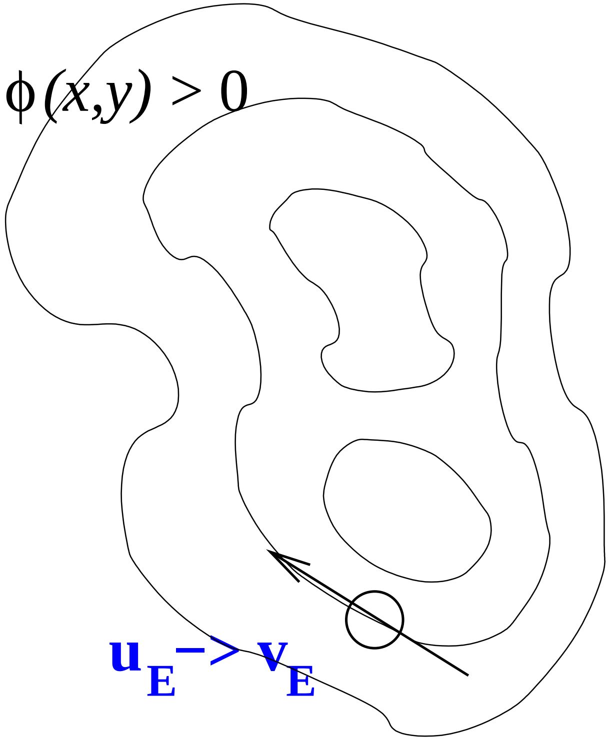
(nonorthogonal, takes advantage of slowly varying B)

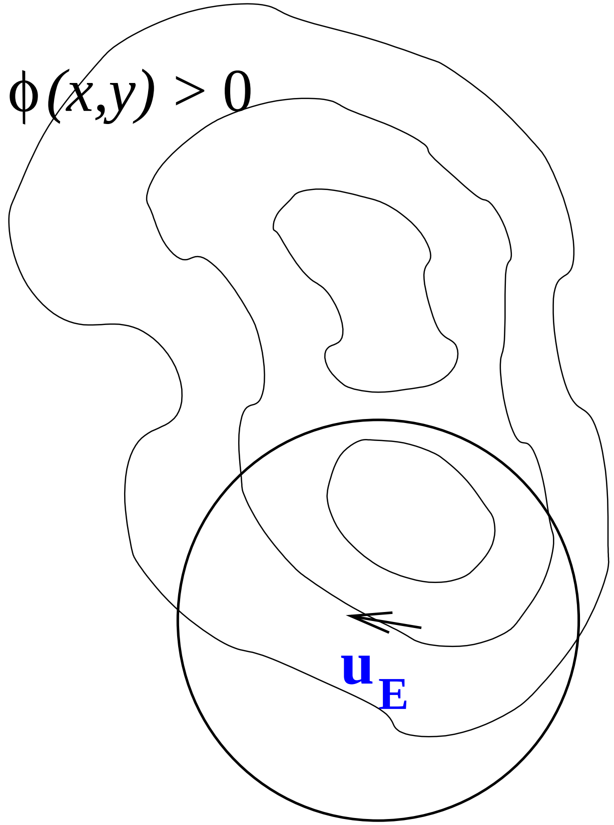
(S Cowley et al Phys Fluids B 1991, B Scott Phys Plasmas 1998, 2001)

ExB Drift at Finite Gyroradius

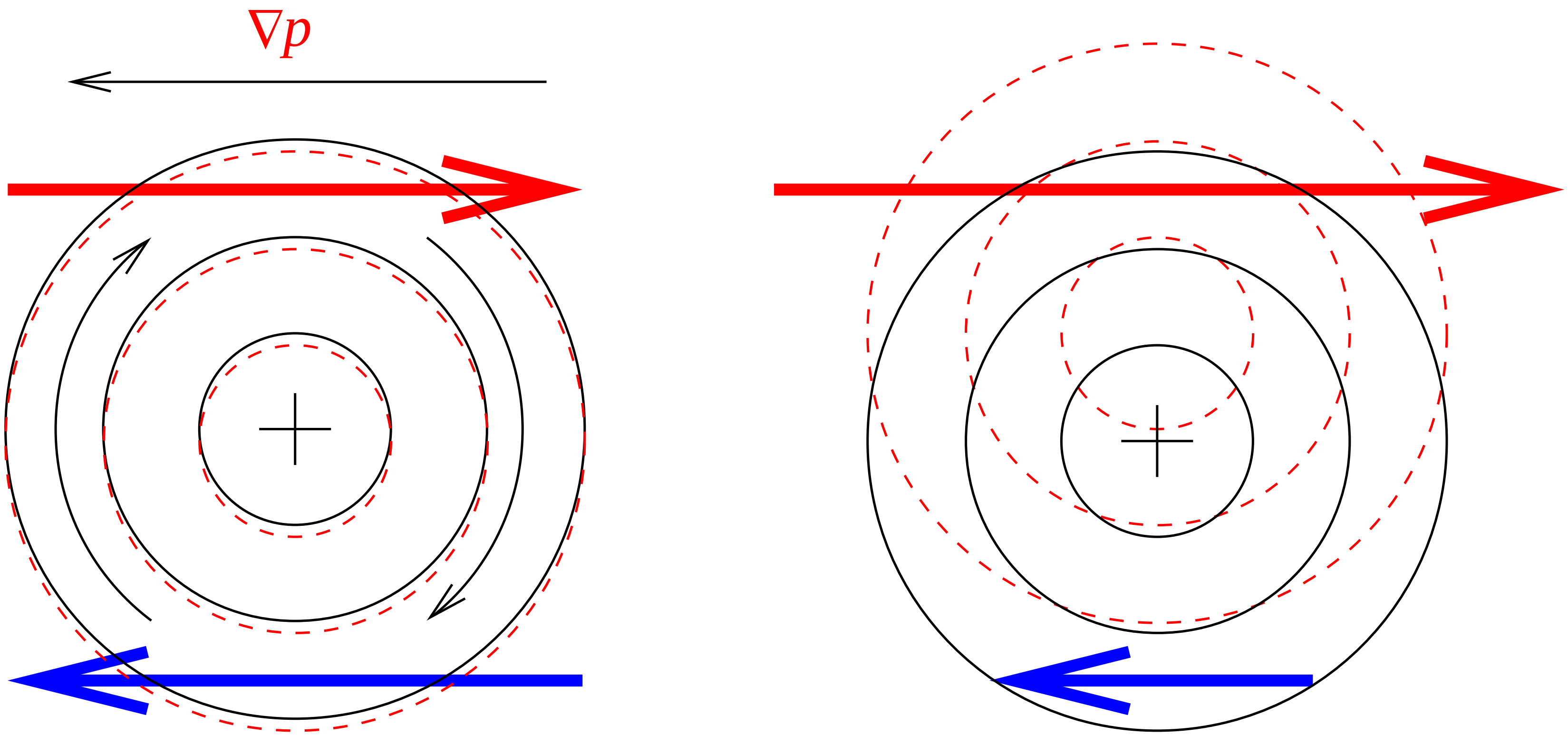
$$v_E = \frac{c}{B^2} \mathbf{B} \times \nabla \phi \qquad u_E = \frac{c}{B^2} \mathbf{B} \times \nabla J_0 \phi$$

$k\rho \ll 1 \qquad k\rho \sim 1$





Phase Shifts and Transport



p and phi in phase

--> no net transport

phase shift --> net transport

phase shift --> net transport down gradient

--> free energy drive

Role of Parallel Forces on Electrons

equation of motion for electrons parallel to B

$$n_e e \left(\frac{1}{c} \dot{A}_{\parallel} + \nabla_{\parallel} \phi + \eta_{\parallel} J_{\parallel} \right) = \nabla_{\parallel} p_e + \text{inertia}$$

Alfven (MHD) coupling

adiabatic (fluid compression) coupling

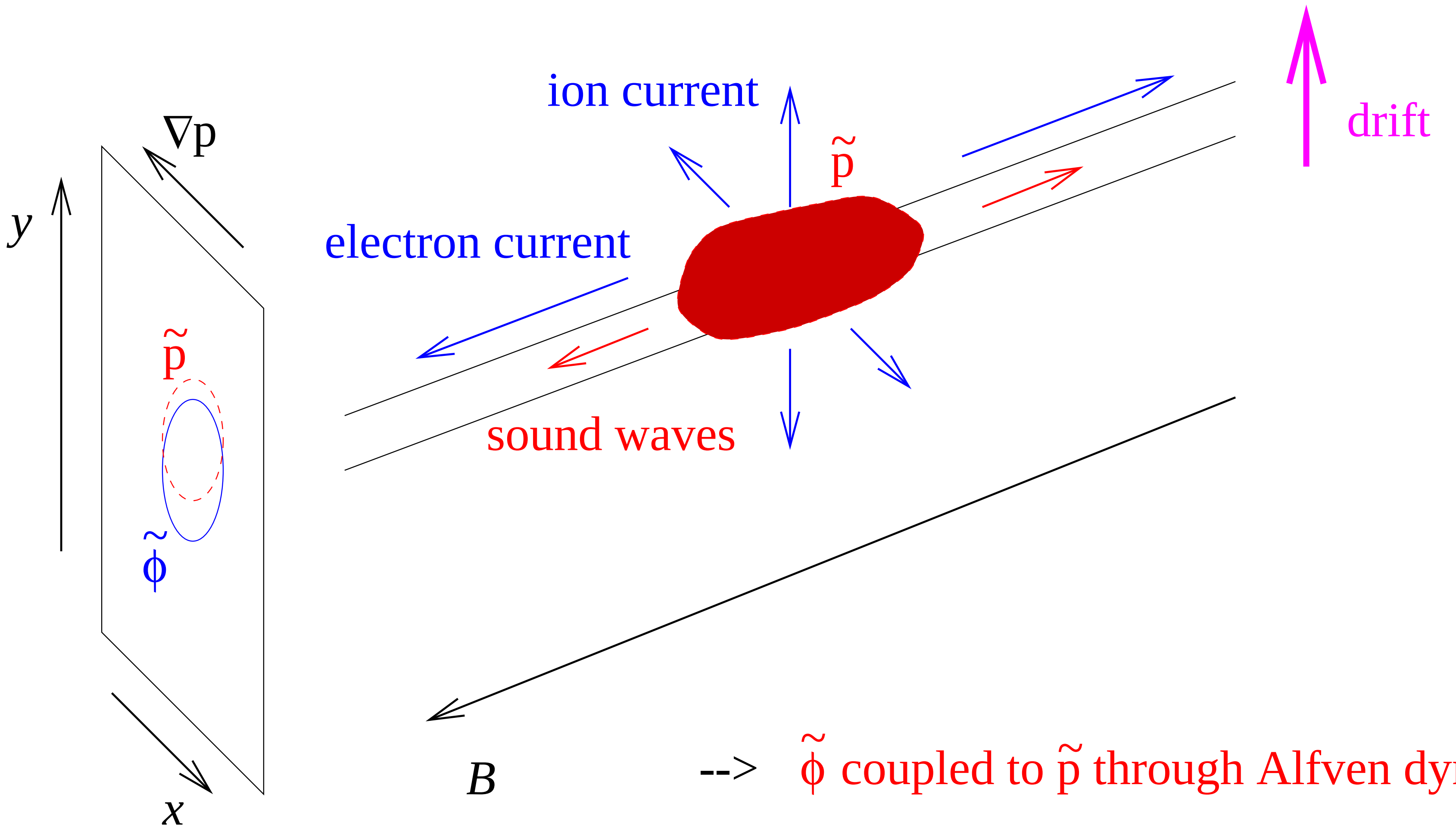
a ‘‘two fluid’’ effect

static balance of gradients --> ‘‘adiabatic electrons’’

general: response of currents to static imbalance

controls possible phase shifts $\tilde{p}_e$ <--> $\tilde{\phi}$

Drift (Alfven) Wave Dynamics



--> $\tilde{\phi}$ coupled to $\tilde{p}$ through Alfven dynamics

--> $\tilde{\phi}$ continually excites $\tilde{p}$ in the gradient

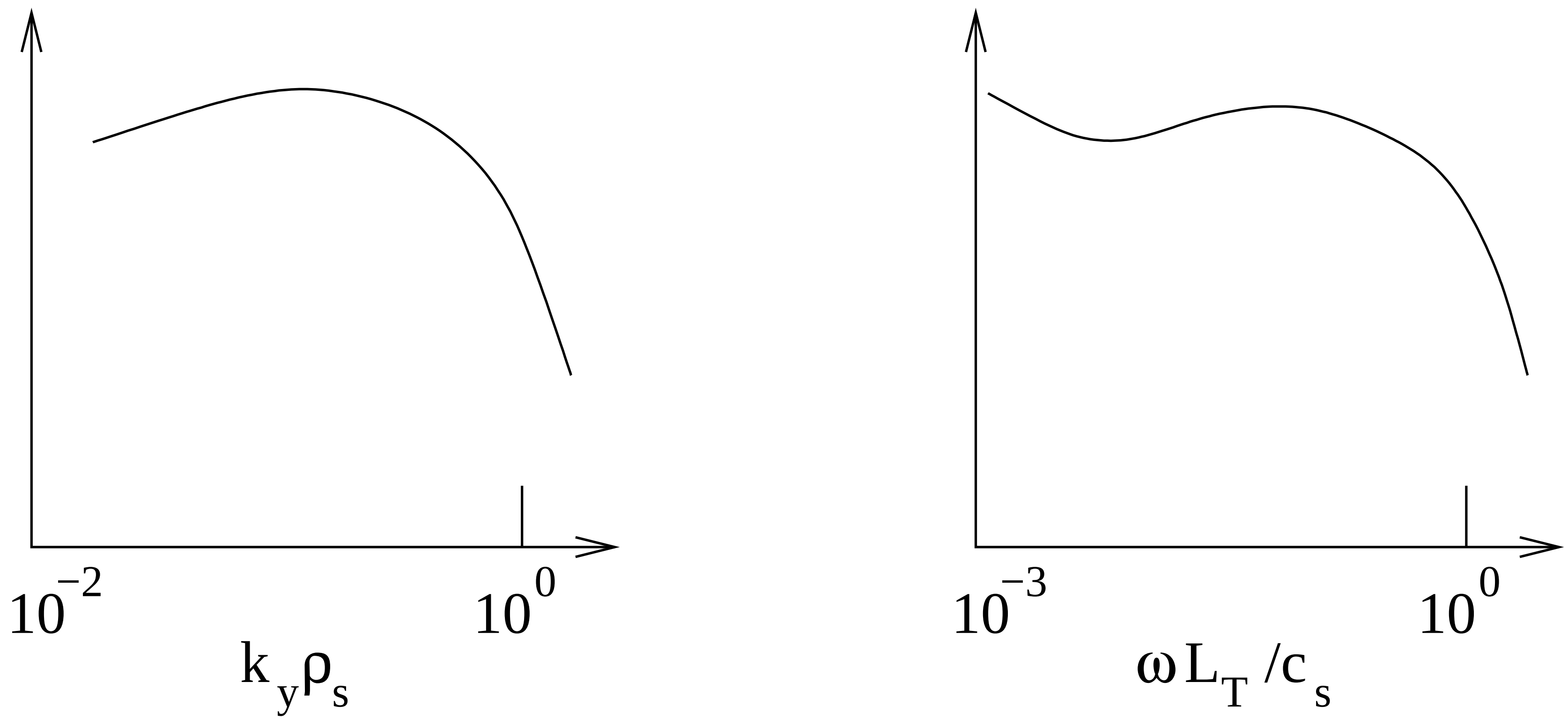
--> structure drifts

(M Wakatani A Hasegawa Phys Fluids 1984)

(B Scott Plasma Phys Contr Fusion 1997)

Scales of Motion

broad range of both time and space scales −− to ion gyroradius



slowest time scale reflect flow/equilibrium component

for equal temperatures, space scale range includes ion gyroradius

high resolution, long runs (> 1000 "gyro−Bohm" times) are necessary

(B Scott Plasma Phys Contr Fusion 2003, 2006)

Numerical Methods

- nonlinearities have the form of brackets

$$\frac{\partial f}{\partial t} + [\psi, f]_{xy} + \cdots = 0 \qquad \text{with} \qquad [\psi, f]_{xy} = \frac{\partial \psi}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial f}{\partial x}\frac{\partial \psi}{\partial y}$$

- spatial discretisation:
  centered-diff for linear terms, Arakawa (J Comput Phys 1966) scheme for brackets
  ◦ basic properties of bracket satisfied to machine accuracy

$$[\psi, f]_{xy} = \frac{1}{3}\left(J_+ + J_0 + J_\times\right)$$

- temporal discretisation:
  "stiffly stable" form (Karniadakis et al J Comput Phys 1991), stable for waves
  ◦ both sides expanded $\Longrightarrow$ all mixed terms in Taylor expansion present
  ◦ one evaluation per time step
  ◦ tested on turbulence and coherent vortices (Naulin and Nielsen, SIAM J Math 2003)

$$\frac{\partial f}{\partial t} = S \qquad \text{with} \qquad \sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{j\,\Delta t} = \sum_{j=1}^{3} \beta_j\, S_j$$

Field Aligned Coordinates

- general axisymmetric Clebsch representation (Dewar/Glasser *Phys Plasmas* 1983)
  ∘ global consistency, shifted metric (B Scott *Phys Plasmas* 1998, 2001)

$$\mathbf{B} = \nabla\chi\times\nabla y_k \qquad y_k = q(\theta-\theta_k) - \zeta - \Delta\alpha_k \qquad s = \theta$$

- Hamada definitions — choose $\Delta\alpha_k(V)$ such that $g_k^{xy} = 0$ at $\theta = \theta_k$

$$\chi = \chi(V) \qquad B^V = B_k^y = 0 \qquad B^s = \chi'(V)$$

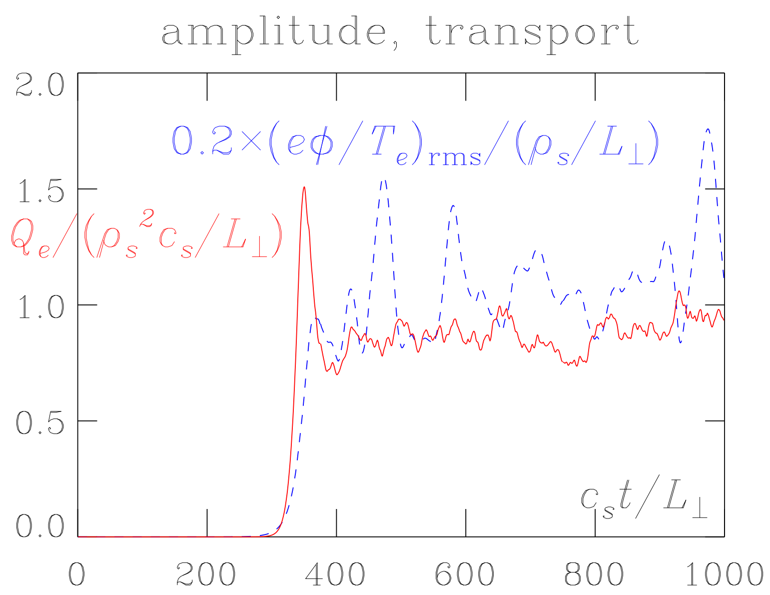
- derivative combination in ExB bracket at $\theta = \theta_k$

