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Plasma-wall transition in magnetized plasmas: impact on wall sputtering and erosion

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Motivations

• **Magnetically confined fusion plasmas**
  – Limiters and divertors in tokamaks
  – Problems: erosion, sputtering, recycling

• **Diagnostics in plasmas**
  – Langmuir probes, retarding field analyzers (RFA), …
  – Plasma-probe interaction can lead to significant errors in the measurements

• **Plasma-assisted surface treatment**
  – Etching, thin-film deposition

• **Sound understanding of plasma-wall interactions is crucial to control the above effects**
Experimental conditions

“Mirabelle” linear device (University of Nancy, France)

✓ Argon plasma
✓ Pressure: $10^{-4}$ torr
✓ Density (ions, electrons): $5 \times 10^9$ cm$^{-3}$
✓ Ion temperature: $T_i \sim 0.04$ eV
✓ Electron temperature: $T_e \sim 1$ eV
✓ $\lambda_D \sim 0.3$ mm; $r_L \sim 6$ mm; $\lambda_{coll} \sim 9$ cm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e / T_i$</td>
<td>$1 \leftrightarrow 35$</td>
</tr>
<tr>
<td>$\omega_{ci} / \omega_{pi}$</td>
<td>$0.1 \leftrightarrow 0.01$</td>
</tr>
<tr>
<td>$\nu / \omega_{pi}$</td>
<td>$10^{-3} \leftrightarrow 10^{-5}$</td>
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</table>
Sheath formation - unmagnetized case

Debye sheath (DS):
- nonneutral region
- width $\sim \lambda_{De}$
- Bohm criterion: ions velocity at DS edge $> C_s$

Collisional presheath (CP):
- quasi-neutral area
- width $\sim \lambda_{coll}$: mean free path
- ion acceleration towards the wall
Debye sheath — Bohm’s criterion

- Criterion for the stability of the electrostatic Debye sheath (DS).
- **Q.** what is the minimum velocity at the entrance of the DS sheath?
- Simple fluid model:

\[
\begin{align*}
\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} &= 0 \\
\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} &= -\frac{e}{m_i} \frac{\partial \Phi}{\partial x}
\end{align*}
\]

\[
\frac{\partial^2 \Phi}{\partial x^2} = -\frac{e}{\varepsilon_0} (n_i - n_e) \\
n_e(x,t) = n_0 \exp \left( \frac{e\Phi}{k_B T_e} \right)
\]

- We look for a stationary solution: \( \partial/\partial t = 0 \)
- The ion Euler equation can be solved for \( n_i \)
- Substituting into Poisson’s equation and linearizing, one obtains

\[
\frac{d^2 \Phi}{dx^2} = \frac{1}{\lambda_D^2} \left( 1 - \frac{k_B T_e}{m_i u_0^2} \right) \Phi \quad \Rightarrow \quad u_0 > \sqrt{\frac{k_B T_e}{m_i} \equiv c_s}
\]
Collisional presheath

- Fluid equations for the ions with collision frequency $W$

$$\frac{d}{dx} (n_i u_i) = 0 \quad \quad \quad u_i \frac{d u_i}{dx} = -\frac{e}{m_i} \frac{d \Phi}{dx} - W u_i$$

- Assuming quasineutrality:

$$\ln \frac{n_e}{n_0} = \frac{e \Phi}{k_B T_e} = \ln \frac{n_i}{n_0}$$

- We obtain:

$$\left( u_i^2 - c_s^2 \right) \frac{du_i}{dx} = -W u_i^2$$

- **Singularity at** $u_i = c_s$, which defines the Debye sheath edge (DSE)
- At the DSE, the assumption of quasineutrality breaks down
  - Need to use full Poisson’s equation
Kinetic modelling I.

- Phase-space distribution function
  \[ f(\vec{r}, \vec{v}, t) \cdot d\vec{r}d\vec{v} = \text{Number of particles in the phase-space volume} \ d\vec{r}d\vec{v} \ 	ext{centered on} \ (\vec{r}, \vec{v}) \]

- Velocity moments of the distribution function

  \[ n(\vec{r}, t) = \int f(\vec{r}, \vec{v}, t) \cdot d\vec{v} \quad \text{Spatial density} \]

  \[ \bar{u}(\vec{r}, t) = \frac{1}{n} \int \vec{v} f(\vec{r}, \vec{v}, t) \cdot d\vec{v} \quad \text{Mean velocity} \]

  \[ nk_B T(\vec{r}, t) = m_i \int w^2 f(\vec{r}, \vec{v}, t) \cdot d\vec{v} \quad \text{Temperature} \]

\[ \vec{w} = \vec{v} - \bar{u} \]
Kinetic modelling II.

