International Workshop on the Frontiers of Modern Plasma Physics

14 - 25 July 2008

New Directions in Plasma Physics.

R. Sagdeev
University of Maryland, College Park, U.S.A.
\[ m \frac{dV}{dt} = e \sum E_i \exp i(\omega_i - kv)t \]
\[ |V - \omega/k| \leq \left( \frac{e\varphi}{m} \right)^{\frac{1}{2}} \]
ADD MORE WAVES

\( \left( \frac{\omega}{k} \right)_2 \)

\( \left( \frac{\omega}{k} \right)_1 \)
\[ \left| V - \frac{\omega}{k} \right| \leq \left( \frac{e\varphi}{m} \right)^{1/2} \]

**Width of resonance**

vs.

\[ \left( \frac{\omega}{k} \right)_{n+1} - \left( \frac{\omega}{k} \right)_n \]

**Distance between resonances**
\[ \left( \frac{e \varphi}{m} \right)^{\frac{1}{2}} \text{ much less than } \left( \frac{\omega}{k} \right)_{n+1} - \left( \frac{\omega}{k} \right)_n \]

This limit corresponds to KAM (Kolmogoroff-Arnold-Mozer) case.

**KAM-Theorem :**

As applied to our case of Charged Particle – Wave Packet Interaction –

“Particle preserves its orbit “
\[
\left(\frac{e\varphi}{m}\right)^{1/2} \quad \text{greater than} \quad \left(\frac{\omega}{k}\right)_{n+1} - \left(\frac{\omega}{k}\right)_n
\]

That means - overlapping of neighboring resonances

Repercussions:

- "collectivization" of particles between neighboring waves;

- particles moving from one resonance to another – “random walk”? And if yes

- what is **Diffusion Coefficient** ? (in velocity space)
\[ m \frac{dV}{dt} = e \sum E_i \exp \left( i(\omega_i - kv) t \right) \]

\[
V = \frac{e}{m} \sum E_i \exp i(\omega - kv)t \frac{1}{i(\omega - kv)}
\]

\[
V \times \frac{dV}{dt} =
\]

\[
e^2 \frac{1}{m^2} \sum \sum EE^* \exp i(\omega_i - \omega_j - k_i v + k_j v)t \frac{1}{i(\omega - kv)}
\]

\[
V^2 \propto Dt
\]
\[ D = \frac{\pi e^2}{m^2} \sum |E|^2 \delta(kv - \omega) \]

\[ \sum_{k} = \frac{1}{2\pi} \int dk \]

Repercussions: Quasilinear Theory, Plateau Formation, Beam + Plasma Instability Saturation etc.
General Conclusions

- Kolmogoroff: Application of KAM theory to the Dynamics of Planetary System
- Plasma case: Application to the Dynamics of Charged Particles
  - more applications:
    - Waves-Particles interaction at Cyclotron Resonance
    - Magnetic Surfaces Splitting? (Trieste, 1966)
    - Advection in Fluids (+20 years)
\[ B_z = B_0; B_y = \frac{x}{L_s} B_0 \]

\[ b_x = b_\perp \cos \left( k_z z + k_y y \right) \]

\[ \frac{dx}{dl} = \frac{b_\perp}{B_0} \cos \left( k_z z + k_y y \right) \]

\[ dy / dl = x / L_s \]

\[ k_z = -k_y \left( \frac{x_0}{L_s} \right) \]

\[ y = x_0 l / L_s + 1 / L_s \int_{x_0}^{x} x \, dl \]

\[ dx / dl = b_\perp / B_0 \cos \left( \frac{k_y}{L_s} \int x \, dl \right) \]
\[ \frac{dv}{dt} = e / mE \cos(k \int v dt) \]

\[ \frac{b_\perp}{B_0} \propto \left( \frac{e}{m} \right) E \]

\[ \frac{k_y}{L_s} \propto k \]

\[ \delta v = (e \varphi / m)^{1/2} \propto \delta x = \frac{b_\perp}{B_0} \left( \frac{L_s}{k_y} \right)^{1/2} \]