International Workshop on the Frontiers of Modern Plasma Physics

14 - 25 July 2008

Physics behind Modern Gyrokinetic Theory, Simulation, and Comparisons to Tokamak Experiments.

T.S. Hahm

Princeton University, Plasma Physics Lab. U.S.A.
Physics behind Modern Gyrokinetic Theory, Simulation, and Comparisons to Tokamak Experiments

T.S. Hahm

Princeton University, Plasma Physics Laboratory

Acknowledgment:

P.H. Diamond, G. Rewoldt, W. Wang, Z. Lin,…
TFTR Team, NSTX Team, DIII-D Team,…

International Workshop on the Frontiers of Modern Plasma Physics
ICTP, Trieste, Italy, 17 July, 2008
Outline

Properties of Tokamak Core Turbulence
Nonlinear Gyrokinetic Description
Some Examples in Tokamak Confinement Physics:

Role of Zonal Flows
Turbulence Spreading

Emphasis:
Conservation Laws
Study of New Physics Mechanisms
Synergism among theory, simulation, and experiment
Outline

Properties of Tokamak Core Turbulence

Nonlinear Gyrokinetic Description

Some Outstanding Confinement Physics Issues:

Role of Zonal Flows
Turbulence Spreading
Microinstabilities in Tokamaks

- Tokamak transport is usually anomalous, even in the absence of large-scale MHD

- Caused by small-scale collective instabilities driven by gradients in temperature, density, ...

- Instabilities saturate at low amplitude due to nonlinear mechanisms

- Particles $\mathbf{E} \times \mathbf{B}$ drift radially due to fluctuating electric field
Amplitude of Tokamak Microturbulence

- Relative fluctuation amplitude $\frac{\delta n}{n_0}$ at core typically less than 1%
- At the edge, it can be greater than 10%
- Confirmed in different machines using different diagnostics
k-spectra of tokamak micro-turbulence

$$k_\theta \rho_i \sim 0.1 - 0.2$$

- from Mazzucato et al., PRL '82 (μ-wave scattering on ATC)
  Fonck et al., PRL '93 (BES on TFTR)

- similar results from

TS, ASDEX, JET, JT-60U and DIII-D
Properties of Tokamak Core Microturbulence

• $\delta n / n_0 \sim 1\%$
• $k_r \rho_i \sim k_0 \rho_i \sim 0.1 - 0.2$
• $k_{||} < 1/qR << k_\perp$: Rarely measured
• $\omega - \mathbf{k} \cdot \mathbf{u}_E \sim \Delta \omega \sim \omega_{*\pi}$

Broad-band $\Rightarrow$ Strong Turbulence

Sometimes Doppler shift dominates in rotating plasmas
# Electrostatic Microinstabilities in Tokamaks

<table>
<thead>
<tr>
<th>Classification: Free energy</th>
<th>Spatio-temporal Scales (wavelength, frequency direction, rough mag.)</th>
<th>Accessibility Mechanism for Instability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapped Ion Mode ( n, T_e, (ITG-TIM) T_i )</td>
<td>( \sim \rho_\theta \quad \sim \omega_\ast_e )</td>
<td>Trapped ion precession resonance (coll-less) Collisions btwn trapped and passing ions (dissipative)</td>
</tr>
<tr>
<td>Ion Temp. Grad. Mode ( T_i )</td>
<td>( &gt; \rho_i \quad &lt; \omega_\ast_{pi} )</td>
<td>Bad curvature or Negative compressibility</td>
</tr>
<tr>
<td>Trapped Electron Mode ( n ) or ( T_e )</td>
<td>( \sim \rho_i \quad &lt; \omega_\ast_e )</td>
<td>Trapped electron precession resonance (coll-less) Collisions btwn trapped and passing e(^-)'s (dissipative)</td>
</tr>
<tr>
<td>Electron Temp. G Mode ( T_e )</td>
<td>( &gt; \rho_e \quad &lt; \omega_\ast_{pe} )</td>
<td>Bad curvature or Negative compressibility</td>
</tr>
</tbody>
</table>
Heuristic Estimation of Diffusion Coefficient

\[ \gamma = \gamma_{\text{lin}} - k_{\perp}^2 D_{\text{turb}} \rightarrow 0 \]

• Nonlinear coupling induced dissipation leads to saturation (B. Kadomtsev '65)

\[ D_{\text{turb}} \sim \gamma_{\text{lin}} / k_{\perp}^2 \sim (v_{Ti} / a) \rho_i^2 \sim (\rho_i/a) (c_{Ti}/eB): \text{GyroBohm scaling; since } \gamma_{\text{lin}} \propto \omega_\ast \propto (v_{Ti}/a), \text{ and } k_{\perp} \propto \rho_i^{-1}. \]
Spectral Transfer in $k_\perp$

