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Intermittency in Plasma Turbulence.

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Intermittency in Plasma Turbulence

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Outline

- Introduction: Intermittency - presence of coherence
- Influence of coherent structures on transport.
- Experimental evidences and numerical studies on the presence and formation of coherent structures.
- Comparison and contrast with the strict quantitative notion of intermittency adopted in the context of hydrodynamic fluids.
- Plasma intermittency: present understanding - some unique aspects

Turbulence

Seemingly erratic flow interspersed with patterns of various sizes.

Presence of structures linked with the concept of intermittency.

Spatial and Temporal Randomness

Intermittency

(A first glimpse)

Qualitatively:

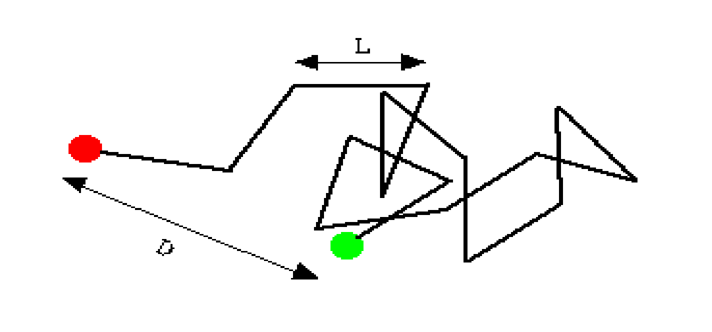
- Intermixing of randomness and coherence in the turbulent state.
- Deviations from Gaussian statistics.

Connection with transport

Structures can lead to particle trapping and/or increased transport by effectively increasing the de-correlation length.

Transport

$$\vec{X} = \vec{x}_1 + \vec{x}_2 + \vec{x}_3 + \ldots;$$
$$\vec{X} = (\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots)\Delta t;$$
$$\langle X^2 \rangle = [\langle \vec{v}_1^2 \rangle + \langle \vec{v}_2^2 \rangle + \ldots 2\langle \vec{v}_1 \bullet \vec{v}_2 \rangle + \ldots]\Delta t^2;$$



- For totally uncorrelated velocities and using the fact that the velocities at individual steps are identically distributed

$$\langle X^2 \rangle = n\Delta t^2 \langle v^2 \rangle = \left(\langle v^2 \rangle \Delta t \right) t = Dt$$

Diffusive transport: mean square displacement proportional to **t**

For Non Diffusive transport velocity correlations are finite

$$\langle X^2 \rangle = t^{\alpha};$$

$\alpha > 1;$ Super-diffusive

$\alpha < 1;$ Sub-diffusive

Influence of coherent structures

Transport in Fusion Devices

- Long time correlations persist!
- Intermittent transport events.
- Some evidences of non diffusive scaling with ‘t’.. Ballistic transport $X^2 \sim t^2$.
- Transport anomalous ‘D’ much greater than the expected neo-classical values. This can be viewed as an effective enhancement of the correlation step size.
- Formation of transport barriers.
- Structures play an important role!

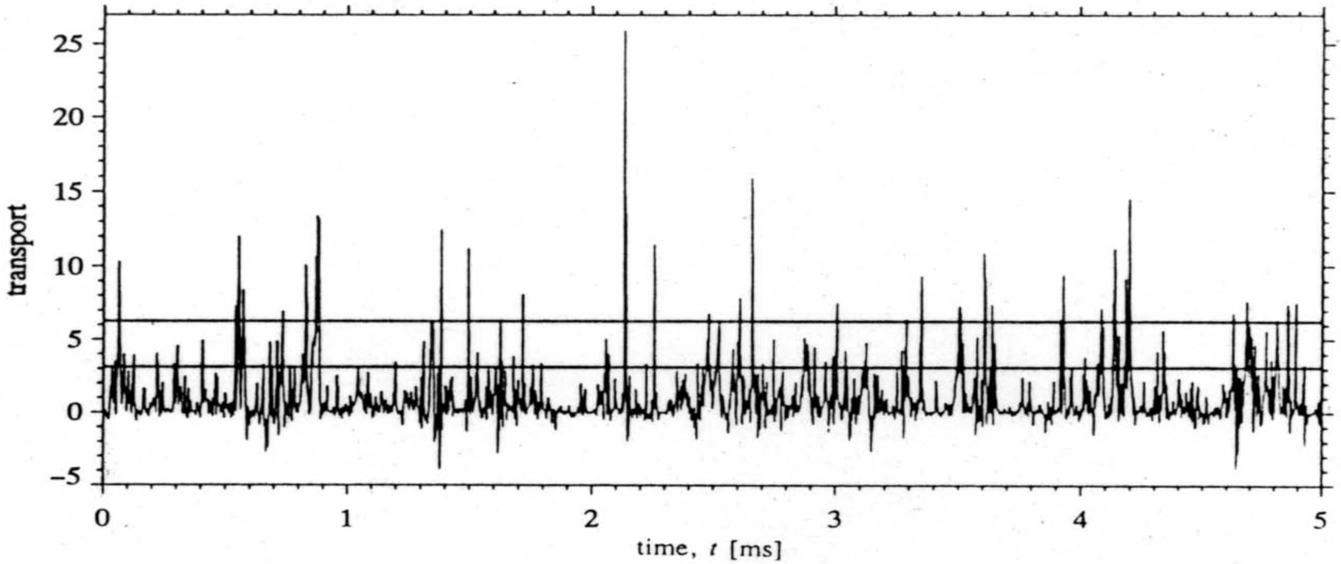


FIG. 7. Time signal of the radial particle flux Γ_r due to radial $\tilde{\mathbf{E}} \times \mathbf{B}$ drift at the midplane of ASDEX (normalized to its mean value). Temperature fluctuations were neglected when calculating Γ_r from floating potential and ion saturation current fluctuations. The data were taken 1 to 2 cm outside the separatrix. The transport is directed radially outwards during 75% of the time, and the integrated transport in the outwards direction is by a factor of 15 to 20 larger than the integrated transport in the inwards direction. Of the transport directed outwards 20% (50%) is accomplished within 2.5% (10%) of the time (intervals for which Γ_r is above the upper (lower) horizontal line).

BURSTY TRANSPORT

Endler et al. NF 1995

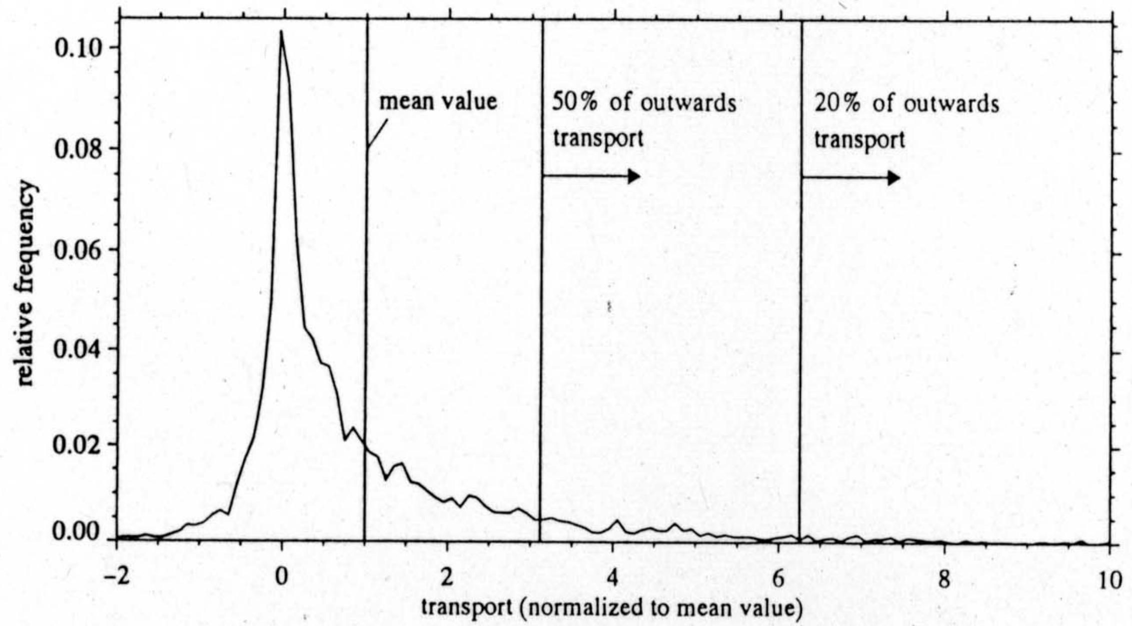
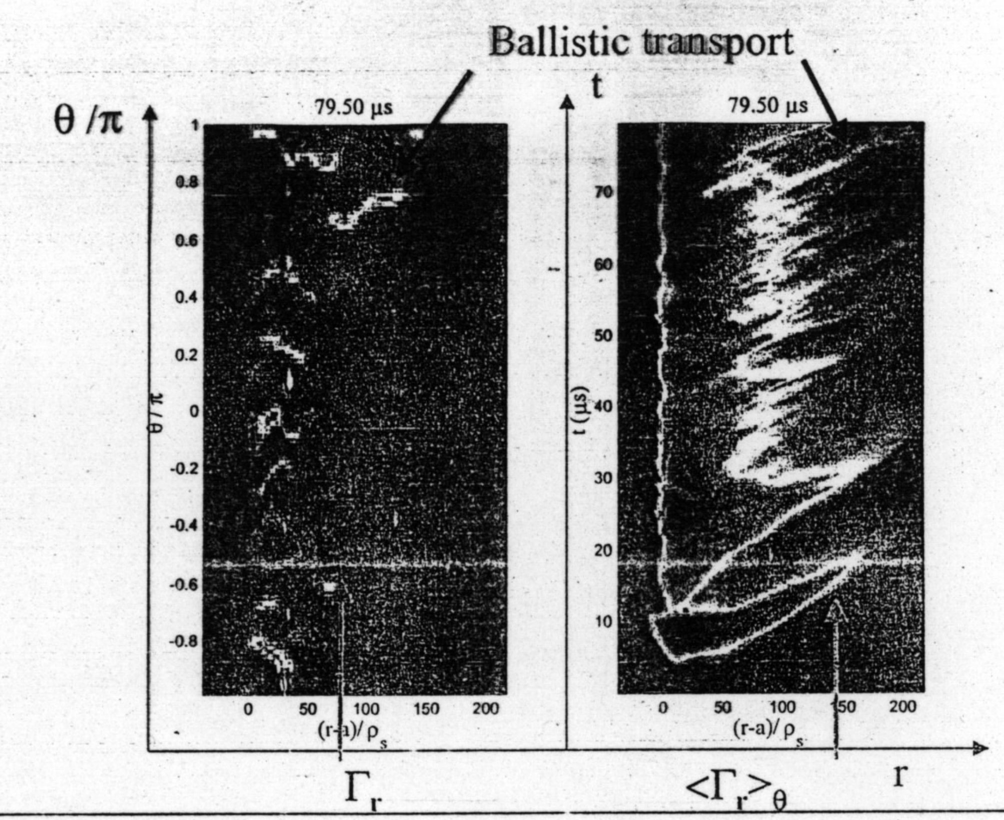


FIG. 8. Distribution function of the time signal of the radial particle flux Γ_r shown in Fig. 7.

Endler et al. NF

Avalanches



Euratom cea
L.P.I.I.M.
Ballistic transport
θ /π
t
79.50 µs
79.50 µs
θ / π
t (µs)
(r-a)/ ρ
(r-a)/ ρ
Γ_r
$<\Gamma_r>_\theta$
r

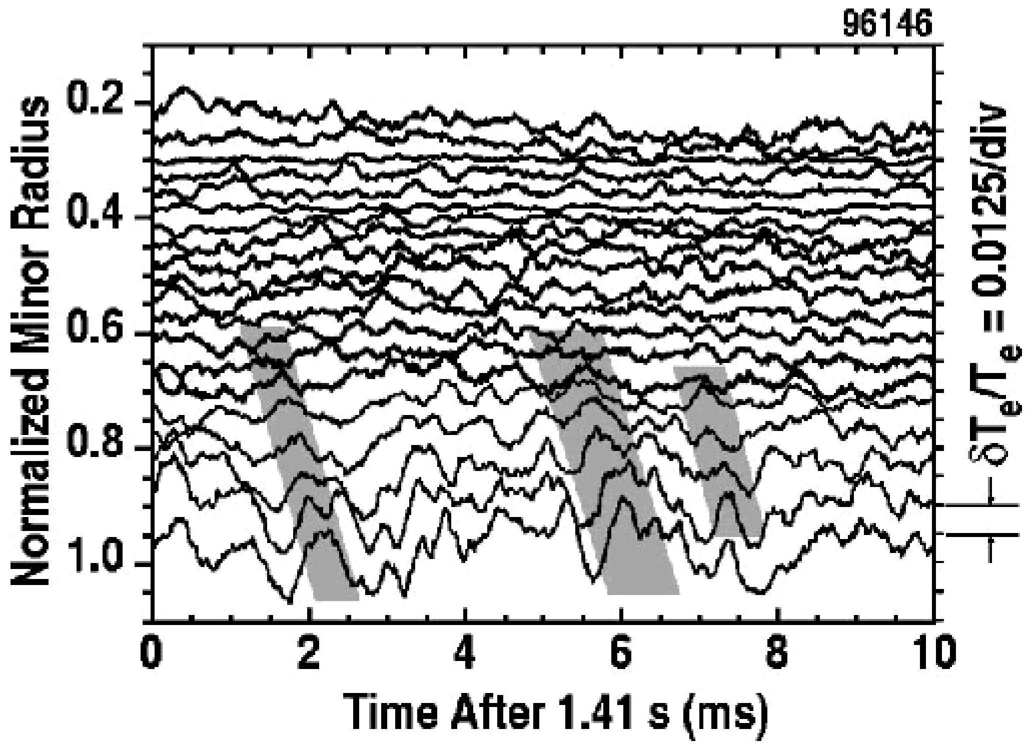


FIG. 4. The normalized ($\delta T/T$) electron temperature fluctuations for discharge 96145 versus time for $\rho=0.24$ to 0.97. Each curve is displaced vertically by an amount proportional to the normalized minor radius for that channel as indicated by the left ordinate scale. The highlighted bands indicate examples of events moving at ~ 300 m/s.

Politzer et al. PoP **9** (2002) 1962

Coherent Structures

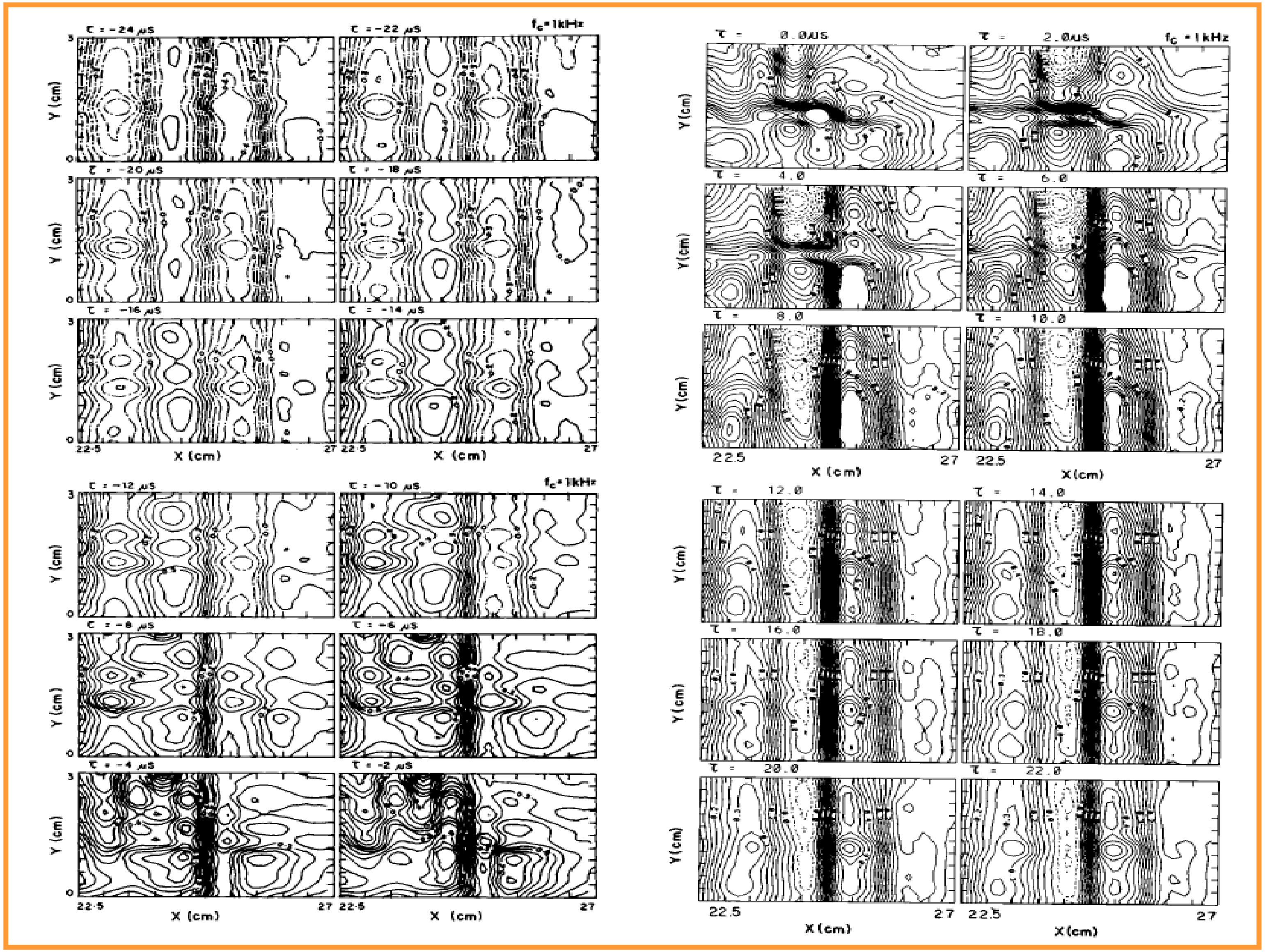
Clearly coherent structures are playing an important role!

Do we have direct evidences for the presence of coherent structures ?

YES !