- **Vlasov equation for the ions**

\[ \frac{\partial f_i}{\partial t} + \bar{v} \cdot \frac{\partial f_i}{\partial \bar{r}} + \frac{e}{m_i} \left( \vec{E} + \bar{v} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \bar{v}} = \left( \frac{\partial f_i}{\partial t} \right)_{\text{coll}} = -v (f_i - f_0) \]

- **Conservation equation in the phase space**
- **Collisions ⇒ relaxation to Maxwellian \( f_0(\nu) \)**
- **\( \nu^{-1} : \text{typical relaxation time} \)**

1D space (x coordinate : normal to the wall) + 3D velocity

- **Boltzmann equilibrium for the electrons**

\[ n_e = n_0 \exp \left( \frac{e\phi}{k_B T_e} \right) \quad \Leftrightarrow \text{the electrons thermalize much faster than the ions} \]

- **Poisson’s equation**

\[ \frac{\partial^2 \phi}{\partial x^2} = -\frac{e}{e_0} \left( \int f_i \, dv - n_e(\phi) \right) \]

Self-consistent system closed by Poisson’s equation ⇒ nonlinearity
Kinetic modelling III.

- **Numerical method**: Vlasov Eulerian code
  - Meshing of the full phase space
  - Low level of numerical noise

- **Strategy**
  - Initialize homogeneous Maxwellian distribution for ions: \( f_i = f_M(v) \)
  - Let it evolve self-consistently until stationary state appears
    - Check that spatial profiles do not vary anymore
  - Corollary: no need for very accurate time-stepping technique

- **Disparate spatial scales**
  - \( \lambda_{De} \ll \lambda_{coll} \)
  - Use inhomogeneous grid:
    \[
    dx = g(s)ds,
    \]
    \[
    g(s) = \Delta x_1 + \frac{\Delta x_2 - \Delta x_1}{2} \{1 + \tanh[c(s - s_0)]\},
    \]
  - \( \Delta x_2 \approx \lambda_{coll} \)
  - \( \Delta x_1 \approx \lambda_{De} \)
Unmagnetized plasma-wall transition

Typical case: $T_e/T_i = 25$; $\nu / \omega_{pi} = 10^{-4}$

$\lambda_{coll} \approx 10^3 \lambda_{De}$

Fluid Bohm's criterion:

$$\langle v \rangle |_{x_0} \geq \left( \frac{k_B T_e}{m_i} \right)^{1/2}$$

Kinetic Bohm's criterion:

$$\langle v^{-2} \rangle |_{x_0} = \int \frac{1}{v^2} f dv \leq \frac{m_i}{k_B T_e}$$
Ion temperature profile

Competition between two effects:

- Electric field: acceleration towards the wall
- Collisions: back to distribution $f_0$
Comparison to experimental results

- Series of temperature measurements in the presheath
  *Oksuz and Hershkowitz, Plasma Sources Sci. Technol. 14, 201 (2003)*
- Experimental conditions: $T_e/T_i = 25$; $\phi_{wall} = -30V$; $v/\omega_{pi} = 10^{-4}$
Sheath formation in a magnetized plasma

**Ordering:**

\[ \lambda_{De} << r_L << \lambda_{coll} \]

- Debye length
- Ion Larmor radius
- Ion–neutral mean free path

**Magnetic presheath (MP):**
- quasi-neutral
- width \( \sim r_L \)
- ion redirection toward the wall

**Collisional presheath (CP):**
- quasi-neutral
- width \( \sim \lambda_{coll} \)
- ion acceleration along magnetic lines
Magnetized plasma-wall transition: phase-space

\[ \alpha = 40^\circ \quad \omega_{ci}/\omega_{pi} = 0.01 \quad T_e/T_i = 10 \quad v/\omega_{pi} = 10^{-3} \]

\[ x = 12000 \lambda_{Di} \quad \lambda_{Di} \]

\[ x = 631 \lambda_{Di} \]

\[ x = 196 \lambda_{Di} \]

Wall

\[ V_x \quad V_y \quad V_z \]

\[ V_x(\theta_{thi}) \]

Plasma

\[ V_x \quad V_y \quad V_z \]

\[ V_x(\theta_{thi}) \]

Debye sheath \[ \sim \lambda_D \]

Magnetic presheath \[ \sim r_L \]