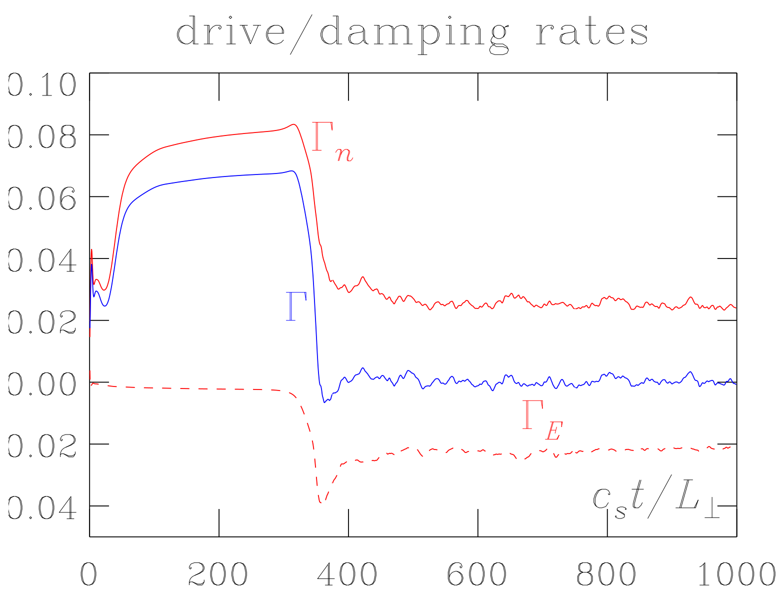
$$\mathbf{v}_E\cdot\nabla f = \nabla\phi\cdot\widehat{\mathbf{F}}\cdot\nabla f \rightarrow F_0^{xs}\frac{\partial\phi}{\partial x}\left(q\frac{\partial f}{\partial y}+\frac{\partial f}{\partial s}\right) - (\leftrightarrow) \equiv [\phi, f]$$

- all derivatives — tensor transformation rules
  ∘ divergence-free F_0 chosen from $\widehat{\mathbf{F}} = (c/B^2)\epsilon\cdot\mathbf{B}$, conserves free energy

Nonlinear Saturation

basic feature of any instability –– transition to turbulence





linear drive (n) ––> linear growth

moment of saturation –– growth rate (T) drops to zero

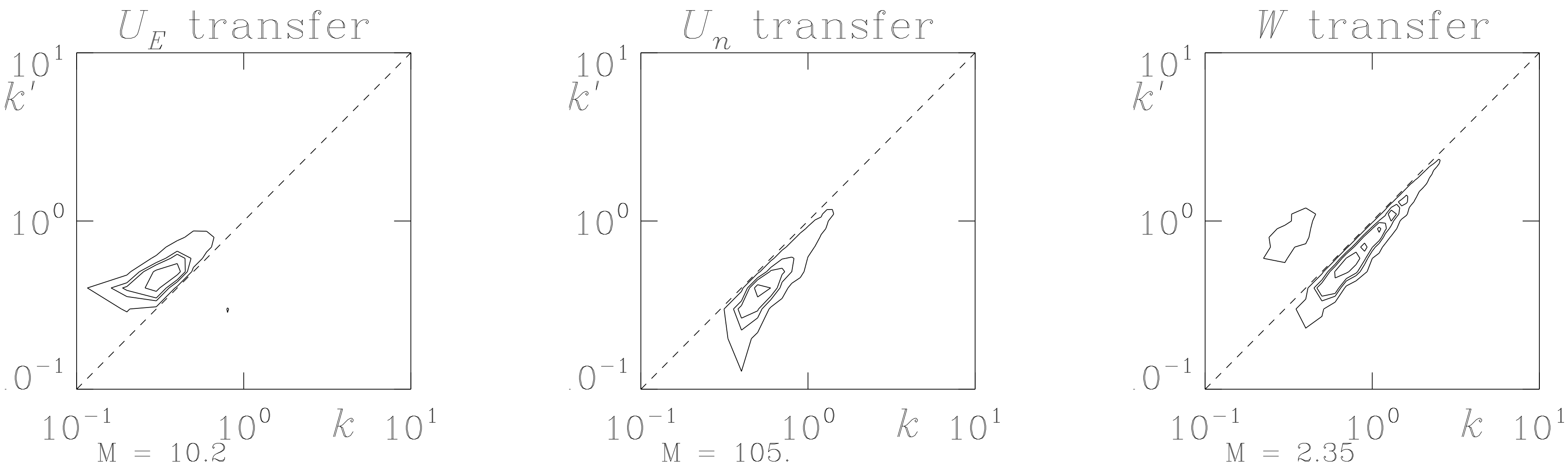
saturation maintained –– nonlinear transfer to subgrid scale dissipation (E)

transport (Q) overshoots, finds saturated balance

(B Scott Phys Plasmas 6/2005)

Nonlinear Cascade in Turbulence

basic statistical character of three wave energy transfer



transfer between wavenumber magnitudes −− from k' to k

all activity near the k' = k line −−> cascade character

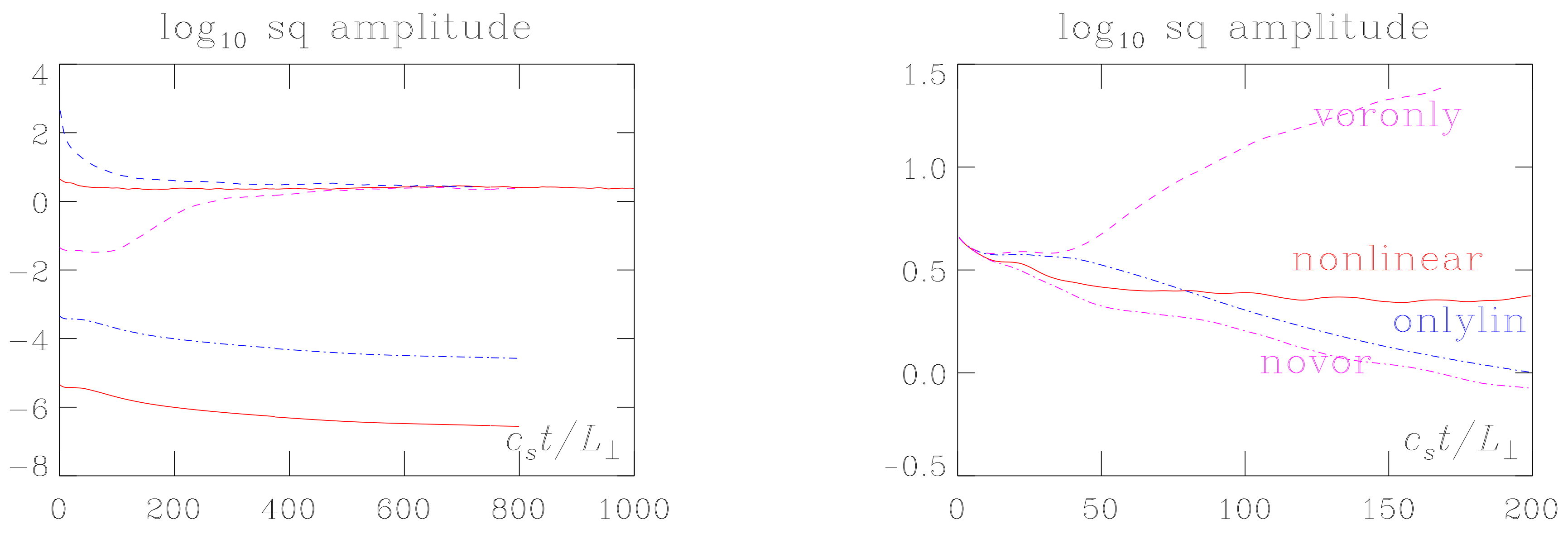
ExB energy is inverse, while other quantities are direct (to higher k)

dominant transfer is through the thermal free energy (n), others also active

(S Camargo et al Phys Plasmas 1995, 1996)

Nonlinear Instability

basic feature of drift wave turbulence (edge turbulence test case)



amplitude threshold −−> linear stability

vorticity nonlinearity −−> damped eigenmodes destabilise each other

role of pressure advection nonlinearity −−> saturation

edge turbulence −−> washes out microinstabilities in toroidal magnetic field

(B Scott Phys Rev Lett 1990, Phys Fluids B 1992, New J Phys 2002)

Energy Transfer

part of energy theorem governed by vorticity equation

$$-\phi_{-k}\left(\dot{\Omega} + v_E \cdot \nabla \Omega + \mathrm{FLR} = \nabla_\parallel J_\parallel + \nabla \cdot \frac{c}{B^2} B \times \nabla p\right)_k$$

Fourier mode k

vorticity $\Omega = (n_e - n_i)\, e$

currents: polarisation parallel diamagnetic

free energy: source in pressure equation, transfer in to vorticity equation

pathways: over parallel dynamics or toroidal compression

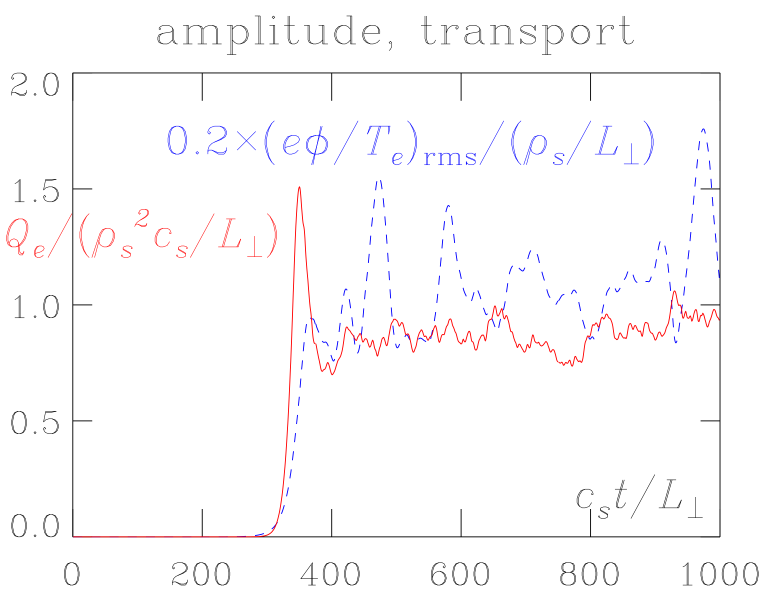
between modes within ExB energy −− nonlinear advection

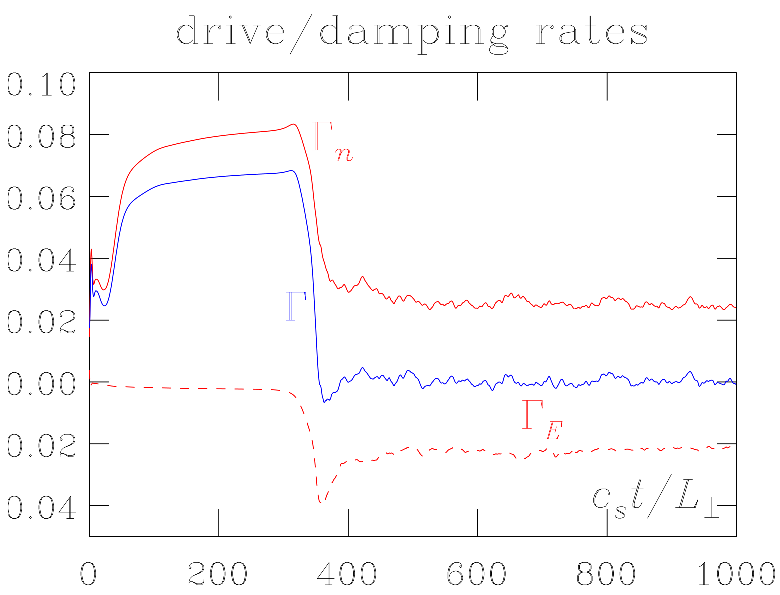
direct, in−context measurement of physical mechanism supporting turbulence

(B Scott Phys Plasmas 2000)

Nonlinear Saturation

basic feature of any instability –– transition to turbulence





linear drive (n) ––> linear growth

moment of saturation –– growth rate (T) drops to zero

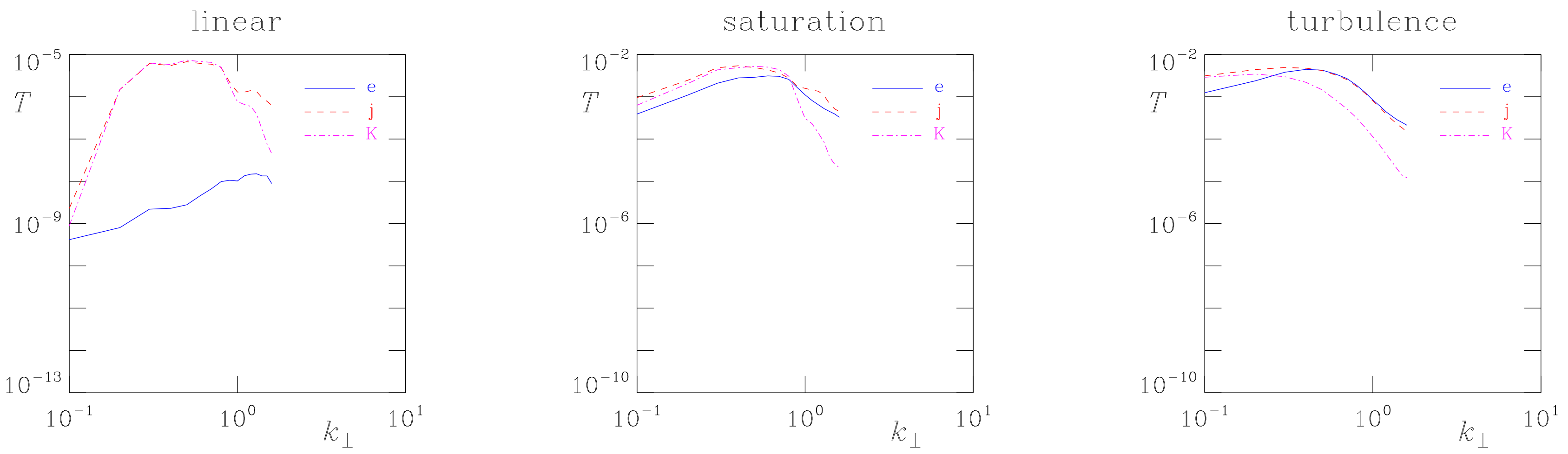
saturation maintained –– nonlinear transfer to subgrid scale dissipation (E)

transport (Q) overshoots, finds saturated balance

(B Scott Phys Plasmas 6/2005)

Vorticity Energetics –– Transition to Turbulence

turbulence imposes its own mode structure on dynamics



linear interchange mode –– balance between diamagnetic/parallel currents

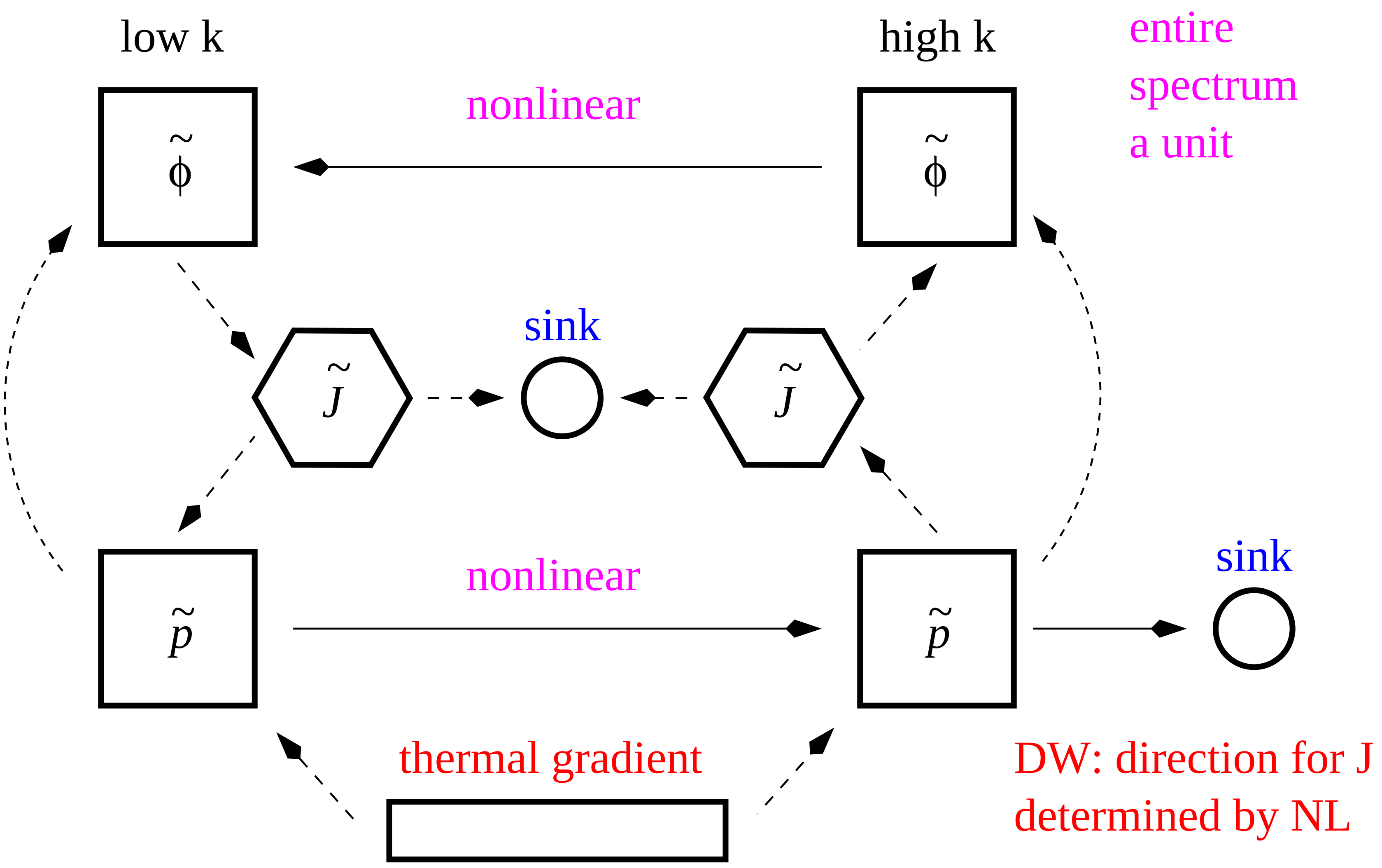
turbulence –– emergence of nonlinear ExB vorticity advection

developed turbulence –– balance between polarisation/parallel currents

basic mechanism supporting eddies in turbulence differs from linear instability

(B Scott Plasma Phys Contr Fusion 2003)

