The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1953-19

International Workshop on the Frontiers of Modern Plasma Physics

14 - 25 July 2008

Unity Beta -Building a Better Bottle?

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Unity Beta -- Building a Better Bottle?

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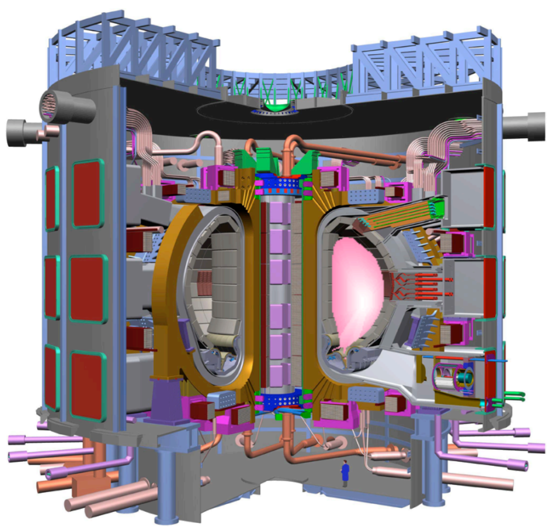

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Better Bottle?

We have a magnetic configuration that will take us to burning plasmas in ITER. This will probably be the configuration of the first generation of fusion reactors.

Can we improve this? What would that mean?

Better confinement? ………………. τ_E
Higher pressure -- lower B? ………… β
More reliable? ………… stability.
Less heat flux? ………. Divertor.

Has every configuration been tried? In 2D?

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Requirements For Fusion.

$$Fusion \quad Power \propto n_D n_T T^2 \propto \beta^2 B^4$$

$$10\,keV < T < 20\,keV.$$

Rough criterion for ignition. $\boxed{nT\tau_E > 3 \times 10^{15} cm^{-3} keV\ s}$

Physics limits the achievable values of these quantities.

n :Density "Greenwald" limit.

$nT = \beta \dfrac{B^2}{8\pi}$: Beta Limit. $\beta = \beta_N \dfrac{I}{aB}$

τ_E : Turbulence.

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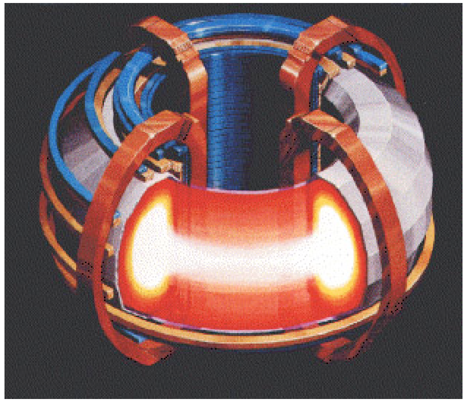
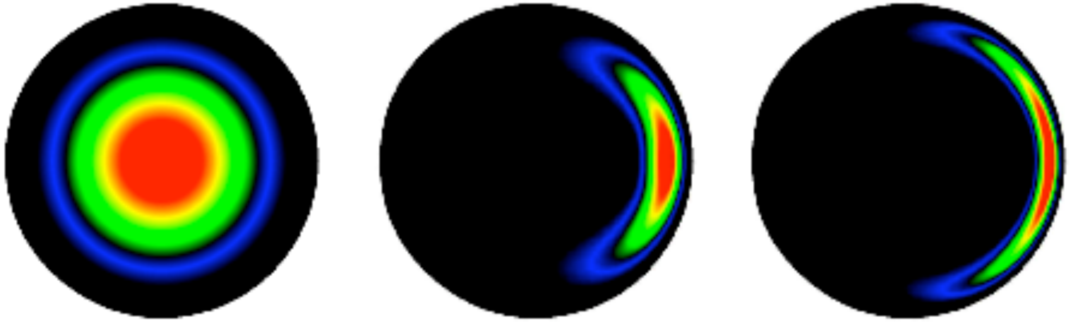
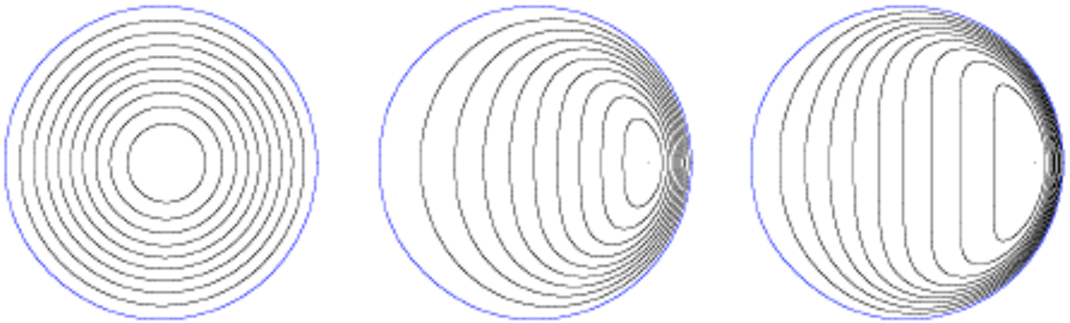
Raising Beta