- $\gamma_{\text{lin}}$ usually peaks at high $k$
- Spectrum peaks at lower $k$ at nonlinear saturation

from Lin et al., IAEA/TH/8-4 (2004)
→ Dual cascade of Hasegawa-Mima type system or
→ Compton Scattering (Nonlinear Landau Damping)
Outline

Properties of Tokamak Core Turbulence

Nonlinear Gyrokinetic Description

Some Outstanding Confinement Physics Issues:

Role of Zonal Flows
Turbulence Spreading
L’aspect Cinématique de la Théorie Gyrocinétique

GTS simulation of ITG Turbulence: S. Ethier, W. Wang et al.,
Minimum number of ordering assumption

- \( \omega/\Omega_i \sim k_\parallel/k_\perp \sim \varepsilon_{k,\omega} \ll 1 \); from spatio-temporal scales of fluctuations

- \( k_\perp \rho_i \sim 1 \) for generality:
  Short wavelength modes (with higher \( \gamma_{\text{lin}} \)) can affect the modes at NL peak (\( k_\perp \rho_i \sim 0.1 \sim 0.2 \)) through NL coupling.
  \( \omega \sim k_\parallel v_{Ti} \) for wave-particle resonance

  i.e., Landau damping

- \( \delta f/f_0 \sim e\delta \phi/T_e \sim 1/k_\perp L_p \sim \varepsilon_\phi \ll 1 \); from small relative fluctuation amplitude

  - \( k e\delta \phi/T_e \sim 1/L_p \): \( \mathbf{E} \times \mathbf{B} \) Nonlinearity \( \sim \) Linear Drive

  - \( \delta n/n_0 \sim \rho/L \sim \) roughly experimental values.
Standard Nonlinear Gyrokinetic Ordering II.

• While the physics origins of $\varepsilon_{k,\omega}$ and $\varepsilon_\phi$ are different, the maximal ordering for NL GK corresponds to $\varepsilon_{k,\omega} \sim \varepsilon_\phi$

• $\varepsilon_{k,\omega} \gg \varepsilon_\phi$ leads back to the Linear Gyrokinetics:

  Taylor-Hastie, Plasma Phys. 10, 419 '68
  Rutherford-Frieman, Phys. Fluids 11, 569 '68
  Tang, Nuclear Fusion 18, 1089 '78
  Antonsen-Lane, Phys. Fluids 23, 1205 '80
  Horton, Rev. Mod. Phys 71, 735 '99

• With $\varepsilon_{k,\omega} \ll \varepsilon_\phi$, one cannot recover the linear dispersion relation of instabilities:

  Self-sustained Turbulence, BS from BDS

  Scott, Phys. Rev. Lett. 65, 3289 '90
Conventional Nonlinear Gyrokinetic Equation

[eg., Frieman and Chen, Phys. Fluids 1982]

- Foundations of Tokamak Nonlinear Kinetic Theory
  for analytic applications, ballooning codes...
- Number of assumptions minimum
- Based on direct gyro-phase average of Vlasov equation
  Lots of algebra and book keeping
- Direct expansions in $\varepsilon$: Self-consistent up to $O(\varepsilon^2)$ →
  Should be fine for linear phase and saturation due to $\mathbf{E}\times\mathbf{B}$
  nonlinearity
- Velocity space nonlinearity: $\nabla_{\parallel} \delta \phi \partial_{v_{\parallel}} \delta f \sim O(\varepsilon^3)$
  Energy, phase space volume **not** conserved.
- May not be able to describe long term behavior accurately

**Topic of Current Research:** [Villard, Hatzky, Sorge, Lee, Wang, Ku]
→ Physics responsible for the difference?
Conventional Nonlinear GK Derivation: Heuristic

• Transforming to guiding center variables, $R = x + \rho$, $\mu = v_{\perp}^2/2B$, $v = v_{||} b + (e_1 \cos \theta + e_2 \sin \theta)$, one can write the Vlasov equation as

$$\frac{\partial}{\partial t} f + v_{||} b \cdot \frac{\partial}{\partial R} f + \frac{E \times b}{B} \cdot \frac{\partial}{\partial R} f + (q/m) E_{||} \frac{\partial}{\partial \nu_{||}} f - \Omega \frac{\partial}{\partial \theta} f = 0$$

• Since $\Omega \gg \omega$, to the lowest order $\Omega (\partial / \partial \theta) f = 0$

• Writing $f = \langle f \rangle + \tilde{f}$, with $f = \langle f \rangle \gg f$ in which $\langle \ldots \rangle$ indicates gyrophase average,

$$\frac{\partial}{\partial t} \langle f \rangle + v_{||} b \cdot \frac{\partial}{\partial R} \langle f \rangle + \frac{E \times b}{B} \cdot \frac{\partial}{\partial R} \langle f \rangle + (q/m) E_{||} \frac{\partial}{\partial \nu_{||}} \langle f \rangle - \Omega \frac{\partial}{\partial \theta} f = 0$$

which is a solubility condition for $\langle f \rangle$.