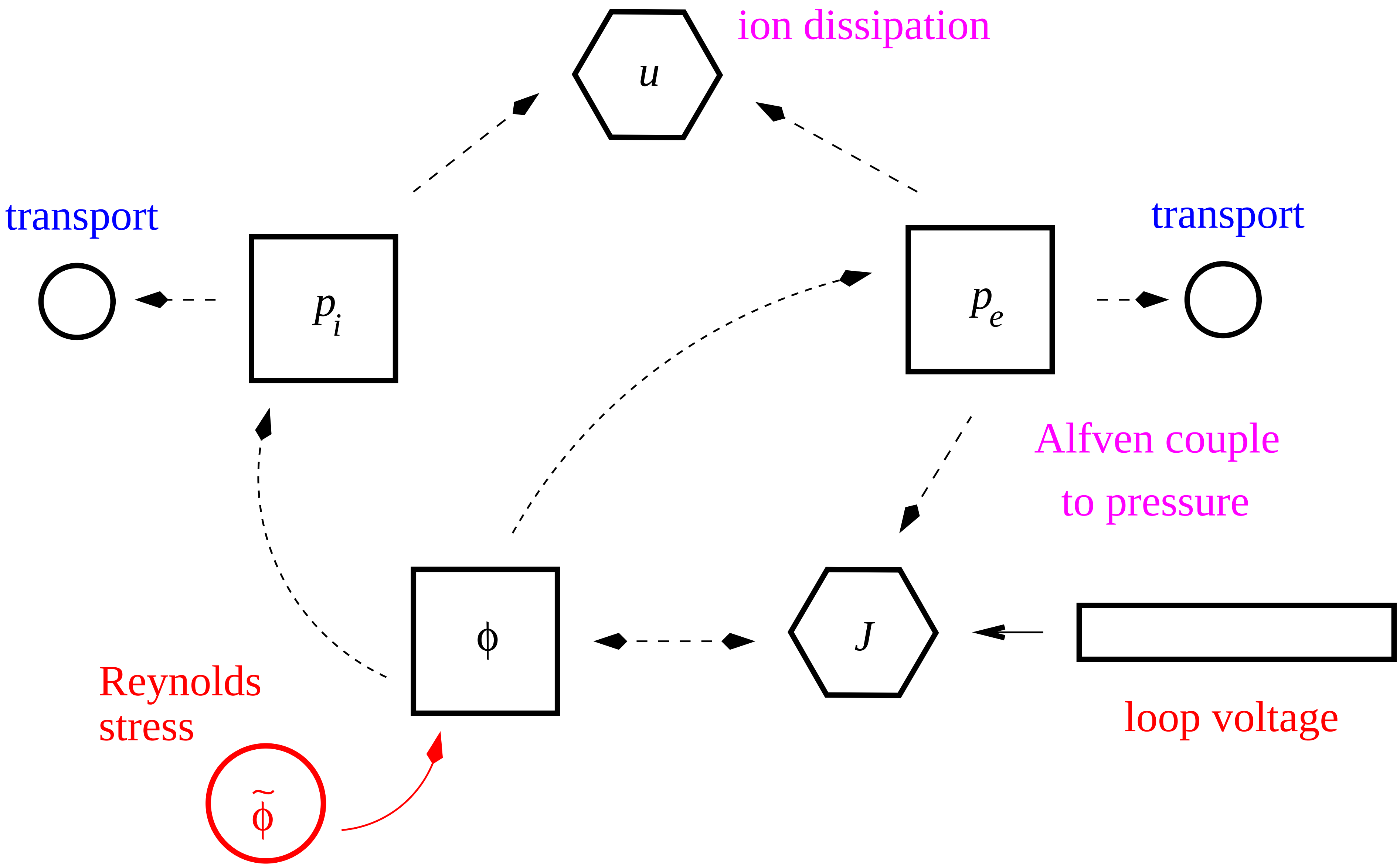
Energy Transfer: electromagnetic turbulence



(B Scott Phys Fluids B 1992, Plasma Phys Contr Fusion 1997)

(S Camargo et al Phys Plasmas 1995 and 1996)

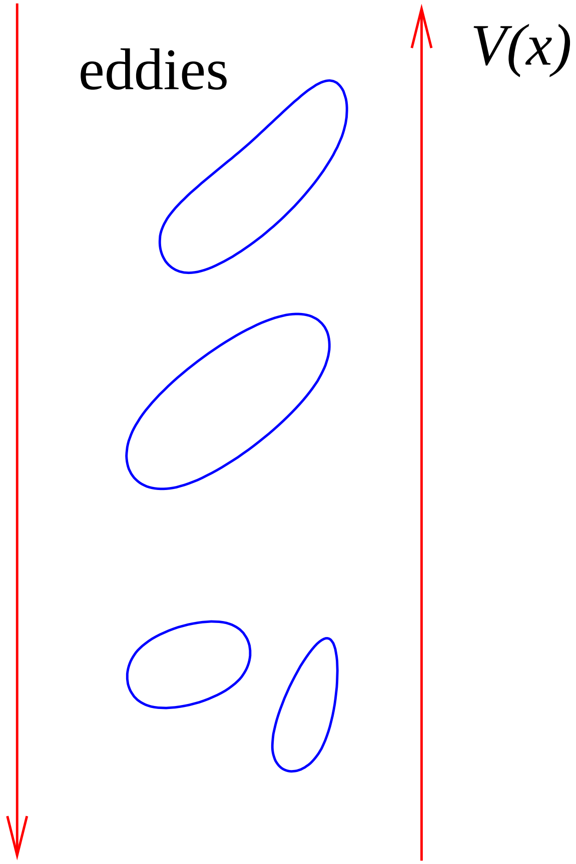
Energy Transfer: equilibrium

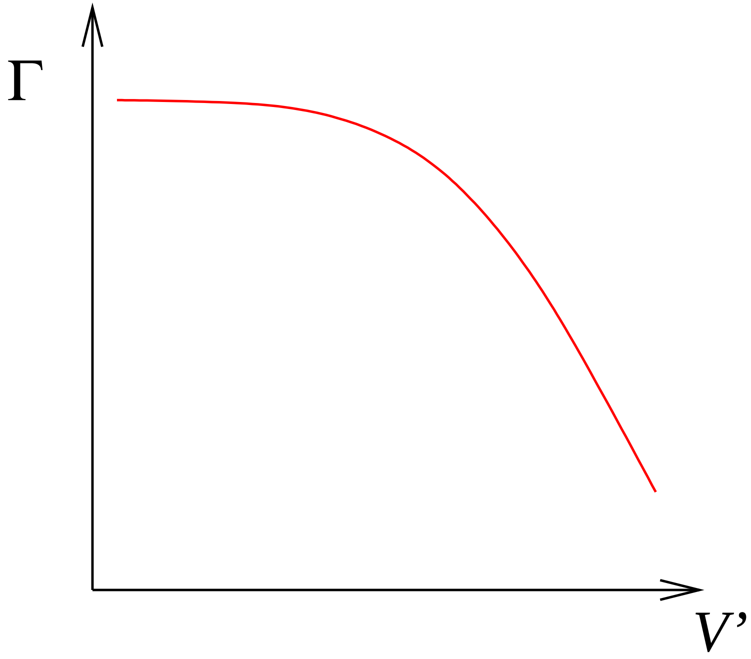


(B Scott Phys Plasmas 2003)

Suppression of Turbulence by Flows

(Biglari Diamond Terry, Phys Fl B 1991)



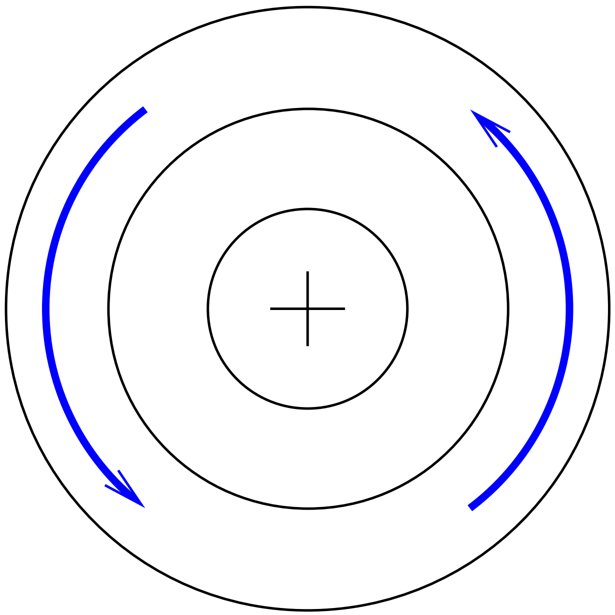


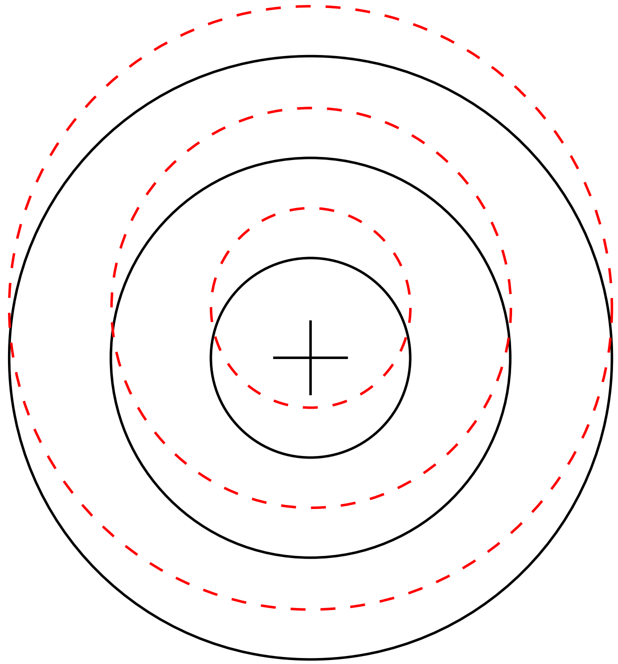
eddies tilted into energy−losing relationship to flow vorticity

−−> same process as in self generation

Zonal Flow, Toroidal Compression

(Winsor et al Phys Fl 1968, Hahm et al Plasma Phys Contr Fusion 2002, 2004)



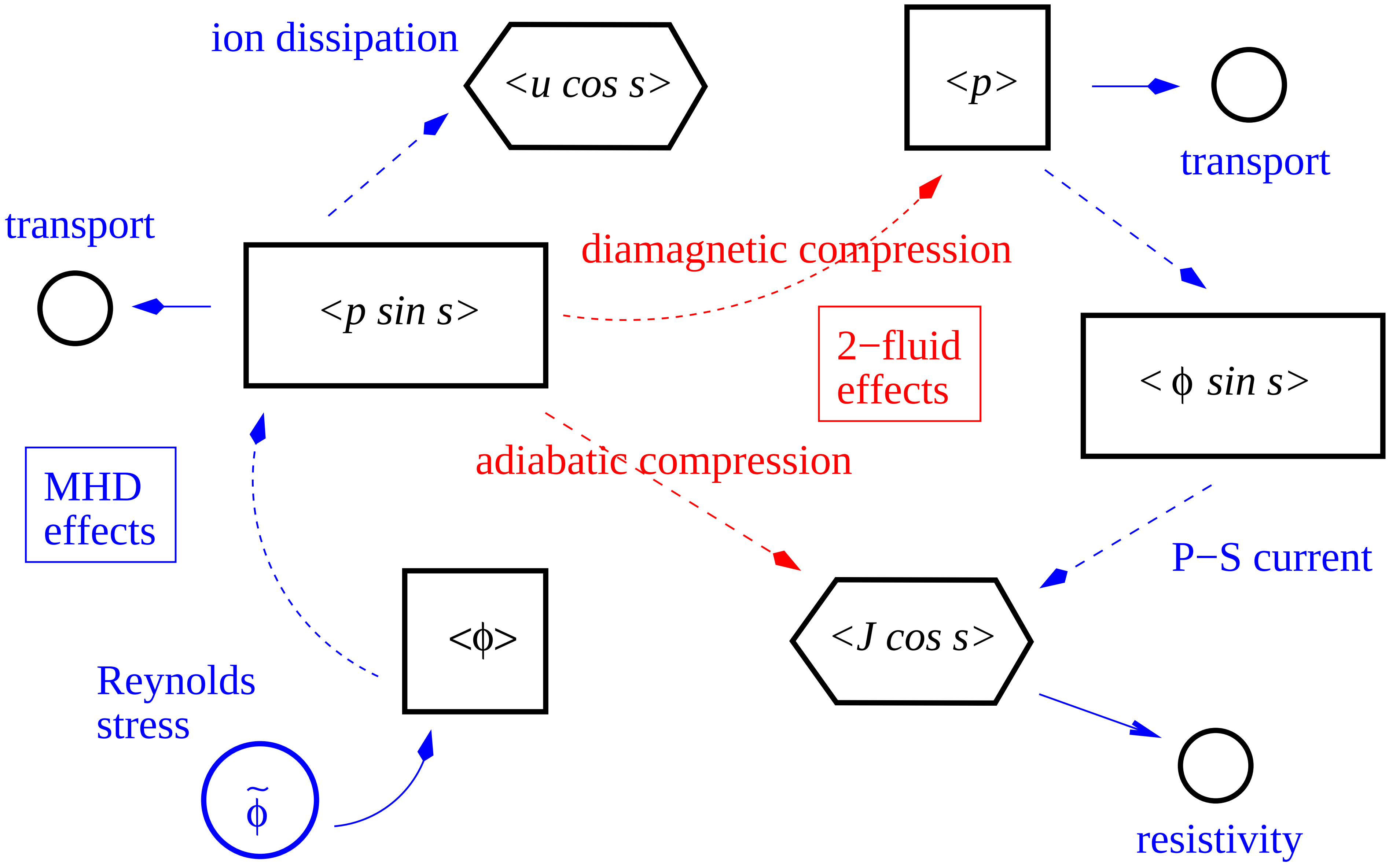


zonal flow	compression at top	pressure sideband
$< \phi >$	divergence at bottom	$<p \sin\theta >$

zonal flow exchanges conservatively with pressure sideband

−−> transfer pathway, equipartition

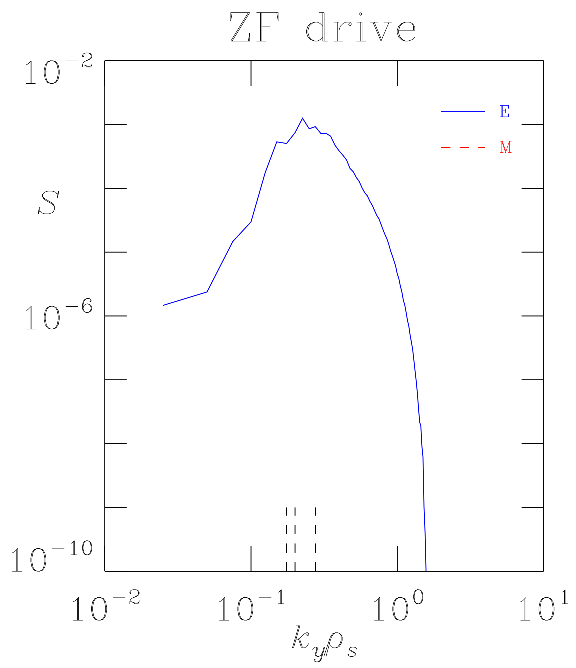
Energy Transfer: flows and currents

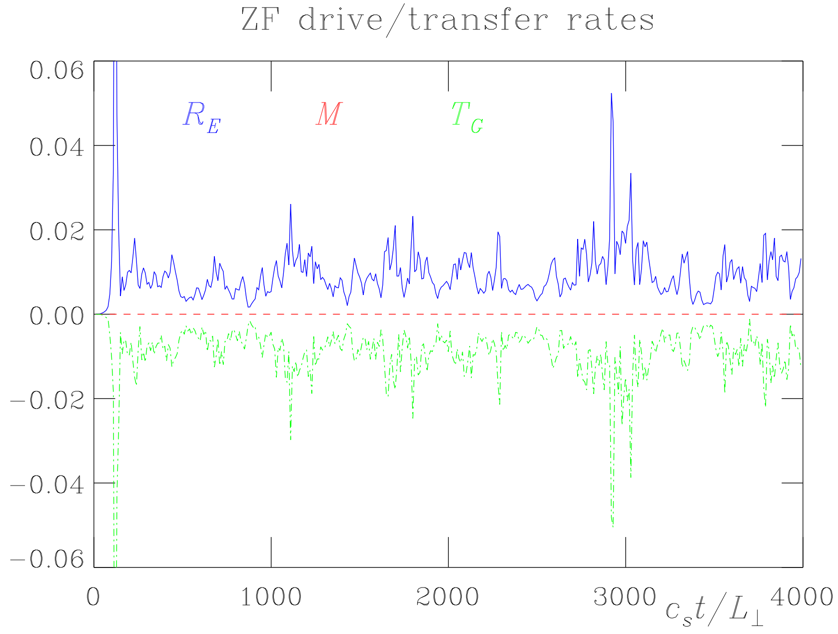


(B Scott Phys Lett A 2003, New J Phys 2005)

Coupling to Zonal Flows

turbulence regulated by flows, regulated by toroidal compression





eddy Reynolds stress −−> energy transfer from turbulence to flows

turbulence moderately weakened but not suppressed

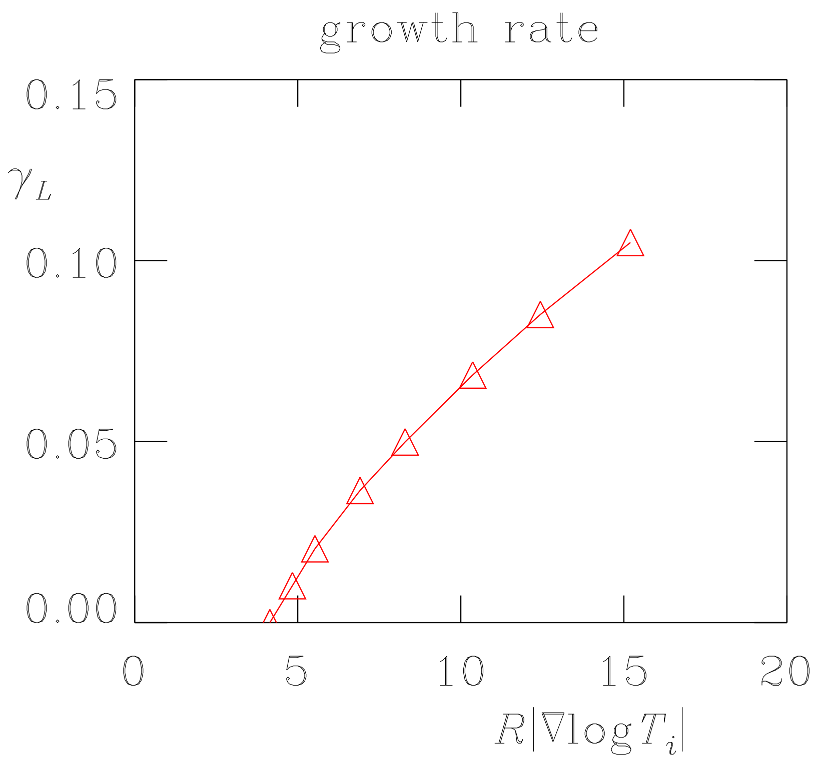
toroidal compression −−> energy loss channel to pressure, turbulence

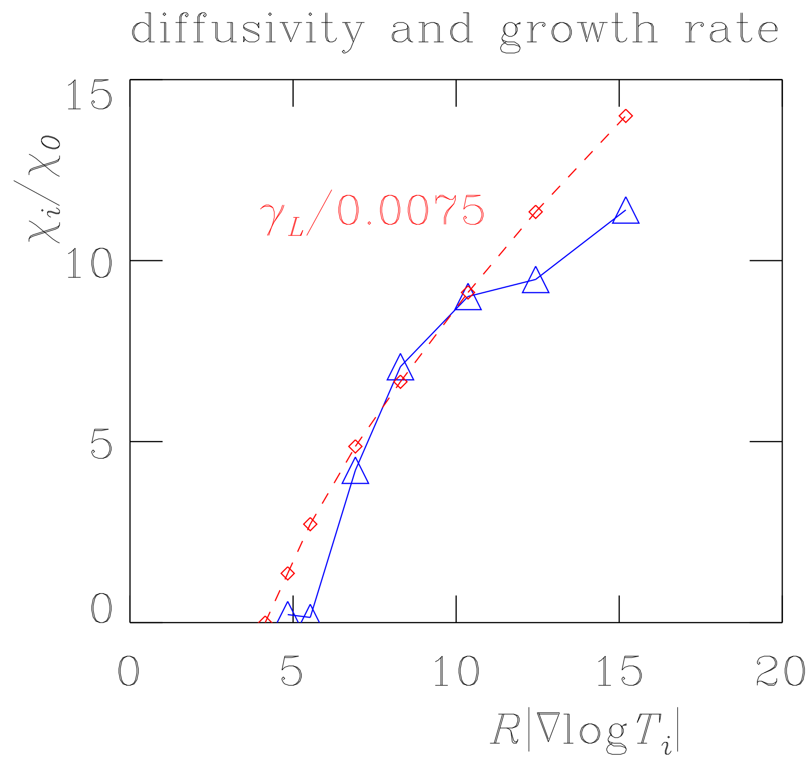
entire system in self regulated statistical equilibrium (turb, flows, mag eq)