$$\mathbf{J} \times \mathbf{B} = \nabla p$$



Radial Force Balance

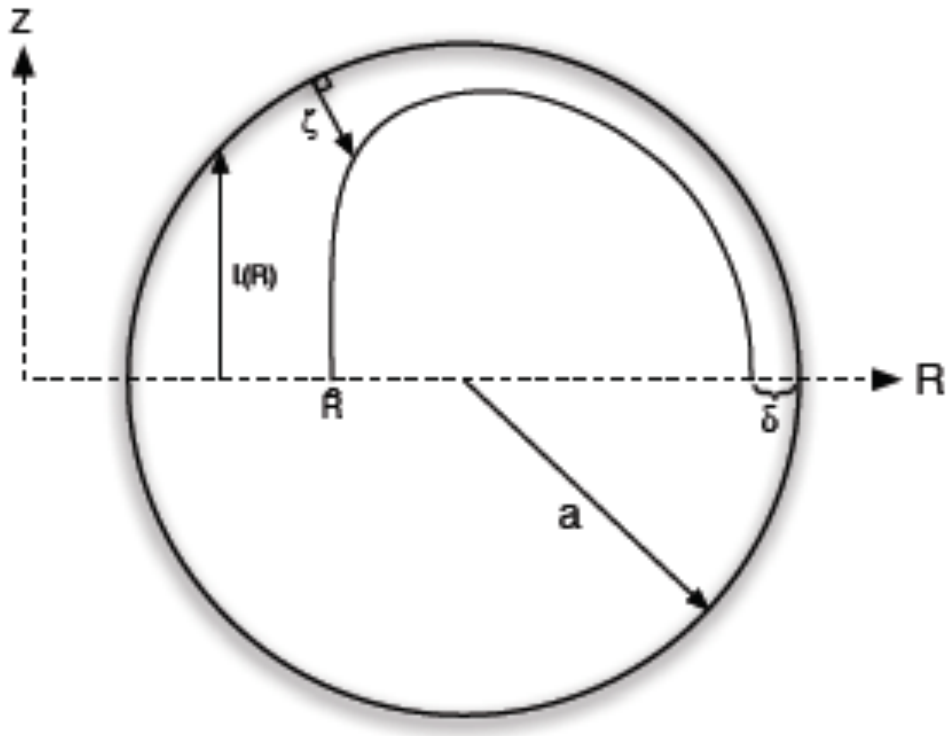
$$\nabla\psi \cdot [\mathbf{J} \times \mathbf{B} = \nabla p]$$

$$\left[R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{\partial^2}{\partial Z^2}\right]\psi = -\mu_0 R^2 \frac{dp}{d\psi} - F\frac{dF}{d\psi}$$

Grad-Shafranov Equation.

where

$$\mathbf{B} = \frac{\nabla\psi \times \mathbf{e}_T}{R} + \frac{F(\psi)}{R}\mathbf{e}_T$$

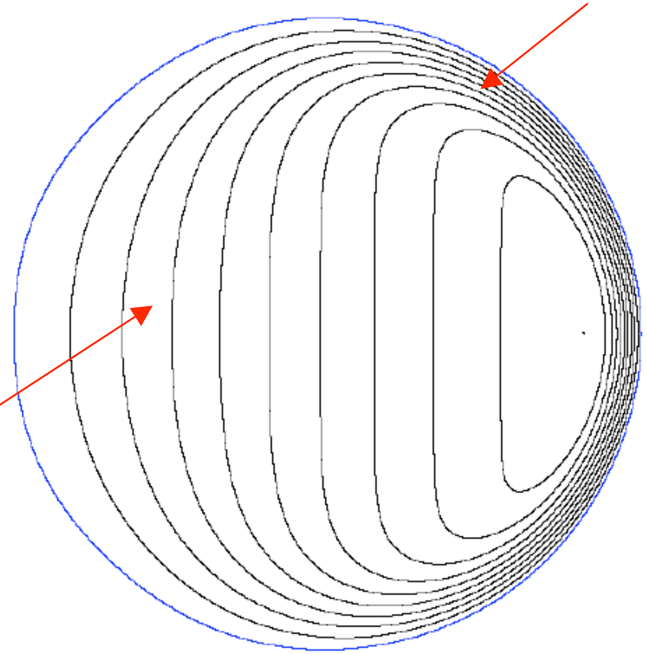


The Small Parameter

$$\frac{\epsilon}{q^2} = \frac{a}{q^2 R} \ll 1$$

$$\mathbf{B}_p \ll \mathbf{B}_T = F/R$$

Surfaces shift and
Squash against edge.



$$\underbrace{\nabla p}_{\mathcal{O}(\beta)} + \underbrace{\frac{\nabla B_{\mathrm{T}}^2}{2}}_{\mathcal{O}(1)} + \underbrace{\frac{\nabla B_{\mathrm{P}}^2}{2}}_{\mathcal{O}(\epsilon^2/q^2)} + \underbrace{\frac{B_{\mathrm{T}}^2 \nabla R}{R}}_{\mathcal{O}(\epsilon)} + \underbrace{B_{\mathrm{P}} \cdot \nabla B_{\mathrm{P}}}_{\mathcal{O}(\epsilon^2/q^2)} = 0$$

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CORE

$$\boxed{\left[R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right)+\frac{\partial^2}{\partial Z^2}\right]\psi} = -\mu_0 R^2\frac{dp}{d\psi} - F\frac{dF}{d\psi}$$

small $\mathcal{O}(\epsilon)$

$$\mu_0 R^2\frac{dp}{d\psi} = -F\frac{dF}{d\psi}$$

Toroidal Field Confinement

$$\boxed{R = R(\psi) \quad or \quad \psi = \psi(R)}$$

Straight vertical flux surfaces.

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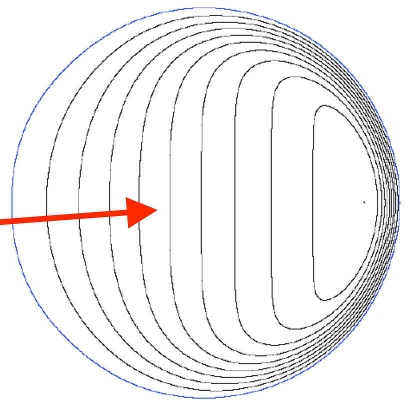
CORE

$$F(\hat{R}) = \sqrt{2\left(C - \mu_0 \int_{R_{\min}}^{\hat{R}} \hat{R}'^2 \frac{dp}{d\hat{R}'} d\hat{R}'\right)}$$

C = constant & p increases and F decreases towards the axis

$$\boxed{R = R(\psi) \quad or \quad \psi = \psi(R)}$$

Straight vertical flux surfaces in core



Boundary Layer -- BL

Gradients are large perpendicular to wall ξ = distance to wall.