• Gyro-phase averaging, one gets an electrostatic NL GK equation in a uniform $B$ field:

$$\frac{\partial}{\partial t} \langle f \rangle + v_{||} b \cdot \frac{\partial}{\partial R} \langle f \rangle + \langle E \rangle \times b \cdot \frac{\partial}{\partial R} \langle f \rangle + (q/m) \langle E_{||} \rangle \frac{\partial}{\partial \nu_{||}} \langle f \rangle = 0$$

• Frequency-wave number expansion and amplitude expansion, and geometric expansion (if it were included) are all lumped together in this procedure. If one modifies an ordering, needs to do the derivation all over again.
Nonlinear Gyrokinetics for Large Scale Computation

• Direct simulation of actual size fusion plasmas in realistic geometry using the primitive nonlinear plasma equations (Vlasov-Maxwell), is far beyond the computational capability of foreseeable future.

• For turbulence problems in fusion plasmas, the temporal scales fluctuations much longer than the period of a charged particle's cyclotron motion, while the spatial scales and gyro-orbits are much smaller than the macroscopic length scales: details of the charged particle's gyration motion are not of physical interest. Develop reduced dynamical equations which capture the essential features

• After decoupling of gyro-motion, gyrokinetic equation describes evolution of gyro-center distribution function, independent of the gyro-phase, $\theta$, defined over a five-dimensional phase space ($\mathbf{R}, v_\parallel, \mu$). save enormous amounts of computing time by having a time step greater than the gyro-period, and by reducing the number of dynamical variables.

• In gyrokinetic approach, gyro-phase is an ignorable coordinate, magnitude of the perpendicular velocity enters as a parameter in terms of an adiabatic invariant $\mu$

• Nonlinear gyrokinetic equations are now widely used in turbulence simulations.
Modern Nonlinear Gyrokinetics

• Starting from the original Vlasov-Maxwell system (6D), pursue "Reduction of dimensionality" for both computational and analytic feasibility.

• Keep intact the underlying symmetry/conservation of the original system.

• Perturbation analysis consists of near-identity coordinate transformation which "decouples" the gyration from the slower dynamics of interest in the single particle Lagrangian, rather than a direct "gyro-phase average" of Vlasov equation.

• This procedure is reversible: The gyro-phase dependent information can be recovered when it is needed.
Phase Space Lagrangian Derivation of Nonlinear Gyrokinetics

[since Hahm, PF 31, 2670 '88, followed by Brizard, Sugama,…]

• **Conservations Laws are Satisfied.**
  • Various expansion parameters appear at different stages
    → Flexibility in variations of ordering for specific application

• Guiding center drift calculations in equilibrium field $B$:
  Expansion in $\delta_B = \rho_i / L_B \sim \rho_i / R$.

• Perturbative analysis consists of near-identity transformations to new variables which remove the gyro-phase dependence in perturbed fields $\delta A(x), \delta \phi(x)$ where $x = R + \rho$:
  Expansion in $\varepsilon_\phi = e[\delta \phi - (v_{||}/c)\delta A_{||}] / T_e \sim \delta B_{||}/B_0$.

• Derivation more transparent, less amount of algebra
Single Particle Phase Space Lagrangian

[Littlejohn, Cary '83,...]

- Fundamental 1-form (phase space Lagrangian in non-canonical variables)
  \[ \gamma \equiv (eA(x) + mv) \cdot dx - (m/2)v^2 dt \]

- Transformation to guiding center variables:
  \[ x \equiv R + \rho, \quad \mu \equiv v_\perp^2/2\Omega, \quad \theta \equiv \tan^{-1}(\frac{v \cdot e_1}{v \cdot e_2}), \ldots \]

- The zero-th order phase space Lagrangian for guiding center:
  \[ \gamma_0 = (eA(R) + mv_\parallel b(R)) \cdot dR + \frac{\mu B}{\Omega} d\theta - H_0 dt \]

angle variable \( \theta \) is ignorable
action is an adiabatic invariant \( \mu \)

\[ H_0 = \mu B + (m/2)v_\parallel^2 \]
Euler-Lagrange Equation

- From variation of phase space Lagrangian:

\[ \frac{d\theta}{dt} = \Omega, \quad \frac{d\mu}{dt} = 0 \]

Decoupling of gyromotion, adiabatic invariant

\[ -eB^* \times \frac{dR}{dt} - mb \frac{dv}{dt} = \mu \nabla B \]

where \( B^* \equiv \nabla \times (A + \frac{m}{c} v_b) = B + \frac{m}{c} v_b \nabla \times b \)

- Decompose via \( b \times \) and \( B^* \), to get

\[ \frac{dR}{dt} = v \frac{B^*}{B^*} + \frac{\mu b}{e B^*} \times \nabla B, \]

and

\[ \frac{dv}{dt} = -\frac{\mu B^*}{m B^*} \cdot \nabla B \]
More on Guiding Center Drift