(B Scott Phys Lett A 2003, New J Phys 2005, PPCF 2006   Naulin et al Phys Plasmas 2005)

Nonlinear Threshold Upshift

cyclone ITG, adiabatic electrons, periodic S-alpha, $100 \times 256\rho_s$



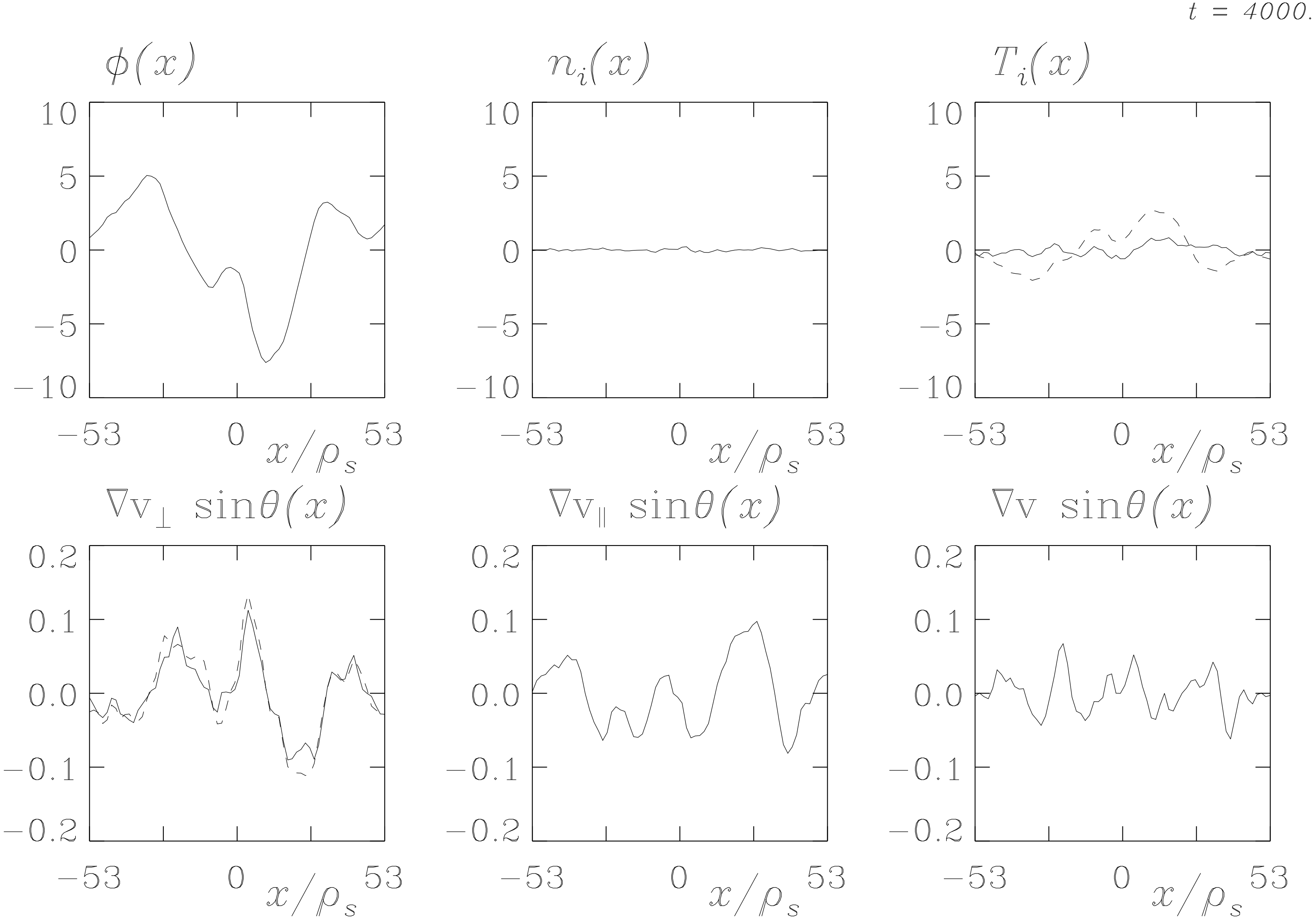


growth rate max values for each case, zero point by extrapolation
transport diffusivity curve shows threshold upshift to 6.0 (B Scott PPCF 2006)

captures standard gyrokinetic result (Dimits et al Phys Plasmas 2000)

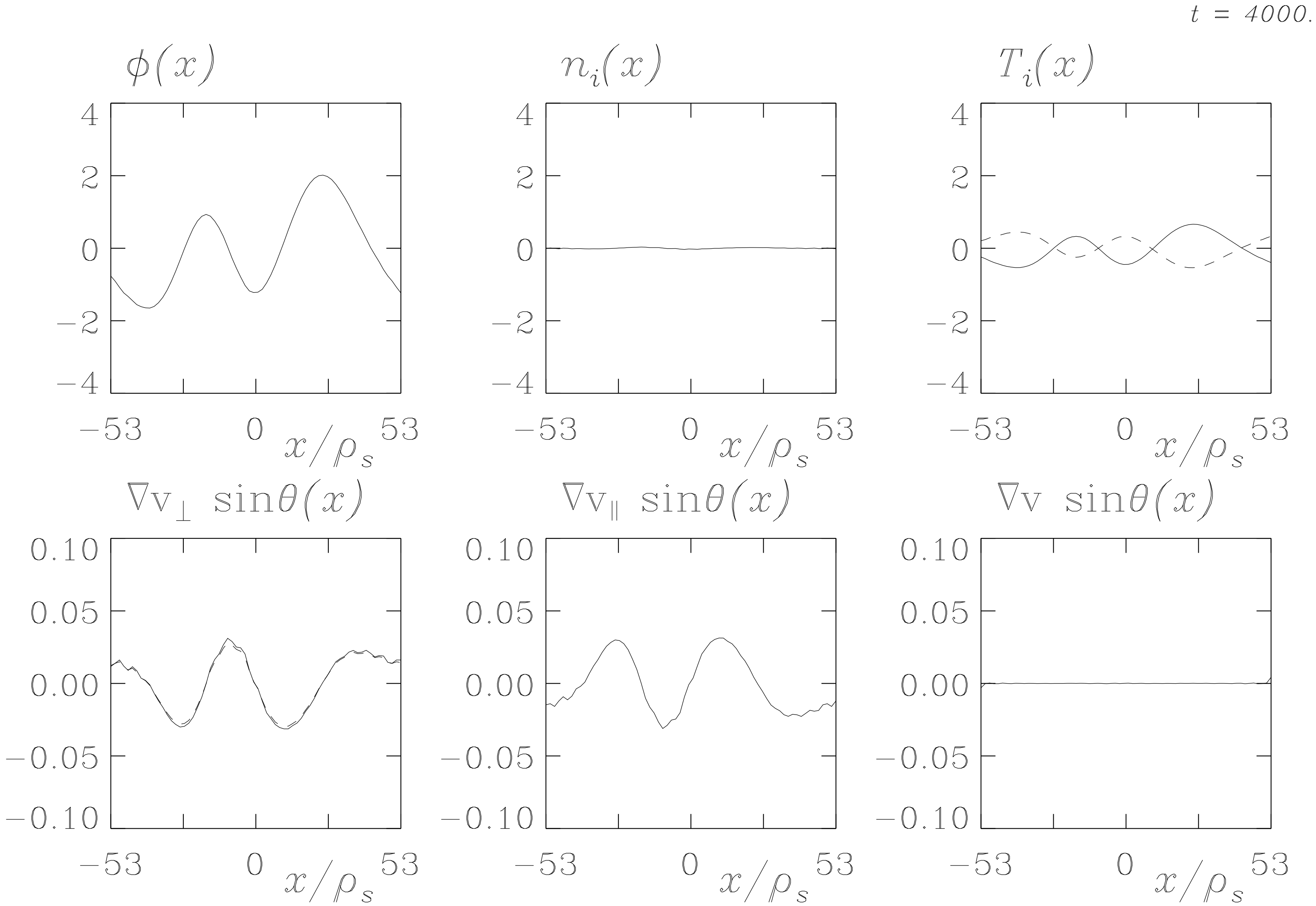
perturbed equilibrium and ion flow divergence profiles

cyclone ITG, periodic S-alpha, $R/L_T = 6.91$ $\hat{\beta} = 0$ $100 \times 256\rho_s$



perturbed equilibrium and ion flow divergence profiles

cyclone ITG, periodic S-alpha, $R/L_T = 4.83$ $\hat{\beta} = 0$ $100 \times 256\rho_s$



electromagnetic cases — notes

- nominal value of beta

$$\hat{\beta} = \frac{4\pi p_e}{B^2}\left(\frac{qR}{L_\perp}\right)^2 = 0.465$$

- very strong “flutter” effects

$$\nabla_\parallel = b^s \frac{\partial}{\partial s} - \hat{\beta}[A_\parallel, \,] \qquad\qquad -\nabla_\perp^2 A_\parallel = J_\parallel \leftrightarrow \nabla_\parallel(p_e - \phi)$$

- as $\hat{\beta}$ rises from zero, transport drops
  - complete stabilisation for $\hat{\beta} = 0.465$ (flows) and 0.52 (flutter)
  - onset of kinetic ballooning (no saturation in periodic S-alpha) for $\hat{\beta} = 0.6$

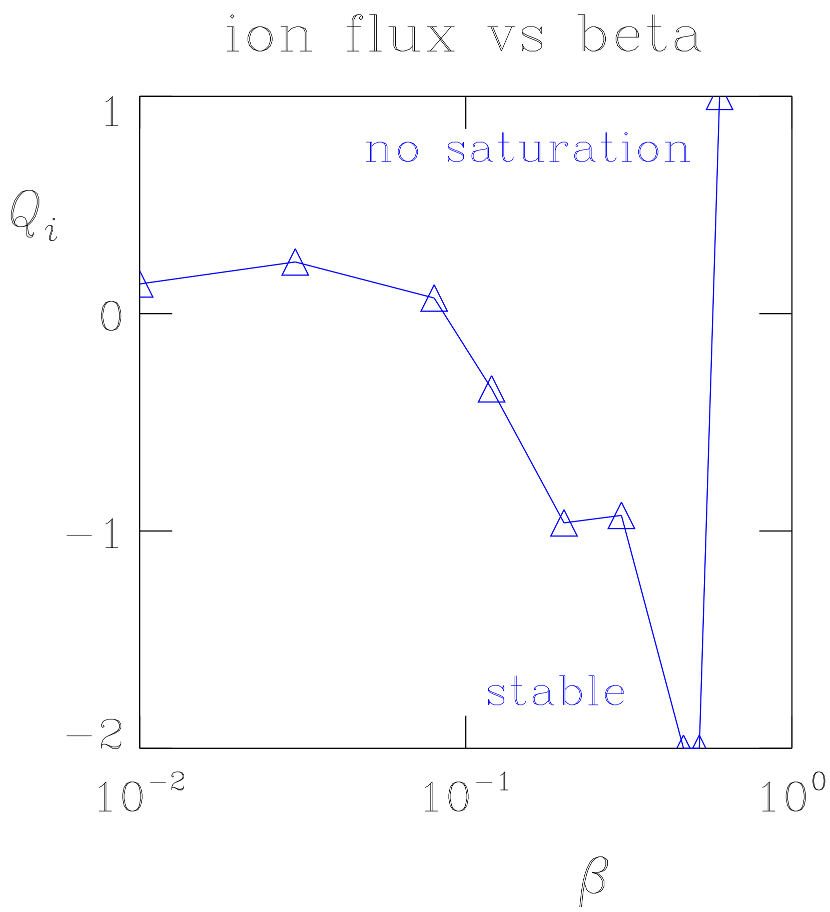
- very important resolution consideration — require $h_x/h_y = 1/4$

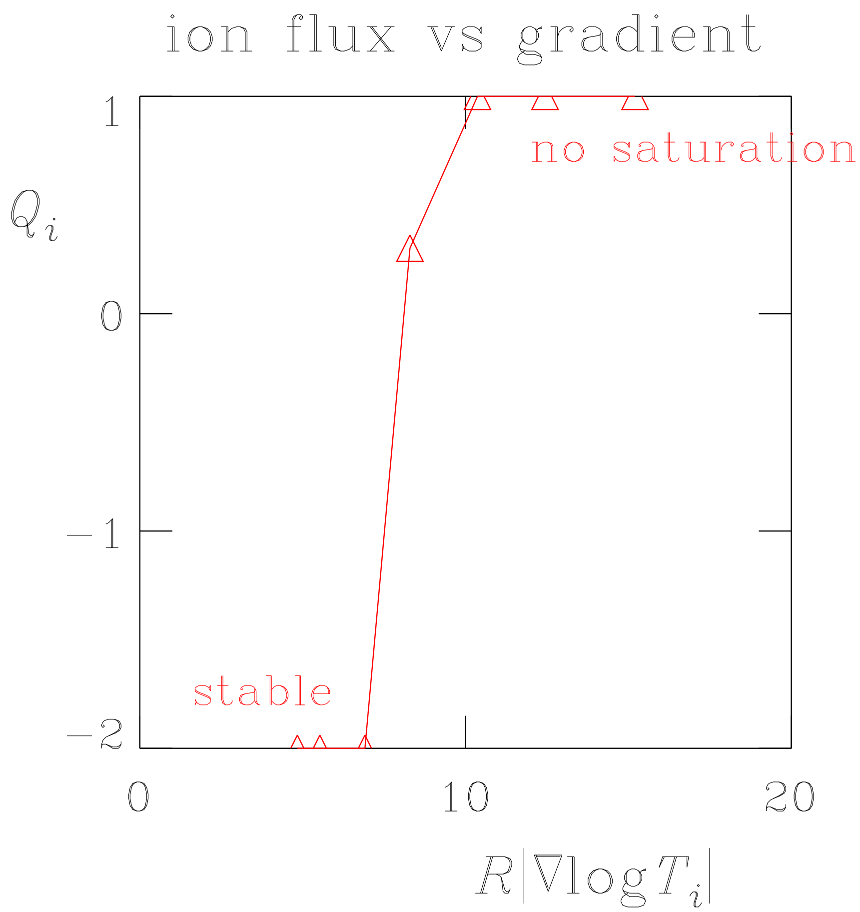
$$\Delta_{\rm rs} = \frac{1}{\hat{s}k_y} \qquad \text{hence} \qquad h_x < \frac{h_y}{\pi\hat{s}}$$

otherwise short wavelength electron response doesn’t see magnetic shear

Electromagnetic Effect on Transport

cyclone ITG, periodic S-alpha, $100 \times 256\rho_s$



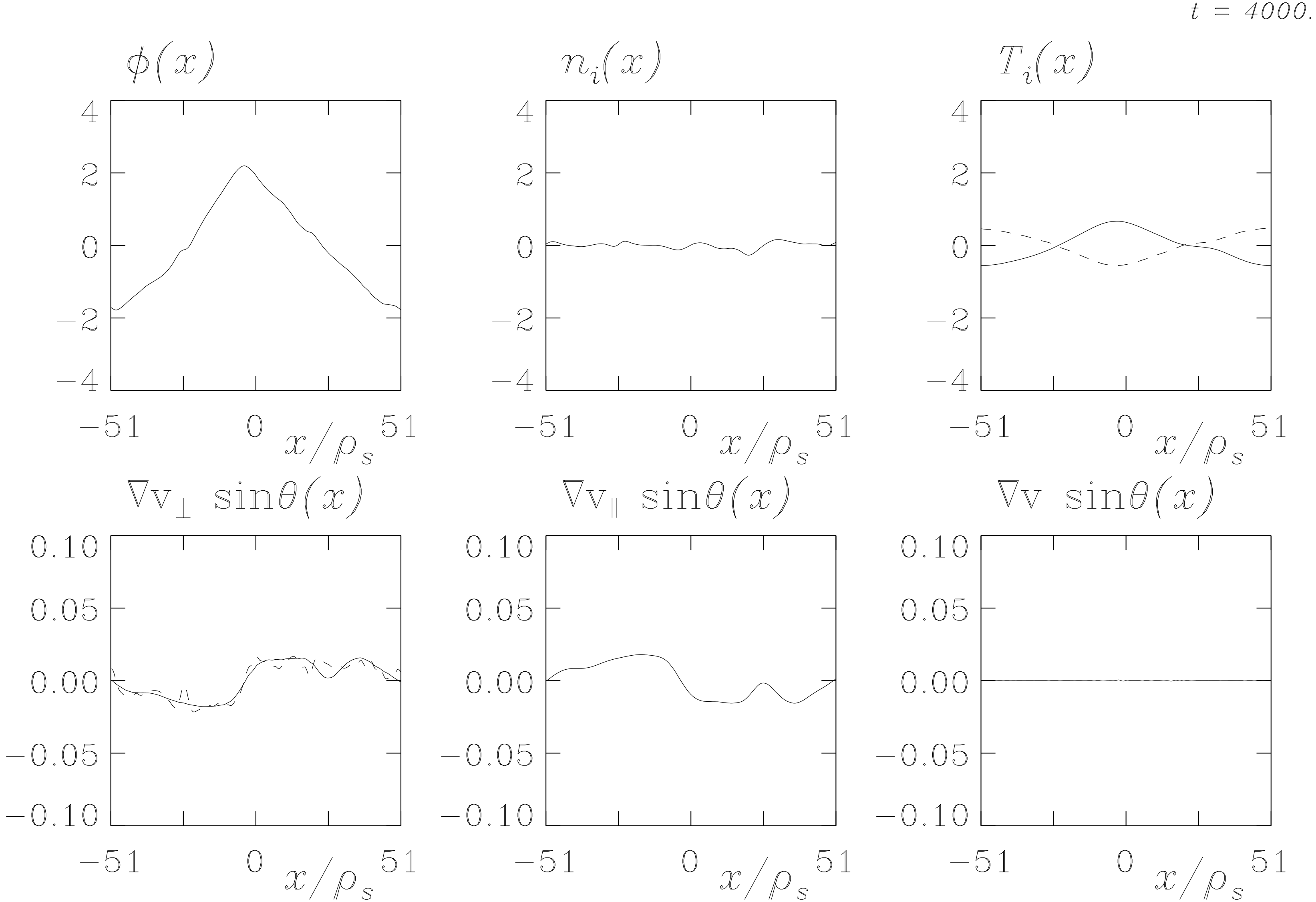


beta stabilises due to both flows and linearly (next slides)

gradient destabilisation directly to kinetic ballooning regime (no saturation)