$$\frac{\partial^2 \psi}{\partial \xi^2} = -\mu_0 (R^2 - \hat{R}^2(\psi)) \frac{dp}{d\psi}$$

Width of Boundary Layer is small and Poloidal Field is strong

$$\left(\frac{\partial \psi}{\partial \xi}\right)^2 = -2\mu_0 \int_R^{\hat{R}} (R^2 - \hat{R}''^2) \frac{dp}{d\psi} \frac{\partial \psi}{\partial \xi} d\xi$$

Poloidal field pressure forces balance the residual force from
Lack of cancellation of pressure and toroidal field forces.

$$|\mathbf{B}_p| \sim \sqrt{\epsilon p} \ll |\mathbf{B}_T|$$

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Boundary Layer --- BL

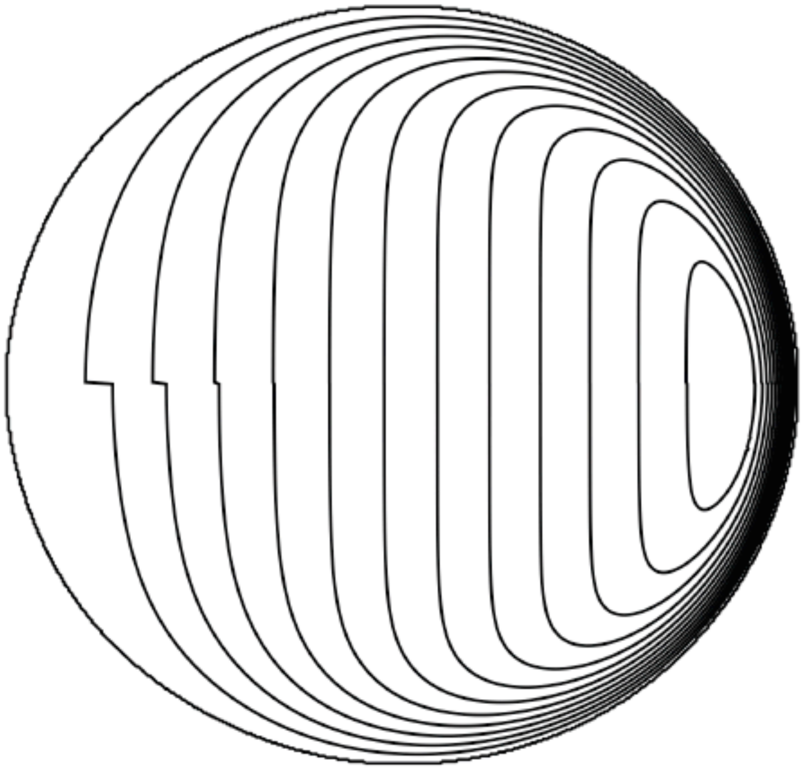
$$\xi(R,\hat{R}) = \int_{R_{\min}}^{\hat{R}} \frac{d\hat{R}' \frac{\partial\psi}{\partial\hat{R}'}}{\sqrt{-2\mu_0 \int_R^{\hat{R}'} d\hat{R}'' \frac{dp}{d\hat{R}''}(R^2 - \hat{R}''^2)}}$$

$$\delta = a\sqrt{\frac{\epsilon}{q^2\beta}}$$

Boundary layer width.
Expansion works if $\delta < a$

Poloidal field increases outwards in Boundary Layer.

Comparison



Agreement gets better as we increase beta

FIG. 3: Comparison of a equilibrium soluton computed in CUBE (top) and the same solution calculated using the analytic theory (bottom).

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Good properties.
1. Good Average Curvature