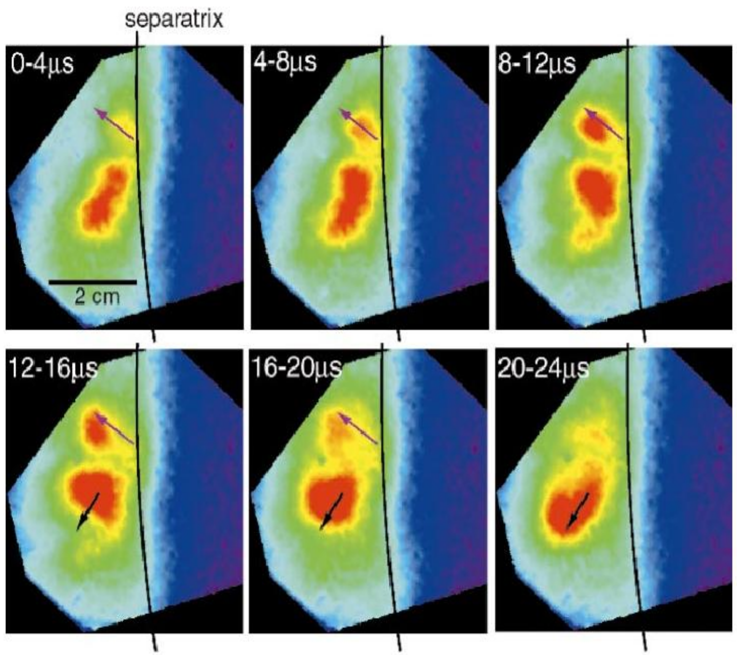
By carrying out conditional statistics measurements, ADITYA showed that the non Gaussianity in edge- SOL turbulence is due to presence of coherent structures
(IAEA Seville Conf 1994 , Phys Plasmas 4 ,4292, 1997 ; 4, 2982, 1997)

Coherent Potential Structures in ADITYA edge plasma.
Joseph et al Phys Plasmas (1997)



Seen also in ASDEX, Caltech Tok, NSTX, Alc CMOD

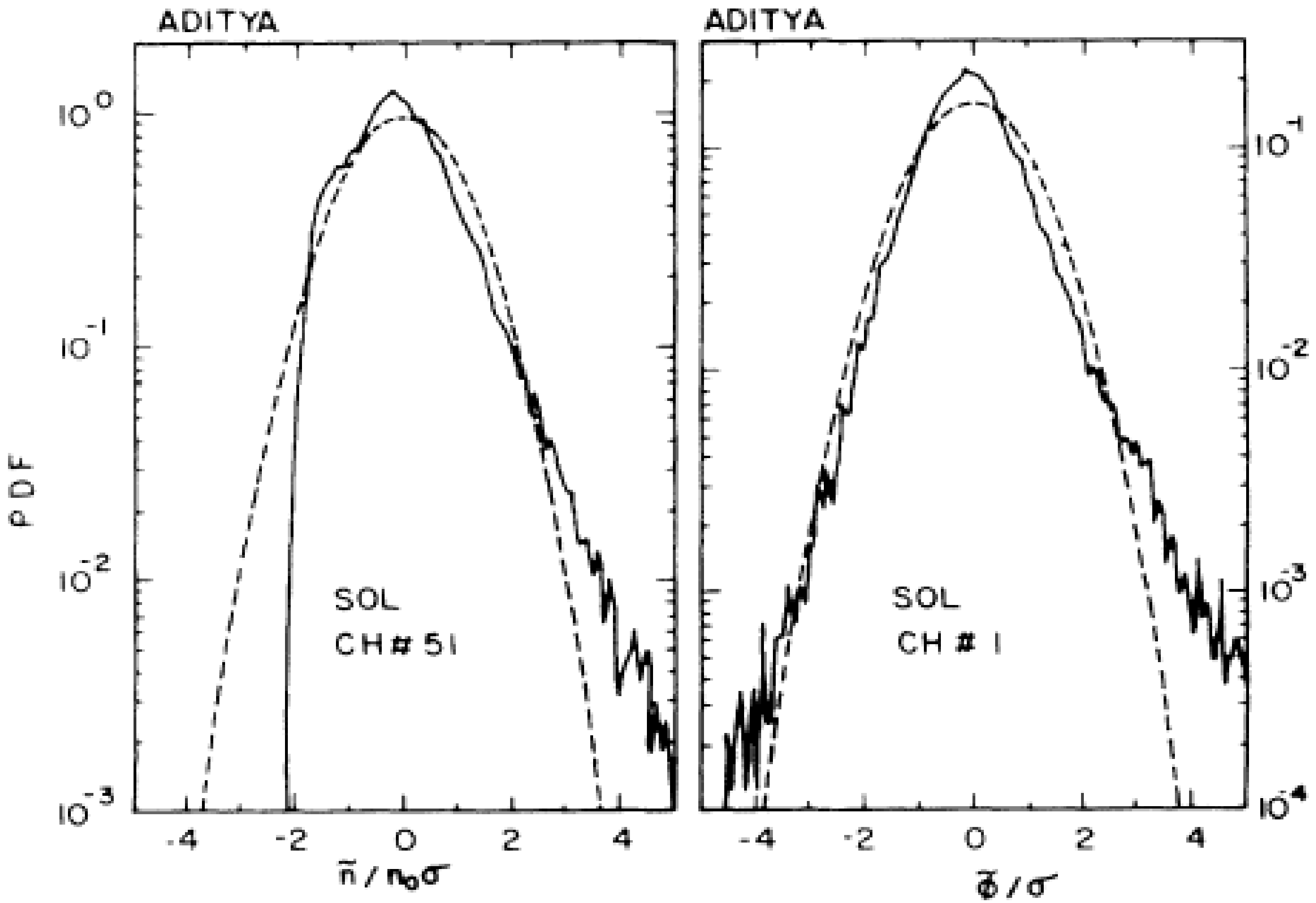
Coherent Structures (continued)



Blob motion and splitting are observed in structures outside the separatrix as seen in optical emissions

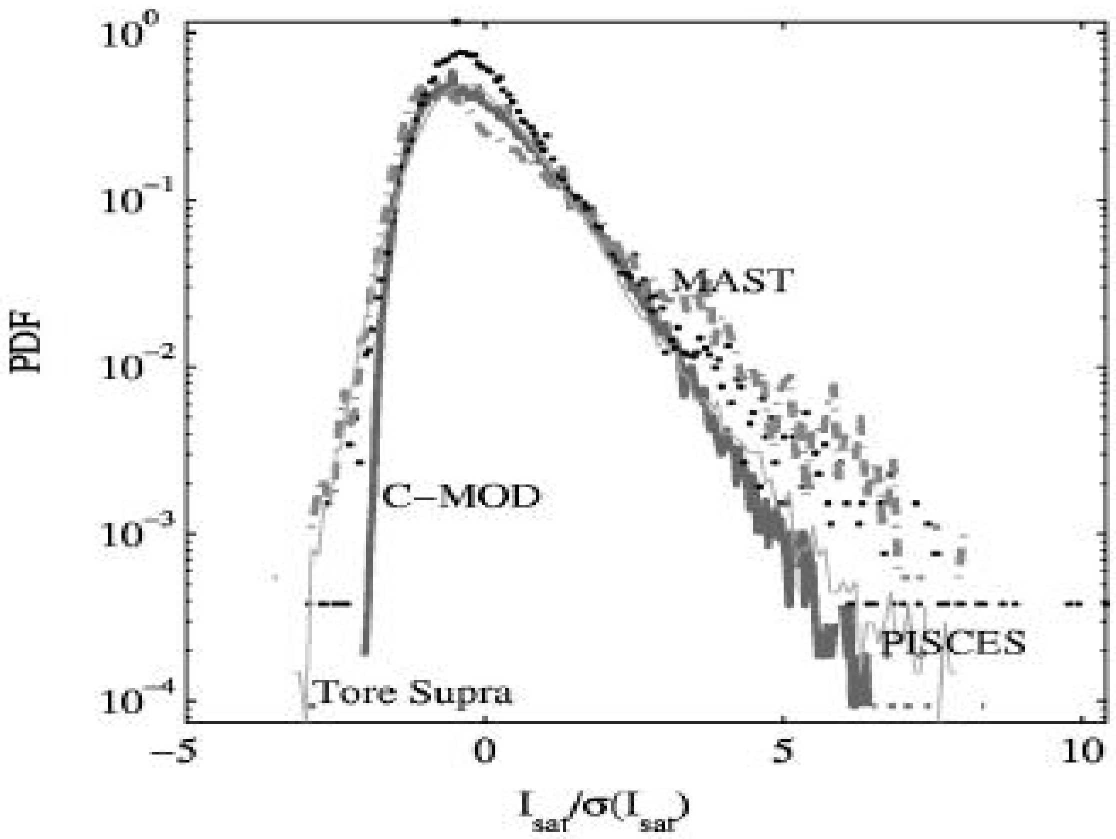
Zweben *et al* POP 9, 1981 (2002)

PDF of density and potential fluctuations in ADITYA
Jha et al Phys Rev Letters (1992)



Also seen in Alcator CMOD, TJ II, TS, ATF, DIIID etc

Experimental evidence for Intermittency



- Universal nature
- Deviation from Gaussian PDF.
- Positive tail

Antar et al POP 10, 419 (2003)

Turbulent heat and particle Transport in Tokamaks

Observation: Transport episodic and bursty!

Evidences:

1. Langmuir Probe data from edge !

- Intermittent coherent structure observations

  ADITYA (Jha et al. 1994, 1997)

  ASDEX (Endler et al. 1995)

- Probablity Distribution function (PDF)

  (long tails & power laws)

  TJ-II,W7-AS etc. (Hidalgo et al. 1994-99)

Turbulent heat and particle Transport in Tokamaks (Contd.)

Evidences:

2. ECE measurements from large Tokamaks show radial propagation of avalanche like events with speeds few hundred meters/ sec. **JET, D-III-D (Politzer 1998)**

Present understanding of formation coherent structures and their influence on transport

P. H. Diamond, S. Itoh, K. Itoh and T. S. Hahm PPCF **47** R35 – R161(2005).

- Transport inhibiting zonal patterns
- Transport enhancing streamer structures.
- Numerical evidences through gyrokinetic simulations as well as simple fluid models

Large Scale Computer Simulations

- Gyrokinetic and Gyrofluid Simulations
  * ITG (Lin et al. , Hammet et al.)
  * ETG/ITG (Dorland et al.)

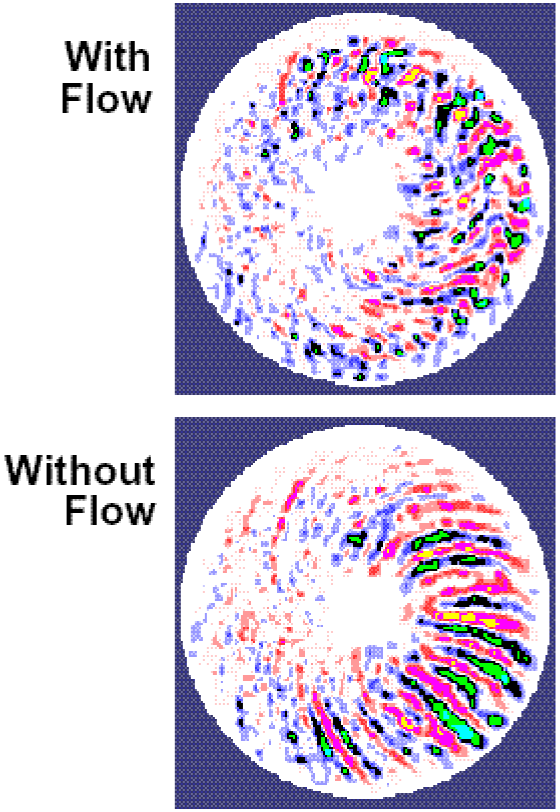
- Resistive fluid Simulations (Edge/ SOL)
  (Carreras et al., Sarazin et al. 2000, Drake et al.)

Have shown transport features associated with the distinct structures

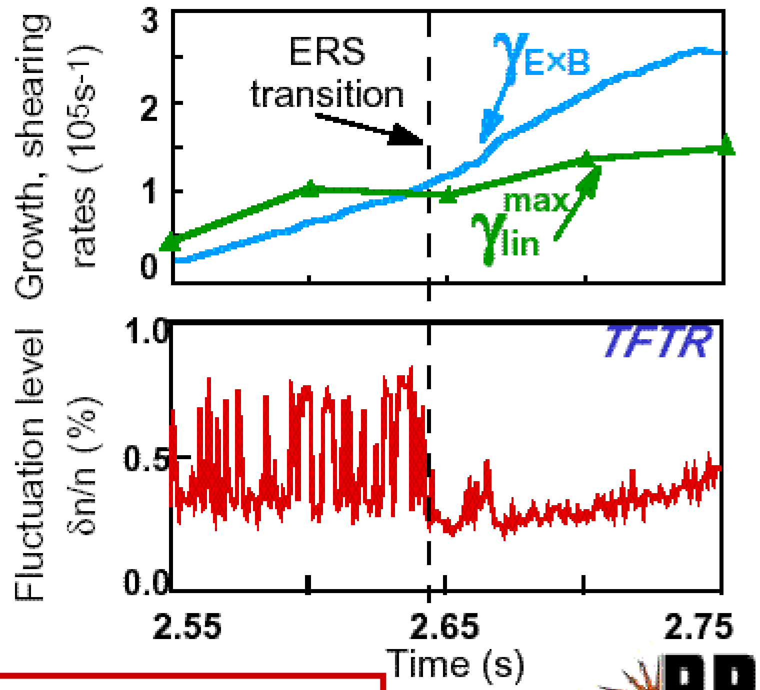
The underlying Physics very complex and nonlinear

Turbulent Fluctuations Suppressed When Flow Shearing Rate Exceeds Maximum Linear Growth Rate of Instabilities

Simulations show turbulent eddies disrupted by strongly sheared plasma flow



Turbulent fluctuations are suppressed when shearing rate exceeds growth rate of most unstable mode



Z. Lin (1998) Science

PPPL PRINCETON PLASMA PHYSICS LABORATORY

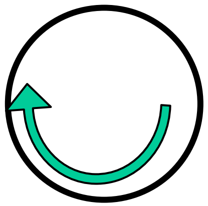
Self regulation of turbulence induced transport

- Importance has been realized in magnetic confinement fusion devices.
- Geometry of fusion devices pretty complicated.
- Added complexity due to magnetic shear effects.
- Difficulty in appreciating the physics of self regulation for such a complex system.

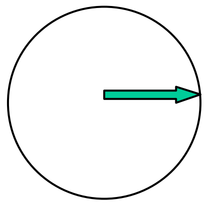
Minimal Physics for generation for such flow patterns ?

Two distinct flow patterns
- Poloidally symmetric shear flow patterns (Zonal Flows)
- Radially extended shear flows (Streamers)

Zonal Flows
(transport inhibiting structures)

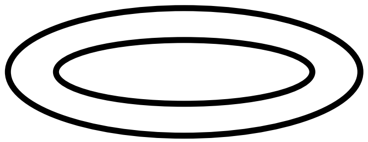


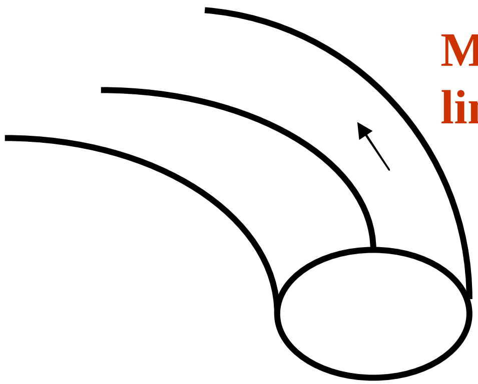
Streamer Flows
(transport enhancing structures)



It is of interest to seek the possibilities of self consistent generation of these flow patterns in the context of a minimal model!!

MCDRT: Magnetic curvature driven RT turbulence

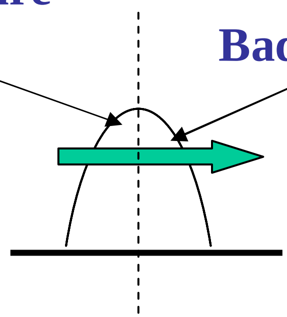




Magnetic field lines are toroidal

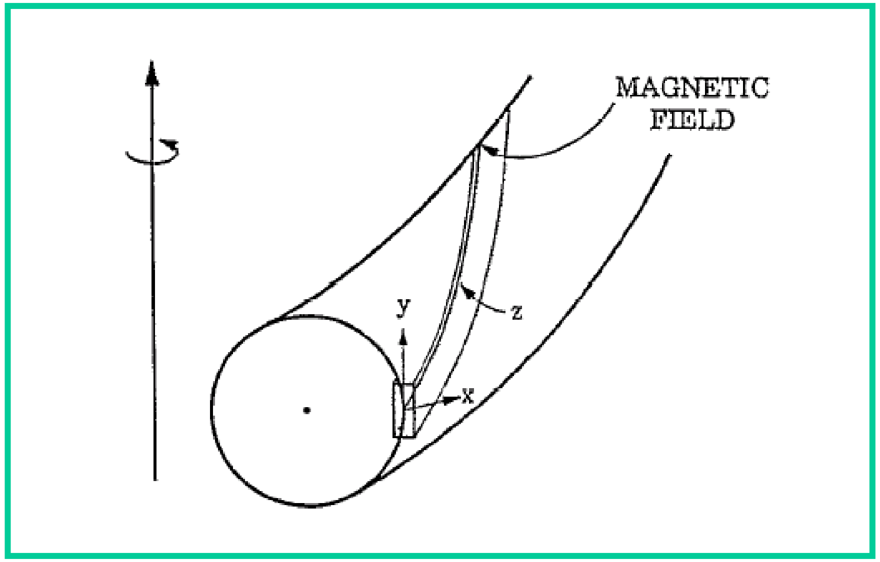
Good curvature region

Bad curvature region



Centrifugal force (effective 'g')

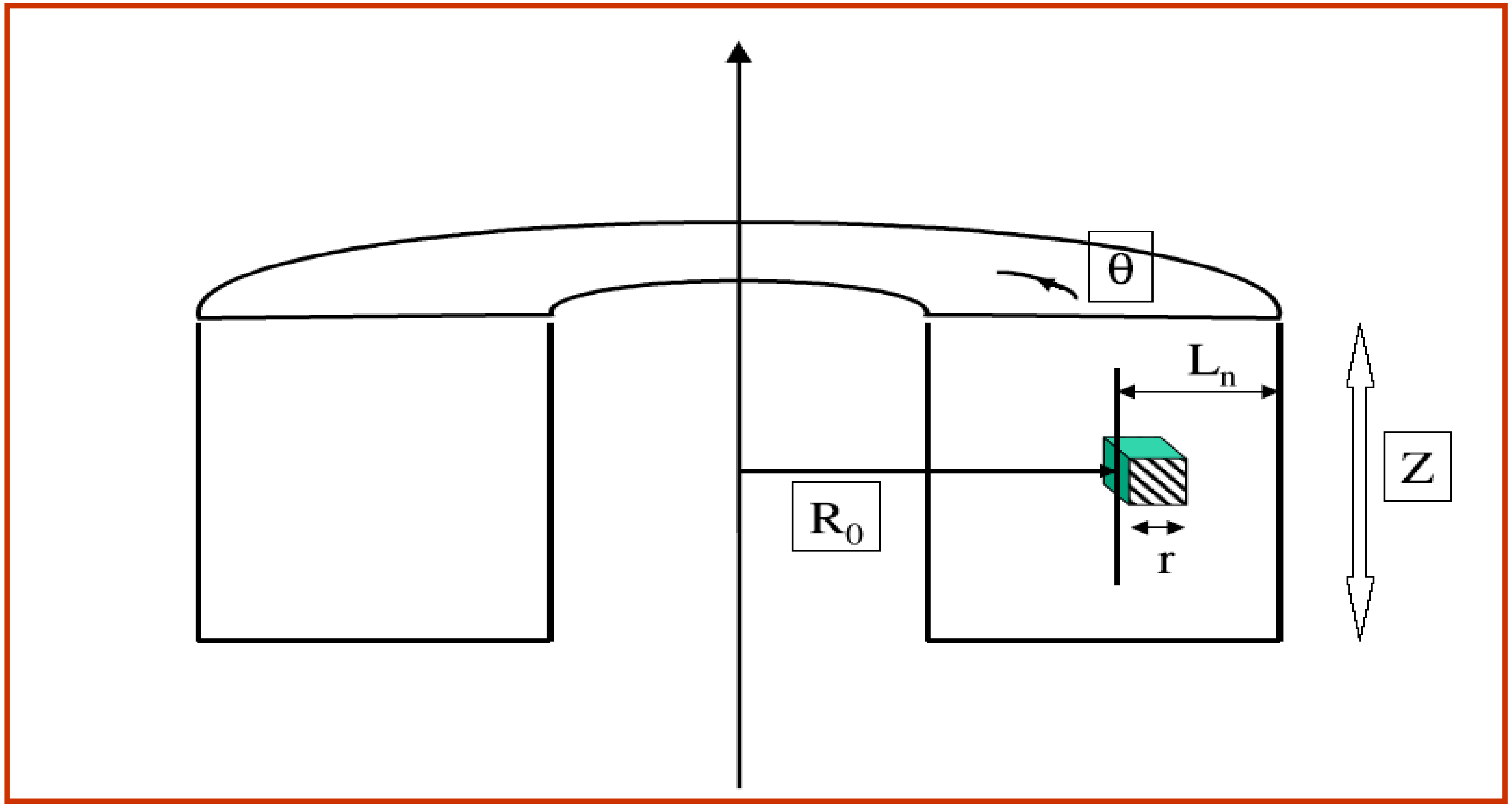
Currentless toroidal devices



Tokamak

Spatial Domain for model representation.

