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Collective effects in plasmas due to the electron spin.

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Collective effects in plasmas due to the electron spin.

By

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Outline

- Why are spin and collective effects of interest?
- A brief background: The general theory of quantum plasmas.
- Spin fluid models
- Linear and nonlinear results.
- Results from improved fluid models
- Spin kinetic theory
- Conclusions and outlook

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Why are collective spin effects of interest?

- It is of theoretical interest to see to what influence basic quantum mechanics may have on collective behavior, which to a large extent has been dominated by classical physics.
- Spin effects have a potential to be of importance not only in high density low temperature plasmas (where the importance of collective effects are reduced), but in more general situations.
- Spin effects introduce qualitatively new physics in plasmas. Thus it can be important even when the magnitudes of the new terms that are introduced are small.
- Coupling to spintronics, an emerging field with potentially very important applications.

Some general aspects of quantum plasmas

Some basic properties of quantum plasmas includes

- *Particle dispersion:* The general tendency for a wave function to spread out (dispersively), which prevents the density to be too well localized (violating the uncertainty relation).

- *The Fermi pressure:* The fact that electrons are Fermions obeying Fermi-dirac statistics introduced a zero temperature pressure (from the Pauli exclusion principle), which is significant in low temperature plasmas.

- ***Spin properties:*** **The magnetic dipole moment of the electron associated with the spin introduces a magnetic dipole force (and higher order spinor effects), as well as new current sources in Ampere's law.**

Source for the basic equations

The effects described above can be dealt with using the Pauli equation:

$$i\hbar\frac{\partial\Psi_\alpha}{\partial t}=\left[\frac{\hbar^2}{2m}\left(\nabla+\frac{ie\hbar}{\hbar c}\mathbf{A}\right)^2+\mu_B\mathbf{B}\cdot\boldsymbol{\sigma}-e\Phi\right]\Psi_\alpha$$

where $\boldsymbol{\sigma}$ consists of the Pauli spin matrices. Factorization of the wave function and a decomposition of the spinor according to

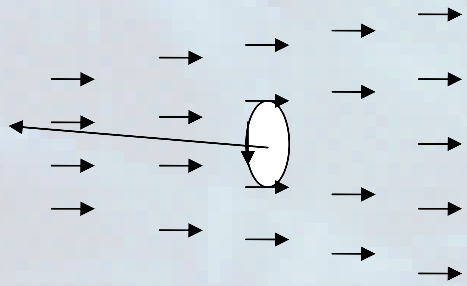
$$\Psi_\alpha=\sqrt{n}\exp(iS_\alpha/\hbar)\varphi_\alpha$$

where φ_α is a normalized two-spinor, leads to several new terms in the momentum equation, an evolution equation for the spin vector, as well as current sources in Ampere’s law due to the spin (See Marklund and Brodin, PRL, **98** (2007) 025001).

The general theory is quite involved (see Mattias Marklunds talk on Monday for more details) but a simple picture might suffice to give the basic idea…

$\mathbf{F}_{es} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Lorentz force



$\mathbf{F}_{md} = (\mathbf{m}_d \cdot \nabla)\mathbf{B}$

magnetic dipole force

The magnetic dipole moment due to the electron spin is

$$\mathbf{m}_d = -\frac{\mu_B}{(\hbar/2)}\mathbf{s}$$

where μ_B is the Bohr magneton and **s** is the spin vector (i.e. a three-vector derived from the expectancy value of the spin operator)

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A simple MHD model involving spin

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\mathbf{V} = -\nabla\left(\frac{B^2}{2\mu_0} - \mathbf{M}\cdot\mathbf{B}\right) + (\mathbf{B}\cdot\nabla)\left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}\right) - \nabla p$$

and

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{V} \times \mathbf{B}) = 0$$

with the magnetization **M** given by

$$\mathbf{M} = \frac{\mu_B \rho}{m_i} \tanh\left(\frac{\mu_B B}{k_B T}\right)\hat{\mathbf{B}}$$

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Some predictions from the spin MHD model

- In linear theory, the Alfvén velocity is modified according to

$$\widetilde{C}_A = \frac{C_A}{\left[1 + (\hbar\omega_{pe}^2/2m_e c^2 \omega_{ce}^{(0)})\tanh(\mu_B B_0/k_B T)\right]^{1/2}}$$

- For perpendicular propagation (to the external magnetic field), the acoustic velocity is modified by

$$\widetilde{V}_A^2 = V_A^2 - \frac{\hbar\omega_{ce}}{m_i}\tanh\left(\frac{\mu_B B_0}{k_B T}\right)$$

[Important in highly magnetized environments, for example pulsars and magnetars]

- For sufficiently low temperatures and high density, a homogeneous magnetized plasma is unstable, similar to the Jeans instability but without gravitation.
(G. Brodin and M. Marklund, Phys. Rev. E, 76, 055403R 2007)

- New nonlinear couplings for shear Alfvén wave propagation are introduced, leading to soliton formation. (G. Brodin and M. Marklund, Phys. Plasmas, 14, 112107 2007)

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Some notes on the above spin MHD model

- The spin magnetization is significant in the low-temperature high density regime (Alfvénic type of effects) *or* in the strongly magnetized low temperature regime (acoustic type of effects)

- For specific problems, spin effects may be of importance even when the relative magnitude of the spin terms are small. In particular the spin properties introduce nonlinear couplings for shear Alfvén waves that don't exist in the absence of spin magnetization.

