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Plasma-wall transition in magnetized plasmas:  
impact on wall sputtering and erosion

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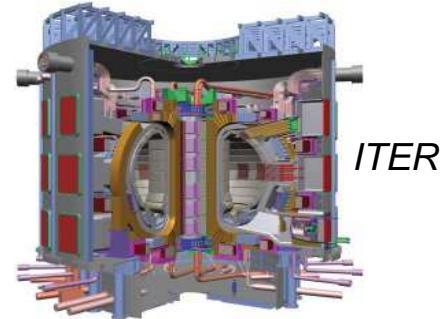
# Plasma-wall transition in magnetized plasmas: impact on wall sputtering and erosion

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# Motivations

- **Magnetically confined fusion plasmas**
  - Limiters and divertors in tokamaks
  - Problems: erosion, sputtering, recycling
- **Diagnostics in plasmas**
  - Langmuir probes, retarding field analyzers (RFA), ...
  - Plasma-probe interaction can lead to significant errors in the measurements
- **Plasma-assisted surface treatment**
  - Etching, thin-film deposition
- **Sound understanding of plasma-wall interactions is crucial to control the above effects**



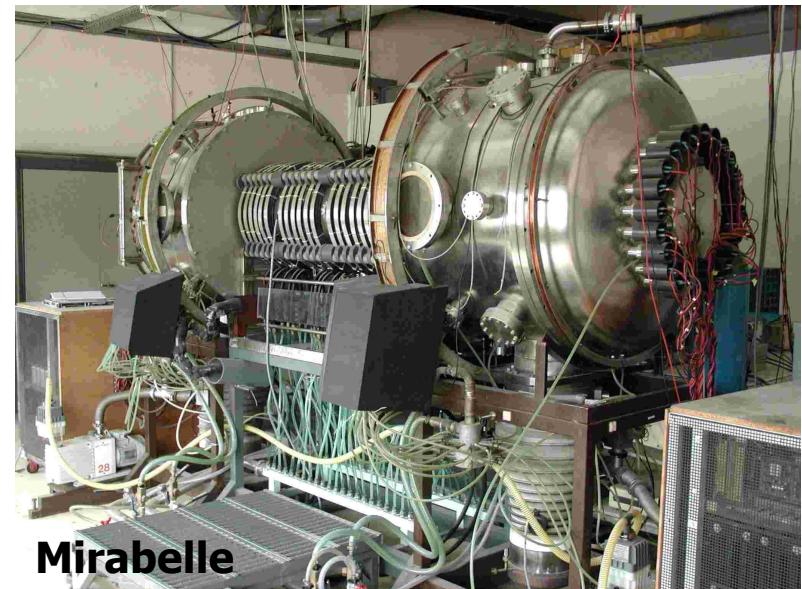
*Langmuir probe*

# Experimental conditions

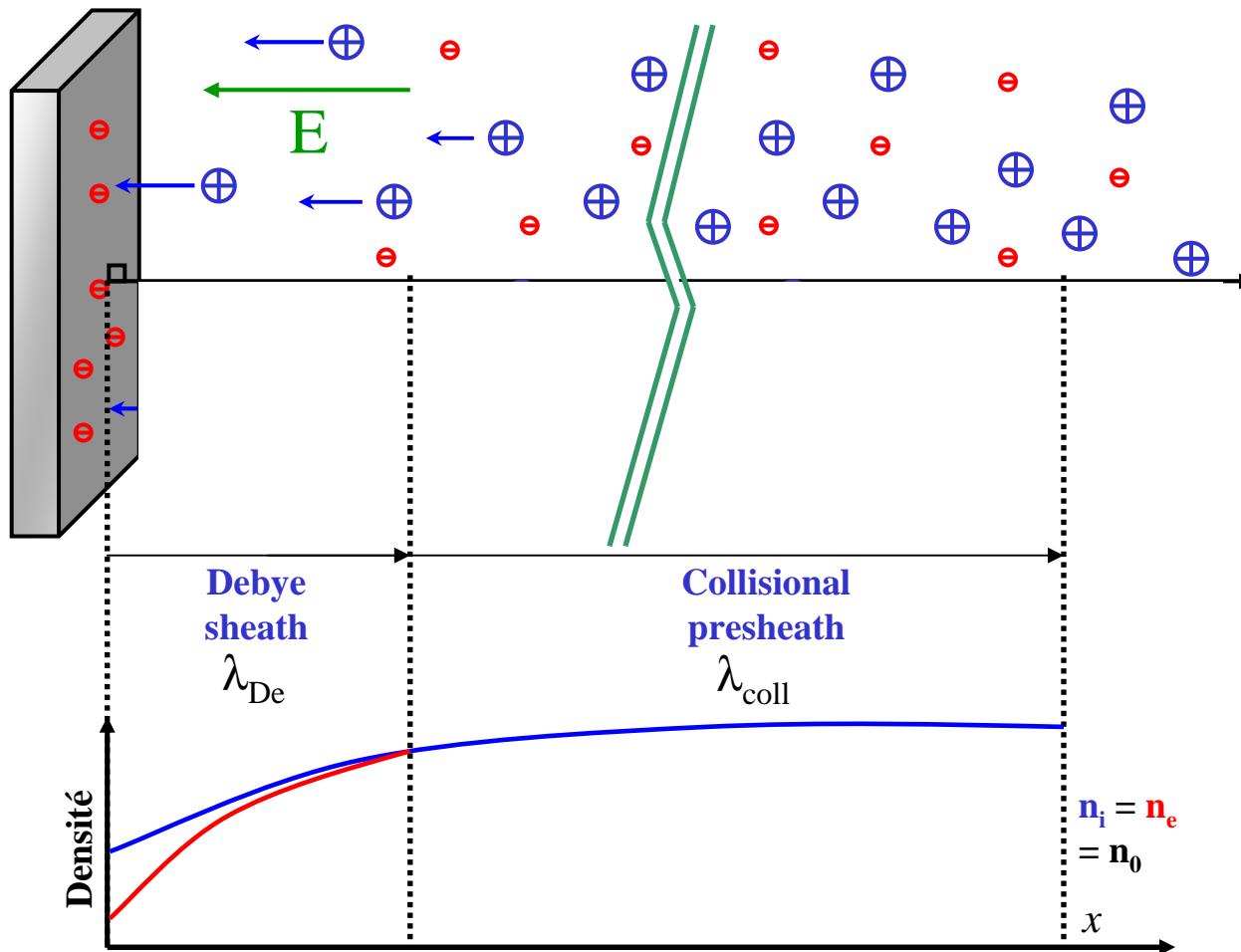
## “Mirabelle” linear device (University of Nancy, France)

- ✓ Argon plasma
- ✓ Pressure :  $10^{-4}$  torr
- ✓ Density (ions, electrons) :  $5 \times 10^9 \text{ cm}^{-3}$
- ✓ Ion temperature:  $T_i \sim 0.04 \text{ eV}$
- ✓ Electron temperature:  $T_e \sim 1 \text{ eV}$
- ✓  $\lambda_{De} \sim 0.3 \text{ mm}$  ;  $r_L \sim 6\text{mm}$  ;  $\lambda_{coll} \sim 9\text{cm}$

Parameter	Value
$T_e / T_i$	$1 \leftrightarrow 35$
$\omega_{ci} / \omega_{pi}$	$0.1 \leftrightarrow 0.01$
$v / \omega_{pi}$	$10^{-3} \leftrightarrow 10^{-5}$



## Sheath formation - unmagnetized case



### Debye sheath (DS):

- nonneutral region
- width  $\sim \lambda_{De}$
- Bohm criterion: ions velocity at DS edge  $> C_s$

### Collisional presheath (CP):

- quasi-neutral area
- width  $\sim \lambda_{coll}$ : mean free path
- ion acceleration towards the wall

## Debye sheath – Bohm's criterion

- Criterion for the stability of the electrostatic Debye sheath (DS).
- **Q. : what is the minimum velocity at the entrance of the DS sheath?**
- Simple fluid model:

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0 \quad \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = - \frac{e}{m_i} \frac{\partial \Phi}{\partial x}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = - \frac{e}{\epsilon_0} (n_i - n_e) \quad n_e(x,t) = n_0 \exp\left(\frac{e\Phi}{k_B T_e}\right)$$

- We look for a stationary solution:  $\partial/\partial t = 0$
- The ion Euler equation can be solved for  $n_i$
- Substituting into Poisson's equation and linearizing , one obtains

$$\frac{n_i}{n_0} = \left(1 - \frac{2e\Phi}{m_i u_0^2}\right)^{-1/2}$$

$$\frac{d^2 \Phi}{dx^2} = \frac{1}{\lambda_D^2} \left(1 - \frac{k_B T_e}{m_i u_0^2}\right) \Phi \quad \Rightarrow$$

$$u_0 > \sqrt{\frac{k_B T_e}{m_i}} \equiv c_s$$

## Collisional presheath

- Fluid equations for the ions with collision frequency  $W$

$$\frac{d}{dx}(n_i u_i) = 0 \quad u_i \frac{d u_i}{dx} = -\frac{e}{m_i} \frac{d \Phi}{dx} - W u_i$$