Collisionnal presheath \[ \sim \lambda_{coll} \]
Magnetized Bohm's criterion

\[ \langle V_x \rangle > \sqrt{\frac{k_B T_e}{m_i}} \equiv C_s \]

at DS edge

Bohm’s criterion **not** satisfied for:
- Large magnetic fields
- Grazing incidence (\( \alpha \) small)
Magnetic presheath (MP) edge: Chodura’s criterion

\[ \langle v_{\parallel} \rangle > \sqrt{\frac{k_B T_e}{m_i}} \equiv c_s \]

at MP edge

- Chodura’s criterion **not** satisfied for:
  - Weak magnetic fields \( (\omega << \omega_{\text{pi}}) \)
  - Large collision rate \( (v >> \omega_{\text{pi}}) \)
- Not very reliable to estimate the MP width
Magnetic presheath width I.

- Magnetic presheath width $\propto$ ion Larmor radius $\propto 1/B$
- Criterion for the magnetic presheath edge
  - MP edge: ions start being collected at the wall
  - Therefore, velocity perpendicular to B becomes nonnegligible

MP width decreases with increasing B
Magnetic presheath width II.

Theoretical estimate: \( \lambda_{MP}^{th} \approx \sqrt{6 \cos \alpha} \frac{\tau^{1/2}}{\omega} \lambda_{Di} \).

\[ \omega = \omega_{ci} / \omega_{pi} \]

Good agreement between numerical, theoretical, and exp. results

\[ \tau = \frac{T_e}{T_i} \]
Magnetic presheath: temperature profiles

- Temperature increases near the wall
- Same mechanism as in unmagnetized plasma

\[
\begin{align*}
V_{//} \\
V_x \\
V_z
\end{align*}
\]
The E X B drift is directed along the z direction

\[ v_E = \frac{E \times B}{B^2} = \frac{E_x(x) \cos \alpha \hat{z}}{B}. \]

- \( V_z \) and \( V_E \) coincide in the collisional presheath, but start diverging in the magnetic presheath
- Guiding-center approach invalid in the MP and DS

Profile of the velocities \( V_z \) and \( V_E \)
Wall sputtering and erosion

Sputtering yield $Y$ depends on:
- Angle of incidence on the wall: $\theta$
- Kinetic energy: $E_{\text{kin}}$

$$Y(\alpha, \omega) \propto \int_0^\infty \int_0^{\pi/2} \frac{E_{\text{kin}}}{\sin \theta} F(\theta, E_{\text{kin}}) d\theta \, dE_{\text{kin}}$$

$\alpha = \text{angle of incidence of the magnetic field}$
$F(\theta, E_{\text{kin}}) = \text{distribution function in angle/energy variables}$
$\omega = \omega_{ci} / \omega_{pi}$
Phase-space distribution at the wall

\[ \mathbf{V} = \mathbf{V}_{\perp} + \mathbf{V}_{\parallel} \]

\[ \mathbf{V}_{\perp} = \mathbf{V}_{x} \perp \mathbf{B} \]

\[ \mathbf{V}_{\parallel} = \mathbf{V}_{y} + \mathbf{V}_{z} \]

\[ \alpha \]

\[ \mathbf{B} \]
**Results:** angle of incidence and kinetic energy on the wall

Average angle of incidence on the wall, $<\theta>$

$$\omega = \omega_{ci} / \omega_{pi}$$

Average kinetic energy on the wall, $<E_{\text{kin}}>$
Results: angle of incidence and kinetic energy on the wall

\[ \theta > \alpha ! \]
Sputtering yield on the wall

\[ Y(\alpha, \omega) \propto \int_0^\infty \int_0^{\pi/2} \frac{E_{\text{kin}}}{\sin \theta} F(\theta, E_{\text{kin}}) \, d\theta \, dE_{\text{kin}} \]

\[ \omega = \omega_{\text{ci}} / \omega_{\text{pi}} \]

\( \alpha = \text{angle of incidence of magnetic field} \)
Heat and particles fluxes on the wall

- For large enough magnetic field, the fluxes follow a ‘sin $\alpha$’ law
**Conclusions**

- Kinetic model for ion population – Vlasov code
- Full description of the magnetized plasma-wall transition
  - Collisisonal presheath, magnetic presheath, Debye sheath
- Experimental validation
- Applications: computed distributions allow calculation of:
  - Energy and angle of incidence of ions on wall
  - Sputtering yield
  - Particles and heat fluxes on wall
- Perspectives
  - Dynamical formation of sheaths
  - Two-species plasmas (DT, impurities, …)

\[\begin{align*}
&\text{S. Devaux, G. Manfredi, Phys. Plasmas 13, 083504 (2006).} \\
\end{align*}\]