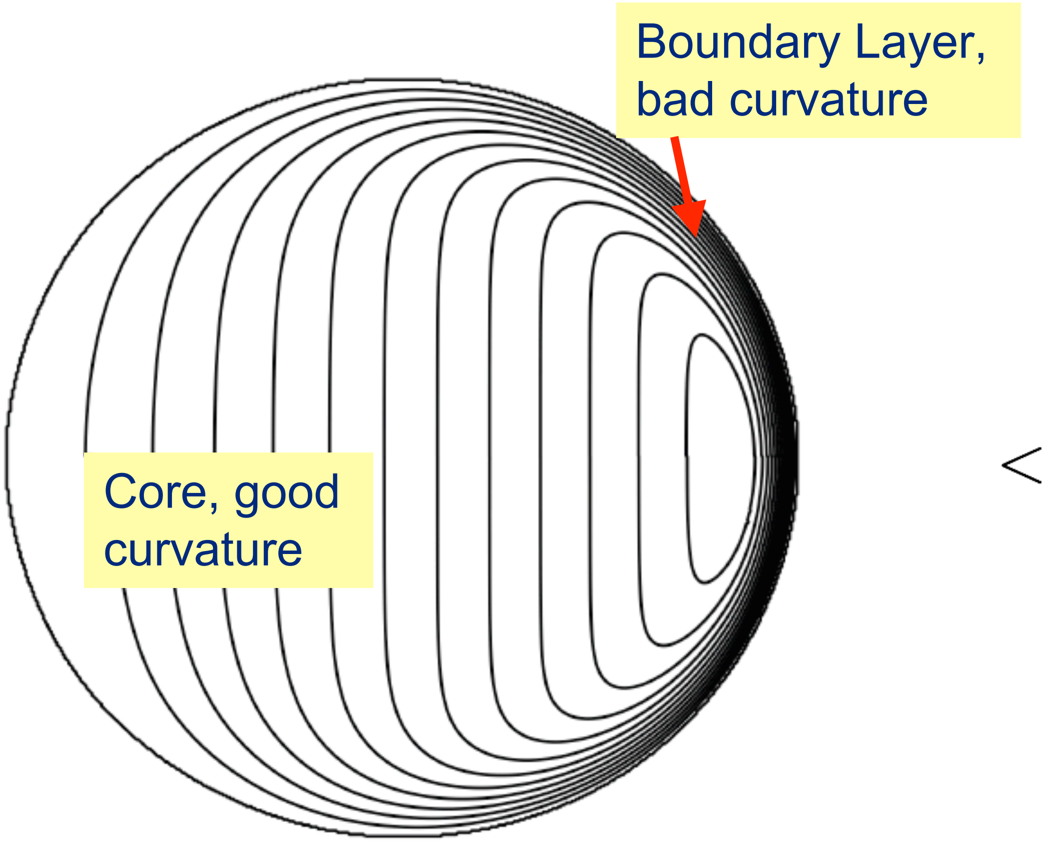


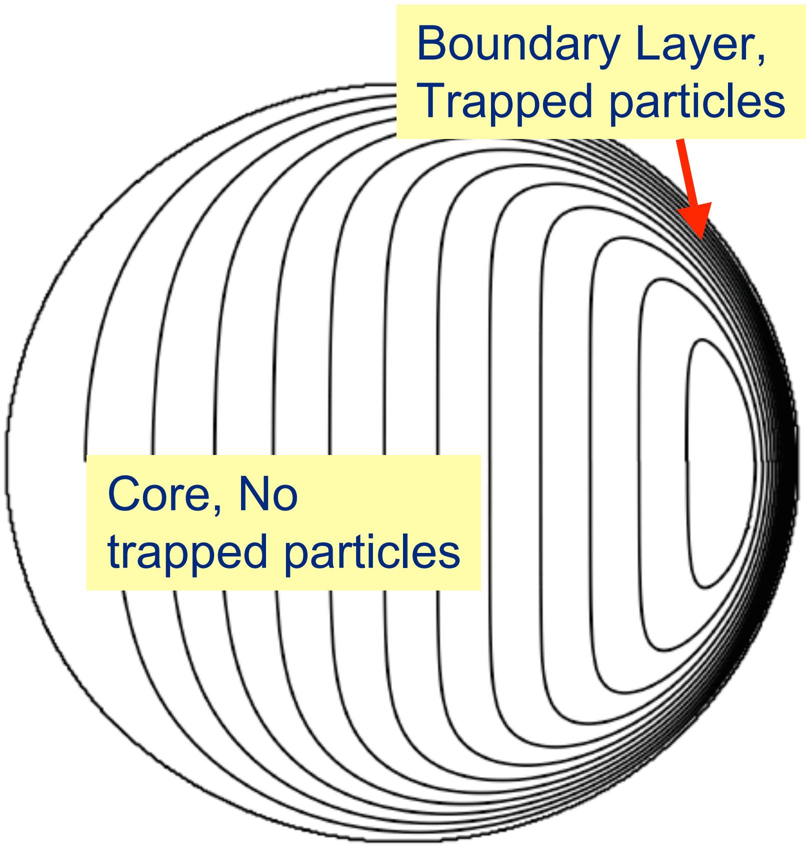
FIG. 3: Comparison of a equilibrium soluton computed in CUBE (top) and the same solution calculated using the analytic theory (bottom).

Bad field line curvature in the boundary layer only. Core dominates average

$$< \nabla p \cdot (\mathbf{b} \cdot \nabla \mathbf{b}) > \sim -\frac{p}{aR}$$

Mercier stable and tearing mode stable.

Good properties.
2. Small trapped particle fraction



|B|constant on flux surface in core $\Rightarrow$ no bounce points in core. Trapped particle fraction…..

$$f_T \sim (1 - \frac{B_{min}}{B_{max}})^{1/2} \frac{\Delta V}{V}$$

As beta increases both factors decrease. **|B|**constant on flux surface in BL too **(omnidigeneity)**. The volume fraction in BL is

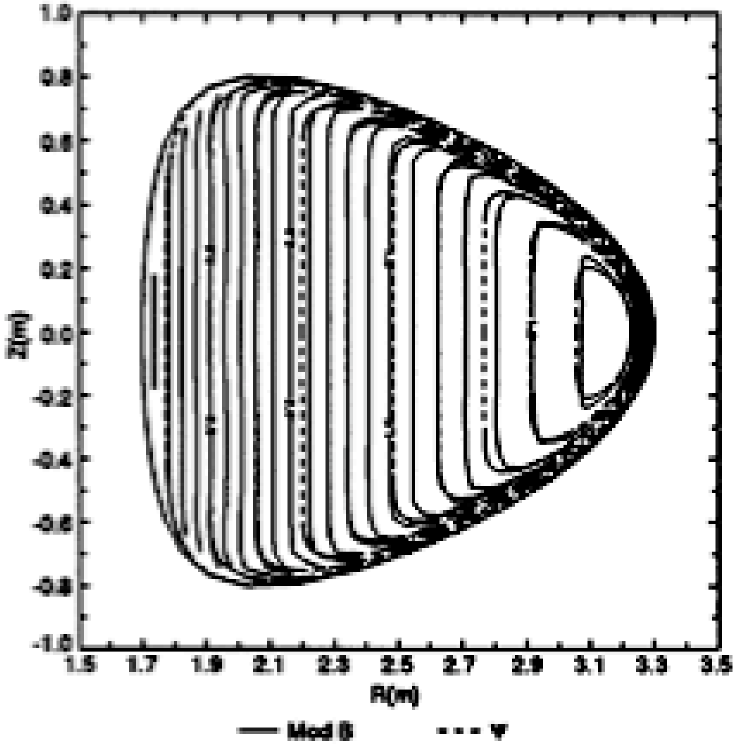
$$\frac{\Delta V}{V} \sim \sqrt{\frac{\epsilon}{q^2 \beta}} \ll 1$$

Banana width is squeezed by strong B_p. Neoclassical transport reduced by more than $\sim \sqrt{\frac{\epsilon}{q^2\beta}}$

Good properties.
3. Magnetic well

$$\mathbf{B}^2 = \frac{F^2}{R^2} + \frac{|\nabla\psi|^2}{R^2}$$

$$p + \frac{\mathbf{B}^2}{2} = constant$$



|B| is small in the center of
The plasma.