- Assuming quasineutrality:  $\ln \frac{n_e}{n_0} = \frac{e\Phi}{k_B T_e} = \ln \frac{n_i}{n_0}$

- We obtain:

$$(u_i^2 - c_s^2) \frac{du_i}{dx} = -W u_i^2$$

- **Singularity at  $u_i = c_s$ ,** which defines the Debye sheath edge (DSE)
- At the DSE, the assumption of quasineutrality breaks down
  - Need to use full Poisson's equation

# Kinetic modelling I.

- ❖ Phase-space distribution function

$f(\vec{r}, \vec{v}, t).d\vec{r}d\vec{v}$  = Number of particles in the phase-space volume  $d\vec{r}d\vec{v}$  centered on  $(\vec{r}, \vec{v})$

- ❖ Velocity moments of the distribution function

$$n(\vec{r}, t) = \int f(\vec{r}, \vec{v}, t).d\vec{v}$$

**Spatial density**

$$\vec{u}(\vec{r}, t) = \frac{1}{n} \int \vec{v} f(\vec{r}, \vec{v}, t).d\vec{v}$$

**Mean velocity**

$$nk_B T(\vec{r}, t) = m_i \int w^2 f(\vec{r}, \vec{v}, t).d\vec{v}$$

**Temperature**

$$\vec{w} = \vec{v} - \vec{u}$$

# Kinetic modelling II.

## ❖ Vlasov equation for the ions

$$\underbrace{\frac{\partial f_i}{\partial t} + \vec{v} \cdot \frac{\partial f_i}{\partial \vec{r}} + \frac{e}{m_i} (\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_i}{\partial \vec{v}}}_{\text{Conservation equation in the phase space}} = \underbrace{\left( \frac{\partial f_i}{\partial t} \right)_{coll}}_{\text{Collisions} \Rightarrow \text{relaxation to Maxwellian } f_0(v)} = -\nu (f_i - f_0)$$

Conservation equation  
in the phase space

Collisions  $\Rightarrow$  relaxation to Maxwellian  $f_0(v)$   
 $v^{-1}$  : typical relaxation time

**1D space** ( $x$  coordinate : normal to the wall) + **3D velocity**

## ❖ Boltzmann equilibrium for the electrons

$$n_e = n_0 \exp \left( \frac{e\phi}{k_B T_e} \right) \Leftrightarrow \text{the electrons thermalize much faster than the ions}$$

## ❖ Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = -\frac{e}{\epsilon_0} \left( \int f_i d\vec{v} - n_e(\phi) \right)$$

Self-consistent system closed by  
Poisson's equation  $\Rightarrow$  nonlinearity

# Kinetic modelling III.

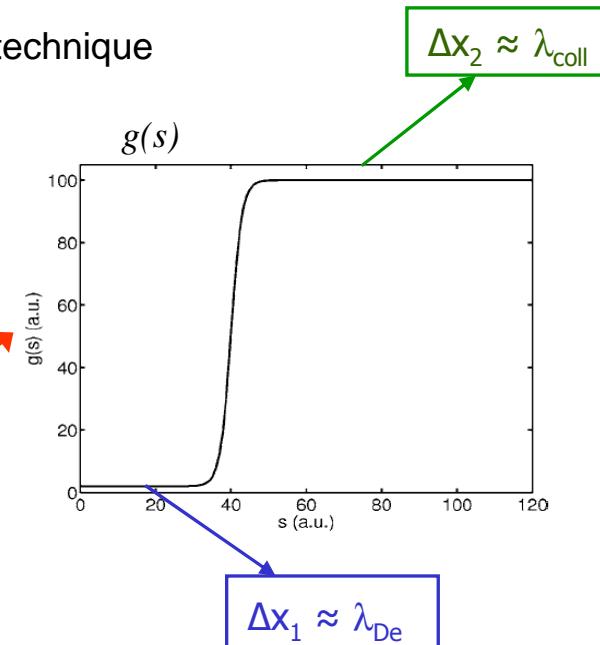
- **Numerical method: Vlasov Eulerian code**
  - Meshing of the full phase space
  - Low level of numerical noise
- **Strategy**
  - Initialize homogeneous Maxwellian distribution for ions:  $f_i = f_M(v)$
  - Let it evolve self-consistently until stationary state appears
    - Check that spatial profiles do not vary anymore
  - Corollary : no need for very accurate time-stepping technique

- **Disparate spatial scales**

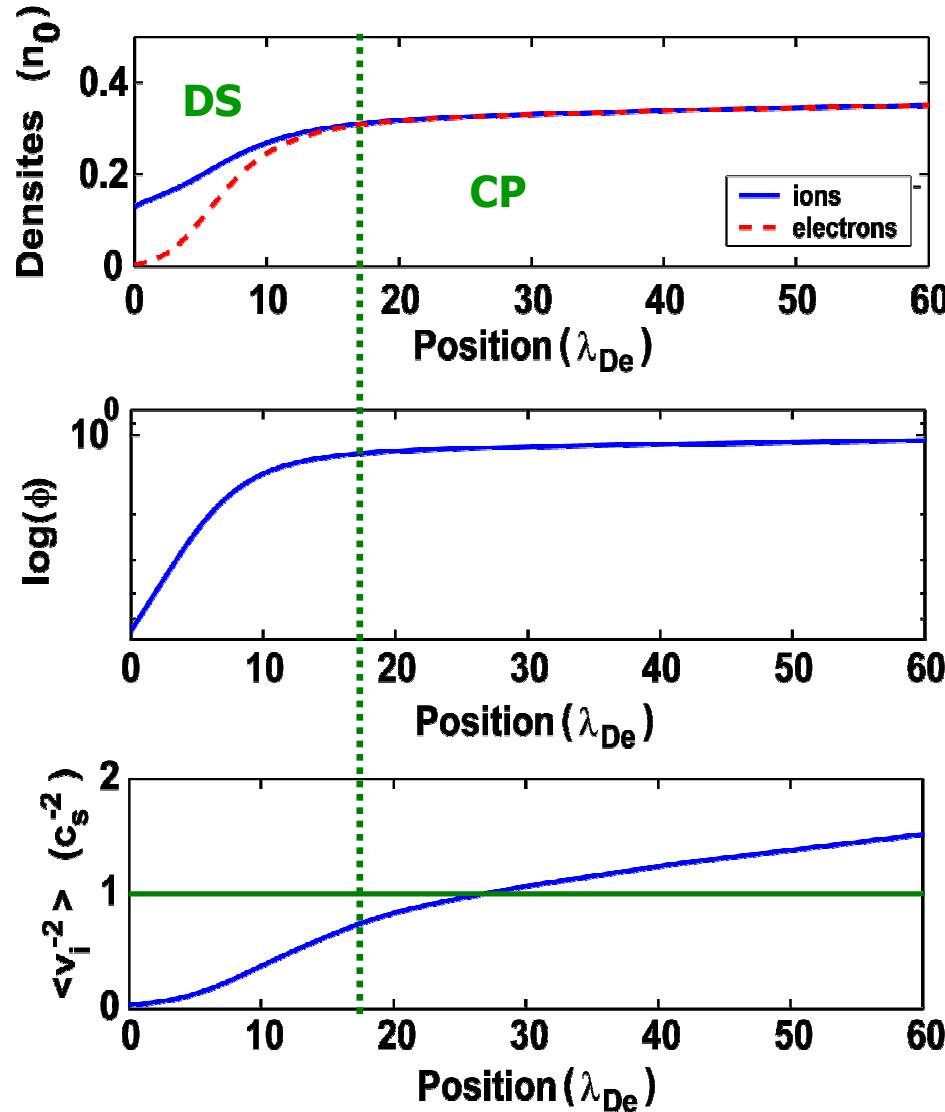
- $\lambda_{De} \ll \lambda_{coll}$
- Use inhomogeneous grid:

$$dx = g(s)ds,$$

$$g(s) = \Delta x_1 + \frac{\Delta x_2 - \Delta x_1}{2} \{1 + \tanh[c(s - s_0)]\},$$