• Frequently asked question:
  “Where is the curvature drift?”
Using an identity \( B^* = B^*b + \frac{m}{e} v\parallel b \times (b \cdot \nabla)b \):

\[
\frac{dR}{dt} = v\parallel B^*b + \frac{m}{e} v\parallel b \times (b \cdot \nabla)b + \frac{\mu}{e B^*} b \times \nabla B
\]

• Infrequently asked question: “Do conventional guiding center drifts conserve energy?”

\[
\frac{dR}{dt} = v\parallel b + v_{curv} + v_{gradB}, \quad \frac{dv\parallel}{dt} = -\frac{\mu}{m} b \cdot \nabla B
\]
do not conserve energy exactly, while our E-L eqns do.

• \( B^* \) is a manifestation of Hamiltonian structure
• \( B^* \) is the density of phase-volume, \( d^6Z = B^*d\mu d\theta dv\parallel d^3R \)
Usual Practice **Violates** Conservation Laws

- Frequently committed mistakes in simulation community:
  - *“I have a freedom to ignore small terms!”*
    - e.g., \( B^* \rightarrow B \)
    - Error in the phase density volume: Artificial dissipation
  - *“I am using equations from a text book.”*

\[
\frac{dR}{dt} = v_{\parallel} b + v_{\text{curv}} + v_{\text{grad}B}, \quad \frac{dv_{\parallel}}{dt} = -\frac{\mu}{m} b \cdot \nabla B
\]

- Error in single particle kinetic energy
- **VERDICT**: Did Little Physics out of Advanced Theory

- **SENTENCE**: Life-time Community Service
Lie Perturbative Analysis I.

[from Hahm, PF 31, 2670 '88]

- Consider electrostatic fluctuation only (for illustration): \( \delta \phi(x) = \delta \phi(R + \rho) \)
- While gyromotion has been decouple in the zero-th order phase space Lagrangian, it appears again in the perturbation. Since it is \( O(\epsilon_\phi) \), we can remove it via near-identity, phase-space preserving Lie transform.
- In addition to zero-th order \( \gamma_0 \), \( \gamma_1 = -e\delta \phi(R + \rho) dt \)
- Perform Lie-perturbation:
  \[
  \Gamma_1 = \gamma_1 - L_1 \gamma_0 + dS_1
  \]
where \( (L_1 \gamma)_\mu = g^\nu_1 (\frac{\partial \gamma_\mu}{\partial z^\nu} - \frac{\partial \gamma_\nu}{\partial z^\mu}) \), transformation of 1 form
Lie Perturbative Analysis II.

- One can choose the gauge function $S_1$ and the generator $g_1$ such that the gyrophase is removed from $\Gamma_1$
- $\Omega \frac{d}{d\theta} S_1 - \frac{\partial S_1}{\partial t} - \frac{dR}{dt} \cdot \nabla S_1 - \frac{dv}{dt} \frac{\partial}{\partial v} S_1 = \frac{e}{\Omega} (\delta \phi - < \delta \phi >)$
- Using $\epsilon_{k,\omega} << 1$, we obtain
  
  $$dS_1 = \frac{e}{\Omega} (\delta \phi - < \delta \phi >) d\theta$$

- $\Gamma_1 = -e < \delta \phi > dt$

  where $< \ldots >$ is the gyrophase average $\frac{1}{2\pi} \int (...)$

- Note that decoupled gyrophase information is kept in $S_1$ and $g_1$ to be used later when necessary.
Lie Perturbative Analysis III.

- Now, $\Gamma = \Gamma_0 - e < \delta \phi > dt$,
  
  $H = H_0 + H_1 = \mu B + (m/2)v^2 + e < \delta \phi >$

- Euler-Lagrange Equation
  
  $$\frac{dR}{dt} = v_\parallel \frac{B^*}{B^*} + \frac{b}{B^*} \times (\frac{\mu}{e} \nabla B + \nabla < \delta \phi >),$$
  
  and
  
  $$\frac{dv_\parallel}{dt} = -\frac{1}{m B^*} (\mu \nabla B + e \nabla < \delta \phi >)$$

- $B^*$ correction in the last term crucial for momentum pinch

- The second order perturbation in $\epsilon_\phi \sim \rho/L_p$ is necessary for energy conservation.
Gyrokinetic Vlasov-Poisson System

- With Euler-Lagrange Eqns, the gyrokinetic Vlasov equation for gyrocenter distribution function $F(R, \mu, v_\parallel, t)$ is:

$$\frac{\partial F}{\partial t} + \frac{dR}{dt} \cdot \nabla F + \frac{dv_\parallel}{dt} \frac{\partial F}{\partial v_\parallel} = 0$$

Note reduction of dimensionality achieved by $\frac{\partial F}{\partial \theta} = 0, \frac{d\mu}{dt} = 0$