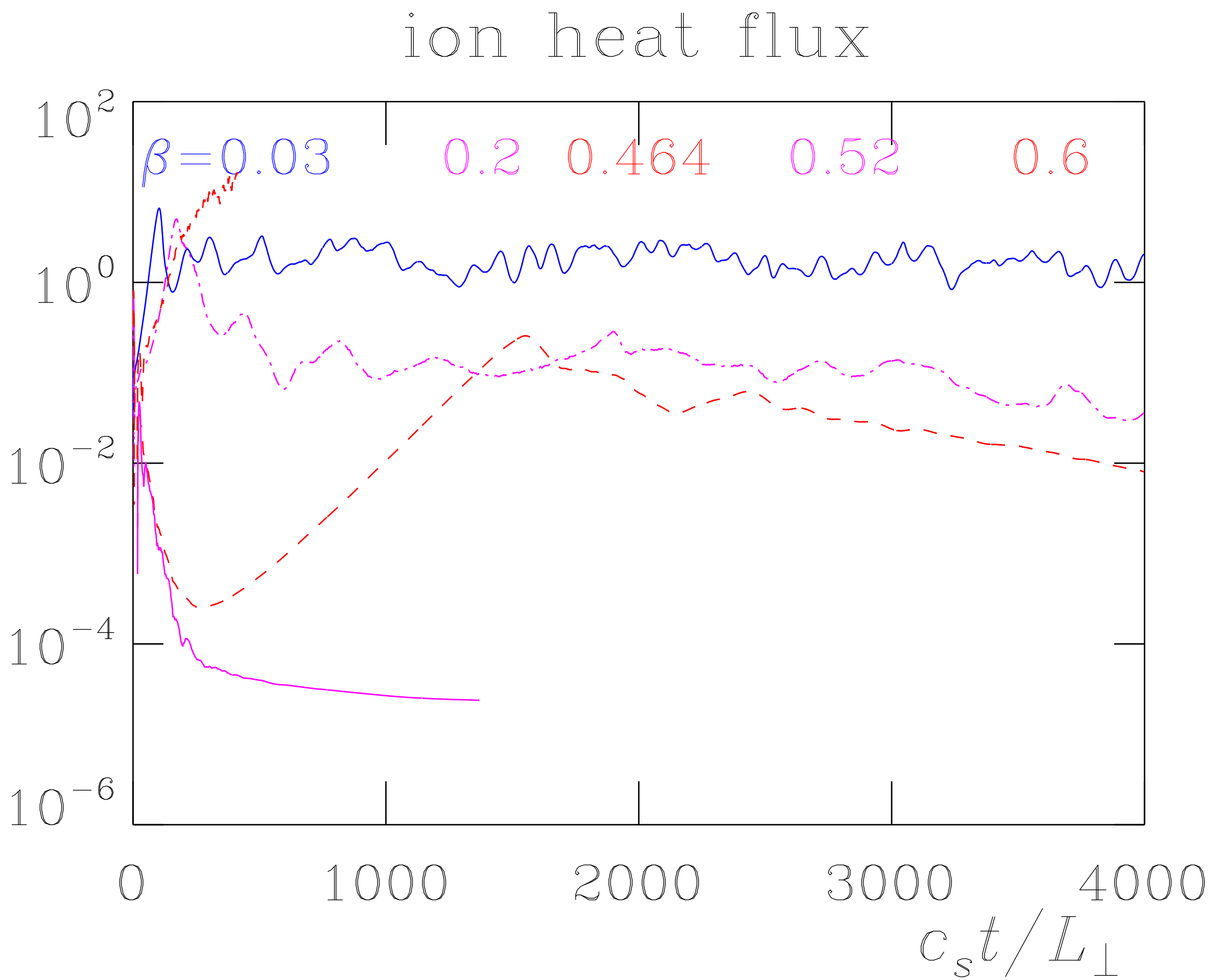
perturbed equilibrium and ion flow divergence profiles

cyclone ITG, periodic S-alpha, $R/L_T = 6.91$ $\hat{\beta} = 0.465$ $100 \times 256\rho_s$



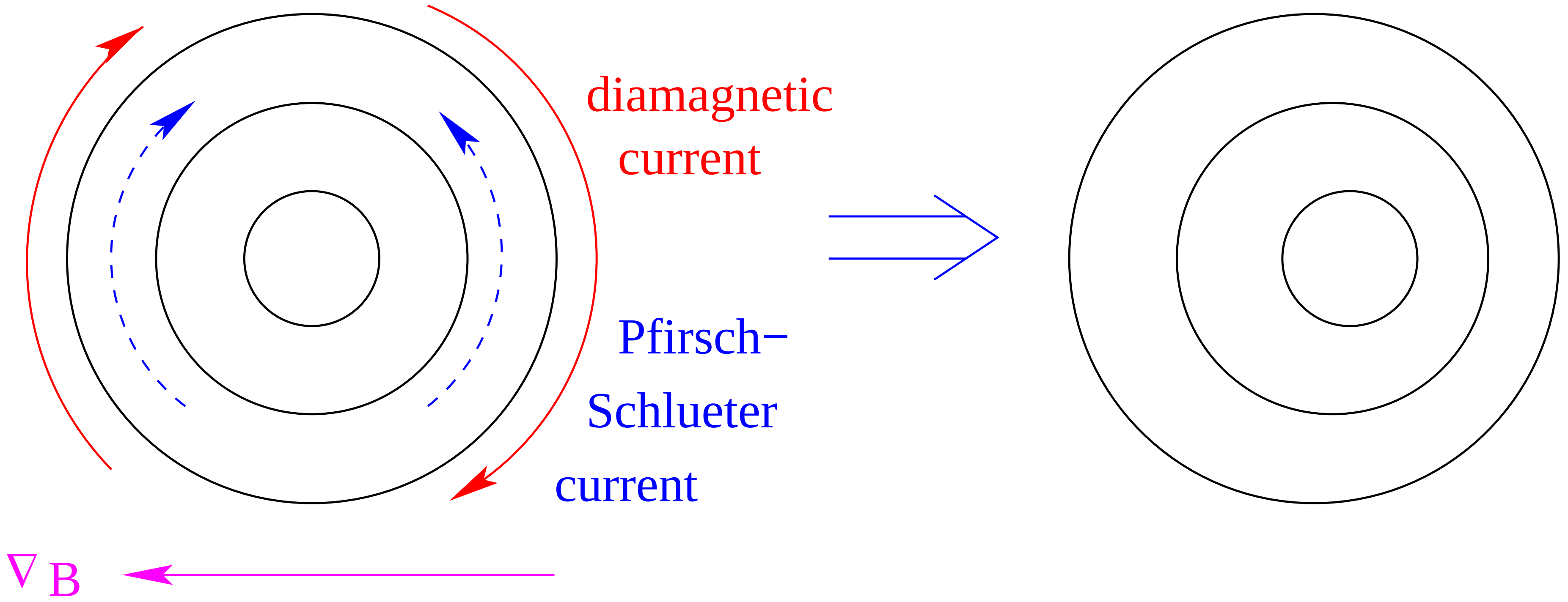
time traces, electromagnetic

cyclone ITG, periodic S-alpha, $100 \times 256\rho_s$



Incorporation of Magnetic Equilibrium

toroidal equilibration current <−−> Shafranov shift



P−S current equilibrates toroidal diamagnetic compression

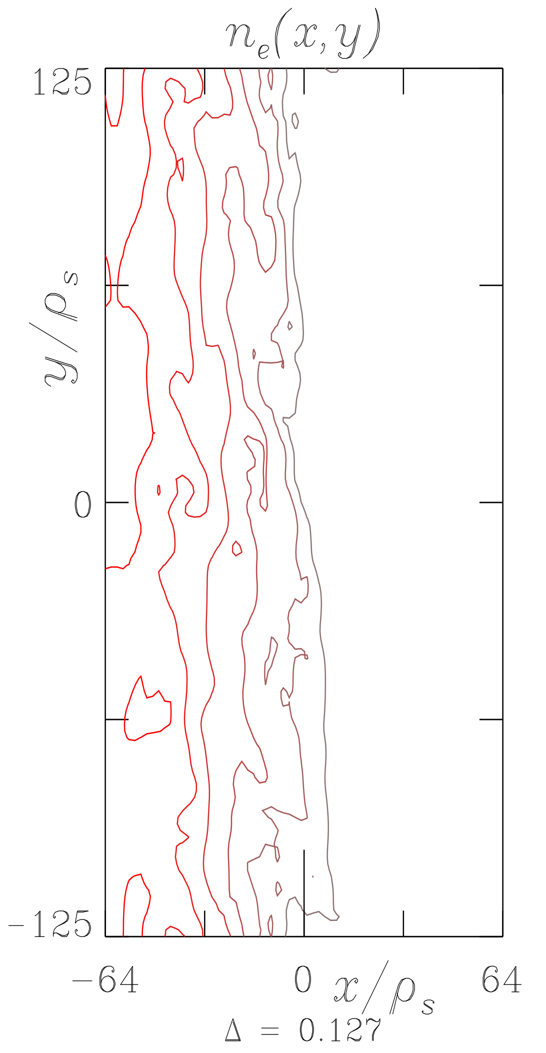
Ampere's Law −−> ''Pfirsch−Schlueter magnetic field'' −−> toroidal shift

current stays in moment variables, magnetic field in coordinate metric

Global Electromagnetic Gyrofluid (GEM):

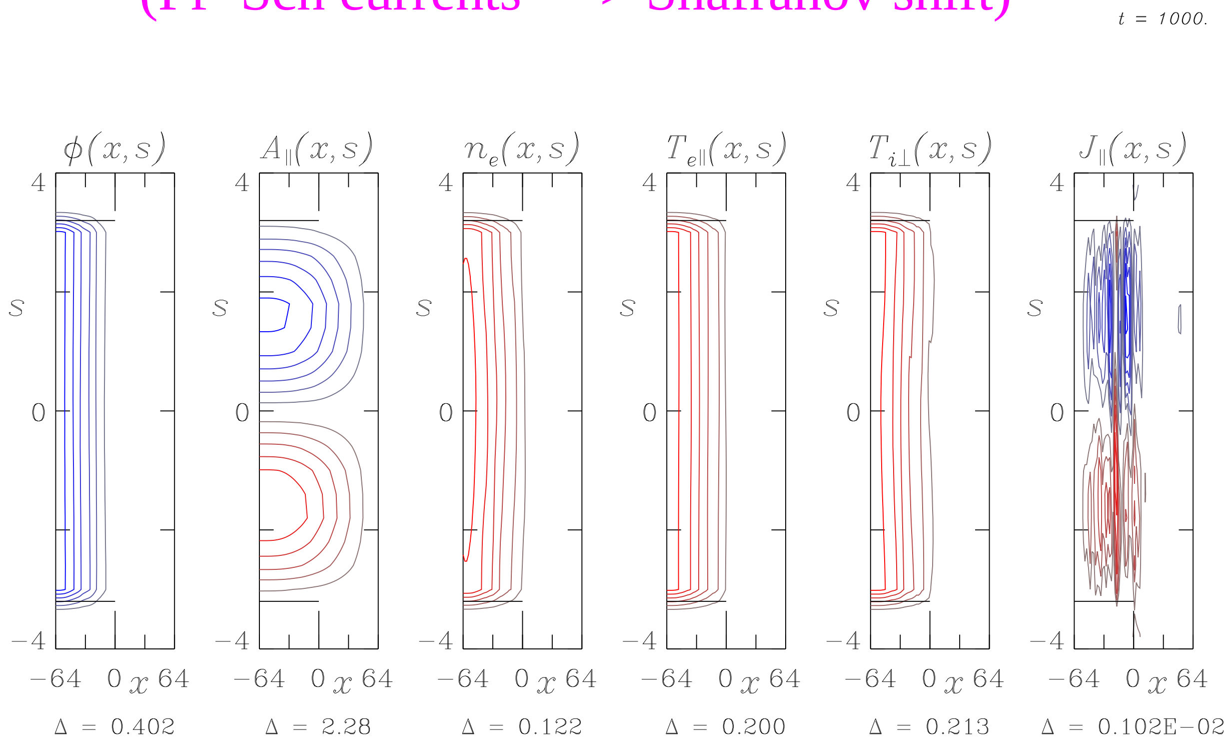
turbulence and transport

(profile + disturbances)



self consistent magn eq, geometry

(Pf−Sch currents −−> Shafranov shift)



L−Mode Base Case (ASDEX Upgrade generic)

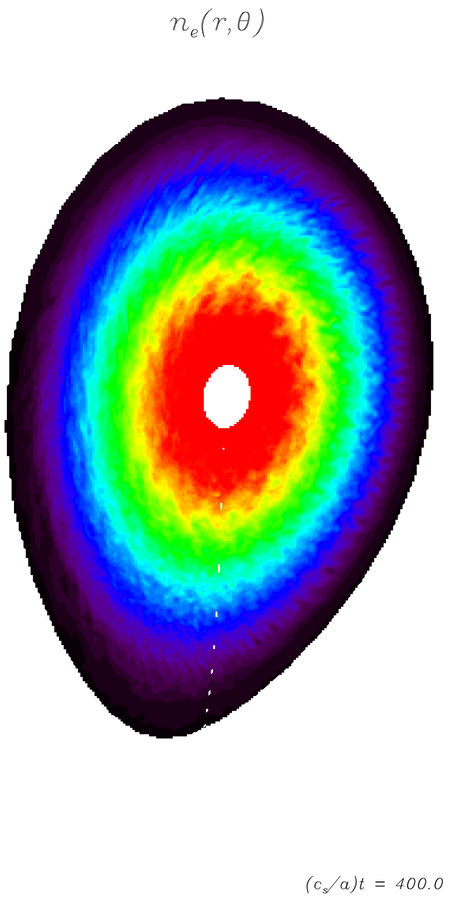
correct mass ratio, gyroradius

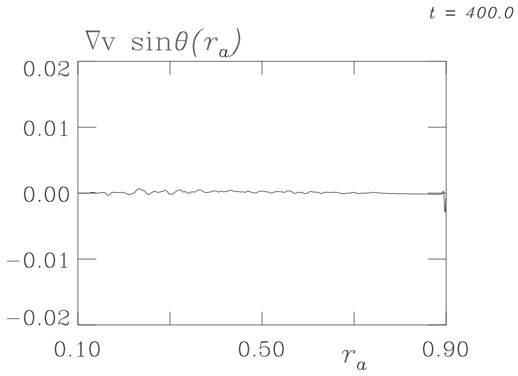
closed/open flux surfaces, separatrix topology

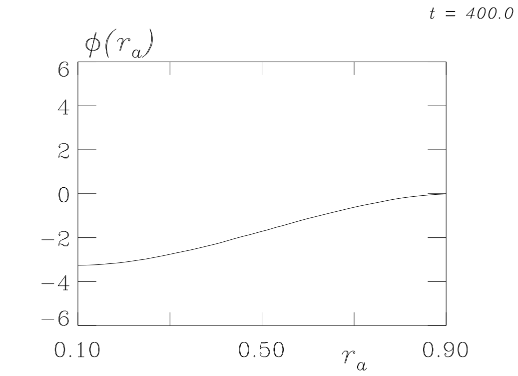
(B Scott Contrib Plasma Phys 2006)

Global Computation in Divertor Geometry

study of turbulence vs rotation scale separation







Gyro-Bohm Convergence and Large Tokamaks

- global drift parameter $\rho_* = \rho_s/a$

- shape of spectrum
  - follows gyroradius, not profile scale length
  - long wavelength side serving as sink must be wide enough

- transport flux level
  - scales as square of ρ_*, converges when spectrum does

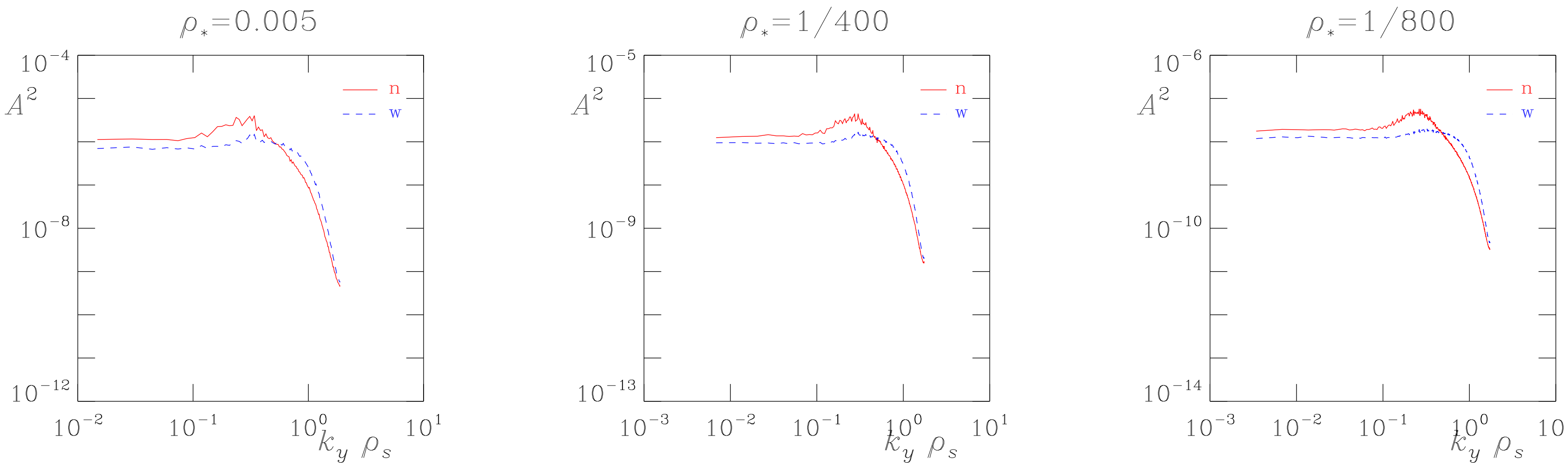
- toroidal flow ("neoclassical") equilibrium
  - drifts: forcing scales as ρ_*
  - turbulence: forcing scales as square of ρ_*
  - flow profile converges when turbulence forcing drops out

- time scale separation
  - both neoclassical and turbulence effects scale as square of ρ_*
  - large tokamak regime reached when source or decay effects can be ignored

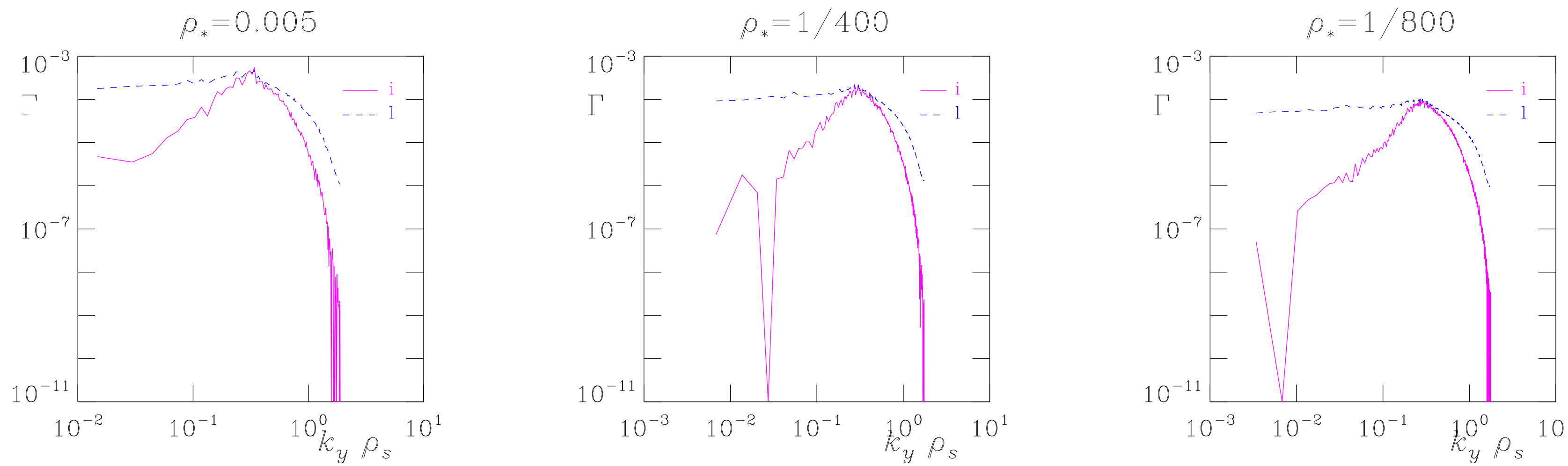
- large tokamak regime is reached generally for $a/\rho_s = 200$ (AUG size)
  - and for $a/\rho_s = 400$ (JET size) for profiles with structure (e.g., ITBs)