# Unmagnetized plasma-wall transition



Typical case :  $T_e/T_i = 25$  ;  $v / \omega_{pi} = 10^{-4}$

$$\lambda_{coll} \approx 10^3 \lambda_{De}$$

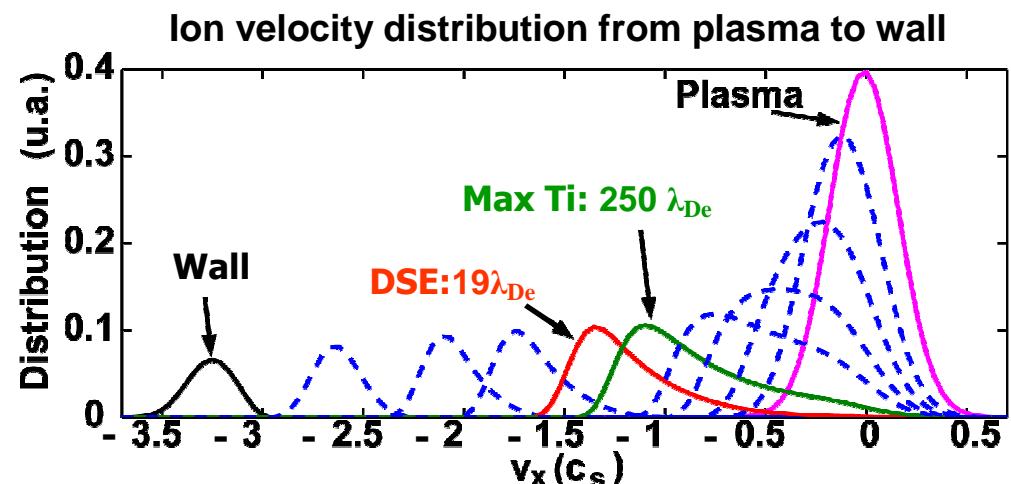
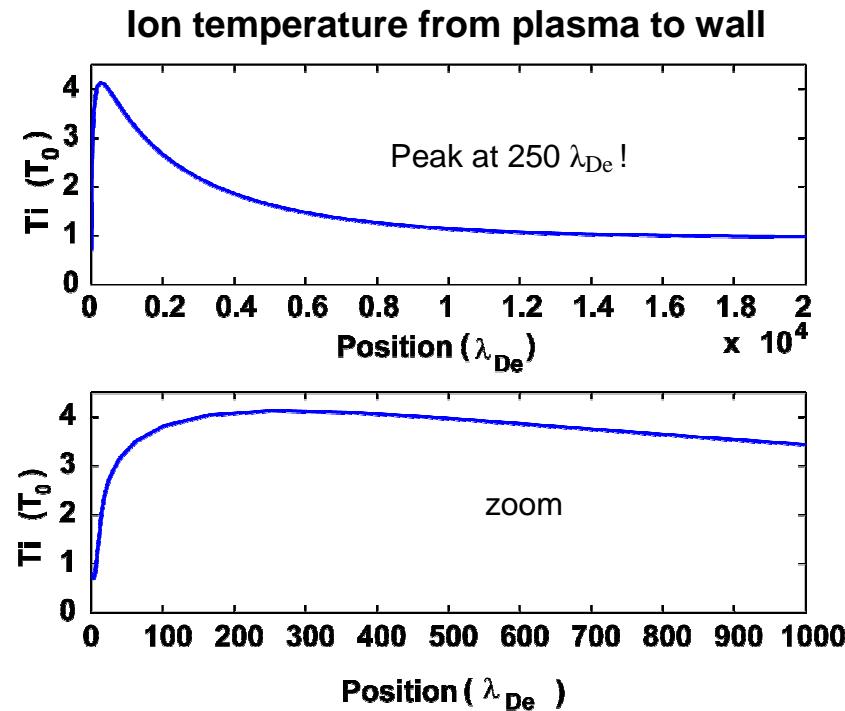
Fluid Bohm's criterion:

$$\langle v \rangle |_{x_0} \geq \left( \frac{k_B T_e}{m_i} \right)^{1/2}$$

Kinetic Bohm's criterion:

$$\langle v^{-2} \rangle |_{x_0} = \int \frac{1}{v^2} f \cdot dv \leq \frac{m_i}{k_B T_e}$$

# Ion temperature profile

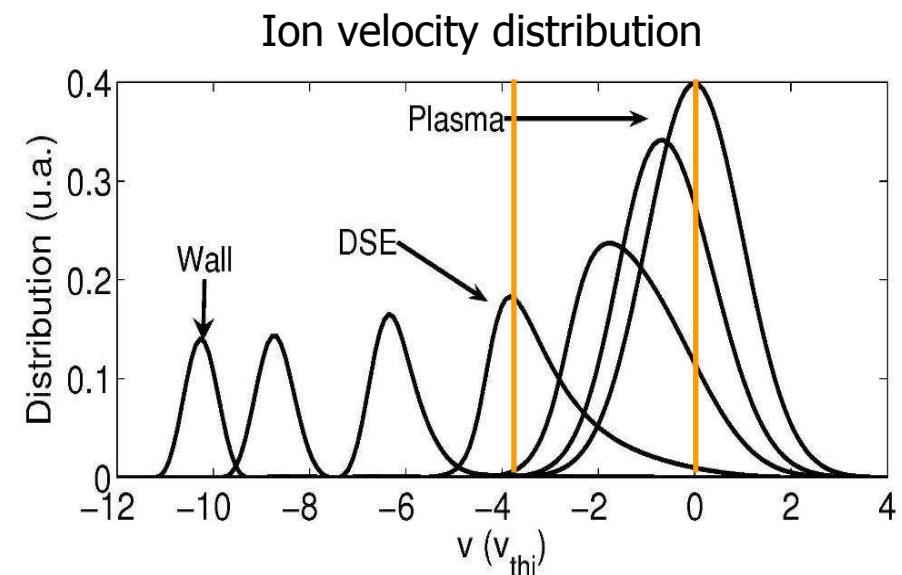
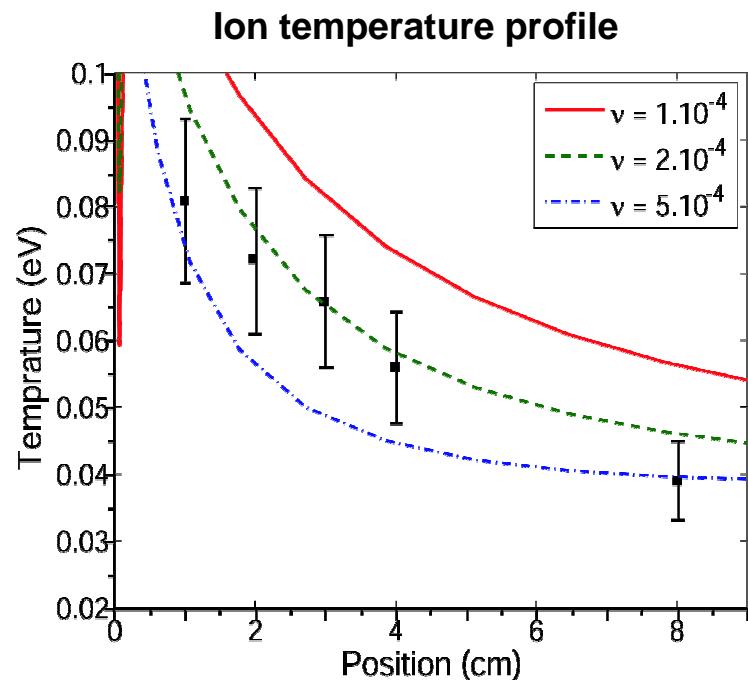


Competition between two effects:

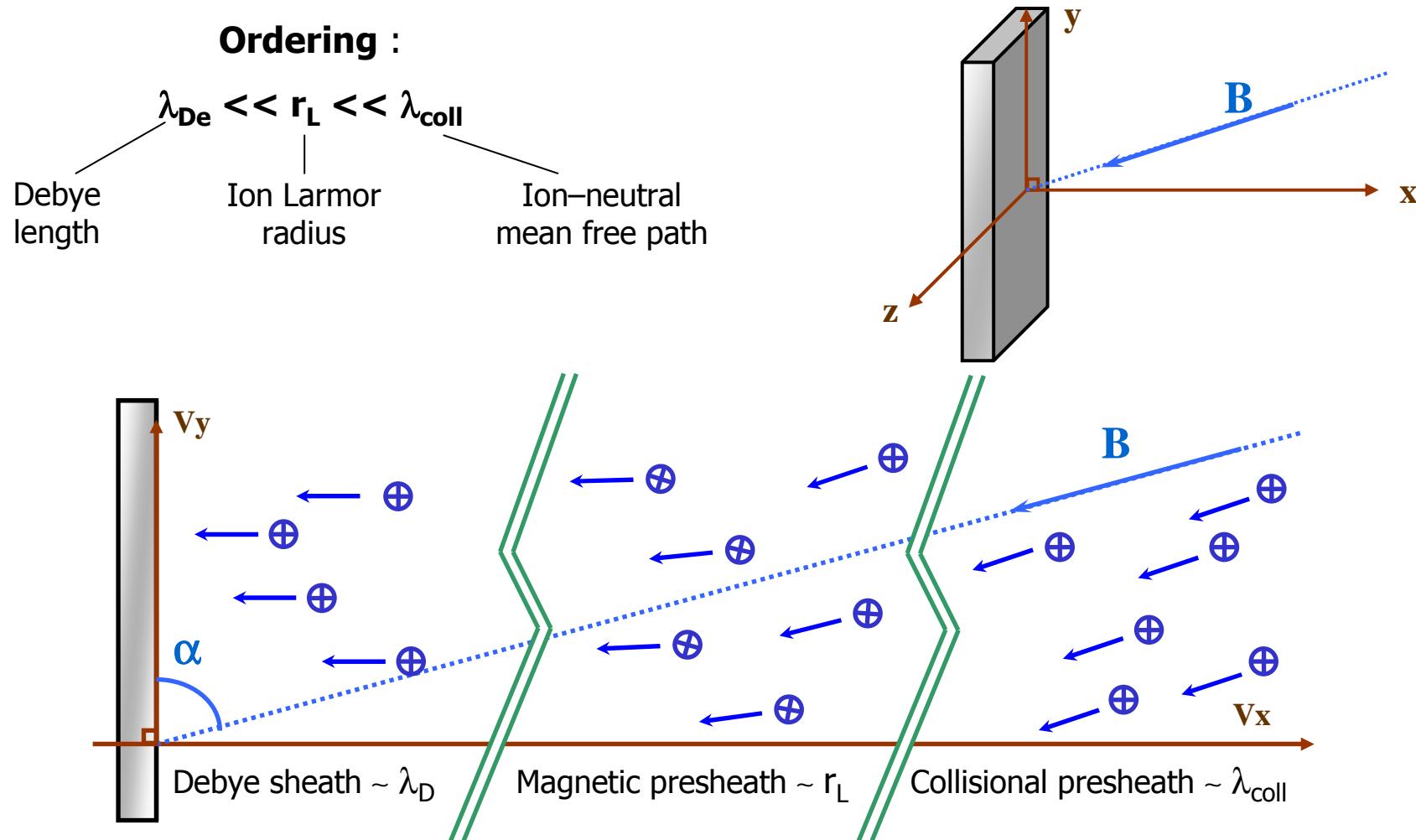
- Electric field : acceleration towards the wall
- Collisions : back to distribution  $f_0$