- Self-consistency is enforced by the Poisson’s equation. Debye shielding is typically irrelevant, one must express the ion particle density $n_i(x)$ in terms of the gyrocenter distribution function $F(R, \mu, v_\parallel, t)$

- Lee [PF 26, 556 '83] has identified the polarization density (in addition to the guiding center density). It was a key breakthrough in advances in GK particle simulations.

$$\delta n_i(x) = \delta n_{gc} + \rho_i^2 \nabla_\perp \cdot N_0 \nabla_\perp (e\delta \phi/T_i)$$
Pullback Transformation

- Widespread Misconception:
  "Gyrokinetic theory throws away the gyrophase dependent part of $F$."
- The gyrophase dependent information is kept in the gauge function $S_1$ or a generator $g_1$.
- This can be used \textit{reversibly} whenever one wants to calculate a quantity in the particle frame from the gyrocenter distribution function.

\[
\int d^6\mathcal{Z} \left( T_G^* F(Z) \right) K(R) \delta^3(R - \mathbf{x} + \mathbf{\rho}) \rightarrow K(x)
\]
- Examples include the polarization density, diamagnetic current, and other quantities related to finite Larmor radius effects.
Polarization Density

- More systematic derivation of GK Poisson’s eqn started since Dubin et al.,[PF 26, 3524 '83] via pullback transformation:

\[
\nabla^2 \delta \phi = -4\pi e \left[ \int d^6 \mathcal{Z} \left( T^*_G \delta f \right) \delta^3 \left( \mathbf{R} - \mathbf{x} + \mathbf{\bar{p}} \right) - \delta n_e(\mathbf{x}, t) \right],
\]

where

\[
T^*_G \delta f \equiv \delta f + \left( \frac{\partial S_1}{\partial \theta} \right) \frac{\partial F_0}{\partial \mu} + \left[ \frac{1}{\Omega} \left( \nabla S_1 \right) \times \mathbf{b} \right] \cdot \nabla F_0
\]

- Contribution to the ion particle density which involves \( S_1 \) is the general form of polarization density. After linearization,

\[
\{ k^2 \lambda_0^2 \} \frac{e \delta \phi_k}{T_{\perp}} n_0 + \{ 1 - \Gamma_0(b) \} \frac{e \delta \phi_k}{\bar{T}_{\perp}} n_0 = \delta \overline{\mathcal{N}}_k - \delta n_{ek}
\]

- It is obvious that the polarization density satisfies

\[
\frac{\partial}{\partial t} \delta n_{pol} + \frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{n}_0 \mathbf{v}_{pol} = 0
\]
Conservation of Energy
and Phase-Space Volume

- It is straight-forward to show the Liouville’s theorem:

\[
\nabla \cdot \left( B^\parallel \frac{d\mathbf{R}}{dt} \right) + \frac{\partial}{\partial \mathbf{v}^\parallel} \left( B^\parallel \frac{d\mathbf{v}^\parallel}{dt} \right) = 0
\]

- The invariant energy for GK Vlasov-Poisson system is obtained by transforming the energy constant of the original Vlasov-Poisson system [Dubin et al.,'83]

\[
E = \int d^6z F_i(\mu B + \frac{M}{2}v^2) + \int d^6z f_e(\mathbf{z})\frac{1}{2} m_e v^2
\]

\[
+ \frac{1}{8\pi} \int d^3x |\mathbf{E}|^2 + \frac{e^2}{2\Omega} \int d^6z F_i \left( \frac{\partial}{\partial \mu} \langle \delta \tilde{\Phi}^2 \rangle + \frac{1}{\Omega} \langle \nabla \delta \tilde{\Phi} \times \mathbf{b} \cdot \nabla \delta \tilde{\Phi} \rangle \right)
\]

Note that the sloshing energy (last term) can be obtained from perturbation up to \(O(\epsilon^2_\phi)\).
Extensions to Edge

[for core transport barriers → Hahm, Phys. Plasmas 3, 4658, '96]

Expansion in $\epsilon_B \sim \rho_i/L_E \sim \frac{B_0}{B}$:

- From $\rho_{ip} \sim L_P \sim L_E$,
  $$u_E \sim u_\ast \sim \frac{\rho_i}{L_p} v_T, \quad \frac{\epsilon\Phi(0)}{T_e} \sim 1.$$ 
- $|S - 1| \sim 1$ (banana orbit distortion), $\frac{\omega_E}{\Omega_i} \sim \epsilon_B^2$ (circular gyro-orbit)
  where $\omega_E \equiv \frac{(RB_\theta)^2}{B} \frac{\partial}{\partial \psi}(\frac{E_r}{RB_\theta})$ [Hahm-Burrell, PoP '95]
  $$S \simeq 1 + \left(\frac{B}{B_\theta}\right)^2 \frac{\omega_E}{\Omega_i}$$ [Hinton-Kim, Furth-Rosenbluth, Shaing,...]