Slab: x for r
y for Z
and z for θ



Equilibrium and ordering scheme

$B_{eq} = B_0(1-x/R)e_z$
$N_{eq} = n_{00}\exp(-x/L_n)$

Electron diamagnetic drift produces equilibrium current J_{eq}

Have retained terms upto order ε^2 where the ordering scheme is:

$$\frac{\omega}{\omega_{ci}} \sim n \sim \phi \sim \frac{\psi}{\sqrt{\beta}} \sim \frac{\rho_s}{l_z} \sim \frac{\rho_s}{L_n} \sim \frac{\rho_s}{R} \sim \varepsilon$$

Perpendicular drifts considered

Ions: E x B and polarization.

Electrons: E x B and diamagnetic

$V_A^2 = 1/\beta$

$V_g = 2\rho_s/R$
$V_n = \rho_s/L_n$

3 dimensional electromagnetic

- Governing equations obtained from electron continuity, quasi-neutrality condition and the inertia-less parallel momentum equation for electrons.
- Variations along the parallel direction is incorporated.
- Electromagnetic effects due to field line bending considered.

Governing equations

Slab approximation

Radial – x
Poloidal – y
Toroidal - z

Normalizations:
Magnetic field: B_0
Time : ω_{ci}^{-1}, length: ρ_s

$$\frac{\partial n}{\partial t} + V_g\frac{\partial n}{\partial y} + (V_n - V_g)\frac{\partial \varphi}{\partial y} + \hat{z}\times\vec{\nabla}\varphi\cdot\vec{\nabla}n - V_A^2\left\{\frac{\partial}{\partial z}\nabla_\perp^2\psi + \hat{z}\times\vec{\nabla}\psi\cdot\vec{\nabla}\nabla_\perp^2\psi\right\} = D\nabla^2 n$$

$$\frac{\partial}{\partial t}\nabla^2\varphi + V_g\frac{\partial n}{\partial y} + \hat{z}\times\vec{\nabla}\varphi\cdot\vec{\nabla}\nabla^2\varphi - V_A^2\left\{\frac{\partial}{\partial z}\nabla_\perp^2\psi + \hat{z}\times\vec{\nabla}\psi\cdot\vec{\nabla}\nabla_\perp^2\psi\right\} = \mu\nabla^4\varphi$$

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial z}(n - \varphi) + V_n\frac{\partial \psi}{\partial y} - \hat{z}\times\vec{\nabla}\psi\cdot\vec{\nabla}(\varphi - n) = \eta_s V_A^2\nabla_\perp^2\psi$$

Invariances

The equations remain invariant under the following scaling transformations

$$z \rightarrow \frac{\bar{z}}{a}; \quad \eta_s \rightarrow a^2 \overline{\eta}_s; \quad \psi \rightarrow a\overline{\psi}; \quad V_A^2 \rightarrow \frac{\overline{V}_A^2}{a^2}$$

Thus note that combinations such as

$$\eta_s V_A^2; \quad k_z V_A; \quad \eta_s k_z^2$$

remain invariant under the transformation.

Scalings help is establishing equivalence in a wide class of phenomena.

2d limit

$$\boxed{\frac{\partial}{\partial z} = 0} \quad \Longrightarrow$$

$$\frac{\partial n}{\partial t} + V_g \frac{\partial n}{\partial y} + (V_n - V_g)\frac{\partial \varphi}{\partial y} + \hat{z} \times \vec{\nabla} \varphi \cdot \vec{\nabla} n = D \nabla^2 n, \qquad (1)$$

$$\frac{\partial}{\partial t} \nabla^2 \varphi + V_g \frac{\partial n}{\partial y} + \hat{z} \times \vec{\nabla} \varphi \cdot \vec{\nabla} \nabla^2 \varphi = \mu \nabla^4 \varphi. \qquad (2)$$

Linearly growing modes with growth rate

$$\boxed{\gamma = \frac{k_y}{k_\perp} \sqrt{V_n V_g}}$$

3d electrostatic limit

Perpendicular drift produces charging of the field lines. Finite k_z implies differential charging and promotes parallel currents.

$$\boxed{\eta_s V_A^2 \nabla^2 \psi >> \partial\psi/\partial t} \Longrightarrow \boxed{\nabla_\perp^2 \psi = \frac{1}{\eta_s V_A^2}\frac{\partial}{\partial z}(n-\varphi)}$$

$$\boxed{\frac{\partial n}{\partial t} + V_g \frac{\partial n}{\partial y} + (V_n - V_g)\frac{\partial \varphi}{\partial y} + \hat{z}\times\vec{\nabla}\varphi\cdot\vec{\nabla}n - \frac{1}{\eta_s}\left\{\frac{\partial^2}{\partial z^2}(n-\varphi)\right\} = D\nabla^2 n}$$

$$\boxed{\frac{\partial}{\partial t}\nabla^2\varphi + V_g \frac{\partial n}{\partial y} + \hat{z}\times\vec{\nabla}\varphi\cdot\vec{\nabla}\nabla^2\varphi - \frac{1}{\eta_s}\left\{\frac{\partial^2}{\partial z^2}(n-\varphi)\right\} = \mu\nabla^4\varphi}$$

Objective

2 dim		3 dim		3 dim
Electrostatic Role of V_n, V_g, D, μ, box size	→	**Electrostatic** Additional parameter : k_{par}, η	→	**Electromagnetic** Additional parameter: plasma beta

- Seek possibility of self regulation in the various models.
- Study the role of various parameters in the detailed process of self regulation. (e.g. pattern formation, its characteristics and the resultant saturation levels etc.).

Nonlinear Simulations

- 2d simulation.
- 3d electrostatic simulation.
- 3d electromagnetic simulation

1. A. Das, S. Mahajan, P. Kaw, A. Sen, S. Benkedda and A. Verga Phys. Plasma, **4** 1018 (1997).
2. A. Das, A. Sen, S. Mahajan and P. Kaw. Phys. Plasma, **8** 5104 (2001).
3. A. Das, A. Sen, P. Kaw, S. Benkedda and P. Beyer. Phys. Plasma **12** 032302 (2005).

2d results (contd.)

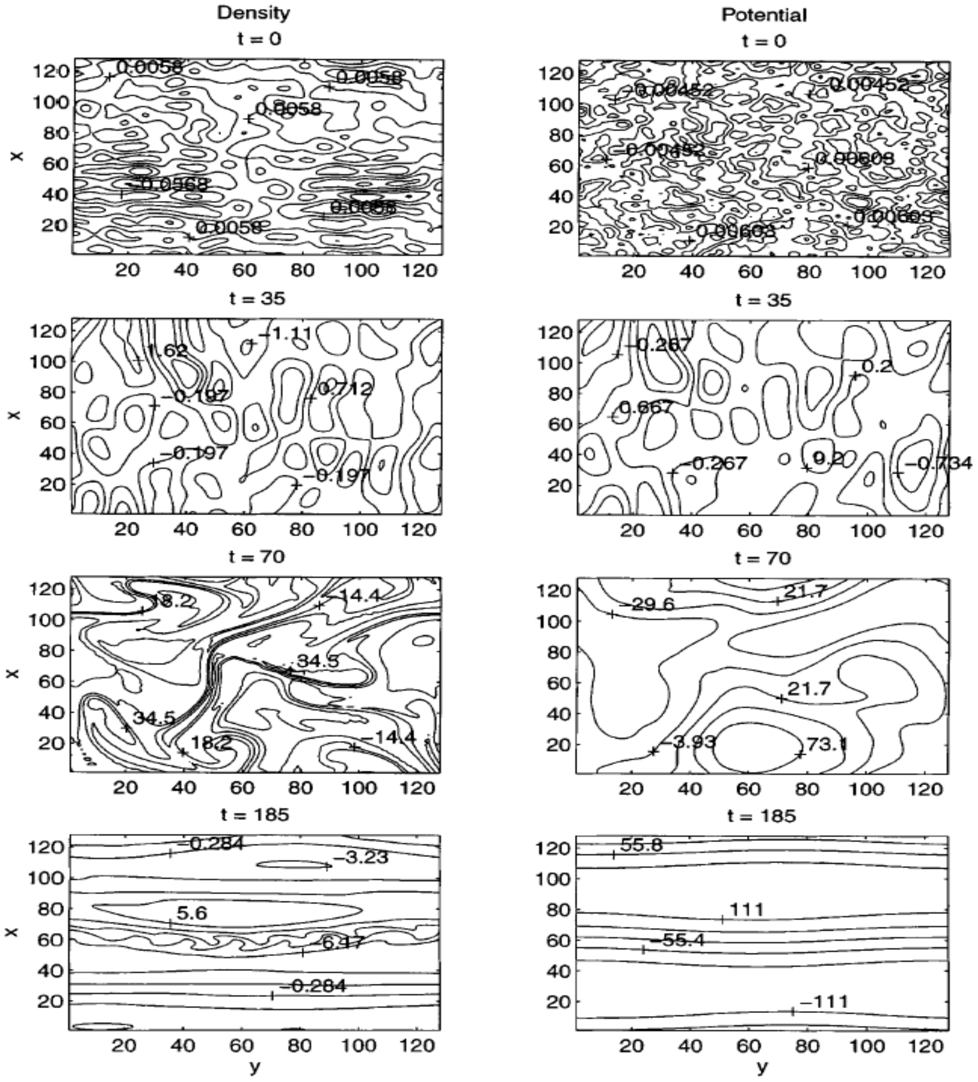


FIG. 3. Contour plot of density and potential at various times.

2d results (contd.)

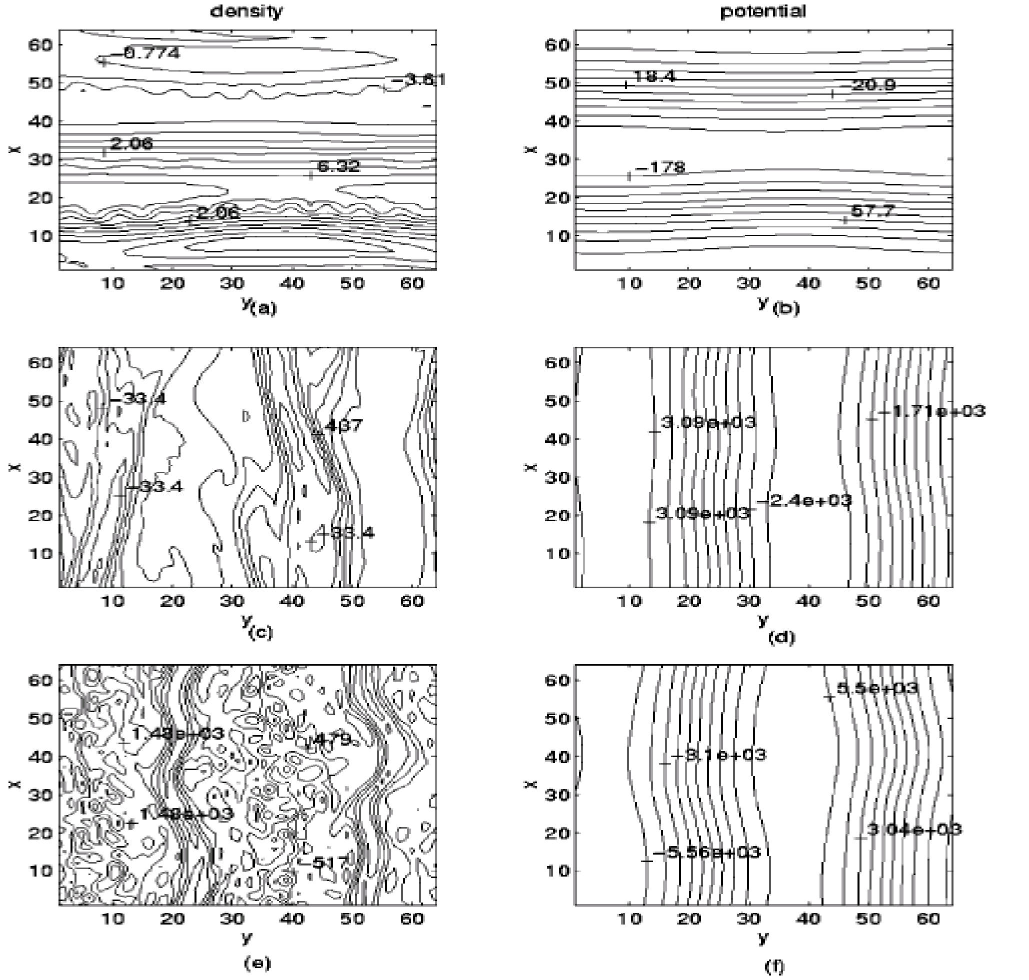


FIG. 2. The iso-contour plots of density and potential at $t=100$ for three different sets of dissipation parameters D and μ. $(D,\mu)=(0.1,0.1)$ in (a),(b); $=(4.0,0.1)$ in (c),(d) and $=(0.1,4.0)$ in (e),(f).

Pattern selection

No initial and/or boundary related anisotropies.
Dynamics governs the asymmetry of the final state.

Dynamics in linear regime prefers formation of patterns having structure in ‘y’ (poloidal) direction. Yet final saturated state comprises of poloidally symmetric shear flow patterns!

For low values of dissipation saturated states with zonal symmetry are obtained, whereas for high values growing radially extended patterns are observed!

2d results (contd.)

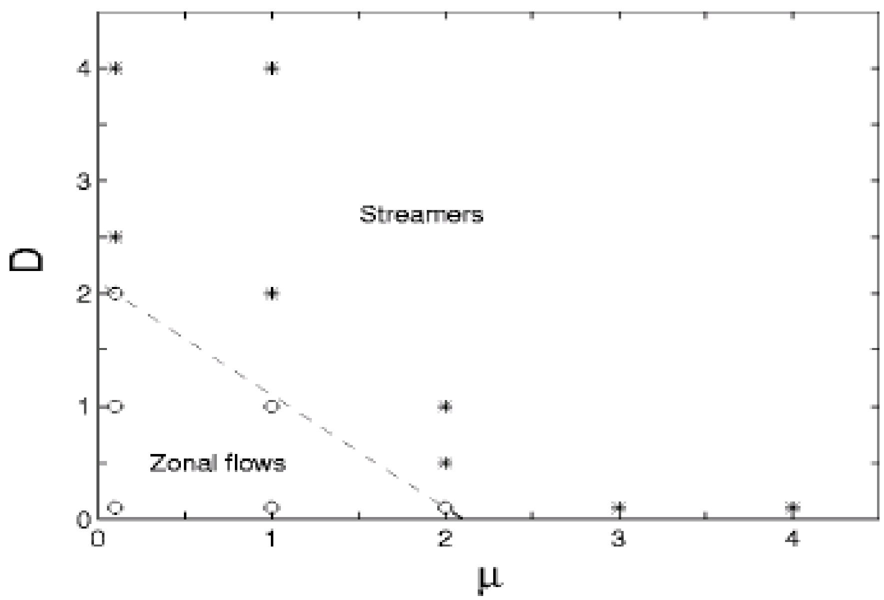


FIG. 3. A consolidated phase diagram in D, μ space delineating regions corresponding to the predominance of zonal and streamers structures in the final state. The circles and asterisks represent the values of D and μ where the numerical simulation was actually carried out leading to the formation of zonal and streamer structures, respectively.

Pattern selection by dynamics

Physics of bifurcation

Cascade towards long scale governed by properties of polarization drift nonlinearity.

$$\boxed{\hat{z} \times \nabla \phi \bullet \nabla \nabla^2 \phi} \longleftarrow$$

Polarization drift nonlinearity.
Potential evolution

$$\boxed{\hat{z} \times \nabla \phi \bullet \nabla n} \longleftarrow$$

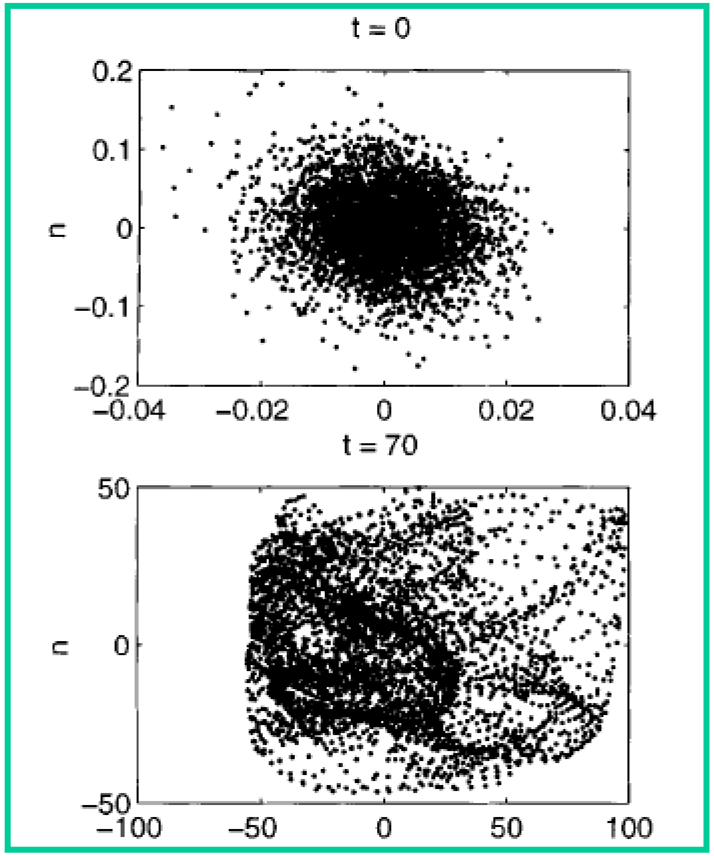
Advective nonlinearity
Density evolution

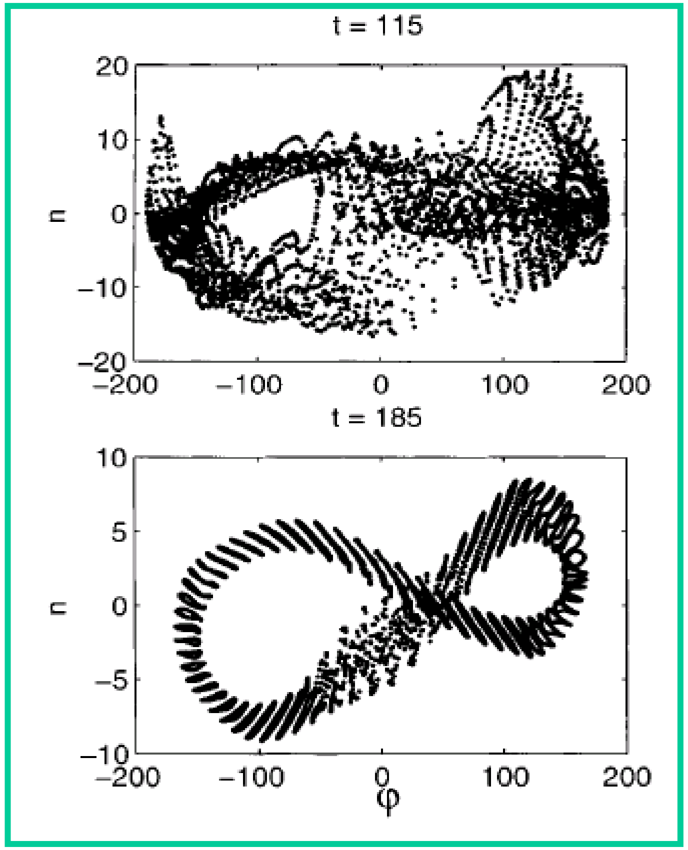
- Density gets slaved to potential; cascade towards long scale due to polarization drift in potential equation dominates.
- Low D and μ small scale RT fluctuations grow. Power cascade towards long scales in nonlinear regime generates streamer and zonal patterns.

Physics of bifurcation (contd.)