## Comparison to experimental results

- Series of temperature measurements in the presheath  
*Oksuz and Hershkowitz, Plasma Sources Sci. Technol. **14**, 201 (2003)*
- Experimental conditions:  $T_e/T_i = 25$  ;  $\phi_{\text{wall}} = -30V$  ;  $v / \omega_{pi} = 10^{-4}$



# Sheath formation in a magnetized plasma



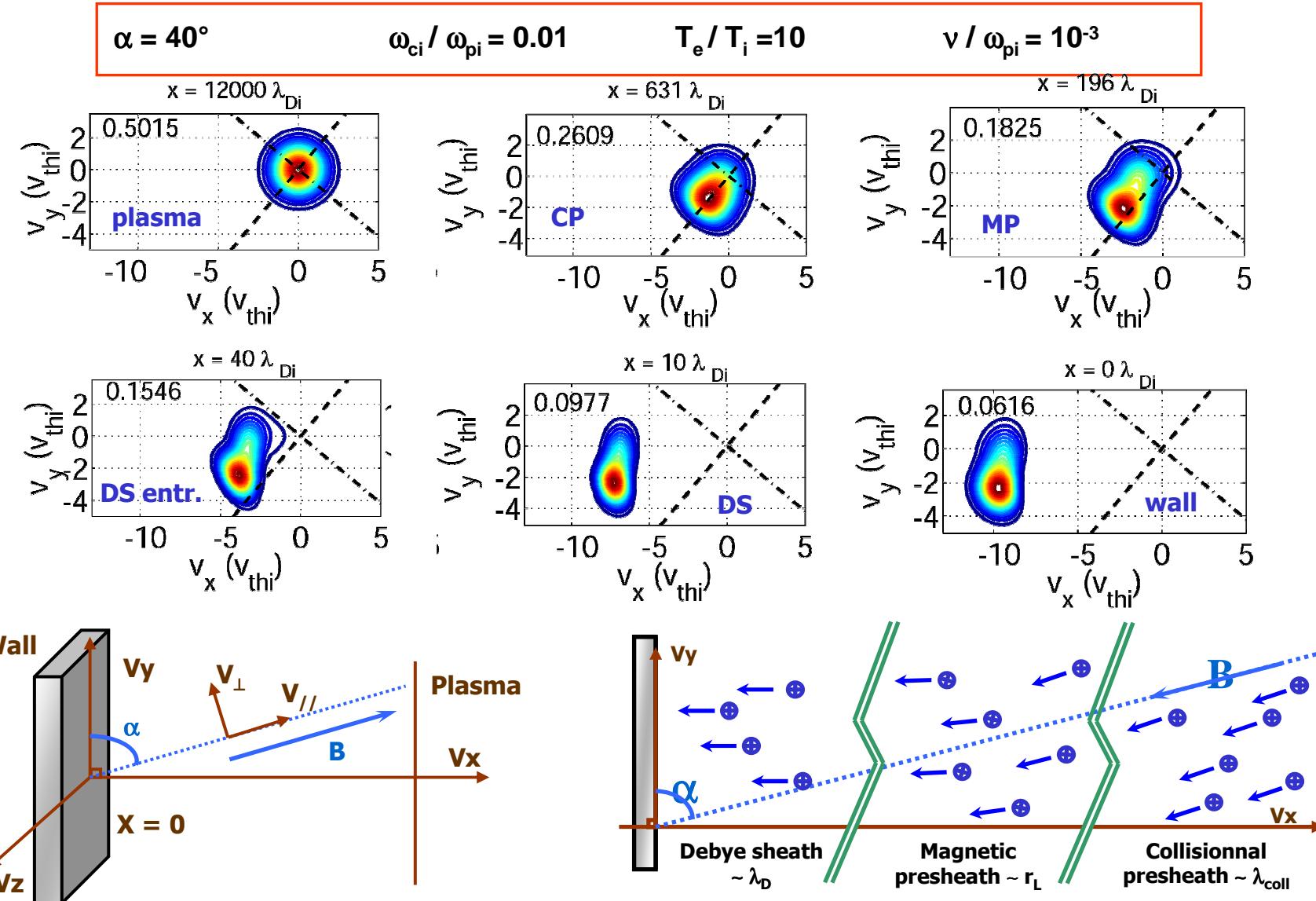
## Magnetic presheath (MP):

- quasi-neutral
- width  $\sim r_L$
- ion redirection toward the wall

## Collisional presheath (CP):

- quasi-neutral
- width  $\sim \lambda_{coll}$
- ion acceleration along magnetic lines

# Magnetized plasma-wall transition: phase-space

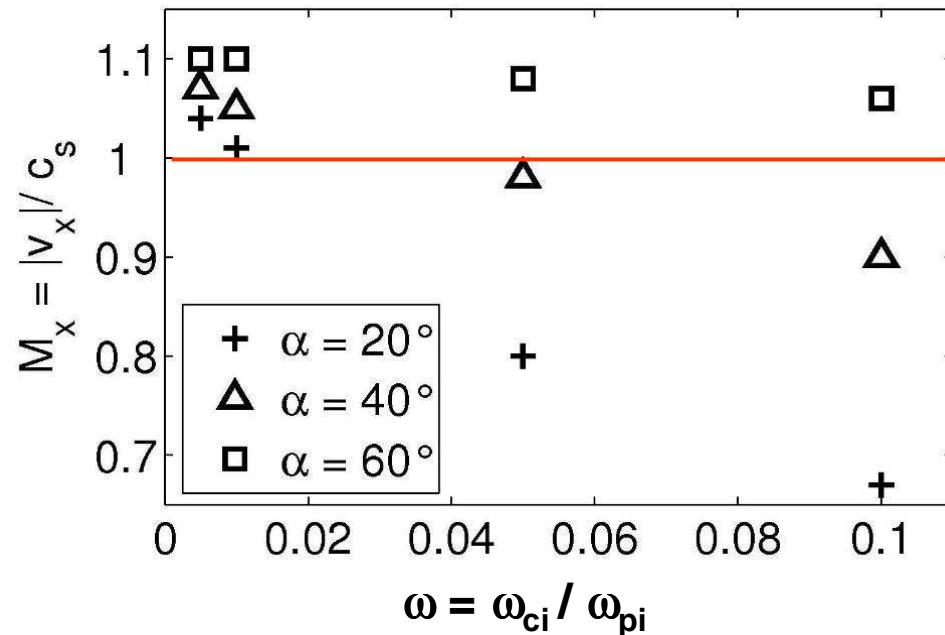


## Magnetized Bohm's criterion

$$\langle v_x \rangle > \sqrt{\frac{k_B T_e}{m_i}} \equiv c_s$$

at DS edge

### Mach number Debye sheath edge



Bohm's criterion not satisfied for:

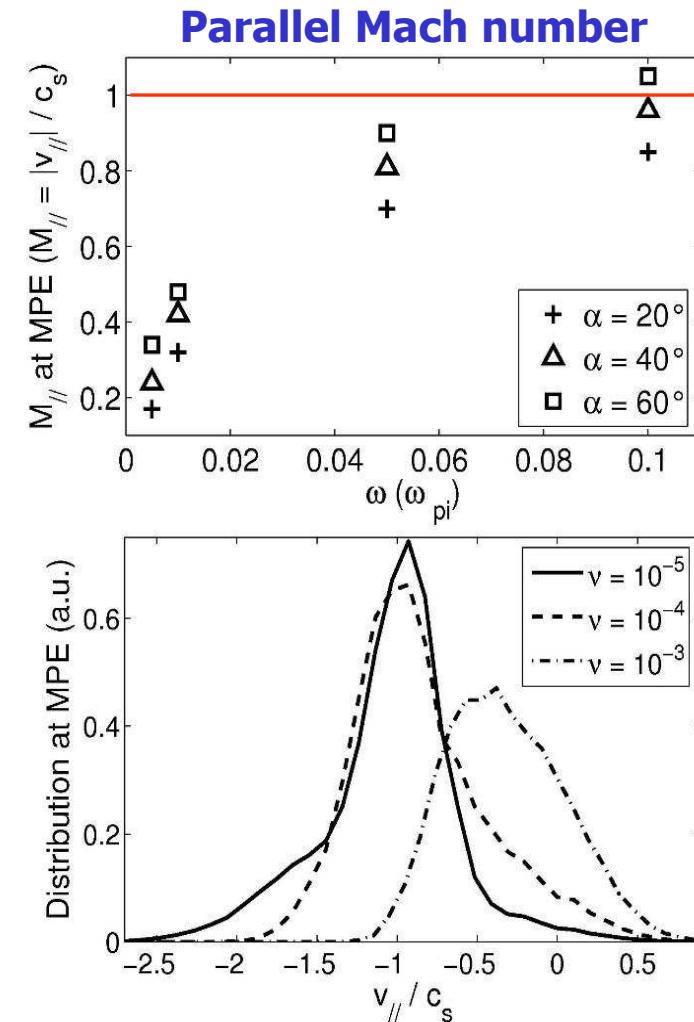
- Large magnetic fields
- Grazing incidence ( $\alpha$  small)

## Magnetic presheath (MP) edge: Chodura's criterion

$$\langle v_{\parallel} \rangle > \sqrt{\frac{k_B T_e}{m_i}} \equiv c_s$$

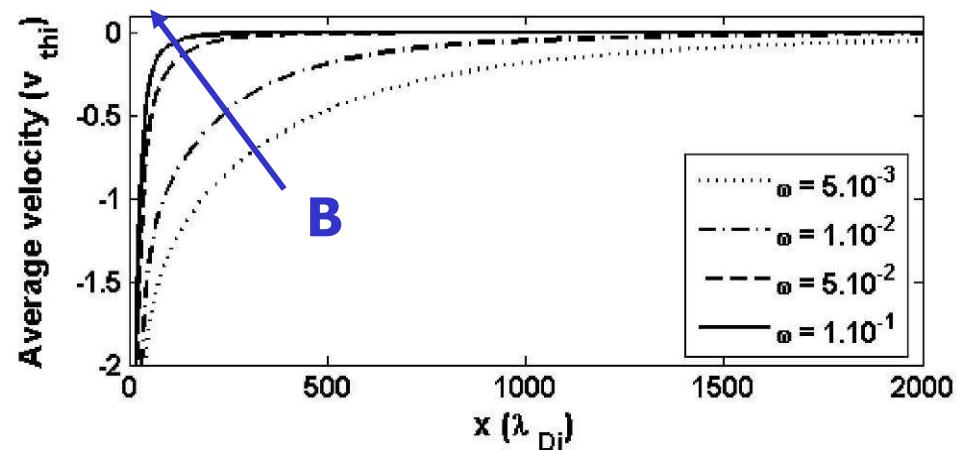
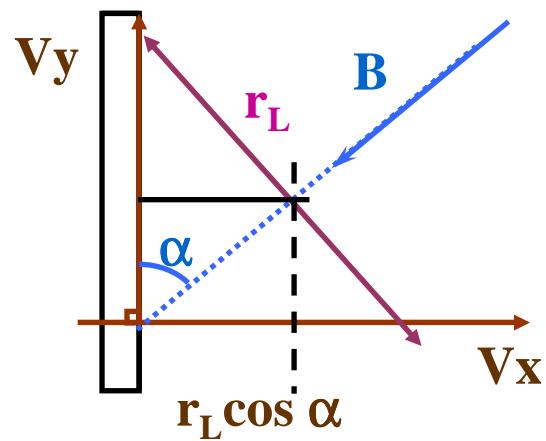
at MP edge

- Chodura's criterion not satisfied for:
  - Weak magnetic fields ( $\omega \ll \omega_{pi}$ )
  - Large collision rate ( $v >> \omega_{pi}$ )
- Not very reliable to estimate the MP width



## Magnetic presheath width I.

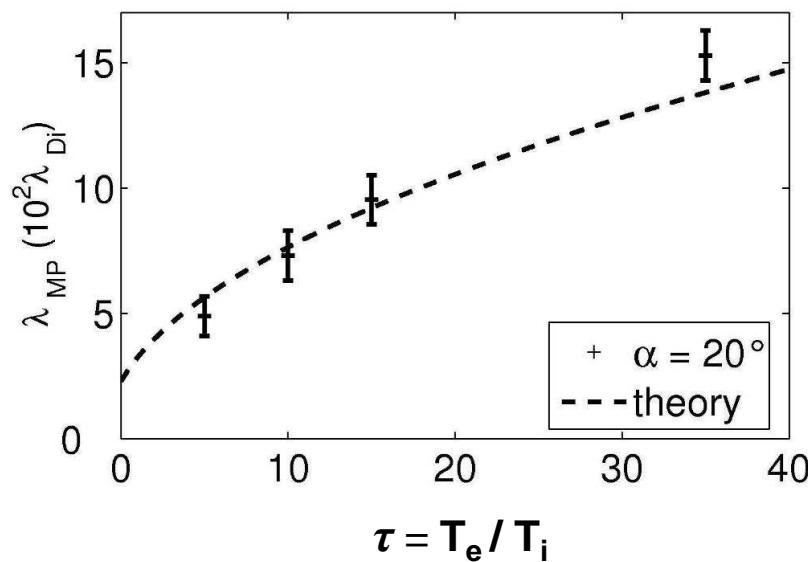
- Magnetic presheath width  $\propto$  ion Larmor radius  $\propto 1/B$
- Criterion for the magnetic presheath edge
  - MP edge: ions start being collected at the wall
  - Therefore, velocity perpendicular to B becomes nonnegligible



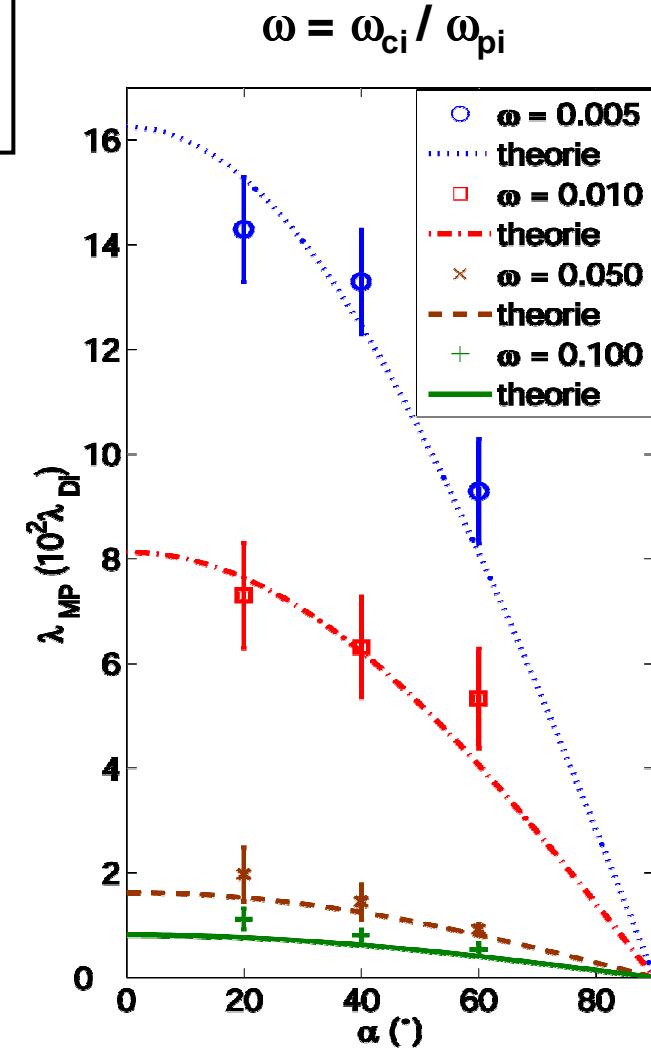
MP width decreases with increasing  $B$

## Magnetic presheath width II.

Theoretical estimate:  $\lambda_{MP}^{\text{th}} \approx \sqrt{6} \cos \alpha \frac{\tau^{1/2}}{\omega} \lambda_{Di}$