- The zero-th order phase space Lagrangian
  $$\gamma_0 \equiv (eA + mu_E + mv \parallel b) \cdot dR + \frac{\mu B}{\Omega} d\theta - H_0 dt$$
  with a guiding-center Hamiltonian
  $$H_0 = e\Phi + \mu B + (m/2)(v_\parallel^2 + u_E^2) + \frac{\mu B}{2\Omega} b \cdot \nabla \times u_E.$$
Summary

- Modern Nonlinear Gyrokinetic Theory has provided a firm theoretical foundation for recent remarkable advances in gyrokinetic simulations and associated theories.

- Its elegance and relative simplicity have contributed to deeper understanding of the gyrokinetic system, not only improving treatment of familiar ones, but also indentification of novel physics effect.

- Significant example: Turbulent Convective Pinch of Toroidal Momentum

- It should be useful for even more complicated systems where several expansion parameters exist.
References on Nonlinear Gyrokinetic Theory I.

- Theoretically-oriented Recent Review
  Brizard and Hahm, Rev. Mod. Phys. 79, 421 '07

- Pioneering paper on conventional NL GK
  Frieman and Chen, PF 25, 502 '82

- NL GK for particle simulation:
  Lee, PF 26, 556 '83

- Proto-type Modern NL GK using Hamiltonian method:
  Dubin, Krommes, Oberman, and Lee, PF 26, 3524 '83
  Hagan and Frieman, PF 28, 2641 '85
  Hahm, Lee, and Brizard, PF 31, 1940 '88
References on Nonlinear Gyrokinetic Theory II.

- Modern NL GK using phase-space Lagrangian Lie perturbation method:
  Hahm, PF 31, 2670 ’88 (General geometry, electrostatic)
  Brizard, J. Plasma Phys. 41, 541 ’89
  (General geometry, electromagnetic)

- NL GK for strongly rotating plasmas:
  Hahm, PF-B 4, 2801 ’92 (in slab)
  Brizard, PoP 2, 459 ’95 (in terms of toroidal rotation)
  Hahm, PoP 3, 4658 ’96 (in terms of $E_r$)

- Energy conservation theorem:
  Brizard, PoP 7, 4816 ’00
  Sugama, PoP 7, 466 ’00 (introduction of field theory)
References on Topics related to Modern NL GK using phase-space Lagrangian Method

- Bounce-averaged Nonlinear Kinetic equation
  Fong and Hahm, PoP 6, 188 ’99 (electrostatic)
  Brizard, PoP 7, 3238 ’00 (electromagnetic)

- High frequency linear gyrokinetic theory:
  Qin and Tang, PoP 11, 1052 ’04 (recovery of compressional Alfven wave, elucidation of differential geometrical meaning of pullback transformation)
Flux-Gradient Relation

Generalization of Fick’s Law:

\[
\begin{pmatrix}
Q_i \\
Q_e \\
\Gamma \\
\Gamma_\phi
\end{pmatrix}
= -
\begin{pmatrix}
\chi_i & \cdots & \cdots & \cdots \\
\cdots & \chi_e & \cdots & \cdots \\
\cdots & \cdots & D & \cdots \\
\cdots & \cdots & \cdots & \chi_\phi
\end{pmatrix}
\begin{pmatrix}
\nabla T_i \\
\nabla T_e \\
\nabla n \\
\nabla U_\phi
\end{pmatrix}
\]
Outline

Properties of Tokamak Core Turbulence

Nonlinear Gyrokinetic Description

Some Outstanding Confinement Physics Issues:

 Ion Thermal Transport with Zonal Flows
Ion Thermal Transport

* Better understood, compared to other transport channels
* In Aux-heated plasmas, typically $\chi_i \gg \chi_{i,\text{Neo}}$ (cf. ITBs)
* Ion Temperature Gradient (ITG) Turbulence: Best Candidate
* With recent advances in gyrokinetic codes, simulation results begin to converge for simple cases, not only in numbers, but also in underlying physics.

The effective upshift of onset condition for large ion heat flux is caused by Zonal flows

[Dimits et al., Phys. Plasmas, 2000] from Cyclone project
What is a zonal flow?