$$\mu = \frac{v_\perp^2}{B} = constant$$

Got to give particles energy
To get them out. Helps
Stability, *Taylor 1963*

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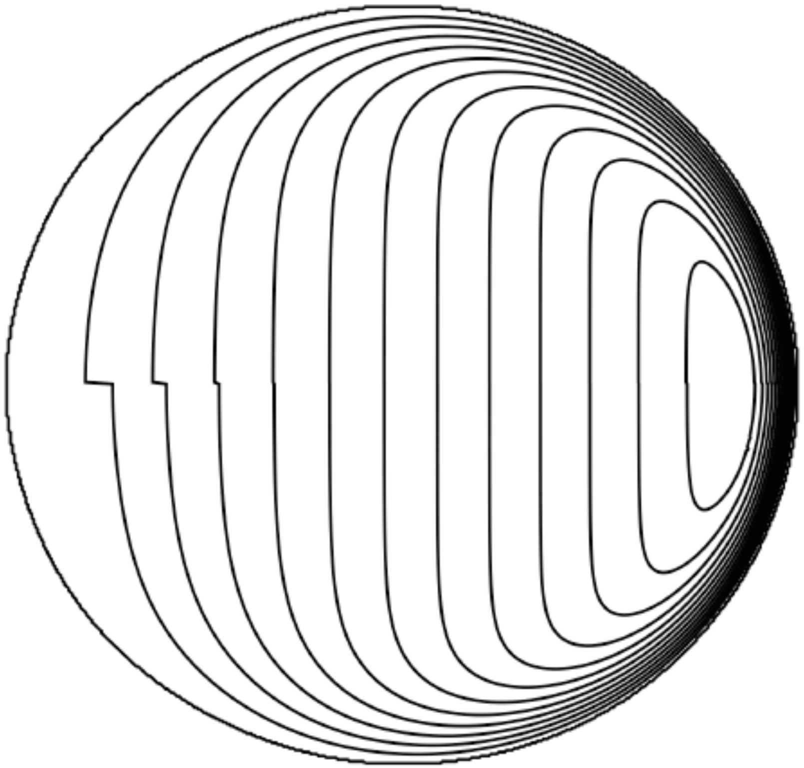
Good properties.
4. Short Connection length

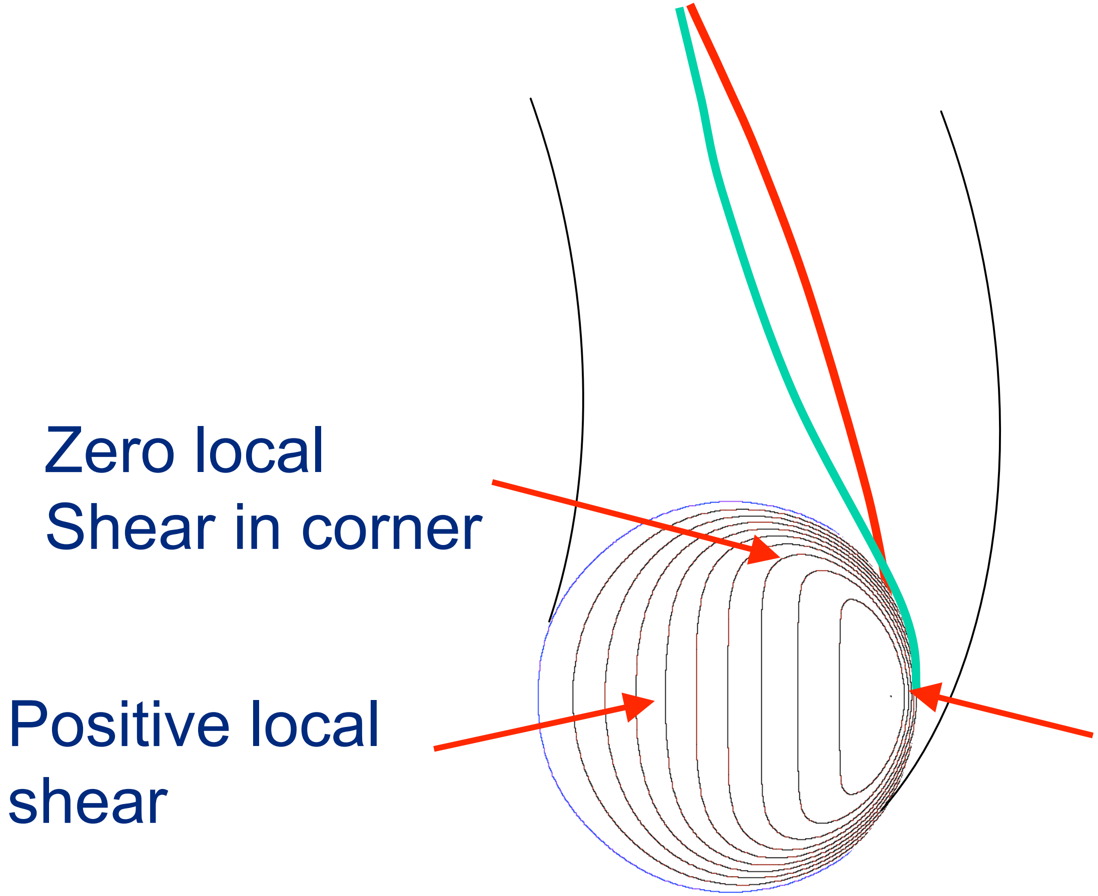


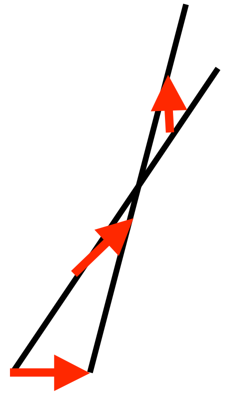
B_p is large in BL so distance along field from bad to Good curvature is

$$L_C \sim qR\sqrt{\frac{\epsilon}{q^2\beta}}$$

Stabilizing.