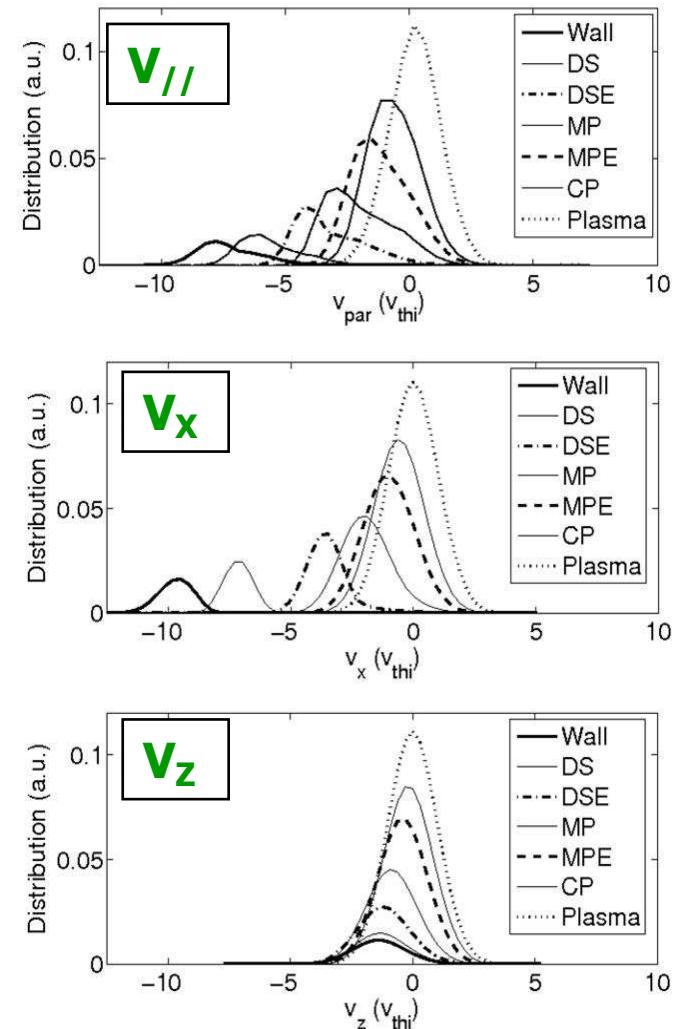
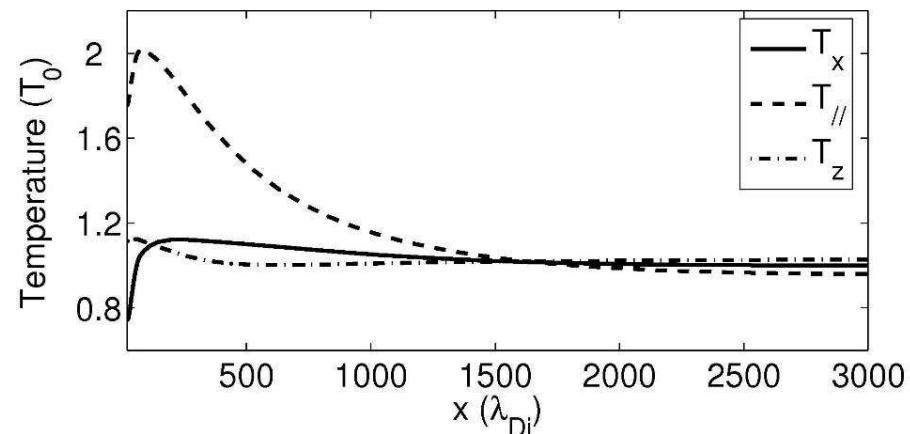


Good agreement between numerical,  
theoretical, and exp. results



## Magnetic presheath: temperature profiles

- Temperature increases near the wall
- Same mechanism as in unmagnetized plasma

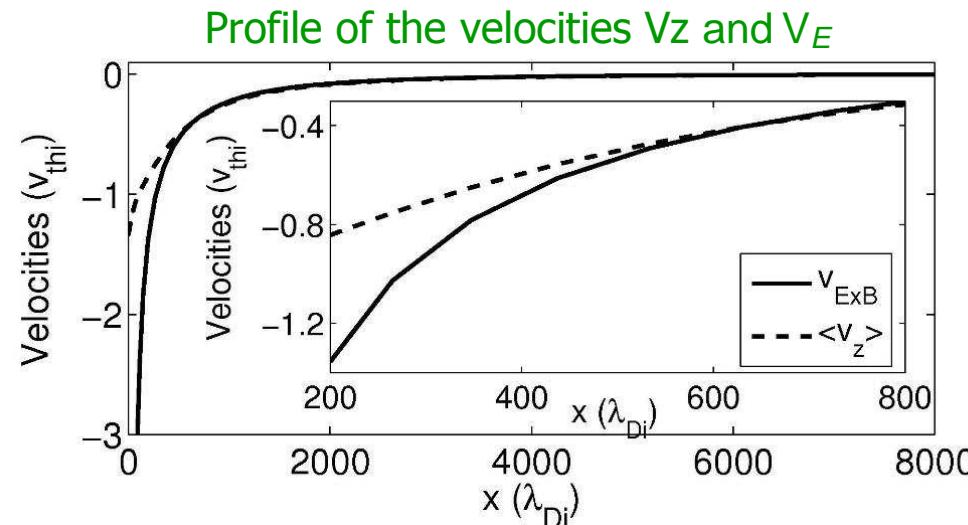
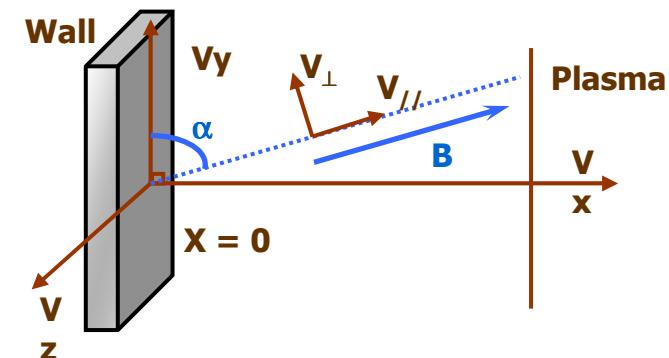


# E X B drift

- The E X B drift is directed along the z direction

$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{E_x(x) \cos \alpha}{B} \hat{z}.$$

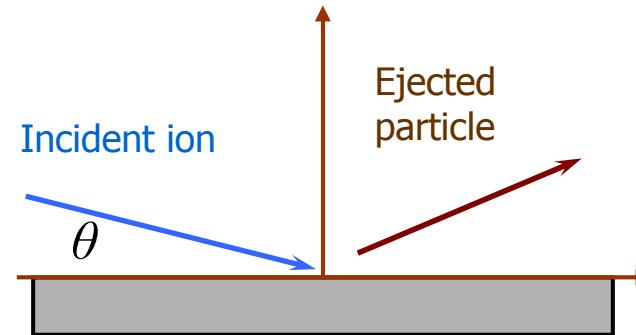
- $V_z$  and  $V_E$  coincide in the collisional presheath, but start diverging in the magnetic presheath
- Guiding-center approach invalid in the MP and DS



## Wall sputtering and erosion

Sputtering yield  $Y$  depends on :