- The form of the nonlinearity is such that a short scale spectrum biased at high k_y leads to larger nonlinear growth of zonals!
- The shear in zonal flow then stabilizes the growing streamers.
- For large D and μ the short scale fluctuations are damped. No nonlinear growth of zonal flow. Linear growth of streamers cannot be contained.

2 – dim scatter plots of ‘n’ vs. φ





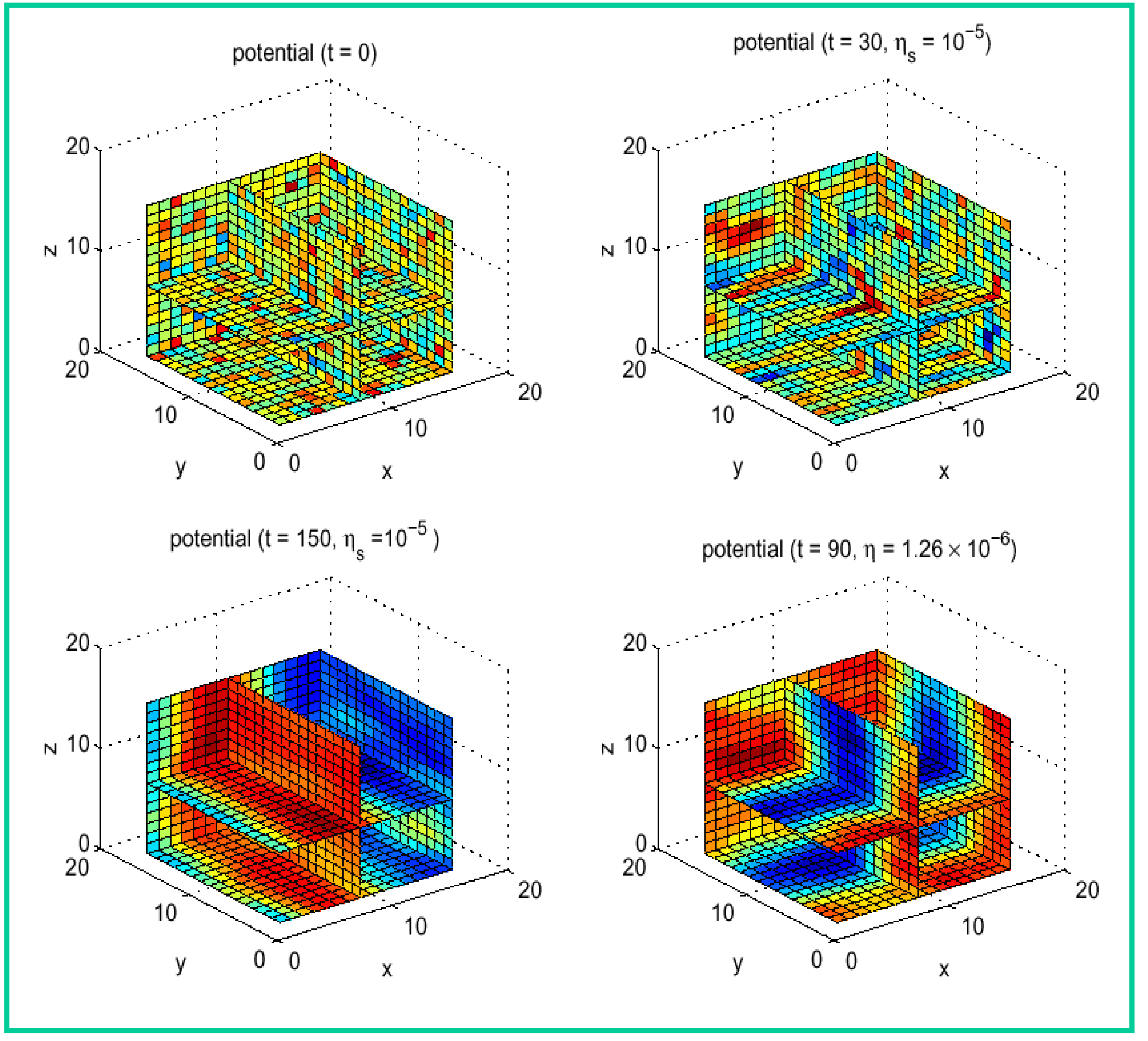
Evidence of slaving!!

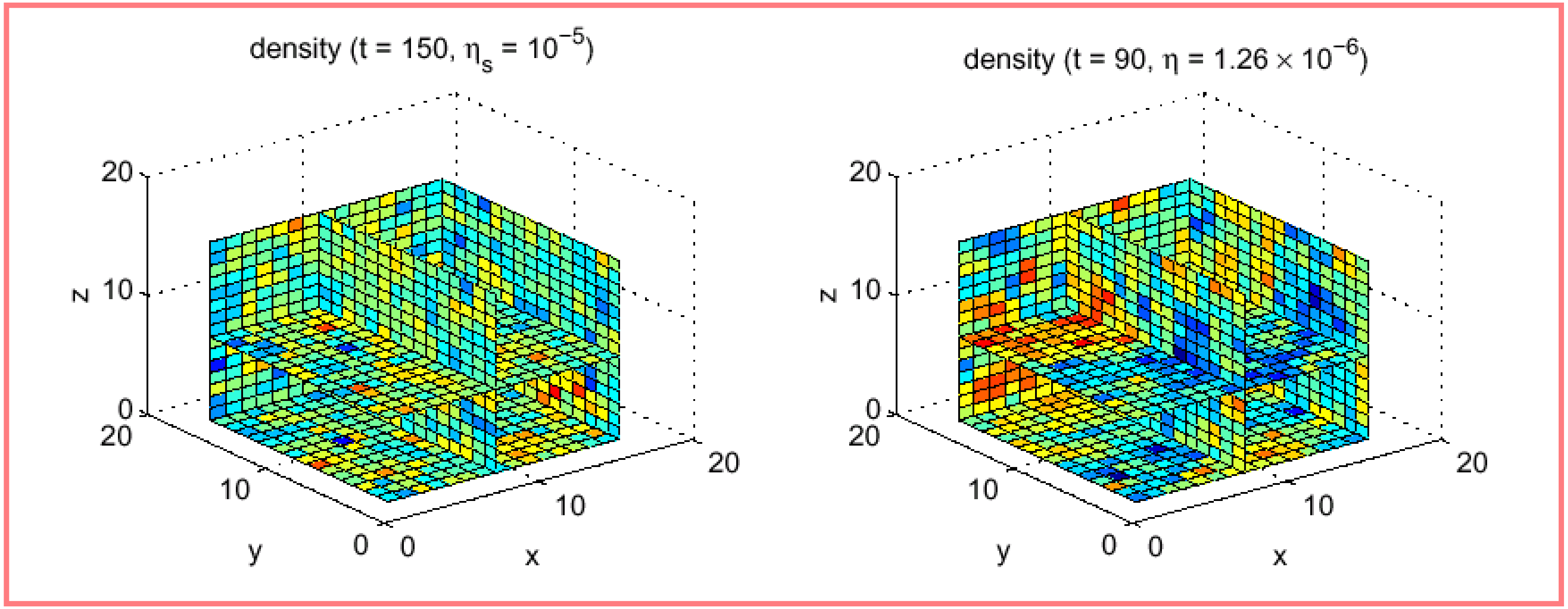
3d Simulations (electrostatic model)

Additional dissipation parameter η_s

Effect of k_z

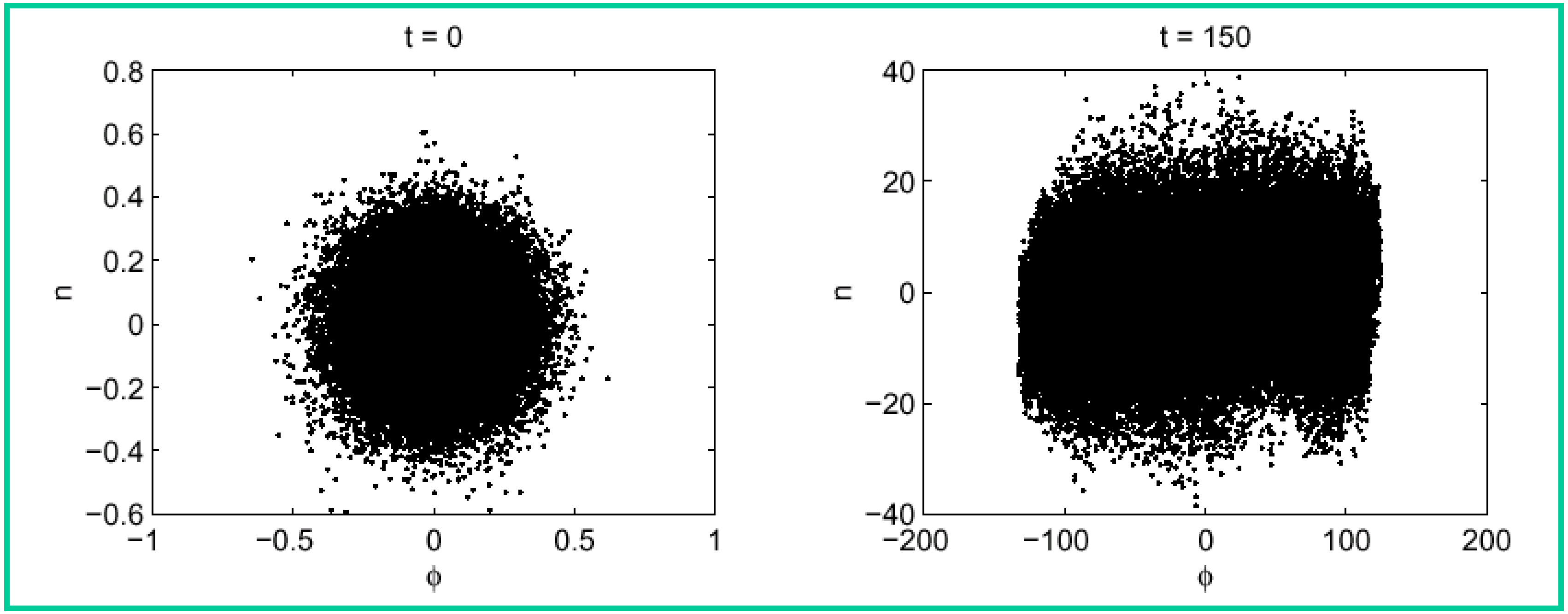
High resistivity forms zonal patterns and yields saturation.



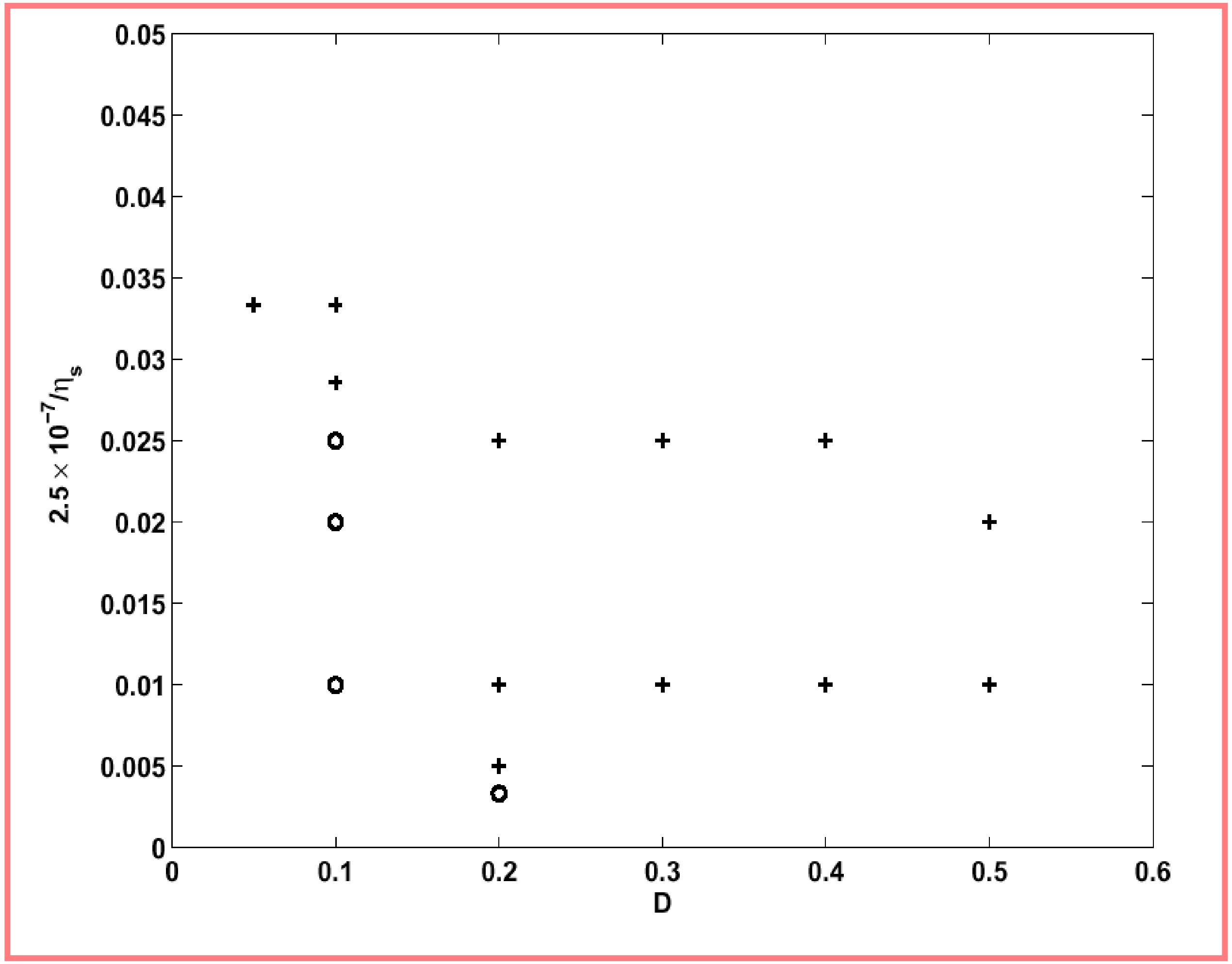


Density shows dominance of short scale structures in both cases. No slaving to potential field.

Evidence for non slaving



3 –dim simulations scatter plots of density vs. φ



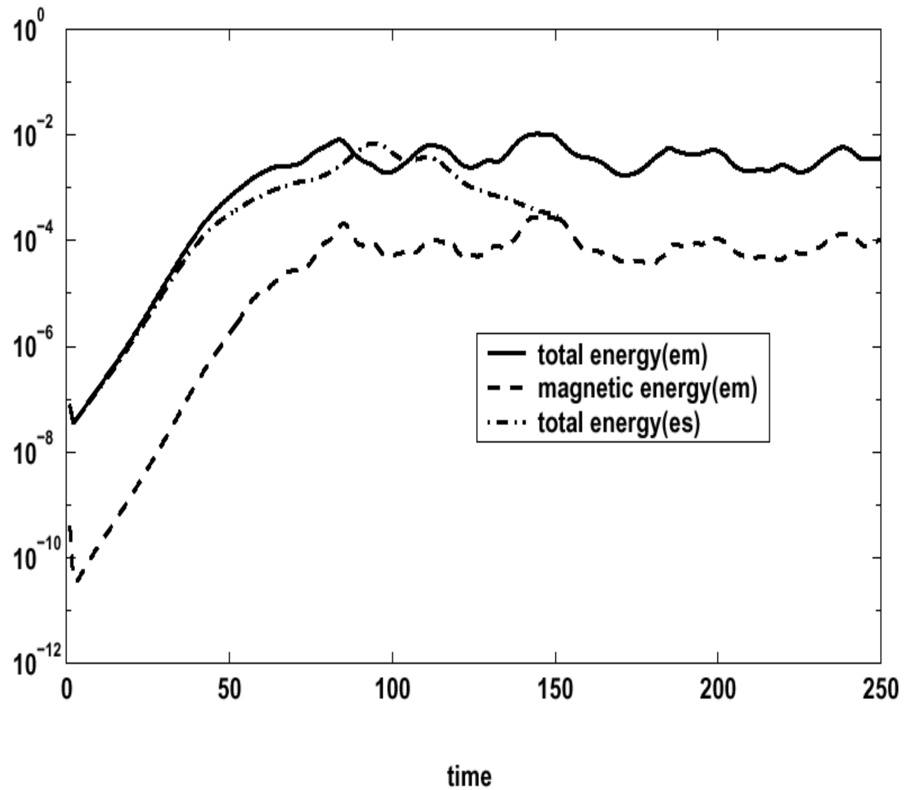
3-dim electrostatics

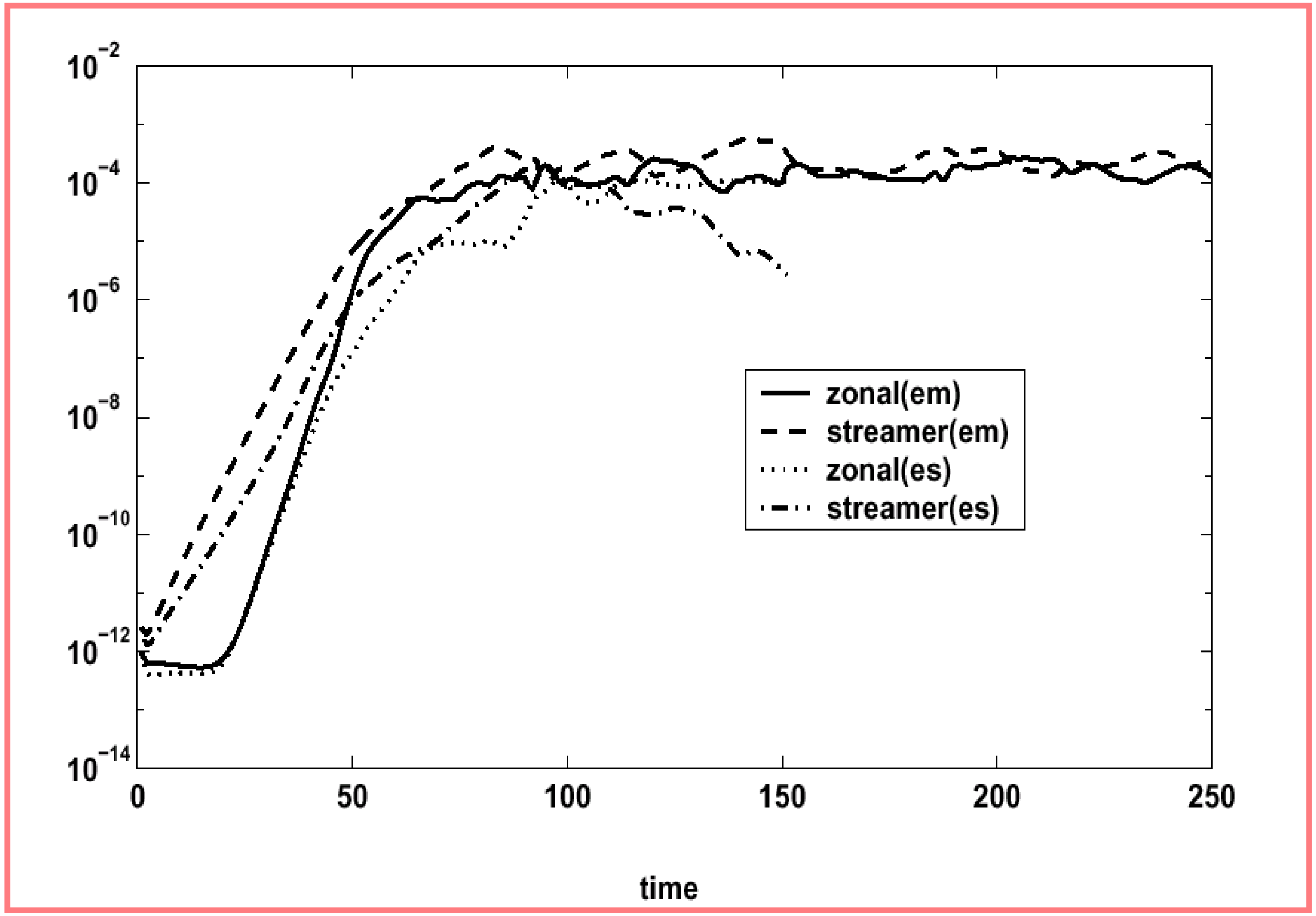
Circles indicate stable zonal pattern and + sign growing streamers.