Spectra for Medium to Large Tokamak Cases

- density and vorticity spectra for the three cases

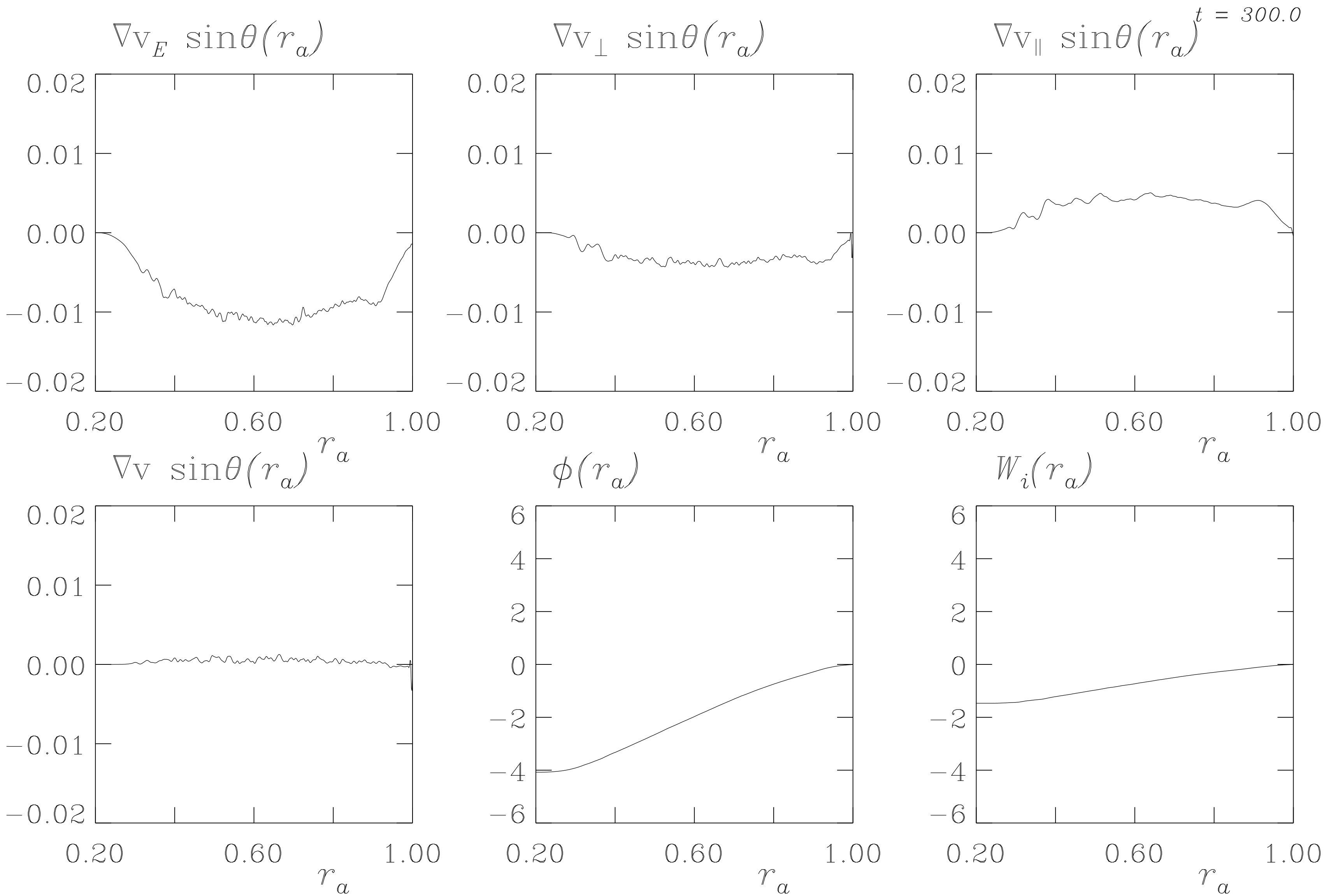


- ion heat source and sink spectra for the three cases



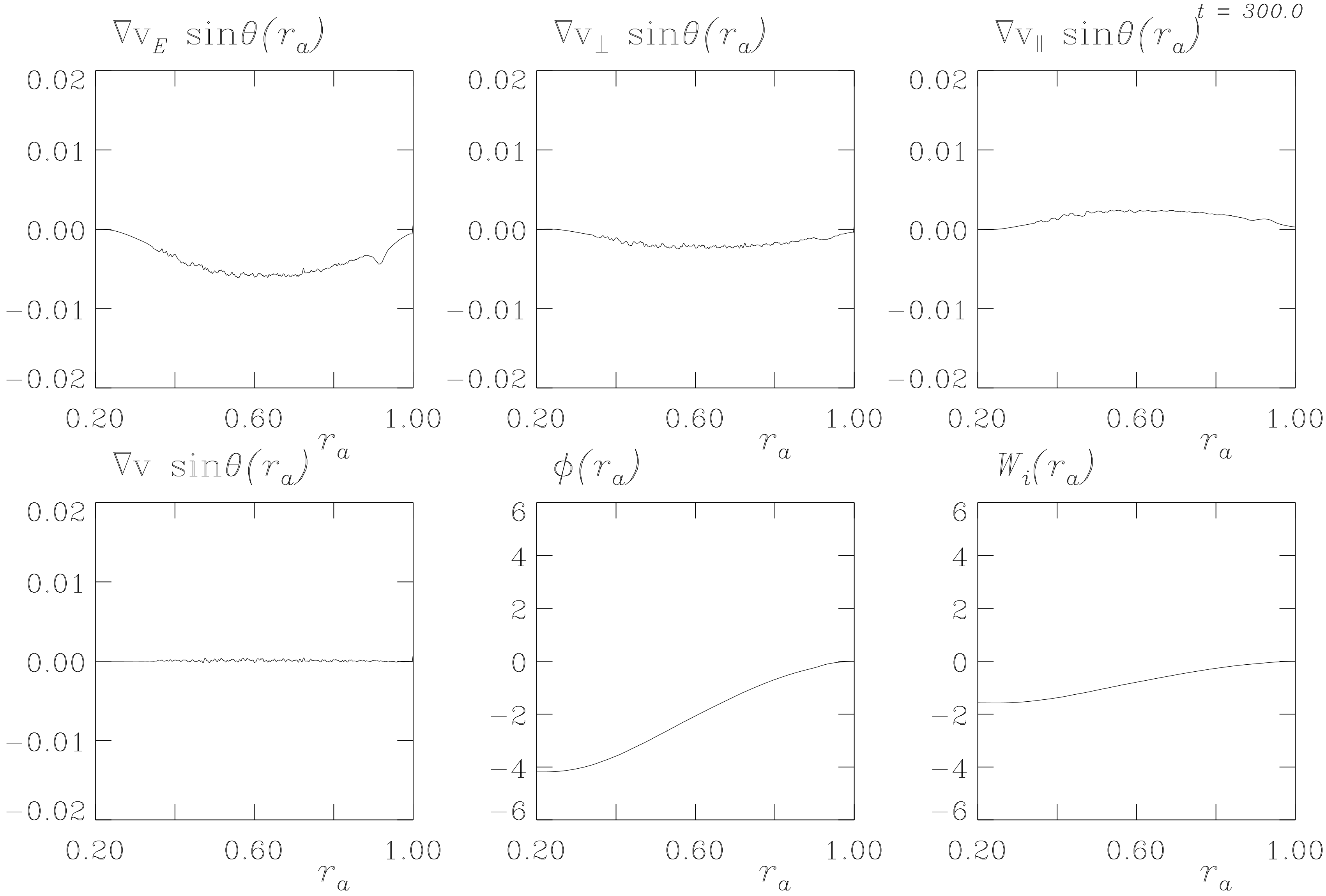
Ion Flow Sideband Divergences — AUG Case

- flow divergence pieces balance closely, slight ZF activity visible



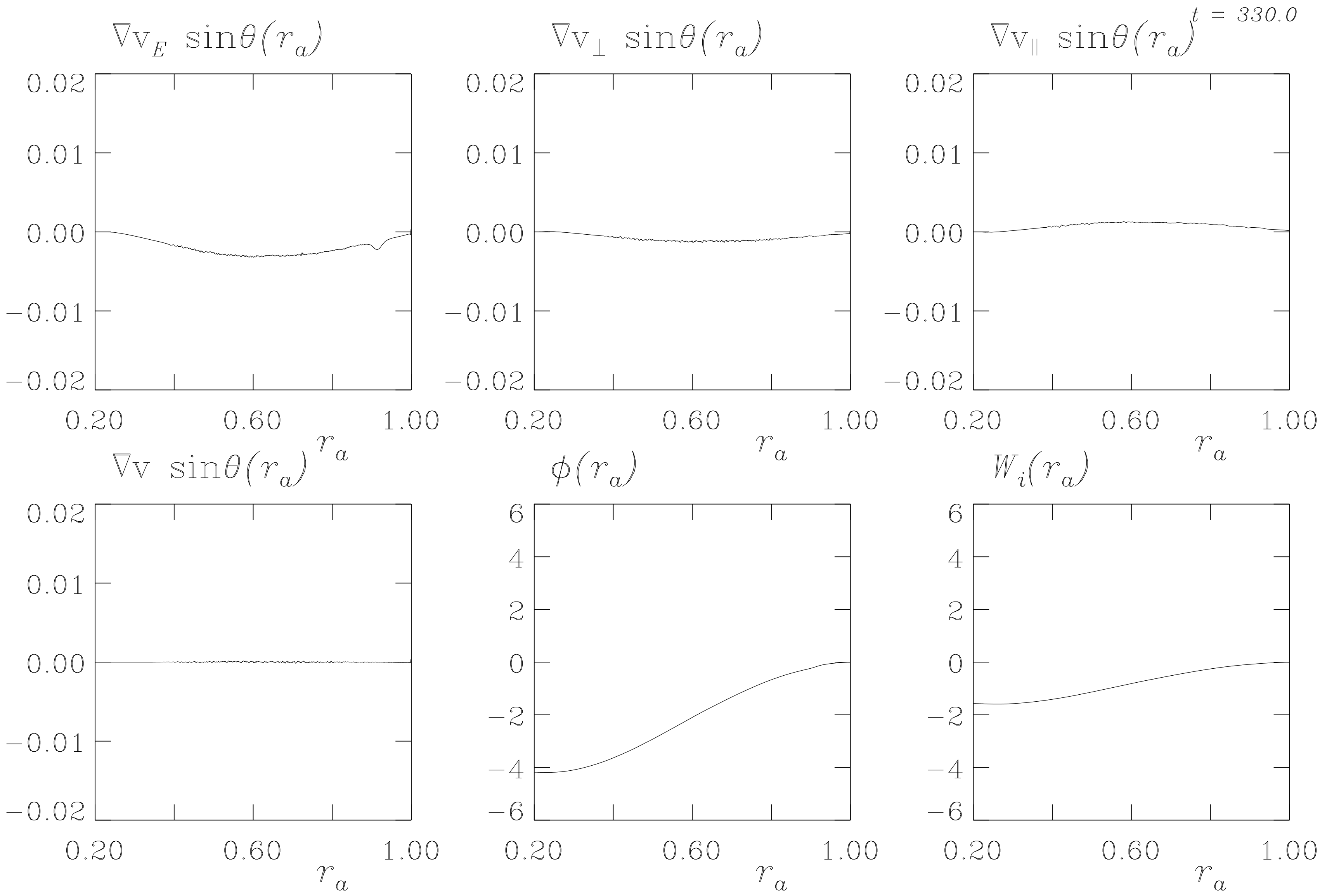
Ion Flow Sideband Divergences — JET Case

- signal of ZF activity now very weak



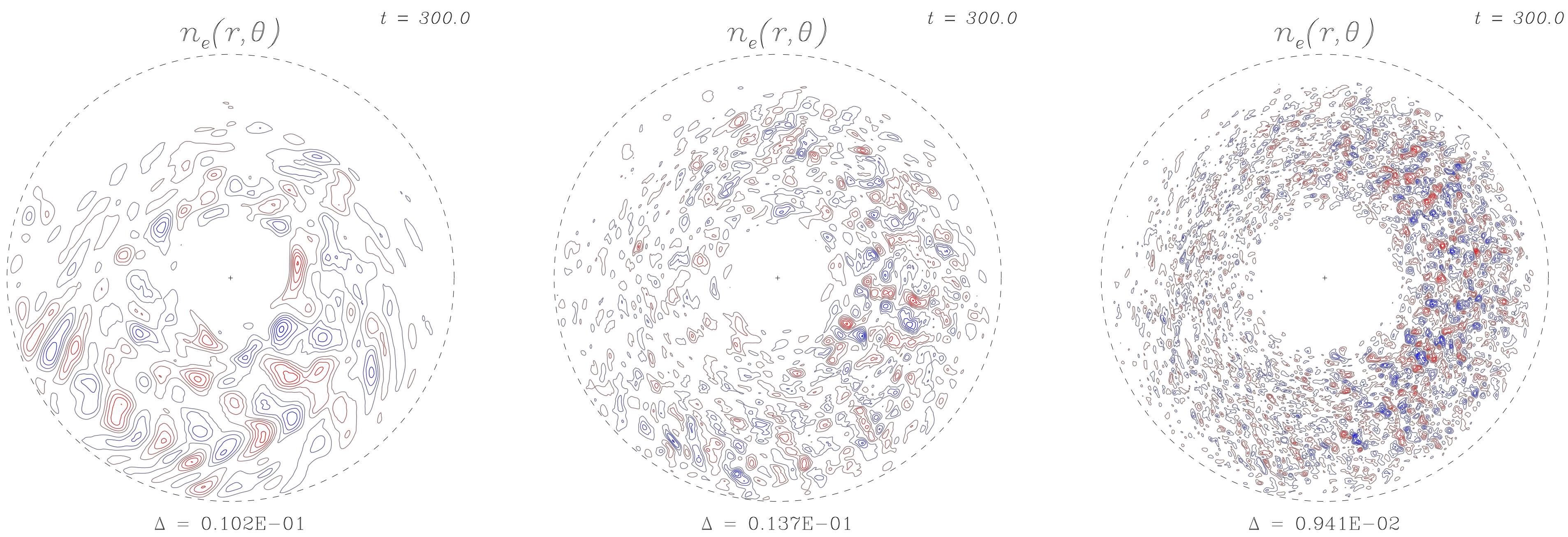
Ion Flow Sideband Divergences — ITER Case

- signal of ZF activity practically nonexistent, divergences are smaller



Look and Feel of Scale Separation

electromagnetic core cases with a/ρ_s of 50, 100, and 200, non-axisymmetric part



- if you can see the eddies on a global plot they're too large!

IBM Blowout Studies using GEM

A Kendl and B Scott 2007/8

- main aim: study of ELM blowout
  - actually just a ballooning instability transitioning into turbulence
  - study physical mechanisms and scalings first, then experimental issues

- GEM: electromagnetic 6-moment gyrofluid for both electrons, ions

(B Scott, Phys Plasmas Oct 2005)

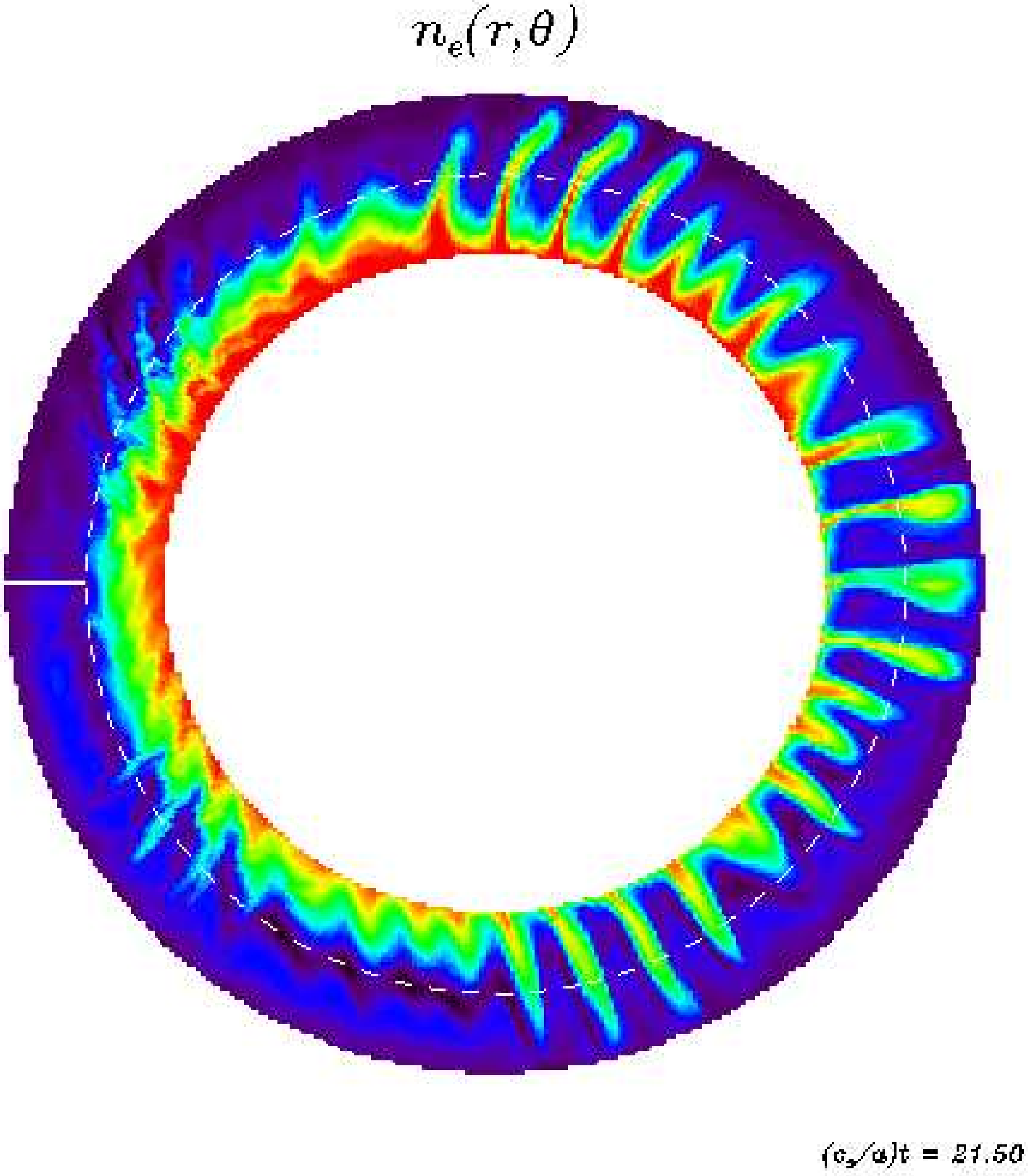
- global geometry, self consistent $q(r)$ and Shafranov shift from $J_{\|}$