- An observation: A single fluid treatment tends to limit the significance of the spin magnetization (in particular if we are far from the strong coupling regime). The reason is that the contributions from the spin-up and down populations cancel to leading order.

The two fluid electron model

- Electrons with spin-up and down are treated as different fluids.
  (G. Brodin, M. Marklund, G. Manfredi, Phys. Rev. Lett, **100**, 175001 (2008))

- For dynamics slow compared to the gyrofrequency, but shorter than the spin relaxation frequency, the electron spin state is preserved.

- The spin force in the electron momentum equation can be written

$$\mathbf{F_s} = \frac{2\mu_B}{\hbar} S_{\pm}^{a} \nabla B_a$$

- For dynamics slow compared to the gyro-frequency, the spin force reduces to

$$\mathbf{F_s} = \pm \mu_B \nabla B$$

- The magnetization currents introduced by each species is

$$\mathbf{j}_m = \pm \nabla \times \left(\mu_B n_{\pm} \hat{\mathbf{B}} \right)$$

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Predictions from the two-fluid electron model

- Linearly the predictions agree with the single electron model.
- Nonlinearly, when an electromagnetic perturbation enters the system, the spin ponderomotive force separates the two populations. This in turn modifies the magnetic field since spin-magnetization no longer cancels.
- When the spin-populations are separated, clearly a two-fluid electron model is needed. At this stage the significance of the spin magnetization is much enhanced.

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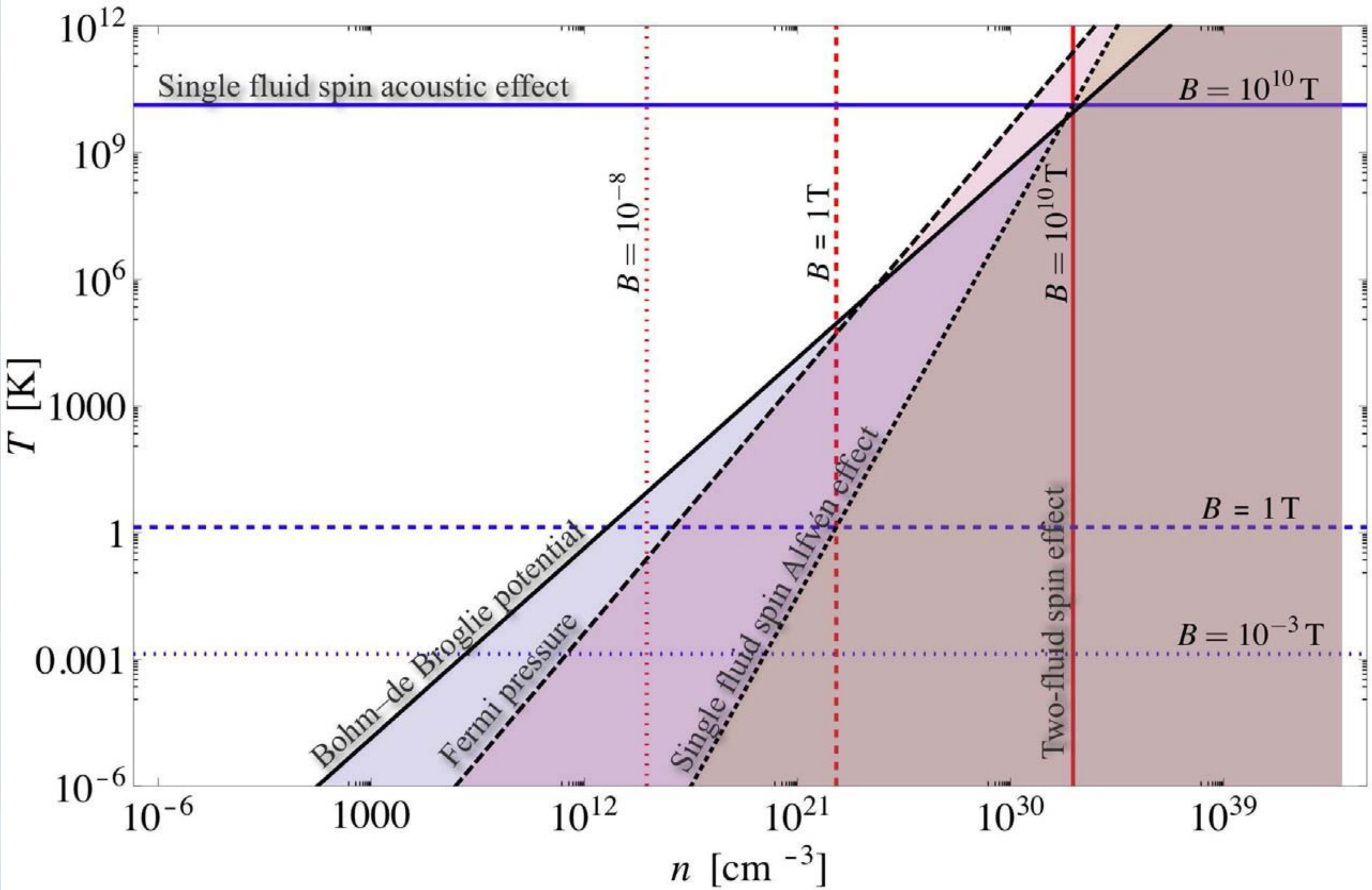
A comparison of the importance of different quantum effects:



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Spin kinetic theory

From the Pauli Hamiltonian and the relation dF/dt = ∂F/∂t+(1/iℏ)[F,H], where [,] is the Poisson bracket and F is any operator, we have the following evolution equations for the momentum and the spin in the Heisenberg picture

$$m_e \frac{d\mathbf{v}}{dt} = -e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) - \frac{2}{\hbar}\mu_B \nabla(\mathbf{B} \cdot \mathbf{s})$$

and

$$\frac{d\mathbf{s}}{dt} = \frac{2}{\hbar}\mu_B \mathbf{B} \times \mathbf{s}$$

Introducing a distribution function in an extended phase space $f = f(t,\mathbf{r},\mathbf{v},\mathbf{s})$ and noting that the phase space density in this increased phase space is preserved along the particle orbits, we find a generalized Vlasov type of equation for electrons

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \nabla f + \frac{d\mathbf{v}}{dt} \cdot \nabla_{\mathbf{v}} f + \frac{d\mathbf{s}}{dt} \cdot \nabla_{\mathbf{s}} f = 0$$

which gives

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \left[-\frac{e}{m_e}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{2}{\hbar}\mu_B \nabla(\mathbf{B} \cdot \mathbf{s}) \right] \cdot \nabla_{\mathbf{v}} f + \frac{2}{\hbar}\mu_B \nabla\left(\mathbf{B} \times \mathbf{s}\right) \cdot \nabla_{\mathbf{s}} f = 0$$

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Spin kinetic theory

The current density in this model is written

$$\mathbf{j} = -e\int \mathbf{v} f\, d\Omega + \frac{2}{\hbar}\mu_B \nabla \times \int \mathbf{s} f\, d\Omega$$

where the first term is the free current and the second term is the magnetization current. The integration is performed over the five dimensional spin-velocity volume element (three velocity dimensions and two spin dimensions).

The spin Vlasov model together with Maxwell's equations obeys an energy conservation law of the form

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{P} = 0$$

where the total energy density W and energy flux **P** are

$$W = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} + \frac{m_e}{2}\int v^2 f d\Omega - \frac{2}{\hbar}\mu_B \mathbf{B} \cdot \int \mathbf{s} f\, d\Omega$$

and

$$\mathbf{P} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} - \frac{2}{\hbar}\mu_B \mathbf{E} \times \int \mathbf{s} f\, d\Omega + \frac{m_e}{2}\int v^2 \mathbf{v} f d\Omega + \frac{2}{\hbar}\mu_B \int \mathbf{v}(\mathbf{B}\cdot\mathbf{s}) f\, d\Omega$$

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Spin kinetic theory

Some features of the spin kinetic model are (G. Brodin, M. Marklund, J. Zamanian, Å. Eriksson, P. L. Mana, Submitted)

- New wave modes due to the spin coupling appear.

- New wave particle resonances appear, with denominators

$$\omega - k_z v_z - n\omega_c + (g/2) m\omega_c$$

  where n is an integer, $m = 0, \pm 1$ and g is the factor associated with the electron magnetic moment, such that g/2=1.0016…

- In general, although the spin terms in the model can be small during various conditions, certain spin effects survive (in particular those mentioned above) independently of the plasma temperature and density.

Conclusions and Outlook

- There is very rich physics associated with the electron spin
  (see Mattias Markunds talk for more examples).
- Single fluid MHD models can incorporate electron spin effects in a straightforward manner.
- Multifluid electron models show that in the nonlinear regime, the significance of spin effects can be enhanced, and the spin effects can survive even in high temperature plasmas
- Within kinetic theory, the spin effects can be even more pronounced, giving raise to lots of new physics, including new wave modes, new types of wave particle resonances, etc.
- Much work remains to be done ….

Thank you for your attention!

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