3 dim Electromagnetic

Plasma $\beta = c_s^2/v_a^2 = 1/V_A^2$; is an additional parameter

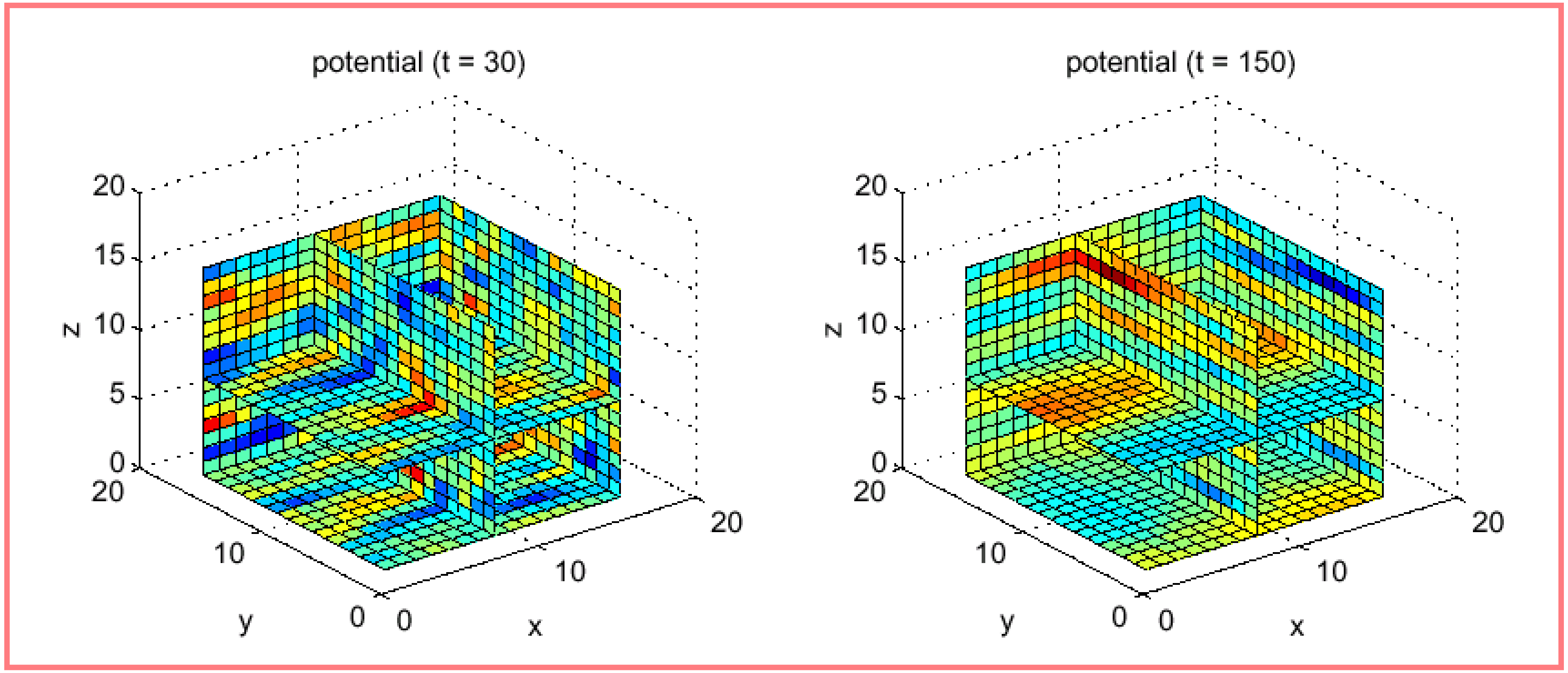
In electromagnetic case saturation occurs at a higher value of the total energy!





Novel state with identical zonal and streamer intensity!

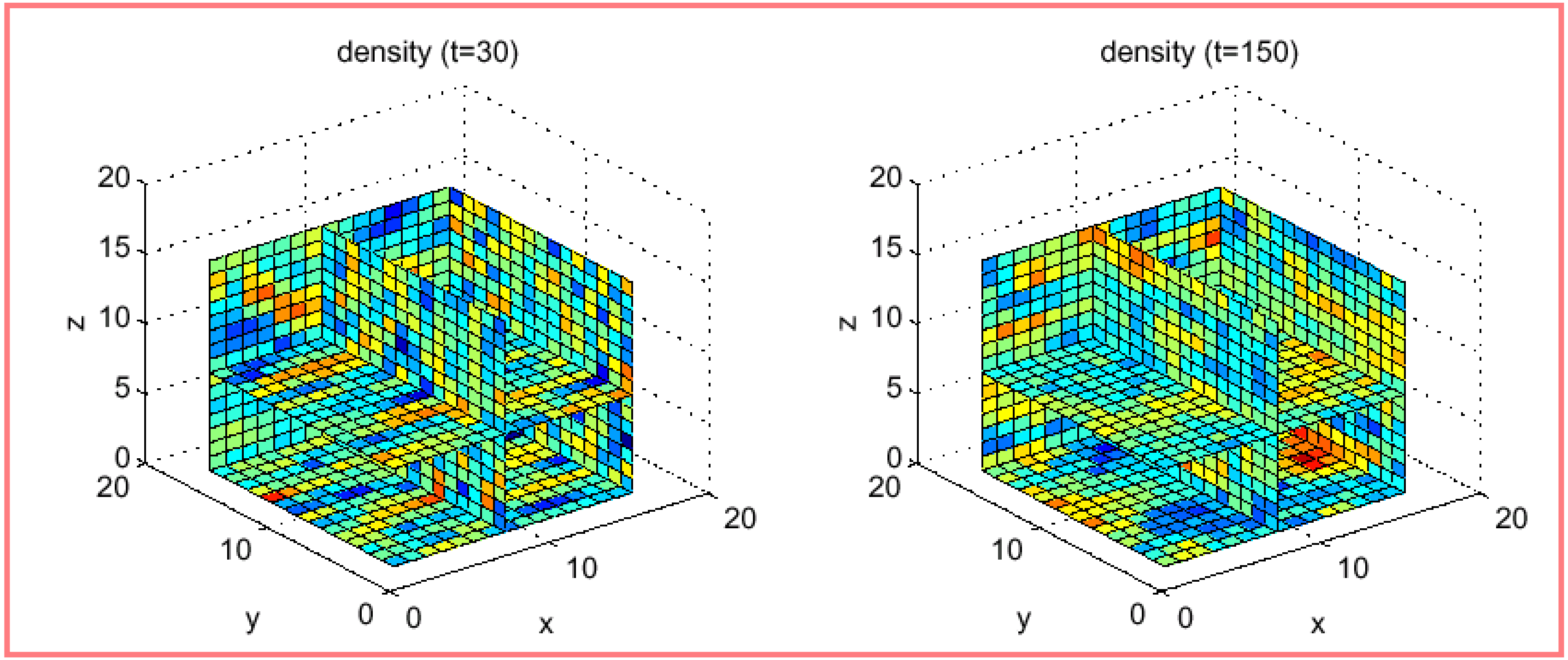
Electromagnetic effects inhibit zonal formation and hence stabilization becomes difficult in this case.



Linear regime

Nonlinear regime

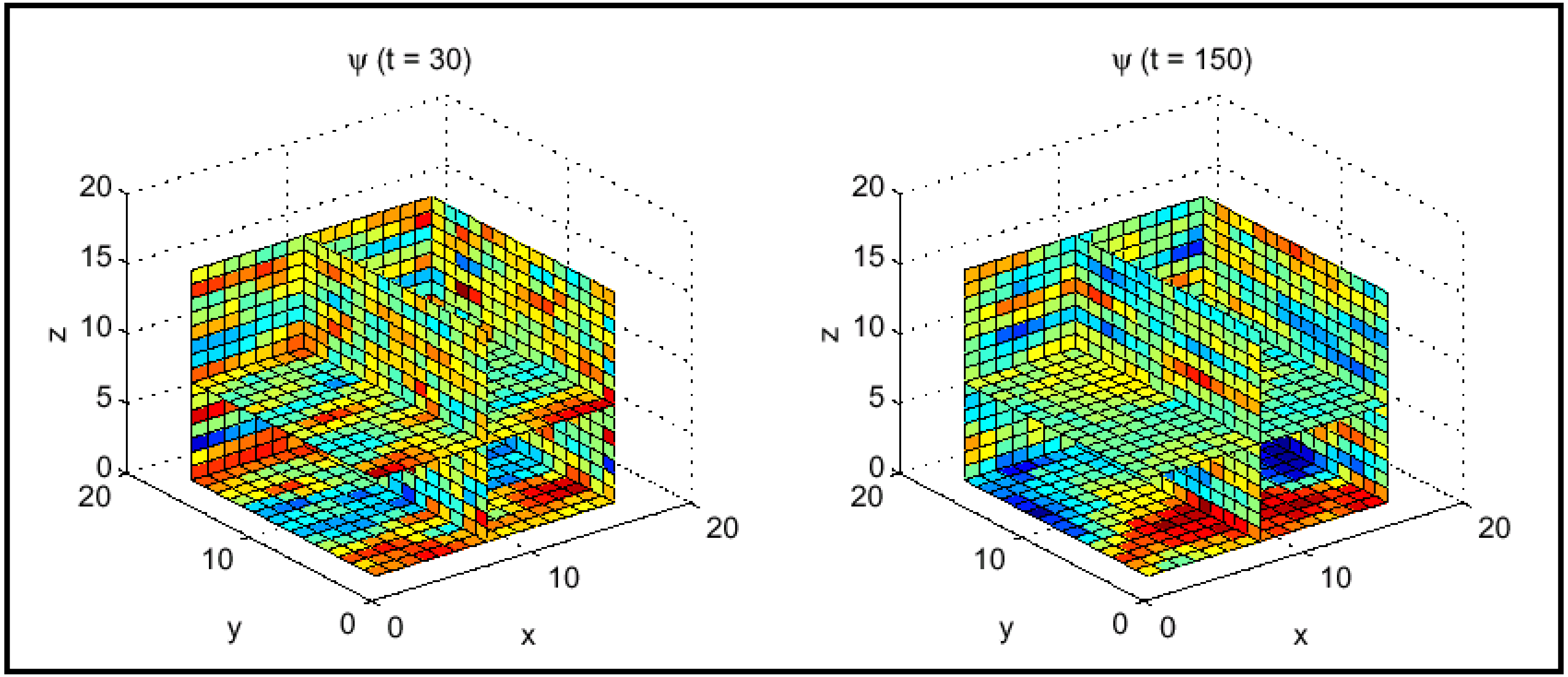
Shows no predominance of zonal power.
Significant structure along ‘z’



Linear regime

Nonlinear regime

Similar to 3-dim electrostatic simulations density has significant spectral power in short scales. No slaving to potential.



Linear regime

Nonlinear regime

Minimal Physics

- Polarization nonlinearity (Reynolds stress) - long scale structure formation.
- Anisotropy of short scale spectrum (characteristic of the instability) decides the pattern symmetry.
- Electromagnetic effects weaken the formation of long structures.

Plasma Turbulence : problem of many scales

Structures and fluctuations with a wide range of scales are present

How do we describe such a system ?

Successful in the context of critical phenomena of phase transition.

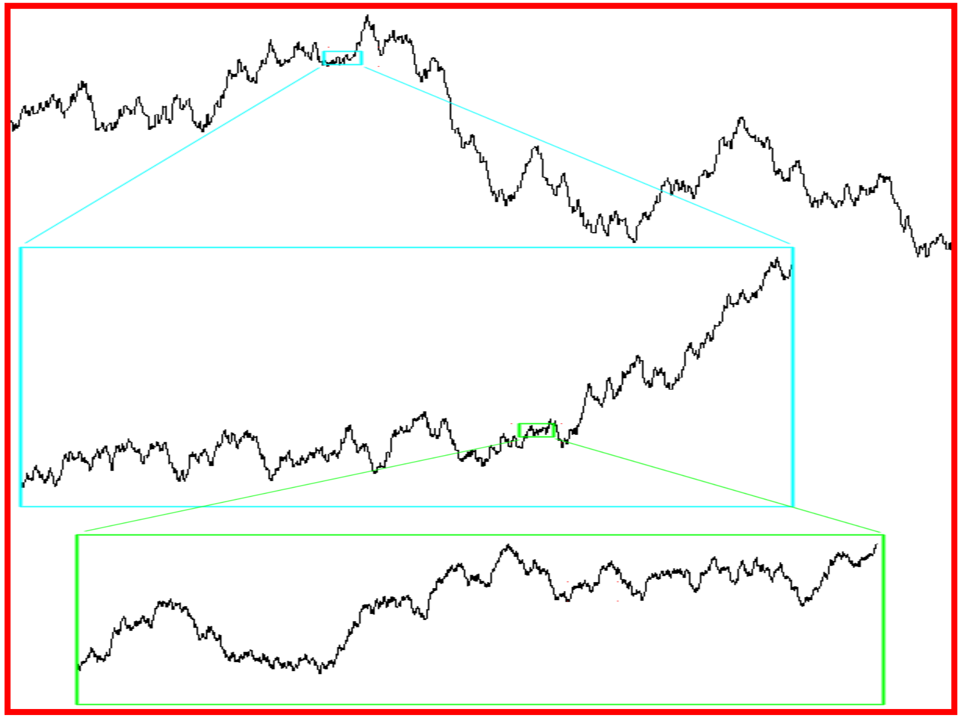
- Self similarity of scales.
- Rules connecting various scales.can be constructed.

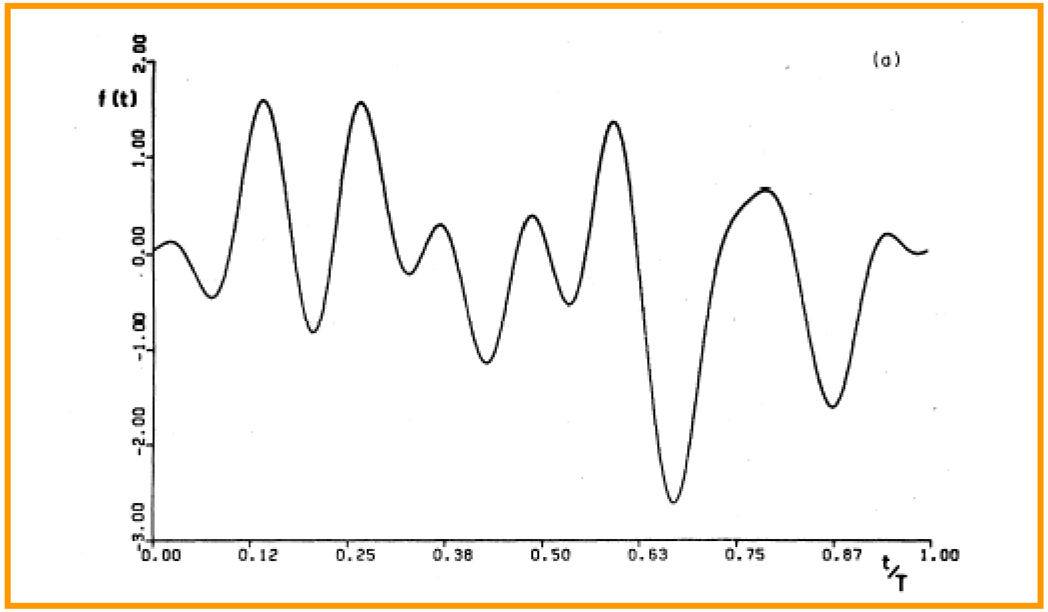
Many scales still …

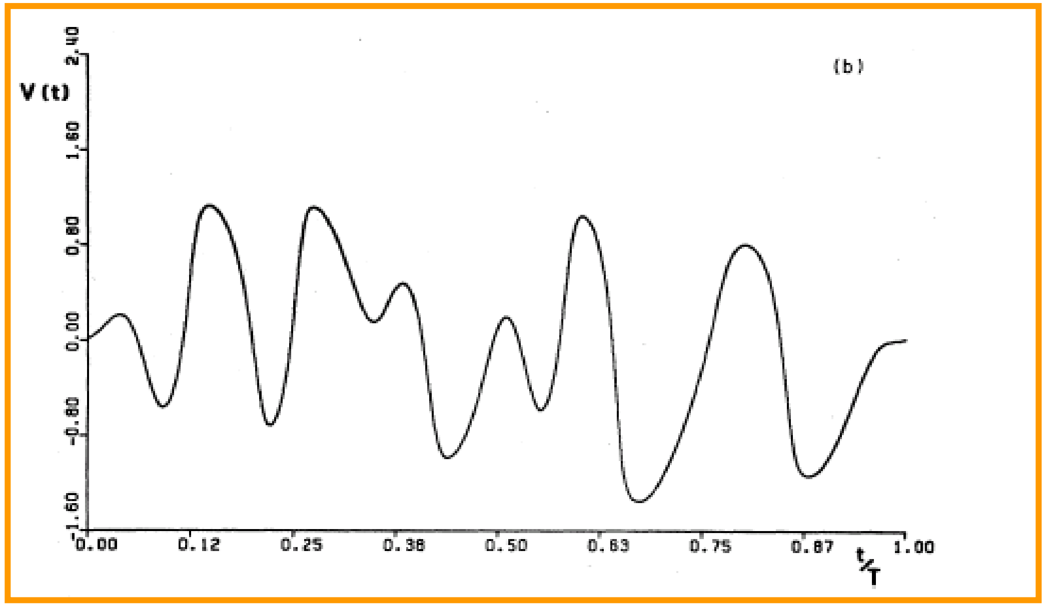
- Simple mathematical rules connect them.
- No special scale. (Scale invariant system)

•SOC system : Self similar

•Intermittency implies lack of this self similarity.
•Fluid turbulence is intermittent !