Drift Waves

Drift waves + Zonal flows

Paradigm Change

ZF are "modes", but:

1. No direct radial transport
2. No linear instability
3. Turbulence driven

ITER

ExB flows

m=n=0, k_r = finite

From GTS

Made in Japan, edited in USA

Zonal flow on Jupiter

From GTS

Made in Japan, edited in USA
Basic Physics of a Zonal Flow

from Diamond, Itoh, Itoh, and Hahm, “Zonal Flows in Plasma—a Review” PPCF ’05

Suppression of DW by Shearing

V_{ExB}

Generation by Vortex Tilting

Damping by Collisions

VT, \n...\n
Drift wave turbulence

\langle \hat{v}_x \hat{p} \rangle

Transport

\langle \hat{v}_x \hat{v}_y \rangle

Zonal flows

Collisional flow damping

Nonlinear flow damping

energy return

\\supseteq

\\supseteq
Role of E x B Shear in Reducing Turbulence

• ExB shear decorrelation in cylinder [Biglari-Diamond-Terry, PF-B ’90]

\[ \omega_E > \Delta \omega_T \]

• Turbulence quenching in gyrofluid simulation [Waltz-Kerbel-Milovich, PoP ’94]

\[ \omega_E > \gamma_{lin} \]

• ExB Shearing Rate in General Toroidal Geometry [Hahm-Burrell, PoP ‘95]

\[ \omega_E = \frac{\Delta r_0}{\Delta \ell} \frac{(RB_\theta)^2}{B} \frac{\partial}{\partial \psi} \left( \frac{E_r}{RB_\theta} \right) \]

• Made possible by developments of experimental diagnostics for \( E_r \) and \( B_\theta \) (MSE, CHERS, …)

• Useful Rule of Thumb for Indication of the importance of ExB shear

• Widely used for experimental results analysis
  (TFTR, DIII-D, JET, JT60-U, AUG, TEXTOR, NSTX, MAST, LHD, W7-AS, …)
  with linear growth calculations via
  FULL [Rewoldt,…]
  Gryffin [Beer, …]
  GS2 [Kotschenreuther,…]
  Synakowski et al., PoP ’97 ---->
Key Physics Mechanisms behind Size Scaling

- **Global Toroidal ITG eigenmode**
  [Horton-Choi-Tang, PF ‘81] [Cowley-Kulsrud-Sudan, PF B’ 91]
  [Romanelli-Zonca, PF B’ 93][Parker-Lee-Santoro, PRL’93]

  Bohm Scaling ?

- **Self-regulation by Zonal Flows:**
  [Cast of Thousands][Review: Diamond et al., PPCF ‘05]

  GyroBohm scaling ?

- **Turbulence spreading**
  [Garbet et al., NF 94, PoP 08]
  [Hahm, Diamond, Lin, et al., PPCF ‘04, PoP’05]
  [Gurcan, Diamond, Hahm et al, PoP ’05, 06]
  [Naulin et al., PoP ’05] [Waltz et al., PoP ’05],…

Deviation from GyroBohm ?

- [Lin, et al., PRL ’02]

Density fluctuations from a GTS simulation of a shaped plasma with typical DIII-D core parameters
[Wang, Hahm, Lee et al., PoP ‘07]
Characterization of Zonal Flow Properties
Motivated Experimental Measurements

[Hahm, Burrell, Lin, Nazikian, Synakowski, PPCF ‘00]

From GTC simulations:
- \( n=0, m=0 \), broad \( k_r \), potential fluctuation
- Broad-band zero-freq ZF & GAM
- Properties of associated density fluctuations

Experiments:
- DIII-D (BES, PCI, Langmuir Probe)
- TEXT, JFT-2M, JIPPT-IIU (HIBP), AUG, (Doppler Reflectometry)

McKee, et al., BES on DIII-D
[presented by Burrell, IAEA-TM, Sept. ‘05]
E x B Shearing by time-dependent Zonal Flow

[Hahm, Beer, Lin, et al., PoP ‘99]

- Gyrofluid Simulations observed that instantaneous $\omega_E(t) >> \gamma_{\text{lin}}$ while turbulence was at L-mode level and transport was anomalous.

- Effective E x B shearing rate has been analytically derived to take into account the time dependence of zonal flows

- From Gyrofluid simulation data analysis, has been observed:

$$\omega_E^{\text{eff}} \sim \gamma_{\text{lin}}$$

-- Shearing due to GAM is predicted to be ineffective for core turbulence.

- Gyrokinetic simulations demonstrated broadening of $k_z$ of ITG turbulence (a symptom of eddy breaking-up) due to zonal flows quantitatively.

-- On the other hand, GAMs can affect edge turbulence, not only by shearing, but also by reducing zonal flows

[B.D. Scott, New Journal of Phys. (2005)]
Ion Thermal Transport scales with $I_p$ in NSTX and Tokamaks

While not inconsistent with neoclassical theory in NSTX,…

Zonal flow characteristics depend on $q$ values:

GAMs can exist only in high $q$ region.
In low $q$ region, Stationary Zonal Flows persists.

GAMs are less effective in reducing turbulence than Stationary ZFs, due to its high frequency
[Hahm, Beer, Lin et al., Phys. Plasmas ‘99]

Transport is consequently lower for lower $q$ value
[Miyato et al., IAEA ‘04, Angelino et al., PPCF ‘06]

Can Experimental Relevance be tested on DIII-D with BES,…?

on NSTX with poloidal CHERS, Reflectometry,…?

and on TS?