Good properties.
5. Strong negative local shear in BL



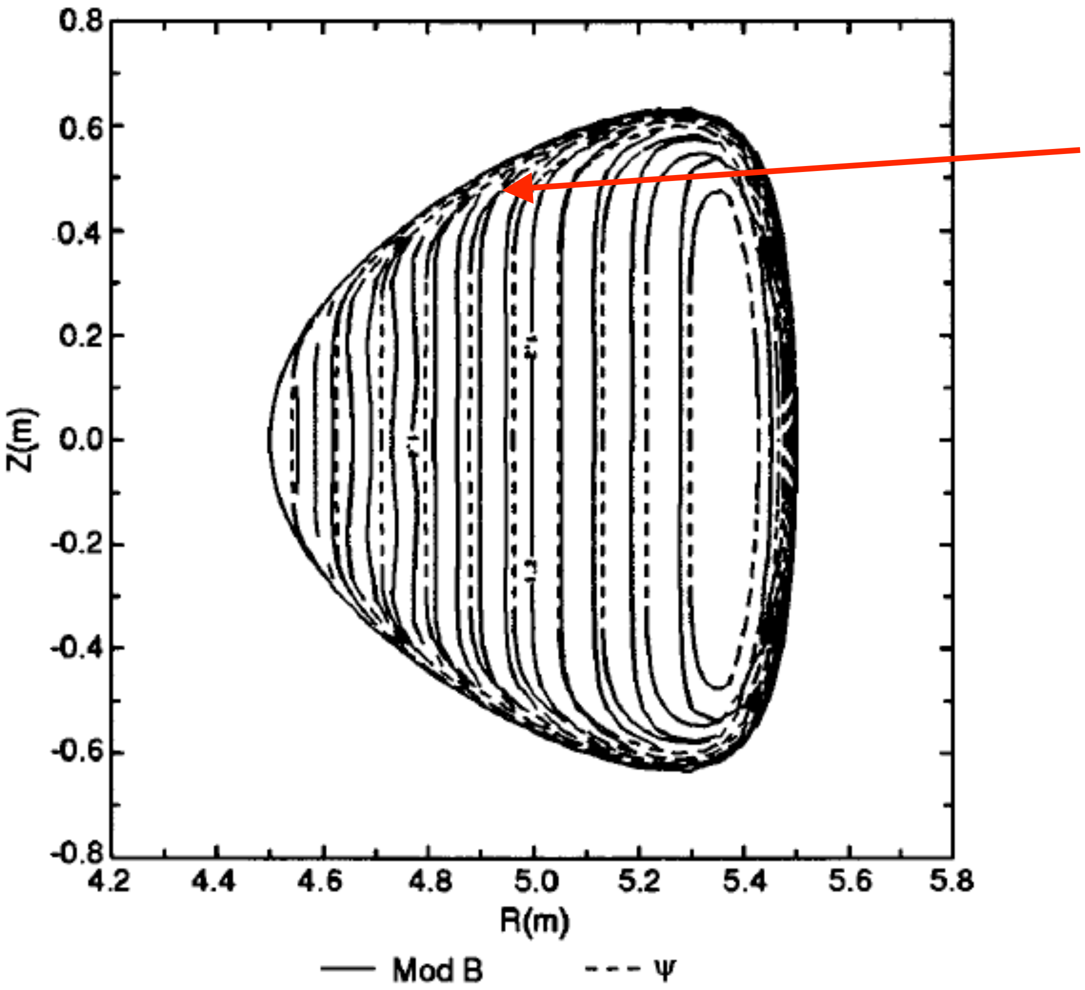


Shear length in boundary layer =

$$L_s \sim \mathcal{O}(a/q\beta) \ll L_c$$

Instabilities are heavily sheared by Magnetic shear in BL

Negative Triangularity - Reverse D.



Zero local Shear in Corner firmly in good Curvature region.

Particles drift reversed

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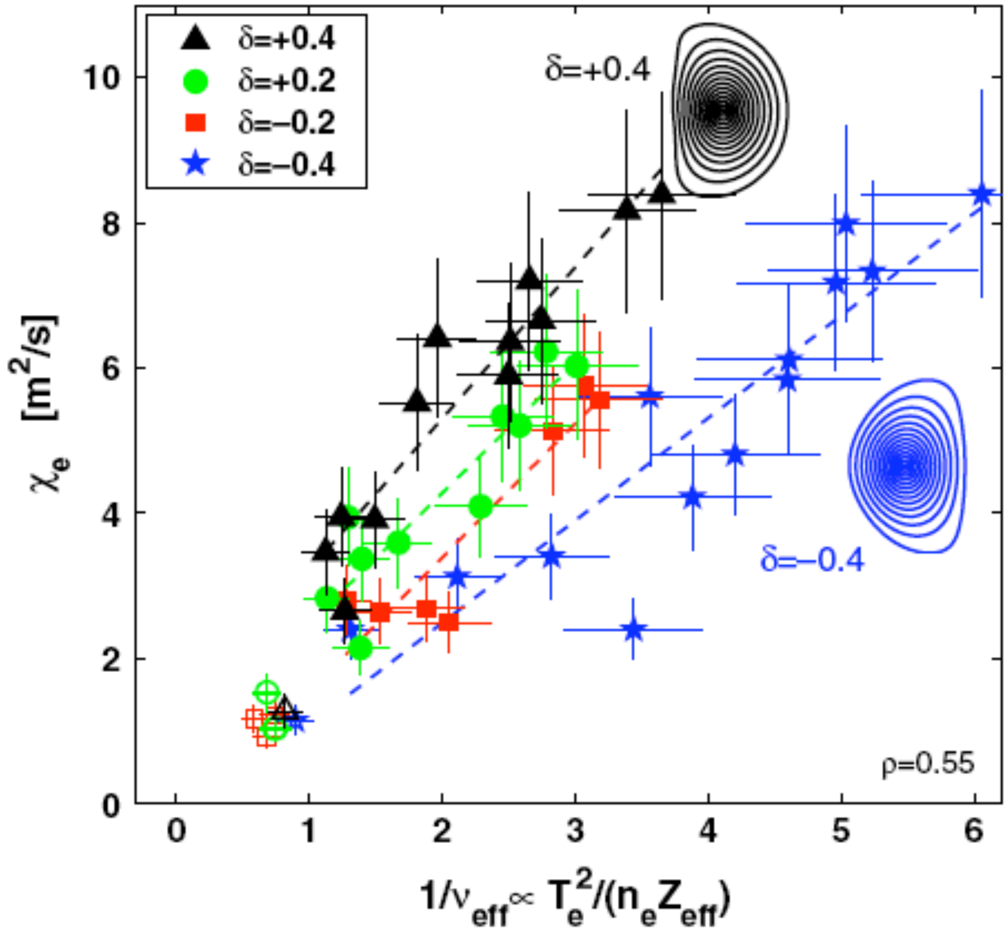
Negative Triangularity - Reverse D. TCV