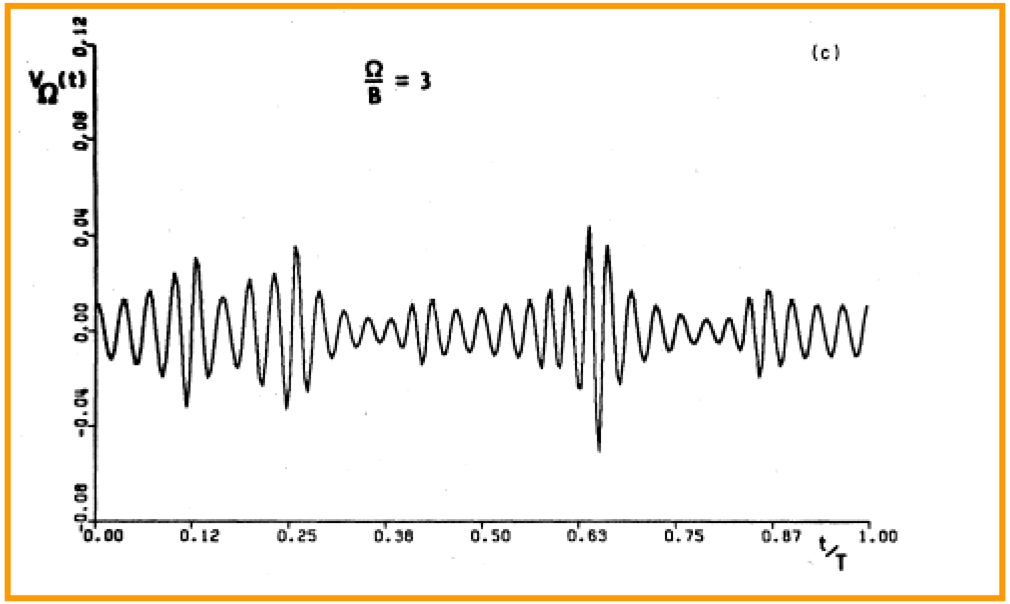


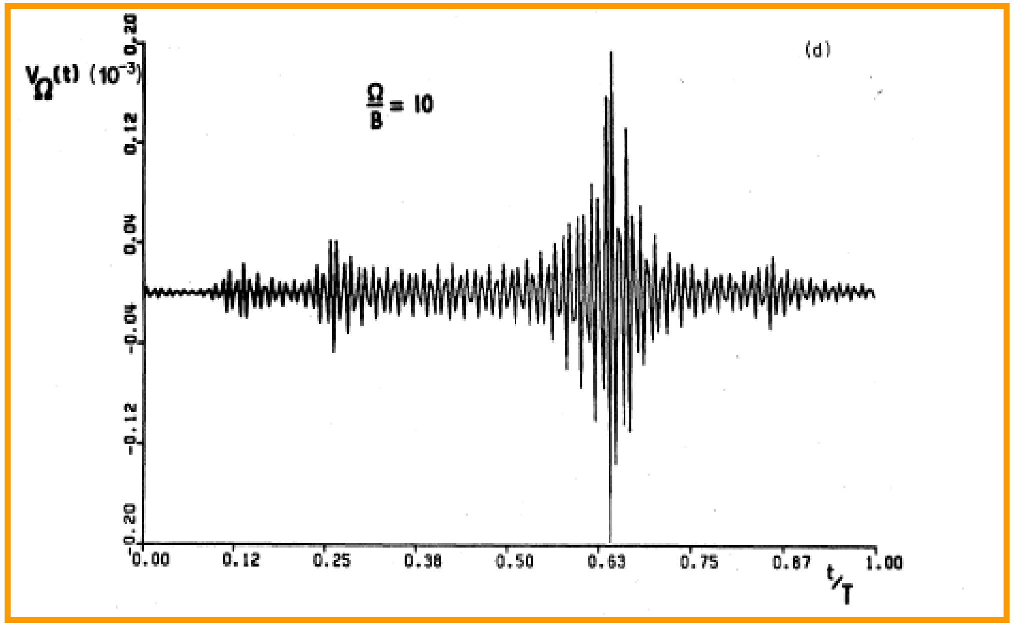


Numerical studies in turbulence

(a) One period of the forcing function.

(b) Unfiltered periodic solution for velocity.





Numerical evidence in turbulence (contd.)

(c) High pass filtered solution. Filter frequency 3 times the maximum excited mode in (a).

(d) Filter frequency is 10 times.

No Self Similarity

Intermittency: increased flatness of PDF

Observation of quiescent period. Activity gets restricted over limited time regime.

Let γ be the fraction time for which the signal is on.

$$\langle v_\gamma^2 \rangle = \gamma \langle v^2 \rangle$$
$$\langle v_\gamma^4 \rangle = \gamma \langle v^4 \rangle$$

$$F_\gamma = \frac{\langle v_\gamma^4 \rangle}{\langle v_\gamma^2 \rangle^2} = \frac{1}{\gamma} \frac{\langle v^4 \rangle}{\langle v^2 \rangle^2}$$

Clearly $\boxed{F_\gamma > F}$

Kolmogorov's similarity hypothesis

Navier Stokes Equation for fluids:

$$\boxed{\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V} = -\nabla P + \nu \nabla^2 \vec{V} + F}$$

Characterized in terms of dimensionless Reynold's number **Re=VL/ν**; large **Re** $\Longrightarrow$ implies nonlinear turbulent state.

K41:

- Energy input at large length scales L.
- Energy dissipation at small length scales l_d.
- Energy cascade in the intermediate inertial range due to nonlinear terms.

K41 (Contd.)

- Inertial range: energy transfer through local interaction of wavenumbers with a rate that is same for all scales.
- $\varepsilon = \varepsilon_k = v_k^2/\tau_k$ = external forcing rate = energy transfer rate in each scale. (ε homogeneous in space and time)

$$E = \int E(k)\, dk;$$

$$E \sim L^2 T^{-2};\ E(k) \sim L^3 T^{-2};\ \varepsilon \sim L^2 T^{-3}$$

$$\varepsilon = dE/dt$$

$$E(k) = \varepsilon^{\alpha} k^{\beta} \rightarrow \alpha = \frac{2}{3};\ \beta = -\frac{5}{3}$$

K41 (Contd.)

- Observations indicate deviations from 5/3rd law. $E(k) \sim k^{-5/3+\mu}$, μ is known as the intermittency coefficient.
- Deviations prominent in higher order structure functions.

Structure functions are defined as

$$S_p(r) = \left\langle \left[\delta v(\vec{x}, t, r)\right]^p \right\rangle$$

$$\delta v(\vec{x}, t, r) = \left[\vec{v}(\vec{x}, t) - \vec{v}(\vec{x} + \vec{r}, t)\right] \bullet \vec{r} / r$$

Structure functions and Intermittency

Note that

$$\boxed{\left\langle (\delta v_r)^2 \right\rangle = \frac{\int (\delta v_r)^2 d^3x}{\int d^3x} = \int |v_k|^2 k^2 dk = \int E(k)dk}$$

$$\Rightarrow \boxed{\left\langle (\delta v_r)^2 \right\rangle \sim |v_k| k^3 \sim E(k)k \sim k^{-2/3} \sim r^{2/3}}$$

$$\boxed{\left\langle (\delta v_r)^p \right\rangle \sim \left\langle \left[(\delta v_r)^2 \right]^{p/2} \right\rangle \sim r^{p/3}}$$

Structure fn of order p

Possible only for a Gaussian ensemble

Thus any deviation from linear p/3 scaling of p^{th} order structure function is a measure of Intermittency

Relationship between bivariate gaussian and scaling of S_p

Let $\phi_1 = \phi(x)$ and $\phi_2 = \phi(x+r)$; their joint bivariate gaussian PDF is given by: (here $\langle \phi_1^2 \rangle = \langle \phi_2^2 \rangle = a$ and $\langle \phi_1 \phi_2 \rangle = b$)

$$\boxed{P(\phi_1, \phi_2) \sim exp\left\{-\phi^T \Sigma^{-1} \phi\right\} = exp\left\{-\frac{\left[a\left(\phi_1^{\,2} + \phi_2^{\,2}\right) - 2b\phi_1\phi_2\right]}{\left(a^2 - b^2\right)}\right\}}$$

$$\boxed{\left\langle (\delta\phi)^2 \right\rangle = \left\langle (\phi_1 - \phi_2)^2 \right\rangle = 2(a-b) = f(r) = r^{\beta}}$$

Self similarity

Homogeneity

Gaussianity and scaling of S_p

$$P(\delta\phi) = \int P(\phi_1, \phi_1 - \delta\phi)\, d\phi_1 = exp\left\{ - \frac{(\delta\phi)^2}{2(a-b)} \right\}$$

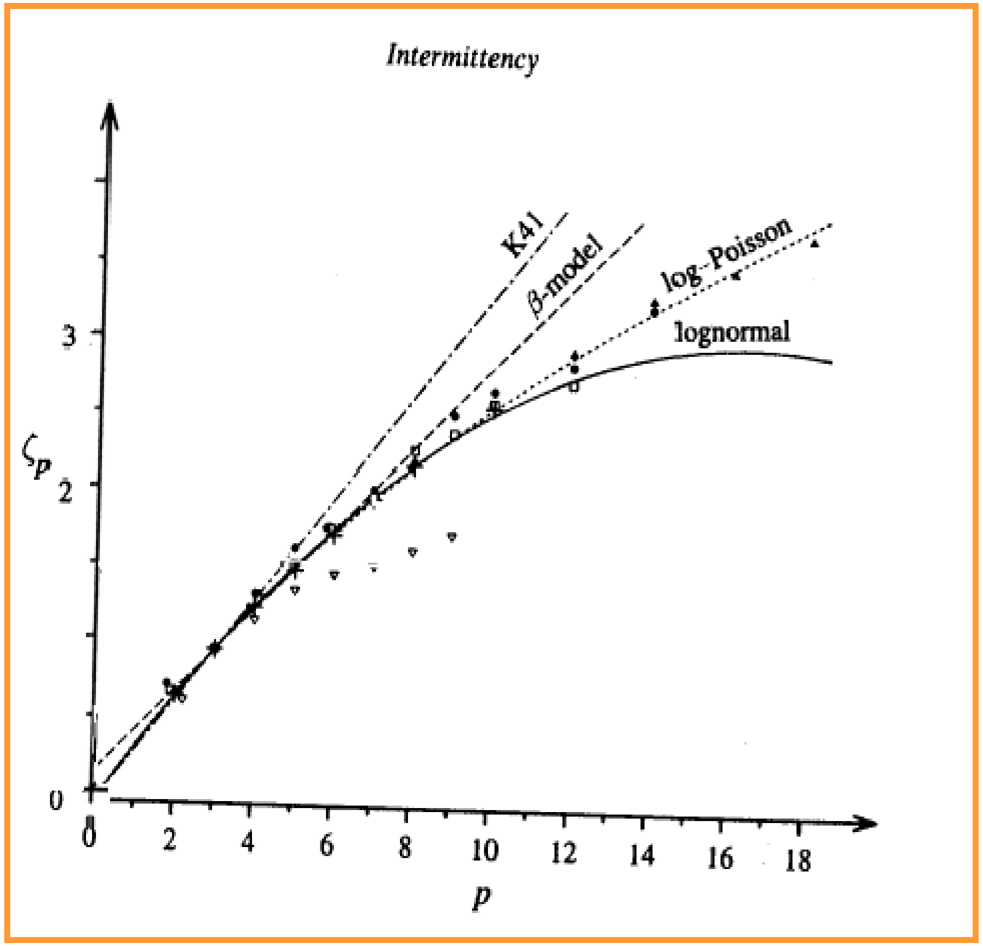
$$S_p = \left\langle (\delta\phi)^p \right\rangle = \left\langle (\delta\phi)^2 \right\rangle^{p/2} \sim \left(r^{\beta} \right)^{p/2}$$

Complete solution specification of a multivariate PDF and the study of deviations from multivariate gaussianity. Structure functions captures the essence.

Quantitatively

Intermittency: is quantified by deviation of ς_p from p/3

$$S_p(r) \sim r^{\varsigma_p}$$



Physical Mechanism of Intermittency

In reality the dissipation ε is a statistical quantity

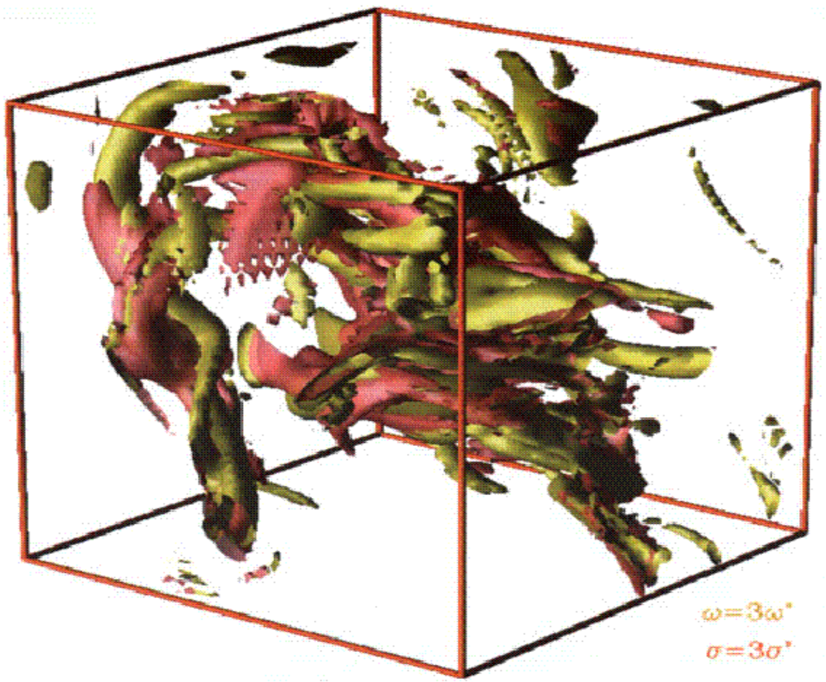


FIG. 5. (Color) Demonstration that vorticity at large amplitudes, say greater than 3 standard deviations, organizes itself in the form of tubes (shown in yellow), even though the turbulence is globally homogeneous and isotropic. Large-amplitude dissipation (shown in red) is not as organized, and seems to surround regions of high vorticity. Smaller amplitudes do not possess such structure even for vorticity. In principle, the multifractal description of the spiky signals of Fig. 4 is capable of discerning geometric structures such as sheets and tubes, but no particular shape plays a central role in that description. The dynamical reason for this organization of large-amplitude vorticity is unclear. The ubiquitous presence of vortex tubes raises a number of interesting questions, some of which are mentioned in the text. At present, elementary properties of these tubes, such as their mean length and scaling of their thickness with Reynolds number, have not been quantified satisfactorily; nor has their dynamical significance.

Concept of Intermittency in short

- In reality ε is a statistical quantity. Can be diagnosed by the presence of patchy, bursty dissipation and transport. $\longrightarrow$ <u>"Intermittency"</u>
- Departure from maximal randomness $\longrightarrow$ Non Gaussian statistics and presence of structures. Extreme events are more probable than gaussian.
- Presence of structures $\longrightarrow$ strong self interaction, local in physical space and non- local in k space.
- Leads to deviation from Kolmogorov's scaling. Deviations are more pronounced for higher order structure functions.

Some Exact Results

- From symmetry considerations of Navier Stokes equation it has been possible to obtain the exact value for $\zeta_3=1$.
- The scaling exponents ζ_p of the structure functions S_p of a passive scalar advected by a velocity field which is self similar, gaussian white in time (δ correlated) has been obtained exactly for all 'p' for space dimension d of 2 and above and for velocity scaling exponent ξ lying between 0 and 2.

$$\left\langle \left(\delta u\left(r,t\right)\right)^{2} \right\rangle \sim r^{\xi}$$

Models for intermittency

Incorporate statistical fluctuations of ε.

- Log normal distribution.

  Kolmogorov, JFM 13, 82 (1962).

Geometrical structure of dissipation region

- Beta model.

  Novikov and Stewart; U Frisch JFM 87, 719 (1978).

- Multifractal model.

  Meneveau and Sreenivasan, JFM, 224, 429 (1991).

Both features and curently the most favoured

- Log Poisson process.

  She and Levegue, PRL 72, 336 (1994)

Lognormal Model

Kolmogorov and Obukhov assumed that ε being a statistical positive definite quantity has a log normal distribution with

$$\boxed{\langle \varepsilon(x)\varepsilon(x+r)\rangle \sim r^{-\mu}}$$

Leading to

$$\boxed{\varsigma_p = \frac{p}{3} - \mu\,\frac{p(p-3)}{18}}$$

Note the value of ς_3 as unity has been obtained exactly from the equations.
Any model ought to satisfy this constraint !!

A good fit to observed numerical and experimental data was obtained upto p = 10 for a value of μ = 0.2

Beta Model

Each level of cascade an eddy of scale l_n splits into $2^D\beta$ eddies of scale $l_{n+1} = l_n$. Here D diemsionality of space and β lies in between 0 and unity defining the fraction of space which is filled at each subsequent scale by the turbulent activity.

$$\boxed{\varsigma_p = \frac{p}{3} - \frac{\delta}{3}(p-3)}$$

Here $\beta = 2^{-\delta}$; and so D-δ is the fractal dimension of the region of activity

Greatest drawback :Linear scaling does not agree with observations.

Multifractal Model

More than one fractal dimensions for the active region.

$$\zeta_p = \left(\frac{p}{3} - 1 \right) D_p + 1$$

$$D_p = log \left[n^p + (1-n)^p \right]^{\frac{1}{(1-p)}}$$

Agreement with experimental data pretty good for n = 0.7

She Levegue Log Poisson Model

- Involves hierarchy of fluctuating structures.
- Requires no adjustable parameter
- Good agreement with data
- Wide acceptance

$$\boxed{\zeta_p^{SL} = \frac{p}{9} + 2\left[1 - \left(\frac{2}{3}\right)^{\frac{p}{3}}\right]}$$

Transport behaviour in tokamaks : Intermittency or self similarity as in SOC?

- SOC : scale similarity
- Intermittency: Lack of strict scale invariance.
- Opinion divided, experimental evidences ‘for’ and ‘against’ both seem to exist.
- More work necessary.

Turbulent heat and particle Transport in Tokamaks (Contd)

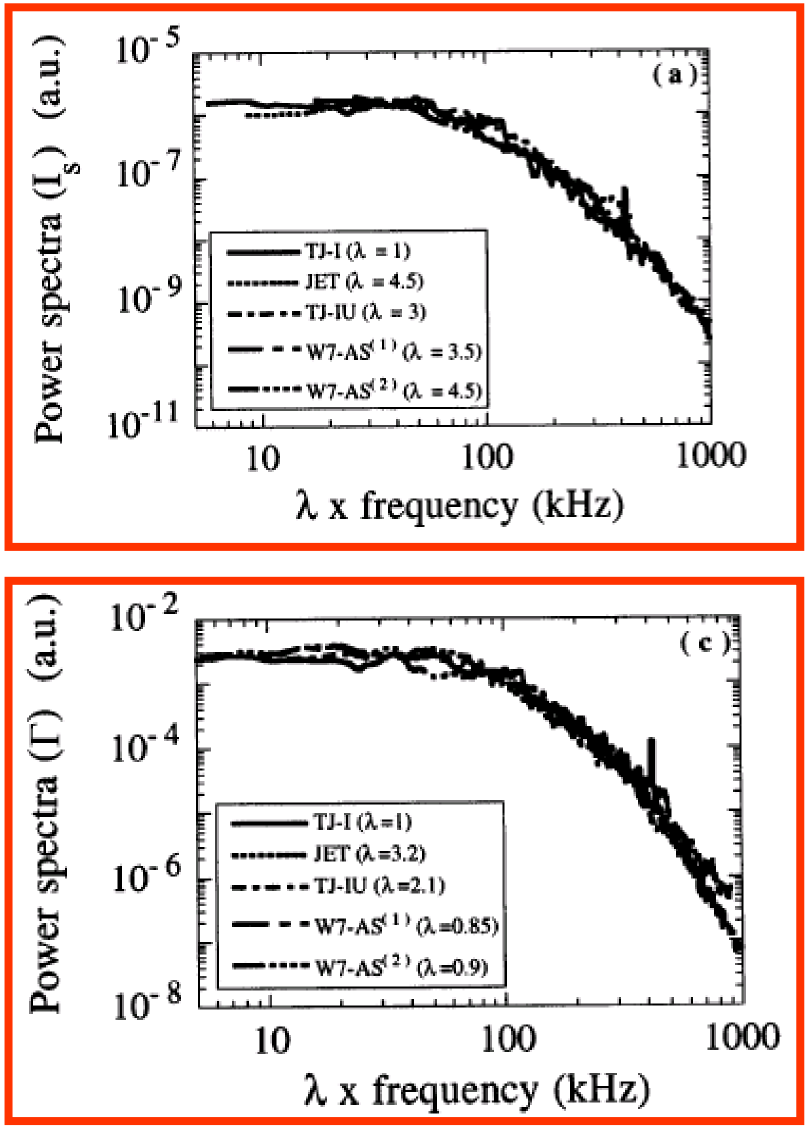
Aspects in favour of SOC:

- Scaling character of frequency spectra

  (Pedrosa et al. 1999)

- Strong profile resilience shown by Thomson scattering measurements. **Tore Supra (Hoang et al 2001)**

- Submarginal profiles **(F. Ryter et al. PPCF 2001)**



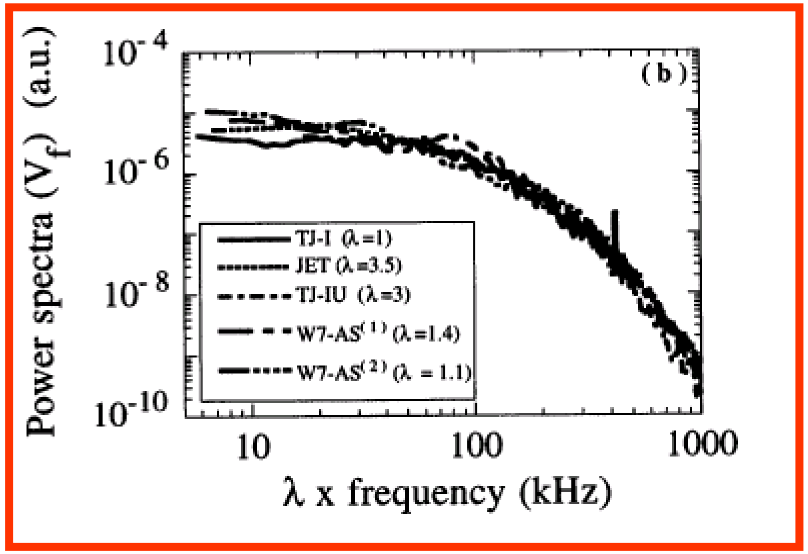


FIG. 3. Rescaled frequency spectra of fluctuations for the ion saturation current (a), floating potential (b), and turbulent transport (c).