- Base Case: AUG #17151 H-mode deuterium

(L Horton et al, Nucl Fusion 2005)

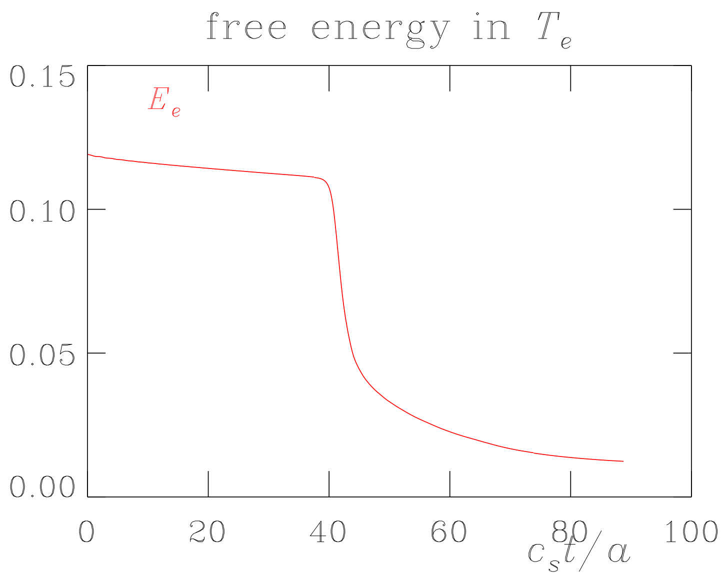
  - main linear mode near toroidal mode $9-10$
  - violent overshoot, cascade, crash (no nonlinear instability)
  - then segue into remnant turbulence

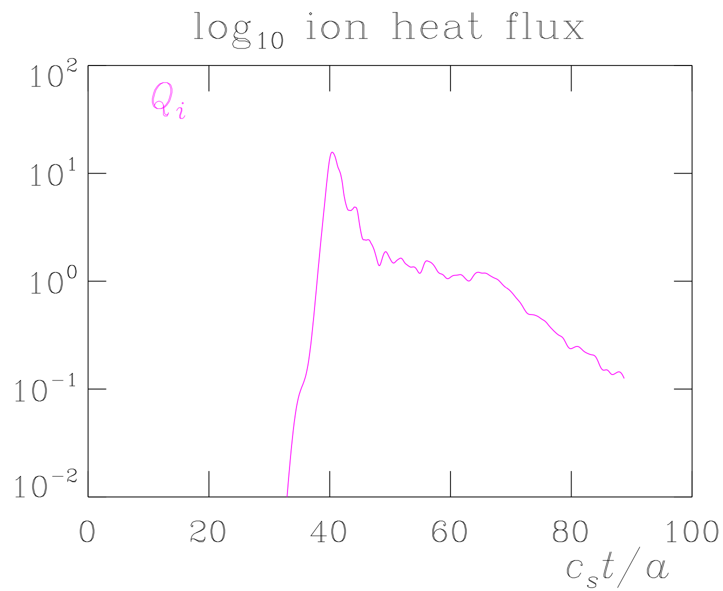
IBM Blowout

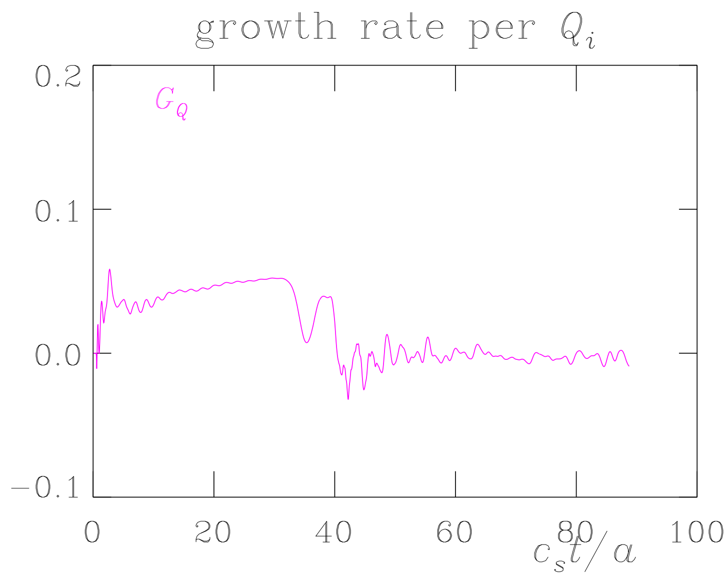


- 20 μsec after apparent quiet
- profile blown away
- finger structure obvious (and trivial)

Blowout Time Traces



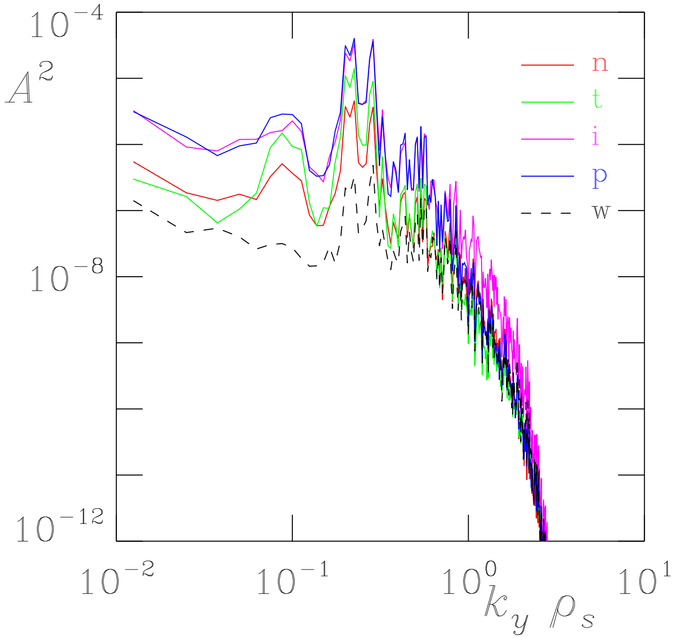


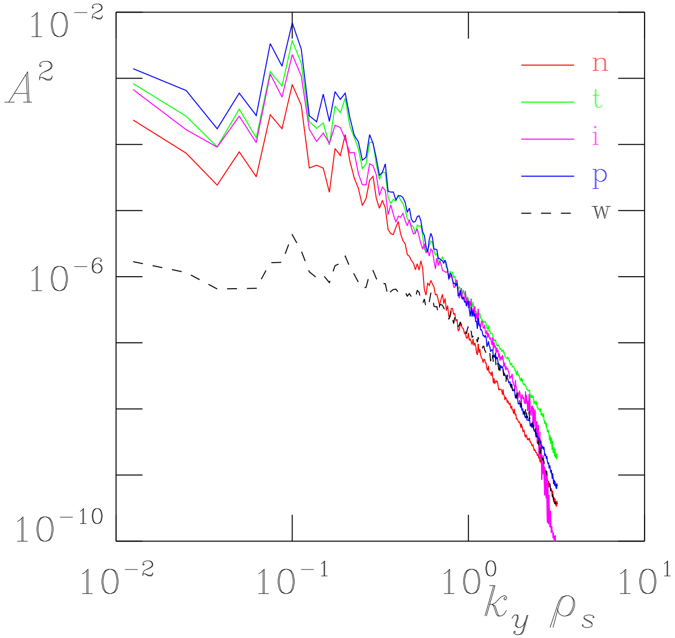


- energy, *ca.* 2/3 lost in blowout
- flux, short event ($< 40\,\mu$sec)
- growth rate as per flux, linear then segue into turbulent aftermath

Blowout Spectra

- linear growth (left) and peak-flux (right) phases





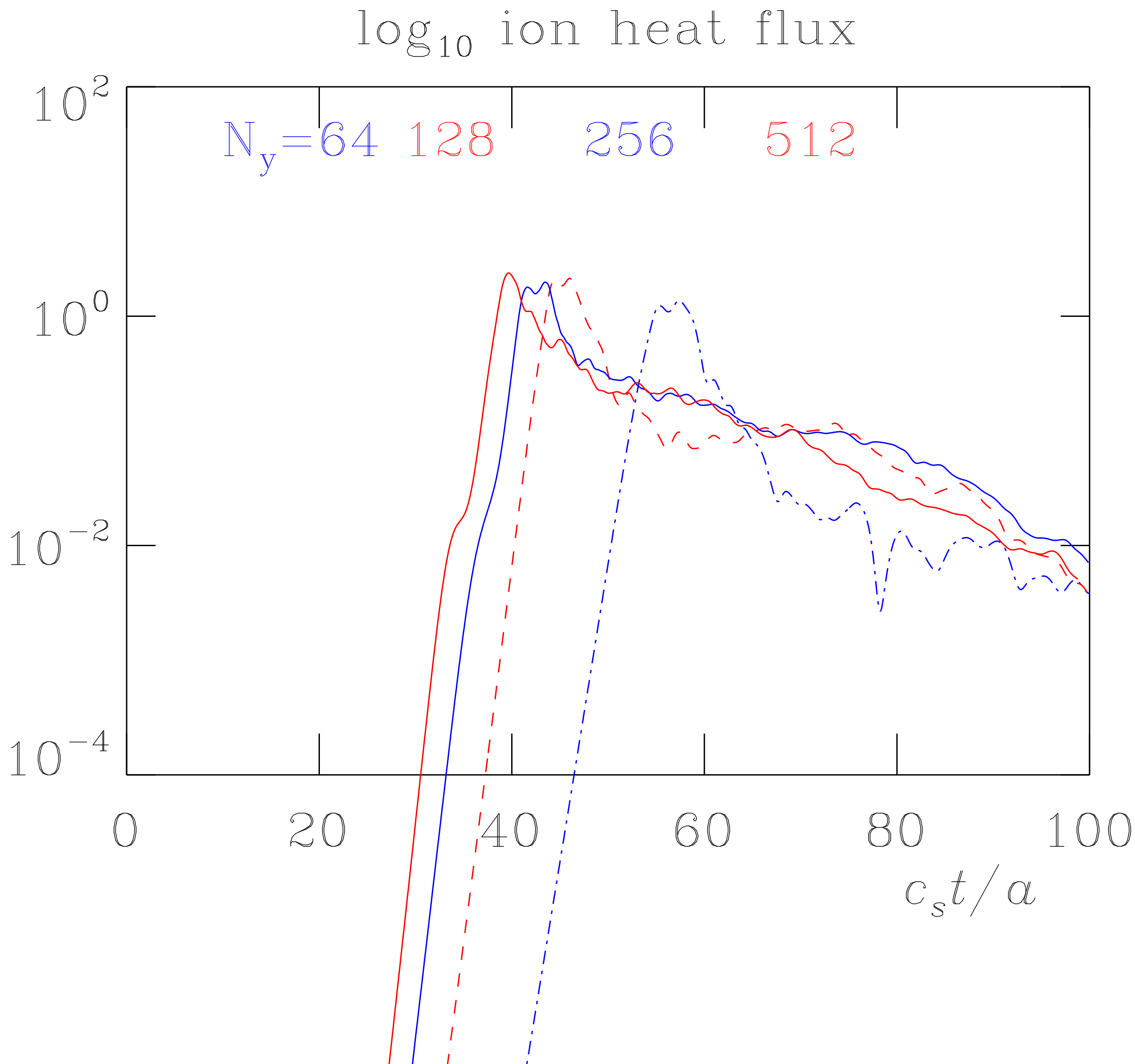
- linear growth: peak modes are $n = 9$ and 10 (MHD: $e\widetilde{\phi}/T_e$ largest)

- peak-flux: vorticity already flat to ion gyroradius scale

crash phase is outside not only MHD but also Braginskii regime

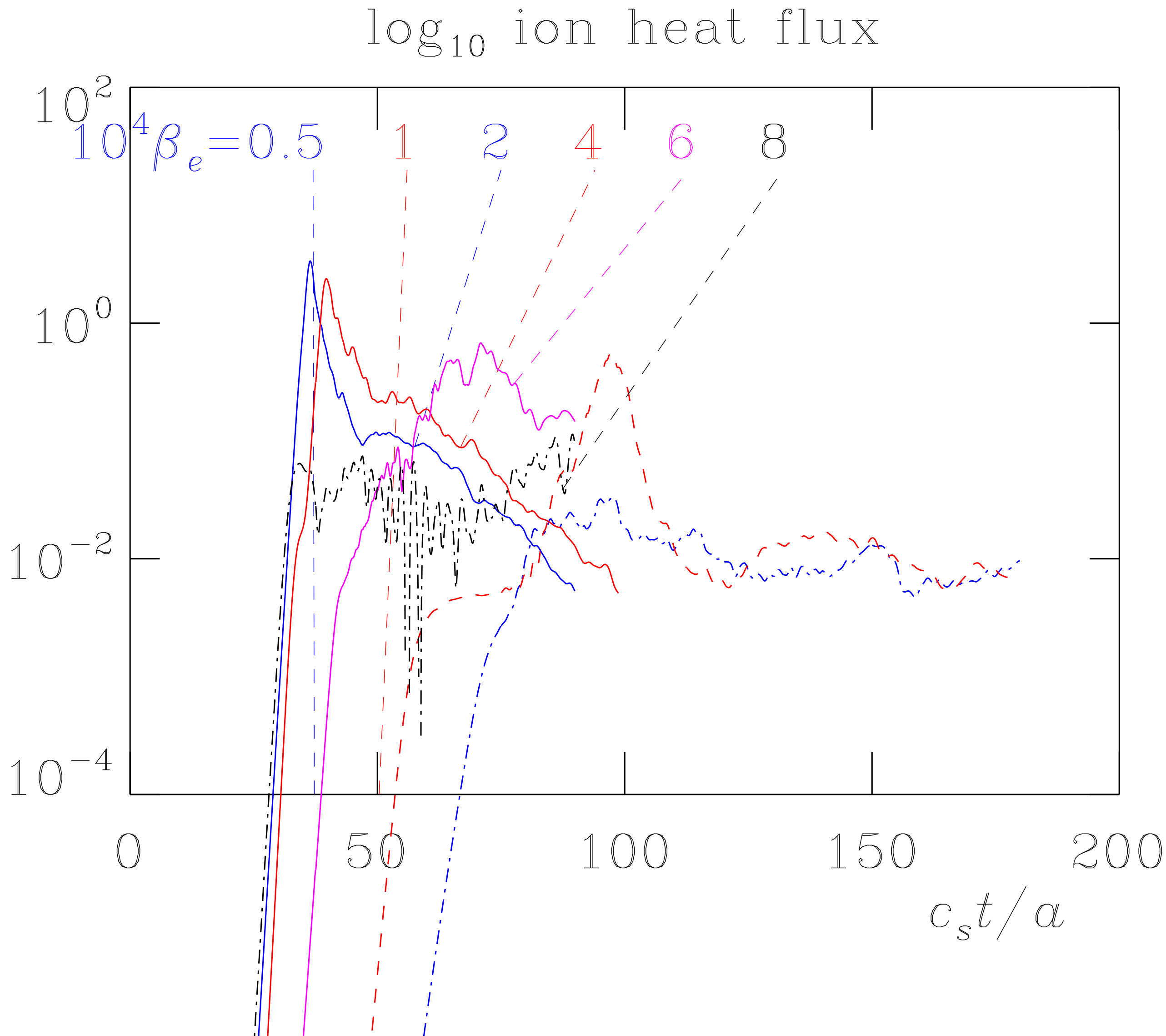
Blowout — Resolution in Drift Angle

- various N_y correspond to max $k_y \rho_s = 0.7 \quad 1.5 \quad 3 \quad 6 \quad (>1$ is required)



Blowout — Various Beta Values

- crossing the threshold ... very high β_e saturates earlier
  - ○ note the sound speed normalisation — growth rates are near γ_I



ELM crash scenario — notes

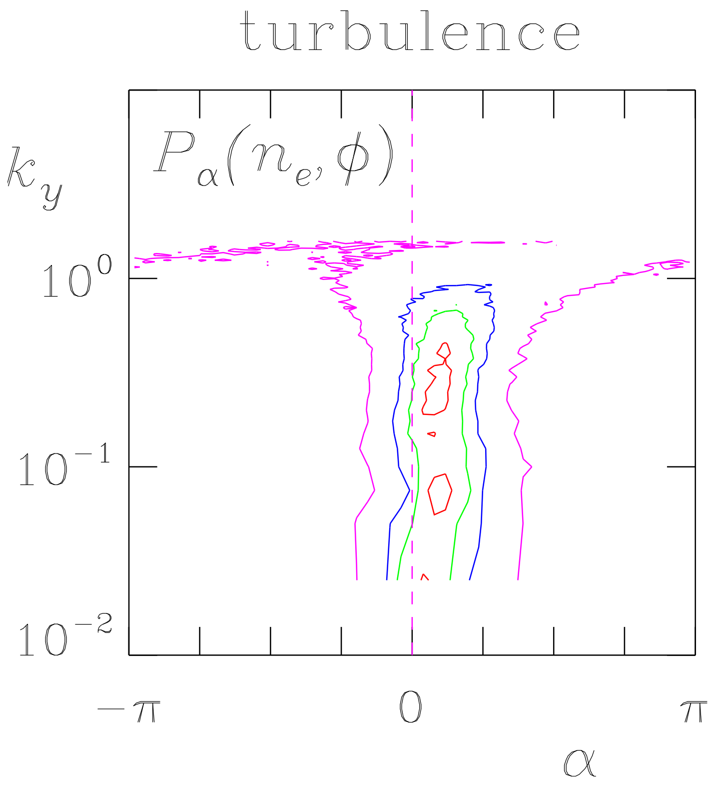
- nonlinear aftermath leaves MHD regime very quickly
  ◦ saturates on its own nonlinearly developed ITG turbulence

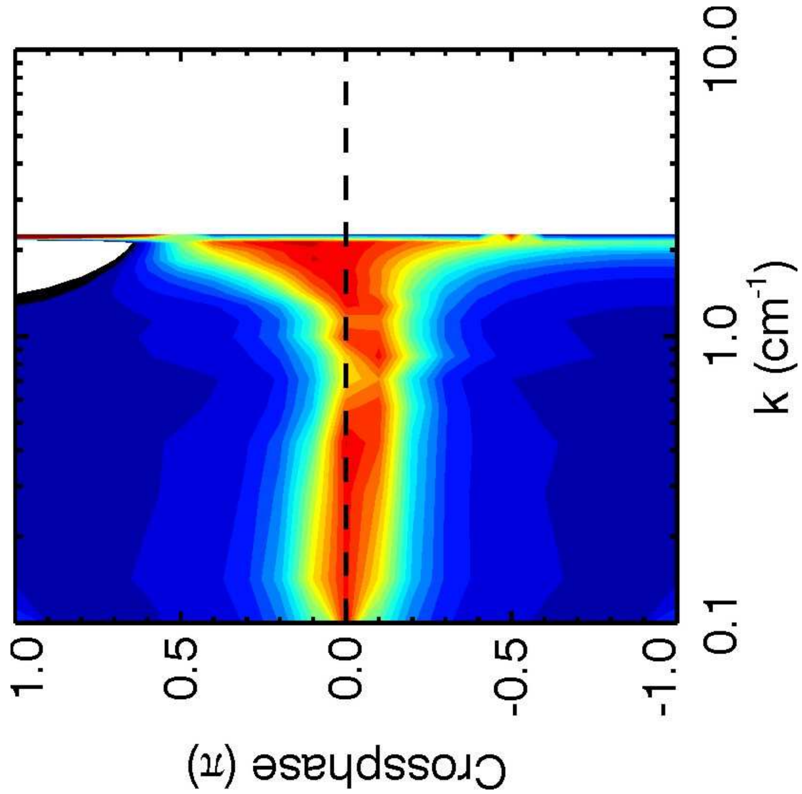
- nonlinear convergence requires ion gyroradius ($10^{-2} < k_\perp \rho_i < 6$)