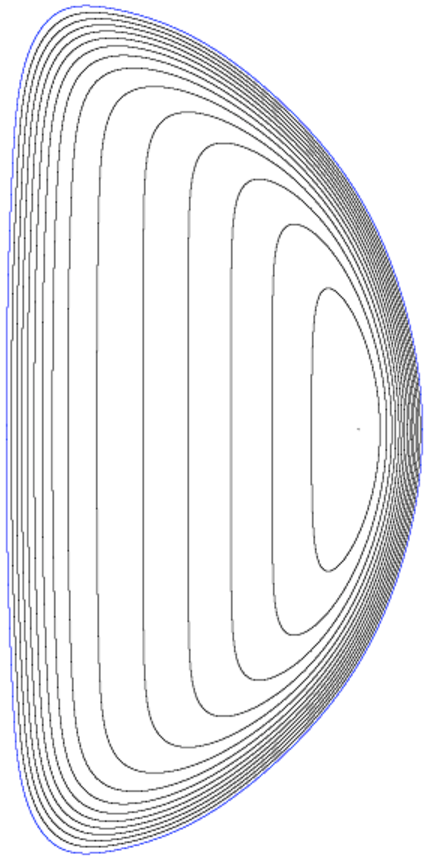
Less transport
In reverse D

Camenen et. al. Nucl Fus. 2007

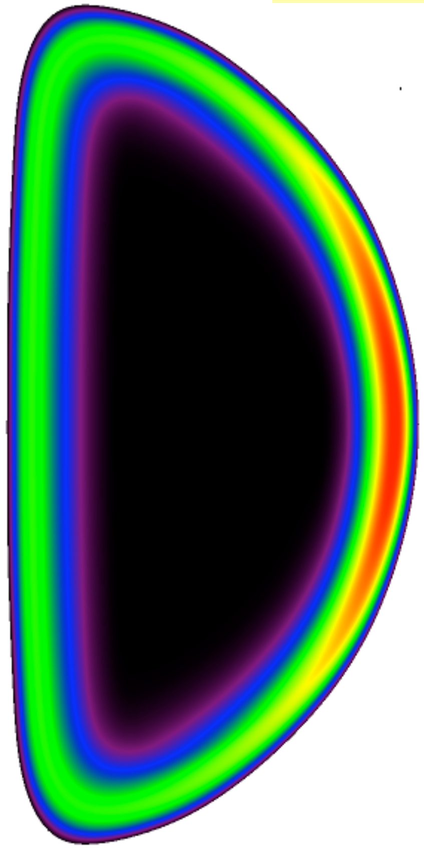
Unity beta current hole equilibrium

This equilibrium is stable to all ideal MHD criteria including internal and external modes for n =1, 2 and 3… ***Note that the β_N is "small" despite the large value of beta.***