M.A. Pedrosa et al.
PRL**82**(1999)3621

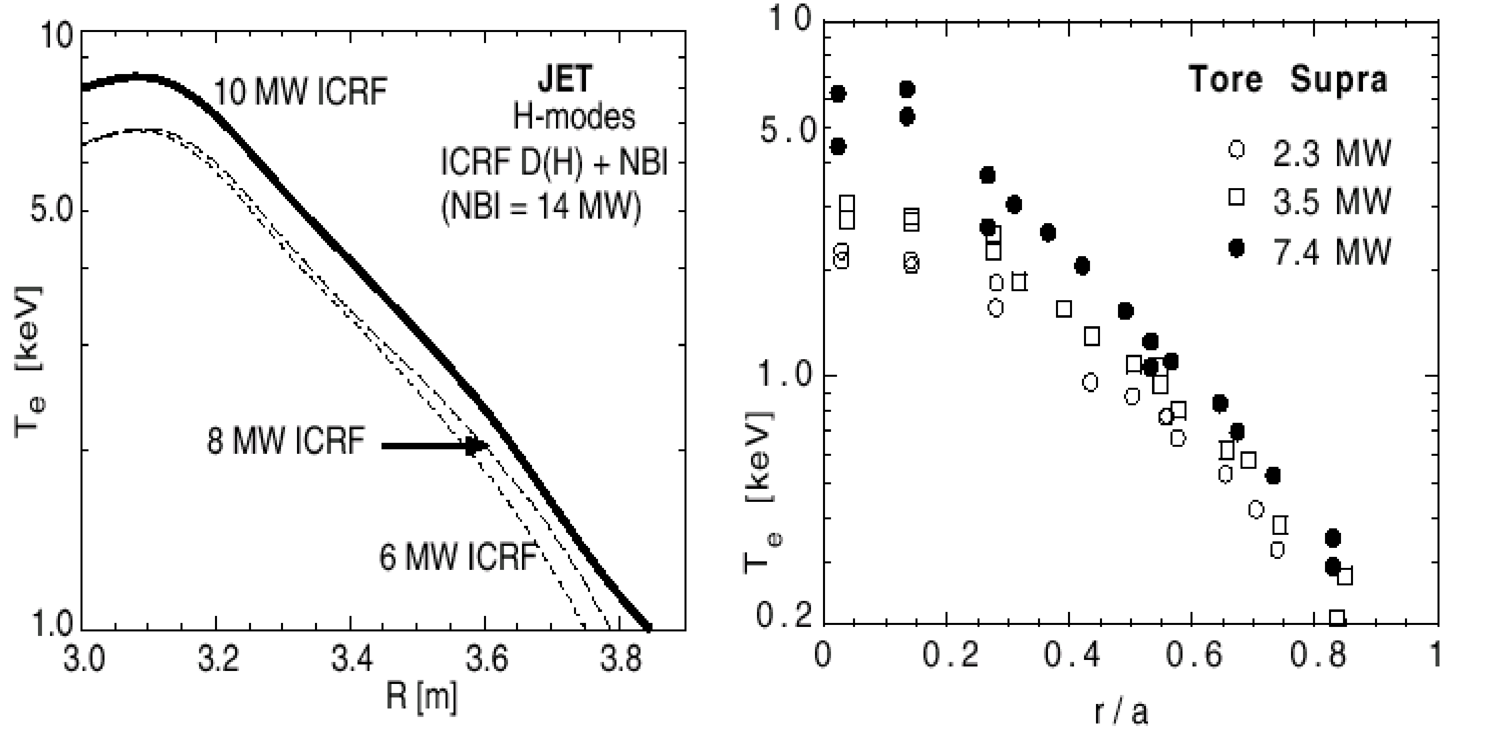


Figure 2. T_e profiles for central heating for ASDEX Upgrade, RTP, FTU TCV, JET and Tore Supra. The regions with constant slope corresponding to constant $\nabla T_e / T_e$ can be clearly seen. (Courtesy of Mantica and Hogeweij for RTP, Jacchia for FTU, Angioni for TCV, Hoang for Tore Supra and Suttrop for JET.)

F.Ryter et al. PPCF**43**(2001) A323-336

Turbulent heat and particle Transport in Tokamaks (Contd)

Aspects in favour of Intermittency:

- PDF of transport does not collapse to a universal scale similar form.
- Short scale and long scale transport features are distinctly different.
- Fluid models are useful as minimal model description for tokamak transport.
- Similarity of plasma fluid model with hydrodynamic fluids (Which are known to be intermittent)

Clearly …

- Tokamak data is very limited to discern any quantitative evidence for intermittency.
- Existence of natural length and time scales results in characteristic structures associated with such scales.
- Highly unlikely that a system plagued by natural length and time scales would be truly self similar.

What Simulations have to say !

Comparison: Plasma with neutral fluid

- Physics analogies: many scales excited, strong mixing, cascade..and so on.
- Fluid models of plasma: MHD, EMHD, Hasegawa Mima etc.
- Existence of waves in addition to eddy like motion.
- Additional fields: Electric and magnetic fields, currents etc.

Strong anisotropy in turbulence.

At times leads to a reduction in dimensionality (3d to 2d).

Intermittency studies in Plasma

(Fluid like Approach)

- Most extensive study on MHD model.
- MHD Model supports Alfven waves.

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \bullet \nabla \vec{v} = \mu \nabla^2 \vec{v} - \nabla p + \vec{J} \times \vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{v} \bullet \nabla \vec{B} = \vec{B} \bullet \nabla \vec{v} + \eta \nabla^2 \vec{B}$$

Controversy: Over Alfven effect!!!
Modifies the turn over time from τ_l to τ_l^2 / τ_{Alf}

Change in Kolmogorov scaling $k^{-5/3}$ to $k^{-3/2}$ (IK scaling)?
Difference difficult to gauge numerically.

Generalized SL scaling

$$\varsigma_p = \frac{p}{g}(1-x) + C\left[1-(1-x/C)^{p/g}\right]$$

Biskamp et al. claim to have solved the controversy reigning Alfven effect.

Here: g defines the basic scaling of the relevant field $\delta z_r \sim r^{1/g}$.
x energy transfer time $\tau_{eff} \sim r^x$
C=3-D; D dimension of dissipative structures.

Neutral Fluids
z = v, g =3, x =2/3,
D=1 so C = 2

MHD Fluid
(w/o Alfven effect)
z = Elsasser fields
g =3, x =2/3, D=2
so C = 1

MHD Fluid
(with Alfven effect)
z = Elsasser fields
g =4, x =1/2, D=2
so C = 1

Comparison with simulation

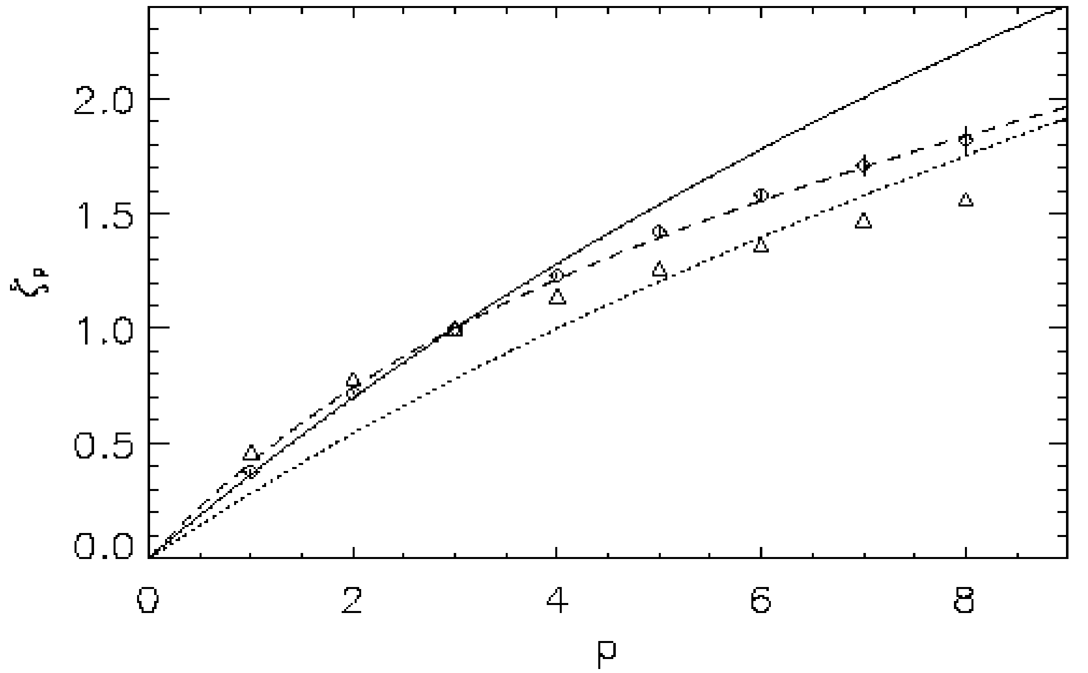


FIG. 4. Scaling exponents ζ_p^+ for 3D MHD turbulence (diamonds) and relative exponents ζ_p^+/ζ_3^+ for 2D MHD turbulence (triangles). The continuous curve is the She-Leveque model ζ_p^{SL}, the dashed curve the modified model ζ_p^{MHD} (7), and the dotted line the IK model ζ_p^{IK}.

Studies on EMHD model

- Presence of special characteristic scale $d_{e.}$.
- Supports dispersive whistler waves.
- Controversy: presence of Whistler effect on scaling

Kolmogorov analysis

- $E_k = \varepsilon^{2/3} k^{-5/3}$ ($kd_e >> 1$)
- $E_k = \varepsilon^{2/3} k^{-7/3}$ ($kd_e << 1$)

IK Analysis (whistler effect)

- $E_k = \varepsilon^{2/3} k^{-5/3}$ ($kd_e >> 1$)
- $E_k \sim k^{-2}$ ($kd_e << 1$)

- Biskamp et al. rule out whistler effect merely on the basis of energy spectra scaling!!
- Dastgeer, Das, Kaw and Diamond PoP **7** 571 (2000) show that cascade is influenced by whistlers though it may not perhaps influence the scaling of spectra directly.

Studies on EMHD model (contd.)

- Intermittency in 2d EMHD have been studied recently by Germaschewski et al.
- The structure function index ζ_p for both b and ψ fields (upto p = 14) show deviations from linearity.
- Fitting parameters (in terms of x, g and C) employed to fit the result with generalized SL expression.
- The fitting parameters were different for the two fields.
- No justification as given in the context of MHD.

Other evidence for Intermittency in EMHD

Boffetta et al.(PRE vol 59 3724(1999)) measured the scaling exponent of various powers of energy dissipation function numerically and showed a nontrivial scaling of the exponent τ_p with p.

$$\left\langle \varepsilon(r)^p \right\rangle = \left\langle \left[\frac{1}{V(r)} \int d^3x \varepsilon(x) \right]^p \right\rangle \sim r^{\tau_p}$$

Other fluid models

(Electrostatic models)

- Hasegawa Mima (HM), ITG, ETG etc.
- HM has a characteristic scale ~ larmor radii.
- Waves : Drift waves.

$$\boxed{\frac{\partial}{\partial t}\left(\phi - \nabla^2 \phi\right) - \hat{z} \times \nabla \phi \bullet \nabla \nabla^2 \phi + v_d \frac{\partial \phi}{\partial y} = \mu \nabla^2 \nabla^2 \phi}$$

- Supports two invariants.
- Hence energy (inverse) as well as vorticity (direct) cascade regimes.
- Two wavevector regimes $k\rho_s << 1$; and $k\rho_s << 1$

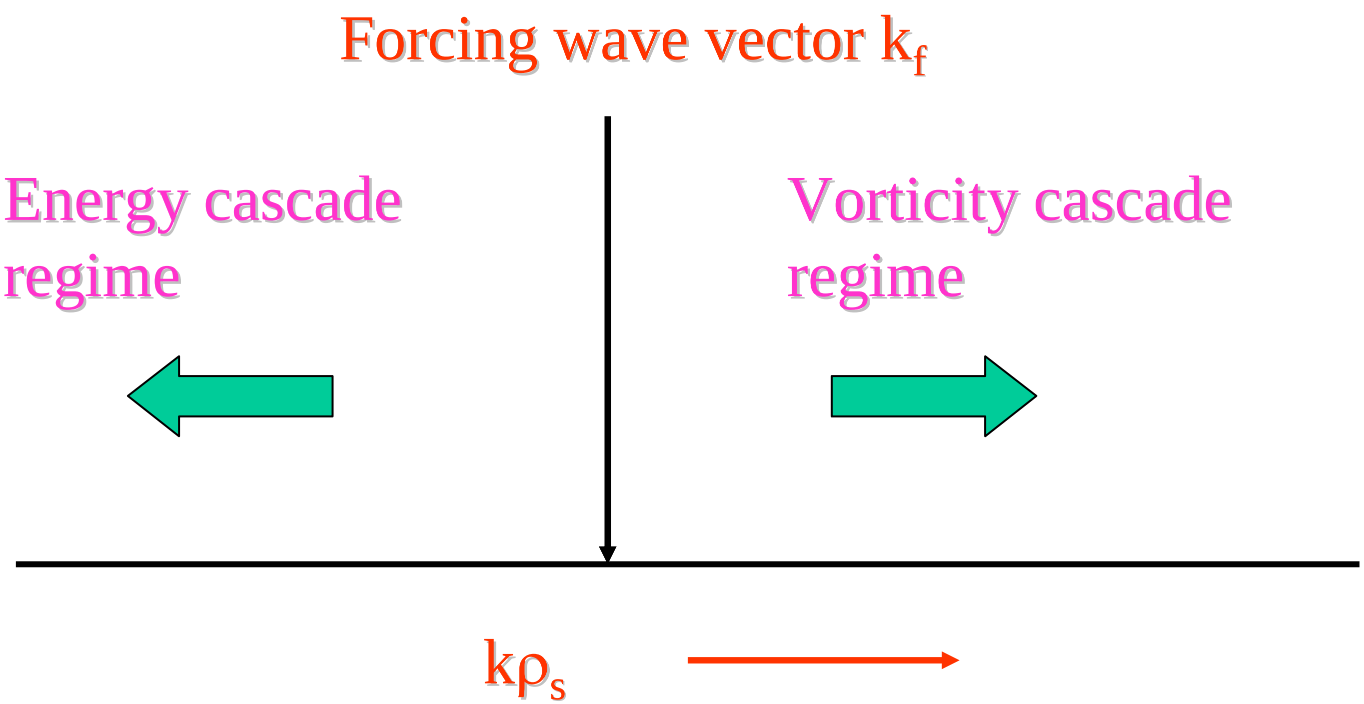
Four Possibilities

Energy cascade: $k\rho_s > 1$

$k\rho_s < 1$

Vorticity cascade: $k\rho_s > 1$

$k\rho_s < 1$



Location of $k\rho_s = 1$ with respect to k_f

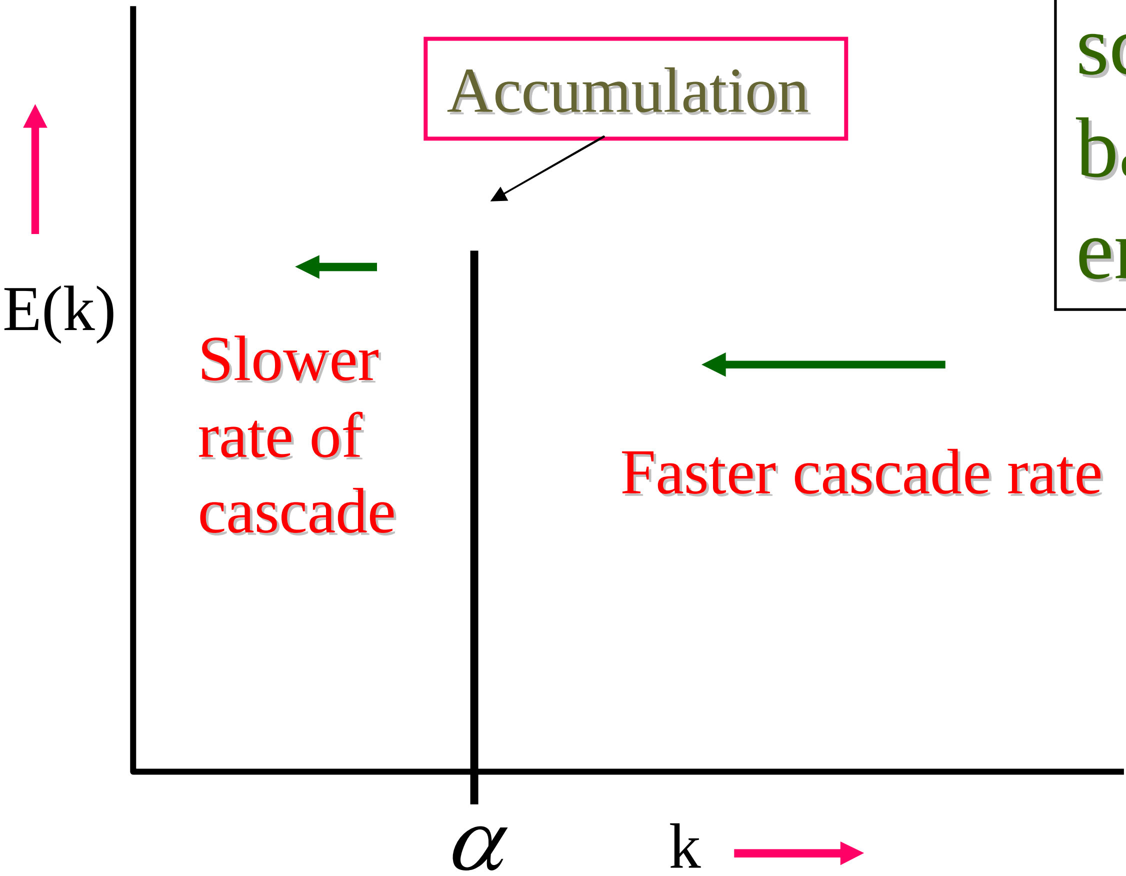
Scalings

- $k\rho_s >> 1$, equations identical to 2d hydrodynamic fluid $E_k \sim k^{-5/3}$ in energy cascade and $E_k \sim k^{-3}$ in vorticity cascade.
- $k\rho_s << 1$, $E_k \sim k^{-11/3}$ in energy cascade and $E_k \sim k^{-5}$ in vorticity cascade.
- Interesting feature in $k\rho_s << 1$ regime, reduced eddy turn over time.

$$\boxed{\hat{z} \times \nabla \phi \bullet \nabla\nabla^{2}}$$

Novel feature: Reduction in eddy turn over time results in accumulation of power in the boundary and formation of quasistationary crystalline structures. Kukharkin et al. PRL (Oscillatory structure functions). ~ 'Intermittency like'