- beta dependence: continuous transition MHD $\leftrightarrow$ ITG turbulence
  ◦ filament size reflects MHD or ITG sides (smaller for ITG)
  ◦ strength of overshoot/bursts follows (smaller for ITG)

underlying character of actual burst events in
L-Mode and ELMs may be similar (S Zweben et al PPCF 2007)

Comparison -- Fluctuation Statistics





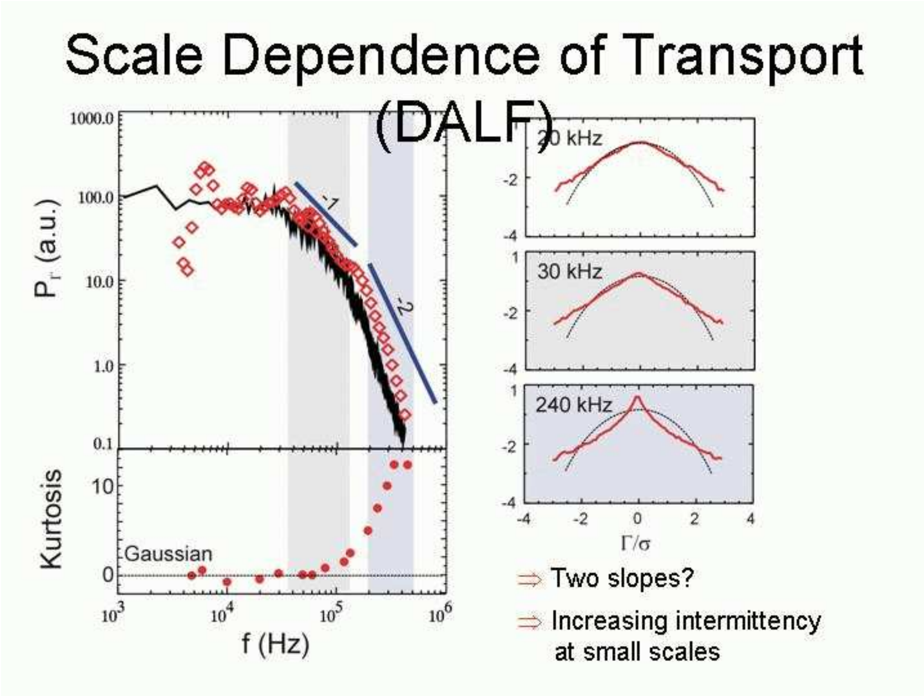
probability distribution of cross phase for each Fourier mode

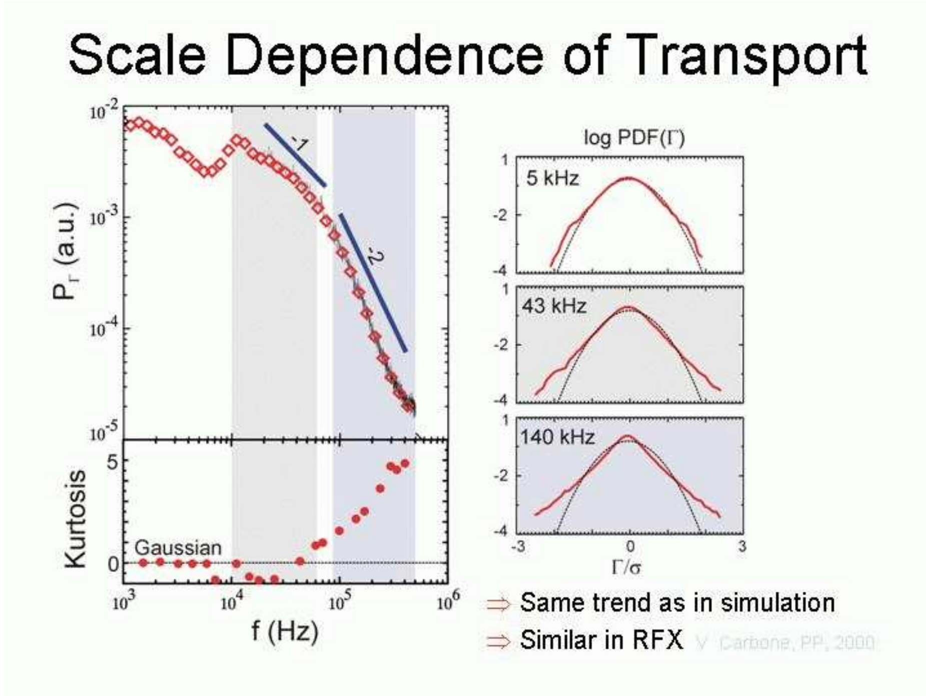
unified spectrum, phase shifts between 0 and $\pi/4$, in code and TJK experiment

basic signature of drift wave mode structure (parallel current dynamics)

(B Scott Plasma Phys Contr Fusion 2003) (U Stroth F Greiner C Lechte et al Phys Plasmas 2004)

Comparison -- Fluctuation Statistics





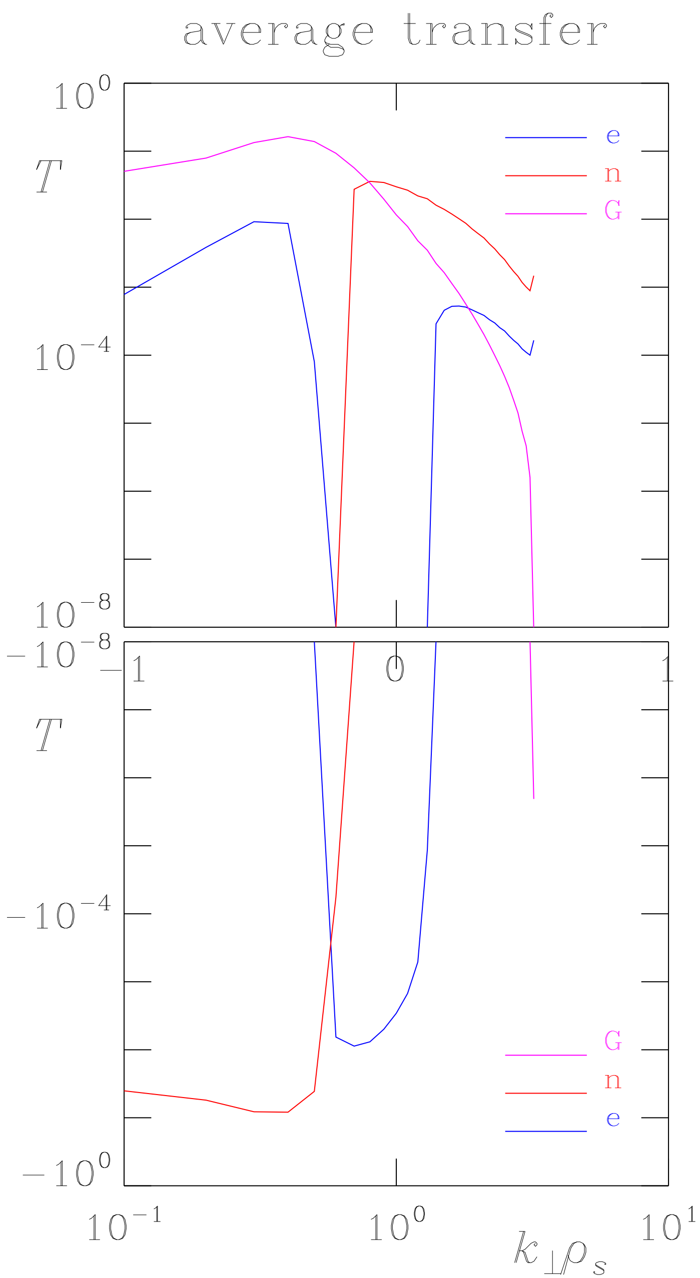
wavelet analysis of fluctuation induced transport in code and TJK experiment

both results show same phenomenology: regime break in spectrum

evidence of nonlinear cascade overcoming drive?

(N Mahdizadeh et al Phys Plasmas 2004)

Nonlinear Free Energy Cascade



direct cascade

--> nonlinear drive at small scales

==> passive scalar regime

frequency/scale correlation

matches with frequency break

evidence for onset of

passive scalar regime

Gyrokinetic Edge Turbulence

- "total-f" version (in development)

$$B_\parallel^* \frac{\partial f}{\partial t} + \nabla H \cdot \frac{c}{e}\frac{\mathbf{F}}{B} \cdot \nabla f + \mathbf{B}^* \cdot \left(\frac{\partial H}{\partial p_z} \nabla f - \frac{\partial f}{\partial p_z} \nabla H \right) = C(f)$$

$$\mathbf{F} = \epsilon \cdot \mathbf{B} \qquad \mathbf{B}^* = \mathbf{B} - \nabla \cdot p_z \frac{c}{e}\frac{\mathbf{F}}{B} \qquad B_\parallel^* = \mathbf{b} \cdot \mathbf{B}^*$$

- "delta-f" version (from which results shown)

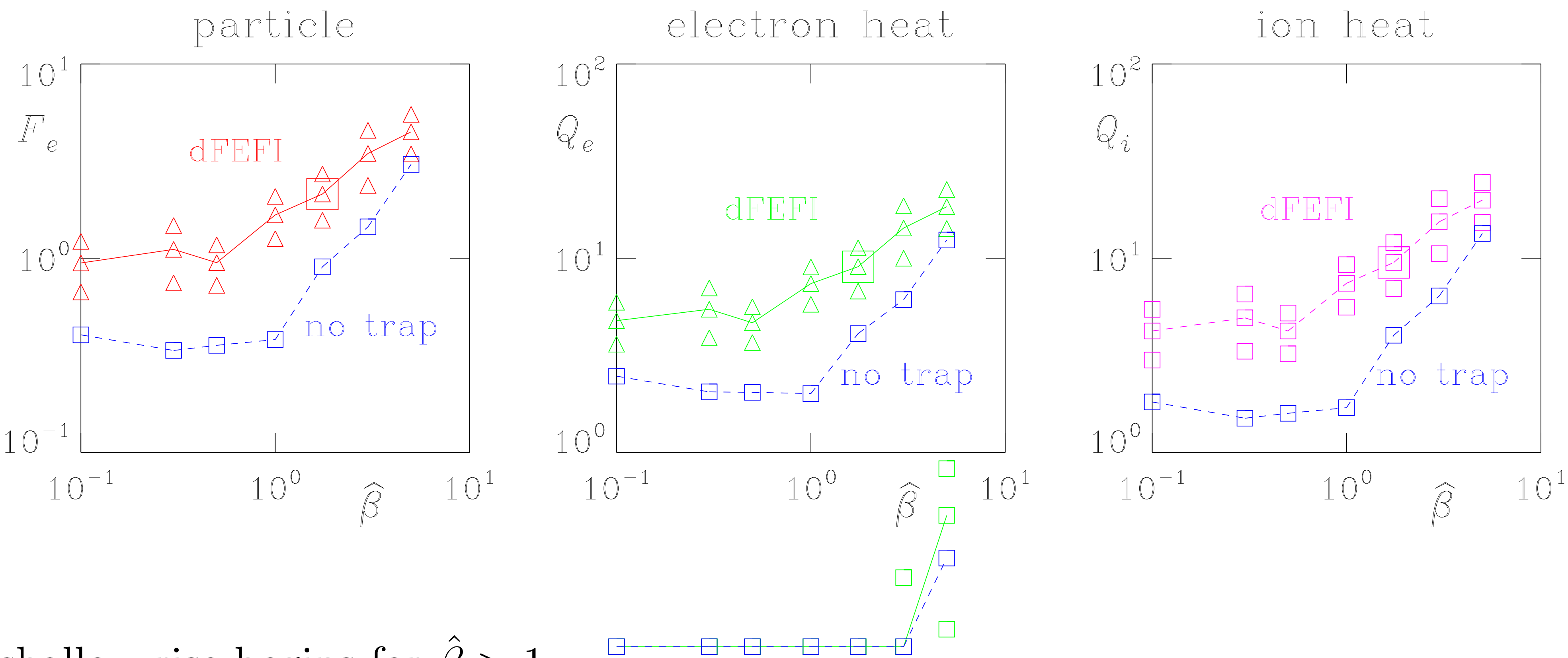
$$\frac{\partial \widetilde{g}}{\partial t} + \frac{cF^{xy}}{eB^2}[\widetilde{H}, \widetilde{h}]_{xy} + \frac{B^s}{B}[H_0, \widetilde{h}]_{zs} + \mathcal{K}(\widetilde{h}) = C(\widetilde{f})$$

$$\widetilde{h} = \widetilde{f} + eJ_0\widetilde{\phi}\frac{F^M}{T} \qquad \widetilde{g} = \widetilde{f} + e\frac{v_\parallel}{c}J_0\widetilde{A}_\parallel \frac{F^M}{T} \qquad \mathcal{K} = \nabla \frac{\mu B - mv_\parallel^2 \log R}{e} \cdot \frac{c}{e}\frac{\mathbf{F}}{B}\nabla_{\{x,y\}}$$

- H is Hamiltonian, with unperturbed and perturbed parts H_0 and $\widetilde{H}$
  - C is collision operator

GK Edge Transport Scaling versus Beta

effects of trapping in equilibrium magnetic field

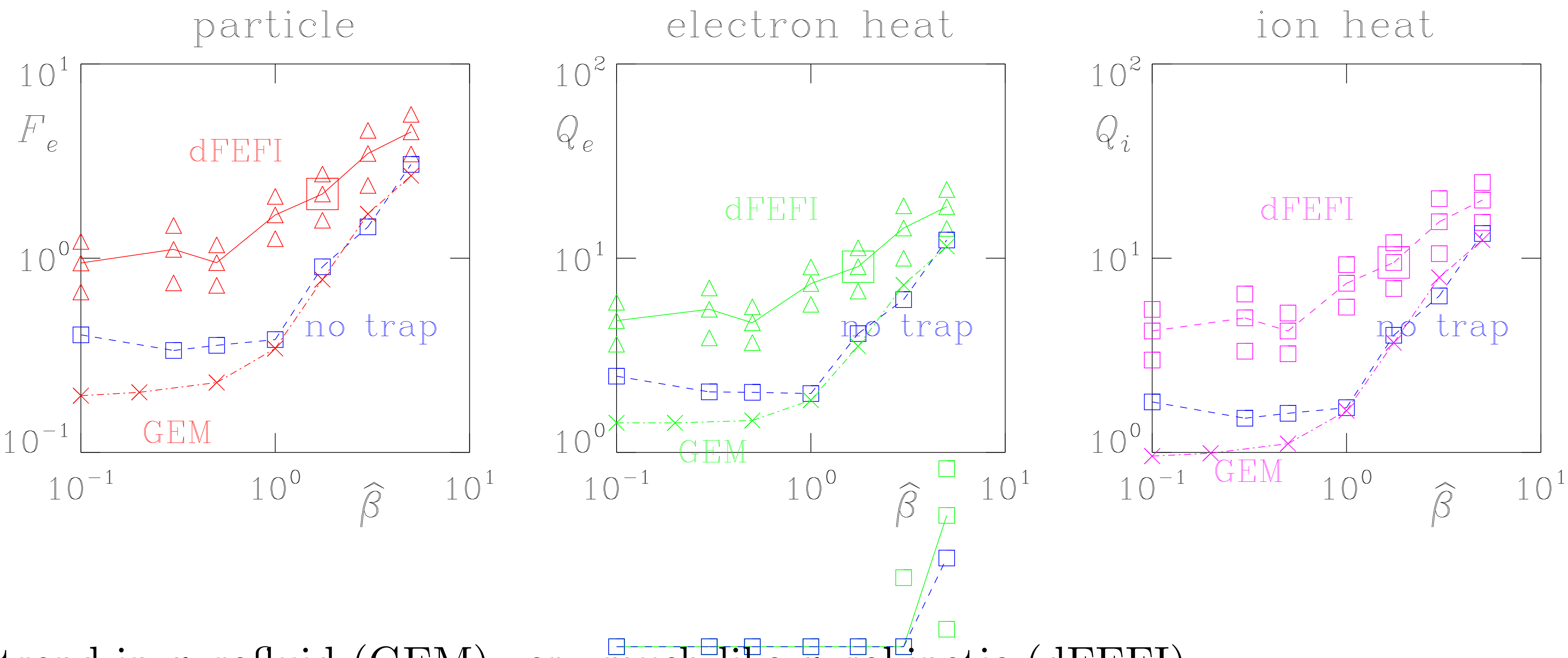


- shallow rise begins for $\hat{\beta} > 1$
  - all ExB transport channels follow each other
  - “magnetic flutter” becomes positive for $\hat{\beta}$ in transition to MHD

- trapping in equilibrium enhances transport (long-wave MHD component, see below)

Gyrokinetic and Gyrofluid Transport Compared

nominal and no-trap GK models versus gyrofluid model

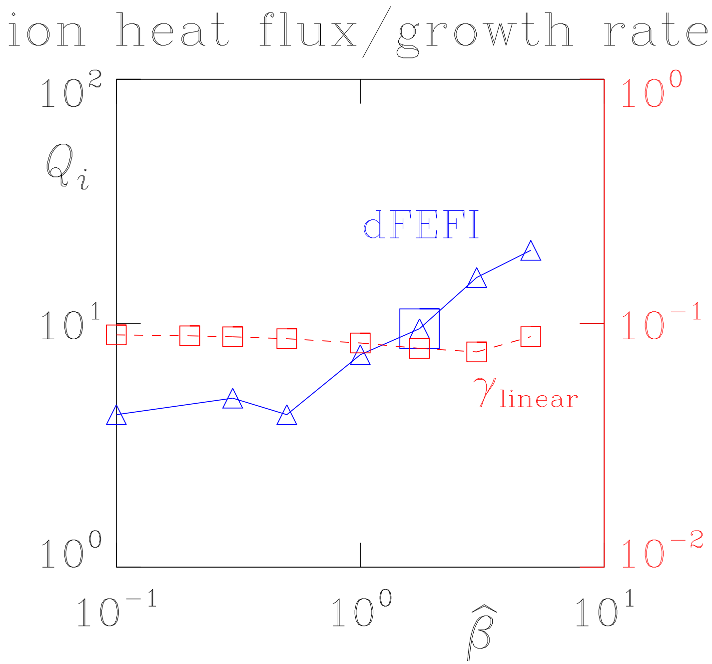


- trend in gyrofluid (GEM) very much like gyrokinetic (dFEFI)
  - especially no-trap version, where models agree on upturn position

- exposes the rising beta trend as general
  - mode structure analysis: nonlinear drive of long wavelength MHD component

Basic Nonlinearity of Edge Turbulence

gyrokinetic turbulence vs linear growth rates



- exposes the rising beta trend as nonlinear-only in this range
  - long-wave component unimportant in linear stage (low growth rates)
  - ExB energy transfer (see above) gives it extra strength

- transport level determined as much by saturation as by drive
  - self consistency determines overall level

Main Points

basics of energetics a central theme for physical understanding

wide overlap between gyrokinetic and gyrofluid models

temperature anisotropy and resolution of ion gyroradius are required

coupling of turbulence to flows extends to the magnetic equilibrium

self consistency: do the magnetic background inside the turbulence model

new physics themes:

✷ global electromagnetic computation

✷✷✷ stable reconnection and equilibration currents

incorporation of trapping effects in fluid codes (may be hopeless)

✷✷ nonlocal gyrofluid/gyrokinetic models −−> edge/core transition

one should expect surprises affecting design of high performance devices