Pierre Gourdain's work



Flux Surfaces



Toroidal Current Distribution

R	6 m
a	2 m
B_T	2.5 T
β	100%
$<\beta>$	12%
β_N	4.6
q_{min}	1.5

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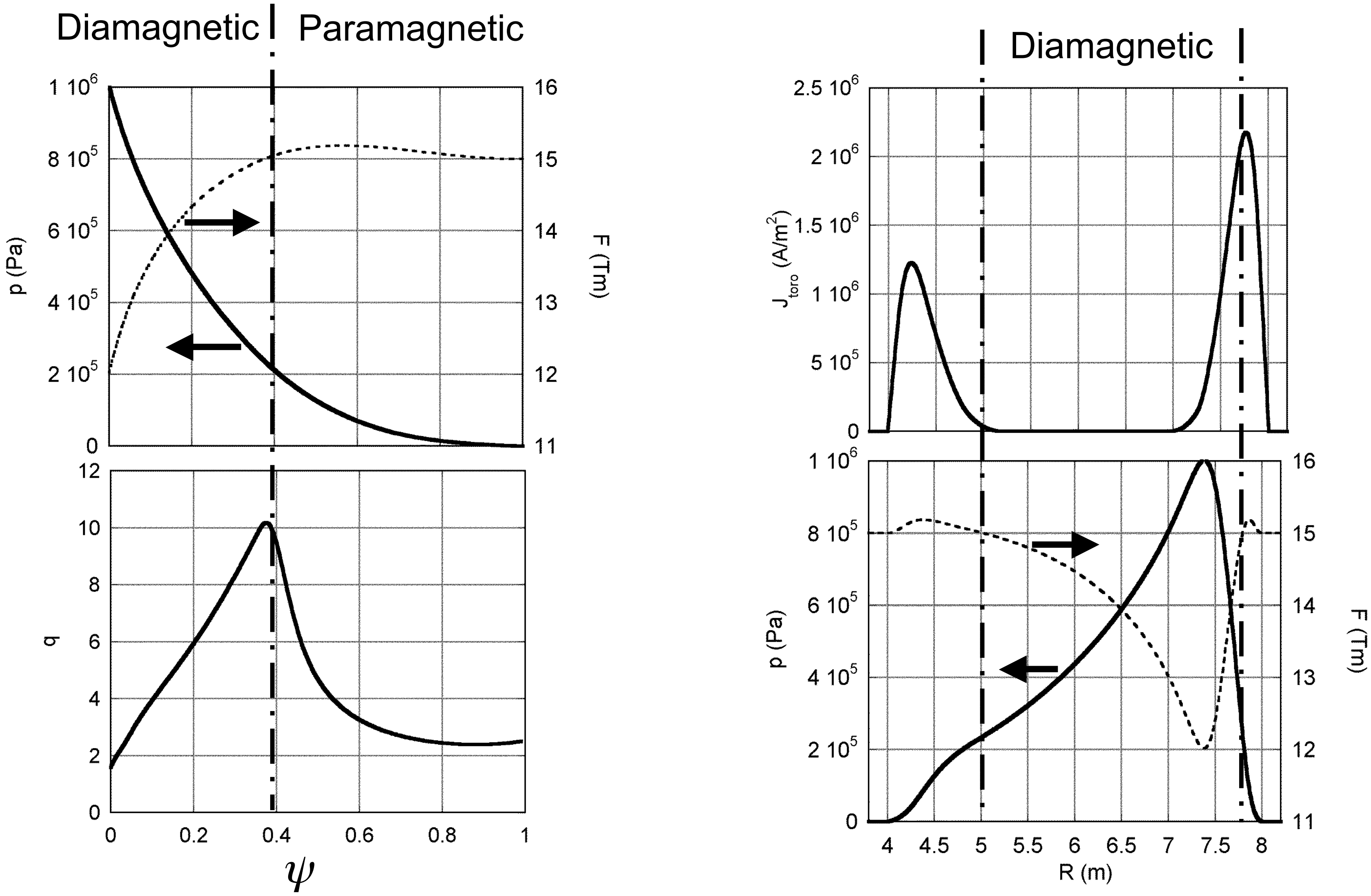
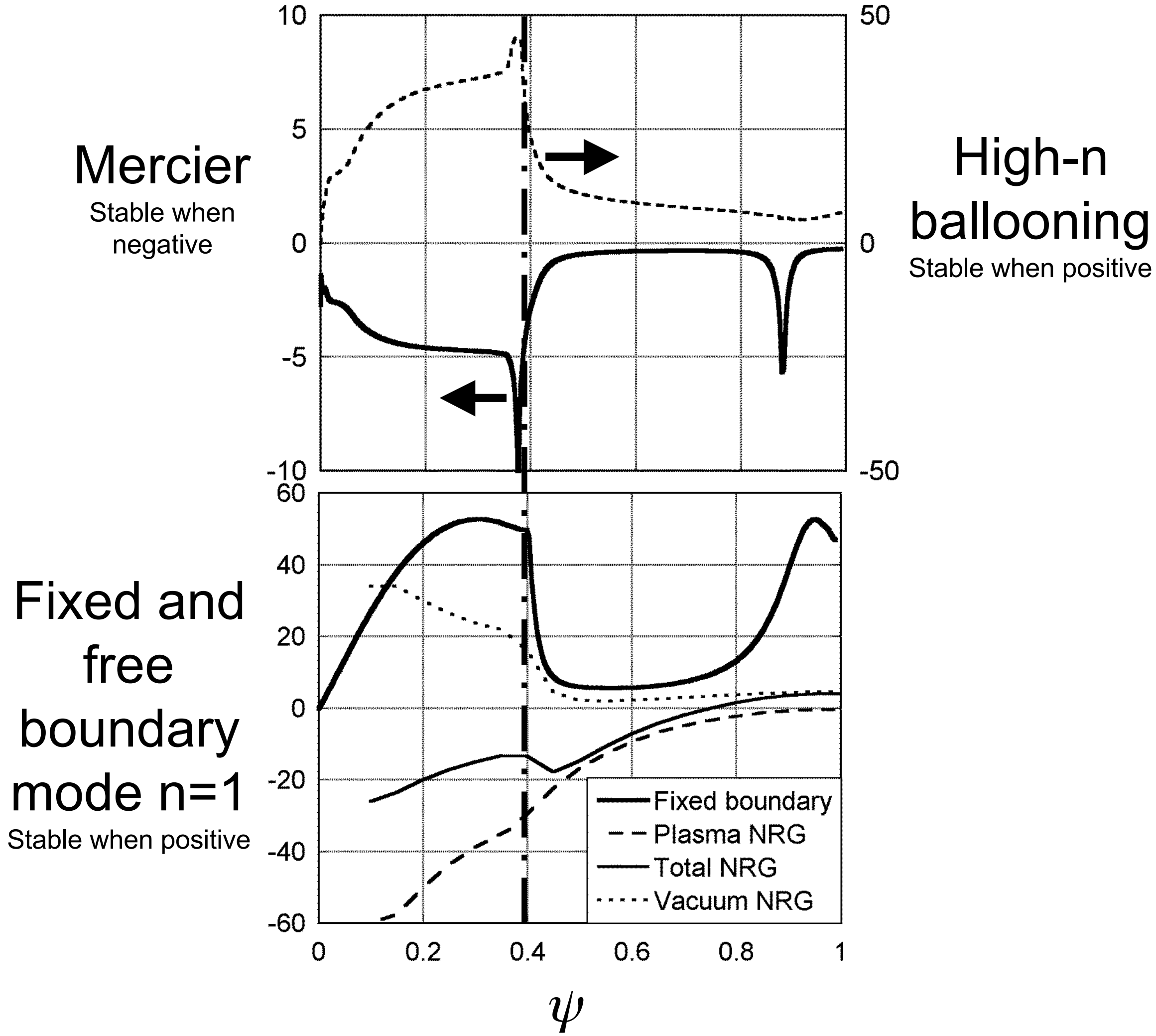
Unity beta current hole profiles



Internal and external kink stability



DCON finds stability for Mercier, high-n ballooning as well as fixed boundary kink modes (n=1).

The free boundary mode n=1 is also stable (stability criteria obtained for $\psi = 1$).

Stability for n=2 and n=3 was also demonstrated.

Better Bottle?

It certainly isn't clear that we can find a better bottle.   But we should use our best tools to look hard.