Plasma flow system

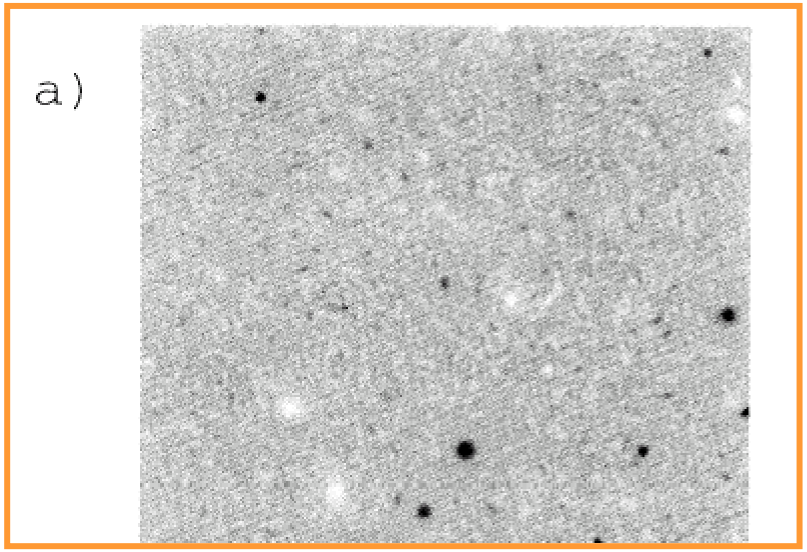


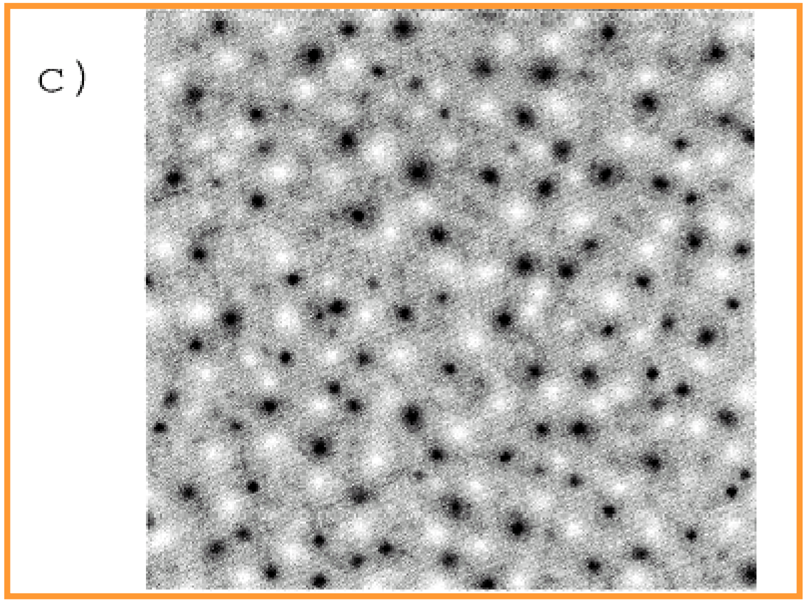
Natural length scales acts as a barrier for energy cascade.

$$\frac{\partial}{\partial t}\left(\nabla^2\phi-\alpha^2\phi\right)+\hat{z}\times\nabla\phi\bullet\nabla\nabla^2\phi=0$$

Numerical studies

- For HM system numerical simulations show a self consistent spontaneous formation of several distinct regions of intense vorticity for decaying as well as randomly driven systems (forcing at scales shorter than the natural length scale).
- These structures appear as quasi–crystalline pattern in 2-D spatial domain and were first identified by Kukharkin et al. PRL **75** 2486 (1995).
- Formation of such structures was attributed due to the existence of a barrier in the inverse energy cascade rate at the natural length scale.





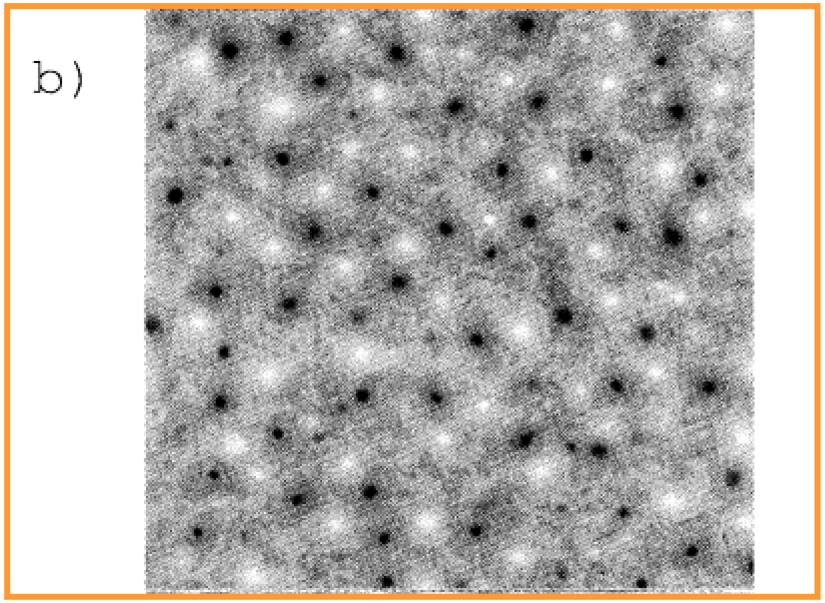


FIG. 1. The potential vorticity, $\xi = \nabla^2\phi - \lambda^2\phi$, field at $N_\lambda = 400$ for the NS and the HM equations: (a) $\lambda = 0$ (NS), (b) $\lambda = 20$, and (c) $\lambda = 40$.

$$\frac{\partial}{\partial t}(\nabla^2\phi - \lambda^2\phi) + J(\phi, \nabla^2\phi) = D + F,$$

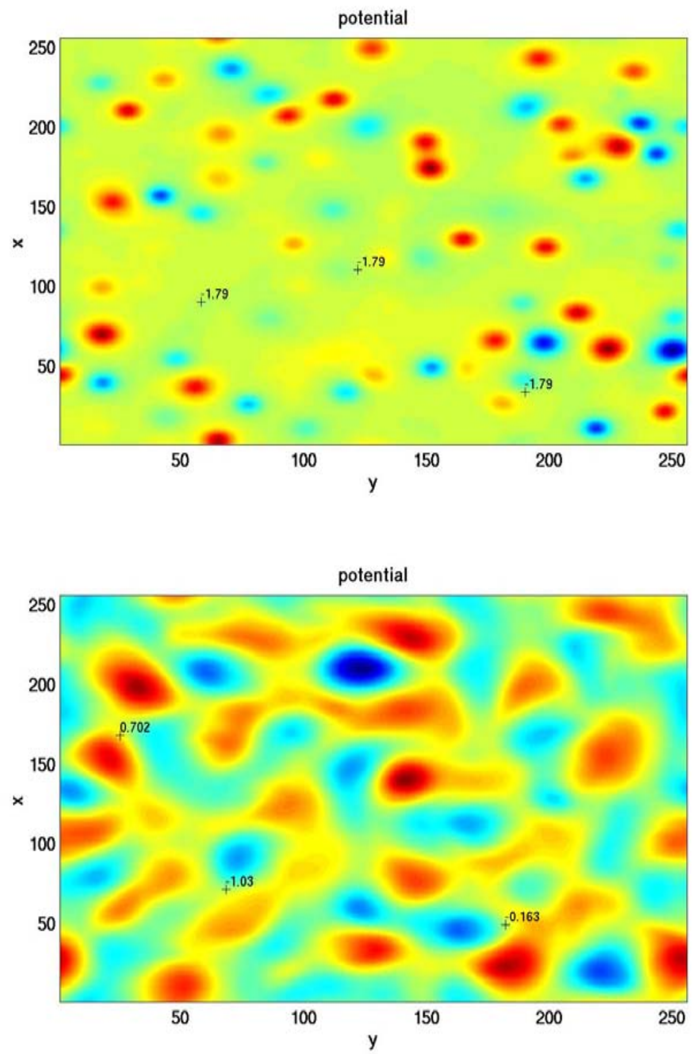
Influence of wave excitations !!

Study by making V_n finite during the course of evolution.

Ref:

- A. Das, Phys. Plasmas (2007) ;
- A. Das, Phys. Plasmas (2008)

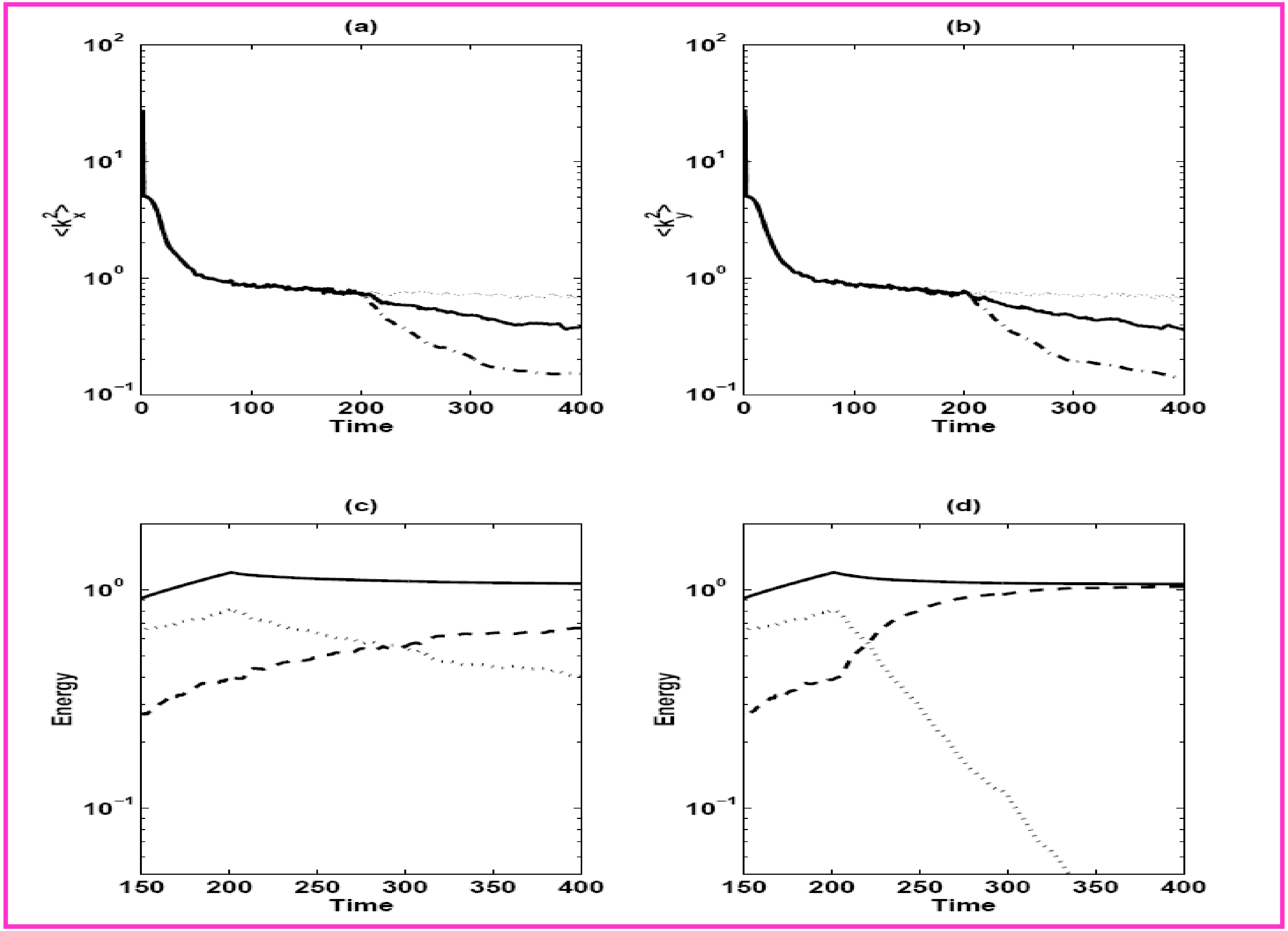
Melting transition in the presence of waves



- In the presence of waves with finite V_n the quasi – crystalline structure undergoes a melting transition
- Transition occurs only when V_n exceeds a certain threshold.

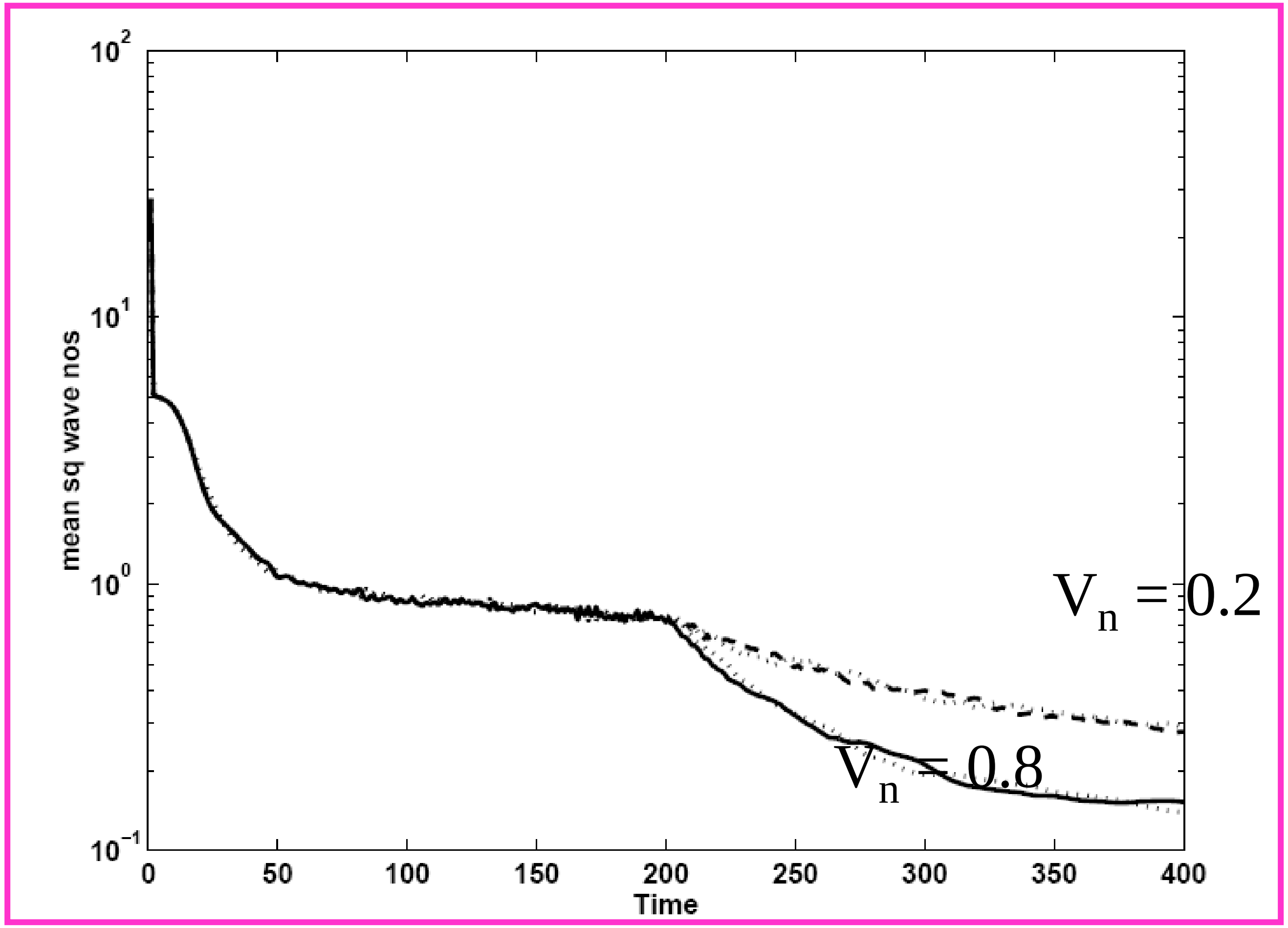
$$V_n \frac{\partial \phi}{\partial y} \sim \hat{z} \times \nabla \phi \bullet \nabla \nabla^2 \phi$$

An increased cascade thru' the barrier in the presence of waves

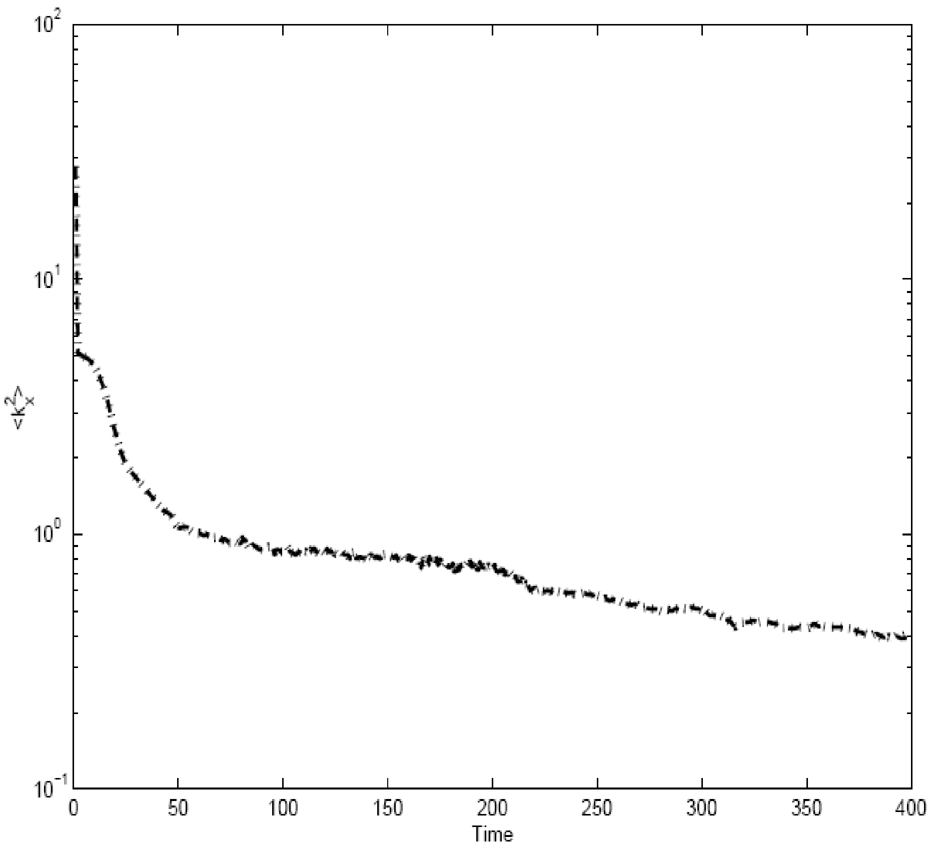


(a)
(b)
(c)
(d)
$<k_x^2>$
$<k_y^2>$
Energy
Time
10^{2}
10^{1}
10^{0}
10^{-1}
0
100
200
300
400
150
250
350

Isotropy of the spectrum



No induced transparency for Doppler shifted translational invariant system



Modified HM equation with

$$\boxed{V_n \frac{\partial \phi}{\partial y} \rightarrow V_n \frac{\partial\left(\nabla^2 \phi - \alpha^2 \phi\right)}{\partial y}}$$

Causes no induced transparency

Numerical Results

- Nonlinear scaling of the ζ_p with p both for EMHD and MHD : signature for Intermittency
- Strong influence of natural length and time scales on cascade and structure formation: Clear demonstration in the context of the Hasegawa Mima system.

Intermittency description : Non Gaussian PDF

Hydrodynamic fluids: Kraichnan – Mapping Closures

- Mapping Closures: A. Das and Predhiman Kaw, Phys. Of Plasmas **2,** 1497 (1995).
- Instanton approach: Kim and Diamond, Phys. Of Plasmas **9**, 71 (2002).

Finally …

- Plasma turbulence fertile area.
- Standard intermittency studies have been limited. Most conclusions derived on the basis of indirect evidences.
- Presence of natural scales and quasi-structures: Influence on S_p – unknown.