- Angle of incidence on the wall:  $\theta$
- Kinetic energy:  $E_{\text{kin}}$



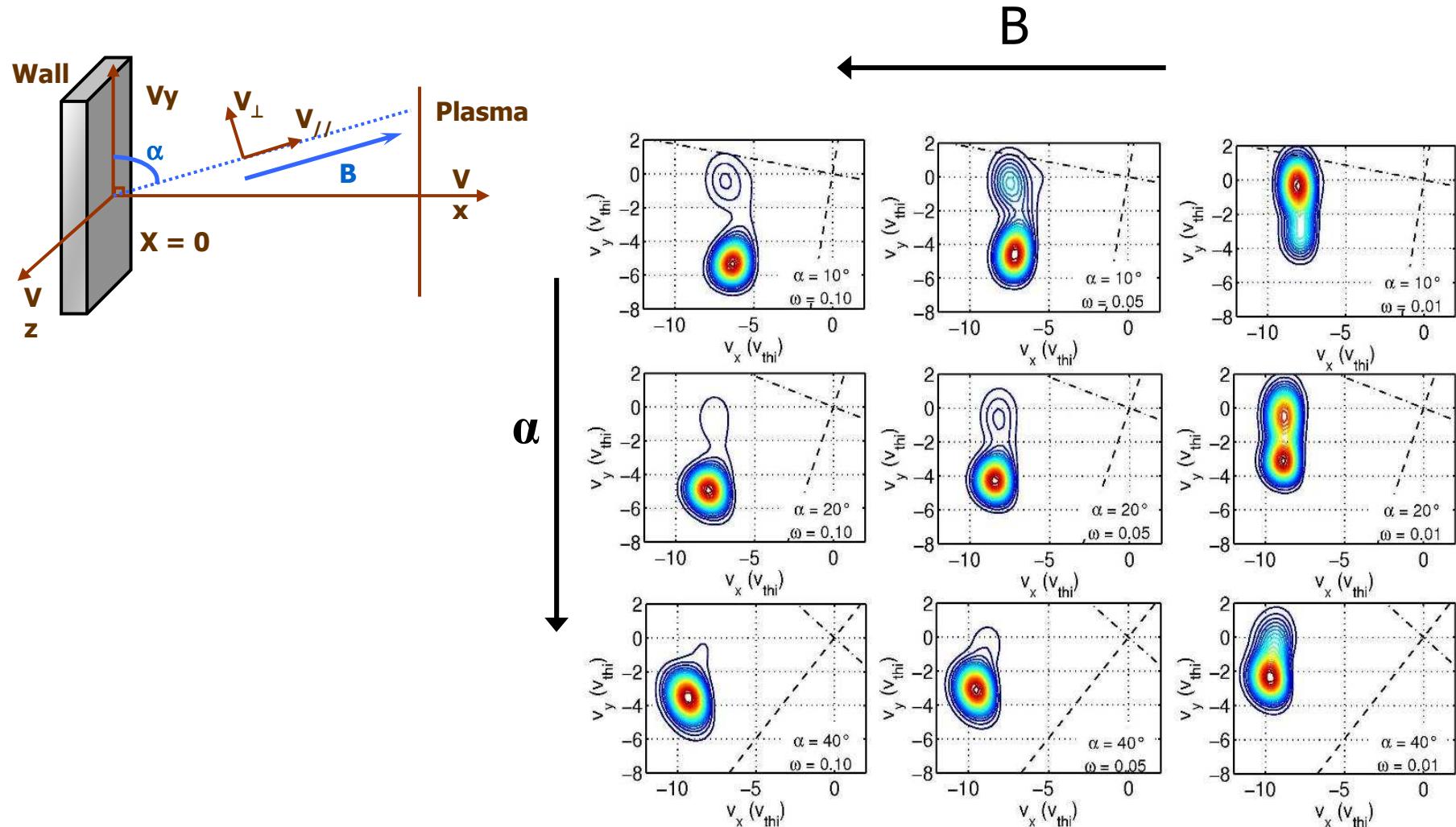
$$Y(\alpha, \omega) \propto \int_0^\infty \int_0^{\pi/2} \frac{E_{\text{kin}}}{\sin \theta} F(\theta, E_{\text{kin}}) d\theta dE_{\text{kin}}$$

$\alpha$  = angle of incidence of the magnetic field

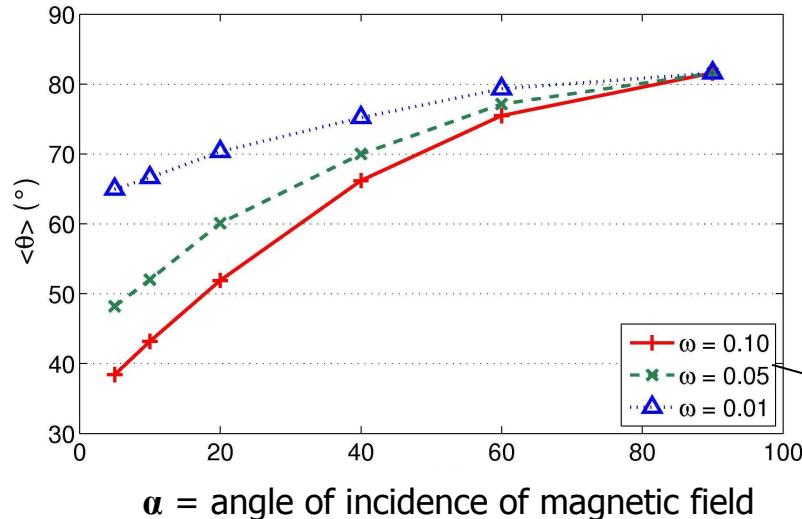
$F(\theta, E_{\text{kin}})$  = distribution function in angle/energy variables

$$\omega = \omega_{ci} / \omega_{pi}$$

# Phase-space distribution at the wall

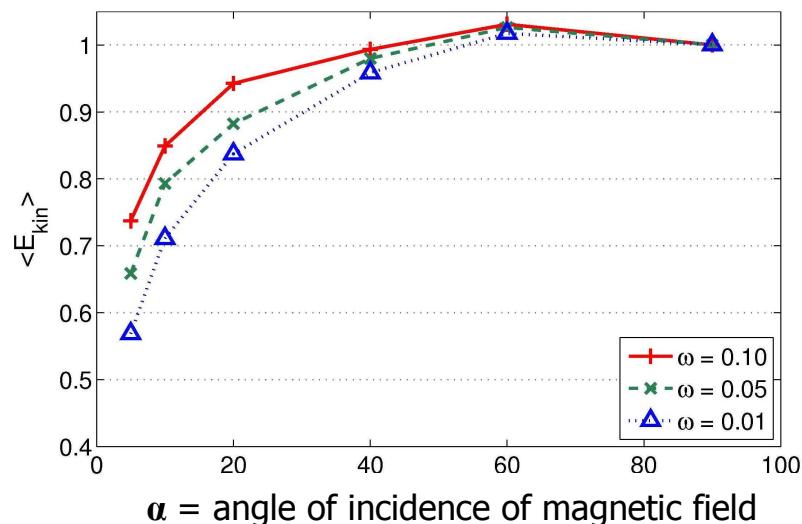


## Results: angle of incidence and kinetic energy on the wall



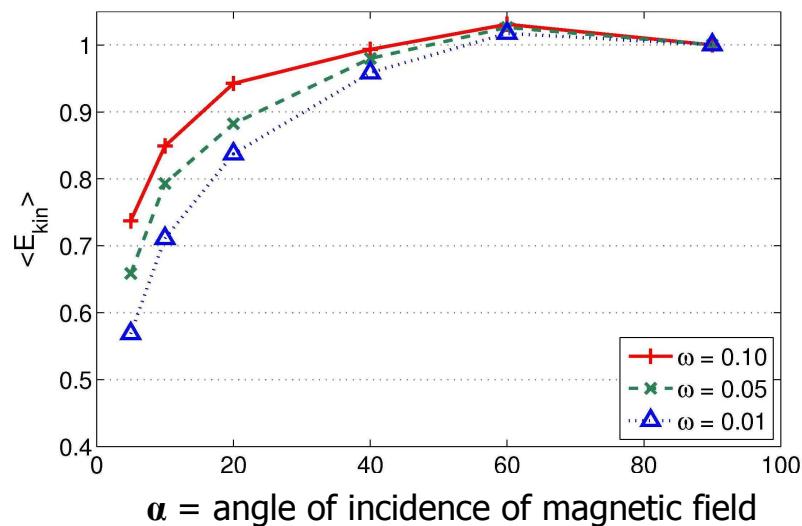
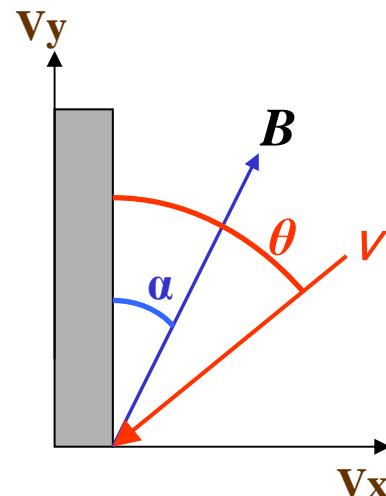
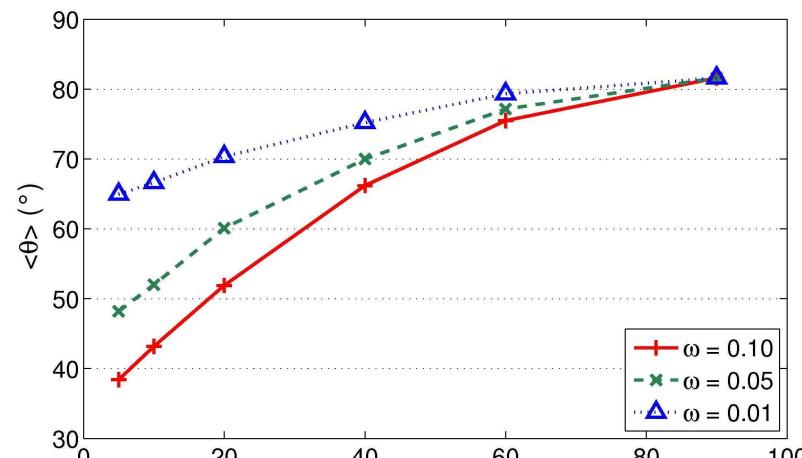
Average angle of incidence  
on the wall,  $\langle \theta \rangle$