GTS simulation by W. Wang
Key Physics Mechanisms behind Size Scaling

• **Global Toroidal ITG eigenmode**
  [Horton-Choi-Tang, PF ‘82] [Cowley-Kulsrud-Sudan, PF B’ 91]
  [Romanelli-Zonca, PF B ’93][Parker-Lee-Santoro, PRL’93]
  **Bohm Scaling ?**

• **Self-regulation by Zonal Flows:**
  [Cast of Thousands][Review: Diamond et al., PPCF ‘05]
  **GyroBohm scaling ?**

• **Turbulence spreading**
  [Garbet et al., NF 94, PoP 08]
  [Hahm, Diamond, Lin, et al., PPCF ‘04, PoP’05]
  [Gurcan, Diamond, Hahm et al, PoP ’05, 06]
  [Naulin et al., PoP ’05] [Waltz et al., PoP ’05],…

Deviation from **GyroBohm ?**

[Lin, et al., PRL ‘02]

Density fluctuations from a GTS simulation of a shaped plasma with typical DIII-D core parameters
[Wang, Hahm, Lee et al., PoP ‘07]
Deviation from GyroBohm Scaling due to Turbulence Spreading

[Hahm,Diamond,Lin,Itoh,Itoh, PPCF ‘04]

- Garbet et al., ‘94 discussed spreading in the context of transient transport.
- Widely observed in many global gyrokinetic simulations [Sydora, Parker,Lin,…]
- Theoretical Research motivated by GK simulations reporting deviation from “gyroBohm” scaling for moderate system size

[Lin, Ethier, Hahm, Tang, PRL ‘02]

Range of fluctuation spreading into linearly stable zone: GK simulation: $\Delta \approx 25 \rho_i$

Nonlinear diffusion model: $\Delta \approx 18 \rho_i$
Turbulence Spreading from Edge to Core

[Hahm,Diamond, Lin, Rewoldt, Gurcan, Ethier, PoP ‘05]

• A model nonlinear diffusion equation

$$\frac{\partial}{\partial t} I = \gamma(x) I - \alpha I^2 + \chi_0 \frac{\partial}{\partial x} \left( I \frac{\partial}{\partial x} I \right)$$

• **Ballistic Front Propagation a la**
  Fisher-Kolmogorov [Gurcan, Diamond, PoP’06]

$$U_x = \gamma^{1/2} \times \left( \frac{\chi_0 I}{2} \right)^{1/2}$$

• From simulation, initial turbulence growth at the edge is followed by ballistic front propagation into core

• Front speed increases with \( R/L_T \)

--- Fast response of core after L-H transition [JET, JT-60U, DIII-D…]?

Turbulence Spreading through a Transport Barrier

- Often, transport is anomalous where pressure gradient is weak and linear instabilities are absent e.g., NSTX [Kaye et al., NF ‘07]

- Non-zero fluctuations and anomalous transport observed in linearly stable zone of JT-60U reversed shear plasma [Nazikian et al., PRL ‘05]

From GTS simulation [W. Wang et al.,] and simple analytic theory, study effects of E x B flow shear on turbulence spreading by placing E x B shear layer next to linearly unstable zone as a barrier
Turbulence spreading is reduced by E x B shear

- GTS nonlinear simulation of ITG turbulence exhibits significant turbulence spreading into the linearly stable zone
  [Wang et al., PoP 14, 072306 ’07]

With ExB shear, spreading extent is reduced as expected from nonlinear diffusion model
[Hahm, Diamond, Lin et al., PPCF 46, A323 ‘04]:

\[
\Delta / V_s \sim 1 / |\gamma'| \Delta \quad \omega_{\text{ExB}} \text{ increases } |\gamma'|
\]

\[
\Delta \sim (V_s / |\gamma'|)^{1/2}
\]

Look for \( \omega_{\text{ExB}} \) effects on \( V_s \)
Spreading Speed is controlled by Local Value of $\omega_{\text{ExB}}$

- Spatio-temporal evolution of front propagation:

$V_s$ increases with fluctuation intensity
[Hahm, Diamond, Lin et al., PPCF ‘04
Gurcan, Diamond, Hahm, PoP ‘05]

$\omega_{\text{ExB}}$ (not $E_r$) reduces fluctuation intensity locally

$\omega_{\text{ExB}}$ decreases $V_s$ locally

$\Delta \sim (V_s / |\gamma'|)^{1/2}$

Rough agreement in trend
Rotation Plays a Central Role in Magnetic Confinement

**Microturbulence**
- Generation via Reynolds stress
- $E_x B$ shear suppression

**Rotation**
- $E_r$ generation
- $E_r$ induced orbit distortion
- Stabilization of RWM,...
- Braking via non-axi-symm. fields
- Generation via Maxwell stress

**Energetic Particles**
- Excitation

**MHD Instabilities**
- MHD-induced transport