$$\omega = \omega_{ci} / \omega_{pi}$$

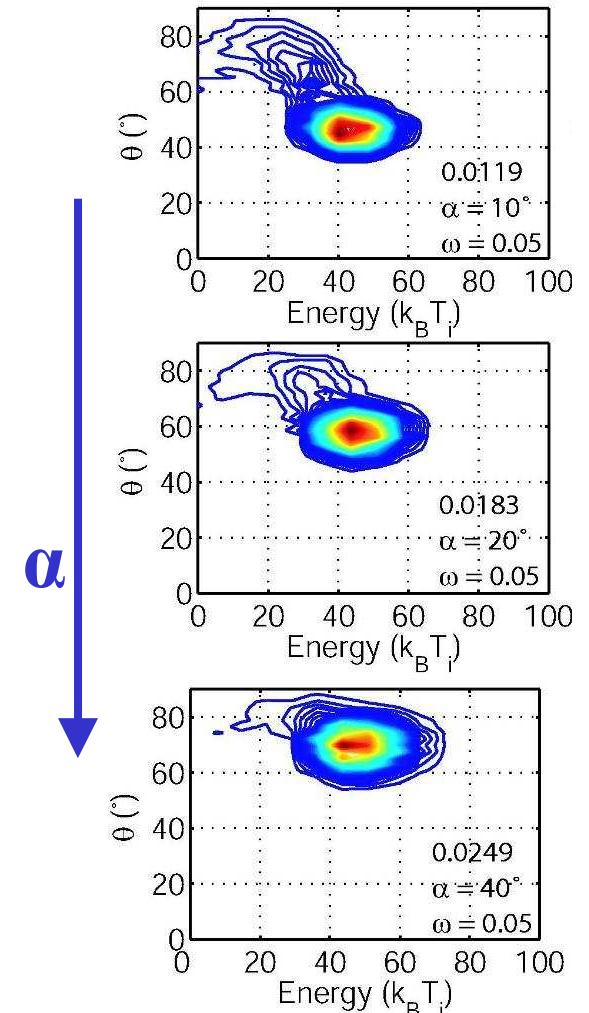


Average kinetic energy on the  
wall,  $\langle E_{kin} \rangle$

## Results: angle of incidence and kinetic energy on the wall

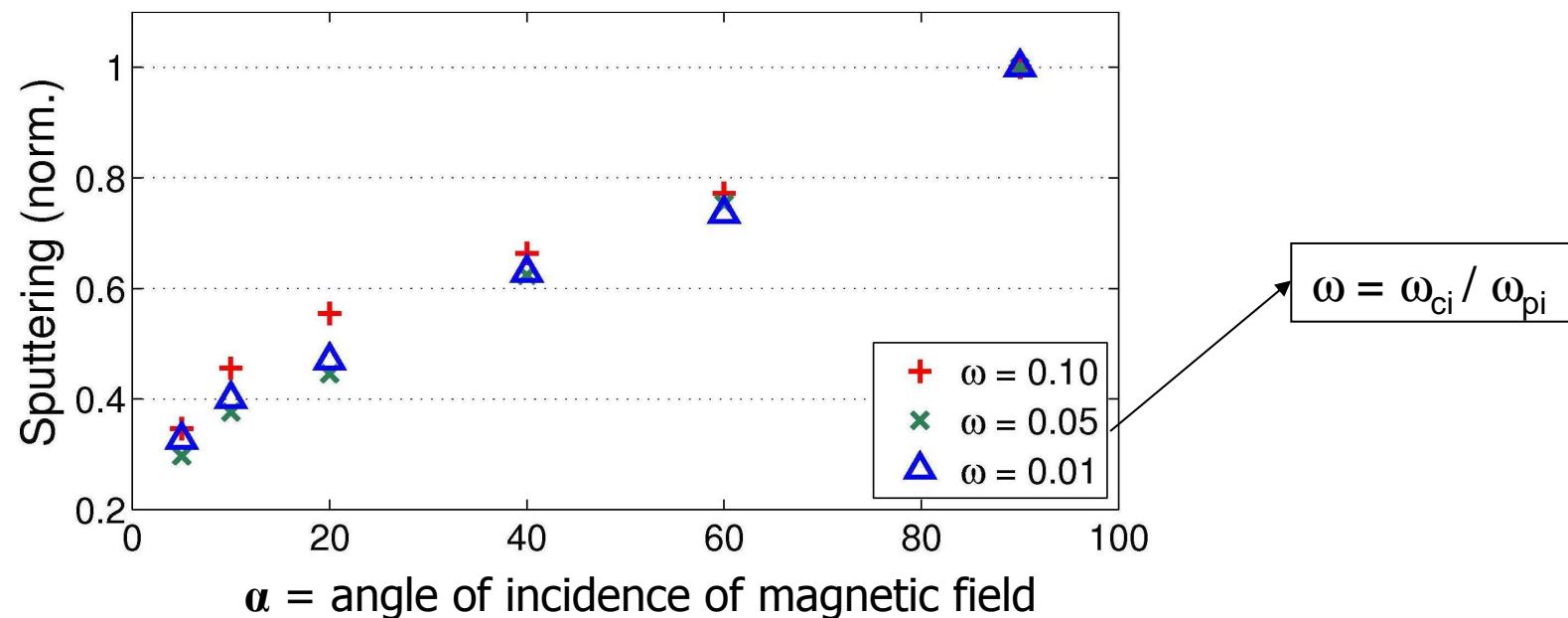


$\theta > \alpha !$



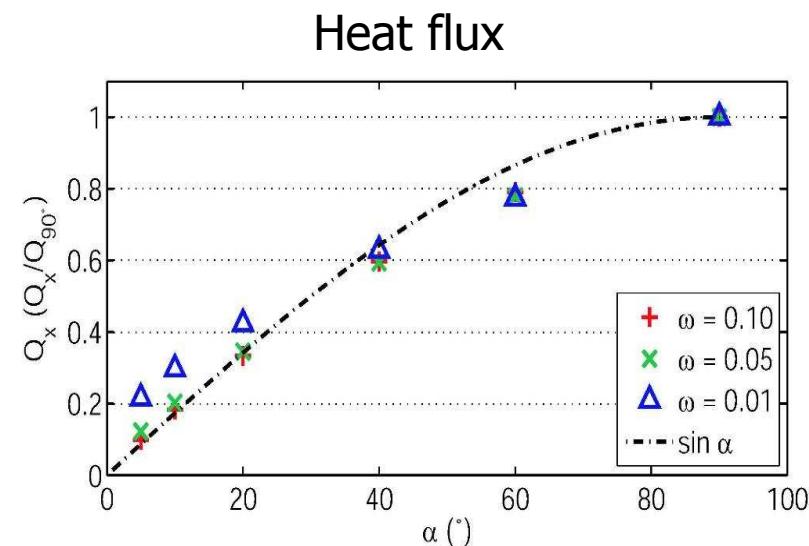
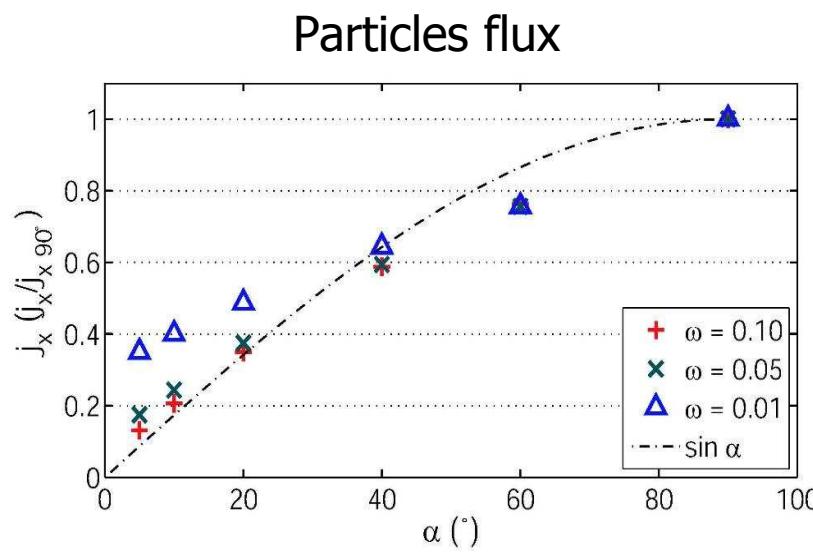
## Sputtering yield on the wall

$$Y(\alpha, \omega) \propto \int_0^\infty \int_0^{\pi/2} \frac{E_{\text{kin}}}{\sin \theta} F(\theta, E_{\text{kin}}) d\theta dE_{\text{kin}}$$



## Heat and particles fluxes on the wall

- For large enough magnetic field, the fluxes follow a ' $\sin \alpha$ ' law



# Conclusions

- Kinetic model for ion population – Vlasov code
- Full description of the magnetized plasma-wall transition
  - Collisional presheath, magnetic presheath, Debye sheath
- Experimental validation
- Applications: computed distributions allow calculation of:
  - Energy and angle of incidence of ions on wall
  - Sputtering yield
  - Particles and heat fluxes on wall
- Perspectives
  - Dynamical formation of sheaths
  - Two-species plasmas (DT, impurities, ...)

- { • S. Devaux, G. Manfredi, Phys. Plasmas **13**, 083504 (2006).  
• S. Devaux, G. Manfredi, Plasma Phys. Control. Fusion **50**, 025009 (2008).